HAPTICS SYMPOSIUM '16, APRIL 8–11, 2016, PHILADELPHIA, PENNSYLVANIA, USA
978-1-5090-0903-9/16/$31.00 ©2016 IEEE

# Exploring Haptic Working Memory as a Capacity-Limited Information Channel

Rachel A. Lerch*, Hengjun Cui†, Soham Patwardhan‡, Yon Visell § and Chris R. Sims¶
*Department of Psychology-Drexel University, Philadelphia PA: Email: Rachel.A.Lerch@drexel.edu
†Department of Electrical Engineering-Drexel University, Philadelphia PA, Email: hengjuncui@gmail.com
‡Department of Electrical Engineering-Drexel University, Philadelphia PA, Email: sohampatwardhan@gmail.com
§Department of Electrical and Computer Engineering-UC Santa Barbra, Santa Barbra CA, Email:yon@mat.ucsb.edu
¶Department of Psychology-Drexel University, Philadelphia PA, Email: Chris.Sims@drexel.edu

***Abstract*—This paper focuses on a particularly crucial aspect of haptic perception: the ability to temporarily store and manipulate haptic sensory information–a capacity termed haptic working memory (HWM). Despite the importance of HWM, the extent and nature of its limitations are largely unknown. Recent research however, has demonstrated that an information-theoretic analysis is able to provide a quantitative definition of working memory capacity for other sensory modalities (visual), and is able to yield novel predictions for human performance. Here we apply this framework to the results of a psychophysical experiment on HWM for object width. This analysis is framed around rate–distortion theory, a branch of information theory that provides optimal bounds on the accuracy of information transmission subject to a fixed information capacity. We demonstrate that a simple model developed from this framework provides an excellent account of the empirical data.**

## I. Introduction

The hands are crucial biomechanical instruments to the human species, through which we regularly perceive and interact with objects in the environment. Part of the challenge in understanding the perceptual foundations of hand function can be traced to the essential way in which haptic perception depends on both active motor behavior and sensation [1], and involves a diverse variety of perceptual tasks, ranging from proprioception and kinesthesia to tactile sensation, and the estimation of object attributes (size, weight, roughness, hardness, etc) [2], [3].

In this paper we focus on one particularly crucial aspect of haptic perception: the ability to temporarily store and manipulate haptic sensory information for the purpose of accomplishing task-relevant goals—a capacity termed haptic working memory (HWM). From a computational perspective, HWM requires the interpretation and storage of noisy sensory signals stemming from an environment that is constantly changing as we interact with objects around us. Consequently, this results in haptic representations which must be frequently updated and continually compared to recent sensory input. Although perception and memory are often depicted as separate and distinct neural processes, evidence suggests that sensory working memory—the maintenance of a working memory (WM) repository of sensory information for the online guidance of behavior—plays an important role in both the short-term storage and encoding of sensory stimuli [4]. Necessarily, perception and memory are two intimately intertwined processes as perceiving an object would be meaningless without the ability to recall and connect it to corresponding memories.

A handful of prior studies have provided evidence for a role of WM in haptic perception; for a review, see [5]. However, the extent and nature of fundamental limitations on haptic perception imposed by encoding, retention, and recall from WM is largely unknown. This stands in contrast with research in visual working memory (VWM), where limits on capacity and processing have been extensively studied (for reviews, see [6], [7]). Recent research suggests that an *ideal observer analysis* [8] based on information theory is able to provide a quantitative definition of VWM capacity, and is able to yield novel predictions for human performance in task-independent ways [9]. The goal of the present paper is to explore the application of this information-theoretic framework to the domain of HWM.

The basis of this analysis stems from the understanding that HWM may be formally studied as an information transmission channel (for a more complete description of this framework see [10]). In particular, much of the present analysis stems from a branch of information theory known as rate–distortion theory which concerns the design and analysis of optimal but capacity-limited communication channels [11], [12]. This approach yields a model of HWM according to which memory representations of haptic features are noise-corrupted versions of the original sensory signals. The capacity available to memory determines the amount of noise that corrupts each representation: If capacity is high, memory representations are accurate and minimally affected by noise. If capacity is low, each memory representation consists of a highly noise-corrupted version of the original signal, and memory accuracy is expected to be poor. The goal of the channel is to minimize the expected cost of communication (memory) error, where costs are defined according to a given *cost function*.

More formally, according to this mathematical framework HWM can be defined as a communication channel with inputs $x$ drawn from a source distribution specified by $p(x)$. In the general case $x$ can refer to a vector of haptic features, but in the present case we will consider haptic memory for a single unitary dimension, namely the width of an aperture felt by the thumb and index finger. The distribution $p(x)$ describes the statistics of the features in a given task or context. The communication channel is defined by the conditional probability distribution $p(y \mid x)$, which describes the probability of an input signal $x$ being transmitted (remembered) as a different signal $y$. The goal for the channel is to minimize the cost of communication error; this cost is specified by the cost function, $\mathcal{L}(x, y)$. For example, one possible cost function is the squared error, $(y-x)^2$. For a given information source, channel, and cost function, the expected cost, or channel *distortion* is given by:

$$D = \mathrm{E}\left[\mathcal{L}(x, y)\right] = \sum_x \sum_y \mathcal{L}(x, y) p(y \mid x) p(x). \qquad (1)$$

Any physical communication channel is necessarily limited to transmitting information at a finite rate. This capacity limit is defined by the mutual information between the channel input and output:

$$I(x, y) = \sum_x \sum_y p(y \mid x) p(x) \log \frac{p(y \mid x)}{p(y)} \qquad (2)$$

With channel distortion and information rate defined, an optimally efficient communication channel is one that minimizes distortion subject to a constraint on information rate and capacity (indicated by *C*):

$$\begin{aligned} \text{Goal: } & \text{Minimize } \mathrm{E}\left[\mathcal{L}(x, y)\right] \text{ with respect to } p(y \mid x), \\ & \text{subject to } I(x, y) \leq C \end{aligned} \qquad (3)$$

This describes an optimization problem under constraint, and the field of rate–distortion theory is concerned with the optimal solution to this problem [11]. This optimization problem can be solved by a variety of methods including convex optimization; in the present paper we rely on a particularly efficient algorithm due to Blahut [13]. For a more detailed treatment of this approach, see [10]. The question we explore in this paper is whether this mathematical framework can yield novel insights regarding the nature or operation of human HWM. If HWM approximates an efficient communication channel in this formal sense, then human performance should be closely approximated by the channel $p(y \mid x)$ that results from solving the above optimization problem.

In particular, the mathematical framework of rate–distortion theory yields three predictions that we test experimentally:

- First, rate–distortion theory predicts that as available channel capacity decreases, memory error must necessarily increase. One means of experimentally inducing changes in available memory capacity is to ask experimental participants to maintain multiple pieces of information in memory simultaneously. Intuitively, if HWM has a fixed capacity, then storing multiple haptic features in memory will leave less capacity available for encoding or transmitting each one. This should translate to an increase in the error with which features can be recalled from memory. This *set size effect* has been widely studied in the context of VWM [7] but has not previously been examined in the case of HWM.
- Second, an efficient communication channel must be sensitive to the statistics of afferent signals. Hence, there is a strong link between memory and statistical learning [14]. In order to minimize expected costs, memory accuracy should be higher for signals that are more likely to be encountered, and less accurate for signals that are unlikely. This prediction is tested by generating haptic stimuli (aperture widths) from a given probability distribution and then examining memory performance as a function of the specific stimulus presented.
- Third, the mathematical framework also predicts that memory representations should be biased. This prediction is shared by Bayesian models of perception [15], where noisy and uncertain sensory signals are combined with prior expectations, or beliefs. The result of combining sensory signals with prior beliefs is that perception is biased or shifted towards the prior. Information theory extends this by also predicting how the magnitude of this bias should vary depending on available capacity. As capacity decreases (when set size is manipulated), the reliability of memory decreases; hence more weight should be given to the prior.

These predictions were tested in a psychophysical experiment on HWM for object width. Object width was chosen as the studied haptic feature as it is simple to generate opposed surfaces requiring grip apertures of varying widths as experimental stimuli, and it can be expected that every human has extensive experience in grasping and haptically perceiving objects of varying width or thickness.

## II. Methods

### A. Participants

Four participants (three female) recruited from the Drexel University Campus took part in this study in exchange for monetary compensation totaling $45 for completing four 1-hour sessions.

### B. Apparatus

Subjects were seated in front of a computer interface consisting of a novel device referred to as the UGrip haptic display (see Figure 1), described fully in a prior publication [16]. This device was used to render object widths that were presented to subjects by manipulating the distance between the plates. The interface consists of a pair of anodized aluminum plates with rubberized surfaces that are grasped in opposition by the index finger and thumb. In addition, the subjects were not able to see the widths presented to them as the device was occluded from sight by a steel box closed on three sides, with the front facing side enclosed with dark fabric. This ensured that only the haptic sensory modality was being used to remember stimuli properties.

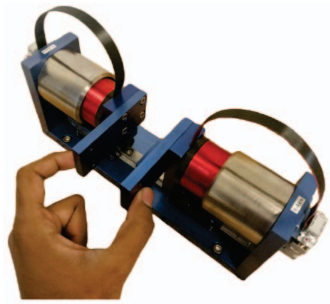

Fig. 1. The haptic display apparatus used for generating object widths, described in [16]. The apparatus controls the separation between two metal plates. Subjects feel the aperture width using their thumb and index finger.

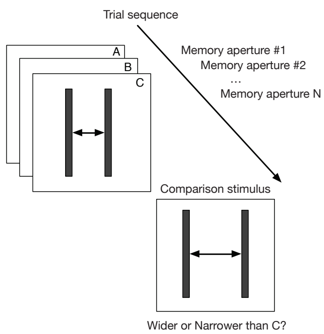


Fig. 2. Sequence of events on each trial. Subjects were sequentially presented with and asked to remember 1–4 memory widths. A probe stimulus was then presented; subjects were asked to respond whether the probe stimulus was narrower or wider than a cued memory width.

### *C. Procedure*

During each experimental trial the subjects were instructed to remember a set of presented memory widths. The widths were drawn from a non-uniform distribution ranging from 45mm to 81mm with a mean memory width of 63mm (the distribution is illustrated in Figure 6). On different experimental sessions (conducted on different days), the number of widths to be remembered varied, using set sizes of n = (1, 2, 3, or 4 items). Widths were presented sequentially, requiring the subject to press a key after feeling each width and to cue the presentation of the next. After the memory width(s) were presented, subjects experienced a comparison stimulus. The task for the subject was to decide whether the comparison width was wider or narrower than a randomly probed memory width previously felt from the stimulus set as illustrated in 2. The width of the comparison stimulus was determined by adding a perturbation value ($\Delta$) to the memory width by drawing from the set $\Delta$ =(–8, –4, –2, +2, +4 , or +8) mm, where positive perturbations correspond to enlarging the width.

Subjects completed 300 trials in each of the four set size conditions. The trial order of the different perturbations was randomized, with the constraint that each perturbation value was presented an equal number of times (50). Following the presentation of the comparison stimulus, subjects responded by pressing one of two keys on a standard keyboard, depending on the perceived width of the perturbation (i.e. wider or narrower) compared to the probed memory item. Subjects were not given feedback regarding the correctness of their choice during the experimental trials.The presentation of both memory stimuli and the comparison probe proceeded at the subject's own pace, as they needed to initiate a keyboard press in order to proceed to the next item. In between each width presentation, subjects placed their hand flat on the table in front of the device. This self-paced progression design, though assuring the physical safety of the participants, sacrificed experimental control over the temporal presentation of stimuli.

Subjects completed 50 trials in each of 24 conditions (4 set sizes $\times$ 6 perturbation magnitudes), for a total of 1200 trials per subject over the course of 4 approximately 1-hr sessions on separate days. One participant completed 1153 trials due to a technical problem in which the device lost power during the set size 3 block; for this session only 253 trials were completed.

### *D. Model predictions*

We developed a computational model of an efficient HWM system based on the mathematical framework of rate–distortion theory [11]. The model assumes that memory is a capacity-limited information channel, where the capacity limit is given by the parameter $C$, measured in bits. The goal of the channel is to minimize the cost of communication error according to a particular cost function, subject to the constraint on available capacity. Memory is defined by a conditional probability distribution $p(y \mid x)$ that describes the probability that a width $x$ is (mis-)remembered as a different width $y$. The optimal memory channel $p(y \mid x)$ was defined according to Equation 3 and computed using the Blahut algorithm [13].

We assume that participants attempted to maximize accuracy on the task, which defines a particular cost function. For a trial with a memory stimulus $x$, perturbation $\Delta$, and remembered stimulus $y$, the model will report "Wider" whenever ($y < x + \Delta$). When $\Delta$ is positive this response is correct (zero cost); otherwise the response is incorrect and defined to have a fixed cost of 1. Following this logic, the task-defined cost function for memory error is given by:

$$\mathcal{L}(x, y \mid \Delta) = \begin{cases} 0, & (y > x + \Delta) \text{ and } (\Delta < 0), \\ 0, & (y < x + \Delta) \text{ and } (\Delta > 0), \\ 1, & \text{otherwise} \end{cases} \quad (4)$$

$$\mathcal{L}(x, y) = \sum_{\Delta} \mathcal{L}(x, y \mid \Delta) p(\Delta) \quad (5)$$

The first equation describes the cost of error conditioned on a particular value of $\Delta$. Since the magnitude of the perturbation is not known at the time of encoding the stimulus in memory, the goal is to minimize the expected cost of error, averaging over all possible values of $\Delta$. For our experiment all perturbations were equally likely, so $p(\Delta) = \frac{1}{6}$.

The other component of the information-theoretic model is the probability distribution over stimuli, quantified by $p(x)$. In our model we allowed for the possibility that the subject's understanding of the distribution of widths may differ from

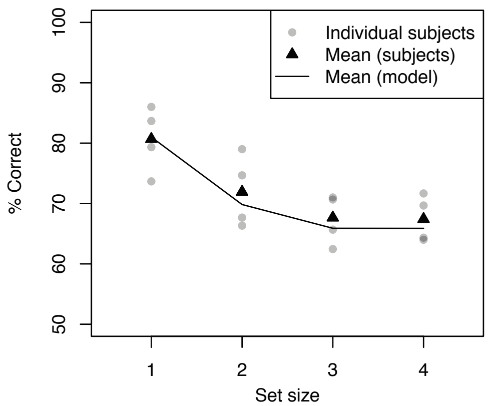


Fig. 3. Proportion of trials correct as a function of set size.

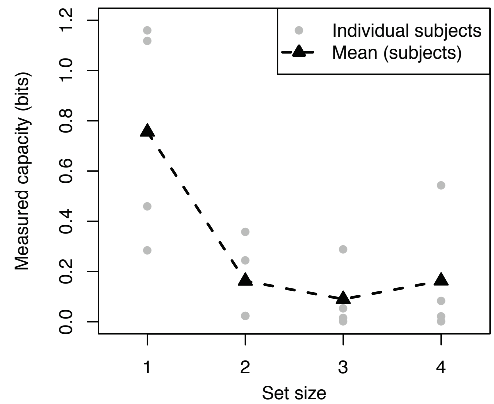


Fig. 4. Estimated memory capacity (C) in bits as a function of set size.

the experimentally defined distribution (i.e. the potential for bias). To allow for this possibility, we treat $p(x)$ as a Gaussian distribution, where the parameters $\mu$ and $\sigma$ are fit to the experimental data from each subject. Hence, the model has three parameters total, $C$, $\mu$, and $\sigma$. For each combination of parameters, rate–distortion theory defines an optimal communication channel $p(y \mid x)$ as described by Equation 3. This distribution in turn generates predictions for performance in the experiment. The three parameters of the model were fit separately to the data from each subject and set size condition via maximum likelihood estimation.

## III. Results

Figure 3 illustrates the proportion of trials answered correctly as a function of set size. Most notable in the data is that performance decreased as the number of stimuli in the set was increased. We conducted an ANOVA on mean percent correct as a function of set size, which confirmed a significant difference in performance, $F(3, 9) = 42.00, p < .001$, with lower performance at higher set sizes. An important limitation of the current study is that it utilized a sequential presentation paradigm. Preliminary analysis revealed that items presented later in the sequence were better remembered. Such recency effects are well established in the literature, however in the current analysis we omit the role of time.

The information-theoretic model closely captures human performance, as illustrated by the solid black line in Figure 3. The model is able to fit the decrease in performance by separately estimating channel capacity for each set size. The resulting estimates of memory capacity are illustrated in Figure 4. For remembering a single width, HWM capacity is approximately 0.8 bits and this value declines substantially at larger set sizes. An ANOVA on the capacity estimates revealed a significant main effect of set size, $F(3, 9) = 12.63, p = .0014$. Capacity was significantly higher in the set size 1 condition, but there were no significant differences in capacity between the remaining three set size conditions. Notably, capacity estimates demonstrated in Figure 4 are substantially lower than estimates previously seen in VWM [9], [10]. However, the results confirm our prediction that increasing the set size should decrease the capacity available for encoding each stimulus item in memory.

Figure 5(a) compares psychometric performance of the human observers against the fit of the information-theoretic model. In each panel, the size of the perturbation ($\Delta$) is plotted against the proportion of trials that subjects reported as a larger width. Correct response requires reporting "Wider" whenever delta is positive. Performance at each set size is plotted in a separate panel. The figure demonstrates that the precision of HWM declines with increasing set size. That is to say, as set size or memory load increases, a larger perturbation ($\Delta$) is necessary to achieve the same level of performance. A similar trend is illustrated by the model.

In addition, human performance shows a bias towards responding "Wider", and the magnitude of this bias is increased at higher set sizes, illustrated by a vertical shift in the psychometric curve in the larger set size conditions. This bias can be explained by the assumption that the subject's implicit model or understanding of the width statistics differed from the veridical distribution of the stimuli. In the framework of Bayesian inference, noisy estimates are biased in the direction of prior knowledge. In exactly the same way, the information-theoretic model exhibits a bias towards the assumed prior, and the magnitude of the bias increases as the quality of memory evidence declines in the higher set sizes.

The biasing role of prior or top-down knowledge in HWM is illustrated more dramatically in Figure 5(b). This figure plots response proportion against deciles of the stimulus distribution. Each decile averages over positive and negative delta trials, so performance should be unbiased (close to 50 percent responding "Wider"). At a set size of one this is observed. However, as the set size increases, the reliability of memory decreases (i.e. added noise in representations) and subjects exhibit an increasing bias that is best explained as a regression-to-the-mean effect. For example, unusually large memory widths are remembered as closer to the mean; hence when presented with a comparison stimulus subjects are biased towards believing that the comparison is larger than the

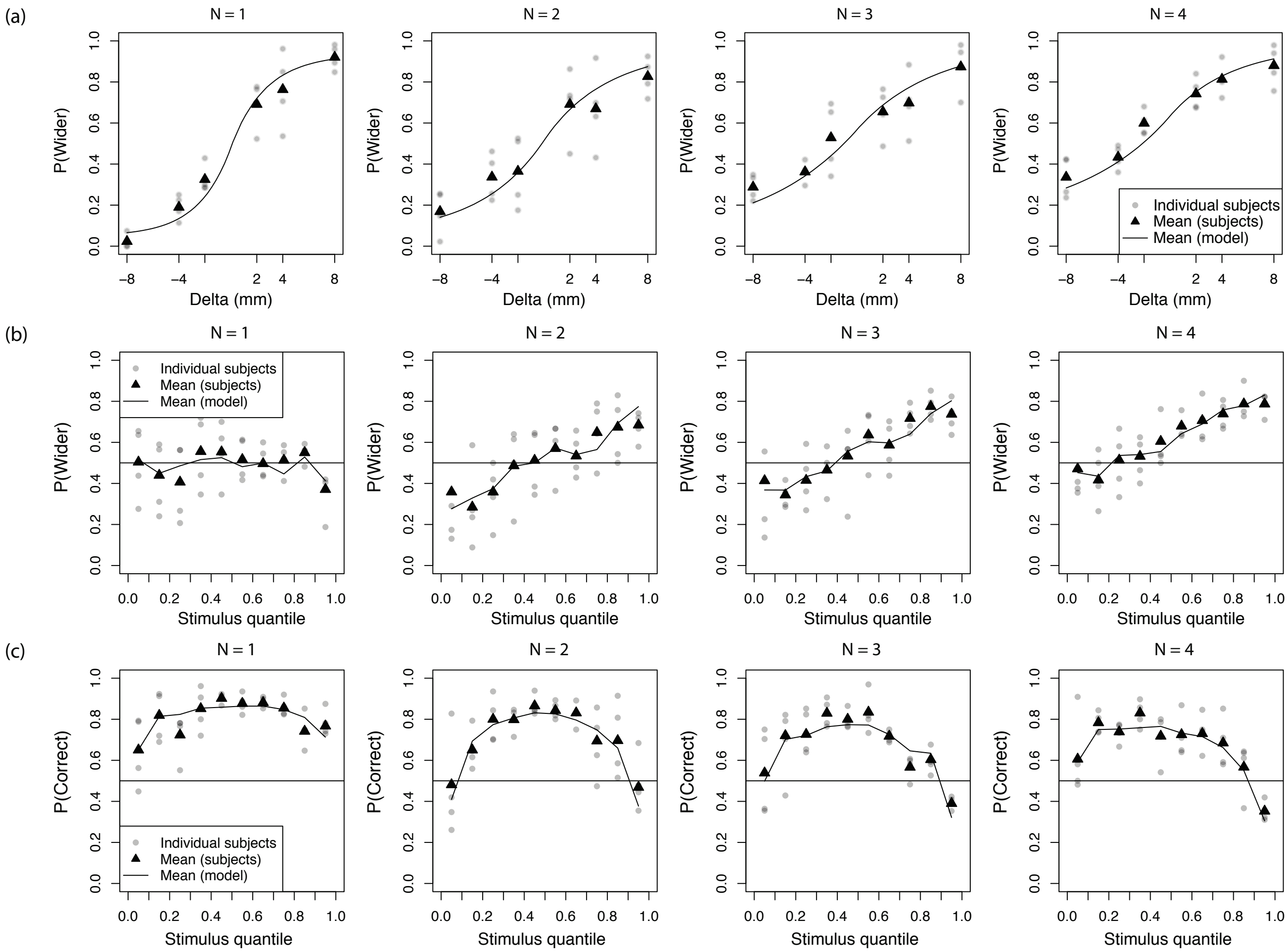


Fig. 5. (a) Proportion of trials judged as "Wider" (perturbed positively) as a function of true perturbation per condition. Mean human data are indicated by triangular markers. Model fit is illustrated by the solid black lines. (b) Response bias, defined as the tendency to report a wider perturbation, as a function of set size and stimulus quantile. Values are binned into 10 equal quantiles (deciles), and response bias is computed separately for each subject, quantile, and condition as well as for the empirical and model averages. (c) Accuracy bias, defined as a higher proportion of correct responses for stimuli which follow the subjects implicit distribution, as a function of deciles of the stimulus distribution.

(mis-)remembered memory stimulus. Both the presence of the bias, and the dependence of its magnitude upon the set size, are closely matched by the information-theoretic model. This demonstrates that HWM is intricately connected to statistical learning and inference.

To account for the bias, it is necessary for the model to assume that subjects' implicit prior over object widths differs from the true stimulus distribution. The parameters of this implicit prior were fit by maximum likelihood estimation, assuming that the implicit prior is a Gaussian distribution with unknown mean and standard deviation. Figure 6 compares the true stimulus distribution with the "fit" distribution for Subject 1 across all four set size conditions. Similar results are obtained for the remaining three participants; the estimated parameters are reported in Table I.

To investigate the extent to which the response bias affected performance, we plotted accuracy according to deciles of the stimulus distribution. Presumably, performance (proportion of correct responses) should be higher for stimuli which are more probable according to the subject's implicit understanding of the stimuli distribution. Figure 5c captures this, whereby the highest performance aligns with the peak of the implicit distribution seen in Figure 6. The subjects' implicit prior assumes narrower widths on average than the actual stimulus distribution, particularly as the set size increases.

TABLE I

AVERAGE MEAN ($\mu$) AND SD ($\sigma$) OF THE IMPLICIT PRIOR OF THE STIMULUS DISTRIBUTION ILLUSTRATED IN FIG 6 (PARAMETERS AVERAGED ACROSS THE FOUR SUBJECTS

| Condition | Set Size 1 | Set Size 2 | Set Size 3 | Set Size 4 |
|---|---|---|---|---|
| **Mean $\mu$** | 62.83 | 62.26 | 60.89 | 57.26 |
| **Mean $\sigma$** | 13.52 | 12.45 | 17.9 | 20.19 |

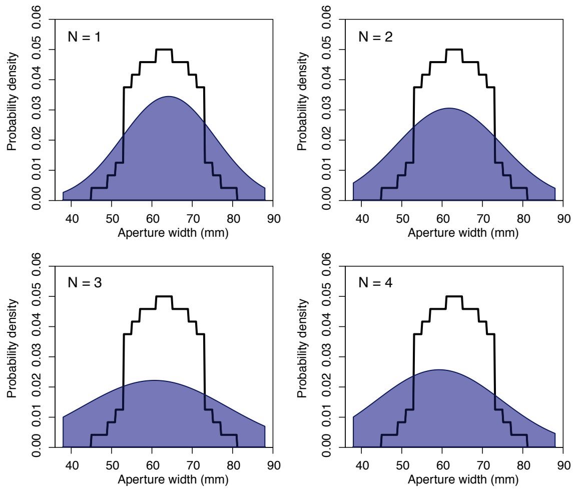


Fig. 6. Comparison of the distribution used to generate stimuli (unshaded region) to the estimated implicit distribution used by a single subject across all set size conditions (shaded region).

## IV. Conclusion

In this paper we focused on a particularly crucial aspect of haptic perception: the ability to temporarily store and manipulate haptic sensory information. A small body of work has examined the role of WM in haptic perception, but has yet to explore the fundamental limitations of the system. As such, the present work is the first application of rate–distortion theory to the study of HWM we are aware of.

Rate–distortion theory chiefly predicts that as available channel capacity decreases, memory error must necessarily increase. Our results fall in line with this prediction, as a significant decline in HWM performance was observed when more than one width was held in memory. Combined with our model capacity estimations, this suggests the HWM system to be of highly fixed capacity and to be poorly suited to the simultaneous storage of multiple items.

Secondly, our theoretical and mathematical framework predict that memory representations, in the presence of sensory uncertainty (i.e. noise) should be biased and this bias should shift towards the prior as the level of uncertainty increases [17]. Notably, subjects adopted an implicit understanding of the stimulus distribution that differed from the actual distribution whereby the observed implicit prior assumed narrower widths on average than the actual stimulus distribution, particularly as the set size increases. More interestingly, the magnitude of this observed bias depended upon the level of memory uncertainty. The relationship between the level of uncertainty and degree of bias towards the prior robustly demonstrates HWM to be intimately connected to statistical learning and inference.

In order to minimize expected costs, memory accuracy should also be more accurate for signals that are more likely to be encountered and less accurate for signals that are less likely. Here, subjects' learned statistics of the stimuli did not match the laboratory-defined distribution. Given the limited exposure to the task environment (approximately 4 hours in total), subjects may not have observed enough sensory evidence to form relatively accurate estimates of the true distribution. In place of adapting to the veridical distribution however, subjects demonstrated a bias for smaller object widths. It could be that this stems from a more global bias towards narrower haptic widths, derived from object properties most commonly encountered in the environment.

This being the case, a productive approach to understanding the nature of capacity limits in HWM may lie in treating memory as a communication channel that has evolved to be able to efficiently encode and store the most commonly encountered haptic object properties (e.g. widths). Continued research might seek to explore the nature of "natural" priors for haptic object size (or grip aperture size) in order to better explain the origin of the biases observed and the inefficiency of HWM for storing multiple stimuli. Lastly, the present analysis does not account for the effect that time or temporal decay may have on memory. As a distinct limitation of this paper, future research should explore the possible effect of this on memory accuracy.

## References


[1] J. M. Loomis and S. J. Lederman, "Tactual perception," *Handbook of perception and human performances*, vol. 2, p. 2, 1986.

[2] S. J. Lederman and R. L. Klatzky, "Haptic perception: A tutorial," *Attention, Perception, & Psychophysics*, vol. 71, no. 7, pp. 1439–1459, 2009.

[3] M. Craddock and R. Lawson, "The effects of size changes on haptic object recognition," *Attention, Perception, & Psychophysics*, vol. 71, no. 4, pp. 910–923, 2009.

[4] T. Pasternak and M. W. Greenlee, "Working memory in primate sensory systems," *Nature Reviews Neuroscience*, vol. 6, no. 2, pp. 97–107, 2005.

[5] A. L. Kaas, M. C. Stoeckel, and R. Goebel, "The neural bases of haptic working memory," in *Human haptic perception: Basics and applications*. Springer, 2008, pp. 113–129.

[6] S. J. Luck and E. K. Vogel, "Visual working memory capacity: From psychophysics and neurobiology to individual differences," *Trends in cognitive sciences*, vol. 17, no. 8, pp. 391–400, 2013.

[7] W. J. Ma, M. Husain, and P. M. Bays, "Changing concepts of working memory," *Nature neuroscience*, vol. 17, no. 3, pp. 347–356, 2014.

[8] W. S. Geisler, "Contributions of ideal observer theory to vision research," *Vision Research*, vol. 51, pp. 771–781, 2011.

[9] C. R. Sims, R. A. Jacobs, and D. C. Knill, "An ideal observer analysis of visual working memory." *Psychological review*, vol. 119, no. 4, p. 807, 2012.

[10] C. R. Sims, "The cost of misremembering: Inferring the loss function in visual working memory," *Journal of vision*, vol. 15, no. 3, pp. 1–27, 2015.

[11] T. Berger, *Rate distortion theory: A mathematical basis for data compression*. Prentice-Hall, 1971.

[12] C. E. Shannon and W. Weaver, *The mathematical theory of communication*. University of Illinois Press, 1949.

[13] R. E. Blahut, "Computation of channel capacity and rate–distortion functions," *IEEE Transactions on Information Theory*, vol. IT-18, no. 4, pp. 460–473, 1972.

[14] A. E. Orhan, C. R. Sims, R. A. Jacobs, and D. C. Knill, "The adaptive nature of visual working memory," *Current Directions in Psychological Science*, vol. 23, no. 3, pp. 164–170, 2014.

[15] K. P. Körding and D. M. Wolpert, "Bayesian integration in sensorimotor learning," *Nature*, vol. 427, no. 6971, pp. 244–247, 2004.

[16] Y. Visell, K. Duraikkannan, and V. Hayward, "A device and method for multimodal haptic rendering of volumetric stiffness," *Proc. of Eurohaptics Conference*, 2014.

[17] K. Körding, "Decision theory: what "should" the nervous system do?" *Science*, vol. 318, no. 5850, pp. 606–610, 2007.