Design optimization of a two-stage compound gear train

Team #3

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Abstract

Gear train is pertinent part of the majority of mechanical power transmission system. It has to be robust enough to sustain the transmitted power over the prolong period of time and also light enough to reduce the overall weight of the system and wastage of material. But since to increase the power to be transmitted by the gear train also causes increase in weight and vice versa, these two objectives rather generates contradicting solutions. Thus, the optimization of gear becomes very significant in order to have a good trade-off between these two entities.

The paper involves the optimization of a two-stage compound gear-train wherein minimization of weight and maximization of power are considered as the two objective functions. The decision variables considered in this report are number of teeth, modules and material. The gear-train system is subjected to constraints such as tooth strength, dynamic loading, minimum module and wear strength. Since it is multi-objective function with contradicting objective function, it becomes difficult to optimize using the conventional optimization techniques. Hence, we will use non-traditional Non-dominated Sorting Genetic Algorithm-II (NSGA-II) which is much more proficient to solve this kind of problem. MATLAB code will be used to implement this algorithm.
Problem Statement

Design and optimize a two-stage compound gear train considering the following multi-objective functions:

- Minimization of the overall weight (f1)
- Maximization of Power transmitted (f2)

For illustration purpose, we have considered a two-stage compound gear-train having the input power as 15 KW, reduction ratio as 2.5 and speed as 1440 rpm.

Certain assumptions for gear parameters:-

- Type of gear tooth profile= Involute
- The Pressure angle of the gear= 20 degrees
- Minimum Number of teeth on the spur gear = 17 (to avoid interference)
- Young's Modulus of the materials = 2.1*10^5 N/mm^2
- Carefully cut gears are considered
- Value of c in Buckingham's Dynamic Load = 11860*e where e-error is expected error in tooth profile
- Ψ (Ratio of Face Width by module) = 0.3

Mathematical Model

Objection function:

1) Minimize - Overall weight of the gears (f1)
   \[ f_1 = \left[ \frac{\pi}{4} \right] (m*Z_1)^2 b \rho + \left[ \frac{\pi}{4} \right] (m*Z_2)^2 b \rho \]

2) Minimize - Power Transmitted (f2)
   \[ f_2 = \frac{15}{100} \left( \frac{(100-50*f^2 (H_s^2 + H_t^2))/\cos(\theta) (H_s + H_t)}{\Psi} \right) \]

Definition of design variables:

<table>
<thead>
<tr>
<th>Decision variable</th>
<th>Notation</th>
<th>Variable type</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module</td>
<td>m</td>
<td>Discrete</td>
<td>{1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6}</td>
</tr>
<tr>
<td>Number of teeth on first gear</td>
<td>Z1</td>
<td>Discrete</td>
<td>{18, 20, 22, 24, 26, 28, 30, 32}</td>
</tr>
<tr>
<td>Material selection</td>
<td>B</td>
<td>Discrete</td>
<td>{0, 1}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0 for Material 1 - 40Ni2Cr1Mo28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 for Material 2 - 40Cr1</td>
</tr>
</tbody>
</table>

Table 1
Available Materials:
1) 40Ni2Cr1Mo28
   Crushing strength [$\sigma_c$] - 11000 kgf/cm²
   Tensile strength [$\sigma_b$] - 4000 kgf/cm²
   Density = 7800 kg/m³

2) 40Cr1
   Crushing strength [$\sigma_c$] - 9750 kgf/cm²
   Tensile strength [$\sigma_b$] - 2375 kgf/cm²
   Density = 7100 kg/m³

Definition of equations and constraints:
- Tooth strength $F_s = [\sigma_b] * b^2 * Y * m$
  where $Y = (0.154 - \frac{0.912}{Z}) * \pi$
  $\Psi = b/a = 0.3$ (Taking recommended value of $\Psi$ as 0.3)
- Number of teeth on a gear is given by $Z_2 = i * Z_1$
- The minimum module for gear is given by
  $$m_{\text{min}} = (1.26)^{\frac{3}{2}} \sqrt{\frac{M_t}{Z^2 \Psi_m * Y \sigma_b}}$$
  [mm]
  where $\Psi_m = b / m = 10$ (Taking recommended value of $\Psi_m$ as 0.3)
- Twisting torque $[M_t] = 60 * 1000 * P / (2 * \pi * N)$ [Nmm]
- Pitch line velocity $V_m = \pi * d_1 * N / 60$ [mm/s]
  The module m obtained from optimization process should be greater than $m_{\text{min}}$.
  This is given by the following constraint $m_{\text{min}} \leq m$......................................................... (1)
- Diameter of pinion $d_1 = m * t_1$ [mm]
- Diameter of Gear $d_2 = m * t_2$ [mm]
- Tangential force on gear tooth $F_t = P / V_m$ [N]
  The constraint for this equation is given by $F_t \leq F_s$......................................................... (2)
- Buckingham’s dynamic load is given by
  $$F_d = F_t + \frac{0.164 * V_m * (C * b + F_t) + Ft}{0.164 * V_m + 1.485 * \sqrt{C * b} + Ft}$$
  Where $C = 11860 * e$
  Where Expected error in tooth profile (e) varies according to module

<table>
<thead>
<tr>
<th>Module (mm)</th>
<th>Error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upto 4</td>
<td>0.025</td>
</tr>
<tr>
<td>Upto 5</td>
<td>0.025</td>
</tr>
<tr>
<td>Upto 6</td>
<td>0.030</td>
</tr>
<tr>
<td>Upto 7</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Table 2
The constraint for Buckingham’s dynamic load is given by $F_d \leq F_s$......................................................... (3)
- Wear strength of the gear is given by
\[ F_w = d1 \cdot Q \cdot b \cdot K, \quad [N] \]
where \( Q = 2 \cdot i / (1 + i) \)
\[ K = [\sigma_c]^2 \cdot \sin (\alpha) \cdot ((1/E_1) + (1/E_2))/1.4 \]
The constraint for this equation is given by \( F_d \leq F_w \)……………………………………….. (4)
• Available choices of the design variables:
  \( m, Z1, Z2, B \in \{ \text{as specified in Table 1} \} \)
• Power Loss
Friction Factor : - 0.08
\( \theta = 20 \text{ degrees involute gear} \)
\( R_o = R + \text{one addendum} \)
Addendum of 20 degrees involutes system = one module (m)
\( R_o = R + m \quad [\text{mm}] \)
\( r_o = r + m \quad [\text{mm}] \)
\( H_t = ((i + 1)/i) \cdot ((R_o/R)^2 - (\cos \theta)^2 2)^{(1/2)} - \sin \theta \)
\( H_s = (i + 1) \cdot ((R_o/R)^2 - (\cos \theta)^2 2)^{(1/2)} - \sin \theta \)
Power Loss percentage (PL) = 50*f*(H_s^2 + H_t^2)/(\cos(\theta)*(H_s+H_t))
Efficiency(\eta) = 100 - PL
Power Transmitted= \eta* 15 KW

**Optimization method**

The task of multi-objective optimization is different from that of single-objective optimization because in multi-objective optimization, there is usually no single solution which is optimum with respect to all objectives. Since our two objective functions are conflicting it is expected to generate an pareto-front. It is only known that none of the generated solutions dominates the others. We are going to use the Non-dominated Sorting Genetic Algorithm-II (NSGA-II).

**Choice of algorithm**

The standard methods for solving multi-objective problem by using the GA is through weighted sum method. The weighted sum method cannot find certain Pareto-optimal solutions in the case of a non-convex objective space. Single-objective optimization algorithm with equal-spaced weight vectors does not always guarantee a uniformly distributed Pareto-optimal front. This implies the limitation of weighted sum method for real world problem.

On the other hand, NSGA-II produces the Pareto-optimal solutions in a single run only. It is capable of handling discrete and real valued decision variables for complex versions of gearbox problem. It is an improvement over the original NSGA algorithm with less computational complexity, consideration of elitism and does not require any parameter calibration.

Hence, we have chosen NSGA-II for solving our problem.
Final Implementation:
We implemented NSGA-II using the code available on the Mathworks website. It includes separate Matlab files for objective function, initializing variables, tournament selection, replacing chromosomes and non-domination sort. It solves for minimization of objective functions. The algorithm requires input optimization parameters, constraints and the objective function. In our case it will minimize the weight and power loss (i.e. increases output power transmitted).

Steps for implementation :
• The lower bound and upper bounds of decision variables are given input as vectors. We set the algorithm to run for integer type of decision variables. However since module used are standard values they are discrete choices. Changes were made in the algorithm for integer to discrete transformation for module selection.
• Mutation fraction and Cross-over fraction are tuned till we get a proper pareto-front
• We run the algorithm for two runs, for stage one and stage two of compound gear sets, with different population size and generations to get pareto fronts. The first run will give pareto-optimal solution values of decision variables for stage one. The input conditions for stage two are then calculated. We assume the gear ratio of each stage remains the same. The second run then gives a pareto-optimal solution values for second stage.

Results

Incumbent solution
Considering the problem as optimization of spur gear train with decision variables and objective function as discussed above, we solve the problem by traditional method using hand calculations. We obtain the following results for the two stages:

<table>
<thead>
<tr>
<th>Material 1 - 40Ni2Cr1Mo28</th>
<th>Material 2- 40Cr1</th>
<th>Material 1 - 40Ni2Cr1Mo28</th>
<th>Material 2-40Cr1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module - m (mm)</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Teeth on pinion -z1(mm)</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Tooth strength - Fs (N)</td>
<td>20776</td>
<td>19275</td>
<td>20776</td>
</tr>
<tr>
<td>Dynamic load - Fd (N)</td>
<td>9851.2</td>
<td>11093</td>
<td>11284</td>
</tr>
<tr>
<td>Wear strength - Fw (N)</td>
<td>11583</td>
<td>14219</td>
<td>11583</td>
</tr>
<tr>
<td>Weight -W (kg)</td>
<td>9.2097</td>
<td>16.373</td>
<td>9.2097</td>
</tr>
<tr>
<td>Power transmitted - P (kW)</td>
<td>14.43</td>
<td>14.43</td>
<td>13.882</td>
</tr>
</tbody>
</table>

Table 2
Optimization results

The implementation of NSGA-II algorithm was performed in Matlab. The stopping condition is the number of generations. Starting with an initial population of 50 and number of generations as 100, the code was executed. However, the solution gave optimal points with many pareto fronts. On increasing the initial population to 150 and number of generations to 400, the following pareto optimal points were obtained.

For stage 1, the plots are as follows:

As expected from the graphs, our goal of reducing weight is conflicting with the objective of maximizing the transmitted power. The optimal points satisfy all the constraints and bounds.

For stage 2, the plots are as follows:
Example of an optimal solution obtained from NSGA-II

<table>
<thead>
<tr>
<th></th>
<th>Stage 1</th>
<th>Stage 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1 - 40Ni2Cr1Mo28</td>
<td>Material 2- 40Cr1</td>
</tr>
<tr>
<td>Module - m (mm)</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Teeth on pinion - z1 (mm)</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Tooth strength - Fs (N)</td>
<td>13291</td>
<td>12941</td>
</tr>
<tr>
<td>Dynamic load - Fd (N)</td>
<td>8708.9</td>
<td>9652.9</td>
</tr>
<tr>
<td>Wear strength - Fw (N)</td>
<td>9049.1</td>
<td>10111</td>
</tr>
<tr>
<td>Weight - W (kg)</td>
<td>7.49491</td>
<td>10.35</td>
</tr>
<tr>
<td>Power transmitted - P (kW)</td>
<td>14.4565</td>
<td>14.44</td>
</tr>
</tbody>
</table>

Table 3

**Discussions**

**Comparison of initial (incumbent) solution and optimized solution**

In the incumbent solution it is difficult to consider all combinations of module and teeth, so the general procedure is to increase the module to satisfy strength criteria because increasing teeth will improve strength by lower margin. But this often leads to over-structure design leading to a higher weight (Table 2).

From Table 3, it is seen that the module and teeth of pinion are optimized resulting in adequate amount of strength to withstand the dynamic loads and wear.

Table 3 represents one of the optimal solution from the Pareto front obtained. The designer will have the option of choosing the best solution out of these based on the priority. For example if weight is the most important criterion, then material 1 can be chosen for both stages.

The optimized solution for stage 1 and material 1 gives 22.8% weight reduction, for stage 1 and material 2 gives 36%, for stage 2 and material 2 gives 23.5%.

However if cost is relatively more important, material 2 can be chosen (material 1 is stronger so in general it would be more expensive).

Also, if power transmitted is important (higher efficiency required) then there has to be a trade-off between the power transmission required and the weight.
Conclusion

The paper has successfully implemented NSGA-II on a conflicting multi-objective function of maximizing power transmitted and minimizing the overall weight of the gear train. The results obtained a pareto-front which gave an optimal set of solution for the multi-objective function. So, a particular value from the available set of solution is selected based on the trade-off between power transmitted and weight. Also, complex mechanical power transmission problems can be effectively modeled using this algorithm. However, some of the weakness of the algorithm are not considering the center distance between the input and output shaft. Also, gear tooth deflection and vibrations were neglected. In future, this algorithm can be easily extended to include cost as another objective function which also plays a crucial role in design and fabrication of gear train system. The gear train casing dimensional constraint can also be introduced to design the gear train for a fixed value of center distance. Also, it can be further extended to a multi-stage gear train system where the algorithm can even give the optimized number of stages that should be selected for a particular constrained and center distance. Moreover, other gears trains consisting of helical, bevel and worm gears can also be optimized by including the static, dynamic, wear and other constraint.
References


2. Deb K., Jain S., “Multi-Speed Gearbox Design Using Multi-Objective Evolutionary Algorithms,” Kanpur Genetic Algorithms Laboratory (KanGAL)


8. Stephen P. Radzevich, Dudley’sHandbook ofPractical GearDesign andManufacture