ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

Living in Infamy: Bad Reputations in Emerging Markets

Authors	Zachary R. Stangebye, and Mark L. J. Wright
Working Paper Number	2026-012A
Creation Date	June 2026
Citable Link	https://doi.org/10.20955/wp.2026.012
Suggested Citation	Stangebye, Z.R., Wright, M.L.J., 2026; Living in Infamy: Bad Reputations in Emerging Markets, Federal Reserve Bank of St. Louis Working Paper 2026-012. URL https://doi.org/10.20955/wp.2026.012

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Living in Infamy: Bad Reputations in Emerging Markets

Zachary R. Stangebye* Mark L. J. Wright†‡

June 15, 2026

Abstract

We present a model of sovereign borrowing in the presence of default risk and asymmetric information. Optimizing sovereigns come in two persistent types with different levels of patience and hence different proclivities to default and borrow. In a stylized model, we construct a pooling equilibrium that is 'infamous' in the sense that the patient sovereign is constrained to borrow like the impatient sovereign on the equilibrium path. We argue that this provides an explanation for the observed lack of private capital markets for many low- and middle-income countries, and use the model to evaluate proposed policy interventions aimed at building a good reputation for such countries.

*University of Notre Dame

†Federal Reserve Bank of St. Louis

‡The authors would like to thank Satyajit Chatterjee, Dean Corbae, Kyle Dempsey, Yan Bai, Naryana Kocherlakota, Aart Kraay, Radek Paluszynski, Charles Engels, V.V. Chari, Daniel Neuhann, Manuel Amador, and Cesar Sosa-Padilla for useful comments and feedback. Any remaining errors are our own.

1 Introduction

Many sovereign nations have little access to *private* international capital markets, such as bank loans and bond issuance, and instead borrow largely from other governments and the multilateral development banks (MDBs).[1] However, both traditional (Paris Club) and new (China and gulf states) government creditors are scaling back their sovereign lending, while MDBs are close to their lending limits. This has raised concerns in international policy circles about the future availability of development finance, as well as proposals for increasing private lending. For example, based on the presumption that private creditors were deterred by debt overhang and weak institutions in these countries, the Highly Indebted Poor Countries initiative (HIPC) and its successor the Multilateral Debt Relief Initiative (MDRI) both offered debt relief in return for institutional and policy reforms. Unfortunately, the response of private creditors has been modest.

In this paper, we propose an alternative explanation for the lack of access to private capital. We hypothesize that these sovereigns are caught in a bad reputational trap in which good credit risks are unable to borrow enough to establish their ability to repay higher levels of debt. Instead, they are stuck in an 'infamous' pooling equilibrium in which they are constrained to borrow the small amounts available to worse credit risks. What is worse, bad reputations are partially self-fulfilling in that negative investor perceptions are causal in generating worse behavior.

We begin by presenting a stylized model of sovereign borrowing in the presence of default risk as in Arellano (2008), augmented by the presence of asymmetric information about a sovereign's type. There are two optimiz-

[1] At any point in time, there are about 70 countries on the Poverty Reduction and Growth Trust (PRGT) eligible list. Countries not on the PRGT list are governed by the Sovereign Risk and Debt Sustainability Framework for Market Access Countries.

ing types of sovereigns who are assumed to be risk-neutral with different discount factors that are not known to private investors. Everything else equal, the more impatient (bad) type prefers to borrow more and defaults more frequently; the more patient (good) type prefers to borrow less and defaults less frequently. Both types are more impatient relative to private investors and seek to borrow against their future (constant) endowment in order to consume more in the present. An unobserved shock that affects the desirability of default is symmetric across the types and is the only form of uncertainty. The support of this shock is finite so that equilibrium revelation of types is possible.

In this environment, we construct a pooling Markov Perfect Equilibrium that is 'infamous' in the sense that the good type is constrained to borrow at the maximum level available to the bad type on the equilibrium path. That is, access to private capital markets is severely limited. The good type also defaults with the same frequency (never) as the bad type on the equilibrium path. This infamous equilibrium exists under mild restrictions on out of equilibrium beliefs: They are only assumed to be monotone in borrowing, in that borrowing more than the equilibrium amount cannot improve a sovereign's reputation. The infamous equilibrium is also unique in the class of equilibria wherein both types borrow the same amount at the risk-free rate. This implies that it can only happen that the bad type drags the good type down. The good type can never raise the bad type up, because the bad type always default when they borrow beyond their debt limit regardless of the possible future rewards.

The intuition is the following. When there is perfect information, a powerful feedback loop exists that rewards the good type for its good behavior. The good type defaults less often and so can issue bonds at better prices. These better prices lower debt service costs, which raises the value of repayment relative to default, which fosters even better behavior on the part of the sovereign. Conversely, when types are hidden, investors cannot condition prices on those types but only on their beliefs regarding them. Conditional on beliefs, the good and bad types face the same budget set. The good type is thus not rewarded for being the good type and, in equilibrium, his behavior worsens to the point where on the equilibrium path he is constrained to borrow as if he were the bad type. Off the equilibrium path his (default) behavior would diverge from the bad type's, but this does nothing to change equilibrium dynamics.

To understand why the patient sovereign cannot signal its type through its borrowing decisions, suppose they were to borrow more than the equilibrium amount. Since higher levels of borrowing are risky, this causes the price of the sovereign bonds to fall, so that the sovereign receives less resources from borrowing on both the marginal unit of borrowing as well as on all inframarginal borrowing up to that point. This cannot directly improve the reputation of the sovereign given our weak monotonicity assumption on out of equilibrium beliefs. However, with some probability, the unobserved taste shock is realized which gives the patient sovereign the opportunity to earn a better reputation by repaying when the impatient sovereign would default. For low enough risk-free interest rates, this gain is guaranteed to be smaller than the lost resources from borrowing at the lower bond price, regardless of the level of the sovereign's initial reputation. That is, any (strictly positive) level of investor pessimism can sustain an infamous equilibrium given a sufficiently small risk-free rate.

If a good type is caught in an infamous equilibrium, there is nothing that he can do to escape from it. All deviations from equilibrium borrowing and repayment decisions can only worsen his situation. However, there is room for a third-party to improve his circumstances. In the spirit of the Poverty Reduction and Growth Trust framework of the World Bank, we consider whether a benevolent third-party such as an MDB or foreign government can induce separation and, in doing so, free the country from its bad reputation.

We find that this is possible as long as the third-party is willing to shoulder some expected costs. In particular, they may be wrong in their conjecture that the country is the good type and discover the hard way that the country is in fact the bad type. To mitigate this risk, we show that the third-party can employ a gradualistic approach. First, it must lend a relatively small amounts at sub-market yields. In contrast to the infamous equilibrium, each repayment of this loan improves the country's reputation and reduces expected default risk in the next period. Eventually, it becomes safe enough to lend a larger loan that induces separation.

We characterize the set of reputations for which this strategy can be effectively applied as a function of the losses the third-party is willing to endure: The more expected losses the third-party is able to incur, the worse can be the initial reputation of the country to be 'saved.' Worse reputations are more expensive to address not because the sub-market-yield loans are different in size, but because (1) the third-party is more likely to be wrong in its conjecture that the type is good, and (2) separation takes longer as reputations take longer to build. We also argue that loan guarantees result in an equivalent separation of types.

1.1 Related Literature

Information frictions have long been known to plague markets for international debt e.g. Cole and Kehoe (1998), Chari and Kehoe (2003), Angeletos and Werning (2006), Cole et al. (2016), Carlson and Hale (2006), Van Nieuwerburgh and Veldkamp (2009), or Gu and Stangebye (2023) to name only a few. Investors typically do not have access to all of the information that they would like to have at the time they make the investment decision, largely because the borrowing country is foreign in nature.

In our work, we present a recursive Markov signaling game between a sovereign government and foreign investors. The extant literature on asymmetric information tends to explore models in which default alone as a signal has been extensively studied, though there has been some variation (see Cole and Kehoe [1998], Sandleris [2008], Phan [2017a, 2017b], Amador and Phelan [2021]), or Morelli and Moretti [2023]).

D'Erasmo (2008) is an important quantitative contribution in which default and borrowing both serve as signals in a reputational model in the vein of Arellano (2008) or Aguiar et al. (2016a). We differ from D'Erasmo (2008) in a couple of important ways. First, our primary contribution is theoretical rather than quantitative, establishing the existence and uniqueness of the infamous equilibrium. Second, our underlying assumptions regarding the sources of asymmetric information are different in relevant ways.

More recent and related papers include Amador and Phelan (2021) and Fourakis (2023). In the former, the authors demonstrate that when one type (the good type) has perfect commitment and follows a prescribed borrowing rule, the (bad) optimizing type will find it optimal to repay (with some probability) to get better prices right up until the point where he defaults and resets expectations. Their equilibrium features strategic default and pooling at the level of the *good type*. In our environment, where both types are optimizing along the default and borrowing margins, we find the opposite, namely that the equilibrium features pooling via borrowing at the level of the *bad type*. The bad type in Amador and Phelan (2021) will also eventually default in equilibrium, whereas in our framework neither type ever defaults as they never engage in risky borrowing.

Fourakis (2023) proposes a similar default model with two optimizing types differentiated by their discount factors among other things. The environment is fundamentally different in that the author proposes a discrete environment with specialized and hidden i.i.d. taste shocks of infinite support such that all possible actions are chosen with some probability in every period. This obviates the need to specify off-equilibrium beliefs. This strategy is also employed by Chatterjee et al. (2023), who explore similar questions in consumer credit. In contrast, our model is fully continuous with well-specified off-equilibrium beliefs that are important for existence and equilibrium dynamics. In fact, a key result in our work is the set of restrictions on off-equilibrium beliefs required to sustain a perfect Bayesian equilibrium.

The remainder of the paper is organized as follows. In Section 2 we describe the benchmark model and define the equilibrium concept. In Section 3 we derive the benchmark results and explore their ramifications. In Section 4 we consider the policy consequences for an altruistic third-party such as an MDB or foreign creditor government.

2 Analytic Model

2.1 Sovereign Government

Consider an infinitely-lived sovereign country that has a constant endowment, y. The country has a risk-neutral flow utility function and discounts the future at rate $\beta_u < 1$. $u \in \{L, H\}$ is a permanent state that is unobserved by the lenders but known to the sovereign. We assume that $\beta_L < \beta_H$ and refer to the former as the 'bad' type and the latter as the 'good' type.

In any period, the country enters the period with short-term debt obligations, b. It can choose at this time to default on its debt. If it does so, it sets the default variable, $d = 1$. Defaulting entails a loss of access to credit markets as well as a permanent output cost, $\phi \geq 0$.

We restrict attention to Markovian dynamics so that time-subscripts are unnecessary. This implies that the default value is given by

$$V_D(u) = \frac{y - \phi}{1 - \beta_u} \tag{1}$$

If the country does not default, it maintains access to credit markets and can issue new debt, b', at an equilibrium price to be discussed momentarily.

There is a single, i.i.d. additive utility shock to defaulting in every period, $\tilde{m}$.[2] Like the sovereign's type, it is observed only by the sovereign and not the lenders. Unlike the sovereign's type, though, it is iid over time. Its transitory nature introduces a plausible signal-noise structure to the investors' problem. Whenever sovereign behavior is observed, investors will attempt to infer from it the extent to which the behavior was driven by the persistent inclinations of the hidden type as opposed to transitory exigencies induced by some realization of a particular utility shock.

We assume further that this shock is binary, satisfying, $\tilde{m} \in \{0, m\}$, where $m > 0$. The probability distribution of this Bernoulli variable is time and type invariant and characterized by $Pr(\tilde{m} = m) = p \in (0, 1)$.

Lenders' beliefs regarding the country's unobservable type can be summarized by $\rho = Pr(\tilde{u} = L)$. Note that because all agents are Bayesian, ρ will operate as a sufficient statistic to describe lender beliefs and their evolution over time. Beliefs regarding the realization of $\tilde{m}$ will exist, but their specification will serve no additional purpose in either inferring $\tilde{u}$ or forecasting $\tilde{m}'$, and so for brevity we do not assign them any notation.

Following Eaton and Gersovitz (1981), we assume that if the country

[2]Throughout the paper, random variables are denoted with a tilde to contrast them from their potential realizations.

chooses to repay its debt it can subsequently issue new debt at an equilibrium price $q(b'|\rho, b)$. We also make the critical assumption that, just as it cannot commit to repaying extant debt, the government *cannot commit* to a particular level of debt issuance at the time it is making its default decision at the beginning of the period. This is an additional intra-period limited commitment friction similar to Cole and Kehoe (1996), though the timing of the debt auction and default decision are reversed here relative to their work.

This structure obviates the need to consider multi-dimensional signaling spaces in a plausible way, as at any given point in time the sovereign is choosing only a single action and taking all future decisions as given. Multi-dimensional signaling structures introduce significant complications (see Battaglini [2002] or Chakraborty and Harbaugh [2007]) that, for our purposes, are unnecessary. It also implies that the sovereign today takes all future policy functions for default and borrowing as given.

The presence of b' in the pricing function reflects the assumption that the sovereign is a monopolist in its own debt market and internalizes the impact of debt issuance on equilibrium prices. The presence of ρ implies that beliefs and reputations can influence prices.

The presence of b is needed because investors can use it in conjunction with sovereign issuance (b') to infer information about the sovereign's type. It implies that investors can potentially learn about the sovereign's type twice in a period: First through its default decision, i.e., d, and second through its issuance decision, i.e., b'.

We will use the following notation to distinguish between beliefs at these different points in a period. Beliefs at the beginning of the period i.e., prior to any decisions being made, will be denoted by ρ. Beliefs *after* repayment but *before* debt issuance will be denoted by ρ^R.

Using a prime to denote future values of a state, we can write out a repayment Bellman as follows.

$$\begin{aligned} V_R(b, \rho^R|u) = \max_{b' \in [0,\bar{b}]} \ & y - b + q(b'|\rho^R, b)b' + \qquad (2) \\ & \beta_u \mathbb{E}\left[(1 - d(b', \tilde{m}', \rho'|u))\, V_R(b', G^R(b', \rho')|u) + \right. \\ & \left. d(b', \tilde{m}', \rho'|u)\,(V_D(u) + \tilde{m}')\right] \\ \text{where} \quad & \rho' = G(b', b, \rho^R) \end{aligned}$$

where G and G^R are belief-updating functions for debt issuance and repayment respectively that will be discussed momentarily.

Note that we assume debt levels lie on a continuous interval from $[0, \bar{b}]$, where $\bar{b} < \infty$. This last assumption rules out potential Ponzi schemes, though we will assume that $\bar{b}$ is so large as to never impose a binding constraint on equilibrium debt issuance.

Because the sovereign lacks commitment, he will default on his extant debt stock whenever the value of doing so is strictly greater than the value of repaying it. This condition defines the default decision rule as follows.

$$\begin{aligned} d(b, m, \rho|u) &= \mathbf{1}\{V_R(b, G^R(b, \rho)|u) < V_D(u) + m\} \qquad (3) \\ \text{and} \quad d(b, 0, \rho|u) &= \mathbf{1}\{V_R(b, G^R(b, \rho)|u) < V_D(u)\} \end{aligned}$$

Notice that the sovereign accounts for the fact that repaying his debt will influence lender beliefs, as evidenced by the passing of them through G^R. Defaulting would also influence them, but because the sovereign never again interacts with foreign investors, these beliefs are irrelevant to the sovereign's value of defaulting and thus are omitted to simplify notation.

Because investors do not observe $\tilde{m}$ directly, it will be notationally useful for us later to define default probabilities that average over these shocks, i.e.,

$$\bar{d}(b,\rho|u) = (1-p)d(b,0,\rho|u) + pd(b,m,\rho|u) \tag{4}$$

2.2 Foreign Investors

Foreign investors are risk-neutral, deep-pocketed, and have access to an outside option that yields a gross risk-free return, R. Their optimal behavior thus implies an equilibrium pricing condition given by

$$q(b'|\rho^R, b) = \frac{\mathbb{E}\left[1 - d(b', \tilde{m}', G(b', b, \rho^R))|\tilde{u})\right]}{R} \tag{5}$$

The only thing that distinguishes this condition from its standard counterparts in Arellano (2008) or Aguiar and Gopinath (2006) is that investors must take expectations not only over fundamental shocks (here, $\tilde{m}'$), but also over the potential types (here, $\tilde{u}$) according to their belief distribution.

We assume that investors are more patient than either type of sovereign, i.e., $0 < \beta_L < \beta_R < 1/R < 1$.

2.3 Belief-Updating Functions

In each period, investors have two opportunities to infer payoff-relevant information from sovereign behavior: First at repayment and, conditional on repayment, at issuance.

We begin with the repayment decision. On the equilibrium path, we assume that investors update their beliefs via Bayes' rule. Off the equilibrium path, we will allow for a flexible formulation with the core assumption that repayment improves beliefs. In particular, we assume that off-equilibrium beliefs are dictated by some function $\nu_R(b,\rho) \in [0,\rho]$. This implies that beliefs are always weakly improved following a repayment when equilibrium play dictates default by all types. In the worst case scenario, beliefs remain at the status quo and in the best case scenario investors assign probability one to the sovereign being the good type.

Writing this out explicitly[3] yields

$$G^R(b,\rho) = \tag{6}$$
$$\begin{cases} \frac{\left[1-\bar{d}(b,\rho|L)\right]\rho}{\left[1-\bar{d}(b,\rho|L)\right]\rho+\left[1-\bar{d}(b,\rho|H)\right](1-\rho)}, & d(b,m,\rho|u)=0 \text{ or } d(b,0,\rho|u)=0 \text{ for some } u \\ \nu_R(b,\rho) & o/w \end{cases}$$

Next is the issuance decision conditional on repayment. We denote equilibrium debt issuance by $a(b,\rho^R|u)$ for a given hidden type u, which is to say that this function satisfies the maximization specified in Equation 2.

On the equilibrium path, beliefs are updated in accordance with Bayes' rule. The country can deviate from the equilibrium path in one of two ways: Over-issuance or under-issuance. As we did with the repayment decision, we model off-equilibrium beliefs in a general way and make only the following intuitive restriction: Over-issuance tends to worsen beliefs while over-issuance tends to improve them. To capture the over-issuance case, we posit a general, decreasing function, $\nu_+ : \mathbb{R}^+ \to [0,1]$ that operates as follows

$$\rho' = \nu_+(b' - \max_u a(b,\rho|u)) \times \rho^R + \left[1 - \nu_+(b' - \max_u a(b,\rho|u))\right] \times 1$$

which is to say that updated beliefs are a convex combination of the status quo and the worst possible beliefs. The fact that ν_+ is decreasing implies that additional issuance beyond the equilibrium cap can only worsen beliefs; never improve them.

We posit another, general decreasing function for beliefs following under-

[3]Note we do not need to write out the belief-updating function (and its associated off-equilibrium beliefs) for the case when the country defaults, as in this case the country is excluded from credit markets forever and reputation will never again matter.

issuance, $\nu_- : \mathbb{R}^+ \to [0,1]$ that operates similarly, i.e.,

$$\rho' = \nu_-(\min_u a(b,\rho|u) - b') \times \rho^R + \left[1 - \nu_-(\min_u a(b,\rho|u) - b')\right] \times 0$$

Here we have a convex combination between the status quo and zero. The decreasing nature of the function implies that beliefs must (weakly) improve as the country issues less and less debt off the equilibrium path.

Spelling this out formally, we get

$$G(b', b, \rho^R) = \tag{7}$$
$$\begin{cases} \frac{\mathbf{1}\{b'=a(b,\rho^R|L)\}\times\rho^R}{\mathbf{1}\{b'=a(b,\rho^R|L)\}\times\rho^R+\mathbf{1}\{b'=a(b,\rho^R|H)\}\times(1-\rho^R)}, & a(b,\rho^R|u) = b' \text{ for some } u \\ 1 - \nu_+(b' - \max_u a(b,\rho|u))(1-\rho^R) & b' > a(b,\rho^R|u) \;\; \forall u \\ \nu_-(\min_u a(b,\rho|u) - b')\rho^R & o/w \end{cases}$$

2.4 Equilibrium Definition

We will consider equilibria that may not hold over the entirety of the state space and it will be important to define them as such. In particular, we will construct equilibria in which incentives align for all parties when reputation levels are moderate-to-low. We also know that in this situation that the reputation will never improve along the equilibrium path. However, the incentive constraints may not bind for off-equilibrium reputation levels that are too high.

To deal with this issue, we will consider equilibria that only hold for certain levels of reputation. The equilibrium construction itself will then ensure that the reputation never departs from this subset of the whole state space. The state space of the model can be expressed as $S \in \left[0, \bar{b}\right] \times \left[0, \bar{b}\right] \times \{0, m\} \times [0,1]$, where the states are, respectively, current debt levels, current debt issuance, the value of the default utility shock, and reputation.[4] Not all

[4]Notice we do not include the hidden type as a state. This is because all equilibrium conditions on

agents observe the full state space, e.g., the lenders do not observe the value of default utility shock, though they may infer it from sovereign behavior in some cases. Nor is it the case that the entire state space is the domain for every function. But this state space is sufficient to uniquely pin down the point in the domain of every relevant function described in the previous section.

Our restricted state space will be $\hat{S} \subset S$. We define it as $\hat{S} = \left[0, \bar{b}\right] \times \left[0, \bar{b}\right] \times \{0, m\} \times \left[\underline{\rho}, 1\right]$ where $\underline{\rho} > 0$. The only difference between these state spaces is that $\hat{S}$ excludes some high-reputation (low-ρ) states. The choice of $\underline{\rho}$ *does not need to be disciplined by other parameter restrictions.* Instead, we will define the relevant equilibrium objects with respect to it.

We can now define our equilibrium concept. A **Markov Perfect Bayesian Equilibrium** is a collection of functions, $\{d, a, q, G^R, G\}$ defined on the relevant subsets of $\hat{S}$ such that

- The sovereign policies, $\{d(\cdot|u), a(\cdot|u)\}$ for $u \in \{L, H\}$ satisfy the Bellman Equation 2 taking as given the pricing and belief-updating functions, $\{q, G^R, G\}$.
- The pricing function, $q(\cdot)$, satisfies Equation 5 taking as given the default and belief-updating functions, $\{d, G\}$.
- The belief-updating functions satisfy Equations 6 and 7 taking as given the sovereign policies, $\{d, a\}$ for both types.
- All possible future states allowed by the implied map given by the equilibrium functions remain in the restricted state space, i.e., $s' \in \hat{S}$ for all potential future s'.

We assume that if an equilibrium with interior beliefs, i.e., $\rho \in \left[\underline{\rho}, 1\right)$ cannot exist, then the economy must coordinate on a full-information equi-

the sovereign's side must hold for both types and all equilibrium condition on the lenders' side can be expressed through the reputation, which is already a state.

librium with a non-changing $\rho \in \{0,1\}$.[5] These beliefs must be correct in that the good type must be met with $\rho = 0$ and the bad type with $\rho = 1$.

3 Analytic Model: Findings

We begin with a few mild and reasonable assumptions.

Assumption 1. $\phi - m(1-\beta_u) \geq 0$ *for* $u \in \{L, H\}$

The first assumption ensures that flow default costs net of the effective flow benefit are large enough to sustain foreign borrowing.

Assumption 2. $\frac{p(\phi - m(1-\beta_u))}{R-1} \geq \frac{(1-p)}{R} m\left(1 - \beta_u R\right)$ *for* $u \in \{L, H\}$

The second assumption will wind up ensuring that the risk of $\tilde{m} = m$ realizing is sufficiently large that, regardless of its type, the sovereign will not borrow into the risky region where default is a possibility. The left-hand-side will wind up being the effective cost of doing this and the right-hand-side will be the benefit.[6] This assumption can be easily satisfied either by setting p to be fairly large, shrinking R toward unity, or both.

One final assumption regarding the relative impatiences of the two types is required.

Assumption 3. $\beta_H \leq .5 \times 1 + .5 \times \beta_L$

These assumptions dictate that the good type is not too patient relative to the bad type. They help ensure the shape of the infamous equilibrium characterized below and also with precluding certain possible over-issuance deviations. These assumptions are fairly weak relative to the dynamics they engender, as we'll observe in a simple numerical example soon.

[5] We verify in Appendix A that such an equilibrium always exists.

[6] The cost will correspond to the lost revenue on the issuance of potentially risk-free debt that is given up when risky debt is issued. The benefit is the extra revenue net of extra expected future debt service in some states.

Before we exposit the principal result, it will be useful to provide one more definition. Let θ be a vector of model parameters in a parameter space, Θ and define $\Theta_\rho = \Theta \times [\underline{\rho}, 1]$ be the Cartesian product of the parameter space and the restricted portion of the reputation space. In Appendix A, we carefully define a non-empty subset, $\bar{\Theta}_\rho \subset \Theta_\rho$ that is useful for the explaining the proposition. In broad terms, parameter-reputation pairs in this subset ensure potential off-equilibrium deviations would not adjust reputation by so much that other assumptions would be violated *in the future.*

We use this object to construct the following map. Let θ be a parameter vector. The function $\rho^* : \Theta \to \mathcal{P}([\underline{\rho}, 1])$ maps[7] parameters into a subset of $[\underline{\rho}, 1]$ such that $(\theta, \rho^*(\theta)) \in \bar{\Theta}_\rho$.

With these preliminaries established, we can begin to lay out the results. Proofs of all results can be found in Appendix A.

Proposition 1 (**Infamous Equilibrium**). *If the following conditions hold on the parameter vector, θ, for some $\rho \in [\underline{\rho}, 1]$*

1. *Assumptions 1-3.*

2. $R > 1$ *is sufficiently close to one*

3. $\nu_-\left(\frac{R(\phi - m(1-\beta_L))}{R-1}\right) \in \rho^*(\theta)$

4. $\nu_R(b, \rho) \in \rho^*(\theta)$ *for any* $b \in [0, \bar{b}]$

and then a Markov Perfect Bayesian Equilibrium with interior beliefs of the following kind always exists. The pricing function is decreasing with non-overlapping intervals on the debt issuance domain given by

1. $\mathcal{B}_1 = \left[0, \frac{R(\phi - m(1-\beta_L))}{R-1}\right]$, *with price* $q(b'|\rho^R, b) = \frac{1}{R}$

[7] We use the power set notation here for $\mathcal{P}$, meaning the set of all subsets of the object in question.

2. $\mathcal{B}_2 = \left(\frac{R(\phi - m(1-\beta_L))}{R-1}, \frac{R(\phi - m(1-\beta_L))}{R-1} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)\right]$, *with price* $q(b'|\rho^R, b) = \frac{1-p\left[1-\nu_+\left(b' - \frac{R(\phi-m(1-\beta_L))}{R-1}\right)(1-\rho^R)\right]}{R}$

3. $\mathcal{B}_3 = \left(\frac{R(\phi - m(1-\beta_L))}{R-1} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right), \frac{R(\phi - m(1-\beta_L))}{R-1} + m\right]$, *with price* $q(b'|\rho^R, b) = \frac{1-p}{R}$

4. $\mathcal{B}_4 = \left(\frac{R(\phi - m(1-\beta_L))}{R-1} + m, \frac{R(\phi - m(1-\beta_L))}{R-1} + m\left[1 + \frac{\beta_H - \beta_L}{1-\beta_H}\right]\right]$ *with price* $q(b'|\rho^R, b) = \frac{(1-p)\nu_+\left(b' - \frac{R(\phi-m(1-\beta_L))}{R-1}\right)(1-\rho^R)}{R}$

5. $\mathcal{B}_5 = \left(\frac{R(\phi - m(1-\beta_L))}{R-1} + m\left[1 + \frac{\beta_H - \beta_L}{1-\beta_H}\right], \bar{b}\right]$, *with price* $q(b'|\rho^R, b) = 0$

We call this equilibrium the "infamous equilibrium."[8]

There is much to unpack in this key result, but it is perhaps easiest to first observe what the infamous equilibrium looks like visually in a simple example. Consider the following calibration, which satisfies all of the assumptions required for Proposition 1 to hold. We set $R = 1.01$, $p = .3$, $m = 1.5$, $y = 1.0$, and $\phi = .16$. The two values of β are 0.9 and 0.93, both of which are empirically relevant in the quantitative literature (see Aguiar et al. [2016b]).

Figure 1 depicts the full-information pricing schedules[9] as well as a collection of infamous equilibrium pricing schedules differentiated by their specifications for off-equilibrium beliefs. In all cases, we set investors' prior to maximum agnosticism, i.e., $\rho = \rho^R = 0.5$.[10] The best infamous equilibrium maintains status quo beliefs upon any over-issuance (ν_+ best), the worst infamous equilibrium assumes immediate assignment of beliefs to the bad type upon any over-issuance (ν_+ worst), and the intermediate infamous equilibrium assumes that additional over-issuance gradually worsens beliefs.

[8] The infamous equilibrium obviously constitutes all equilibrium objects and not just the pricing function. We describe the pricing function in more detail primarily because much of the intuition for how the equilibrium behaves can be derived from the pricing function.

[9] The full-information equilibrium is unique. See Appendix A for an argument to this effect.

[10] We will validate momentarily that the reputation never changes.

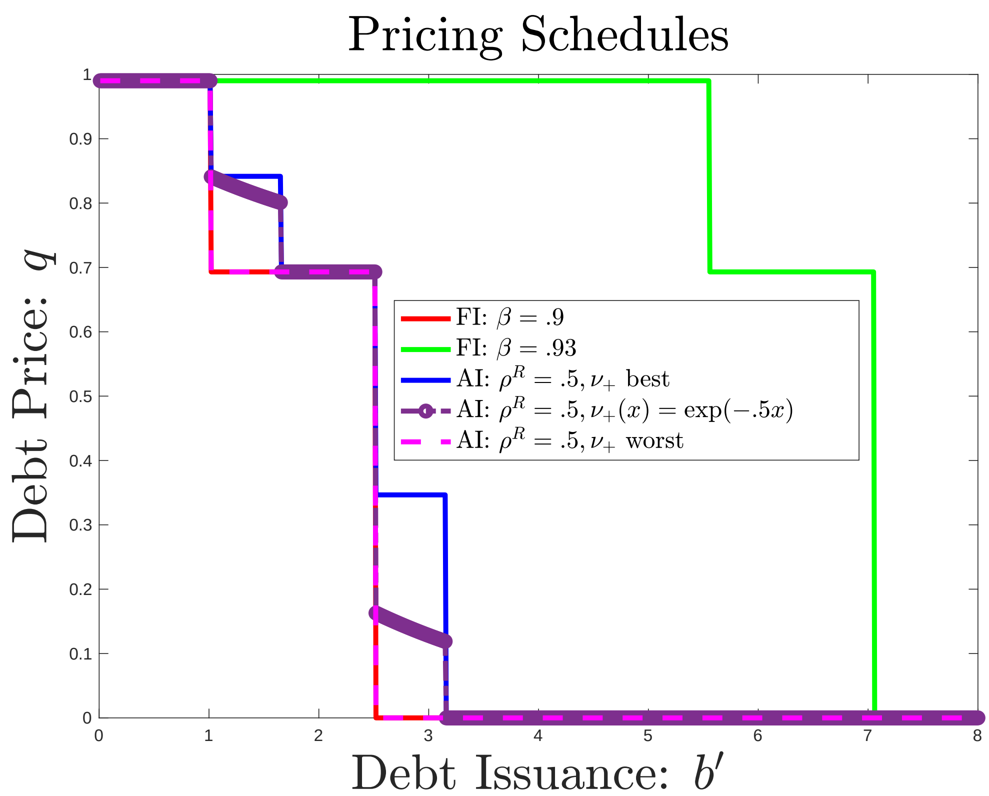


Figure 1: Infamous Equilibrium Examples

We observe, not surprisingly, that the pricing schedule in the worst infamous equilibrium literally overlaps with the full-information schedule of the bad type. The reason is because all over-issuance immediately assigns beliefs irrevocably to the bad type. What is more surprising is just how far the best infamous equilibrium is from the full-information pricing schedule of the good type. Even under these rosy assumptions for off-equilibrium beliefs, i.e., status quo beliefs following over-issuance, there is a significant contraction of the budget set. This is the sense in which the equilibrium is always 'infamous.'

Significant consequences of the infamous equilibrium will be explored in the corollaries that follow.

Corollary 1. *The infamous equilibrium features on-equilibrium pooling along* *both borrowing and default margins. In particular,*

1. *Both types always repay for any realization of* $\tilde{m}$.

2. *For any* $b \in [0, \bar{b}]$ *and any* $\rho^R \in [\underline{\rho}, 1]$*, the country borrows* $a(b, \rho^R|u) = \frac{R(\phi - m(1-\beta_L))}{R-1}$ *for either* u*. This is the full-information risk-free debt limit for the bad type.*

Corollary 1 tells us about the dynamics of the infamous equilibrium. It asserts that both types borrow as much as is allowed at the risk-free rate and repay it in every period. They are constrained in this issuance by the amount specified in the corollary. Interestingly, this amount ($R(\phi - m(1 - \beta_L))/(R-1)$) is exactly equal to this risk-free limit for the *bad* type in the full-information case.

This is true regardless of investor beliefs and the reason is as follows. The good type would indeed repay more than the bad type, but there are two forces working against this taking place. First, off-equilibrium beliefs for over-issuance are met with a *worsening* of beliefs, not an improvement. Thus, if the good sovereign tried to over-issue with the hopes of distinguishing himself by paying more back, it would have the counterproductive effect of worsening his reputation today. Second, over-issuance itself at the equilibrium price schedule is sub-optimal regardless of reputational considerations. The drop in prices the good type would receive *on his entire stock of debt* is too large to justify issuing a marginal amount more into risky territory to distinguish himself.

This is why the condition on R being close to one is pivotal for existence. The closer R is to one, the greater is the capacity for *both* types to issue risk-free debt. R has no bearing on the *additional* issuance allowed in the risky region, though; this is determined entirely by m and the two discount factors. Thus, a low R implies that price reductions from risky borrowing are very expensive relative to the modest additional quantities that can be issued. This is precisely the reason for Corollary 2.

Corollary 2. *As R decreases toward unity, the range of initial beliefs (ρ) consistent with an infamous equilibrium expands.*

Corollary 2 is noteworthy because it says something about the sorts of environments in which infamous equilibria can arise. If no infamous equilibrium exists, then no equilibrium that in any way resembles it can exist at all[11]. In high interest rate environments, then, the only allowable equilibria are full-information outcomes where investors assign probability one beliefs to the correct type. As interest rates come down, though, the region of beliefs that can support infamous dynamics expands. This suggests that the secular reduction in interest rates from the developed world that has been observed over the past several decades (Rachel and Smith [2017]) may have had the effect of inducing infamous equilibrium dynamics in more emerging markets.

Despite the fact that both types are constrained to issue debt at the risk-free price of the bad type under full-information, the pricing schedule off-equilibrium reflects the differences in default behavior across the types and accounts for investor beliefs. And there are significant differences for the good type in default behavior between the infamous equilibrium and its full-information counterpart. This is made clear in Corollary 3.

Corollary 3. *In the infamous equilibrium, the bad type defaults exactly as he would under full information but the good type defaults at lower debt levels relative to the full-information equilibrium.*

The first part of this corollary follows because the bad type receives exactly the same continuation value in the infamous equilibrium as he does

[11] This is established later in Proposition 3.

under full information. Namely, he services his debt limit forever at the risk-free price. This implies that equilibrium default behavior is identical between these cases.

The second part of this corollary is more interesting. The good type receives a substantially worse pricing schedule in the infamous equilibrium relative to full-information. What this does in equilibrium is *reduce the value of repaying extant debt*. A lower value of repayment implies more frequent default. Thus, default behavior for the good type occupies significantly more of the state space in the infamous equilibrium. For instance, the good type ceases to repay with probability one at $R(\phi - m(1-\beta_H))/(R-1)$ under full-information; in the infamous equilibrium, though, this occurs at the supremum of $\mathcal{B}_2$, which is significantly less. In Figure 1 for example, the latter is nearly a quarter the size of the former.

The pooling of borrowing and default behavior implies the next result, Corollary 4, which highlights the hopelessness of the infamous equilibrium for the good type.

Corollary 4. *Beliefs never improve in the infamous equilibrium, i.e.,* $\rho' = \rho^R = \rho$.

As the investors will never observe any behavior except for repayment at the prescribed equilibrium debt limit, the investors will never get an opportunity to update their beliefs. As such, their beliefs following both repayment and debt issuance will always be constant. The good type will thus never be able to 'break out' of the budget set of the bad type barring some sort of third party intervention outside the scope of the current model.

The final conditions worth discussing are those referring to the off-equilibrium beliefs that may improve reputation. The conditions in Proposition 1 on these belief specifications, ν_- and ν_R are rather technical and can be found in the proof in Appendix A. Their intuitive function, however, is easy to understand. They simply ensure that if the beliefs and parameters are such that an infamous equilibrium holds today, then an off-equilibrium deviation via under-issuance or repayment does not improve beliefs sufficiently to rule out the equilibrium tomorrow. They hold trivially for status quo beliefs and continue to hold by continuity in the region specified by the proposition.

The infamous equilibrium has consequences for macroeconomic objects of interest, for instance capital inflows. This is established in Proposition 2.

Proposition 2. *In the infamous equilibrium, the good type faces a reduction in capital inflows (relative to full information) of*

$$\frac{Rm(\beta_H - \beta_L)}{R-1}$$

which is increasing as R decreases toward unity.

By reducing the effective debt limit, the infamous equilibrium reduces capital inflows significantly. The magnitude of this reduction increases as gross interest rates converge to unity from above. So not only are infamous equilibria more likely to emerge in low interest rate environments, but they will also be more damaging in such environments.

At a fundamental level, infamous equilibria have the property that the presence of the bad type drastically shrinks the budget set of the good type and worsens his default behavior as a consequence. It is natural to wonder whether the reverse is possible: Does an equilibrium exist where the good type 'lifts up' the bad type, expanding his budget set and reducing his propensity to default? The next result answers this negatively.

Proposition 3. *The infamous equilibrium is the unique Markov Perfect Bayesian Equilibrium in which both types always repay and thus borrow at* *the risk-free rate.*

The reason why the infamous equilibrium is unique in its class is because the bad type's full-information risk-free debt limit, which is that point at which all types borrow in the infamous equilibrium, is the maximum amount that the bad type would ever repay given risk-free prices. If a greater reputation ever improved his budget set further at risk-free prices, he would simply borrow into this range and then default with strictly positive probability. But this would be inconsistent with risk-free prices and thus cannot constitute an equilibrium.

The reason why the reverse is possible is because the good type is constrained to issue at debt levels that are *less* than he would repay given risk-free prices. This implies a reduction in the value of repayment, which in turn reduces the amount he is willing to repay to retain access to credit markets.

4 Policy Consequences: Third-Party Assistance

When a country finds itself in an infamous equilibrium, there is nothing they can do on their own to escape it. All deviations from equilibrium play serve only to make a bad situation worse.

However, it is possible for an altruistic third-party, such as a multilateral development bank (MDB), to induce separation between the types. In this section, we outline how this can be accomplished in three ways. First, we consider concessional loans in which the supranational is prepared to take a finite loss in expected value. Second, we explore a subsidized investment guaranty. Lastly, we discuss a trial-by-fire approach in which lending occurs at a penalty rate. In all three cases, the environment satisfies the assumptions underpinning Proposition 1, so that the good type cannot signal his own way out of an infamous equilibrium by altering his repayment, default, or issuance decision.

We assume that the third-party has linear utility, derives a strictly positive utility benefit, $S > 0$, once separation is achieved,[12] and is willing to expend (in expectation) up to some monetary amount $D > 0$. To begin, the third-party lender momentarily supplants private creditors, such that the country is never dealing with more than one creditor at a time.

4.1 A Separating Concessional Loan Package

Concessional loan packages are often extended to low income countries at below-market interest rates by the International Development Association (IDA) of the World Bank. Unlike IDA concessional loans, which include grace periods of up to a decade or more in which no repayment from the borrower is due, we consider loans with no grace period.[13] This is important because it is the act of repayment that signals a borrower country's type. For concreteness, we assume that these loans are made at the risk free rate so that the IDA absorbs all default risk. We characterize dynamic trajectories of such lending that serve to foster eventual separation.

We begin by observing that there are two types of concessional loans that can be extended to the country:

1. Purchasing $\hat{b}_B \in (b_{S,L}, b_{R,L}]$ at the risk-free rate. This strategy will induce the bad type to default (but not the good type) when a default taste shock arrives. If no such shock arrives, both types repay. Repeated application of such lending would have the following effect: If the sovereign is in fact the bad type, then eventually he will reveal

[12] We assume this benefit is achieved even if the type is bad, but the results are robust to it being only attained if the type is revealed to be good.

[13] For example, IDA 40-year credits have an 11-year grace period during which no interest or servicing fees are charged.

himself via default. If the sovereign is the good type, he will never fully reveal himself, but he will *build* his reputation (hence the 'B').

2. Purchasing $\hat{b}_S \in (b_{R,L}, b_{S,H}]$ at the risk-free rate. This strategy induces immediate separation. The bad type will default on this loan with probability 1 and the good type will repay it with probability 1 as doing so reveals his type and thus grants him the continuation value associated with the (vastly better) full-information pricing schedule (hence the 'S' for *separation*).

Given this, we consider the following policy proposal:

- Lend a relatively small amount, $\hat{b}_B \in (b_{S,L}, b_{R,L}]$ repeatedly to build the country's reputation. To minimize losses, we will set $\hat{b}_B = b_{S,L} + \epsilon$ for some small but strictly positive ϵ.[14]

- After a finite amount of time, increase the size of the loan to $\hat{b}_S = \hat{b}_B + m$, which will by construction be $> b_{R,L}$. This will induce full separation.

This strategy reduces expected losses for a couple of reasons:

- Losses in the 'reputation-building' stage will be smaller because the quantity of debt lent is smaller *and* the risk of incurring a loss is lower (it only happens with probability p).

- By the time the third-party lends the separation-inducing larger amount, $\hat{b}_B + m$, the reputation is sufficiently established that expected losses are small.

With this in mind, we can characterize the third party's optimal strategy. First, we can say something about stopping time.

[14] Because the default set of the bad type is open here, a loss minimizing strategy does not technically exist. So we assume that the third-party must lend at least some arbitrarily small ϵ more than the relevant thresholds.

Proposition 4. *Under the gradual lending policy, a good type will separate in finite time (and a bad type will eventually default).*

If we express the expected (net present) cost associated with this policy as given a reputation, ρ as $EL(\rho)$, we can also say the following

Proposition 5. *$EL(\rho)$ exists and is unique, bounded, and strictly increasing in ρ.*

Given that EL is strictly increasing, it can be inverted. We define $\pi_{salvageable}(D) = EL^{-1}(D)$. This function then traces out the worst set of beliefs under which a country can be 'saved' given a finite amount of resources devoted to expected losses, $D > 0$. It is easy to see that $\pi_{salvageable}(D)$ is increasing, so the more resources the third party is willing to allocate to this cause, the worse can be the prior. This relationship can be observed in Figure 2, which assumes the sample parameters in the benchmark model and additionally that $S = \hat{b}_B$.

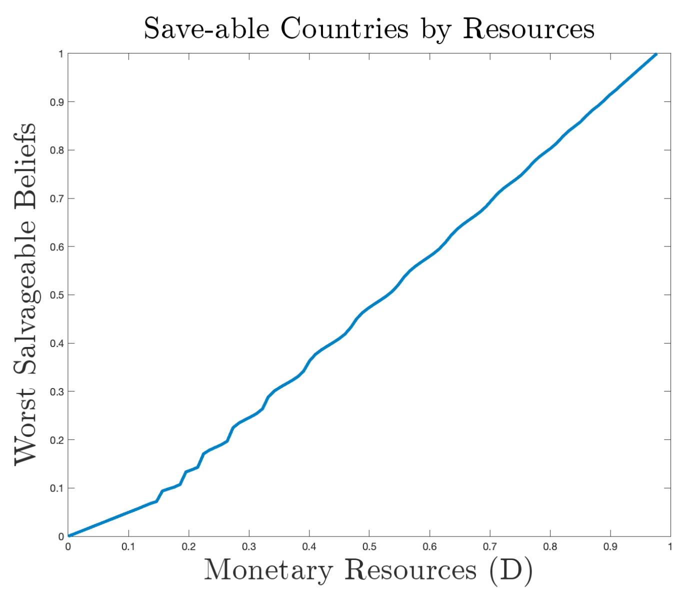
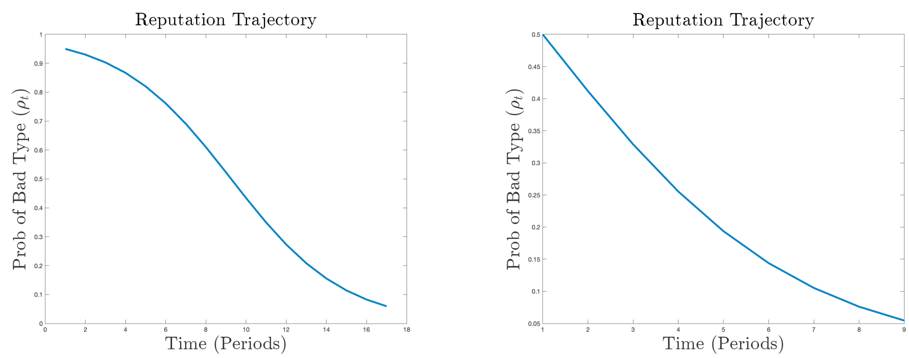


Figure 2: $\pi_{salvageable}(D)$

We can trace out the dynamics of this lending policy and the associated beliefs under different initial degrees of pessimism. These trajectories can be found in Figure 3, which plot the beliefs during the period of third-party intervention until the last (separating period). We observe that it takes more than twice as long to induce separation when $\pi = 0.95$ relative to the case when $\pi = 0.5$. The expected cost is also significantly higher and can be computed from the inverse of Figure 2.



(a) High Initial Pessimism

(b) Low Initial Pessimism

Figure 3: Sample Reputation Trajectories

4.2 Loan Guarantees

It is worth noting that our assumption that all private lending is supplanted by third-party lending can be relaxed considerably. In particular, if the MDB is willing to provide guarantees to private creditors to shield them from some or all of the losses, then private credit can be brought to bear to help facilitate the concessional loan policy we proposed above. Various forms of guarantee are provided by the World Bank Group under its Guarantee Platform, although typically only in the context of project finance. Here we consider a guarantee applied to general government borrowing.

To see this, note that, if the MDB was willing to reimburse the lenders fully in the event of a default, then the lenders would clearly be willing to provide private credit at the risk-free rate, as they now bear no consequences of default. Thus, the lenders can provide all initial funds for the gradual, reputation-building policy described above.

The MDB, however, must still set the terms in terms of the *quantity* of such loans made. They cannot provide guarantees for debt levels above $\hat{b}_B$ during the reputation-building phase or $\hat{b}_S$ during the separation phase. This will automatically limit the flow of private credit to these limits and thus achieve the same policy end.

Note, however, that these guarantees do not change the scale of resources required for the MDB. The expected losses they would incur if they were to provide all of the funding themselves are the same as the expected losses they would incur if guaranteed private debt were to trigger a loss event.

4.3 Cost-Free Separation

We finally consider another policy exercise in which the MDB adjusts their credit based on debt *rates* rather than debt *levels*. The basic idea is that, if they are the only lender, the MDB can put the country through something of a 'trial by fire' by raising the lending rates they offer to a country. Recall that Corollary 2 told us that lower rates expand the set of infamous equilibria. It follows as well that higher rates shrink the set of infamous equilibria until, potentially, they do not exist anymore and only full-information equilibria remain.

In practice, the basic idea is that the good type will eventually start servicing debt levels that the bad type will not, and this will eventually induce separation. In the model, this would happen as follows. Lending of all types would shrink with higher rates until, eventually, the primary cost of over-issuance, which is the foregone revenue of equilibrium debt issuance, becomes diminuitive. At this time, both types will over-issue. The bad type will eventually default on this over-issued debt before the good type and separation will follow.

The benefit of such a policy is that it does not cost the MDB anything. There are two significant drawbacks, though. The first is that it is a painful process for the country regardless of type, as the country faces both higher rates and restricted borrowing opportunities. The benefits of separation for the good type may be worthwhile for some parameterizations, but for others they may not be.

The second drawback is that it may not be feasible for the MDB to force the sovereign to borrow at a higher rate. Contractual provisions that work to ensure that MDB lending is senior to all private lending may work to limit private sector lending. If this fails, and if private creditors are unwilling to cooperate with the program, then the country may instead borrow from them at the market rate, which is lower, in which case there is no chance for future separation.

A Proofs of Analytic Findings

The proofs will not follow exactly the order laid out in the body of the paper. Rather, we will first characterize the equilibrium in the full-information case (in which investors can perfectly observe both u and $\tilde{m}$) and use it to construct the infamous equilibrium in the case of asymmetric information.

A.1 Proof of Proposition 1: Preliminaries

We will drop the u-subscript on the β for this section as all results will apply to both types so long as they satisfy Assumptions 1 and 2. The dependency on ρ or ρ^R also disappears for the relevant functions.

We first observe that the sovereign's scalar default value, in the absence of the utility shock, can be expressed in the following terms

$$V_D = \frac{y - \phi}{1 - \beta}$$

The next thing we can immediately observe as a result of lenders being risk-neutral and deep-pocketed is that the equilibrium pricing expression has the following form:

$$q(b') = \frac{\mathbb{E}\left[1 - d(b', \tilde{m}')\right]}{R} \tag{8}$$

Notice that not only has the dependency on ρ^R disappeared, but so too has the dependency on b. The only reason for its inclusion in the pricing function in the first place was that its difference with b' could be used to infer some information about the sovereign type that could be relevant for prices. In the full-information case, though, the pricing function will always be flat in b and so we drop this argument for brevity.

With these preliminaries, we can assert the following proposition:

Lemma 1. *$q(b')$ is a decreasing step function with three non-overlapping regions.*

1. *If $b' \in \mathcal{B}_1$, then $q(b') = \frac{1}{R}$*

2. *If $b' \in \mathcal{B}_2$, then $q(b') = \frac{1-p}{R}$*

3. *If $b' \in \mathcal{B}_3$, then $q(b') = 0$*

It is further the case that $\mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3 = [0, \bar{b}]$.

Proof. Since V_D is constant in b, we need only establish that V_R is decreasing in b. This is easy to see by the principle of optimality. Let $b < \hat{b}$ and let $a(\hat{b})$ be the optimal policy for b' given $\hat{b}$. Given that the flow payoff is decreasing in b, if we evaluate the payoff given an initial level of debt b using the feasible policy $a(\hat{b})$ (instead of optimal policy), we will find that the value is at least as large as $V_R(\hat{b})$. Further, the optimal strategy given an initial debt b must yield a higher overall value then any other feasible alternative (including $a(\hat{b})$). This implies that $V_R(b) \geq V_R(\hat{b})$ and thus V_R is a decreasing function.

The proposition follows from the fact that the expected default frequency will be increasing in b' as a consequence. It will be stepped as the proposition asserts because of the binary nature of the shock. In $\mathcal{B}_1$, the shock $\tilde{m}$ will never sway the sovereign to default; in $\mathcal{B}_2$, the sovereign will only default if the shock is realized $\tilde{m} = m$; and in $\mathcal{B}_3$, the sovereign will default no matter what. The probabilities of these events determine the values of the prices laid out in Proposition 1. The thresholds on the regions will be determined shortly. □

It is worth noting that the step-function will feature right-hand continuities and left-hand discontinuities as a consequence of the assumption that the country repays when indifferent as spelled out in Equation 3.

We use Proposition 1 to define some new terms: Let $b_S = \sup \mathcal{B}_1$ and $b_R = \sup \mathcal{B}_2$. With these in hand we can say the following:

Lemma 2. *The optimal debt issuance policy function is constant and satisfies $a(b) \in \{b_S, b_R\}$ for any $b \in [0, \bar{b}]$.*

Proof. To see this, we will conjecture that the optimal policy lies outside of this set and show that the sovereign can always improve by moving to an object in this set.

Let us consider a candidate for optimal policy $\hat{b} \in \mathbf{int}(\mathcal{B}_1)$ for some arbitrary initial b. If this were the case, then the price of issuance would be $1/R$, as was established in Proposition 1. In this case, the sovereign could issue some $\hat{b} + \epsilon \in \mathcal{B}_1$ for some small but positive ϵ. The utility gain in the current period would be ϵ/R and the discounted utility loss in the next period would be (at most) $\beta\epsilon$. We say 'at most,' as this would be the utility loss accrued by not changing the issuance strategy tomorrow, which would serve as a lower bound for the payoff implied by the optimal policy tomorrow. Because $1/R > \beta$, the sovereign's payoff is strictly improved by this deviation and thus $\hat{b}$ cannot be optimal. Such a deviation is always possible until the sovereign issues at b_S, in which case $b_S + \epsilon$ is accompanied by a discrete drop in the price for any $\epsilon > 0$. Thus, b_S is a candidate for optimal issuance and is preferred over any issuance in $\mathcal{B}_1$.

Let us consider alternatively a candidate level of issuance $\hat{b} \in \mathbf{int}(\mathcal{B}_2)$ for some arbitrary initial b. If this were the case, then the price of issuance would be $(1-p)/R$, as was established in Lemma 1. In this case, the sovereign could issue some $\hat{b} + \epsilon \in \mathcal{B}_2$ for some small but positive ϵ. The utility gain in the current period would be $\epsilon(1-p)/R$ and the discounted utility loss in the next period would be (at most) $\beta\epsilon(1-p)$, as the sovereign will only be required to honor this debt in the state in which he does not default, which happens with probability $1-p$ in this region. We say 'at most,' as this would be the utility loss accrued by not changing the issuance strategy tomorrow, which would serve as a lower bound for the payoff implied by the optimal policy tomorrow and because, if he does default, it will be because he is strictly better off by doing so. Because $1/R > \beta$, the sovereign's payoff is strictly improved by this deviation. This deviation is always possible until the sovereign issues at b_R, in which case $b_R + \epsilon$ is accompanied by a discrete drop in the price (to zero) for any $\epsilon > 0$. Thus, b_R is a candidate for optimal issuance and is preferred over any issuance in $\mathcal{B}_2$.

Finally, we show that the country would never issue in $\mathcal{B}_3$. If he issued some $\hat{b} \in \mathcal{B}_3$, then there would be no intertemporal transfer of resources: Debt would command a zero price today, which generates no revenue, and there is no repayment tomorrow. Further, the sovereign faces a certain future default cost of ϕ tomorrow. The sovereign could do strictly better than this by issuing some small level of debt $\epsilon \in \mathcal{B}_1$, which would engender a utility gain of ϵ/R, which is larger than the utility loss of $\beta\epsilon$. The sovereign also avoids default costs this way, so it is clearly better. And since we have already established that b_S dominates all issuance levels in $\mathcal{B}_1$, b_S must dominate any potential issuance in $\mathcal{B}_3$.

The constancy of the policy function follows from the linearity of the utility (and thus the constancy of the marginal utility).

$\square$

We can now begin to sketch out exactly what the equilibrium pricing schedule and borrowing/default behavior look like in the full-information case.

Lemma 3. *The equilibrium is unique. Further, the budget set is increasing in β. In particular, the non-overlapping sub-intervals of the debt domain* *are given by*

1. $\mathcal{B}_1 = \left[0, \frac{R(\phi - m(1-\beta))}{R-1}\right]$
2. $\mathcal{B}_2 = \left(\frac{R(\phi - m(1-\beta))}{R-1}, \frac{R(\phi - m(1-\beta))}{R-1} + m\right]$
3. $\mathcal{B}_3 = \left(\frac{R(\phi - m(1-\beta))}{R-1} + m, \bar{b}\right]$

Further, for any $b \in [0, \bar{b}]$*, the optimal issuance policy is* $a(b) = \frac{R(\phi - m(1-\beta))}{R-1} = b_S$*.*

Proof. We begin by determining the thresholds of the sub-intervals. To determine the thresholds of the sub-intervals, we need to know exactly the value functions of repayment and default. Thus, we will need to know future optimal borrowing behavior. Utilizing Lemma 2, we take a guess-and-verify approach. We will suppose that country finds it optimal to borrow b_S in the future and find conditions under which he finds it optimal to do so today as well.

If he is borrowing b_S in the future, then he will never default in any period after tomorrow. Under this assumption, we can compute both b_S and b_R via the equalities $V_R(b_S) = V_D + m$ and $V_R(b_R) = V_D$. The first equality implies

$$\begin{aligned} \frac{y - b_S\left(\frac{R-1}{R}\right)}{1-\beta} &= \frac{y-\phi}{1-\beta} + m \\ b_S &= \frac{R(\phi - m(1-\beta))}{R-1} \end{aligned}$$

The second equality is

$$y - b_R + \frac{1}{R}b_S + \frac{\beta}{1-\beta}\left(1 - b_S\left(\frac{R-1}{R}\right)\right) = \frac{y-\phi}{1-\beta}$$

Notice the assumption that b_S is optimal issuance on the left-hand side. After mild manipulation and substitution, we arrive at a similar expression

$b_R = b_S + m$, and so we refer to this quantity as b_R. These two are sufficient to trace out the budget set as described in Lemma 3.

But we still need verify that borrowing b_S is indeed optimal in all states. Since this is a limited commitment model, we can do this by ensuring that if the country finds it optimal to borrow b_S in all future periods he will also find it optimal to do so today. Thus, we need only prevent a one-shot deviation from b_S to b_R in the current period. The payoffs from following the assumed b_S strategy (always) and deviating (only once and only today) to the b_R strategy for any arbitrary b today are given respectively by

$$y - b + \frac{1}{R} b_S + \frac{\beta}{1-\beta}\left(y - b_S\left(\frac{R-1}{R}\right)\right) \tag{9}$$

$$y - b + \frac{1-p}{R} b_R + \beta \left[\begin{array}{c} (1-p)\left(y - b_R + \frac{1}{R} b_S + \frac{\beta}{1-\beta}\left(y - b_S\left(\frac{R-1}{R}\right)\right)\right) \\ +p\left(\frac{y-\phi}{1-\beta} + m\right) \end{array} \right] \tag{10}$$

Subtracting Equation 10 from Equation 9 and requiring that the difference be positive yields the condition[15]

$$\frac{p(\phi - m(1-\beta))}{R-1} \geq \frac{(1-p)}{R} m\,(1 - \beta R) \tag{11}$$

The left-hand-side is the reduction in consumption resources the sovereign would receive as a result of the price drop (which pertains to the entire debt stock) were he to issue at b_R. The right-hand-side is increase in utility the sovereign receives from front-loading an additional m resources at the risky price $(1-p)/R$ were he to issue b_R. One-shot deviations will be precluded provided that the left-hand-side is greater.

Assumption 2 ensures that this is the case. Thus, existence is established and Lemma 3 holds. □

[15] The algebra is simplified using the observation that $V_R(b_S) = V_D + m$ and its substitution in the continuation value.

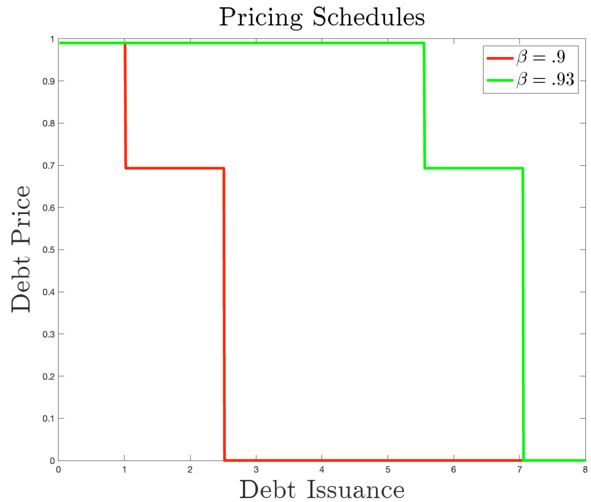


Figure 4: Full-Information Example

We now provide a simple calibration to illustrate the environment here. We set $R = 1.01$, $p = .4$, $m = 1.5$, $y = 1.0$, and $\phi = .16$. We consider two different values of β (both of which are empirically relevant) that, importantly, will satisfy one of the later assumptions required in the asymmetric information analysis. The resulting pricing functions can be observed in Figure 4.

It is worth noting the substantial difference between the two pricing schedules in Figure 4 despite their relative similarity in discount factors, e.g., the good type is allowed to borrow roughly 4x as much as the bad type. This is because a feedback loop is operative here. The good type is inclined to default less often than the bad type, which expands his budget set. But in equilibrium, this expansion of the budget set *also increases the value of repaying debt obligations relative to the value of defaulting* as credit market access becomes more lucrative. Thus the budget set expands further.

We will see this break down in the asymmetric information case: When the good type is not rewarded with better prices, he does not behave nearly as nicely as he would in the full-information case.

A.2 Proof of Proposition 1 Continued

Having characterized the full-information case in the preliminary section, we now construct the infamous equilibrium in the asymmetric information case. The first thing we observe is that the sovereign will never have an incentive to repay/default to manipulate public perception of $\tilde{m}$, unless (potentially) it is necessary to do so to manipulate perception of β_u. This follows from its iid nature and obviates many potential strategic considerations.

We employ a guess-and-verify approach in establishing the infamous equilibrium. In particular, we will build it up under the assumption that both types borrow at $b_{S,L}$, which is the largest possible safe level of borrowing for the bad type under full information. After building such an equilibrium, we will be able to verify its validity, structure, and uniqueness.

We proceed in three steps: (1) Trace out the pricing schedule and implied default behavior under the assumption that both types borrow $b_{S,L} = \frac{R(\phi - m(1-\beta_L))}{R-1}$ in equilibrium, (2) verify that over-borrowing is precluded in equilibrium, and (3) verify that under-borrowing is precluded in equilibrium.

Step 1 (Tracing out the prices and equilibrium default behavior): If both types are constrained to issue $b_{S,L}$ forever, we know from the full-information case that the low-β type will be content with this arrangement by the definition of $b_{S,L}$. That is, he will choose to borrow this maximal allowed amount and repay his debt in every period just as happened in the full-information case for any level of reputation, i.e.,

$$V_{R,t}(b_{S,L}, \rho_t^R | L) = \frac{y - \phi}{1 - \beta_L} + m, \ \forall \rho_t^R$$

Reputation similarly does not matter for the construction of $b_{R,L}$ or its associated value function because initial debt states are publicly observed and thus we get the same expression as the full-information case as follows.

$$\begin{aligned}
V_{R,t}(b_{R,L}, \rho_t^R | L) &= \frac{y-\phi}{1-\beta_L} \\
V_{R,t}(b_{S,L}, \rho_t^R | L) + (b_{R,L} - b_{S,L}) &= \frac{y-\phi}{1-\beta_L} \\
b_{R,L} &= b_{S,L} + \left(\frac{y-\phi}{1-\beta_L} - V_{R,t}(b_{S,L}, \rho_t^R | L) \right) \\
b_{R,L} &= b_{S,L} + m
\end{aligned}$$

Thus, we have the upper bounds of $\mathcal{B}_1$ and $\mathcal{B}_3$, both of which are time and reputation-invariant. Since these bounds will be used to construct all other relevant objects, we declare time-invariance more generally.

We now turn to the good type. Under the assumption that the good type is also relegated to issuing $b_{S,L}$ forever, we can construct the value of repayment, $V_{R,t}(b_t, \rho_t^R | H)$. To determine the threshold at which the good type would stop repaying with probability one, we need only equate $V_{R,t}(b_{S,L} + \epsilon_1, \rho_t^R | H) = V_{D,t}(H) + m$. Doing so yields

$$\begin{aligned}
y - b_{S,L} - \epsilon_1 + \frac{1}{R} b_{S,L} + \beta_H \left(\frac{y - b_{S,L}\left(\frac{R-1}{R}\right)}{1-\beta_H} \right) &= \frac{y-\phi}{1-\beta_H} + m \\
\frac{y - b_{S,L}\left(\frac{R-1}{R}\right)}{1-\beta_H} - \epsilon_1 &= \frac{y-\phi}{1-\beta_H} + m \\
\implies \epsilon_1 &= m \left(\frac{\beta_H - \beta_L}{1-\beta_H} \right)
\end{aligned}$$

where the second line follows from substituting in the known expression for $b_{S,L}$ and some simple manipulation.

It is not hard to observe via simple algebra that $b_{S,L} + \epsilon_1 < b_{S,H}$, which corresponds to the safe borrowing limit under full information for the good type, if and only if $\beta_H < 1/R$, which is assumed. This is established as follows

$$
\begin{aligned}
b_{S,L} + \epsilon_1 &= \frac{R}{R-1}(\phi - m(1-\beta_L)) + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right) \\
&< \frac{R}{R-1}(\phi - m(1-\beta_L)) + m\left(\frac{\beta_H - \beta_L}{1-1/R}\right) \times \frac{R}{R} \\
&= \frac{R}{R-1}(\phi - m(1-\beta_H)) = b_{S,H}
\end{aligned}
$$

This implies that the good type is made worse off by the fact that he is constrained to the (on-equilibrium) budget set of the bad type establishes Corollary 3.

A symmetric argument follows for determining the threshold at which the good type 'just' repays when the shock is good, i.e., $V_{R,t}(b_{R,L} + \epsilon_2, \rho_t^R | H) = V_{D,t}(H)$.

$$
\begin{aligned}
& y - b_{R,L} - \epsilon_2 + \frac{1}{R} b_{S,L} + \beta_H \left(\frac{y - b_{S,L}\left(\frac{R-1}{R}\right)}{1-\beta_H}\right) = \frac{y-\phi}{1-\beta_H} \\
\implies & y - b_{S,L} - m - \epsilon_2 + \frac{1}{R} b_{S,L} + \beta_H \left(\frac{y - b_{S,L}\left(\frac{R-1}{R}\right)}{1-\beta_H}\right) = \frac{y-\phi}{1-\beta_H} \\
\implies & y - b_{S,L} - \epsilon_2 + \frac{1}{R} b_{S,L} + \beta_H \left(\frac{y - b_{S,L}\left(\frac{R-1}{R}\right)}{1-\beta_H}\right) = \frac{y-\phi}{1-\beta_H} + m \\
& \implies \epsilon_1 = \epsilon_2
\end{aligned}
$$

Where the last line follows from the derivations of ϵ_1.

It is worth noting that $b_{S,L} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)$ will be less than $b_{S,L} + m$, which is the bad-type's repayment with probability $1-p$ threshold, only under Assumption 3. We maintain this assumption such that the pricing schedule has the shape of Figure 1.

Thus, while the bad type repays with probability one up until $b_{S,L}$, the good type repays in this way up until $b_{S,L} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)$. In the region $\left(b_{S,L} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right), b_{S,L} + m\right]$, both types repay with probability $1-p$.

And while the bad type repays with probability $1-p$ up until $b_{R,L}$, the good type repays in this way up until $b_{R,L} + m\left(\frac{\beta_H-\beta_L}{1-\beta_H}\right)$. Beyond this threshold, all types default with probability one. Noting this behavior and applying Equation 8 together with the prior beliefs delivers the equilibrium price in each of these regions that we find in Proposition 1. It is important to note that we are assuming (to be verified later) that investor beliefs never change over time.

The optimal default behavior just described is summarized Table 1. Default behavior does not change with reputation, but only with the debt levels, provided reputation levels are such that the infamous equilibrium exists.[16] Any deviations from this default policy function are met with the off-equilibrium beliefs described in the body of the paper.

Type	$\mathcal{B}_1$	$\mathcal{B}_2$	$\mathcal{B}_3$	$\mathcal{B}_4$	$\mathcal{B}_5$
β_L	R	D w.p. p	D w.p. p	D	D
β_H	R	R	D w.p. p	D w.p. p	D

Table 1: Default Behavior

Equilibrium prices in these intervals are just given by this default behavior combined with on or off-equilibrium beliefs regarding the country's type. Working this out gives the pricing schedule in Proposition 1.

It is worth observing just how different this schedule can be from the β_H type under full-information, as this is the crux of the Infamous Equilibrium. In particular, we observe that the full-information threshold is determined by a multiple of β times $\frac{R}{R-1}$. When R is close to unity, as it typically is in the quantitative literature, this multiplier becomes enormous. As such, small changes in β can result in large increases in the implied budget set (see Figure 4 for instance).

The asymmetric information budget set, however, never allows borrowing

[16]Reputation will affect the debt price across these intervals, but not the thresholds of the intervals.

beyond $m\left(\frac{\beta_H-\beta_L}{1-\beta_H}\right)$ of the bad type. Importantly, this difference is *not* scaled by $\frac{R}{R-1}$ since it is determined only by sovereign preferences and not by the discrepancy between lender discount values $(1/R)$ and the sovereign's discount value (β). As such, the asymmetric information budget set is required to 'hug' the β_L set rather closely.

Step 2 (Ruling out over-issuance): We derived the budget set in the previous step from the default behavior of the two types under the assumption of borrowing $b_{S,L}$ for each type forever. Thus, default behavior is, by construction, in keeping with equilibrium conditions. What we now show is that on-equilibrium borrowing behavior is equally satisfied, first by ruling out the temptation to over-issue debt.

We rely on the restriction imposed by limited commitment here, as the country cannot deviate by selecting an alternative trajectory of debt issuance/default, but can only select debt issuance instead. Default/repayment behavior in the future will always be taken as described in the previous section.

First, we note that the budget set is increasing in beliefs and that over-issuance can only weakly worsen beliefs on impact. This does not preclude over-issuing to improve reputation eventually (a strategy we will consider), but the reputation would have to get worse before it could get better.

A second cost of overborrowing is the drop in price. In equilibrium, the sovereign can issue at the risk-free price, but overborrowing will necessarily entail a costly drop in that price. It is this force primarily that is operative in the proof. There will be many distinct parameter restrictions that will be required to preclude over-issuance, but they can *all* be satisfied by requiring that R goes to one from above. The reason is that both types can issue more in equilibrium as this happens; however, the potential gains from over-issuing do not increase in like manner. This implies that the revenue drop becomes significantly more costly (potentially up to infinite) as R goes to one, which is why this restriction is mentioned in the proposition.

Step 2.1 (Ruling out over-issuance for the low type): We will need to preclude all possible deviations for both types. We begin with the low type, β_L. Under Assumptions 1 and 2, we know that the β_L type would strictly prefer to borrow $b_{S,L}$ over $b_{R,L}$. This is because (1) he prefers this under full information and (2) the reputational considerations only hurt him, i.e., he will enter the future with a worse reputation and thus with a weakly smaller budget set than otherwise.

At $b_{R,L}$ both types employ the same default strategy, i.e., default only when the shock hits. This implies that Bayesian updating will preserve the bad reputation generated by off-equilibrium borrowing and thus imply a worse future reputation than he currently has.

We'll need to check five other over-issuance deviations as well: β_L must prefer $b_{S,L}$ over anything in $\mathcal{B}_2$ or $\mathcal{B}_4$, both of which are new options in the asymmetric information case; and β_H must prefer $b_{S,L}$ to all three regions of interior price.

We do so by precluding a one-shot deviation into $\mathcal{B}_2$, much like we did in the full-information case. To fully establish the general result, we will preclude these deviations given the *best* possible off-equilibrium beliefs, which is the case where $\nu_+ = 1$ and over-issuance is met with nothing but status quo beliefs. All other allowable off-equilibrium beliefs will generate worse prices, and so if deviations are precluded in this case they are precluded in all other cases.

Thus, we'll take the difference of the following equations for any initial

b.

$$y - b + \frac{1}{R} b_{S,L} + \frac{\beta_L}{1-\beta_L}\left(y - b_{S,L}\left(\frac{R-1}{R}\right)\right) \tag{12}$$

$$\sup_{\Delta b \in \left(0, m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)\right]} \begin{array}{l} y - b + \frac{1-p\rho}{R}\left(b_{S,L} + \Delta b\right) + \\ \beta_L \left[\begin{array}{c} (1-p)\left(y - b_{S,L} - \Delta b + \frac{1}{R} b_{S,L} + \frac{\beta_L}{1-\beta_L}\left(y - b_{S,L}\left(\frac{R-1}{R}\right)\right)\right) \\ +p\left(\frac{y-\phi}{1-\beta_L} + m\right) \end{array} \right] \end{array} \tag{13}$$

Interestingly, if the bad type repays tomorrow, his reputation will improve. However, this does not wind up impacting equilibrium payoffs, which are determined by borrowing at $b_{S,L}$ forever at the reputation-independent risk-free price.

Simplifying the difference and requiring it to be greater than or equal to zero, we arrive at the condition.

Assumption 4.

$$\frac{\rho p(\phi - m(1-\beta_L))}{R-1} \geq \sup_{\Delta b \in \left(0, m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)\right]} \Delta b\left[\frac{1-p\rho}{R} - \beta_L(1-p)\right]$$

This expression dictates that the costs of switching from $b_{S,L}$ to the first ‘cliff’ are always bigger than the benefits. The left-hand side (a cost), is the revenue impact that the reduction in price has on the equilibrium issuance; the right-hand side (a gain), is the additional utility gained from the marginal debt issuance beyond equilibrium play. Note that it is always positive by virtue of the fact that the price is always weakly better than the actual repayment odds and the fact that $\beta_L < 1/R$.

Notice that this condition can be satisfied by ensuring that R is larger than but very close to one, as it increases $b_{S,L}$ (embedded in the first term) to infinity without increasing any other terms significantly. This is because the loss (LHS) from the price drop increases to infinity as R decreases to one, but the utility benefit (RHS) changes only slightly.

Now we preclude issuance of the bad type to the supremum of $\mathcal{B}_4$. In this region, the bad type will default with probability one in the next period, but could perhaps gain by significant of issuance of debt at rock-bottom prices beforehand, since investors hold out some hope that he may be the good type and offer positive prices. We preclude this as before by demanding that equilibrium play be more desirable than this alternative.

$$y - b + \frac{1}{R} b_{S,L} + \frac{\beta_L}{1-\beta_L}\left(y - b_{S,L}\left(\frac{R-1}{R}\right)\right) \tag{14}$$

$$\sup_{\Delta b \in \left(m, m\left(1+\frac{\beta_H-\beta_L}{1-\beta_H}\right)\right]} \quad y - b + \frac{(1-p)(1-\rho)}{R}\left(b_{S,L} + \Delta b\right) + \beta_L\left(\frac{y-\phi}{1-\beta_L} + pm\right) \tag{15}$$

The continuation value in the deviation follows from the fact that the low-type will always default in the next period but will only receive the added benefit with probability p. If we take the difference between these expressions, we arrive at another condition, which is

Assumption 5.

$$\left[\frac{1-(1-p)(1-\rho)}{R-1}\right]\left(\phi - m(1-\beta_L)\right) \geq$$
$$\sup_{\Delta b \in \left(m, m\left(1+\frac{\beta_H-\beta_L}{1-\beta_H}\right)\right]} \quad m(1-p)\left[\frac{1-\rho}{R}\left[\frac{\Delta b}{m}\right] - \beta_L\right]$$

This condition can also be interpreted as the prior one. Namely, the left-hand-side is the loss in revenue on equilibrium issuance resulting from the deviation. The right-hand-side is the potential intertemporal transfer of utility which, as in the last case, is maximized at $\sup \mathcal{B}_4$. Again, it can be attained by taking R to one from above for the same reasons as described above.

A deviation anywhere in $\mathcal{B}_5$ is immediately precluded for the low type. Issuing into this region delivers no revenue at all today, as the price is zero. Further, it implies that the bad type will default with probability one tomorrow. This implies that the bad type will receive the expected continuation value of default by issuing into this region, i.e., $(y-\phi)/(1-\beta_L)+pm$, which is strictly less than he attains by equilibrium play, where debt levels are chosen such that the continuation value is $(y-\phi)/(1-\beta_L)+m$. Thus, the bad type would never over-issue into $\mathcal{B}_5$.

Step 2.2 (Ruling out over-issuance for the high type): We need to consider the deviations for β_H separately, as the necessary parameter restrictions will be slightly different.

We begin by precluding a deviation to the supremum of $\mathcal{B}_2$ for the β_H type. We'll take the difference of the following equations for any initial b.

$$y-b+\frac{1}{R}b_{S,L}+\frac{\beta_H}{1-\beta_H}\left(y-b_{S,L}\left(\frac{R-1}{R}\right)\right) \tag{16}$$

$$\sup_{\Delta b\in\left(0,m\left(\frac{\beta_H-\beta_L}{1-\beta_H}\right)\right]} \quad y-b+\frac{1-p\rho}{R}\left(b_{S,L}+\Delta b\right)+$$
$$\beta_H\left[y-b_{S,L}-\Delta b+\frac{1}{R}b_{S,L}+\frac{\beta_H}{1-\beta_H}\left(y-b_{S,L}\left(\frac{R-1}{R}\right)\right)\right] \tag{17}$$

Notice a couple of things here. First, in contrast to β_L, the β_H type knows he will never default in this region but instead must pay back the additional debt in all states of the world. Second, reputation first decreases (because of over-issuance) and then increases (because of repayment when the bad type might default with some probability), but neither of these effects manifests themselves in the continuation value because the sovereign must return to risk-free pricing in the subsequent period (as is demanded by limited commitment as well as the principle of one-shot deviation).

Subtracting the payoff from the deviation from that of equilibrium play and manipulating mildly, we arrive at the marginal consideration of this particular strategy

Assumption 6.

$$\underbrace{\frac{p\rho}{R-1}(\phi - m(1-\beta_L)}_{\textit{Revenue loss from price drop}} \geq \underbrace{\sup_{\Delta b \in \left(0, m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)\right]} \Delta b \left[\beta_H - (1 - p\rho)\right]}_{\textit{Marginal utility from extra borrowing}}$$

Unlike the bad type, here it is less clear whether there is positive utility to be gained from issuing in this region or not. It will depend on both the reputation as well as the probability of the shock. If the right-hand-side term is positive, the sovereign would optimally choose $\Delta b = \sup \mathcal{B}_2$; if the right-hand-side is negative, he would optimally choose $\Delta b = 0$. This latter deviation technically does not exist, but ruling it out precludes all other, infinitesimally small deviations that he might make in this region.

Either way, though, as before this condition can always be satisfied as R approaches unity from above as $b_{S,L}$ (and thus the lost revenue) grows to infinity in this case but not the rest of the terms in the inequality.

A deviation of the good type to the supremum of $\mathcal{B}_3$ is precluded already by Assumptions 1 and 2, which ensure that the marginal benefit of issuing m more is not worth the drop in the price for the prior $b_{S,L}$.[17] As we discussed with the bad type when considering this deviation, there is no increase in reputation for repaying debt, as both types repay debt with the same frequency. Thus, the reputation only decreases going into the next period. A deviation to $\mathcal{B}_5$ is precluded also for the same reason as it was precluded for the bad type earlier.

[17] It does not matter for this condition that the initial debt level is $b_{S,L}$ and not $b_{S,H}$ because utility is linear and only the difference of size m matters.

Finally, we must preclude a deviation to $\mathcal{B}_4$.

The two payoffs from this deviation are

$$y - b + \frac{1}{R} b_{S,L} + \frac{\beta_H}{1-\beta_H}\left(y - b_{S,L}\left(\frac{R-1}{R}\right)\right) \tag{18}$$

$$\sup_{\Delta b \in \left(m, m\left(1+\frac{\beta_H - \beta_L}{1-\beta_H}\right)\right]} \quad y - b + \frac{(1-p)(1-\rho)}{R}\left(b_{S,L} + \Delta b\right) + \\ \beta_H \left[\begin{array}{c} (1-p)\left(y - b_{S,L} - \Delta b + \frac{1}{R} b_{S,L} + \frac{\beta_H}{1-\beta_H}\left(y - b_{S,L}\left(\frac{R-1}{R}\right)\right)\right) \\ +p\left(\frac{y-\phi}{1-\beta_H} + m\right) \end{array} \right] \tag{19}$$

To preclude this deviation, their difference must be weakly positive. These expressions can be simplified a bit when we recall that, for the good type, the value of repaying $b_{S,L}$ is equal to the value of defaulting with the shock plus $m(\beta_H - \beta_L)/(1-\beta_H)$. Using this expression to substitute out some expressions in the continuation value, we arrive at a simple condition:

Assumption 7.

$$\left[\frac{1-(1-p)(1-\rho)}{R-1}\right]\left(\phi - m(1-\beta_L)\right) \geq \\ \sup_{\Delta b \in \left(m, m\left(1+\frac{\beta_H - \beta_L}{1-\beta_H}\right)\right]} \quad \Delta b(1-p)\left[\frac{1-\rho}{R} - \beta_H\right] - \beta_H p m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)$$

Whether the term attached to Δb is negative or positive is determined by the current reputation. But again, in either case, we can always set R towards unity to ensure this condition is satisfied.

It will be convenient for us to group the four new assumptions laid out here (Equations 4, 5, 6, and 7) into one group that we call the **overissuance conditions**. As we argued in this section, the overissuance conditions can always be satisfied for any feasible set of parameters as R decreases to unity from above. Hence the restriction described in Proposition 1 as well as Corollary 2.

Step 3 (Ruling out under-issuance and off-equilibrium repayment): We have two final types of deviation to consider: Under-issuance and off-equilibrium repayment. These are related because they both improve reputation through off-equilibrium behavior.[18] Precluding these deviations will turn out to be similarly related.

We begin with off-equilibrium repayment. We preclude such a deviation by setting up a sort of cushioned subset of the parameters from which off-equilibrium deviations cannot escape. Let $\hat{\Theta}_\rho \subset \Theta_\rho$ be defined as follows. $[\theta, \rho] \in \hat{\Theta}_\rho$ if and only if $[\theta \backslash \rho, \nu_R(b, \rho)] \in \Theta_\rho \ \forall b \in [0, \bar{b}]$. In words, this means that if a parameter-belief pair lies in $\hat{\Theta}_\rho$, then not only does it currently generate a valid pricing schedule and discourage over-issuance (by virtue of it being in Θ), but it would *continue* to do so following the reputational improvement given by off-equilibrium repayment.

It is easy to see that under status-quo off-equilibrium beliefs we get that $\hat{\Theta} = \Theta$ and the deviation is easily satisfied. But it is also easy to see (by continuity) that $\nu_R < \rho$ clearly exist that ensure $\hat{\Theta}$ is non-empty.

Now for under-issuance. Under full-information, under-issuance will never be optimal because the pricing functions are step-wise flat and both types are less patient than the foreign creditors. However, the temptation to under-issue could arise for reputational considerations, as the sovereign could expand its budget set tomorrow by reducing its debt position today to increase its reputation in accordance with ν_-.

Ruling out under-issuance is done similarly to off-equilibrium repayment. Let $\bar{\Theta}_\rho \subset \hat{\Theta}_\rho$ be defined as follows. $[\theta, \rho] \in \bar{\Theta}_\rho$ if and only if $[\theta \backslash \rho, \nu_-(b_{S,L}) \times \rho] \in \hat{\Theta}_\rho$. In words, this ensures that the off-equilibrium beliefs engendered by the under-issuance today do not improve beliefs so

[18] Off-equilibrium default need not be considered since this both ends the game and worsense beliefs.

much that the pricing schedule no longer maintains or a deviation of over-issuance becomes beneficial tomorrow.

As before, this can be done immediately for status quo beliefs, i.e., $\nu_- = 1$, in which case $\bar{\Theta}_\rho = \hat{\Theta}_\rho$. But it is also easy to see by continuity that $\nu_- < \rho$ clearly exist that ensure $\bar{\Theta}_\rho$ is non-empty.

To achieve the final result, we observe that the set of reputations for which Assumptions 4-7 hold expands arbitrarily as R goes to one from above.[19] Thus, we take this until $\rho^*(\theta)$ includes $\underline{\rho}$. This ensures that all off-equilibrium deviations are precluded both now and following potential off-equilibrium deviations today for the entirety of the restricted state space.

And with this final maneuver we have finally proved Proposition 1. Note that this final condition (including $\underline{\rho}$) is sufficient but not necessary. All off-equilibrium deviations precluded by the proposition will also be precluded at all worse reputation levels.

The various relevant corollaries were either demonstrated in the proof or will be discussed momentarily in the discussion of the implied dynamics.

Dynamics: Given that we have verified all of the equilibrium conditions. We now discuss the dynamics. We know that the equilibrium is pooling along the borrowing margin everywhere (at $b_{S,L}$ in fact). We also know that *on the equilibrium path*, the two types pool in their debt repayment (which happens with probability one). This establishes Corollary 1.

There do exist states in which the two types would separate in the default behavior, but these are always off-equilibrium. Thus, no information is ever revealed in equilibrium. This establishes Corollary 4. The good type is thus stuck in an 'infamous equilibrium,' wherein he defaults more often as he normally would on account of receiving worse equilibrium pricing schedules. Given his inability to commit to future behavior, he is powerless

[19] There would be a discontinuity at $\rho = 0$, but we will not reach that.

to escape it.

A.3 Proof of Proposition 2

This result is much more straightforward. We need only compare the full-information level of borrowing allowed by lenders with the level allowed in the infamous equilibrium under asymmetric information. This difference is $b_{S,H} - b_{S,L}$, which is given by

$$\frac{R(\phi - m(1-\beta_H))}{R-1} - \frac{R(\phi - m(1-\beta_L))}{R-1} = \frac{Rm(\beta_H - \beta_L)}{R-1}$$

which establishes the result. It is manifest that this difference increases (even to infinity) as R decreases toward unity.

A.4 Proof of Proposition 3

To see why the infamous equilibrium is unique in its class, we consider any other alternative. That is, we consider whether any other debt level at which both types pool borrowing behavior at the risk-free rate can be sustained given equilibrium restrictions on beliefs and behavior. Let us call such a candidate threshold $\hat{b}$

We first consider the trivial case where $\hat{b} < b_{S,L}$, i.e., a debt limit less than the full-information limit for either type. Because of the linearity of sovereign utility, this would necessitate a discontinuity in the pricing function at $\hat{b}$. This could never obtain because it would be inconsistent with the market pricing condition (Equation 5). We know from the full-information case that the bad type will repay up to $b_{S,L}$ at the risk-free rate *regardless of reputational considerations*. Thus, the default probability of issuance between $\hat{b}$ and $b_{S,L}$ would have to be zero for both types. But this would violate Equation 5, as we required a kink in the pricing function in this region that could not exist. Thus such an equilibrium could not obtain. A similar argument would hold for $\hat{b} > b_{S,H}$, as the good type would not repay in this region even if it could service that debt at the best possible price (the risk-free price). Thus, the only possible other candidates for $\hat{b}$ are in $(b_{S,L}, b_{S,H}]$.

It turns out that we cannot have $\hat{b}$ in this region either, though. The best possible price that reputation can buy the sovereign is the risk-free price. We also know that by construction $b_{S,L}$ is the greatest amount of debt that the bad type would repay at this optimal price. Thus, even with the best possible reputation, the bad type will never repay more than $b_{S,L}$ with probability one. This implies that the bad type will necessarily default with *some* probability beyond $b_{S,L}$. If $\hat{b}$ is above $b_{S,L}$ then, the bad type would borrow up to it. However, his default probability would be strictly positive in the next period, which contradicts the risk-free price he received for this issuance in accordance with Equation 5. Thus, no $\hat{b}$ in this region could ever be an equilibrium.

The reason why the good type can see his budget set contract but the bad type cannot see his budget set expand is because the bad type's value of repayment in the infamous equilibrium does not change relative to the full-information case. Thus, the threshold for repayment cannot change. However, the good type's value of repayment in the infamous equilibrium *does* change. In fact, it drops significantly enough that default behavior worsens to the point where it consistent with the monotone pricing schedule of Proposition 1.

A.5 Proof of Proposition 4

We begin by expressing the third-party's disutility recursively. We can then use this solution to characterize the expected losses associated with the policy. Because, unlike the benchmark case, this policy will engender changing dynamics over time and eventual separation, it will be more notationally convenient for us to use time subscripts for this section so that the illustration in Figure 3 makes more sense.

We define $ED_t(\rho_t)$ to be the expected disutility to the third party given a prior ρ_t that the country is the bad type.

$$ED_t(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{(1-p)\rho_t + (1-\rho_t)}{R} \times \min\left\{ \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} - S, ED_{t+1}\left(\rho_{t+1}\right) \right\}$$
$$\rho_{t+1} = \frac{(1-p)\rho_t}{(1-p)\rho_t + (1-\rho_t)}$$

We establish that this expected disutility function exists and is unique.

Lemma 4. $ED_t(\rho_t)$ *exists, is unique, is bounded, is strictly increasing in* ρ_t*, and can be found via dynamic programming.*

We use Blackwell's sufficiency to establish that the following operator is a contraction:

$$T(ED)(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{(1-p)\rho_t + (1-\rho_t)}{R} \times \min\left\{ \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} - S, ED\left(\rho_{t+1}\right) \right\}$$
$$\rho_{t+1} = \frac{(1-p)\rho_t}{(1-p)\rho_t + (1-\rho_t)}$$

1. **Monotonicity:** Suppose we had two expected disutility function that satisfied $ED^1(\rho) \leq ED^2(\rho)$ for all $\rho \in [0, 1]$. It is straightforward to observe that $T(ED^1)(\rho) \leq T(ED^2)(\rho)$ as well. The flow disutilities will be equal to one another, but the continuation value of $T(ED^2)$ will be greater than $T(ED^1)$.

2. **Discounting:** We set discount factor to be $\hat{\beta} = 1/R$. We now show that $T(ED + a) \leq T(ED) + \hat{\beta}a$ for any positive constant a. As flow disutilities will play no role here, we restrict attention to the continu-

ation values:

$$
\begin{aligned}
&\frac{1-p\rho_t}{R}\min\left\{\frac{\rho_{t+1}\left[\hat{b}_B+m\right]}{R}-S, ED\left(\rho_{t+1}\right)+a\right\}\\
&\qquad\qquad\leq\frac{1-p\rho_t}{R}\min\left\{\frac{\rho_{t+1}\left[\hat{b}_B+m\right]}{R}-S+a, ED\left(\rho_{t+1}\right)+a\right\}\\
&\qquad\qquad=\frac{1-p\rho_t}{R}\min\left\{\frac{\rho_{t+1}\left[\hat{b}_B+m\right]}{R}-S, ED\left(\rho_{t+1}\right)\right\}+\frac{1-p\rho_t}{R}a\\
&\qquad\qquad\leq\frac{1-p\rho_t}{R}\min\left\{\frac{\rho_{t+1}\left[\hat{b}_B+m\right]}{R}-S, ED\left(\rho_{t+1}\right)\right\}+\frac{1}{R}a
\end{aligned}
$$

$$
\implies T(ED+a)\leq T(ED)+\hat{\beta}a
$$

With these conditions established, we have a that T is a contraction, which implies existence, uniqueness, and the ability to solve via dynamic programming.

Boundedness follows immediately from a the following limiting argument. Let us conjecture that ED_t is increasing in ρ_t (this will be verified later). We can evaluate the Bellman equation in this case to derive:

1. $ED_t(0) = 0$, i.e., if it is believed that the country is the good type, there are no expected losses.

2. $ED_t(1) = \frac{p\hat{b}_B}{R-1+p}$. This will be the case under the assumption that $\frac{p\hat{b}_B}{R-1+p} < \frac{\hat{b}_B+m}{R}$, which we maintain from now on.

Notice that if ED_t is increasing, then $ED_t(1)$ will be its upper bound. We now verify its increasing nature by taking derivatives on a case-by-case depending on which of the continuation values is smaller.

1. If the next period induces separation, we have

$$ED_t(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{1 - p\rho_t}{R} \times \left(\frac{1-p}{1-p\rho_t} \frac{\rho_t \left[\hat{b}_B + m\right]}{R} - S \right)$$

$$\implies ED_t'(\rho_t) = \frac{p\hat{b}_B}{R} + \frac{(1-p)\left[\hat{b}_B + m\right]}{R^2} - \frac{pS}{R}$$

This will be positive provided $S \leq \hat{b}_B$, which we assume from now on.

2. If the next period does not induce separation, we have

$$ED_t(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{1 - p\rho_t}{R} \times ED_{t+1}(\rho_{t+1})$$

$$ED_t'(\rho_t) = \frac{p\hat{b}_B}{R} - \frac{p}{R} ED_{t+1}(\rho_{t+1}) + \frac{1-p\rho_t}{R} ED_{t+1}'(\rho_{t+1}) \left[\frac{1-p}{1-p\rho_t} + \frac{p\rho_t(1-p)}{(1-p\rho_t)^2} \right]$$

...

$$ED_t'(\rho_t) = \frac{p\hat{b}_B}{R} - \frac{p}{R} ED_{t+1}(\rho_{t+1}) + \frac{1-p}{(1-p\rho_t)R} ED_{t+1}'(\rho_{t+1})$$

$$\implies ED_t'(\rho_t) > \frac{p\hat{b}_B}{R} - \frac{p}{R}\frac{p\hat{b}_B}{R-1+p} + \frac{1-p}{(1-p\rho_t)R} ED_{t+1}'(\rho_{t+1})$$

$$\implies ED_t'(\rho_t) > \frac{p\hat{b}_B}{R}\left[1 - \frac{p}{R-1+p}\right] + \frac{1-p}{(1-p\rho_t)R} ED_{t+1}'(\rho_{t+1})$$

This last line establishes combined with the fact that the terminal period, T, (whenever it is) will feature $ED_T'(\rho_T) > 0$ is enough to establish by induction that $ED_t'(\rho_t) > 0$ for any t and any ρ_t. This also verifies the assumption under which $ED_t(1)$ was an upper bound, which delivers boundedness as well and completes the proof.

A.6 Proof of Proposition 4

This is fairly straightforward. $ED(\rho)$ will shrink as time passes because ρ will converge to zero as time passes on. Given that $S > 0$ does not depend on ρ, at some point the costly separation loan becomes a low enough cost strategy that it necessarily becomes optimal at some finite time T. The lower is S, the larger will T be, but $T < \infty$ will exist for any $S > 0$.

If the type is bad, then they will either default with a flow probability p before time T or with probability 1 at time T.

A.7 Proof of Proposition 5

Once we have established the existence of expected disutility, we can write out expected losses recursively like we did for expected disutility as follows.

$$EL_t(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{(1-p)\rho_t + (1-\rho_t)}{R} \times$$
$$\left[\mathbf{1}\left\{ \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} \leq ED_{t+1}\left(\rho_{t+1}\right) \right\} \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} + \right.$$
$$\left. \mathbf{1}\left\{ \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} > ED_{t+1}\left(\rho_{t+1}\right) \right\} EL_{t+1}\left(\rho_{t+1}\right) \right]$$
$$\rho_{t+1} = \frac{(1-p)\rho_t}{(1-p)\rho_t + (1-\rho_t)}$$

Proposition 5 follows from the fact that Blackwell's sufficiency applies equally to the recursive operator defining $EL_t(\rho_t)$. It is thus a contraction. Strict monotonicity also follows.

References

[1] **Aguiar, Mark and Gita Gopinath**, "Defaultable Debt, Interest Rates, and the Current Account," *Journal of International Economics*, 2006, *69* (1), 64–83.

[2] ——, **Satyajit Chatterjee, Harold Cole, and Zachary R. Stangebye**, "Quantitative Models of Sovereign Debt Crises," *Hand-*

book of Macroeconomics, Volume II, 2016.

[3] **___ , ___ , ___ , and ___** , "Quantitative Models of Sovereign Debt Crises," *Handbook of Macroeconomics, Volume II (Forthcoming)*, 2016.

[4] **Amador, Manuel and Christopher Phelan**, "Reputation and Sovereign Default," *Econometrica*, 2021, *89* (4), 1979–2010.

[5] **Angeletos, George-Marios and Ivan Werning**, "Crises and Prices: Information Aggregation, Multiplicity, and Volatility," *American Economic Review*, 2006, *96* (5), 1720–1736.

[6] **Arellano, Cristina**, "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 2008, *98* (3), 690–712.

[7] **Battaglini, Marco**, "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica*, July 2002, *70* (4), 1379–1401.

[8] **Carlson, Mark and Galina B Hale**, "Rating Agencies and Sovereign Debt Rollover," *Topics in Macroeconomics*, 2006, *6* (2).

[9] **Chakraborty, Archishman and Rick Harbaugh**, "Comparative Cheap Talk," *Journal of Economic Theory*, January 2007, *132* (1), 70–94.

[10] **Chari, V.V. and Patrick J. Kehoe**, "Hot Money," *Journal of Political Economy*, 2003, *111* (6), 1262–1292.

[11] **Chatterjee, Satyajit, Dean Corbae, Kyle Dempsey, and Jose-Victor Rios-Rull**, "A quantitative theory of the credit score," *Econometrica*, 2023, *91* (5), 1803–1840.

[12] **Cole, Harold L. and Patrick J. Kehoe**, "Models of Sovereign Debt: Partial Versus General Reputations," *International Economic Review*, 1998, *39* (1), 55–70.

[13] ——— **and Timothy J. Kehoe**, "A Self-Fulfilling Model of Mexico's 1994-1995 Debt Crisis," *Journal of International Economics*, 1996, *41* (3-4), 309–330.

[14] ——— **, Daniel Neuhann, and Guillermo Ordonez**, "Debt Crises: For Whom the Bell Tolls," *NBER Working Paper No. 22330*, 2016.

[15] **D'Erasmo, Pablo**, "Government Reputation and Debt Repayment," *Manuscript, University of Texas Austin*, 2008.

[16] **Eaton, Jonathon and Mark Gersovitz**, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 1981, *48* (2), 289–309.

[17] **Fourakis, Stelios**, "Sovereign Default and Government Reputation," *Mimeo*, 2023.

[18] **Gu, Grace Weishi and Zachary R. Stangebye**, "Costly information and sovereign risk," *International Economic Review*, 2023.

[19] **Morelli, Juan M. and Matias Moretti**, "Information Frictions, Reputation, and Sovereign Spreads," *Journal of Political Economy*, 2023, *131* (11), 2969–3265.

[20] **Phan, Toan**, "A model of sovereign debt with private information," *Journal of Economic Dynamics and Control*, 2017, *83*, 1–17.

[21] ——— , "Sovereign Debt Signals," *Journal of International Economics*, 2017, *104*, 157–165.

[22] **Rachel, Lukasz and Thomas D. Smith**, "Are Low Real Interest Rates Here to Stay?," *International Journal of Central Banking*, September 2017, *13* (3).

[23] **Sandleris, Guido**, "Sovereign Defaults: Information, Investment and Credit," *Journal of International Economics*, 2008, *76* (2), 267–275.

[24] **Van Nieuwerburgh, Stijn and Laura Veldkamp**, "Information Immobility and the Home Bias Puzzle," *Journal of Finance*, 2009, *64* (3), 1187–1215.