Advanced Multiplication

Lecture 11

As I promised, these problems are definitely a challenge! As you saw, doing enormous problems … requires all of the previous squaring and memory skills that we've learned. Once you can do a 4-digit square, even if it takes you a few minutes, the 3-digit squares suddenly don't seem so bad!

In this lecture, we'll look at mental math techniques for enormous problems, such as squaring 3- and 4-digit numbers and finding approximate cubes of 2-digit numbers. If you've been practicing the mental multiplication and squaring methods we've covered so far, you should be ready for this lecture.

To square 3-digit numbers quickly, you must be comfortable squaring 2-digit numbers. Let's start with 108^2. As we've seen before, we go down 8 to 100, up 8 to 116, then multiply 100 × 116 = 11,600; we then add 8^2 (= 64) to get 11,664. A problem like 126^2 becomes tricky if you don't know the 2-digit squares well, because you'll forget the first result (152 × 100 = 15,200) while you try to work out 26^2. In this case, it might be helpful to say the 15,000, then raise 2 fingers (to represent 200) while you square 26.

Here's a geometry question: The Great Pyramid of Egypt has a square base, with side lengths of about 230 meters, or 755 feet. What is the area of the base? To find the answer in meters, we go down 30 to 200, up 30 to 260, then multiply 200 × 260 = 52,000; we then add 30^2 (= 900) to get 52,900 square meters.

To calculate the square footage (755^2), we could go up 45 to 800, then down 45 to 710, or we could use the push-together, pull-apart method: 755 + 755 = 1510, which can be pulled apart into 800 and 710. We now multiply 800 × 710 = 568,000, then add 45^2 (= 2025) to get 570,025 square feet.

One way to get better at 3-digit squares is to try 4-digit squares. In most cases, you'll need to use mnemonics for these problems. Let's try 2345^2. We go down 345 to 2000, up 345 to 2690. We then multiply 2000 × 2690, which is (2000 × 2600) + (2000 × 90) = 5,380,000. The answer will begin with 5,000,000, but the 380,000 is going to change.

How can we be sure that the 5,000,000 won't change? When we square a 4-digit number, the largest 3-digit number we will ever have to square in the middle is 500, because we always go up or down to the nearest thousand. The result of 500^2 is 250,000, which means that if we're holding onto a number that is less than 750,000 (here, 380,000), then we can be sure there won't be a carry.

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How can we hold onto 380,000 while we square 345? Using the phonetic code we learned in Lecture 9, we send 380 to the MOVIES. Now we square 345: down 45 to 300, up 45 to 390; 300 × 390 = 117,000; add 45^2 (= 2025); and the result is 119,025. We hold onto the 025 by turning it into a SNAIL. We add 119,000 + MOVIES (380,000) = 499,000, which we can say. Then say SNAIL (025) for the rest of our answer. We've now said the answer: 5,499,025.

Notice that once you can square a 4-digit number, you can raise a 2-digit number to the 4th power just by squaring it twice. There's also a quick way to approximate 2-digit cubes. Let's try 43^3. We go down 3 to 40, down 3 to 40 again, then up 6 to 49. Our estimate of 43^3 is now 40 × 40 × 49. When we do the multiplication, we get an estimate of 78,400; the exact answer is 79,507.

Finally, we turn to 3-digit-by-2-digit multiplication. The easiest 3-by-2 problems have numbers that end in 0, because the 0s can be ignored until the end. Also easy are problems in which the 2-digit number can be factored into small numbers, which occurs about half the time. To find out how many hours are in a typical year, for example, we calculate 365 × 24, but 24 is 6 × 4, so we multiply 365 × 6, then multiply that result by 4.

The next easiest situation is when the 3-digit number can be factored into a 2-digit number × a 1-digit number. For instance, with 47 × 126, 47 is prime, but 126 is 63 × 2; we can multiply 47 × 63, then double that result. For the most difficult problems, we can break the 3-digit number into two parts and apply the distributive law. For a problem like 47 × 283, we multiply 47 × 280 and add 47 × 3.

In our last lecture, we'll see what you can achieve if you become seriously dedicated to calculation, and we'll consider broader benefits from what we've learned that are available to everyone. ■

Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapter 8.

Doerfler, *Dead Reckoning: Calculating Without Instruments.*

Lane, *Mind Games: Amazing Mental Arithmetic Tricks Made Easy.*

Problems

Calculate the following 3-digit squares.

1. 107^2

2. 402^2

3. 213^2

4. 996^2

5. 396^2

6. 411^2

7. 155^2