The Space Cadet Mission

Annabelle Schmitte, Jeremy Andrews

https://www.statnews.com/2021/06/11/space-radiation-sperm-international-space-station/

Overview

Recap of project

Physics & Math background

The direction of force on the system

How energy affects the system

Kepler’s 1st, 2nd law

Progress Coding in Python

Our Conclusions

Future Work

Recap of Project

Using classical mechanics, investigating how two objects in space interact through gravitational force.

Math and physics oriented

Derived several important equations used throughout stellar mechanics including Kepler’s laws

Used Python programming for numerical integration of differential equations

Physics Background

- All matter in the universe affects space around it through a gravitational force

- Strength of gravity is proportional to the product of the masses and inversely proportional to the square of the distance between the objects →

$$F = G\frac{m_1 m_2}{r^2}$$

F = force
G = gravitational constant
m = mass
r = distance

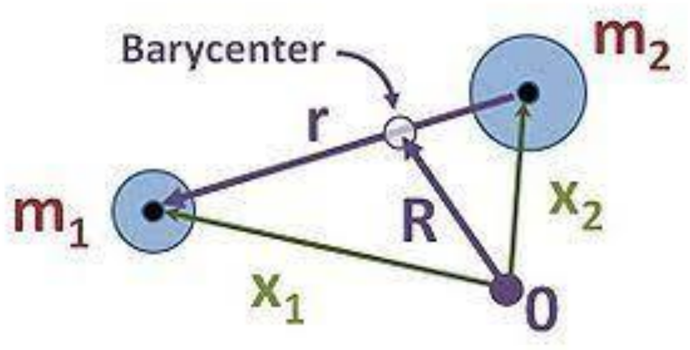
- These natural processes can be described through mathematics

Force and Velocity

- An object does not need a force to maintain velocity
- In a vacuum an object would continue on a straight path, otherwise constant velocity unless acted on by an outside force
- Voyager probes were launched in 1977 and are still traveling today without any force to continue there velocity

The Two Body Problem

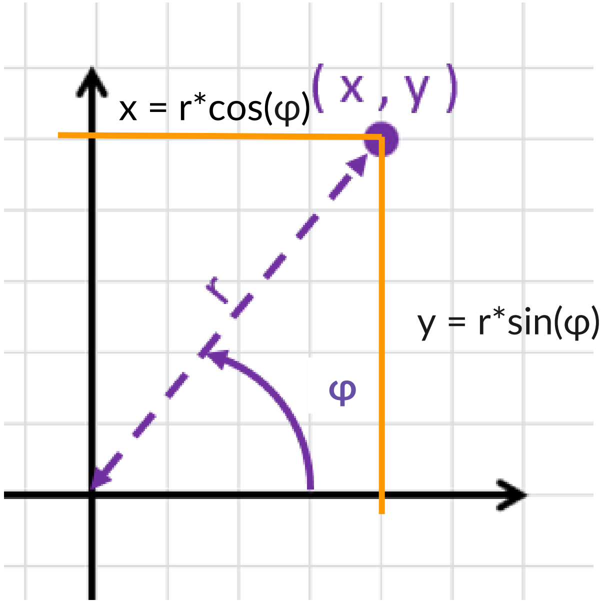
- Two objects can refer to two molecules, two asteroids, or even our sun and Earth
- Both bodies act on each other through a force
- Force and angular momentum allow the two bodies to stay in orbit (Newton's 1st law)



r = distance between the masses
R = center of mass
m_i = mass of a body
x_i= distance from the origin to a body

Polar Coordinates

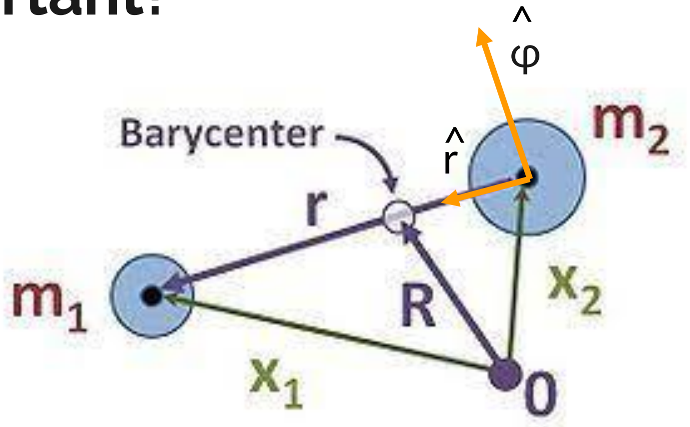
- We started the mathematical process with Cartesian coordinates (x,y) then converted to polar coordinates
- Polar coordinates are a two-dimensional coordinate system where each point on a plane is determined by the distance from a reference point (r) and an angle from a reference direction (ϕ)
- In a circular orbit r would remain constant



Polar vs. Cartesian Coordinates. Dec. 2019.
https://www.101computing.net/polar-vs-cartesian-coordinates/g

Why was this conversion important?

- allows us to recognize the conservative central force as the only force acting upon the two bodies, which is radial
- found the derivative of ϕ and simplified our equation for the force along r
- Angular momentum is conserved



$F_r=(Gm_1m_2)/r^2$

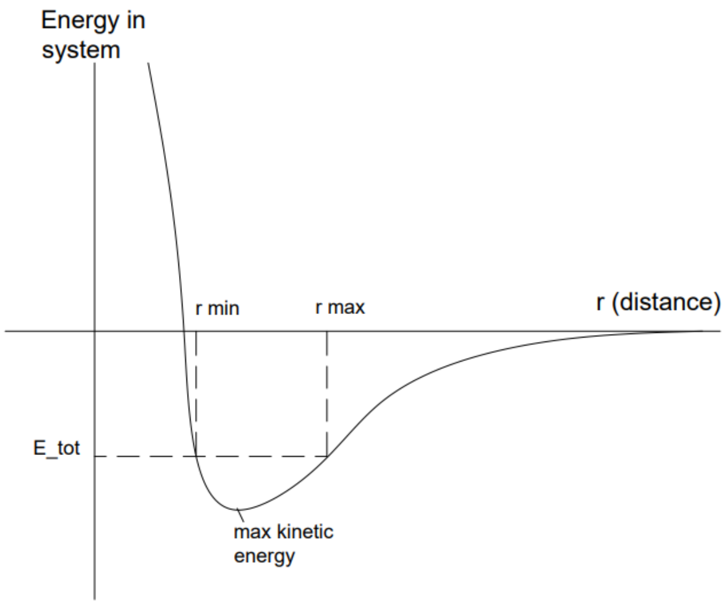
$F_\phi=0$

r = distance between the masses

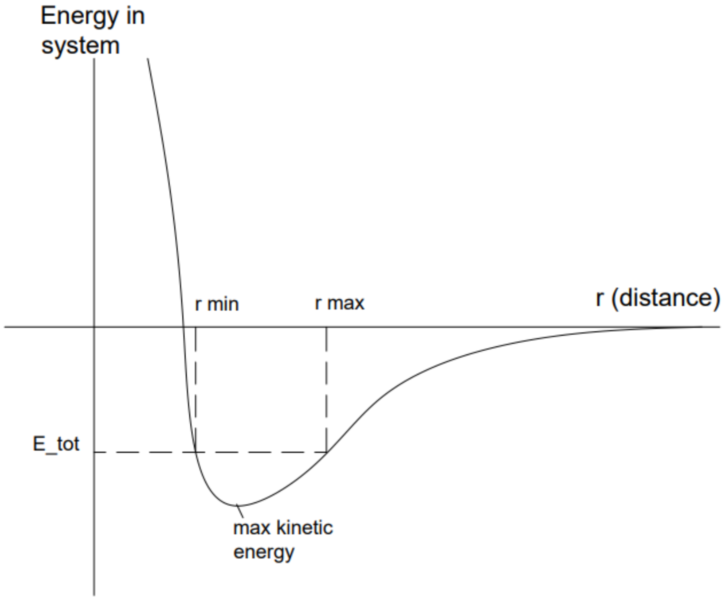
m_i = mass of a body

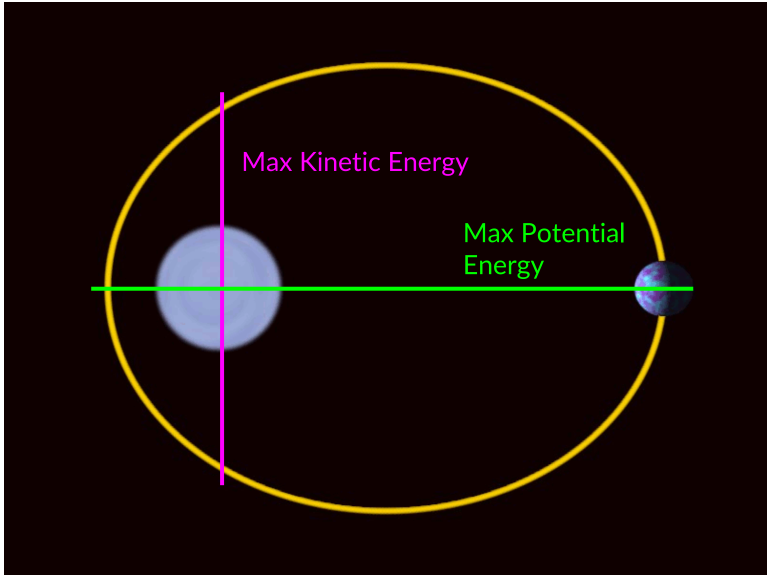
Energy within the two-body system

- Energy is split between kinetic and potential energy
- There is a trade off between these two energies as the position of the orbiting object changes
- Max kinetic energy=max velocity
- Max potential energy=minimum velocity



Kepler's 2nd Law





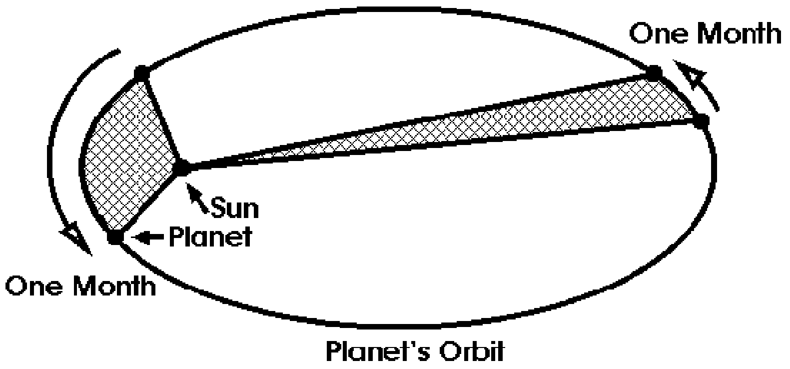
https://commons.wikimedia.org/wiki/File:Classical_Kepler_orbit_120frames_e0.6.gif

Kepler’s 2nd Law

- In a system with periodic motion in orbit, the distance between the two bodies will sweep out an equal area over an equal time
- Area and period are directly proportional

$$A = \frac{1}{2}\frac{L}{\mu}\tau$$

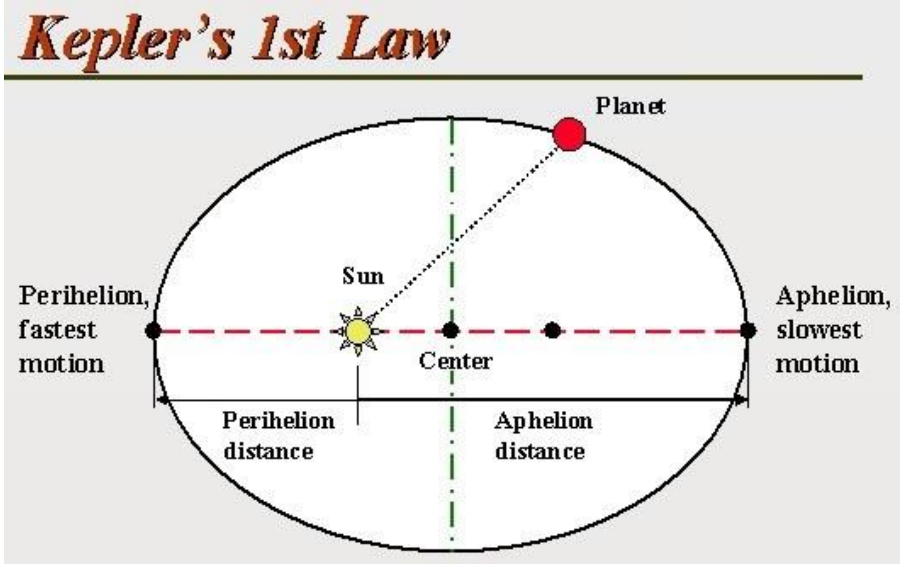
A = area
L = angular momentum
μ = reduced mass
τ = period



Kepler’s 1st Law

This states that each planet's orbit around the sun is an ellipse.

We call this elliptical orbit the eccentricity of the orbit, or how noncircular the orbit is.



http://web.mit.edu/12.004/TheLastHandout/PastHandouts/Chap03.Orbital.Dynamics.pdf

https://www.quora.com/What-is-an-example-of-Keplers-1st-law

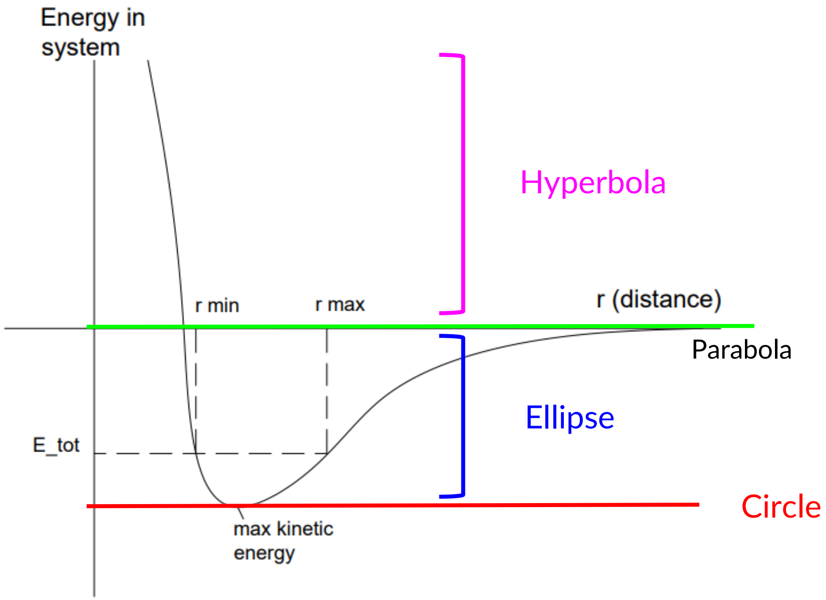
Eccentricity

The eccentricity of an orbit is a ratio that describes the shape of the orbit

$$e = \sqrt{1 + \frac{2E_{tot}L^2}{\mu k^2}}.$$

The orbit type depends on E_{tot} in the following manner:

$E_{tot} = -\frac{\mu k^2}{2L^2}$,	$e = 0$,	circle,
$E_{tot} < 0$,	$0 < e < 1$,	ellipse,
$E_{tot} = 0$,	$e = 1$,	parabola,
$E_{tot} > 0$,	$e > 1$,	hyperbola.



Coding Exercise

Purpose was to enter known orbital data into python to produce plots

Used numerical integration of differential equations obtained through our derivations

Two bodies in this example were Earth and the moon

Produced plots comparing position to velocity, angle to angular momentum.

Data Provided:

Mass of Earth: 5.97*10^(24) kilograms

Mass of Moon: 7.35*10^(22) kilograms

Semi major axis of moon’s orbit: 3.84*10^8 meters

Eccentricity of moon’s orbit: 0.0549 (unitless)

Gravitational constant: 6.67*10^(-11) m^3/(s^2*kg), same everywhere in the universe

Plotting Position and Velocity as a Function of t

r(t)=position

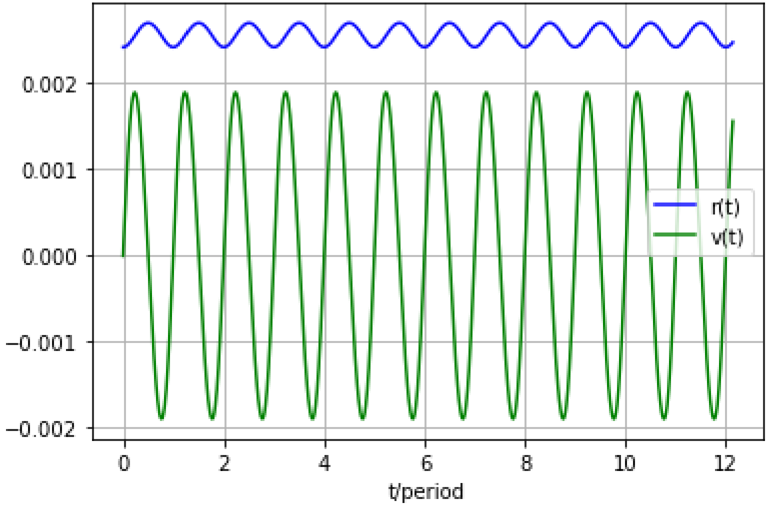
v(t)=velocity

Velocity is how fast the position changes

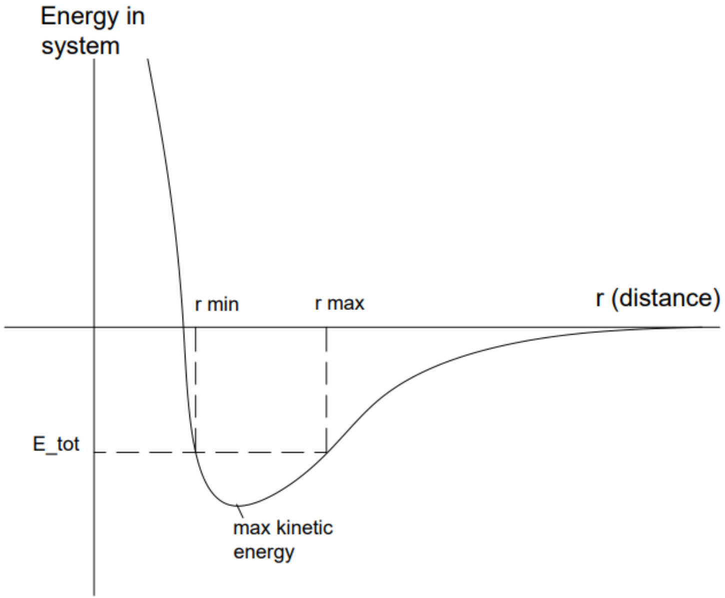
Note that one period of r(t) coincides with one period of v(t)

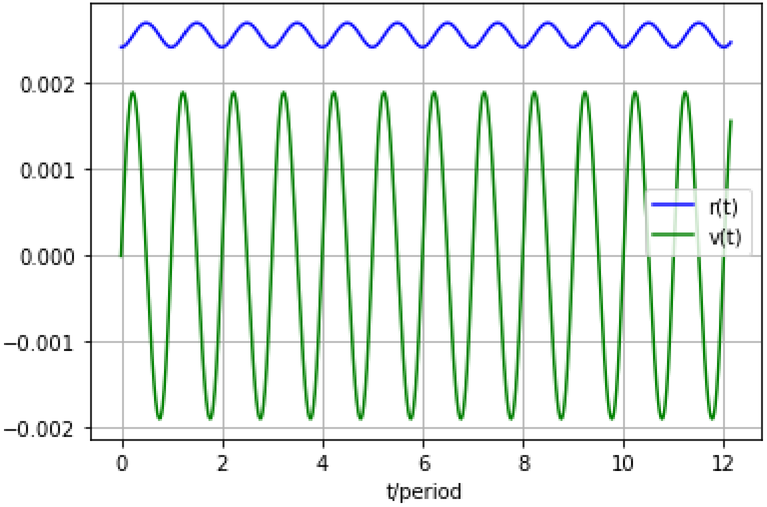
When v(t)=0, our orbiting body is at its turn around point

We can compare this plot to our potential energy curve



Comparing Potential Energy to Position and Velocity





Our Conclusions

Built a theoretical foundation of the physical background and better understanding of the complexity of the two body problem

Gained experience applying higher level calculus to a real world influenced problem

Learned how to use python coding to visualize mathematical and physical processes

Potential Future Work

Exploring the effect that other variables have on two bodies, including atmospheric drag and solar radiation pressure

Refining Python code to model these added variables

Examining the complexities of the three body problem