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Optimal Monetarist Arithmetic or How to Inflate If You Must *

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Abstract

In their celebrated 1981 paper "Some Unpleasant Monetarist Arithmetic," Sargent and Wallace show that when a central bank is required to transfer resources to the fiscal authority, it faces a trade-off: if it chooses to keep inflation low in the short run, then it must be willing to accept higher inflation in the long run. In this paper I characterize the optimal interest rate (and inflation) policy by a central bank faced with a version of the Sargent-Wallace scenario. I explore this question in a setting in which a certain amount has to be transferred for an uncertain period of time. I find that uncertainty makes the optimal policy to deviate from a Barro-like martingale behavior of taxes or a Lucas-Stokey type of solution where the tax (or distortion) inherits the properties of the stochastic process for transfers.

The optimal nominal interest rate (and inflation) is such that the central bank chooses to issue debt (e.g., reverse repos) instead of raising the required amount of seigniorage. Initially inflation is lower than what would be required to raise enough resources to pay for the transfer plus the interest on existing debt. The intuition for this result is that the monetary authority is taking

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advantage of an option: if the transfer period is short, then (relatively) low inflation today can spread the losses over time. Over time, as the debt increases, more revenue has to be raised. Interest rates increase. The time path of inflation depends on whether the monetary authority can "inflate away" part of the debt. If this default is costly, the end of the transfer period is associated with permanently lower inflation. If a sudden increase in the price level (default) is an option (and if it chooses to be exercised), the end of the transfer period is associated with high (and brief) inflation and then stabilization.

Keywords: Optimal monetary policy, inflation, fiscal dominance.

JEL Classification codes: E5, E6.

1 Introduction

In their celebrated 1981 paper "Some Unpleasant Monetarist Arithmetic," Sargent and Wallace (see Sargent and Wallace (1981)) show that when a central bank is required to transfer resources to the treasury —typically described as a case of fiscal dominance or active fiscal policy [1] — the monetary authority faces a trade-off: if it chooses to keep inflation low in the short run, then it must be willing to accept higher inflation in the long run. The Sargent-Wallace analysis, even though it was couched in terms of financing a transfer to the fiscal authority, applies more generally to any situation in which a central bank has to finance a transfer in the absence of "external" support —that is, without relying on external sources to raise resources and sterilize the increases in the money supply.

In this paper I study what the optimal strategy is to finance a temporary transfer. My main finding is that when the (flow) amount of the transfer is known but there is uncertainty about the duration of the transfer period, the optimal monetary policy is one characterized by increasing nominal interest rates and inflation that drop after the transfer period ends. The thought exercise is, in some sense, similar to questions about optimal taxation. The optimal inflation tax deviates from a Barro-like policy (see Barro (1979)) that prescribes a martingale-like behavior taxes. The inflation tax is increasing over time during the transfer phase. The optimal inflation tax does not mimic the exogenous stochastic process for the transfer and can even display non-monotonicities. In this sense, it deviates from the intuition in Lucas and Stokey (1983) that finds that the stochastic process for taxes —in my case the inflation tax— inherits the properties of the exogenous shocks. [2]

When does a central bank have to transfer resources? There are, at least, two different scenarios in which this is the case. The more standard one is the case of a central bank monetizing the deficit. This is the case of fiscal dominance. A second, less common scenario is when a central bank experiences cash flow losses from its assets earning less than its liabilities. This is the situation that several central banks have found themselves in in recent years.

The literature on fiscal dominance finds ample evidence that temporary fiscal shocks are associated with higher inflation. [3] Argentina in the mid 2010's is an example of the type of temporary transfers that I model. In January 2016, the

[1]See Leeper (1991) for the distinction between active and passive fiscal policies.

[2]See Aiyagari et. al. for a qualification of these results and Chien and Wen (2022) for the case of aggregate and idiosyncratic uncertainty

[3]see Cevik and Miryugin (2025) for cross country evidence

newly elected government of Argentina announced a policy of decreasing government budget deficits in the context of a stabilization plan. It also specified what fraction of the deficit would have to be financed by the monetary authority. In the case of Argentina, this was a significant amount. For the year 2016, the Central Bank (BCRA) was required to transfer to the treasury approximately 2% of GDP out of a total government deficit of 4.8% of GDP. The plan specified that the transfers would decrease and, eventually, end.[4] This policy effectively required the BCRA to finance a negative cash flow in the form of a transfer to the treasury with no resources ever going in a different direction.

In a different context —both geographically and institutionally— one outcome of the QE policies pursued by many central banks is that they find themselves facing a negative cash flow.[5] The Federal Reserve Bank (Fed) has recorded negative cash flows since it started raising the policy rate and, at this time (March 2025), the negative balance is about $224 billion, which is a small fraction of GDP. The Swiss National Bank (SNB) lost the equivalent of $143 billion in 2022, which is about 18% of GDP, and the Reserve Bank of Australia has reported a negative equity of 12.4 billion due to the increase in nominal interest rates. [6]

Although the origin of the negative cash flow in the two types of situations is quite different —inflationary finance in one case and losses from interest rate risk in the other— the monetary authorities face the same question: How should this temporary negative cash flow be financed? I assume that there is some uncertainty about the stochastic process for negative cash flow. I concentrate on the case in which the size of the negative cash flow is known but the duration of the negative cash flow phase is random. In the simple version of the model, there are two phases. Phase I corresponds to the time that the central bank has to transfer resources, while phase II includes the time after the transfer ends. As discussed above, the optimal policy displays an increasing inflation tax rate that drops at the end of phase I. The optimal policy always falls short of financing the transfer plus the interest on the debt. Thus, throughout this period, consolidated central bank debt increases. Moreover, the central bank in this exercise engages in open market operations that change the debt-to-money ratio at different points in time.

[4] In addition, the BCRA decided to increase the monetary base by close to another 2.5% to increase the stock of foreign reserves. In March 2023 the BCRA announced a policy of decreasing transfers (Adelantos Transitorios) over the next two years.

[5] Since the 2008 crises central banks have held large portfolios of relatively long dated securities whose value have decreased as a consequence of increases in nominal interest rates. Their liabilities are short run and not affected by changes in interest rates

[6] A good discussion is in Cecchetti and Hilscher (2024)

The results are driven by the uncertainty about the negative cash flow. To see this, consider an alternative where the duration of the period is known. Optimal smoothing considerations imply that distortions —in this case seigniorage— should be constant over time. Thus, the constant level of seigniorage is chosen so that its present value equals the present value of the negative cash flow.[7]

Why is that uncertainty influences the optimal policy? It provides an option value to waiting to raise seigniorage. Since it is optimal to spread the welfare losses over the infinite horizon, it makes sense to raise less resources than necessary to finance the transfer plus any interest on existing debt early on in the process. This policy "gambles" on an early termination of phase I. Because the optimal policy keeps postponing raising all the revenue necessary to finance the cash flow plus the interest on the accumulated debt, it is necessary to raise more resources. This requires higher nominal interest rates, higher inflation, and lower real money balances.

The prescription of the optimal policy for interest rates (and inflation, since the Fisher equation holds in this model) depends on the social cost of high inflations. Formally, I model high inflations as jumps in the price level. In the model, I restrict the times at which the central bank can engineer these (costly) high inflation episodes. Jumps in the price level have the same effect as an outright default given that all liabilities are nominal. I find that the lower the cost of high inflations, the lower the optimal interest rate (and inflation) during phase I. The reason for this is simple: the possibility of (costly) default is equivalent to endowing the central bank with another "technology" to raise revenue. This implies a lower reliance on seigniorage, which in turn results in lower interest rates. The most constrained central bank faces an infinite cost of jumps in the price level and, consequently, follows what Auernheimer (1974) labeled as an "honest" policy: No jumps in the price level.

I then extend the model in a couple of directions. First, I allow for gradual decreases in the amount transferred. This corresponds to a number of phase I events, each with its own uncertain duration. The interesting result in this case is a strong form of non-monotonicity. It is possible that, after the amount of the transfer has decreased (say, the economy is in the second phase I) that under the optimal policy, the economy will experience inflation rates that are higher than during the high transfer period. Inflation can display a seesaw pattern. This possibility makes it

[7] Uribe (2016) (2020) shows that the optimal policy is to keep the inflation rate constant forever even when the transfer is not constant. In the case that the monetary authority can default at the time that net income returns to its normal level, Manuelli and Vizcaino (2017) show that inflation is high for as long as the central bank's net income is negative and it drops to zero when net income is normalized.

difficult to use inflation as the only measure to evaluate whether a central bank is following the announced policy.

The second extension recognizes that raising resources through seigniorage to be transferred to the fiscal authority should allow for the decrease of some other distortionary tax. This, in turn, implies that consumption and real interest rates respond to some measure of cumulative seigniorage. In this case the qualitative features of the optimal policy resemble those of the simple case: seigniorage falls short of the short-term myopic financing needs and it increases over time.

The paper is organized as follows. The next section provides a brief (and incomplete) review of the literature on the impact of negative cash flows for the conduct of monetary policy. Section 3 presents the basic model. Section 4 describes the optimal policy in phase II —that is, after the period of negative cash flow has ended. Section 5 studies the optimal policy in phase I, section 6 contains some extensions, and section 7 offers some concluding comments.

2 Brief Literature Review

The model in this paper applies to situations where there is fiscal dominance as well as circumstances in which a central bank has a negative cash flow that is unrelated to fiscal demands. I will briefly describe some work on these two different problems.

As a result of the 2008 financial crises, many central banks changed how they conducted monetary policy. Probably the most significant change was the expansion of their balance sheets. Central banks purchased large amounts of assets —many were long-dated securities— and financed the purchases issuing short-term interest-earning reserves. The difference in interest rates created an operating profit that was remitted to the treasury. Even before the recent increases in nominal interest rates, many observers noted that the situation could be reversed: if, for policy reasons, the rate paid by the central bank has to be increased while at the same time the income from its portfolio of long-term assets does not change, the monetary authority could potentially sustain operating losses.

In the case of the Fed, Goodfriend (2014) alerted to that possibility and argued that the issue of how to finance such a loss is a delicate and difficult problem.[8]

Del Negro and Sims (2015) describe a model that distinguishes between the central

[8]Goodfriend considers the costs and benefits of financing this shortfall either issuing (non-interest) reserves or borrowing in the RRP market. He views both options as problematic and advocates instead building a buffer stock from earnings.

bank's balance sheet, with its own budget constraint, and the rest of the government budget. They study the conditions under which it is feasible to support a *given* policy that they model as a version of the interest-smoothing version of the Taylor rule. They define fiscal support as a transfer from the treasury to the central bank and show conditions under which a temporary negative cash flow *does not* require fiscal support to implement the given, arbitrary policy. They parameterize a version of their model and argue that, for the U.S., the Federal Reserve is not likely to need fiscal support in the near future since they estimate that the present value of seigniorage is large. Critical to their exercise is the assumption that the economy is *deterministic*. Thus, in the case Del Negro and Sims study, the right measure is to compare present values —using the risk free rate— of resources and liabilities. Alternative assumptions about the demand for money yield different estimates of the present value of resources available to the Fed (assets plus seigniorage) and, hence, potentially different views on the sustainability of a given policy in the absence of fiscal support. There are two major differences between Del Negro and Sims (2015) and this paper. First, they consider a time zero change in a variable —typically an interest rate— that is perfectly predictable. Thus, they solve a non-stochastic model that was not designed to capture the challenges imposed by uncertainty over the size of the negative cash flow. Second, they study the feasibility of implementing a given class of monetary policy rules. By feasibility they correctly impose a transversality condition on the real value of central bank liabilities. In this paper, I am interested in understanding how he central bank should deal with this situation. In other words, what should the central bank do?

Hall and Reis (2015) also explore the sustainability of what they call a "dividend policy", which amounts to a rule that determines a transfer from the central bank to the treasury (which can be negative) as a function of the state of the economy. They emphasize that the "stability of a central bank depends crucially on what happens to its dividends when net income is negative." They study several policies that deal with strategies to manage the Fed's and the European Central Bank's balance sheet, and they emphasize different approaches to designing a financial policy with an emphasis toward what they view as a *financially stable policy* defined as a strategy that rules out explosive paths of interest-paying liabilities issued by the central bank. They point out that they do not view their model as a useful way to study the inflationary consequences of negative net income. They state that "a small loss for the Fed would require very large increases in inflation." They claim that higher seigniorage is associated with a loss of central bank independence.

Hall and Reis (2015) and Del Negro and Sims (2015) emphasize that, given current institutional arrangements, it is important to recognize that the central bank and the treasury have separate budget constraints when the treasury commits to no transfers to the central bank. This case —which Del Negro and Sims (2015) labeled lack of fiscal support— clearly requires formally recognizing the connection between central bank decisions and fiscal policy. Cavallo et al. (2019) discuss the fiscal implications of alternative strategies concerning the Fed's balance sheet normalization. Reis (2015) provides a summary.

The modern literature on monetary policy under fiscal dominance (or "active" fiscal policy in Leeper (1991)) probably begins with Sargent and Wallace (1981). They show that when the central bank has to support the fiscal authority, the choice is between inflation today or inflation tomorrow.[9]

This early work spurred a large literature on whether open market operations are neutral. Benigno and Nisticò (2015) describe a variety of neutrality and non-neutrality results associated with open market operations in a context where the central bank has a large balance sheet. They show that neutrality obtains only under relatively stringent conditions and that, if the central bank faces the risk of a negative net income, it is likely that a traditional monetary policy will require fiscal support in the form of transfers from the treasury to the central bank.

Diaz-Gimenez et al. (2008) show that time consistency limits the ability of the monetary authority to implement optimal policies. The key difference with this paper is that the path of deficits is not decreasing (and random) and that there is coordination between the fiscal and monetary authorities, which implies that taxes other than the inflation tax are used to finance the service of the debt. Martin (2011) has a thorough discussion of the joint determination of monetary and fiscal policy. He shows that, in the absence of commitment, the endogenous choice of government debt affects future monetary policy. Relative to that Martin (2011) I simplify the problem by fixing the transfer exogenously (this is a form of commitment and a strong version of fiscal dominance) but I am able to study the impact of uncertainty on optimal policies. Martin (2023) has a thorough discussion of the conditions under which fiscal dominance can emerge in a class of models. In the context of the New Keynesian models, Leeper et al. (2016) discusses optimal monetary and debt policy imposing time consistency. For a recent survey of monetary models and optimal policies, see Canzoneri (2011). For a survey of central bank credit to governments, see Jacome (2012).

[9]Even before the Sargent-Wallace article, the literature recognize the possibility that fixing some variables would make money endogenous or "passive". See Olivera (1970).

3 The Economy

3.1 The Environment

I study the simplest model that allows me to capture the notion of a temporary negative cash flow of uncertain duration. Net income is given by $-P_t x_t$ where P_t is the price level and the process x_t is

$$x_t = \begin{cases} x & t \in [0, T] \\ 0 & t \geq T \end{cases} , \tag{1}$$

where $x > 0$.

T is an exponentially distributed random variable with parameter η. This implies that the expected duration of the negative net income phase is $1/\eta$. In section 5, I extend the model to allow for multiple phases of decreasing (in absolute value) net income. I label the random period of time in which $x_t > 0$ phase I and the period in which $x_t = 0$ phase II.

There are at least two possible interpretations of x. In the case of inflationary finance, x is simply the amount that the central bank has to transfer to the treasury. In the case of independent central banks, it is the difference between what they earn on assets and what they pay for their liabilities.

I assume a pure exchange economy. Income per period is given by y, and consumption is $c = y - g$, where g is government spending. I simplify the real side of the economy so that I can concentrate on the optimal monetary policy. In the Extensions section I discuss a case in which consumption depends on the accumulated value of the transfer and, hence, consumption and interest rates depend on the policy.

3.2 Households

I assume that there is a representative dynasty that has preferences over consumption and real money balances given by

$$U = E\left[\int_0^\infty e^{-\rho t}\left[u(c_t) + v(m_t)\right] dt\right],$$

where c_t is consumption at time t and $m_t = M_t/P_t$ is real money balances. The functions u and v are assumed strictly concave and twice continuously differentiable. The private sector budget constraint is given by

$$dM_t + dB_t^T + dB_t^M = \left(i_t(B_t^T + B_t^M) + P_t y_t - P_t c_t - P_t \tau_t\right) dt + d\tilde{N}_t, \tag{2}$$

where $\tilde{N}_t$ is the Poisson process that captures the (potential) jump in the price level associated with the end of the transfer period —that is, a potential jump that occurs at time T.[10]

On the income side, the representative dynasty earns interest at the nominal rate i_t on its holdings of treasury-issued debt, B_t^T, as well as the stock of monetary-authority-issued debt, B_t^M.[11] In addition, the representative household earns income, spends resources purchasing consumption, and pays taxes (if $\tau_t < 0$, the household receives a transfer). The notation emphasizes that it is possible to separate the policy decisions made by the treasury —choices of B_t^T and τ_t— from those made by the central bank.

It useful to study this economy during phase I (no jumps in the price level). In this case, the appropriate version of equation (2) is

$$dM_t + dB_t^T + dB_t^M = \left(i_t(B_t^T + B_t^M) + P_t y_t - P_t c_t - P_t \tau_t\right) dt.$$

Let W_t be total financial wealth. Thus,

$$W_t = B_t^T + B_t^M + M_t,$$

and with this notation the budget constraint in phase I is

$$dW_t = (i_t W_t - i_t M_t + P_t y_t - P_t c_t - P_t \tau_t)\, dt.$$

Let lower case letters denote real values of every variable. The real version of the budget constraint is then

$$dw_t = ((i_t - \pi_t)\, w_t - i_t m_t + y_t - c_t - \tau_t)\, dt.$$

What is the potential impact of the arrival of phase II (at time T)? Since the only state variable is wealth, a jump in the price level can affect the real value of wealth. Let the post-jump price level be P_T' and the pre-jump price level be simply $P_T = P_{T^-}$, which corresponds to the left limit of the price process. Real wealth before and after the jump is given by

$$w_T' = \frac{W_T}{P_T'}, \text{ and } w_{T^-} = \frac{W_T}{P_{T^-}},$$

[10] The assumption that money earns no interest is not essential. All that is required is that if i^m is the nominal return on money balances, then $i - i^m > 0$.

[11] If one views B_t^M as interest-earning reserves, then money, M_t, is the set of liabilities that are used in transactions. For a transactions based model that derives a stable demand see Lucas and Nicolini (2013).

This implies that

$$w'_T = w_T \frac{P_{T^-}}{P'_T}.$$

In order to avoid specifically using the (somewhat cumbersome) left limit notation, I will denote

$$w_T = w',\, w_{T^-} = w,\;\; P_T = P' \text{ and } P_{T^-} = P.$$

The stochastic process for wealth is then

$$\begin{aligned} dw_t = &\left(\left(i_t - \pi_t\right) w_t - i_t m_t + y_t - c_t - \tau_t\right) dt \\ &\left(\left(i'_t - \pi'_t\right) w'_t - i'_t m'_t - \left(i_t - \pi_t\right) w_t + i_t m_t\right) dN_t, \end{aligned} \tag{3}$$

where a $'$ over a variable denotes its its value after the (possible) jump in the price level.

The problem faced by the representative consumer is simply

$$U = E\left[\int_0^\infty e^{-\rho t}\left[u(c_t) + v(m_t)\right] dt\right],$$

subject to equation (3), where the expectation is taken over the realization of the time of the first jump of the Poisson process.

Before analyzing the optimal monetary policy, it is necessary to derive the optimal behavior on the part of the private sector, since their decision rules are constraints that the central bank has to incorporate into its optimal policy problem.

The HJB equation corresponding to the household's optimization problem is (ignoring the aggregate state that, in this setting coincides with the individual state)

$$\begin{aligned} \rho H(w) &= \max_{c,m}\left\{u(c) + v(m) + H'(w)\left(\left(i - \pi\right) w - im + y - c - \tau\right)\right. \\ &\quad \left. + \eta\left[H(w\frac{P}{P'}) - H(w)\right]\right\}. \end{aligned}$$

The first-order conditions are

$$u'(c) = H'(w),$$

and

$$v'(m) = H'(w)i.$$

Thus, the demand for money is simply given by

$$\frac{v'(m)}{u'(c)} = i. \tag{4}$$

Equation (4) completely summarizes optimal behavior on the part of the household. As such, it must be imposed as a constraint on the problem solved by the monetary authority.

If consumption is constant (as it will be the case in equilibrium) so is $H'(w)$. This implies that, in equilibrium, $H''(w) = 0$. Differentiating the HJB equation with respect to w and imposing the envelope condition we get that[12]

$$(\rho + \eta)\, H'(w) = H'(w)\,(i - \pi) + \eta H'(w\frac{P}{P'})\frac{P}{P'}.$$

However, if equilibrium consumption is constant, it follows that

$$i = \rho + \eta\,(1 - P/P') + \pi,$$

and, hence, that the demand for money is

$$\frac{v'(m)}{u'(c)} = \rho + \eta\,(1 - P/P') + \pi. \tag{5}$$

In this economy the realized real interest rate *contingent on no arrival* of the phase II (normal net income) is $r = \rho + \eta\,(1 - P/P')$. It follows that the *higher the expected jump in the price level the higher the realized real interest rate.* If, on the other hand, the optimal policy is restricted to continuous paths of the price level (that is, no default), then the real interest rate is equal to the discount factor. Moreover, it also follows that once the economy enters phase II, the real interest rate satisfies $r = \rho$.

3.3 Central Bank

To determine the optimal monetary policy, it is necessary to be explicit about the objective of the central bank as well as the constraints that it faces. What is the monetary authority's budget constraint? Here, as the literature correctly emphasizes, it is important to separate the central bank's balance sheet from the consolidated government's budget constraint. The key difference is that, in this model, the central

[12]This derivation is intuitive. The result formally follows from the relevant stochastic discount factor and the equilibrium level of consumption.

bank's only source of income that can be used to service any interest-paying liability is seigniorage. It is possible, with some complication of notation, to add earnings from the assets in the central bank's portfolio to its budget constraint. In this case, the policy that I describe implies that whenever income is positive it is transferred to the treasury, while negative net income has to be financed using seigniorage. The only qualitative difference with a model that allows for an explicit return on the portfolio is that the limit on the central bank's liabilities must take into account the future earnings on its assets.[13] Thus, in what follows I use a simplified version of the central bank's budget constraint that embeds this transfer policy.

The central bank's budget constraint after time T, that is, in phase II ($x_t = 0$), is

$$dM_t + dB_t^M = i_t B_t^M dt.$$

With a slight abuse of notation I use W_t to denote total liabilities of the central bank. $W_t = M_t + B_t^M$. The budget constraint is then given by

$$dW_t = (i_t W_t - i_t M_t)\, dt,$$

or, in real terms,

$$dw_t = ((i_t - \pi_t)\, w_t - i_t m_t)\, dt.$$

Since $i_t - \pi_t = r_t$ and $i_t = v'(m_t)/u'(c)$, then the phase II budget constraint that incorporates optimal behavior by the private sector is

$$dw_t = \left(r_t w_t + x - \frac{v'(m_t) m_t}{u'(c)} \right) dt, \tag{6}$$

where r_t depends on the particular equilibrium that obtains.

It simplifies the presentation to rewrite the central bank's problem in terms of seigniorage. Let

$$h(m) \equiv v'(m) m.$$

I assume that $h(m)$ has the standard "inverted U" shape. The maximum is attained at $m = m^+$, and at that point $h'(m) = 0$. Let $\hat{m}(z, c)$ be the real money balances associated with raising revenue z when consumption is c.

[13]This possibility is discussed in the Extensions section. A more involved exercise would allow the central bank to accumulate precautionary reserves or to liquidate some good assets to make up the negative net income. However, I ignore this possibility since my objective is to determine how to deal with negative net income rather than how to prevent the possibility of negative cash flow.

$$z_t = \frac{v'(\hat{m}_t)\hat{m}_t}{u'(c)}, \quad \text{or} \quad zu'(c) = h(\hat{m}(z,c))$$

I assume that the central bank operates on the efficient (downward sloping) side of the Laffer curve. Let $L(z,c) \equiv v(\hat{m}(z,c))$. Under standard conditions the function L is decreasing (higher seigniorage requires higher interest rates and lower real money balances) and concave.[14]

In the case that consumption is constant —as is the case in the basic model— I will ignore the extra notation c. In the extension, when c is endogenous, it will be necessary to be explicit about consumption.

4 Optimal Policy: Phase II

Let time T be the realization of the random duration time corresponding to phase I. The central bank is faced with a stock of liabilities, w_T, and wants to maximize the utility of the representative agent. Since consumption is constant and there is no residual uncertainty after time T, the problem faced by the monetary authority is simply

$$\max_{z_t, w'} \int_0^\infty e^{-\rho t} L(z_t) dt - \kappa\Delta(w_T - w'),$$

subject to

$$dw_t = (\rho w_t - z_t)\, dt, \text{ with } w_0 = w' \text{ and } w' \leq w_T,$$

where the function $\kappa\Delta$ captures the notion of a default penalty. Higher values of κ correspond to costlier defaults. In particular, I take the case of the limit as $\kappa \to \infty$ as capturing the situation in which the price level does not jump. I assume that Δ is C^2 and defined on the nonnegative reals. It satisfies

$$\Delta(0) = 0, \ \Delta^{'}(x) > 0, \ \Delta^{''}(x) > 0,$$

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[14] Concavity requires restrictions on the elasticity of the marginal utility of real money balances.

[15] This is satisfied, for example by $\Delta(x) = e^{\lambda x} - 1$

The Default Decision The default decision is made by the central bank at the time that the economy switches from phase I to phase II. [16] Let z^+ be the maximum level of seigniorage that the central bank can raise. Since the real interest rate is constant and equal to ρ, the highest level of central bank liabilities consistent with an equilibrium in phase II is

$$\rho w^{++} = z^+ = z(m^+),$$

where m^+ is the level of real money balances that is associated with the highest level of seigniorage. If w_T is greater than w^+ the central bank must default. Of course it is possible that it would choose to default even if $w_T \leq w^{++}$.

Since in this phase all relevant quantities (z_t and w_t) are stationary, the objective function of the monetary authority is to choose w' —the post-default level of liabilities— to maximize

$$\max_{w' \leq w^{++}} \left(\frac{L(\rho w')}{\rho} - \kappa \Delta(w - w') \right).$$

Let the solution to this problem be denoted $\tilde{w}(w)$. Let w^D be the smallest level of liabilities consistent with zero default. Then, w^D solves

$$L'(\rho w^D) + \kappa \Delta'(0) = 0, \tag{7}$$

which implies that $w^D < w^{++}$.

There are few restrictions imposed on w^D. To see this, note that as $\kappa \to 0$, w^D must converge to w_0, while as $\kappa \to \infty$, w^D must converge to w^{++}. To see this, note that as $\kappa \to \infty$ equation (7) requires $L'(\rho w^D)$ to converge to $-\infty$ and this, in turn, implies that w^D converges to w^{++}.

In the interior case, the higher the κ the higher w^D: Higher default costs enlarge the no-default region. Moreover, differentiation of the first-order condition of the central bank's problem shows that

$$\frac{\partial \tilde{w}}{\partial w} < 1, \quad \text{and} \quad \frac{\partial \tilde{w}}{\partial \kappa} > 0.$$

[16] This is, in some sense, extreme. It basically assumes that the costs of default are lower depending on the state of the economy. In ongoing work I allow for default at any time with a fixed cost in addition to the variable cost captured by Δ. In addition it is necessary to assume some exogenous randomness. For example, adding uncertainty on the value of κ. Let $\kappa \in \kappa_L, \kappa_H$, with κ_H so high that it is never optimal to default. The optimal policy is a stopping time of the form "stop the first time that w_t reaches $\hat{w}$ if, say, $\kappa = \kappa_L$ ". The longer the duration of phase I the more defaults observed in equilibrium.

The value of the central bank maximization problem at the start of phase II (after the default decision has been made) is given by the C^2 function function M is given by

$$M(w) = \begin{cases} \frac{L(\rho w)}{\rho} & w \leq w^D, \\ \frac{L(\rho \tilde{w}(w))}{\rho} \text{-}\kappa\Delta(w - \tilde{w}(w)) & w \geq w^D \end{cases}, \tag{8}$$

The optimal continuation policy is very simple:

- If the level of central bank liabilities is low at the time of the regime switch, that is, if $w_T \leq w^D$, there is no default at T and, consequently, no large inflation.
- If $w_T > w^D$, the optimal policy requires a partial default at T that brings the real value of the nominal liabilities of the central bank from w_T down to $\tilde{w}(w_T)$. This default is engineered by a jump in the price level. After the initial jump, inflation and real money balances are constant.

5 Optimal Policy: Phase I

In this section I study the optimal policy during the high negative cash flow phase (phase I).

5.1 "Honest "Policy

I assume that the monetary authority is constrained to pick policies that rule out jumps in the price level.[17] This implies that $P' = P$ and that the real interest rate is constant and equal to $r = \rho$.

If the central bank pursues an honest policy, this puts restrictions on how much debt it can issue during phase I. In particular, the policy requires the debt level at the time that the economy switches to phase II to be below the maximum, that is, $w_T \leq w^{++}$. To guarantee that this constraint holds, there is a highest possible value of the central bank's liability during phase I —which I denote w^+— that cannot be exceeded. This value is given by

$$\rho w^+ + x = z(m^+) = z^+.$$

If $w > w^+$ then $dw > 0$ and it is unbounded. This implies that, for sufficiently long T, w_T will exceed the maximum feasible level of liabilities consistent with no default

[17]This can obtain as the limit when κ goes to ∞ in the cost of default function.

in phase II (w^{++}). Thus, it is *not feasible* for an honest government to let the level of total liabilities exceed w^+ during the period in which net income is negative. This must be the case even though $w^+ < w^{++}$, that is, the maximum value of liabilities is strictly below the maximum sustainable without default.[18] If the central bank followed a policy that results in $w_t \in (w^+, w^{++})$ at some time t, during phase I, then such a policy has a non-zero probability of requiring a default when the economy switches to the normal, zero net income, phase.

Let $V(w)$ be the value of the central bank's problem when the state is w during phase I. The HJB equation is[19]

$$\rho V(w) = \max_z \{L(z) + V_w(w)(\rho w + x - z) + \eta [M(w) - V(w)]\}, \tag{9}$$

where, in this case, $M(w)$ is given by the first branch of the function in equation (8).

The optimal choice of seigniorage satisfies

$$L^{'}(z) = V_w(w). \tag{10}$$

The key properties of the optimal policy are summarized in the following proposition (all proofs are in the Appendix).

Proposition 1 ("Honest" Optimal Policy) *The optimal honest policy during phase I has the following properties:*

- *If $w < w^+$, then z_t is increasing, which implies that m_t is decreasing and i_t and π_t are increasing.*
- $z_t > \rho w_t$.
- *The liabilities of the central bank, w_t, are increasing and $w_t \to w^+$ (which implies $m_t \to m^+$).*

The proposition characterizes the optimal monetary policy when net income is negative. For a given initial level of liabilities, w_0,[20] the optimal policy determines an initial level of real money balances, m_0, and a path for future seigniorage. Given that

[18] Note that w^+ is independent of any additional resources that the central bank can count on in phase II. This, of course, is the consequence of market incompleteness.

[19] Existence of a solution follows from standard arguments. I assume that the solution is classical.

[20] As in any monetary model, the central bank has an incentive to engineer a large inflation at $t = 0$ to reduce w_0 as much as possible. The w_0 in the model should be viewed as whatever is the value after that initial inflation. I assume this level is small.

seigniorage is increasing, real money balances are decreasing. The nominal interest rate given by

$$\frac{L(z_t)}{\hat{m}_t} = \frac{v^{'}(\hat{m}(z_t))}{u^{'}(c)} = i_t$$

is increasing. The inflation rate is also increasing and given by

$$\pi_t = i_t - \rho.$$

The economy displays the standard Fisherian result: Inflation and nominal interest rates move one for one.

Along the optimal path, total liabilities of the central bank are increasing. Since real money balances are decreasing, the ratio of bonds to money has an upward trend, and the central bank relies increasingly on debt issues to finance the negative cash flow. Thus, along the optimal path, the central bank is actively engaged in open market operations that change the composition of its liabilities between non-interest-bearing money and interest-bearing bonds (or reverse repos).

Why is inflation increasing? When there is uncertainty about the size (or, equivalently, the duration) of the transfer, there is an option value associated with initially picking a "too low" inflation rate, in the sense that seigniorage falls short of financing needs. To see this, let $\tilde{z}(w)$ be the constant level of seigniorage that "satisfies" the financing needs. In this case $\tilde{z}(w)$ is given by

$$\tilde{z}(w) = \rho w + x,$$

and the central bank can meet its obligation and raise enough revenue to service the debt.

The previous proposition shows that the optimal policy is such that $z_t < \tilde{z}(w)$. The reason is that the central bank chooses higher real money balances (and lower interest rates and a relatively smaller debt-money ratio) taking into account that there is a positive probability —constant in the Poisson setting— that the duration of phase I will be short. As times passes, the optimal policy relies on increasing debt issues as the best option to smooth out the distortion associated with non-constant real money balances. Under the honest policy there is an upper bound on the total liabilities of the central bank. The "honest" central bank will never exceed the limit w^+. At this point, inflation is at its revenue-maximizing level and, effectively, $z_t = \tilde{z}(w^+)$

At (random) time T the central bank engages in an open market operation that reverses the previous trend. Since $z_t > \rho w_t$ a switch to phase II is accompanied by a need to raise less seigniorage. This results in lower nominal interest rates and inflation relative to the pre-jump situation. The corresponding increase in the demand for money is met by changing the mix of money and bonds — total liabilities w_T stay constant— associated with one large open market operation. The size of the decrease in the inflation rate depends on the size of the central bank's liabilities at T (this, in turn, is a function of the duration of phase I).

In terms of the scenario raised by Sargent and Wallace (1981), there is a simple answer in the case of temporary fiscal needs: If the monetary authority follows an optimal policy, it will never choose to keep inflation "too low" and plan for higher inflation in the future. The optimal policy has inflation lower than what would be required by some myopic financing rule (the $\tilde{z}(w)$ rule discussed above) but still induces a decrease in inflation in the future. The role of uncertainty is essential for this result: Uribe (2020) shows that a deterministic path for x_t will be associated with a constant level of seigniorage and inflation even though the total liabilities of the central bank will reflect the changes in the process x_t.

5.2 "Non Honest" Policy

In this section I study the general case where the continuation value is $M(w)$ (both branches in (8)). In this case, the central bank will choose to default when the economy switches to phase II if the stock of liabilities at that time is greater than or equal to w^D. Even though the level of w^{++} is independent of the features of the default penalty, the level of w^D is not. As argued above, w^D increases with the cost of default as parameterized by κ. For arbitrarily high values of κ —the case that captures the honest central bank optimal policy— there will be no default.

If $w \geq w^D$, the relevant interest rate during phase I is

$$r(w) = \rho + \eta \left(1 - \frac{\tilde{w}(w)}{w}\right) \tag{11}$$

The following proposition summarizes the results in this case:

Proposition 2 ("Non-Honest"Optimal Policy) *The optimal policy in phase I is characterized by*

- *For all levels of central bank liabilities, w, $z(w)$ is increasing in w, and the stock of debt is increasing over time ($dw > 0$). This implies that nominal interest rates and inflation are increasing as well.*

- *If $w \leq w^D$, then the real interest rate is $r_t = \rho$.*
- *If $w > w^D$, the real interest rate is $r(w) = \rho + \eta\left(1 - \frac{\tilde{w}(w)}{w}\right)$. Since $\tilde{w}(w) < w$, the real interest rate is higher than in the no-default region.*
- *Let $\kappa' \geq \kappa$ and let $z(w, \kappa')$ and $z(w, \kappa)$ be the optimal levels of seigniorage in each case. Then, $z(w, \kappa') \geq z(w, \kappa)$.*
- *There exists a w^* such that*

$$\tilde{w}'(w^*)L_z(r(w^*)w^* + x) = L_z(\rho\tilde{w}(w^*))$$

and there is a phase I steady state in the sense that as $t \to \infty \quad w_t \to w^*$.

Proof. See Appendix ■

Qualitatively, the optimal policy under default resembles the policy followed by the honest government. There are three important differences. First, the non-honest central bank will choose a lower inflation rate (for a given level of liabilities) than the honest central bank. This, in some sense, captures the intuition from the "Unpleasant Monetarist Arithmetic", since the lower inflation is made possible by the option of a large inflation in the future.

Second, an honest government would never choose a policy such that $w_t > w^+$, since this would result in a positive probability of default. On the other hand, if $w^* > w^D \geq w^+$, the non-honest central bank can afford to gamble. For $w \in (w^+, w^D)$ The interest rate stays at ρ because even if there is a switch to phase II the central bank will choose not to default.

Lastly, if $w^D < w^+$, then for $w \in (w^D, w^+)$ real interest rates are low if the central bank is honest, but they are higher if the central bank is not. The reason is that, if given the option (i.e., a switch to phase II), the non-honest central bank will choose to inflate. The higher real interest rate reflects this possibility.

As in the honest policy case, the seigniorage raised by the non-honest central bank falls short of the "financing needs" defined as the value of x plus the interest on the existing liabilities. This implies that liabilities increase over time, and so do interest rates. Qualitatively, this is similar to the optimal policy of the honest central bank.

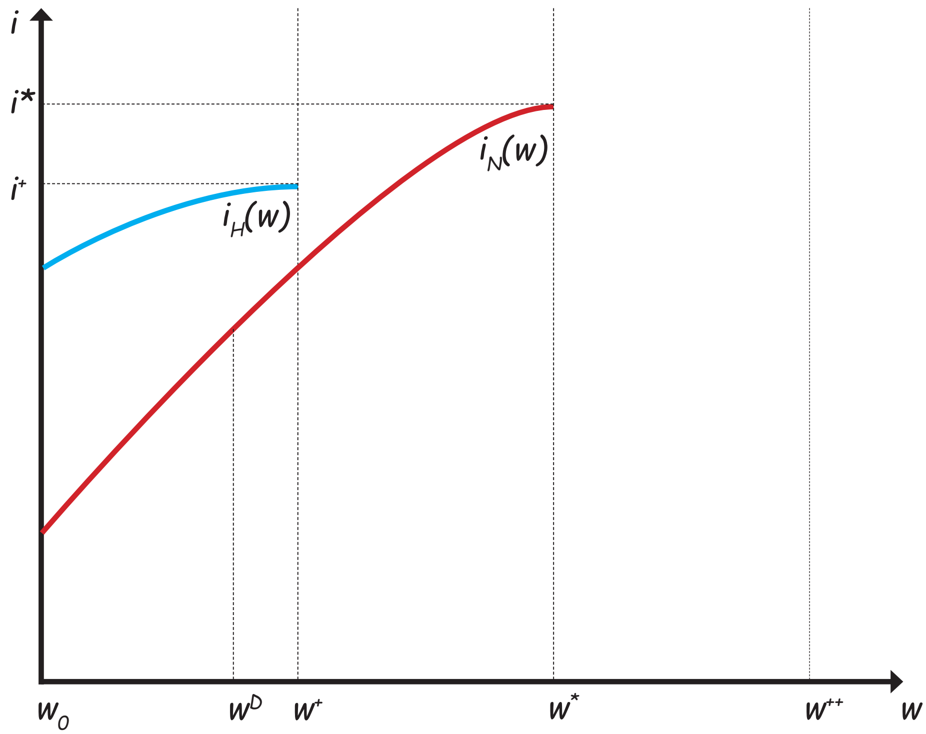

Figure 1: Optimal Policies. $i_H(w)$ (Blue = "Honest") and $i_N(w)$ (Red = "Non Honest")

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Figure 1 displays two optimal functions in (i, w) space. The policy of increasing z and w corresponds to an increase in the nominal interest rate (and inflation), i. It assumes that $w^* < w^+$. The blue locus —the function $i_H(w)$— corresponds to the honest central bank. Along the equilibrium path the movement is in the northeast direction: liabilities are increasing, real money balances are decreasing, and nominal interest rates and inflation increase.

The red locus displays the optimal interest policy for a non-honest government. For any level of liabilities, the nominal interest rate associated with the non-honest policy is lower than the corresponding level for the honest policy. This implies lower inflation.

At this level of generality it is not possible to determine the changes in the real interest rate as w increases. In the limit (when there is no steady state), $r(w) \to \rho + \eta$ as $w \to \infty$.

6 Extensions

In this section I discuss some extensions of the simple model. Since the possibility of default simply shifts the levels but does not have important quantitative effects, I concentrate on the "honest" government case.

6.1 Multiple Phases

How does the optimal policy change when the decrease (in absolute value) of the negative cash flow is gradual? Is it the case that under the optimal policy the nominal interest rate displays a monotonically decreasing path?

To discuss this case I allow for two phases of negative cash flow.[21]

I restrict the presentation to just two levels of negative income, but it is relatively easy to see how to extend the analysis to any finite number of rounds. As before, net income is $-x_t$, where x_t is given by

$$x_t = \begin{cases} x_0 & t \in [0, T_0] \\ x_1 & t \in [T_0, T_1], \\ 0 & t \geq T_1, \end{cases}$$

where $x_1 < x_0$ and, as before, I maintain the Poisson assumption with parameters η_0 and η_1. Thus, the environment is such that an initially high level of transfers (of uncertain duration) will be followed by a lower level (also of uncertain duration) and then a return to the normal (zero) net cash flow.

Let $V_j(w)$ be the central bank's value function when $x_t = x_j$. Then $V_1(w)$ coincides with the value function described in the case of the honest policy with just one level of x (if $x = x_1$). In each phase there is a highest level of liabilities consistent with no jumps in the price level. They are given by w_0^1 and w_1^1 given by

$$\rho w_1^+ = z(m^+) - x_0 < \rho w_2^+ = z(m^+) - x_1.$$

The relevant HJB equation when $x = x_0$ is

$$\rho V_0(w) = \max_z \left\{ L(z) + V_0'(w) \left(\rho w + x_0 - z \right) + \eta_0 \left[V_1(w) - V_0(w) \right] \right\}. \tag{12}$$

[21] In this case there are two instances, the times T_0 and T_1 that, in principle are candidates for a default. This slightly complicates the analysis but does not add much to the stylized properties of optimal policies.

The same arguments that we used before show that the function V_0 is concave and that the level of liabilities is increasing over time. If it is C^2 taking the derivative on both sides of equation (13) one gets that

$$\eta_0 \left[V_0^{'}(w) - V_1^{'}(w)\right] = V_0^{''}(w)\left(\rho w + x_0 - z\right), \tag{13}$$

The right side of equation (13) is negative given the concavity of V_0 and that dw is positive. Thus,

$$V_1^{'}(w) = L^{'}(z_1(w)) \; > \; V_0^{'}(w) = L^{'}(z_0(w)),$$

which given the concavity of the L function implies that $z_0(w)$ is greater than $z_1(w)$. Consequently, for the same level of liabilities, inflation is lower. However, there are paths for the time series of inflation that display higher inflation after T_0 than the max before T_0. This can happen if, for example, $T_0 \ll T_1$ and w_t is close to w_2^+. In this case, inflation is at the highest level in the low negative cash flow stage.

$$m_t \to m^+ \text{ as } w_t \to w_2^+$$

Figure 2 shows one such path. The function $i_J(w)$ corresponds to the nominal interest rate when $x - x_j$ for $j \in 0, 1$. In the Figure, w_S is the level of central bank liabilities corresponding to $t = T_0$.[22] The blue path corresponds to the initial phase (high x), and the green path to the second phase.

It also shows one possible realization (in dotted red) that assumes that the first phase ends when $w_t = w_S = w_{T_0}$ and the second phase ends when $w_t \to w_2^+$.

[22]Recall that since the solution is on the efficient side of the Laffer curve, higher z corresponds to lower m and higher π.

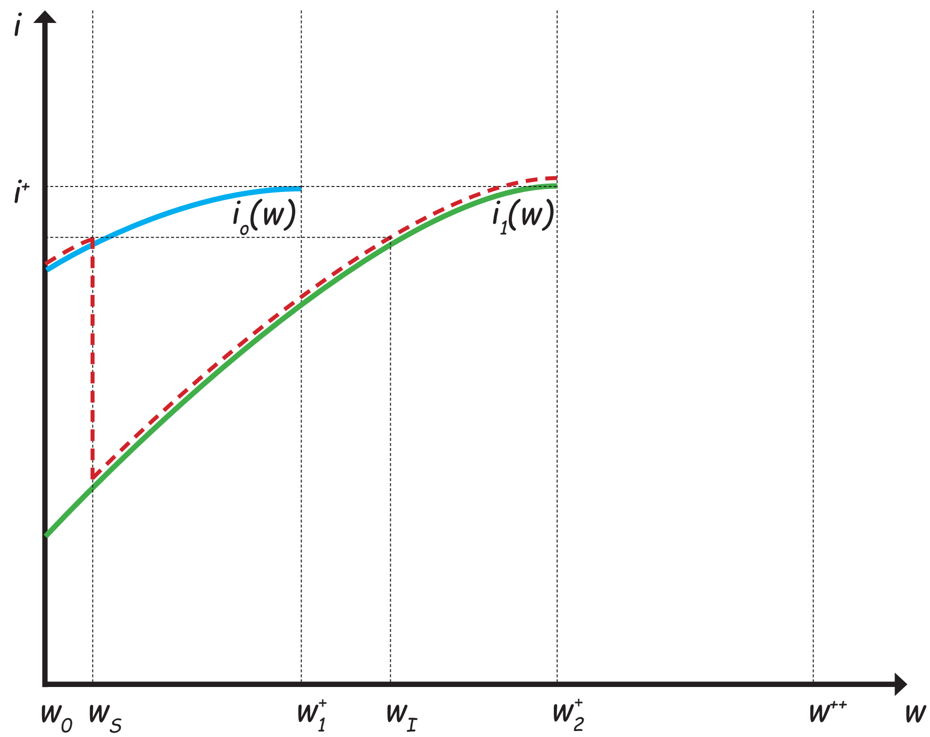

Figure 2: "Honest" Policy with Multiple Phases

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During the first phase, interest rates and total liabilities increase until time T_0. Right before the switch from the $-x_0$ to the $-x_1$ regime, the nominal interest rate is $i_0(w_S)$. At $t = T_0$, the central bank —responding to the good news that the level of transfers has decreased— performs an open market operation that increases real money balances (decreases nominal interest rates) from $i_0(w_S)$ to $i_1(w_S)$. This is associated with a decrease in the bonds-to-money ratio. From then on, the path follows the green locus. If the switch to the long-run state (that is, $x = 0$) happens after the central bank liabilities exceed w_I, then a lower level of transfers (recall that, at this point, the transfer is $x_1 < x_0$) is associated with higher interest rates and inflation.

It is interesting to note that even though net income is in a very clear sense weakly increasing (transfers are weakly decreasing), inflation and interest rates do not display a monotone path.

As in the case of a single stage, the optimal policy initially displays low nominal interest rates and low inflation (relative to full financing). As liabilities increase, the monetary authority chooses a higher nominal interest policy as well as a higher ratio of interest-bearing liabilities to non-interest-bearing money.

6.2 Adjusting Tax Policy

The basic model assumes that the amount of resources transferred by the central bank has no impact on equilibrium consumption. This corresponds to the case in which the fiscal authority has access to lump-sum taxes.[23] A more realistic view is that the transfers of the central bank reduce the need for distortionary taxes; and, hence, as the effective amount of transfers increases, so does equilibrium consumption.

It is useful to consider a new state variable, q_t, that measures the effective amount of resources transferred from the monetary authority to the fiscal authority. The law of motion of q is

$$\dot{q}_t = -\delta q_t + x_t, \quad \text{and} \quad 0 < \delta < 1$$

The reason for the "depreciation" factor δ is that, in its absence, the stock of transfers could grow without bound. In a growing economy δ is just the negative of the growth rate.[24]

One simple way of modeling consumption is to assume that[25]

$$c_t = c(q_t) = c + \rho q_t$$

In this section I consider only the "honest" government case since the option with default is similar. To simplify the presentation I assume that the utility function is of the constant relative risk aversion variety with

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}$$

The real interest rate is

[23] This is also the case when the monetary authority is transferring resources to the private sector directly.

[24] It is possible to dispense with δ and impose a cap on the total value of the transfer. This will make the model more non-stationary with little gain in insight

[25] In the basic version $c = y - g$. This formulation captures the idea that higher transfers reduce distortionary taxes which, in tune, increase consumption by an amount equal to the long run real interest rate (ρ) times the stock of transfers (q_t).

$$r(q) = \rho \left[1 + \theta \frac{x - \delta q}{c + \rho q} \right].$$

Since $lim_{t\to\infty} q_t = \frac{x}{\delta}$ it follows that $r(q)$ is a decreasing function of q and, as $t \to \infty$, $q_t \to \rho$. The reason for the high real interest rate is standard: future consumption is higher than present consumption; hence, the interest rate has to exceed the discount factor.

Note that, in this version, the demand for money depends on the nominal interest rate and q. Formally $m(i, q)$ solves

$$\frac{v'(m(i, q))}{u'(c(q))} = i.$$

The demand for money decreases with the nominal interest rate and it increases with q. This last effect is driven by the income or transaction effect.

Let z be, as before, the real value of seigniorage. Then, let $\hat{m}(zu'(c(q))$ be the level of real money balances consistent with raising z. Recall that

$$z = i\hat{m}(zu'(c(q)) = \frac{v'(\hat{m}(zu'(c(q)))\hat{m}(zu'(c(q))}{u'(c(q))}.$$

As shown before $\hat{m}(zu'(c(q)))$ is the value of m that satisfies

$$zu'(c(q)) = \hat{m}v'(\hat{m}) = h(\hat{m})$$

As before, it is convenient to work backwards and describe the optimal policy during phase II. In this phase the interest rate is constant (equal to ρ) and consumption is constant as well (given by $c(q)$. In this case the optimal policy is fairly simple: it is optimal to raise a constant amount of seigniorage to service the debt. Let T be the random time at which the positive x transfer ends. Then, $z_t^{II} = \rho w_T$ for all $t \geq T$.

The continuation utility, $M(w, q)$, is given by

$$M(w, q) = \frac{v(\hat{m}(\rho w u'(c(q_T))))}{\rho}.$$

Let's now discuss the problem faced by the central bank in phase I. The relevant payoff in terms of seigniorage

$$L(z, q) = v(\hat{m}(zu'(c(q))) \quad \text{and} \quad L_z(z, q) = \frac{v'(\hat{m})}{h'(\hat{m})} u'(c(q)) < 0$$

The key difference with the basic model is that the real interest rate is decreasing over time as increases in the value of the resources transferred allowed the fiscal authority to reduce distortionary taxes; hence, this increases consumption.

The following proposition summarizes the basic results for this version of the model (proof in the Appendix):

Proposition 3 *The optimal policy $z^*(w_t, q_t)$, for $t < T$, has the following properties:*

- $\dot{z} = \frac{\partial z^*}{\partial w}\dot{w} + \frac{\partial z^*}{\partial q}\dot{q} > 0$
- $z^*(w, q) > \rho w$
- $\dot{w} = r(q)w + x - z^*(w, q) > 0.$
- *The nominal interest rate i_t is increasing over time. If the elasticity of $v'(m)$ is —in absolute value— less than 1, then inflation is increasing over time.*

Since $z^*(w, q) > \rho w$, the required level of seigniorage drops at time T. This implies an increase in real money balances and a decrease of the nominal interest rate. Since the real interest rate drops as well, the impact on inflation is ambiguous. However, if T is sufficiently large, $\mid (q_T) - \rho \mid < \epsilon$ for any ϵ, and the change in real rates is arbitrarily small, while the change in nominal rates is much larger. In this case —and this is just an extreme set of conditions— inflation decreases.

As in the base case, nominal interest rates and seigniorage increase over time during phase I. Since real rates decrease, this results in increases in inflation that exceed the increase in nominal rates. This result suggests that inflation will be higher in this case. The same option value of waiting is at work in this case. Liabilities increase over time.

Overall, this alternative view of fiscal dominance in which the transfer from the central bank frees up treasury resources with the consequent impact on interest rates and out has the same qualitative properties as the simple model.

6.3 More Traditionally Unpleasant Monetarist Arithmetic

The basic model assumes that, when the economy switches to phase II, the level of liabilities of the central bank cannot exceed what can be financed by the maximum level of seigniorage, w^+. This choice is, in some sense, arbitrary. If the monetary authority decided that the maximum long-run inflation associated with w^+ is excessive, it can set a lower limit, $w^* < w^+$, not to be exceeded in phase I.[26]

What are the consequences of this choice? Put simply, the lower debt limit results in lower inflation in phase I.

To see this, let $V(w; w^*)$ be the value function of the problem when the debt limit is $w^* < w^+$. Then totally differentiating the HJB equation with respect to w^* and imposing the envelope condition one gets that

$$(\rho + \eta)\frac{\partial V(w; w^*)}{\partial w^*} = \frac{\partial^2 V(w; w^*)}{\partial w \partial w^*}\left[\rho w + x - z\right]$$

Since increasing w^* relaxes the constraints faced by the central bank, it follows that $\partial V(w; w^*)/\partial w^* > 0$. The same argument as in Proposition 1 shows that $\rho w + x - z$ is positive. Thus, $\partial^2 V(w; w^*)/\partial w \partial w^* > 0$. Since

$$L^{'}(z(w; w^*) = \frac{\partial V(w; w^*)}{\partial w}$$

and $L^{''} < 0$, differentiating both sides with respect to w^* shows that $\partial z/\partial w^* < 0$. Tighter limits result in lower inflation in both the short and long run.

6.4 A Central Bank with Resources in Phase II

So far, I have assumed that the central bank has no resources of its own and it does not have fiscal support. In this section I extend the model to allow for some positive income in phase II. There are different possible models that are consistent with this stylized view of resources. One possibility is the case of a central bank that purchases assets in phase I (and hence the negative cash flow) and receives earnings in phase II.[27] An alternative view is that the fiscal authority commits to transferring resources when phase I ends.

[26] In this model it is trivial to show that the optimal w^* is w^+. The argument is simple: Choosing any level of maximum liabilities below the highest feasible adds a constraint to the problem and, hence, lowers the value.

[27] For example, one can view phase I as a "crises" period in which the central bank is purchasing assets (net negative flow) and phase II as a normal period in which the "investments" in assets have a positive payoff. To be more realistic, the central bank has to have some expectation of the size of b which is probably not known during phase I. To the extent that the certainty equivalence of that expectation is positive the same results obtain.

A simple way of modeling this is that if w_T are the liabilities of the central bank at the end of phase I, it receives a "gift" from the treasury that lowers this level to $w_T - b$. Using exactly the same reasoning, it follows that $\partial z/\partial b < 0$.

The same results apply to a central bank that in phase II finds itself earning interest on assets. If $b - w_T > 0$, the optimal policy calls for $i = \rho$ and a transfer to the fiscal authority of $\tau = \rho(b - w_T)$. If If $b - w_T < 0$, then $Z^{II} = \rho(w_T - b)$ and $i > \rho$. [28]

The key insight is that w^+ is independent of b. Thus, for an honest central bank, this constraint on the maximum level of liabilities is binding. The impact of higher resources (higher b) is positive in the sense that it decreases the optimal interest rate in phase I; but, if phase I lasts long enough, the interest rate (and inflation) are independent of b.

This shows the significant difference between resources in good times (phase II) and the option to inflate away the debt. The latter provides the central bank with more flexibility.

7 Concluding Comments

What are the lessons for central banks that could find themselves in a situation in which net income is temporarily negative (deficit) and there is no forthcoming fiscal support? The first lesson is that having the possibility to issue non-interest-earning money (or reserves)[29] as well as other liabilities that earn the market rate of return —reverse repos in the case of the Fed and also some proposals sympathetic to the possibility that the ECB will issue Eurobonds (see, for example, Corsetti et. al. (2016))— is an essential component of the optimal policy mix.

The results show that there is no simple one-to-one correspondence between the negative cash flows and interest rates and that it is always optimal (in phase I) to collect a level of seigniorage that falls short of the cash flow plus interest on existing debt. Thus, during the period of deficits, the optimal policy calls for continuing (and increasing) indebtedness by the central bank.

The intuition for this result is that uncertainty about the duration of the negative cash flow phase creates an option value for keeping inflation low (raising less revenue than necessary and borrowing the difference) since this allows for an optimal

[28]In the Appendix I show that $\partial z/\partial b < 0$.

[29]As discussed before all that is necessary is that this special liability earn less than the market return.

smoothing of the distortions associated with raising seigniorage.

The optimal policy viewed as an interest rate policy does not resemble a Taylor rule. In the case of the honest policy (no surprise inflation), the nominal interest rate is an increasing function of the real value of the total liabilities of the central bank. An outside observer just looking at the data will find a positive relationship between inflation and nominal interest rates. In the case of the honest policy, the model implies that $i = \rho + \pi$ so that the coefficient on the observed policy rule is one.

One interesting finding —mostly in the context of stabilization plans— is that gradual decreases in central bank financed transfers need not be accompanied by steady decreases in inflation. There are realizations of the relevant stochastic process (the timing of reductions) that display a sew-saw pattern of inflation over time.

The "non-honest" optimal policy —one that allows the central bank to engineer large, surprise inflations— implies a lower (relative to the honest policy) nominal interest rate —for a given level of liabilities— and a larger level of maximum central bank debt. Any shock that lowers the social cost of default is associated with decreases in current inflation at the expense of possible larger jumps in the price level when phase II arrives.

Finally, a central bank that has access to other forms of genuine resources in phase II may be able to follow its preferred monetary rule during early stages of phase I. In this case there is no conflict between current (potentially non-optimal) and optimal policies. However, if this phase lasts a sufficiently long period, then the optimal policy will require that interest rates increase and additional seigniorage be raised.

8 Appendix

8.1 An Alternative Formulation of the Penalty Function

Assume that the cost of default is a disruption of production that results in consumption being temporarily lower.

Let post-default consumption be $y-\theta\Gamma(w-\tilde{w}(w))$, and assume that with expected duration $1/\alpha$ the economy goes back to normal.

The continuation utility of the central bank is then

$$\frac{1}{\rho}\left(u(y)+L(z(\tilde{w}))\right)+\frac{1}{\rho+\alpha}\left[u(y-\theta\Gamma(w-\tilde{w}))-u(y)\right]$$

The term

$$\frac{1}{\rho+\alpha}\left[u(y-\theta\Gamma(w-\tilde{w}))-u(y)\right],$$

corresponds to

$$-\kappa\Delta(w-\tilde{w})$$

In this case the real interest rate during phase I has to be adjusted to take into account the effect of the loss of consumption. The appropriate rate is

$$r=\rho+\eta\left[1-\frac{u'(y-\theta\Gamma(w-\tilde{w}))}{u'(y)}\frac{\tilde{w}(w)}{w}\right]$$

Since

$$\frac{u'(y-\theta\Gamma(w-\tilde{w}))}{u'(y)}>1$$

the real interest rate is lower than in the base case (for a given level of default).

8.2 Proofs

This section includes the proofs of Propositions 1, 2, and 3.[30]

Proof. Proposition 1 . The optimal policy problem is the HJB equation corresponding to the optimal policy in phase I under the restriction to a continuous path of the price level is

$$V(w) = \max_{z_t} E\left[\int_0^T e^{-\rho t} L(z_t)dt + e^{-\rho T} M(w_T)\right],$$

subject to

$$dw_t = (\rho w_t + x - z_t)\, dt, \text{ with } w_0 = w$$

where the expectation is taken over the realization of T.

Since $L(z)$ is concave and the constraint linear, the function $V(w)$ is concave.

The HJB equation associated with this problem is

$$\rho V(w) = \max_z \{L(z) + V_w(w)(\rho w + x - z) + \eta [M(w) - V(w)]\}$$

where, in this case, $M(w)$ is given by the first branch of the function in equation (8).

The first-order condition defines $z(w)$ by

$$L'(z(w)) = V_w < 0. \tag{14}$$

Concavity of V and L implies that $z(w)$ is increasing in w.

Next we show that $\dot{w} > 0$. To see this, assume to the contrary that $z \geq \rho w + x$. Totally differentiating the HJB equation and imposing the envelope condition, we get that

$$\eta V_w - M_w = V_{ww}\dot{w} \tag{15}$$

A standard calculation shows that

$$M_w = L'(\rho w).$$

Under the assumption that $z \geq \rho w + x$, we get that $V_w = L'(z) < M_w = L'(\rho w)$. Thus, the left-hand side of (15) is negative. Since strict concavity implies that $V_{ww} < 0$, it follows that $\dot{w} > 0$.

[30] To streamline the presentation, the proofs assume more differentiability than needed. The arguments go through by carefully taking differences and imposing the right bounds

The same argument shows that $z > \rho w$ as the left-hand side of (15) must be negative since we just proved that the right-hand side is negative.

To show that z is decreasing in η, let $V(w;\eta)$ be the value of the problem. Assume to the contrary that $\eta' > \eta$ and that $z(w,\eta') < z(w,\eta)$ in a set of positive measure. This will lead to a contradiction.

Given the assumption, the first-order condition implies that $V'(w;\eta') < V'(w,\eta)$. In any interval where this holds, there must be a lowest value for which it does not. Let this value be denoted $\tilde{w}$. It follows that $V'(\tilde{w},\eta') = V'(\tilde{w},\eta)$ and hence that $z(\tilde{w},\eta') = z(\tilde{w},\eta)$. Note that if there is no such $\tilde{w}$, let $\tilde{w} = w^1$ and it is easy to check that the argument goes through.

Since $V'(w;\eta') < V'(w,\eta)$, integrating both sides from any $w < \tilde{w}$ we get that

$$V(\tilde{w},\eta') - V(w,\eta') < V(\tilde{w},\eta) - V(w,\eta).$$

Since at $w = \tilde{w}$, we have that

$$\begin{aligned}(\rho+\eta')V(\tilde{w};\eta') - \eta' M(\tilde{w}) &= L(z(\tilde{w};\eta')) + V'(\tilde{w};\eta')[\rho\tilde{w} + x - z(\tilde{w};\eta')] \\ &= L(z(\tilde{w};\eta)) + V'(\tilde{w};\eta)[\rho\tilde{w} + x - z(\tilde{w};\eta)] \\ &= (\rho+\eta)V(\tilde{w};\eta) - \eta M(\tilde{w}).\end{aligned}$$

Using these two results it follows that, for all $w < \tilde{w}$,

$$(\rho+\eta')V(w;\eta') - \eta' M(w) > (\rho+\eta)V(w;\eta) - \eta M(w).$$

I now show that this leads to a contradiction. To see this, rewrite the previous condition as

$$L(z(w;\eta') + V'(w;\eta')[\rho w + x - z(w;\eta')] > L(z(w;\eta) + V'(w;\eta)[\rho w + x - z(w;\eta)].$$

Since $L(z)$ is concave, we have that

$$L(z(w;\eta')) \leq L(z(w;\eta)) + L'(z(w;\eta))(z(w;\eta') - z(w;\eta).$$

Using this in the previous inequality and since $L'(z(w;\eta)) = V'(w;\eta)$, we get that strict inequality is equivalent to

$$V'(w;\eta)z(w,\eta') + V'(w;\eta')[\rho w + x - z(w;\eta')] > V'(w;\eta)[\rho w + x]$$

or,

$$0 > (V'(w;\eta) - V'(w;\eta'))[\rho w + x - z(w;\eta')]$$

which leads to a contradiction since, under the assumption, $V'(w;\eta) - V'(w;\eta') > 0$; and we showed already that, under the optimal policy, $\rho w + x - z(w;\eta')$.

We now show convergence of w_t to w^+. Since $dw > 0$, then the process w_t either converges to some $w^* < w^+$ or it converges to w^+. Consider the value of the program at w^*. Since $dw = 0$ it follows that

$$V(w^*) = \frac{\eta M(w^*) + L(\rho w^* + x)}{\rho + \eta}$$

Differentiating the $V(w^*)$ function it follows that

$$V'(w^*) = \frac{\eta L'(\rho w^*) + \rho L'(\rho w^* + x)}{\rho + \eta} > L'(\rho w^* + x)$$

which implies that $z(w^*) = \rho w^* + x$ (as required by the convergence assumption) violates the first-order condition. Since $V'(w^*) > L'(\rho w^* + x)$, the optimal z is strictly less than $\rho w^* + x$, which implies that, at $w = w^*$, $dw > 0$. ■

Proof. Proposition 2. The main difference between the problem in 1 is that the interest rate depends on the stock of liabilities. Let the Hamiltonian for this problem be

$$H(z, w, \lambda) = L(z) + \eta M(w, \kappa) + \lambda(r(w)w + x - z),$$

where the function $r(w)w$ is given by

$$r(w)w = \begin{cases} \rho & w \leq w^D \\ (\rho + \eta)w - \eta\tilde{w}(w) & w \geq w^D \end{cases}$$

The function is continuous but it has a kink at $w = w^D$. This requires stronger concavity properties on the L function to guarantee that the Arrow-Kurz condition holds and the maximized Hamiltonian is concave in w. In this case, the first-order conditions are sufficient for a maximum.

The optimal choice of z satisfies

$$L'(z(w, \kappa)) = \lambda = V_w(w, \kappa).$$

The dynamics of the marginal cost of central bank debt are given by

$$\dot{\lambda} = (\rho + \eta)\lambda - \left[\eta M_w + \lambda \frac{dr(w)w}{dw}\right].$$

The derivative M_w is given by

$$M_w = L'(\rho\tilde{w}(w)).$$

There is no steady state if $w < w^D$. To see this, suppose to the contrary that there is a steady state at $w = \hat{w} < w^D$. Then setting $\dot{\lambda} = 0$, I get,

$$L'(\rho\hat{w} + x) = \frac{\eta}{\rho + \eta} L'(\rho\hat{w}),$$

but this violates strict concavity of L, which implies that $L'(\rho\hat{w} + x) < L'(\rho\hat{w})$.

I now show that, under the condition in the proposition, there is a steady state at $w = w^* > w^D$. As before, set $\dot{\lambda} = 0$ and note that

$$\frac{d(r(w)w)}{dw} = (\rho + \eta) - \eta\tilde{w}'(w).$$

Thus, the steady-state condition is

$$\tilde{w}'(w^*)L'(r(w^*)w^* + x) = L'(\rho\tilde{w}(w^*)).$$

This steady state always exist. To see this, note that $\tilde{w}(w) \leq w^{++}$. Thus, as w grows without bound, the right-hand side is bounded. The term $\tilde{w}'(w)$ is converging to 0 as $L'(\rho w) \to -\infty$ as $w \to w^{++}$. Thus, there is always a steady state in Phase I.

For $w < w^*$, $\dot{\lambda} < 0$. Thus,

$$\left(\rho + \eta - \frac{dr(w)w}{dw}\right)\lambda < \eta M_w$$

.Thus,

$$\left(\rho + \eta - \frac{dr(w)w}{dw}\right)V_w - \eta M_w < 0.$$

I now show that this implies that $\dot{w} > 0$.

The HJB equation for this problem is

$$\rho V(w;\kappa) = \max_z \left\{L(z) + V_w(w;\kappa)\right)\left(r(w)w + x - z\right) + \eta\left[M(w;\kappa)) - V(w;\kappa))\right]\right\}.$$

Totally differentiating with respect to w (and imposing the envelope condition, we get that

$$\left(\rho + \eta - \frac{dr(w)w}{dw}\right) V_w - \eta M_w = V_{ww}\dot{w}.$$

Since we showed that the left-hand side is negative and $V_{ww} < 0$, it follows that $\dot{w} > 0$.

The argument that shows that $z(w;\kappa)$ is increasing in w and that $z(w;\kappa) > \rho\tilde{w}(w)$ mimics the same derivation as in proposition 1.

Finally, I show that if $\kappa' \geq \kappa$ then $z(w;\kappa') \geq z(w;\kappa')$. To see this, assume that for some small interval of time $(t, t^*(\delta))$, where $t^*(\delta) = minT, t+\delta$, for an arbitrarily small δ, the claim is not true. Formally, assume that in this interval $z(w_s;\kappa') < z(w_s;\kappa)$ for $s \in (t, t^*(\delta))$. For $s \geq t^*(\delta)$, $z(w_s;\kappa') = z(w_s;\kappa)$. It follows then that

$$\begin{aligned} V(w_t,\kappa') - V(w_t,\kappa) > & E\left\{\int_t^{t^*(\delta)} e^{-(\rho+\eta)(s-t)}[(L(z(w_s;\kappa') + \eta M(w_s;\kappa') \right. \\ & - L(z(w_s;\kappa) + \eta M(w_s;\kappa)]ds. \\ & \left. + e^{-(\rho+\eta)(t^*(\delta)-t)}[V(w_{t^*(\delta)}(\kappa'),\kappa) - V(w_{t^*(\delta)}(\kappa),\kappa)]\right\} \end{aligned}$$

Here $w_{t^*(\delta)}(\kappa')$ is the level of liabilities at time $t^*(\delta)$ under the alternative z policy. It follows that $lim_{\delta\to 0}[w_{t^*(\delta)}(\kappa') - w_{t^*(\delta)}(\kappa)] = 0$.

Thus, by taking limits as $\delta \to 0$ we have that (under the assumption that $z(w_s;\kappa') < z(w_s;\kappa)$)

$$V(w_t,\kappa') - V(w_t,\kappa) \geq \eta[M(w_t;\kappa') - M(w_t;\kappa)].$$

It is convenient to consider small changes in κ and use the differentiable version of the previous inequality. Thus, we showed that

$$V_\kappa(w;\kappa) - \eta M_\kappa(w;\kappa) > 0.$$

Totally differentiating the HJB equation with respect to κ we get that

$$(\rho + \eta)V_\kappa - \eta M_\kappa = V_{w\kappa}\dot{w}.$$

Since the left-hand side is positive and $\dot{w} > 0$, then $V_{w\kappa} > 0$. The first-order condition for the optimal choice of z is

$$L'(z(w;\kappa)) = V_w(w;\kappa).$$

Differentiating both sides with respect to κ we get that $\partial z/\partial\kappa > 0$. ■

Proof. Proposition 3. The relevant state vector for this problem is (w, q). The HJB equation is

$$\rho V(w,q) = \max_z \left\{ L(z) + V_w(w,q)) \left(r(q)w + x - z\right) + V_q(w,q)(-\delta q + x) + \eta \left[M(w,q) - V(w,q)\right] \right\}.$$

The associated Hamiltonian is

$$H = L(z,q) + \eta M(w,q) + \lambda_w[r(q)w + x - z] + \lambda_q[-\delta q + x] + \mu[w^+(q) - w],$$

where $w^+(q)$ is the maximum level of liabilities consistent with no default in phase I. As $q_t \to q^* = w/\delta\ w^+(q) \to w^+$.

The Hamiltonian satisfies the Arrow-Kurz condition and the first-order conditions are necessary and sufficient. The conditions are

$$L_z(z,q) = \lambda_w$$

$$\dot{\lambda}_w = (\rho + \eta)\lambda_w - [\eta M_w + \lambda_w r(q) - \mu].$$

Note that there is a steady state.[31] The co-state variable $\lambda_w(t)$ is decreasing and it converges to $-\infty$.[32] In the limit,

$$L_z(z^+, \rho) + \mu = L_z(\rho w^+, \rho)$$

I next show that $z(w,q)$ is increasing over time. The first-order condition associated with the Hamiltonian dynamics and $\dot{\lambda}_w < 0$ implies

$$(\rho + \eta - r(q))L_z(z^*(w,q),q) - \eta M_w < 0.$$

Totally differentiating the HJB equation with respect to w, we get that (using the condition that $L_z = V_w$)

$$(\rho + \eta - r(q))L_z(z^*(w,q),q) - \eta M_w = V_{ww}\dot{w} + V_{wq}\dot{q} < 0.$$

[31] I will show later that it cannot hit the lower boundary in finite time.

[32] Formally L_z is converging to $-\infty$ while the Lagrange multiplier μ is converging to ∞ so that the sum remains bounded. An alternative restriction that eliminates this issue is to restrict m^+ so that it falls short of the maximum of the Laffer curve. With this modification, the same arguments apply but all limits are finite.

From the first-order condition

$$L_z(z, q) = V_w$$

it follows that

$$L_{zz}\dot{z} + L_{zq}\dot{q} = V_{ww}\dot{w} + V_{wq}\dot{q} < 0.$$

Since $L_{zz} < 0, L_{zq} > 0$, and $\dot{q} > 0$, it follows that $\dot{z} > 0$.

Next we show that $z(w, q) > \rho w$. Since $r(q) > \rho$, $\rho + \eta - r(q) < \eta$. Thus, since $L_z < 0$,

$$\eta L_z < (\rho + \eta - r(q))L_z < M_w.$$

Since

$$M_w = L_z(\rho w, q)$$

we have that

$$L_z(z^*(w, q), q) < L_z(\rho w, q),$$

which implies $z^*(w, q) > \rho w$.

In order to show that inflation is increasing over time, it is sufficient that the elasticity of the marginal utility of money be, in absolute value, less than one. We impose this condition at this point. Let inflation at time t be

$$\pi_t = i_t - r(q_t) = \frac{v'(\hat{m}(z_t u'(c(q_t))))}{u'(c(q_t))} - r(q_t).$$

Differentiating this expression with respect to time I get that

$$\dot{\pi}_t = \frac{v''(\hat{m})}{h'(\hat{m})}\dot{z}_t - r'(q_t)\dot{q}_t - \frac{\rho u''(c(q_t))}{(u'(c(q_t))^2}\left[\frac{v''(\hat{m})}{h'(\hat{m})} - v'(\hat{m})\right]\dot{q}_t.$$

The first two terms are positive and the condition on the elasticity of the marginal utility of money guarantees that the last term is as well.

Next I show that $\dot{w} > 0$. Assume to the contrary that $\dot{w} > 0$. Since we showed that

$$V_{ww}\dot{w} + V_{qw}\dot{q} < 0,$$

This implies that $V_{qw} > 0$. Let's consider the second derivative of w. It is given by

$$\ddot{w} = r'(q)w\dot{q} + r(q)\dot{w} - \dot{z} < 0.$$

This implies that w decreases. It either converges to some level w_L or it hits zero (a lower bound) in finite time.

I now argue that it cannot converge to w_L. To see this, suppose to the contrary that it converges. Then,

$$0 = r(q_t)w_L + x - z_t,$$

and taking the time derivative on both sides I get,

$$r'(q_t)\dot{q}_t - \dot{z}_t = 0,$$

which is a contradiction, as both terms are negative.

Thus, it must be the case that $w_L = 0$. However, in this case we showed that

$$\dot{w} = x - z(0, q) > 0,$$

which is a contradiction. Thus, it follows that $\dot{w} > 0$.

■

Claim 1 *$z^*(w, b)$ is decreasing in b.*

Proof. Let $z^*(w, b)$ be the optimal seigniorage policy. Let $b' \geq b$. Then it follows that

$$V(w_t, b') \geq \int_t^\infty e^{-(\rho+\eta)(s-t)}[L(z^*(w_s, b) + \eta M(w_s - b')]ds.$$

Thus,

$$V(w_t, b') - V(w_t, b) \geq \int_t^\infty e^{-(\rho+\eta)(s-t)}\eta[M(w_s - b') - M(w_s - b)]ds.$$

Since $M(w_s - b') - M(w_s - b)$ is increasing in w_s (this depends on $b' > b$) and since $\dot{w} > 0$, it follows that

$$\int_t^\infty e^{-(\rho+\eta)(s-t)}\eta[M(w_s - b') - M(w_s - b)]ds \geq \frac{M(w_t - b') - M(w_t - b)}{\rho + \eta}.$$

Putting these results together I get

$$(\rho+\eta)(V(w_t,b')-V(w_t,b))-\eta(M(w_t-b')-M(w_t-b))\geq 0.$$

As before, it is convenient to work with the differentiable version of that inequality. I then use

$$(\rho+\eta)(V_b(w_t,b)+\eta(M_b(w_t-b)\geq 0$$

Differentiating the HJB equation I get

$$(\rho+\eta)(V_b(w_t,b)+\eta(M_b(w_t-b)=V_{wb}\dot{w},$$

which implies that $V_{wb}\geq 0$. From the first-order condition for the optimal choice of seigniorage we get that

$$L'(z^*(w,b))\equiv V_w.$$

Differentiating both sides with respect to b I get

$$L''(z^*(w,b))\frac{\partial z^*(w,b)}{\partial b}=V_{wb},$$

which proves

$$\frac{\partial z^*(w,b)}{\partial b}<0.$$

■

References

[1] Aiyagari, S. Rao, A. Marcet, T. Sargent and J Seppala, 2002, "Optimal Taxation Without State Contingent Debt", *Journal of Political Economy*, Vol 110, No. 6, December, pp: 1220-1254.

[2] Auernheimer, L., 1974, "The Honest Government's Guide to the Revenue from the Creation of Money," *Journal of Political Economy*, Vol. 82, No. 3, pp. 598-606.

[3] Barro, Robert, 1979, "On the Determination of the Public Debt." *Journal of Political Economy,* 87(6): 940-971.

[4] Benigno, P. and S. Nisticò, 2015, "Non-Neutrality of Open Market Operations," CEPR Discussion Paper N0. 10594.

[5] Canzoneri, M., R. and B. Diba, 2011, "The Interaction Between Monetary and Fiscal Policy," In Benjamin M. Friedman, and Michael Woodford, editors: **Handbook of Monetary Economics**, Vol. 3B, The Netherlands: North-Holland, 2011, pp. 935-999.

[6] Cao, Q, 2015, "Optimal Fiscal and Monetary Policy with Collateral Constraints," manuscript.

[7] Cavallo, M., M. Del Negro, W. S. Frame, J. Grassing, B. A. Malin and C. Rosa, 2019, "Fiscal Implications of the Federal Reserve's Balance Sheet Normalization," *International Journal of Central Banking*, December.

[8] Cecchetti, S. and J. Hilscher, 2024, "Fiscal Consequences of Central Bank Losses", NBER working paper No 32478, May

[9] Cevik, S. and F. Miryugin, 2025, "It's Never Different: Fiscal Shocks and Inflation", *Comparative Economic Studies*, Vol. 67, pp:186-220.

[10] Chien Y. and Y. Wen, 2022, "The Determination of Public Debt Under Both Aggregate and Idiosyncratic Uncertainty", *Journal of Economic Theory*, 105474, pp:1-37.

[11] Corsetti, G., L. Dedola, M. Jarocziński, B. Maćkowiak and S. Schmidt, 2016, "Macroeconomic Stabilization, Monetary-Fiscal Interactions, and Europe's Monetary Union," ECB working paper No. 1988.

[12] Del Negro, and C. Sims, 2015, “When Does a Central Bank’s Balance Sheet Require Fiscal Support?” *Journal of Monetary Economics*, 73, pp: 1-19.

[13] Díaz-Giménez, J., Giovannetti, G., Marimon, R. and P. Teles, 2008, “Nominal Debt as a Burden on Monetary Policy,” *Review of Economic Dynamics*, Volume 11, Issue 3, pp: 493–514.

[14] Diercks, A. M., 2017, “The Reader’s Guide to Optimal Monetary Policy,” working paper.

[15] Jacome, L. I., Matamoros-Indorf, M., Sharma, M. and Townsend, S., 2012, “Central Bank Credit to the Government: What Can we Learn from International Practices”, IMF working paper wp/12/16.

[16] Leeper, E.,L. Campbell and L. Ding, 2021, “Optimal Time-Consistent Monetary, Fiscal and Debt Maturity Policy,” *Hournal of Monetary Economics*, 117, pp: 600-617.

[17] Lucas, R. and J. P. Nicolini, 2015 “On the stability of Money Demand”, *Journal of Monetary Economics*, Vol. 73, pp: 48-65.

[18] Manuelli,R. and J.I. Vizcaino, 2017,“Monetary Policy with Declining Deficits: Theory and an Application to Recent Argentine Monetary Policy,” *Federal Reserve Bank of Saint Louis Review*, 99(4), pp: 351-375.

[19] Martin, F., 2011, “On the Joint Determination of Fiscal and Monetary Policy,” *Journal of Monetary Economics*, 58, pp: 132-145.

[20] Martin, F., 2023, “Fiscal Dominance”, Federal Reserve Bank of St. Louis Working Paper 2020-040.

[21] Olivera, Julio H. G., 1970, “On Passive Money”, *Journal of Political Economy*, Vol. 78, No. 4, Part 2, pp: 805- 814.

[22] Reis, R., 2015, “Comment on: ’When Does a Central Bank’s Balance Sheet Require Fiscal Support?’,” *Journal of Monetary Economics*, 73, pp: 20-25.

[23] Sargent, T. and B. Smith, 1987, “Irrelevance of Open Market Operations in Some Economies with Government Currency Being Dominated in Rate of Return,” *American Economic Review*, 77, pp: 78-92.

[24] Sargent, T. and N. Wallace, 1981, "Some Unpleasant Monetarist Arithmetic," Federal Reserve Bank of Minneapolis *Quarterly Review*, Fall, pp:1-17.

[25] Seierstad, A. and K Sydsaeter, 1977, "Sufficient Conditions in Optimal Control Theory," *International Economic Review,* Vol. 18, No. 2, pp: 367-391.

[26] Uribe, Martin, 2016, "Is the Monetarist Arithmetic Unpleasant?," NBER working paper 22866.

[27] Uribe, Martin, 2020, "Financing Covid-19 Deficits in Fiscally Dominant Economies: Is the Monetarist Arithmetic Unpleasant?", *East Asian Economic Review*, Vol. 24, No. 4, December, pp: 417-440

[28] Wallace, N., 1981, "A Modigliani-Miller Theorem for Open Market Operations," *American Economic Review*, 71 (3), pp: 267-74.