ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

The geography of wealth: shocks, mobility, and precautionary savings

Authors	Maximiliano A. Dvorkin, and Brian Greaney
Working Paper Number	2024-033C
Revision Date	September 2024
Citable Link	https://doi.org/10.20955/wp.2024.033
Suggested Citation	Dvorkin, M.A., Greaney, B., 2024; The geography of wealth: shocks, mobility, and precautionary savings, Federal Reserve Bank of St. Louis Working Paper 2024-033. URL https://doi.org/10.20955/wp.2024.033

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

The Geography of Wealth: Shocks, Mobility, and Precautionary Savings*

Maximiliano Dvorkin
FRB of St. Louis

Brian Greaney
University of Washington

September 25, 2025

Abstract

The spatial distribution of wealth in the United States is very heterogeneous. We study the spatial distribution of wealth in a country and how it is shaped by regional earning characteristics and mobility frictions. For this, we develop a tractable model of consumption, savings, and location choice with many regions, incomplete markets, and heterogeneous agents facing persistent and transitory income shocks. Our theory extends a workhorse macroeconomic model of consumption and savings under uncertainty to an economy with multiple labor markets and costly mobility. Despite complex spatial and individual heterogeneity, we characterize the optimal consumption, savings, and mobility decisions of workers in closed form. Mobility frictions increase precautionary savings as workers hedge against consumption fluctuations generated by moving decisions. The spatial distribution of wealth is primarily driven by the interaction between persistent income shocks, saving behavior, and worker sorting over locations. Our results highlight the importance of accounting for worker mobility and regional heterogeneity in earnings dynamics when studying the spatial distribution of wealth.

JEL Classification: R12, R23, E21, J61, F16

Keywords: Mobility, precautionary savings, spatial equilibrium, wealth, inequality.

*We thank Rodrigo Adao, George Alessandria, Klaus Desmet, Elisa Giannone, Pablo Fajgelbaum, Kyle Herkenhoff, Ricardo Reyes-Heroles, Esteban Rossi-Hansberg, Hannah Rubinton, and Marcos Sora for valuable comments. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of St. Louis, or the Federal Reserve System.

1 Introduction

Economic activity varies dramatically across space. In the United States, per-capita earnings in the richest state are roughly 100% larger than in the poorest. Even after adjusting for regional cost-of-living differences, this gap remains substantial at approximately 60%. A large literature examines the causes and consequences of regional income inequality. In contrast, there has been very little analysis of the spatial wealth distribution. Even after controlling for income, average wealth varies tremendously across space. For example, the mean wealth-to-earnings ratio varies across US states by a factor of more than two.

In this paper, we seek to understand the determinants of the spatial wealth distribution. Our analysis focuses on how spatially heterogeneous income processes, mobility frictions, and individual savings decisions interact to shape the allocation of wealth across regions.

Using microdata on earnings and wealth, we document a series of novel facts regarding the spatial distribution of wealth and earnings volatility in the United States. First, there is substantial heterogeneity in overall wealth and non-housing wealth across states. In addition, we find a strong positive correlation between earnings and different wealth measures, with average wealth across states increasing more than proportionately with average earnings. Second, there are also marked differences in the volatility of earnings: across US states, wage volatility varies by over 100%. We also find a strong positive correlation between the volatility of residual wages for individuals and average earnings by states.

Amid substantial regional differences in income levels and volatilities, approximately 2.5% of Americans move across state lines each year. Individual mobility choices are shaped by both economic and personal motivations. For example, a household facing a decline in earnings can improve its well-being by moving to a less expensive location. Migration therefore serves as a form of insurance against changes in individual economic circumstances. On the other hand, households often relocate for personal reasons, such as family obligations. Because earnings vary substantially across space, these geographic transitions can generate large income changes and are therefore a source of risk. In this way, regional differences and migration can shape the consumption and savings decisions of households. This motivates a unified framework that combines economic geography and individual risk to analyze the sources and implications of spatial wealth inequality.

We develop a tractable dynamic quantitative spatial model with heterogeneous agents and incomplete markets. Our model has risk-averse workers who face both uninsurable idiosyncratic productivity shocks and idiosyncratic location preference shocks. Labor markets differ by wage, housing costs, and the volatility of individual productivity shocks. Workers can move across labor markets but face a moving cost to do so. Due to risk aversion, they accumulate savings as a precautionary measure against idiosyncratic shocks.

Our model extends the seminal framework of Caballero (1990) to an economy with many labor markets and mobility frictions. Despite rich heterogeneity and dynamics, we can characterize the consumption, savings, and mobility decisions of workers in closed form. We leverage this tractability to estimate location-specific income processes. There is a large literature that analyzes how idiosyncratic income shocks shape individual saving behavior. This literature typically abstracts from regional differences in income processes. Motivated by our empirical evidence that income volatility varies substantially across space, we allow for spatially heterogeneous income processes. This poses two challenges. First, there are a large number of parameters to estimate. Second, since individuals may migrate in response to individual shocks, failing to account for endogenous spatial selection would yield biased estimates. Due to the tractability of our model, we can estimate a large number of location-specific parameters to match the observed joint distribution of earnings shocks and migration decisions. To our knowledge, we are the first to estimate location-specific income processes with a procedure that accounts for dynamic spatial selection. We estimate a strong positive correlation between persistent income shocks and wages, and a weak correlation between transitory income shocks and wages.

We use our model to decompose the relative importance of wage differences, the volatility of persistent and transitory income shocks, and spatial sorting for generating observed spatial wealth patterns. Despite not using geographic wealth data in our calibration, our estimated model generates key features of the spatial wealth distribution. In particular, it closely matches the observed across-state variance of wealth-to-earnings ratio and the correlation between wealth-to-earnings ratio and wage.

Our model contains three key mechanisms that enable it to generate the observed positive correlation between the wealth-to-earnings ratio and wages. First, the precautionary saving motive is stronger in more volatile labor markets. Since we estimate a positive spatial correlation between persistent income shocks and wages, this force leads to higher saving rates in higher wage locations. Second, due to idiosyncratic location preference shocks, agents in high-wage locations (where the marginal utility of consumption is low) face a risk of being induced to move to a low-wage location (where the marginal utility of consumption is high). Agents respond to this risk by saving more in high-wage locations and less in low-wage locations. Our model delivers a "spatial Euler equation" that formalizes this logic. Third, positive complementarity between individual productivity and wages causes high-productivity workers to sort into high-wage locations. Since individual productivity is mean-reverting, high-productivity households save at higher rates than low-productivity households to protect against individual productivity risk. As a result, this force endogenously leads to higher saving rates in high-wage locations.

Quantitatively, wage heterogeneity is the dominant driver of spatial wealth inequality, accounting for approximately 78% of the variance in log wealth across states. This mechanism primarily affects saving conditional on individual productivity (i.e. the "spatial Euler equation" mechanism) and has less of an impact on spatial sorting. Persistent income volatility differences across states contribute meaningfully to wealth inequality, while transitory volatility differences have small effects on savings and wealth.

Of course, there are other forces that also contribute to spatial wealth heterogeneity. Our approach is to focus on standard channels that are emphasized in the macro and spatial literature. We find that the interaction between individual income risk, heterogeneous local income processes, and spatial frictions generates key features of the spatial wealth distribution. We interpret this as evidence that these channels are key determinants of saving behavior.

Importantly, a version of the model in which migration is not allowed does much worse at matching the spatial wealth data than the main model. In particular, the no-mobility model *overstates* spatial wealth inequality and the relationship between wealth and wages. This may seem surprising, as neither the sorting nor spatial Euler equation mechanisms mentioned above are operational in the no-mobility model. The reason for this result is that the no-mobility model predicts a much stronger effect of income volatility on savings than the full model. Intuitively, in the full model, agents have two ways to react to an individual shock: adjust precautionary savings, or migrate to a more suitable location. In the no-mobility model, only adjusting precautionary savings is possible. When calibrated to match economic geography data, the spatial model predicts an important role for the spatial adjustment mechanism. When this is shut down, household saving responds to individual income risk in a way that is at odds with the spatial wealth data. This suggests that economic geography plays a crucial role in shaping household financial decisions.

Lastly, we use our model to analyze the forces that explain changes in regional wealth levels over time. An existing literature documents moderate convergence in regional income levels over the past several decades. We document two new facts: both wealth and earnings volatility also display modest spatial convergence since 1990. Our model generates wealth convergence patterns that are consistent with those observed between 1990-2010. We find that regional wage changes account for the majority of this convergence. Notably, the no-mobility model fails to generate any meaningful wealth convergence. Whereas local wage changes have a large effect on local wealth in the full model, in the no-mobility model they are isomorphic to permanent income changes and so households do not adjust wealth in response. This result further highlights the importance of geographic mobility and spatial differences in shaping household saving behavior.

Related literature. Our paper is primarily related to three literatures. First, it contributes to the dynamic quantitative spatial economics literature.[1] Many papers in this literature, including the seminal contribution of Artuç et al. (2010), abstract from wealth accumulation. Important examples include Dix-Carneiro (2014), Desmet et al. (2018), Caliendo et al. (2019), and Traiberman (2019). In a recent influential paper, Kleinman et al. (2023) incorporates capital accumulation by assuming that saving decisions are made by immobile rentiers, while workers live hand-to-mouth and can move subject to migration costs. Other notable papers that take this approach include Cai et al. (2022) and Vanhapelto (2022).

A recent strand of the dynamic quantitative spatial literature develops models where agents

[1] See Desmet and Parro (2025) for a recent survey of this literature.

face joint saving and migration decisions. Crews (2023) develops a model where agents make dynamic human capital and location decisions. Greaney et al. (2024), Giannone et al. (2024), Luccioletti (2023), and Sun (2024) develop quantitative spatial models with incomplete markets and two assets: risk-free and owner-occupied housing. These models rapidly become intractable as the number of locations/sectors increases. Our model is simpler in some dimensions as we abstract from binding borrowing constraints and illiquid assets, but our closed-form expressions allow us to obtain a sharp characterization of the forces driving individuals' precautionary savings, mobility, and wealth across space. Dvorkin (2023) develops a tractable model of heterogeneous agents and incomplete markets where individuals accumulate human capital and assets. We abstract from human capital accumulation, but introduce both transitory and (partially) persistent income shocks. Non-permanent shocks create a precautionary saving motive, which we show is an important determinant of individual saving and spatial wealth inequality.

Bilal and Rossi-Hansberg (2021) also use a model with joint location/saving decisions to argue that workers trade current gains in amenities and wages for future earnings potential and capital accumulation. They show, both empirically and theoretically, that individuals use "location as an asset" to smooth consumption in response to individual shocks. This is also true in our model.[2] We focus on two distinct questions: first, what are the forces that shape the spatial wealth distribution we observe in the data; and second, how do mobility frictions, idiosyncratic location preferences, and spatially heterogeneous earning processes shape savings behavior.

A recent strand of the dynamic quantitative spatial literature explores optimal spatial allocations and the possible benefits of place-based policies. Important papers in this literature include Donald et al. (2025a), Donald et al. (2025b), Fajgelbaum and Gaubert (2025), and Mongey and Waugh (2024). Our dynamic model features incomplete markets with income and preference shocks, as well as mobility frictions, leading to savings as a force of self-insurance. Our goal is not to characterize optimal policy, but instead to show how savings motives differ across space

[2] Bilal and Rossi-Hansberg (2021) emphasize the importance of borrowing constraints. We abstract from borrowing constraints to maintain tractability and closed-form expressions, but (in contrast) introduce idiosyncratic location preferences and moving costs. That is, locations in our model are *risky* and *frictional* assets. Intuitively, the introduction of an exogenous borrowing limit would likely increase the correlation we find between wealth-to-income ratios and wages, as it is more likely for individuals near the borrowing limit in the financial asset to relocate to low wage (and low cost of living) states, a mechanism that Bilal and Rossi-Hansberg (2021) emphasize.

and to help explain observed spatial wealth differences.

Another related literature examines spatial income convergence and divergence patterns. Barro and Sala-i Martin (1992) and more recently Ganong and Shoag (2017), Giannone (2017), and Gaubert et al. (2021) analyzed the evolution of average income across US states and found some income convergence.[3] We document two new facts: wealth and earnings volatility differences across states also display a moderate level of convergence over time since 1990. Baum-Snow and Pavan (2013) and Gaubert et al. (2021) document important trends in earnings inequality over time that differ across US cities and states. Our work complements these findings by establishing a series of facts about the wealth distribution and how it relates to the characteristics of economic conditions at the local level. Moreover, Diamond and Gaubert (2022) studies how increased spatial sorting can shape earnings inequality. We show that sorting and differences in earnings risk (or earnings volatility) are forces that affect the distribution of consumption, savings, and wealth across regions.

Last, we connect with a literature that estimates individual income processes. Most of the papers in this literature, including the seminal contributions of Lillard and Willis (1978) and MaCurdy (1982), abstract from differences in volatility across regions and mobility and sorting of workers across space. Meghir and Pistaferri (2011) offers a comprehensive review of this literature. A recent paper by Braxton et al. (2025) is an exception, as it estimates income process parameters by state. However, it abstracts from sorting and spatial selection. After accounting for the dynamic selection of workers across regions, we find that there is considerable heterogeneity across space in the variance of transitory and persistent income shocks.

The paper is organized as follows. Section 2 documents facts about the distribution of wealth across US states. Section 3 develops a dynamic model of consumption, savings, and mobility to help rationalize these facts. In Section 4, we estimate a large set of parameters related to the characteristics of the income process in each state, mobility frictions, preference shocks, and selection. In Section 5, we use our estimated model to understand the forces that shape the distribution of wealth across space. Section 6 concludes.

[3]Some of these works also note that the rate of convergence has slowed down in recent decades compared to the past.

2 Earnings volatility and wealth across US states

US states show important differences in per capita income. The ratio between the highest per-capita income state and the lowest is approximately 2 in nominal terms and 1.6 in real terms. This fact also holds for earnings and even after controlling for differences in worker observables. While per-capita income across states was gradually converging until 1980, this process of convergence appears to have slowed down or perhaps reversed recently, as discussed in Ganong and Shoag (2017), Gaubert et al. (2021), Diamond and Gaubert (2022), and Giannone (2017).

In this section we document a series of new facts: US states show important differences in the levels of wealth per capita and wealth-to-earnings ratios, and in the variance of individual earnings and wage changes, which we refer to as volatility of earnings. Moreover, both the average wealth by state and the volatility of earnings show moderate convergence in recent decades.

2.1 The distribution of wealth across US states

We document several new facts about the spatial distribution of wealth in the United States and its evolution in recent decades. Our analysis focuses on mean and median wealth by state, using a combination of data sources. For more recent years, we use individual-level microdata from the Survey of Income and Program Participation (SIPP). The availability of this data is more limited further back in time, and we complement the analysis with recently published data on wealth by geography from Geowealth-US (Suss et al., 2024).[4]

The SIPP is a household-based survey designed as a series of nationally representative panels. Each panel generally features a large sample of households that are interviewed multiple times over a three or four-year period, depending on the year. From 1996 to 2017, the survey is structured as non-overlapping panels, with surveys every 4 months. Beginning in 2018, the survey switched to having overlapping panels interviewed annually. The SIPP provides comprehensive information on individuals' earnings over time, government transfers, and participation in social

[4]Suss et al. (2024) construct measures of wealth across geographic areas in the US using a sophisticated imputation method with microdata from the public-use samples of the US Decennial Census and the Survey of Consumer Finances. The correlation of mean and median levels of wealth by state between the SIPP and Geowealth-US for the year 2020 is very high.

programs, among other demographic, family, and work-related characteristics, since 1983. The wealth data is part of the "Assets and Liabilities" topical module available in different waves across the SIPPs panels over time.[5] The different asset and liability categories available in the SIPP are not as comprehensive and detailed as those in the Survey of Consumer Finances. However, as shown in Appendix A (Figure 9), the wealth distribution is very similar in both surveys except for the top percentiles.[6]

Our sample includes individuals between 25 and 60 years old. To increase the sample size, we pool data for several years. Our cross-sectional facts use the years 2014 to 2019.[7] In the SIPP (and the SCF) wealth is measured at the level of the household or the family, as it is non-trivial how to assign the value of different assets and liabilities to each family member. In our sample, we keep only the heads of households and assign all of the family wealth to this individual.

We compute moments of total net worth by state. Since housing is an important component of wealth and house prices differ widely across regions, we also construct a measure of non-housing wealth, which excludes the value of real estate, mortgages, and equity lines of credit. While we focus mostly on total net worth, the patterns we highlight are similar when using non-housing net worth, albeit with a different magnitude. When constructing means, we trim the top and bottom 1% of the distribution of wealth and earnings in the sample for each state.

Figure 1 shows the ratio of mean total net worth and non-housing net worth relative to mean annual earnings and its correlation with average annual earnings by state. Across US states, wealth relative to earnings is higher on the East and West coasts and in the North, and lower in much of the South and the Rust Belt. This is true for both measures of wealth, although in some states, such as California and New York, total net worth is relatively higher than non-housing net worth. Nonetheless, the correlation with mean annual earnings in the state is strong, as shown in the right panels of the figure. Note that mean earnings can be up to two times larger in

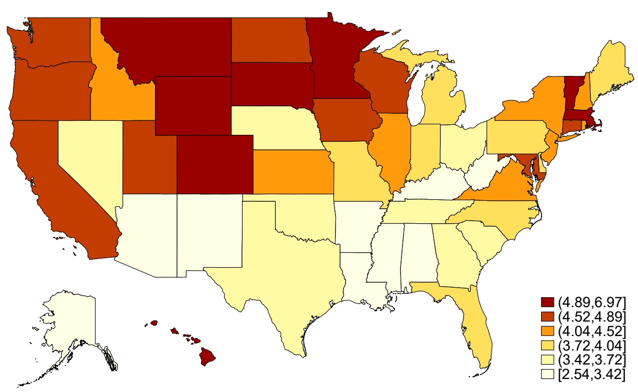
[5] There is a module with information about financial assets in every year from 1994 to 2021, excluding 2006, 2007, 2008, and 2012.

[6] The Survey of Consumer Finances is designed to oversample individuals at the top of the wealth distribution. Thus, it captures in great detail the different asset holdings of the wealthiest individuals in the United States.

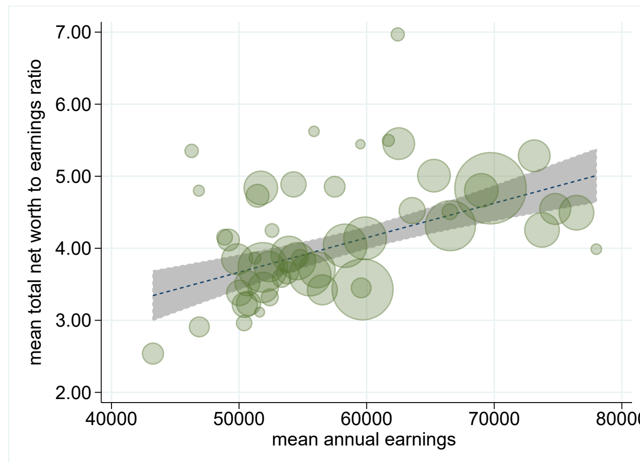
[7] The SIPP was redesigned in 2014 and the measures of wealth before that year are not as comprehensive nor are comparable to the more recent periods. Our sample ends in 2019 to avoid using data from the Covid pandemic years.

states at the higher end of the distribution relative to states at the lower end. The dispersion in the ratio of mean net worth to mean earnings can be larger than a factor of two, which implies that the dispersion in wealth is even larger as the ratio already takes into account differences in income in the denominator.

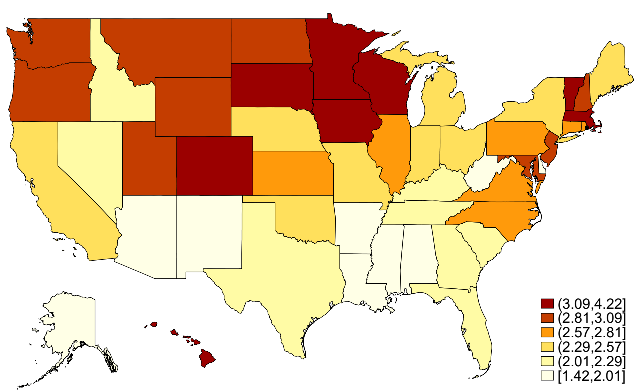
Figure 1: Moments of mean wealth across states



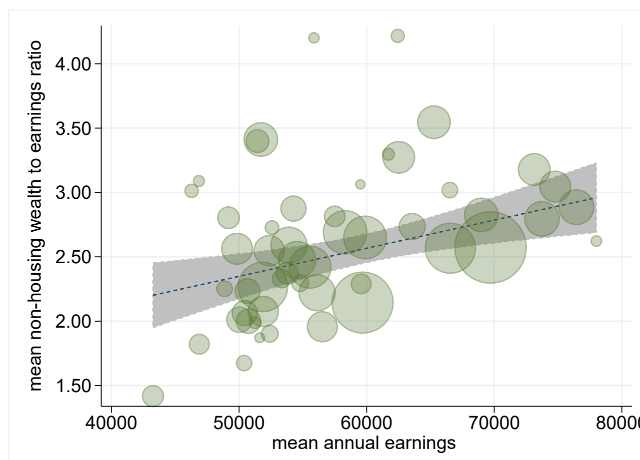
(a) Mean total net-worth to mean earnings ratio



(b) Total net-worth ratio vs earnings



(c) Mean non-housing wealth to mean earnings ratio



(d) Non-housing wealth ratio vs earnings

Note: Panels (a) and (c) show a map of the ratio of mean total net worth and non-housing wealth to mean annual earnings, respectively, in each state for the years 2014-2019. Panel (b) and (d) show the relationship between the ratio of mean total net-worth and non-housing wealth to mean annual earnings, respectively, and mean annual earnings for the state. Sample includes people between 25 and 60 years old heads of households. Sources: authors' calculations using data fom the SIPP, 2014-2019.

To evaluate the evolution of wealth in recent decades we follow Barro and Sala-i Martin (1991) and Ganong and Shoag (2017) and compare how the growth rate of wealth for a state over a period of time is related to the initial level of wealth. If wealth grows faster in states that had initially lower levels of wealth, relative to the mean, then we say that wealth converges over time.[8] Thus, we estimate the regression

$$y_{i,t+h} - y_{i,t} = \alpha + \beta\, y_{i,t} + e_i, \tag{1}$$

where $y_{i,t}$ is a variable of interest, such as log-wealth, for state i in period t, and e_i is the error term of the regression. $y_{i,t+h} - y_{i,t}$ measures the change in the variable from period t to $t+h$. If the coefficient β is negative, then states with initially lower levels of $y_{i,t}$ grew faster than those with higher levels and, thus, wealth tends to converge over this period.

Table 1: Wealth convergence for different periods and data sources

	Geowealth-US 1980-2010		Geowealth-US 1990-2010		SIPP 2000-2010	
	Δlog-mean	Δlog-median	Δlog-mean	Δlog-median	Δlog-mean	Δlog-median
	(1)	(2)	(3)	(4)	(5)	(6)
Initial period log-mean wealth	0.048		-0.053		-0.284**	
	(0.150)		(0.050)		(0.138)	
Initial period log-median wealth		-0.268*		-0.262***		-0.253**
		(0.137)		(0.051)		(0.122)
Observations	51	51	51	51	46	46

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: This table shows the regression coefficient β in equation (1) for different time periods, moments of wealth, and data sources. Columns (1) to (4) use information from Geowealth-US (Suss et al., 2024), columns (1) and (3) use the log of mean wealth for the state and columns (2) and (4) use the log of median wealth. Columns (1) and (2) use the difference between 2010 and 1980 levels of wealth and columns (3) and (4) use the difference between 1990 and 2010. Columns (5) and (6) use data from the SIPP, mean and median, respectively. For the SIPP data, the moments of wealth in 2000 are the average across years 1997 and 2001 for each state for that moment. Similarly, the moments of wealth for 2010 are the average across years 2009 and 2015 for each state for that moment. Regressions include U.S. states plus D.C. as observations. The SIPP data for early years combines state information from small, contiguous states, thus we have fewer observations.

[8]This concept is related to β-convergence in Barro and Sala-i Martin (1991, 1992). We do not directly estimate β-convergence here as the regression for β-convergence in those papers is derived from the neoclassical growth model and applies to convergence in income under the assumptions of that model. Our measures of convergence are simpler and directly use moments in the data that captures the relationship between initial levels of wealth and the growth of wealth over time. We develop a theory of income, savings, and migration in the next section.

Table 1 shows the estimates of the coefficient β from regression (1) for different periods and data sources. Since mean wealth can be disproportionately influenced by a few individuals at the upper end of the wealth distribution, we also report the coefficient using median wealth. The results are drawn from two primary data sources: the Geowealth-US dataset covering 1980-2010 and 1990-2010 periods, and the SIPP for 2000-2010.[9]

The estimates reveal a consistent pattern of wealth convergence over this period, and the point estimate is, in general, negative and significant. The relationship is negative and significant using median wealth in the Geowealth dataset, and for both mean and median in the SIPP. The estimated coefficients show that states with lower initial log-median wealth levels experienced relatively faster wealth growth over the next decades.[10]

2.2 Earnings volatility across space

Economic theory suggests that precautionary motives are an important determinant of savings, as individuals seek to self-insure against the effects of earnings volatility. We now study earnings volatility across US states and its evolution over time.

Using wage data from the Outgoing Rotation Groups of the Current Population Survey (CPS-ORG), we document another novel fact: US states show important differences in the volatility of individual earnings and wages. We document this fact using information on wages for individuals between 25 and 60 that we can link one year apart. Our first set of cross-sectional moments uses information for the years 2010 to 2019. We follow the usual approach in the literature and compute measures of volatility of individual log-hourly wages, log-weekly earnings, and individual arc percent changes in wages or earnings.[11] As is usual in the literature, we drop from our sample individuals with very low levels of weekly earnings.[12] Hourly wages are computed as

[9] To construct means and medians of wealth in 2000, we pool data and use the average across years 1997 and 2001 for each state for each moment. Similarly, for 2010, where we use the average across years 2009 and 2015.

[10] As we discussed previously, the availability of asset data in the SIPP for earlier years is more limited. We extend the period to 2020 and also find negative and significant coefficients in the SIPP for the change between 2000 and 2020.

[11] The arc percent change in a variable x between two periods of time is defined as arc chg.$(x_t) = \frac{x_t - x_{t-1}}{(x_t + x_{t-1})/2}$.

[12] In particular, we drop individuals with weekly earnings lower than five hours a week with an hourly wage of one half the federal minimum hourly wage. In this way, we exclude individuals that work an equivalent of less than 260 hours a year, at one-half the minimum wage. These are sample restrictions similar to Heathcote et al.

weekly earnings divided by the usual weekly hours. In all cases, we deflate nominal wages and earnings using the Consumer Price Index.

As recently documented in Moffitt et al. (2022), imputed earnings data can artificially magnify the amount of wage volatility. In our sample, we exclude observations with imputed earnings. In addition, Moffitt et al. (2022) recommends trimming the top and bottom 1% of the sample of wages when using survey data, as the sample from surveys can fail to properly capture the very low and very high levels of income and magnify measures of volatility. We proceed in this way, trimming the top and bottom 1% of the distribution of wages or earnings by state and year.

Figure 2 shows the distribution across states of the variance of individual weekly earnings changes and hourly wage changes, from one year to the next for different demographic groups. To construct each measure, first we compute the variance of wage or earnings changes across individuals in each state. Then we use each state as an observation and plot the distribution of the selected moment across states. Thus, each box in the figure displays the interquartile range and the median of the volatility of wages, and the lines that extend out from each box are the lower or upper adjacent values.[13]

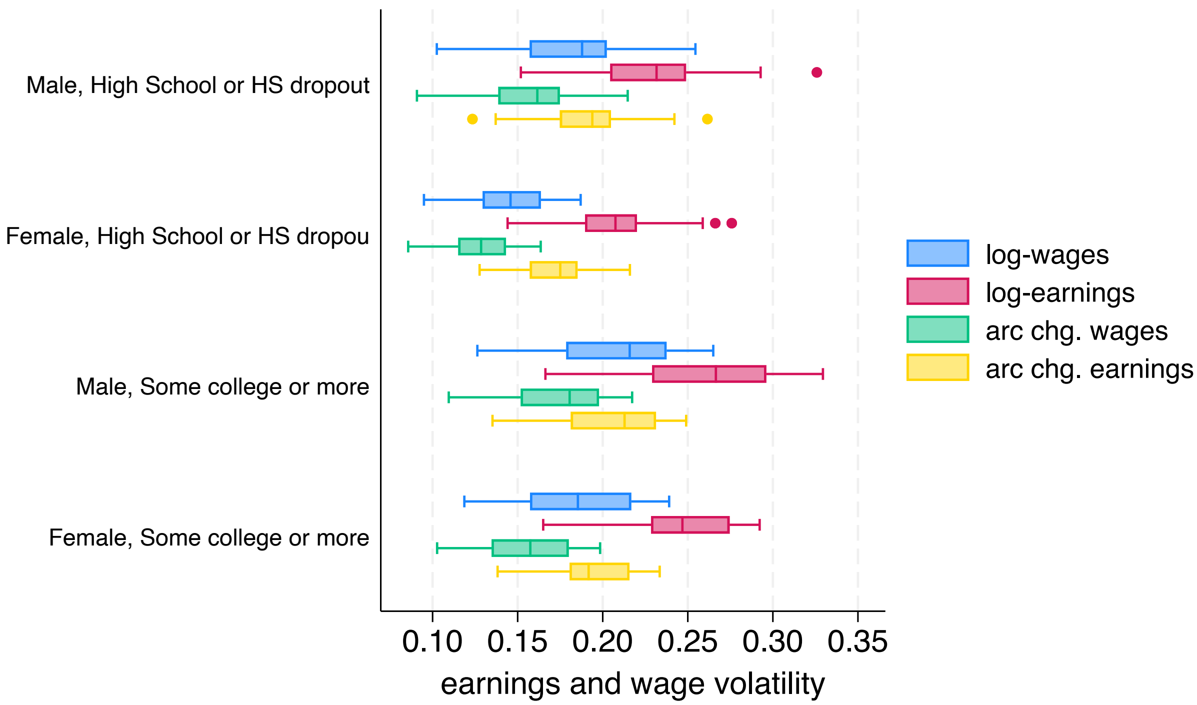
As the figure shows, there is substantial dispersion in earnings and wage volatility across states, irrespective of the measures used or the demographic group. The volatility of wages can be over 50% larger for a state at the 75th percentile of the distribution than for a state at the 25th percentile, and more than two times larger between the state with the minimum and maximum values. The dispersion in the variance across states is larger for more educated individuals, and lower for females with a level of education equal to high school or less.

We follow standard practice in the literature and construct residual log-wages as the deviation of log-wages for an individual from the predicted value from a Mincer regression that uses a rich set of controls for demographic characteristics – including different age groups, industry, and states. We run this regression pooling all years and specify all of these controls as fixed effects that vary by year. That is, our regression has demographic group-year fixed effects, state-year

(2010).

[13]The lower adjacent value is the maximum of the minimum point of the distribution or the 25th percentile minus 1.5 times the interquartile range. Similarly, the upper adjacent value is the minimum between the maximum value in the distribution and the third quartile plus 1.5 times the interquartile range.

Figure 2: Volatility of log-wages across US states



Note: This Figure shows the distribution of the variance of (1) log-hourly wages, (2) log-earnings, (3) arc percent change in log wages, and (4) arc percent change in log earnings across states for different demographic groups. Measures computed using CPS data matched records over one year for individuals between 25 and 60 years old employed at the time of the survey. All in real dollars of 2021. Pooled 2010-2019 monthly data, trimmed at the top and bottom 1% of real hourly wages by state and year. Non-imputed earnings data only.

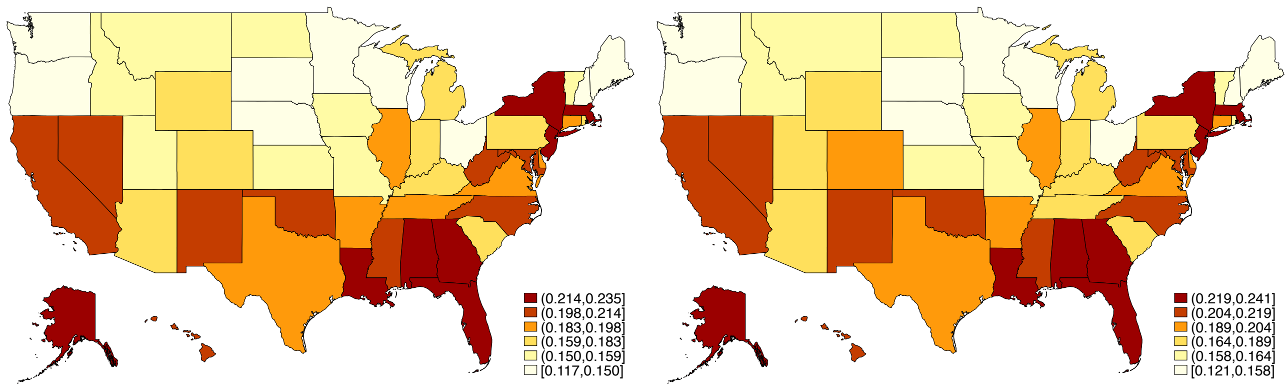
fixed effects, and industry-year fixed effects.[14]

Panel (a) of Figure 3 shows a map with the volatility of log-wages across US states, while panel (b) shows the volatility of residual log-wages. As the figure shows, the volatility of (residual) log-hourly wages is higher in most of the East Coast states, states in the South, and in the West, including Alaska and Hawaii. With the exception of Illinois, wage volatility is lower in the Midwest and the North. These differences are substantial and, as mentioned before, wage volatility can be over 100% larger across different states. Our findings align well with the evidence on earnings volatility in Lamadon et al. (2019), who use restricted-access microdata from the

[14] Our demographic groups (or worker type) are the Cartesian product of gender dummies, race dummies (white, black, and other), education dummies, (high school dropout, high school, and college or more), and four age group dummies (less than 30, between 30 and and 39, between 40 and 49, and between 50 and 60). All of these dummies are interacted with a year dummy.

Internal Revenue Service and show greater earnings volatility in the East and West coasts, and in the South.

Figure 3: Volatility of log-wages across US states



(a) Log-wages (b) Residual log-wages

Note: Variance of log-hourly wage changes and log-residual wages changes for individuals between 25 and 60 years old employed at the time of the survey. Residuals computed from a regression of log-wages on a rich set of demographic-year fixed effects, state-year fixed effects and industry-year fixed effects. CPS data matched records over one year. Pooled 2010-2019 monthly data, trimmed at the top and bottom 1% of real hourly wages by state and year. Non-imputed earnings data only.

Volatility of wages also exhibits convergence over time since 1990. Table 2 show estimates of coefficient β using regression (1) of the growth in the volatility of log-wages or residual log-wages on the initial level of the log-volatility. Similar to wealth and moments of average income in the literature, the volatility of wages has converged in the recent decades.

Table 2: Convergence in wage volatility

	Change between 1990-2010	
	Var(Δ log-wage)	Var(Δ residual log-wage)
	(1)	(2)
log-wage volatility in 1990	-0.169**	-0.168**
	(0.072)	(0.072)

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: this table shows the regression coefficient β in equation (1) for the change in the logarithm of the variance of log wages, column (1), and residual log wages, column (2) and initial period log-volatility. Individual log-wages and residual log-wages computed using matched individuals over a year in the CPS-ORG, pooling data for 1990 to 2000 and 2010 to 2019. Sample restricted to employed individuals between 25 and 60 years old. Wage residuals computed using a Mincer regression with demographic controls, age, time, state, and industry fixed-effects. We trim the top and bottom 1% of the distribution of wages by year and state.

The literature has long recognized differences in average levels of income across states. We document several novel facts: earnings volatility and mean and median wealth differences across states are as large or even larger than average income differences. Moreover, in a way akin to income, earnings volatility and wealth show some moderate convergence over time. In the next section we develop a model to understand how spatial differences in average wages, income volatility, and wealth are linked.

3 Model

We develop a model of consumption and savings in an economy with many regions, where mobility across labor markets is subject to individual preference shocks and frictions, and income processes vary across space. To simplify the analysis, we characterize the dynamic problem of households given exogenous paths for prices, interest rates, and wages across regions. Our model combines the ideas of Caballero (1990) on precautionary savings in a context of volatile earnings with costly mobility across space.

Time is discrete and runs forever. The economy is populated by a continuum of individuals of measure one facing a probability of death d, as in the Blanchard-Yaari perpetual youth model.[15] Each period a new cohort of workers of measure d is born, and the population is constant. The economy consists of J regions, which are also labor markets. Workers start each period attached to labor market $j = \{1, 2, \ldots, J\}$ and have the option to reallocate to a different market at the beginning of the period if they choose, but mobility is costly.

Individuals derive utility from the consumption of a final good and local amenities. The final good is freely traded and serves as the numeraire. Following Caballero (1990), we assume that the utility function is CARA with parameter γ.[16] In addition, all individuals living in region j pay a rental cost $q_{j,t}$, which captures differences in housing costs between regions over time as in

[15] In Appendix E, we extend the model to a life-cycle economy with bequests.

[16] The assumption of CARA utility, along with no borrowing limit and Weibul-distributed idiosyncratic location preferences (discussed below), allows us to derive closed-form expressions for the optimal decision rules of individuals. However, our main insight that mobility shocks generate income fluctuations and a motive to save does not rely on this assumption.

Bilal and Rossi-Hansberg (2021).[17]

Workers are heterogeneous in the efficient units of labor they supply (inelastically) to their labor market. Although workers have on average one unit of labor to supply to the market, each worker's endowment is random and changes over time, which we interpret as idiosyncratic shocks to a worker's labor income. In terms of timing, we assume that innovations to a worker's labor supply are realized after making the mobility and reallocation decisions, but before the consumption-savings decision.[18] The total efficient units of labor of worker i are $(1 + z_i' + \eta_i')$, where $z_i' = \rho\, z_i + \epsilon_i'$. ϵ_i' and η_i' are i.i.d. shocks distributed normally with mean zero and variance $\sigma^2_{\epsilon,\ell}$ and $\sigma^2_{\eta,\ell}$, respectively. Note that we assume the autocorrelation parameter $\rho \in (0, 1)$ is common across all labor markets, but the variances of the shocks can differ across regions. We label z_i the persistent income shock and η_i the transitory income shock.[19]

Markets are incomplete, but individuals can self-insure against shocks by borrowing or saving in a risk-free bond. Bonds can be purchased at the price of $1/(1 + r_t)$ units of final consumption at date t and each bond pays one unit of the final consumption good the next period if the bondholder survives.[20]

Individuals have the option to move to a different labor market at the beginning of each period. Geographic mobility is costly for workers, affecting their utility due to non-pecuniary factors.[21] We capture moving frictions with parameter $\psi_{j\ell}$, which scales utility due to non-pecuniary costs of relocating from labor market j to ℓ, with $\psi_{jj} = 0$, and $\psi_{j\ell} < 0$ for $j \neq \ell$. In addition, mobility and reallocation decisions are influenced by idiosyncratic preference shocks

[17] Since we assume unit housing consumption in each region, we abstract from the consumption of housing in the utility function. We can rescale regional amenities to include housing without impact on the results.

[18] This timing assumption is convenient to get closed-form expressions that we can compare to the literature on precautionary savings, but can be relaxed with minimal changes.

[19] The model can easily accommodate a persistent parameter ρ that varies across regions. However, we prefer the assumption of a common autocorrelation parameter ρ as it prevents workers with low realizations of the persistent shock to prefer regions with a low autocorrelation and those with high realizations to prefer regions with more persistence. In addition, estimation of region-specific ρ would demand panel data beyond the level is publicly available for the US economy.

[20] Borrowing implies that the worker issues these actuarial bond to a global market, and repays the next period if the worker survives.

[21] It is also possible to extend the model to have pecuniary costs of moving affecting the level of savings or wealth. The extension is straightforward, but identification of these frictions separately from non-pecuniary frictions would be more demanding in terms of data.

for different labor markets. We denote these shocks by the random vector $\varepsilon \in \mathbb{R}^J_+$, where each element is distributed i.i.d. Weibull with shape parameter $\nu > 0$ and identical scale parameter $\lambda = \bar{\lambda} \frac{J^{1/\nu}}{\Gamma\left(1+\frac{1}{\nu}\right)}$. We assume a Weibull distribution for ε because (as shown below), this distribution enables closed-form expressions for individual policy functions with CARA utility.

Finally, we assume that workers who die are replaced by a newborn attached to the same labor market. Newborn workers start with an exogenous and identical level of assets $\overline{a}^0$.[22]

Let $V_{j,t}(a, z, \varepsilon)$ denote the value function of a worker with last-period labor market j, persistent income shock z, bond-holdings a, and preference shocks ε. To simplify notation, we omit the individual subscript i with the understanding that assets and shocks vary across individuals. We use sub-index t to represent the aggregate state of the economy at that time.[23] The worker's problem can be written recursively as[24]

$$V_{j,t}(a,z,\varepsilon) = \max_{\ell} \left\{ \varepsilon_\ell e^{\psi_{j\ell}} E_{\eta',z'|z,\ell} \left[\max_{c_{j\ell,t}, a_{j\ell,t+1}} \left\{ \left(-\frac{1}{\gamma} \right) e^{-\gamma \, c_{j\ell,t} - A_\ell} + \beta \left[E_{\varepsilon'} \left[V_{\ell,t+1}(a_{j\ell,t+1}, z', \varepsilon') \right] \right] \right\} \right] \right\} \tag{2}$$

$$\text{s.t.:} \qquad c_{j\ell,t} + \frac{1}{(1+r_t)} a_{j\ell,t+1} + q_{\ell,t} = w_{\ell,t}(1-\tau)\,(1+z'+\eta') + a \tag{3}$$

where (3) is the budget constraint, which links consumption and savings of a worker who moves from j to ℓ. The discount factor β takes into account the discount rate and the survival probability. Individuals pay a proportional income tax τ that is common for the whole economy. Given our timing assumption, workers make the reallocation decision before observing the realization of the labor supply shock η' and ϵ'.

It is important to highlight that the persistent component of labor supply, z', is complementary to the wages of the region. Thus, the value of z influences the mobility and reallocation decisions of individuals, as the expectation for an individual with high z is to have a high value of z' after reallocation. Therefore, due to selection, the distribution of individuals with shock z across regions will in general not align with the unconditional distribution of z.

[22]These assumptions allow for a well-defined and stationary distribution of wealth over time. They can be relaxed at the cost of some cumbersome notation.

[23]In our quantitative application we focus on stationary equilibria, with no variation in aggregate conditions.

[24]For the value function to be well-defined, the transversality condition $\lim_{t\to\infty} \frac{a_{j\ell,t}}{(1+r_t)^t} = 0$ must hold, which we impose.

Finally, note that non-pecuniary costs enter the problem multiplicatively with factor $e^{\psi_{j\ell}}$. Since both period utility and lifetime utility are negative, a higher value of non-pecuniary costs of moving from j to ℓ decreases the value of making that transition. Amenities A_ℓ enter the period utility, generating complementarity with consumption. As a result, the marginal utility of consumption varies with amenities.

3.1 Characterization of the worker's problem

We now discuss the optimal conditions that characterize the solution to the worker's problem in this economy. It is convenient to simplify the notation with the following definitions. Let $\tilde{y}_{\ell,t} = w_{\ell,t}(1-\tau) - q_{\ell,t}$ and $\tilde{w}_{\ell,t} = w_{\ell,t}(1-\tau)$. In addition, denote ex ante lifetime utility by $v_{j,t}(a,z) = E_\varepsilon[V_{j,t}(a,z,\varepsilon)]$.

Proposition 1. *The optimal consumption-savings decisions of a worker, after choosing location, are characterized by,*

$$c_{\ell,t}(a,z',\eta') = \Xi_{\ell,t}(z') + \kappa_t\left[\tilde{y}_{\ell,t} + \tilde{w}_{\ell,t}\,\eta' + a\right], \tag{4}$$

$$a_{\ell,t+1}(a,z',\eta') = (1+r_t)(1-\kappa_t)\left[\tilde{y}_{\ell,t} + \tilde{w}_{\ell,t}\eta' + a\right] + (1+r_t)\left(\tilde{w}_{\ell,t}z' - \Xi_{\ell,t}(z')\right) \tag{5}$$

where κ_t is a scalar that can vary with time, but is the same across locations, and $\Xi_{\ell,t}(z')$ depends on the realization of the shock z', the selected labor market, and period. Moreover, the ex-ante lifetime utility and the share of individuals moving from labor market j to ℓ, $\mu_{j\ell,t}(z)$ are,

$$v_{j,t}(a,z) = \left(-\frac{1}{\gamma}\right)\frac{1}{\kappa_t}e^{-\gamma\kappa_t\,a}\,\bar{\lambda}\left(\frac{1}{J}\sum_{\ell=1}^{J} e^{\nu(\mathbb{B}_{1,j\ell,t})}\left(E_{z'|z,\ell}\left[e^{-\gamma\Xi_{\ell,t}(z')}\right]\right)^{-\nu}\right)^{-(1/\nu)}, \tag{6}$$

$$\mu_{j\ell,t}(z) = \frac{e^{\nu(\mathbb{B}_{1,j\ell,t})}\left(E_{z'|z,\ell}\left[e^{-\gamma\Xi_{\ell,t}(z')}\right]\right)^{-\nu}}{\sum_{m=1}^{J} e^{\nu(\mathbb{B}_{1,jm,t})}\left(E_{z'|z,m}\left[e^{-\gamma\Xi_{m,t}(z')}\right]\right)^{-\nu}} \tag{7}$$

where $\mathbb{B}_{1,j\ell,t} = A_\ell - \psi_{j\ell} + \gamma\kappa_t\tilde{y}_{\ell,t} - (\kappa_t\gamma\tilde{w}_{\ell,t})^2(\sigma^2_{\eta,\ell})/2$, and $\Xi_{\ell,t}(z')$ and κ_t are defined recursively *as,*

$$\kappa_t = \frac{\kappa_{t+1}(1+r_t)}{1+\kappa_{t+1}(1+r_t)} \tag{8}$$

$$\begin{aligned}\Xi_{\ell,t}(z) &= \left(-\frac{1}{\gamma}\right)\left(\frac{1}{1+\kappa_{t+1}(1+r_t)}\right)\Bigg(\log\left(\beta(1+r_t)\right) - \gamma\kappa_{t+1}(1+r_t)\tilde{w}_{\ell,t}z + \\ &\quad + A_\ell + \log(\bar{\lambda}) - \frac{1}{\nu}\log\left[\frac{1}{J}\sum_{m=1}^{J} e^{\nu(\mathbb{B}_{1,\ell m,t+1})}\left(E_{z'|z,m}\left[e^{-\gamma\Xi_{m,t+1}(z')}\right]\right)^{-\nu}\right]\Bigg)\end{aligned} \tag{9}$$

The proof is in Appendix B. Proposition 1 extends the results in Caballero (1990) with persistent and transitory income shocks to an economy with many labor markets and costly mobility. Given the assumption of a common real interest rate across labor markets, the parameter κ_t is identical for all individuals. However, equation (4) shows that consumption differs between individuals in the same labor market ℓ due to differences in idiosyncratic shocks, assets, and their mobility and reallocation choices. Savings decisions have a similar flavor, with wealth influenced by mobility and reallocation frictions through $\Xi_{j,t}(z)$.

In Appendix B, we show that the optimal consumption/savings decision satisfies a *spatial Euler equation*, given by

$$\frac{\partial\, u\left(c_{\ell,t}(a,z',\eta'),A_\ell\right)}{\partial c} = \beta(1+r_t)\,\bar{\lambda}\left[\frac{1}{J}\sum_{m=1}^{J}\left(e^{\nu\,\psi_{\ell m}}\,E_{\eta'',z''|z'}\left[\frac{\partial\, u\left(c_{m,t+1}(a''_{j\ell,t+1},z'',\eta''),A_m\right)}{\partial c}\right]\right)^{-\nu}\right]^{-1/\nu} \tag{10}$$

As usual, the Euler equation links consumption in period t to future consumption through savings and expected future income. The main difference here is that consumption has a spatial component. That is, the consumption-savings decision in period t depends on the disposable income in region ℓ and the expected disposable income in the future, which depends on next period's chosen region. However, since next period's region has not yet been decided, the right hand side of (10) captures the expected marginal utility of future consumption across future labor markets. In this way, mobility frictions, the characteristics of preference shocks, local amenities, and future disposable income in different regions affect the expected value of next period (marginal) utility, thus affecting today's consumption and savings decisions.

Note that this condition is different from Bilal and Rossi-Hansberg (2021). On the one hand, we do not impose a borrowing limit other than the natural borrowing limit (the transversality condition), so our spatial Euler equation always holds with equality. On the other hand, our economy has mobility and reallocation costs, so our individuals may be "constrained in their location asset" ex-post.

Similar to Bilal and Rossi-Hansberg (2021), workers sort across regions due to the complementarity between individual productivity and wages. In particular, equation (7) shows that workers with high values of z will move with higher probability to regions with higher wages, as z is persistent and expected to be high for many periods. Preference shocks, amenities, and mobility frictions affect the patterns of spatial sorting. In addition, the permanent component of income, $\tilde{y}$, depends negatively on the local cost of living, or rents, $q_{\ell,t}$. Thus, workers prefer locations with high wages and low rents, all else equal. Note, however, that locations are a risky asset which differ in their level of income volatility. Since workers dislike volatility, migration and sorting will be influenced also by the volatility of the income process in each region and the likelihood of moving in the future.[25]

In the simplest case, with two periods, two regions, no income shocks ($\eta = z = 0$), and no amenity differences ($A_\ell = 0$), equation (10) becomes

$$\frac{\partial\, u\,(c_{j,t}(a))}{\partial c} = \beta(1+r_t)\,\bar{\lambda}\,\left[\frac{1}{2}e^{-\nu\,\psi_{jj}}\,\left(\frac{\partial\, u\,(c_{j,t+1}(a_{t+1}))}{\partial c}\right)^{-\nu} + \frac{1}{2}e^{-\nu\,\psi_{j\ell}}\,\left(\frac{\partial\, u\,(c_{\ell,t+1}(a_{t+1}))}{\partial c}\right)^{-\nu}\right]^{-1/\nu}.$$

This expression highlights a novel motive for saving in spatial economies: when disposable income differs across locations but is deterministic within a location, individuals will borrow/save to smooth consumption in the event of moving across locations. In other words, preference shocks for different locations can induce income fluctuations, and individuals will have a motive to save to hedge against mobility events.[26] With no mobility frictions and identical disposable income

[25]Note that in our model, the level of asset holdings of an individual does not directly influence the mobility decision. As we relax some of our assumptions, that would no longer be the case, but we lose the sharp characterization of some of our expressions and need to resort to a quantitative analysis.

[26]It is important to highlight that the realization of preference shocks for the current period do not appear in our spatial Euler equation, since these shocks affect equally current and future utility. Thus, the savings motive we highlight is not due to temporary changes in patience or impatience, but rather about the likelihood of moving

across locations, the consumption-savings decision is standard, leading to the usual expression for the Euler equation.[27]

3.2 Risks, local characteristics, mobility, and precautionary savings

The savings policy rule (5) says that workers save a fraction of their current cash-on-hand, plus an additional term that is decreasing in the endogenous variable $\Xi_{\ell,t}(z')$. Given our assumptions, the marginal propensities to consume and save out of transitory income shocks or assets are identical across labor markets. Additional effects related to expected future income and precautionary savings are captured by $\Xi_{\ell,t}(z')$, leading to different levels of savings in different labor markets.

Equation (9) defines $\Xi_{\ell,t}(z)$ recursively. To gain intuition, we use the equations in Proposition 1, assuming $z = \epsilon = 0$. In this case, there are no persistent income shocks, and $\Xi_{\ell,t}(z)$ and $\mu_{j\ell}$ do not depend on z. We can write the negative of $\Xi_{\ell,t}$ as

$$
\begin{aligned}
-\Xi_{\ell,t} \quad = \quad & \underbrace{\left(\frac{1}{\gamma}\right)\sum_{s=1}^{\infty}\left(\frac{1}{\Pi_{k=0}^{s-1}(1+r_{t+k})}\right)\frac{\kappa_t}{\kappa_{t+s}}\log\left(\beta(1+r_t)\right)}_{\text{impatience}} - \underbrace{k_t\sum_{s=1}^{\infty}\left(\frac{1}{\Pi_{k=0}^{s-1}(1+r_{t+k})}\right)\tilde{y}_{\ell,t+s}}_{\text{permanent income}} \\
& + \underbrace{\frac{\gamma\,\kappa_t}{2}\sum_{s=1}^{\infty}\left(\frac{1}{\Pi_{k=0}^{s-1}(1+r_{t+k})}\right)\kappa_{t+s}\left(\tilde{w}_{\ell,t}\right)^2\sigma^2_{\eta,\ell,t+s}}_{\text{precautionary savings}} \qquad (11) \\
& + \underbrace{\left(\frac{1}{\gamma}\right)\sum_{s=1}^{\infty}\left(\frac{1}{\Pi_{k=0}^{s-1}(1+r_{t+k})}\right)\frac{\kappa_t}{\nu\kappa_{t+s}}\left[\log\left(\mu_{\ell\ell,t+s}\right)-\log\left(1/J\right)+\nu\log(\bar{\lambda})\right]}_{\text{mobility \& amenities}}.
\end{aligned}
$$

Equation (11) consists of four terms. The first term captures the effects of impatience on savings. If individuals are more impatient relative to the market, such that $\beta(1+r_t) < 1$, then this

to other regions and the fact that regions differ in their characteristics. Our assumption on the distribution of the preference shock provides a simple expression for the expectation of the maximum utility across regions, but the result we highlight here does not rely on a specific distribution for preference shocks or the functional form we chose for utility.

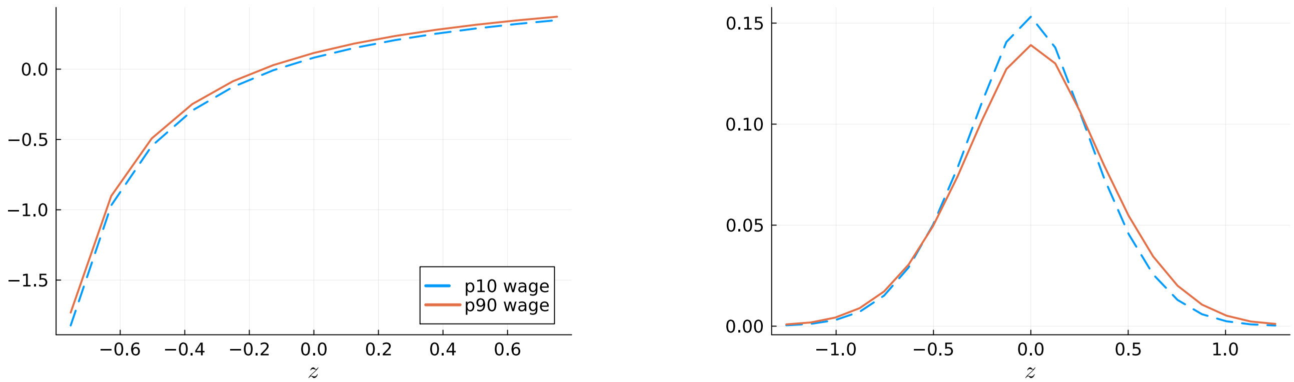
[27]The recent works of Mongey and Waugh (2024) and Donald et al. (2025a) study the role of preference shocks and incomplete markets in economic environments characterized by static or dynamic, respectively, discrete choice decisions. They argue that a planner intervention increases workers' welfare. Our results in this section highlight how access to a simple set of uncontingent financial securities, i.e. savings and borrowing, allows workers to self-insure against the effects of mobility or preference shocks.

first term is negative and savings would be lower. The second term captures future expected permanent income for a worker who does not move. It is easy to show that this term alone does not lead to savings differences after taking into account the first part of equation (5), as for non-movers, consumption increases one-to-one with the level of permanent income. The third term captures the effects of precautionary savings. Workers in regions that have more volatile earnings, leading to higher levels of $\sigma^2_{\eta,\ell,t}$, will save more to self-insure against these shocks. The last term captures how mobility affects consumption and savings. Clearly, $\mu_{\ell\ell,t+s}$ is endogenous and itself depends on the future values of Ξ across all locations. However, we can get some intuition as mobility rates serve as a sufficient statistic for the (relative) expected value of labor markets. First, note that with only one labor market, the last term is zero and $\Xi_{\ell,t}$ would be the same as that in Caballero (1990) for i.i.d. income shocks. Relatedly, if the probability of staying in a labor market is identical to the probability of moving to any other labor market, such that $\mu_{\ell\ell,t} = 1/J$, then the last term is zero. This would happen when all labor markets are identical and workers face no moving frictions, which is essentially a single labor market economy. As mentioned before, with no frictions and identical labor markets, the spatial Euler equation becomes a standard Euler equation and the usual motives to save apply.

With labor markets that differ in their characteristics and with moving and reallocation frictions, workers will stay with high probability in their region due to moving costs, but more so in regions with high levels of wages. Then, $\mu_{\ell\ell} > 1/J$ and the last term would, in general, be positive. Higher values of ν translate into a lower variance of preference shocks, which reduces this precautionary savings motive. Individuals living in a very desirable location will tend to save more as $\mu_{\ell\ell}$ would be higher in this case, all else equal.

In the general case with persistent income shocks, z, there is an additional precautionary saving motive. Specifically, since z is mean-reverting in expectation, individuals save when z is high and dissave when it is low to smooth consumption. Following Wang (2003), we refer to this saving motive as "saving for a rainy day." At the same time, differences across space in wages and the complementarity between wages and z imply that people in high-wage locations will save (borrow) more when their z is higher (lower). Figure 4 uses a numerical example to illustrate this effect. Panel (a) shows the behavior of the savings rate, that is, the change in assets relative

Figure 4: Effects of persistent shock and sorting on savings: numerical example



(a) Saving rate $(a' - a)/[(1+z)w_j]$

(b) z distribution

to earnings, for different values of z for a state with high wages (the 90th percentile of the wage distribution, which we estimate below) and a state with low wages (the 10th percentile). The savings rate function is concave and increasing in z. Agents save when z is sufficiently high and dissave when it is sufficiently low. Note that individuals in high-wage states have higher saving rates than those in low-wage states, reflecting the effects of mobility discussed previously. Panel (b) highlights sorting across space. The distribution of z in high-wage locations is shifted rightward relative to low-wage locations. That is, high-z agents sort into high-wage locations, and vice-versa. The combination of these two forces generates more savings and wealth in high-wage states, a mechanism that aligns with the data. Without mobility and sorting, saving for a rainy day would not lead to differences in saving rates across regions.

For simplicity, we abstract from permanent productivity differences across individuals. It is important to highlight that, with permanent productivity differences, workers will sort across space and take into account how much their income may change when moving across regions. However, as mentioned previously, this feature alone would not generate differences in saving *rates* across space because permanent income changes do not affect savings if the level of permanent income is identical across regions, even if it is different by types of workers. It is the combination of persistent income shocks and dynamic sorting that contributes to differences in saving for a rainy day across regions.

3.3 Aggregation

Despite individual heterogeneity in asset holdings and earnings, we can transparently characterize the evolution of aggregate consumption, earnings, and wealth for workers in each labor market. The reason is that consumption and savings decisions are linear in cash-on-hand, as shown in Proposition 1. Thus, the average consumption depends linearly on the average cash-on-hand in each market, and similarly for savings. As in Dvorkin (2023), we can also use properties of the mixture of distributions to compute the *evolution* of first (and higher-order) moments of the distribution of consumption and wealth. The following proposition summarizes these results:

Proposition 2. *Let $\bar{\Lambda}_{\ell,t}(z')$ be the density of workers with shock z' in labor market ℓ at the end of period time t. If newborns start with an exogenous and identical level of assets, $\bar{a}^0$, and an exiting worker with persistent shock z in region j is replaced by a newborn with an identical $z = 0$ in the same region, then the evolution of average consumption, $\bar{c}_{\ell,t}$, and average asset holdings, $\bar{a}_{\ell,t+1}$, for workers with shock z' in labor market ℓ are characterized by,*

$$\begin{aligned}\bar{c}_{\ell,t}(z') \quad &= \quad \sum_{j=1}^{J} \int \frac{\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_\ell(z'-\rho\, z)}{\bar{\Lambda}_{\ell,t}(z')}(1-\delta)\left(\Xi_{\ell,t}(z') + \kappa_t \tilde{y}_{\ell,t} + \kappa\, \bar{a}_{j,t}(z)\right) \mathrm{d}z \,+ \\ &\quad\quad \sum_{j=1}^{J} \frac{\mu_{j\ell,t}(0)f_\ell(z')}{\bar{\Lambda}_{\ell,t}(z')}\left(\Xi_{\ell,t}(z') + \kappa_t \tilde{y}_{\ell,t} + \kappa\, \bar{a}^0\right)\left(\delta \int \bar{\Lambda}_{j,t-1}(z)dz\right), \qquad (12) \\ \bar{a}_{\ell,t+1}(z') \quad &= \quad \sum_{j=1}^{J} \int \frac{\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_\ell(z'-\rho\, z)}{\bar{\Lambda}_{\ell,t}(z')}(1-\delta)\left[(1+r_t)(1-\kappa_t)\left(\tilde{y}_{\ell,t} + \bar{a}_{j,t}(z)\right) + (1+r_t)\left(\tilde{w}_{\ell,t}\, z' - \Xi_{\ell,t}(z')\right)\right] \mathrm{d}z \\ &\quad\quad + \sum_{j=1}^{J} \frac{\mu_{j\ell,t}(0)f_\ell(z')}{\bar{\Lambda}_{\ell,t}(z')}\left[(1+r_t)(1-\kappa_t)\left(\tilde{y}_{\ell,t} + \bar{a}^0\right) + (1+r_t)\left(\tilde{w}_{\ell,t}\, z' - \Xi_{\ell,t}(z')\right)\right]\left(\delta \int \bar{\Lambda}_{j,t-1}(z)dz\right), \qquad (13)\end{aligned}$$

where δ is the fraction of agents that die in a period (death probability), and $\bar{a}_{j,t}(z)$ is the average asset holding of workers in labor market j and shock z at the end of period $t-1$. Moreover,

$$\bar{\Lambda}_{\ell,t}(z') = (1-\delta) \sum_{j=1}^{J} \int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_\ell(z'-\rho\, z)\, \mathrm{d}z + \sum_{j=1}^{J} \mu_{j\ell,t}(0)f_\ell(z') \left(\delta \int \bar{\Lambda}_{j,t-1}(z)dz\right), \quad (14)$$

where $f_\ell(x)$ is the density of a normal with mean zero and variance $\sigma^2_{\epsilon,\ell}$.

Finally, the economy-wide level of wealth, $\bar{a}_{t+1}$, and consumption, $\bar{c}_t$, are,

$$\bar{a}_{t+1} = \sum_{\ell=1}^{J} \int \bar{\Lambda}_{\ell,t}(z')\, \bar{a}_{\ell,t+1}(z')\mathrm{d}z' \tag{15}$$

$$\bar{c}_{t} = \sum_{\ell=1}^{J} \int \bar{\Lambda}_{\ell,t}(z')\, \bar{c}_{\ell,t}(z')\mathrm{d}z' \tag{16}$$

The proof is in Appendix B. Proposition 2 highlights the ways the model connects with and departs from an economy with a single labor market. With a single market, average consumption depends linearly on average cash-on-hand, as in Caballero (1990). With many labor markets, the average consumption and savings in region ℓ depend on the average income and wealth of all individuals moving into labor market ℓ from all other labor markets j, conditional on z. In addition, equation (13) shows that the distribution of wealth in a region depends on the patterns of sorting of workers with different levels of persistent income across states and their consumption-savings decisions.

4 Estimation of regional income process and structural parameters

We estimate a large set of parameters that characterize the income process by state. These include wages and the variance of transitory and persistent income shocks. In addition, we estimate mobility costs by origin and destination, and the migration elasticity parameter ν. We show that it is feasible to estimate parameters in two steps. First, we obtain estimates for wages, the variances of income shocks, and the persistence parameter ρ, and in a second stage, estimate parameters for mobility costs and elasticity ν.

We assume an interest rate of $r = 0.03$ and set the discount factor to $\beta = 1/(1 + r) = 0.971$. We use regional housing expenditure data from the Bureau of Economic Analysis to calibrate $q_{\ell,t}$. Moreover, we assume $\gamma = 3$, as this parameter captures both absolute risk aversion and absolute prudence in CARA preferences.[28] The parameter $\overline{\lambda}$ is estimated to match the observed economy-wide average wealth-to-earnings ratio across states of 4.09. In the baseline model, we estimate $\overline{\lambda} = 0.372$.

4.1 Estimation of income process with mobility and selection

A large literature, originating with the seminal works of Lillard and Willis (1978), Lillard and Weiss (1979), and MaCurdy (1982), estimates random income processes. By and large, this literature abstracts from differences in the volatility of income across regions and from mobility and self-selection of workers across space.[29] Mobility and self-selection make the estimation challenging as individuals may migrate in response to realizations of idiosyncratic shocks. This creates differences between the conditional distribution of workers in a state and the unconditional distribution in the population. Thus, the estimation needs to take into account selection and estimate parameters for all regions jointly. In this section, we estimate the parameters that characterize the income process in different US states, w_ℓ, $\sigma^2_{\epsilon,\ell}$, and $\sigma^2_{\eta,\ell}$, accounting for the dynamic selection of workers due to unobservable characteristics.

The intuition behind our estimation procedure is simple. Our model tightly links the distribution of workers across labor markets and residual income over time. The joint distribution of workers that at the end of period t were located in labor market ℓ and had a labor supply of z', conditional on not dying,[30] is

$$\Lambda_{j\ell,t-1,t}(z, z') = \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)\, f_\ell(z' - \rho\, z), \tag{17}$$

where $\mu_{j\ell,t}(z)$ is the conditional probability that a worker in j with past shock z moves to ℓ in period t and $f_{\epsilon,j}(z' - \rho\, z)$ is the conditional density that a worker in ℓ with past shock z gets a labor supply shock of z', which, given our assumptions, is the normal density with mean zero

[28] Our value of $\gamma = 3$, representing prudence, lies at the midpoint of estimates in the literature. For example, Cagetti (2003) estimates prudence to be between 4 and 5, while Christelis et al. (2020) estimates it at 2.

[29] The recent work by Braxton et al. (2025) estimate different income processes by state but abstract from mobility and dynamic selection of workers across space.

[30] If workers die or exit the sample, we cannot link them over time. In this way we treat the model and the data symmetrically.

and variance $\sigma^2_{\epsilon,\ell}$. Importantly, note that all the elements in the previous equation depend on the value of the parameters ρ and $\sigma^2_{\epsilon,\ell}$, for all ℓ. In addition, note that $\mu_{j\ell,t}(z)$ also depends on these parameters, plus additional parameters and endogenous variables in our model, as economic conditions in different locations influence the choices of workers.

We can write log-earnings for individual i in the model as[31]

$$\log(earnings_{i,\ell,t}) = \log(\tilde{w}_{\ell,t}) + \log\left(1 + z'_{i,\ell,t} + \eta'_{i,\ell,t}\right).$$

Assuming earnings shocks are not very large, we can approximate the previous expression by

$$\log(earnings_{i,\ell,t}) \approx \log(\tilde{w}_{\ell,t}) + z'_{i,\ell,t} + \eta'_{i,\ell,t}.$$

Since $\tilde{w}_{\ell,t}$ is common for all workers in region ℓ, residual log-earnings in the model is $\zeta'_{i\ell t} = z'_{i,\ell,t} + \eta'_{i,\ell,t}$. Our estimation procedure uses moments of residual log-earnings in the data and those implied by the model, and picks the parameter values that minimize the distance between the two.

For this, note that the marginal density of z' in the model, integrating over j and z, is $\bar{\Lambda}_{\ell,t}(z')$. Although the unconditional mean of z in all locations is zero, due to selection, the mean of z conditional on the selected labor market can be different. Nonetheless, the mean and variance of residual earnings $z' + \eta'$ for labor market ℓ can easily be computed as,

$$E\left[z'_{i\ell t} + \eta'_{i\ell t}|\ell, t\right] = E\left[z'_{i\ell t}|\ell, t\right] = \int z' \, \bar{\Lambda}_{\ell,t}(z')\mathrm{d}z' \tag{18}$$

$$Var\left[z'_{i\ell t} + \eta'_{i\ell t}|\ell, t\right] = Var\left[z'_{i\ell t}|\ell, t\right] + Var\left[\eta'_{i\ell t}\right] = \int \left(z' - E\left[z'|\ell\right]\right)^2 \bar{\Lambda}_{\ell,t}(z')\mathrm{d}z' + \sigma^2_{\eta,\ell} \tag{19}$$

[31]We also construct residual log-earnings in the data. As is usual in the literature, residual log-earnings are obtained as the difference between observed log-earnings for an individual and the projected log-earnings for this individual, where projected earnings come from a Mincer regression on workers' demographic characteristics, which we augment to also include industry and state fixed effects.

In addition, the covariance of residual earnings for two consecutive periods is,

$$\begin{aligned} Cov((z_{i\ell t-1}+\eta_{i\ell t-1}),(z'_{i\ell t}+\eta'_{i\ell t})|j,\ell,t-1,t) &= E\left[z\,z'|j,\ell,t-1,t\right]-E\left[z|j,t-1\right]E\left[z'|\ell,t\right], \quad (20)\\ E\left[z\,z'|j,\ell\right] &= \int\int z'\,z\,\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)\,f_\ell(z'-\rho\,z)\,\mathrm{d}z\,\mathrm{d}z'. \end{aligned}$$

Note that the empirical counterparts for all these moments (variance and covariance) can be easily computed using data on workers' residual earnings and labor market choices over two consecutive periods.

The difficulty in estimation is that the right-hand side of these moments and of (17) depends not only on the autocorrelation and variance of the income process, but also on a large number of model parameters and endogenous variables that define how people make their location and mobility choices in the model, $\mu_{j\ell,t}(z)$. However, if $\mu_{j\ell,t}(z)$ were known, an estimation procedure that seeks to minimize the distance between model-generated moments conditional on parameters and moments in the data would be easy to implement.

Our estimation uses this idea. Our procedure uses the empirical $\mu_{j\ell,t}(z)$ (or conditional choice probabilities) , which can be estimated using data on workers' mobility and residual income.[32] The empirical $\hat{\mu}_{j\ell,t}(z)$ is already (implicitly) evaluated at the population parameter values for ρ, $\sigma^2_{\epsilon,\ell}$ and $\sigma^2_{\eta,\ell}$.[33] In this way, we can construct a minimum distance estimator (or a simulated method of moments estimator) to minimize the distance between the moments of residual log-earnings in the data and the right-hand side of equations (18), (19) and (20), using values for $\hat{\rho}$, $\hat{\sigma}^2_{\eta,\ell}$, and $\hat{\sigma}^2_{\epsilon,\ell}$, and estimated values for $\hat{\mu}_{j\ell,t}(z)$, for all ℓ. Finally, our estimation also uses the covariance of residual log-earnings between period t and $t-2$ in the model and the data to estimate a value for the persistence parameter ρ. In sum, our estimation procedure uses the estimated conditional choice probabilities from the data, $\hat{\mu}(z)$, to account for the dynamic selection of workers across space due to their persistent income shocks.

To implement our estimation, we use data from different sources. First, we estimate the

[32] The idea of using of conditional choice probabilities in the estimation of a subset of model parameters goes back to Hotz and Miller (1993). Here, we extend this idea to the estimation of random income processes.

[33] Appendix C contains additional details on how we link residual income in the data and the model and how we estimate $\mu_{j\ell,t}(z)$.

conditional choice probabilities using data from the American Community Survey, pooling years 2011-2019.[34] We first construct a measure of residual log-earnings using the same controls and sample selection criteria as discussed in Section 2. In addition, we have data on individuals' state of residence in the current year and a year before. Using this information, we estimate the the mobility matrices $\mu_{j,\ell}(z)$.[35]

Conditional on a value for ρ we only need moments on the variance of residual log-earnings and the covariance of residual log-earnings at t with residual log-earnings at $t-1$ at the individual level by US state. For this, we use data from the CPS-ORG pooling all months from January 2010 to December 2019.[36] As in Section 2, we use individuals that we can link from one year to the next and use the same Mincer regression to obtain residual log-earnings. Since in the CPS-ORG we only observe individuals that live in the same residence one year apart, in our estimation procedure we only use model counterparts for the variance and covariance using state stayers when constructing the minimum distance objective function.[37]

Finally, our estimate for ρ uses data on the covariance of residual log-earnings with its first and second-order lag for the whole economy. In the data, we compute the ratio of the covariance of residual log-earnings for individuals at year t and year $t-2$ and the covariance of residual log-earnings for individuals at year t and year $t-1$. For this, we use the Panel Study of Income Dynamics (PSID) and average the years 1991 to 1997. The Mincer regression we use to obtain residual wages is similar to the one used with the CPS-ORG data, except we use decade fixed effects rather than year fixed effects given the sample size in the PSID. The estimated value of this ratio of covariances for individuals who do not move states is 0.934. We use this moment to estimate a value of $\rho = 0.934$, which is close to the estimates of Floden and Lindé (2001),

[34] We access the American Community Survey data via IPUMS (Ruggles et al. (2024)).

[35] When estimating the income process for 1990's, we use mobility estimates for $\mu_{j,\ell}(z)$ using data on yearly interstate mobility from the American Community Survey pooling years 2001-2010, as other datasets either have a too small sample size or lack information on earnings at the individual level. The Census of Population for the year 1990 and 2000 have individual level information on mobility over a period of 5-years, which prevents us from using this data. Since mobility patterns do not change widely between 1990 and 2000, we take this is a reasonable approximation. Note, however, that we only use 1990's data when studying convergence in section 5.2.

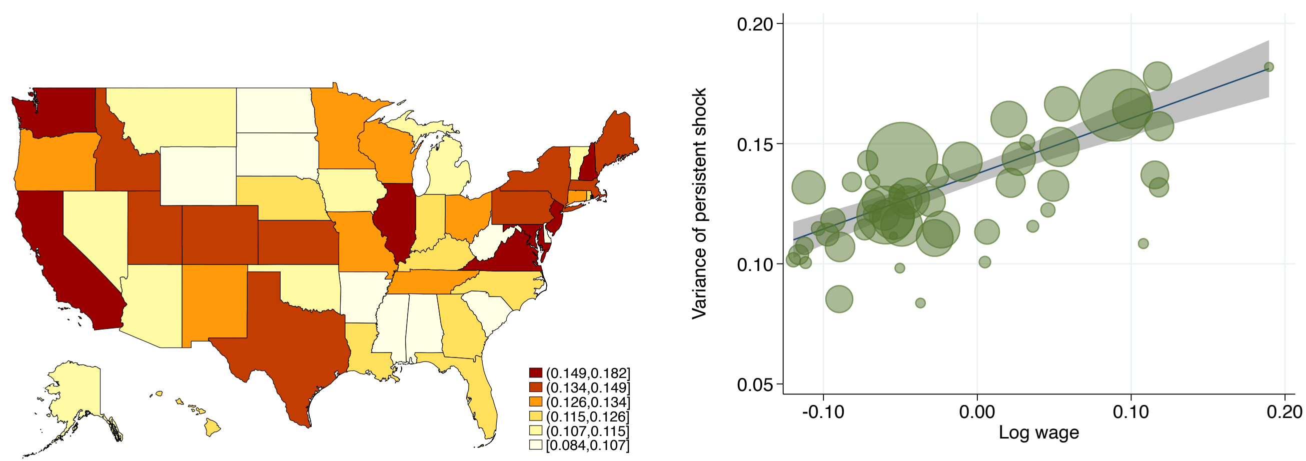
[36] When estimating income process for 1990's, we pool CPS-ORG data for all months in years 1990 to 1999.

[37] While other surveys offer a longer panel dimension, their sample size is too small to adequately estimate the needed variance and covariance for states with few observations.

[Karahan and Ozkan (2013), and Braxton et al. (2025).[38]

Figures 5 and 6 show, respectively, the estimated value of the variance of the persistent and transitory component of residual log-earnings by US state for years 2010-2019, and their correlation with the estimated log-wage for the state. As Figure 5 shows, the variance of the persistent component is larger in the Northeast and West, and in some other states, such as Illinois, Texas, and Minnesota. The variance of the persistent component varies by more than one hundred percent across states. The correlation with the state log wage is quite strong and close to 0.8, and the variance of the persistent component is larger in states where earnings are, on average, higher.

Figure 5: Estimated Variance of Persistent Shock $\sigma^2_\epsilon/(1-\rho^2)$



(a) Geographic distribution (b) Correlation with log wage

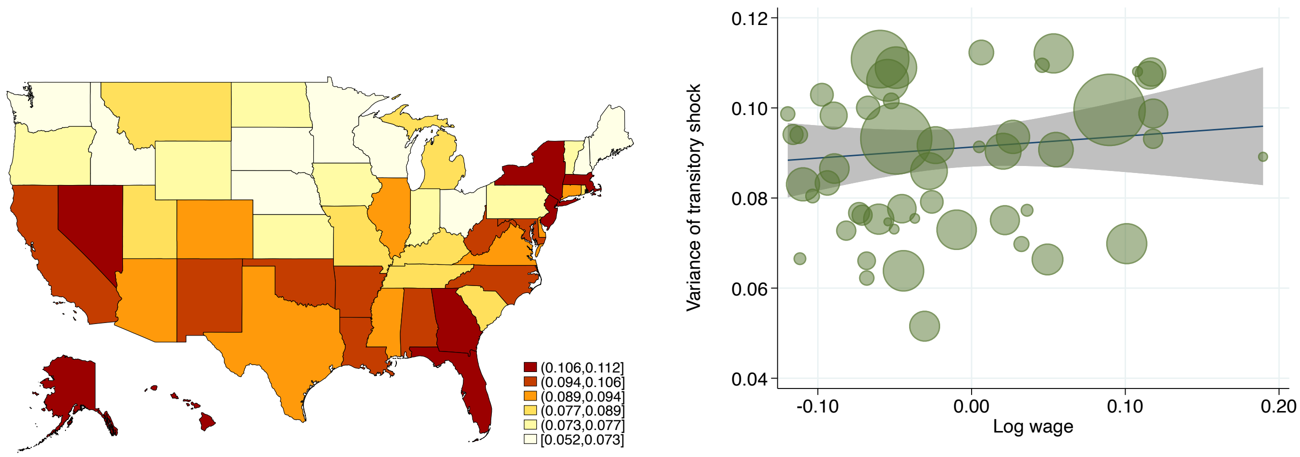
Note: Panel (a) shows the estimated variance of the persistent component of residual log-earnings, z, in each US state, and panel (b) shows its correlation with the estimated log-wage for the state. See the text for estimation details and sample selection.

The variance of the transitory component presents some similarities and some striking differences relative to the persistent component. The variance of the transitory component is larger in the East Coast and parts of the West Coast, but also particularly large in the Southeast. While

[38] The PSID switched to a biennial interview frequency after 1997, which restricts the computation of the covariances to this period. The fact that the ratio of covariances and parameter ρ have an almost identical value should not be surprising given that, as shown in Floden and Lindé (2001), this ratio identifies this parameter given our assumptions on the income process in a model with no migration.

the dispersion across states is somewhat smaller, there is still an important heterogeneity, and the ratio of values for the state with the highest estimated variance to the lowest is around 2. On the other hand, the correlation with the estimated log-wage is weaker, at around 0.2. As the figures implicitly suggest, the correlation between these variances across states is low, with a value slightly above 0.1.

Figure 6: Estimated Variance of Transitory Shock, σ_η^2



(a) Geographic distribution

(b) Correlation with log wage

Note: Panel (a) shows the estimated variance of the transitory component of residual log-earnings, η, in each US state, and panel (b) shows its correlation with the state fixed effect in a Mincer regression of log-earnings that includes also demographic and industry controls. See the text for estimation details and sample selection.

The estimation of unit wages, w_ℓ, uses estimates of the distribution of individuals with shock z across locations, $\bar{\Lambda}_{\ell,t}(z')$, which we obtain with the estimation of the income process. Note that, in the model,

$$E\left[\log(earnings_{i\ell t})|\ell,t\right] \quad = \quad \log(\tilde{w}_{\ell,t}) + E\left[z'_{i\ell t}|\ell,t\right].$$

The last term on the right is the mean value of z in region ℓ, which in general will not be zero due to selection. Then, using estimates of $\bar{\Lambda}_{\ell,t}(z')$, we can compute the expectation $E\left[z'_{i\ell t}|\ell,t\right]$ and using the moments of log-earnings from the data, we can recover $\log(\tilde{w}_{\ell,t})$.[39]

[39]Since worker's demographic characteristics and industry affect earnings, we use the average of the estimated

4.2 Estimation of mobility costs and ν

We estimate mobility costs, amenities, and ν as follows. Recall that $\mathbb{B}_{1,j\ell} = A_\ell - \psi_{j\ell} + \gamma\kappa\,\tilde{y}_\ell - \frac{\kappa^2\gamma^2\tilde{w}_\ell^2}{2}(\sigma^2_{\eta,\ell})$, with $\psi_{jj} = 0$ for all j. Call $\hat{\mathbb{B}}_{1,\ell} = \gamma\kappa\,\tilde{y}_\ell - \frac{\kappa^2\gamma^2\tilde{w}_\ell^2}{2}(\sigma^2_{\eta,\ell})$, such that $\mathbb{B}_{1,j\ell} = A_\ell - \psi_{j\ell} + \hat{\mathbb{B}}_{1,\ell}$. Then, $\hat{\mathbb{B}}_{1,j}$ can be directly pinned down from the estimated values of wages, rents, interest rates, γ, and estimates for $\sigma^2_{\eta,j}$, given the assumption of $\psi_{jj} = 0$.

Using the equilibrium condition for $\mu_{j\ell}$ and Ξ_ℓ from Proposition 1 and assuming a stationary equilibrium, we have,

$$e^{-\gamma\Xi_\ell(z')} = (\tilde{\beta}\bar{\lambda})^{\tilde{r}} e^{-\tilde{r}\,\hat{\mathbb{B}}_{1,\ell} - \tilde{r}A_\ell} e^{\tilde{r}A_\ell}\, e^{-\gamma\kappa\tilde{w}_\ell z'} \left(E_{z''|z',\ell}\left[e^{-\gamma\Xi_\ell(z'')}\right]\right)^{\tilde{r}} (\mu_{\ell\ell}(z'))^{\frac{\tilde{r}}{\nu}}\; J^{\frac{\tilde{r}}{\nu}} \tag{21}$$

where $\tilde{\beta} = \beta(1+r)$ and $\tilde{r} = 1/(1+r)$. Expressed in logs,

$$-\gamma\Xi_\ell(z') = \tilde{r}\log(\tilde{\beta}\bar{\lambda}) - \tilde{r}\,\hat{\mathbb{B}}_{1,\ell} - \gamma\kappa\tilde{w}_\ell z' + \tilde{r}\log\left(E_{z''|z',\ell}\left[e^{-\gamma\Xi_\ell(z'')}\right]\right) + \frac{\tilde{r}}{\nu}\log\mu_{\ell\ell}(z') + \frac{\tilde{r}}{\nu}\log J,$$

which defines a fixed point for $\Xi_\ell(z)$, given values for ν, $\mu(z')$, and $\hat{\mathbb{B}}_{1,\ell}$. Note that amenities do not directly enter this formula.

Take the difference of log-mobility using (7),

$$\begin{aligned}\log(\mu_{j\ell}(z)) - \log(\mu_{jj}(z)) \;&=\; \nu\left(\hat{\mathbb{B}}_{1,\ell} - \hat{\mathbb{B}}_{1,j}\right) + \nu\,(A_\ell - A_j) - \nu\,\psi_{j\ell} \\ &\quad -\nu\left[\log\left(E_{z'|z,\ell}\left[e^{-\gamma\Xi_\ell(z')}\right]\right) - \log\left(E_{z'|z,j}\left[e^{-\gamma\Xi_j(z')}\right]\right)\right].\end{aligned} \tag{22}$$

The left-hand side is known given the estimates of mobility in the previous subsection. Conditional on the value for ν, $\hat{\mathbb{B}}_{1,j}$, and $E_{z'|z,j}\left[e^{-\gamma\Xi_j(z')}\right]$, the right-hand side has $\psi_{j\ell}$, A_ℓ, and A_j, as unknowns.

Then, taking expectations with respect to z conditional on j (as z is associated with the

state-time fixed effect as $E\left[\log(earnings_{i\ell t})|\ell, t\right]$, which we obtain from the Mincer regression we use to obtain residual log-earnings, as discussed in Section 2.

origin labor market), we have,

$$E\left[\log\left(\mu_{j\ell}(z)\right)-\log\left(\mu_{jj}(z)\right)|j\right] = \nu\left(\hat{\mathbb{B}}_{1,\ell}-\hat{\mathbb{B}}_{1,j}\right)+\nu\left(A_\ell-A_j\right)-\nu\,\psi_{j\ell} \\ -\nu E_z\left(\left[\log\left(E_{z'|z,\ell}\left[e^{-\gamma\Xi_\ell(z')}\right]\right)-\log\left(E_{z'|z,j}\left[e^{-\gamma\Xi_j(z')}\right]\right)\right]\Big|j\right). \quad (23)$$

Conditional on a value of ν, we can obtain estimates for $[(A_\ell - A_j) - (1-\tilde{r})\,\psi_{j\ell}]$, which is a composite term that includes amenity differentials and mobility costs.[40] Note that, since we assume that costs and amenities do not vary with z, we cannot perfectly match all mobility flows by z. We only match the average over z.

We estimate the Weibull shape parameter ν to match observed spatial sorting patterns. Since there is complementarity between individual productivity z and wages, high-z households sort into high-wage locations. Higher values of ν correspond to less dispersion in idiosyncratic location preference shocks, so the strength of this sorting pattern is increasing in ν.[41] In other words, the heterogeneity in persistent earnings shocks between individuals and the differences in their incentives to move across regions generate variation in the data that allows us to estimate the parameter ν.[42] Using the empirical migration probabilities $\mu_{j\ell}(z)$ and estimated persistent volatilities $\sigma^2_{\epsilon,j}$ from the first stage of our estimation procedure, we can solve equation (14) to obtain the empirical density of state variables $\overline{\Lambda}_j(z)$. Integrating over this density yields the empirical expected value of z in each location:

$$\mathrm{E}(z_j)=\frac{\int z\overline{\Lambda}_j(z)\mathrm{d}z}{\int \overline{\Lambda}_j(z)\mathrm{d}z}.$$

In other words, since we do not match all $\mu_{j\ell}(z)$ with our estimates of mobility costs, but only the

[40] To separately identify amenities from mobility costs, we require additional data and potentially additional assumptions, as only the composite term $[(A_\ell - A_j) - (1-\tilde{r})\,\psi_{j\ell}]$ is pinned-down with our estimation strategy. One possibility is to regress the composite term to proxies for local amenities, such as weather, crime, or number of restaurants per capita. We do not pursue this here as we do not conduct any exercise that require a separate value for amenities from costs, and thus directly work with the composite term.

[41] As $\nu \to 0$, location decisions are determined entirely by idiosyncratic preferences. In this case, all households choose each location with the same probability, regardless of z. As ν increases, pecuniary considerations become relatively more important and there is stronger sorting by z.

[42] In a model with homogeneous workers, Artuç et al. (2010) use changes in wages and mobility over time to identify the elasticity parameter (in their case, mobility was between industries). We do not use changes over time as heterogeneity in z between workers provides variability in earnings and mobility according to our model.

mean over z, we still have variation in mobility driven by the intensity of sorting. The parameter ν affects the intensity of sorting across locations for workers with different levels of z, and thus the distribution of z over regions.

We pick ν to match the estimated coefficient of a regression of $\mathrm{E}(z_j)$ on log wage for the states. The coefficient of that regression is 0.11 using the estimates of $\mathrm{E}(z_j)$ and log wages from the data. In the model, the value of ν that generates the same regression coefficient is 3.95. In Appendix D (Figure 11), we provide visual evidence that this moment identifies ν. Figure 12 shows the variation in z_j we estimate in the data and the fit of the model given our estimate of ν.

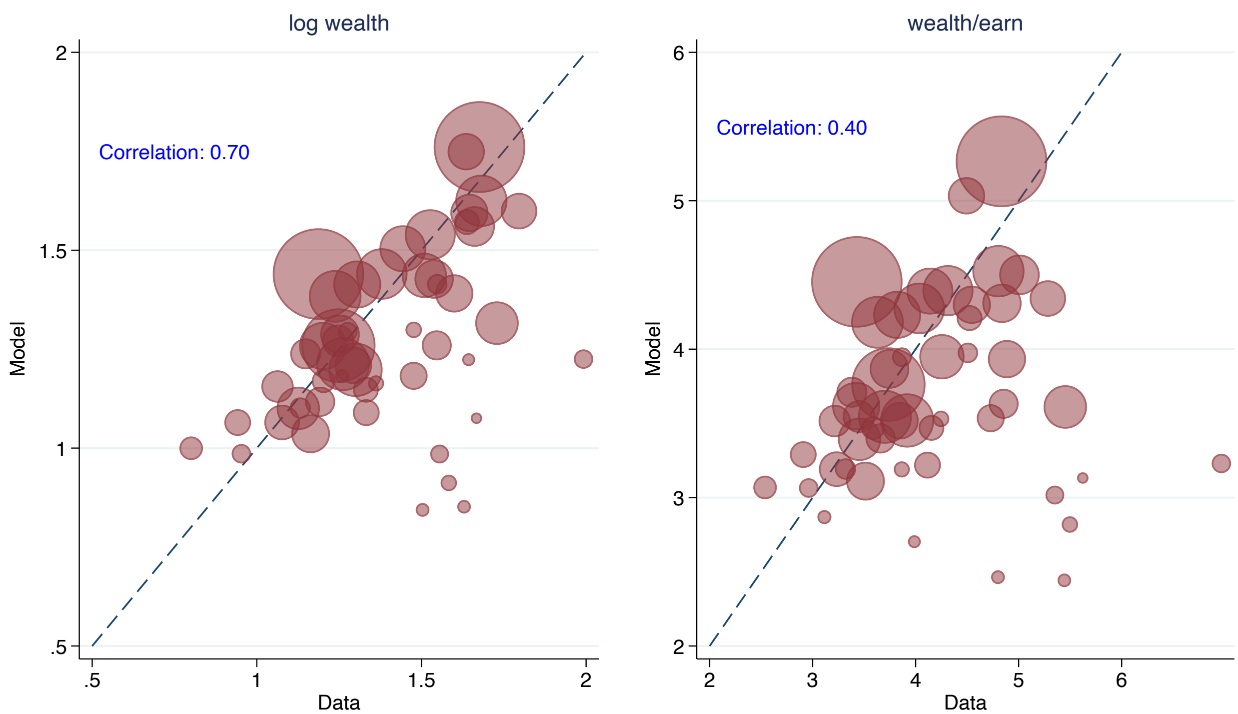
4.3 Model fit

As described previously, we only use earnings and mobility data to estimate most model parameters, while exogenously calibrating rents and the interest rate. We only use data on average wealth to average earnings for the whole nation to calibrate $\bar{\lambda}$, such that the model's economy-wide average wealth matches the level of mean wealth in the data for the US. Here we discuss whether our model is able to match the observed patterns of the state distribution of wealth for the United States.

Figure 7 shows moments of wealth and wealth to earnings by state in the model and the data. A perfect fit of the model would imply that all points lie on the 45 degree line. As the figure shows, the correlation of log mean wealth by state and log mean wealth relative to mean income in the model and the data is very high. The model seems to generate lower levels of wealth than the data for states at the lower end of the distribution. Largely due to this, the dispersion in wealth is larger in the model than in the data (see Table 3). Since we do not use information of wealth by state in our estimation, it is clear that the model captures reasonably well the patterns and magnitudes of the spatial distribution of wealth.

We conclude that our model is able to account for important features of the observed spatial distribution of earnings, sorting, and wealth. In the next section, we analyze the importance of different mechanisms and forces in the model in accounting for the spatial distribution of wealth.

Figure 7: Model fit



Note: The left panel shows log-mean net worth by state in the data for 2014-2019 using the SIPP, and log-mean wealth in our model under the estimated parameter values. Similarly, the right panel shows the mean net worth to mean earnings ratio in the data vis-a-vis the model. The dashed line represents the 45 degree line of perfect fit.

5 Quantitative analysis

As documented in Section 2, the data features substantial variation in wealth and wealth-to-earnings ratios across space. As we have argued, part of this variation can be explained by spatial income differences coupled with mobility shocks or changing preferences for locations. Differences in the volatility of earnings across space also play a role. In this section, we use our model to analyze the forces that shape the spatial wealth distribution.

5.1 Sources of Spatial Inequality

Our theory highlights three key mechanisms that shape the distribution of wealth across space, all of which induce a positive correlation between income and wealth-to-earnings. First, as shown in Figures 5 and 6, we estimate a positive correlation between wage and earnings volatility. Since the precautionary saving motive is stronger in more volatile labor markets, this force induces a positive correlation between wage and wealth-to-earnings. We refer to this mechanism as "correlated risk and wage." Second, as can be seen in the spatial Euler equation (10), idiosyncratic location preference shocks cause individuals in high-wage locations (where the marginal utility of consumption is low) to save against the risk of drawing a strong preference for a low-wage location (where the marginal utility of consumption is high). This mechanism, which we refer to as "spatial Euler equation," also induces a positive correlation between wage and wealth-to-earnings. Finally, complementarity between individual productivity z and wage leads high-productivity individuals to sort into high-wage locations. Due to mean reversion in individual productivity, high-z individuals save at higher rates. This mechanism is illustrated in Figure 4. Similarly to the first two mechanisms, this force induces a positive correlation between wage and wealth-to-earnings. We refer to this mechanism as "sorting."

Table 3 examines the role of various aspects of spatial heterogeneity in shaping the geographic wealth distribution. The first two rows compare the model versus data. We focus on four (untargeted) statistics: the across-state variances of log wealth and wealth-to-earnings ratio, and the estimated coefficients of regressions of log wealth and wealth-to-earnings on log wage.[43] In

[43]The model also makes predictions about the spatial earnings distribution. We match the state earnings data

the data, both wealth and wealth-to-earnings are strongly correlated with wage. The mechanisms described above imply that the model generates these correlations as well. Quantitatively, the model matches all four statistics fairly well.

Table 3: Sources of Spatial Inequality

	Log Wealth		Wealth/Earn	
	Variance	Wage Coefficient	Variance	Wage Coefficient
Data	0.0483	2.79***	0.4407	6.87***
Baseline model	0.0443	2.62***	0.3797	6.27***
One Wage	0.0097		0.1487	
One Persistent Volatility	0.0259	1.71***	0.2019	2.47***
One Transitory Volatility	0.0438	2.59***	0.3755	6.16***
No Mobility	0.0580	3.26***	0.5500	9.51***

Note: This table shows the across-state variances of log wealth, log earnings, and log wealth-to-earnings in the data, estimated model, and various counterfactuals. Observations are weighted by observed population share. In the "one wage" counterfactual, all states have the population-weighted wage. In this counterfactual, the volatilities σ_ϵ^2 and σ_η^2 are adjusted so that earnings volatility are unchanged relative to the estimated baseline. In the "one persistent volatility" and "one transitory volatility" counterfactuals, all states have the population-weighted average persistent and transitory volatilities, respectively. A p value less than 0.10 is denoted by *, less than 0.05 by **, and less than 0.01 by ***. The number of observations for the computation of the variance and in each regression is 51, corresponding to US states plus DC.

Rows three to five of Table 3 show results when various aspects of spatial heterogeneity are eliminated. We successively equalize wages w_j, persistent volatilities $\sigma_{\epsilon j}^2$, and transitory volatilities $\sigma_{\eta j}^2$ across locations. In each case, the relevant variable is replaced by the population-weighted average. For the "one wage" counterfactual, we adjust volatilities $(\sigma_\epsilon^2, \sigma_\eta^2)$ to $(\sigma_{\epsilon j}^2, \sigma_{\eta j}^2) \cdot (w_j/\overline{w})^2$, where $\overline{w}$ is the average wage, so that conditional earnings volatilities $w_j^2(\sigma_\epsilon^2 + \sigma_\eta^2)$ are unchanged. These rows show that wage heterogeneity is the most important source of spatial wealth inequality. When wage differences are eliminated, the variance of log wealth falls by 78% (from 0.0443 to 0.0097) and the variance of wealth-to-earnings falls by 61% (from 0.38 to 0.15). Shutting down differences in persistent volatility has a smaller, but still meaningful, effect. This counterfactual effectively shuts down the "correlated risk and wage" mechanism, leading to

quite closely: the across-state correlation between log earning in our model and the data is 99.7%. This is perhaps not surprising, as our estimation targets both average earnings (with wages) and important aspects of spatial sorting (with ν: see Section 4.2). Here we focus on the spatial wealth and wealth-to-earnings distributions, which are not targeted in our estimation.

less variance and lower correlation between wealth and wage. Eliminating transitory volatility differences also reduces spatial wealth inequality and the correlation between wealth and wage, but the effects are quantitatively small.

The final row of Table 3 shows results when migration is not allowed.[44] This counterfactual over-predicts spatial wealth variation and the correlation between wealth and wages. This is perhaps surprising, as neither the "spatial Euler equation" nor "sorting" mechanisms operate in this scenario. For reasons described below, this is because the no-mobility model predicts a much stronger effect of volatility on saving (the "correlated risk and wage" channel) than the full model.

In Table 4, we further explore the determinants of spatial wealth patterns by computing the elasticity of wealth and wealth-to-earnings with respect to local parameters. To compute the elasticity of a variable y with respect to x, we first compute the elasticity of y_j with respect to x_j for every location j. The table reports the population-weighted average of these elasticities.[45] We further decompose elasticities into two components. The first component is the elasticity when the distribution of workers with different characteristics across states is held constant ("fix $\Lambda(j, z)$"). This component shuts down sorting effects and isolates the effects of a parameter *conditional* on z. The second component is the elasticity when the outcome of interest is held constant ("fix $y(j, z)$"). This component isolates the effects of a parameter via its impact on sorting patterns.

The first panel of Table 4 shows elasticities with respect to local wage.[46] Both wealth and earnings respond more than one-to-one to wage changes. In this case of earnings, this is because complementarity between z and w causes high-z workers to sort into high-wage locations. Both the "spatial Euler equation" and "sorting" mechanisms amplify the effect of wage changes on wealth. As a result, the wealth elasticity is nearly three times as large as the earnings elasticity,

[44] In the no-migration counterfactual, we set all parameter values equal to those from the baseline estimation except $\overline{\lambda}$, which we re-estimate to match the average wealth-to-earnings ratio. We also set the population share of each location equal to the shares from the baseline estimation.

[45] We compute the elasticity of y_j with respect to x_j as follows. First, we compute the counterfactual y'_j when x_j is replaced by $x'_j = 1.01x_j$. The local elasticity is $[\log(y'_j) - \log(y_j)]/\log(1.01)$.

[46] Similarly to the "one wage" counterfactual of Table 3, when computing elasticities with respect to wages we adjust σ_ϵ and σ_η so that earnings volatilities are unchanged.

Table 4: Elasticities

	Wealth	Earning	Wealth/Earning
Wage			
Elasticity	3.28	1.12	2.16
Elasticity, fix $\Lambda(j, z)$	3.14	1.00	2.14
Elasticity, fix $y(j, z)$	0.15	0.12	0.02
Persistent Volatility			
Elasticity	0.22	-0.00	0.22
Elasticity, fix $\Lambda(j, z)$	0.17	0.00	0.17
Elasticity, fix $y(j, z)$	0.05	-0.00	0.05
Transitory Volatility			
Elasticity	0.01	0.00	0.01
Elasticity, fix $\Lambda(j, z)$	0.01	0.00	0.01
Elasticity, fix $y(j, z)$	0.00	0.00	0.00

Note: This table shows the elasticities of local wealth, earnings, and wealth-to-earnings with respect to wages, persistent volatility, and transitory volatility. See the text for details regarding how these elasticities are computed.

implying a wealth-to-earnings elasticity of approximately 2. The final two rows of this panel show that the vast majority of the effects of wages are due to saving responses *conditional* on z. The sorting effect is much smaller in comparison. That is, the "spatial Euler equation" mechanism is quantitatively much more important than the "sorting" mechanism.

The second panel of Table 4 shows elasticities with respect to the persistent volatility parameter $\sigma^2_{\epsilon j}$. As expected, the wealth elasticity is positive because the strength of the precautionary saving motive is increasing in risk. Similar to wage, most of the elasticity is accounted for by changes conditional on z, not sorting. The wealth elasticity with respect to persistent volatility is less than one-tenth of the elasticity with respect to wage. However, it is larger than the elasticity with respect to transitory volatility (panel 3), which is nearly 0. The intuition is that individuals do not need to save a large amount to insure against the effects of transitory shocks. Since the effects of persistent shocks have a longer impact over time, individuals need to save more to effectively insure against them. A similar intuition applies to the spatial Euler equation effect. Shocks that trigger mobility across regions are typically very persistent, as interstate mobility is around 2.5% per year on average. Therefore, savings are very responsive to this type of uncertainty.

In order to understand the differences between the full and no-mobility models, it is instructive to compare their elasticities. The elasticities for the no-mobility model are shown in Table 5. In sharp contrast to the full model, the elasticity of wealth with respect to wage in the no-mobility model is exactly 0. This is consistent with the logic of Caballero (1990). When there is no mobility, a wage change in a location is effectively a permanent income change. Since consumption responds one-to-one to permanent income, permanent income changes do not affect savings or wealth.

Table 5: Elasticities: No-Mobility Model

	Wealth	Earning	Wealth/Earning
Wage			
Elasticity	0.00	1.00	-1.00
Elasticity, fix $\Lambda(j,z)$	0.00	1.00	-1.00
Elasticity, fix $y(j,z)$	0.00	0.00	0.00
Persistent Volatility			
Elasticity	0.86	0.00	0.86
Elasticity, fix $\Lambda(j,z)$	0.86	0.00	0.86
Elasticity, fix $y(j,z)$	0.00	0.00	0.00
Transitory Volatility			
Elasticity	0.04	0.00	0.04
Elasticity, fix $\Lambda(j,z)$	0.04	0.00	0.04
Elasticity, fix $y(j,z)$	0.00	0.00	0.00

Note: This table shows the elasticities of local wealth, earnings, and wealth-to-earnings with respect to wages, persistent volatility, and transitory volatility in the no-mobility model. See the text for details regarding how these elasticities are computed.

On the other hand, the elasticity of wealth with respect to persistent volatility is roughly four times higher in the no-mobility model than the full model. As a result, the “correlated risk and wage” mechanism is much stronger in the no-mobility model. In fact, it is so strong that the no-mobility model over-predicts the relationship between wealth-to-earnings and wage (see Table 3), despite having no role for either the “spatial Euler equation” or “sorting” mechanisms. The elasticity of wealth with respect to transitory volatility is also higher in the no-mobility model, but is quite small in both.

These results demonstrate that space plays an important role in shaping household responses to individual risk. Specifically, saving is less responsive to individual productivity risk than it would be if there were no migration option. Intuitively, households have two ways to insure against an adverse income shock in the full model: draw down savings, or move to a less expensive location.[47] In the no-mobility model, the only adjustment mechanism is a change in savings. As a result, precautionary savings is more responsive to risk in the no-mobility model.

This logic is similar to that highlighted by Bilal and Rossi-Hansberg (2021). They analyze a model with borrowing limits and no mobility frictions.[48] In their model, households respond to a shock by first drawing down wealth and then migrating. Figure 8 shows the response to an individual productivity shock in our model. This figure shows the log difference in the expected value of various outcomes after receiving a negative shock versus the expected outcomes if no shock were realized. We compare responses to the same shock in the state with the 10th percentile wage (Tennessee) versus the state at the 90th percentile (Washington). In our model, households simultaneously adjust both wealth and location in response to a shock. As shown in the top right panel, they draw down wealth in both locations. However, the wealth response is more pronounced in the low-wage location. This is because households there have fewer lower-wage locations to move to, and thus less scope to adjust the "location asset." The bottom left panel shows that expected wage decreases for both households, as they move toward lower-wage locations.[49] The wage response is larger for the household who was initially in the high-wage location. Finally, the bottom right panel shows the probability of being in a different location than the initial one. The household from the high-wage location is more likely to move after a negative shock, as its initial location has become less desirable. The household from the low-wage location is less likely to move, as the shock makes its initial location more desirable.

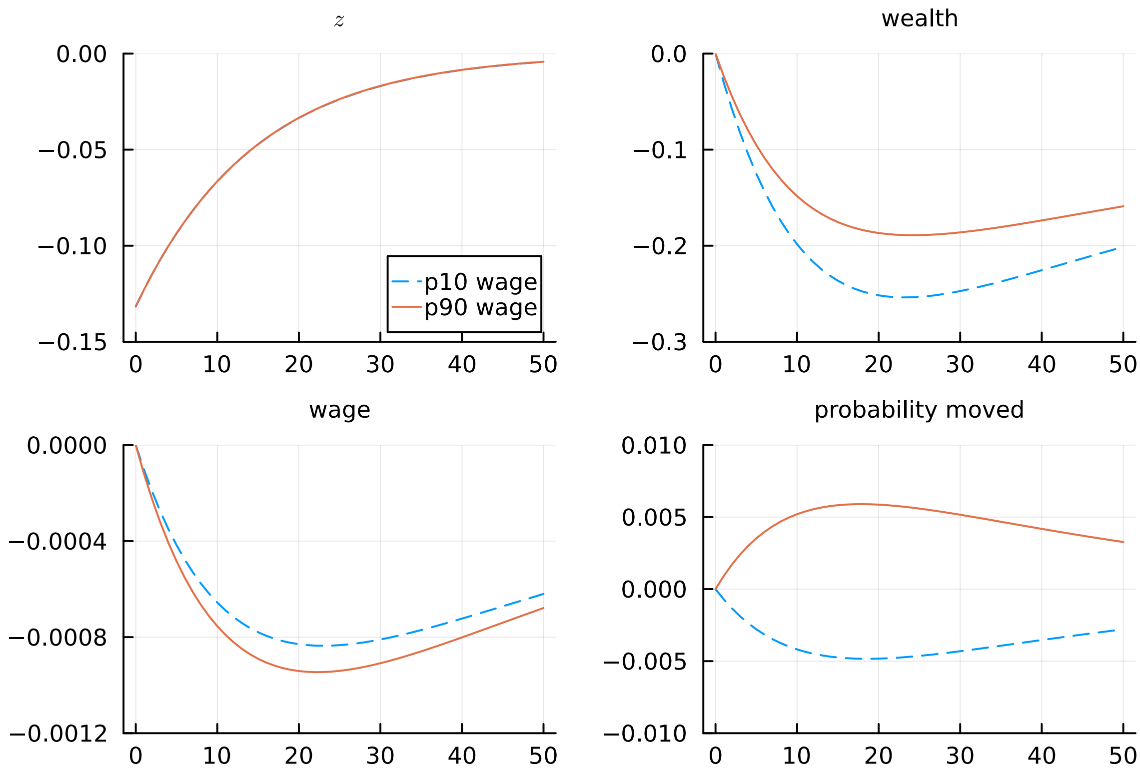
In summary, our model successfully generates key features of the spatial wealth data. Of course, other mechanisms that we abstract from could also be important. However, we view the success of our model as suggesting that the interaction between precautionary saving motives

[47] In the data, wages and house prices are positively correlated. Due to non-homothetic housing demand, low-z households enjoy higher consumption in low-wage, low-price locations than high-wage, high-price locations.

[48] In contrast, our model has no borrowing limit and frictional migration. Another difference is that in our model, risk varies across space.

[49] Note that, due to idiosyncratic location preference shocks, households only move to a lower-wage location *in expectation*. Even after a negative shock, some households will move to higher-wage locations.

Figure 8: Individual response to a shock



Note: This figure shows the log difference between expected outcomes after receiving a negative productivity shock and expected outcomes in the absence of the shock. The shock is a one standard deviation reduction in individual productivity z ($-\sigma_\epsilon$, where σ_ϵ is the population-weighted average standard deviation of innovations to z).

and mobility frictions play a prominent role. Furthermore, our results suggest that economic geography is an important determinant of household saving behavior. In theory, spatial heterogeneity and the option to migrate should affect how much households in different locations save. Our results show that these forces are quantitatively important in a model that is calibrated to match the spatial data.

5.2 Spatial Convergence

There is a large literature that seeks to understand the forces that explain spatial income convergence patterns. In this section, we examine the determinants of spatial wealth convergence patterns through the lens of our model. As documented in Section 2, the data features consistent wealth convergence since 1980. In this section, we focus on the time period 1990-2010, when higher-quality regional mobility, earnings volatility, and wealth data are available.

For this analysis, we estimate an additional steady state using data from 1990-1999. We refer to this steady state as "1990", and the steady state described in Section 4 (which is estimated to match data from 2010-2019) as "2010." We use the same value of ν (3.95) for both steady states. All other parameters are estimated separately for each steady state as described in Section 4.

We start by looking at the convergence of the primitives of our model: wages and volatilities. Table 6 shows the convergence coefficients from regression (1) for wages and volatilities, that is, the intensity and direction of change of state-level wages and volatilities since 1990. There is statistically significant convergence for wages (at the 1% level) and persistent volatility (at the 10% level). The point estimate for transitory persistence is negative but statistically insignificant.

Table 6: Convergence of Spatial Parameters, 1990-2010

Wage	σ_ϵ^2	σ_η^2
-0.19***	-0.20*	-0.13
(0.05)	(0.11)	(0.09)

Note: This table shows the estimated coefficient β from equation (1), which is a regression of the log change of a variable y between 1990-2010 on the log of the variable in 1990. Standard errors are in parenthesis. A p value less than 0.10 is denoted by *, less than 0.05 by **, and less than 0.01 by ***. The number of observations in each regression is 51, corresponding to US states plus DC.

We now seek to explain how much of the changes in these model primitives explain the patterns of convergence in wealth across states. The first row of Table 7 shows earnings and wealth convergence in the estimated model. The model generates statistically significant wealth convergence over this time period, consistent with the empirical evidence presented in Table 1. The next three rows isolate the effects of changes in wages, persistent volatility, and transitory volatility. For each of these exercises, we compute a counterfactual 2010 steady state in which only the relevant variable is changed relative to 1990. These results show that wage changes are by far the most important driver of wealth convergence. In isolation, wage changes deliver a median wealth convergence coefficient of -0.24 (significant at the 1% level). Between 1990-2010, wage convergence has reduced incentives both to save against the risk of moving to a low-wage location and for high-productivity, high-saving workers to sort into high-wage locations. For both reasons, wage convergence leads to wealth convergence in the model. The convergence in persistent volatility shown in Table 6 causes wealth convergence as well. In the case of median wealth, persistent volatility changes generate a convergence coefficient that is nearly half as large as the coefficient caused by wage changes (-0.11, significant at the 1% level). The reason persistent volatility convergence causes wealth convergence in the model is that volatility and wealth are initially correlated. Since the precautionary motive is stronger in more volatile locations, reducing volatility in initially high-volatility locations implies reduced wealth in initially high-wealth locations. For the same reason, transitory volatility changes also generate some wealth convergence. However, this effect is much smaller, both because transitory volatility has a smaller effect on saving behavior and because transitory volatility convergence has been weaker than persistent volatility convergence.

Table 7: Spatial Convergence, 1990-2010

	Δ log-mean earnings	Δ log-mean wealth	Δ log-median wealth
Main Model			
Estimated	-0.17***	-0.15***	-0.21***
Change W	-0.19***	-0.23***	-0.24***
Change σ_ϵ^2	0.02***	-0.04**	-0.11***
Change σ_η^2	0.00***	-0.02***	-0.04***
No-Mobility Model			
Estimated	-0.19***	-0.03	-0.04
Change W	-0.19***	0.00	0.00
Change σ_ϵ^2	0.00	0.05	0.07
Change σ_η^2	0.00	-0.01***	-0.02***

Note: This table shows the estimated coefficient β from equation (1), which is a regression of the log change of a variable y between 1990-2010 on the log of the variable in 1990. We compute medians by simulating a large number of individuals over many periods. A p value less than 0.10 is denoted by *, less than 0.05 by **, and less than 0.01 by ***. The number of observations in each regression is 51, corresponding to US states plus DC.

The bottom panel of Table 7 shows convergence results in the no-mobility model. This model generates similar earnings convergence as the main model. However, in sharp contrast to the full model, it does not generate statistically significant wealth convergence. Whereas wage changes are the most important source of wealth convergence in the full model, they do not generate statistically significant wealth convergence in the no-mobility model. The reason (discussed previously) is that in the no-mobility model, wage changes are isomorphic to permanent income changes and so do not affect savings. Local wage changes only affect saving in the full spatial model, due both to the "spatial Euler equation" and "sorting" channels. Similarly to the full model, changes in earnings volatility have a much smaller effect on convergence patterns than wage changes. The net result is weak, statistically insignificant wealth convergence in the no-mobility model.

In summary, our model is able to generate wealth convergence patterns that are consistent with the data. Most of the convergence generated by the model is due to changes in saving behavior caused by wage changes over time. An alternative version of the model without migration fails to generate statistically significant convergence. This failure of the no-mobility model suggests a key role for migration in shaping the distribution of wealth across space.

6 Conclusion

Our analysis makes several important contributions to understanding the spatial distribution of wealth in the United States. We document novel empirical facts showing substantial variation in wealth-to-earnings ratios across states, with higher ratios concentrated on the East and West coasts and in northern regions, strongly correlated with average earnings by regions. We also establish that both wealth levels and earnings volatility exhibit patterns of convergence over time, similar to the well-documented convergence in average incomes across states. These findings provide new evidence on how geographic factors shape household wealth accumulation beyond simple income differences.

We develop a theoretical framework that extends standard precautionary savings models to incorporate spatial mobility and location choice with heterogeneous agents facing income risk. Our model yields three key mechanisms that generate a positive correlation between wages and wealth-to-earnings ratios: correlated risk and wages, whereby higher volatility in high-wage locations increases precautionary savings; a spatial Euler equation effect, where households in high-wage areas save against the risk of moving to lower-wage locations; and spatial sorting, through which high-productivity workers concentrate in high-wage areas and save more due to mean reversion in their productivity shocks. Despite the model's complexity, we show these mechanisms can be characterized analytically, providing closed-form solutions for consumption, savings, and mobility decisions.

Our quantitative analysis demonstrates that the model successfully matches key features of the spatial wealth data without directly targeting wealth patterns in the estimation procedure. We find that wage heterogeneity emerges as the most important driver of spatial wealth inequality, accounting for the majority of observed variation across states. The spatial Euler equation mechanism proves quantitatively more important than sorting effects in explaining wealth patterns. Notably, we show that a version of our model without mobility fails to generate the observed wealth convergence patterns, highlighting how geographic mobility fundamentally alters household saving behavior compared to standard single-location models.

These results suggest that economic geography plays a crucial role in household financial decisions that have been underappreciated in previous research. We demonstrate that the interaction between precautionary saving motives and mobility frictions creates novel channels through which regional economic conditions affect wealth accumulation. Our findings have implications for understanding both cross-sectional wealth inequality and its evolution over time, as our convergence analysis shows that changes in regional wage differences significantly influence the spatial distribution of wealth. Future research might explore how these mechanisms interact with housing markets and other location-specific factors to further shape the geography of wealth.

References

Artuç, E., Chaudhuri, S., and McLaren, J. (2010). Trade shocks and labor adjustment: A structural empirical approach. *American Economic Review*, 100(3):1008–1045.

Barro, R. J. and Sala-i Martin, X. (1991). Convergence across states and regions. *Brookings Papers on Economic Activity*, pages 107–182.

Barro, R. J. and Sala-i Martin, X. (1992). Convergence. *Journal of Political Economy*, 100(2):223–251.

Baum-Snow, N. and Pavan, R. (2013). Inequality and city size. *Review of Economics and Statistics*, 95(5):1535–1548.

Bilal, A. and Rossi-Hansberg, E. (2021). Location as an asset. *Econometrica*, 89(5):2459–2495.

Blundell, R., Pistaferri, L., and Preston, I. (2008). Consumption inequality and partial insurance. *American Economic Review*, 98(5):1887–1921.

Braxton, J. C., Herkenhoff, K. F., Rothbaum, J. L., and Schmidt, L. (2025). Changing income risk across the us skill distribution: Evidence from a generalized Kalman filter. *forthcoming, American Economic Review*.

Caballero, R. J. (1990). Consumption puzzles and precautionary savings. *Journal of Monetary Economics*, 25(1):113–136.

Cagetti, M. (2003). Wealth accumulation over the life cycle and precautionary savings. *Journal of Business & Economic Statistics*, 21(3):339–353.

Cai, S., Caliendo, L., Parro, F., and Xiang, W. (2022). Mechanics of spatial growth. *NBER Working Paper*.

Caliendo, L., Dvorkin, M., and Parro, F. (2019). Trade and labor market dynamics: General equilibrium analysis of the China trade shock. *Econometrica*, 87(3):741–835.

Christelis, D., Georgarakos, D., Jappelli, T., and Van Rooij, M. (2020). Consumption uncertainty and precautionary saving. *Review of Economics and Statistics*, 102(1):148–161.

Crews, L. (2023). A dynamic spatial knowledge economy. Working Paper.

Desmet, K., Nagy, D. K., and Rossi-Hansberg, E. (2018). The geography of development. *Journal of Political Economy*, 126(3):903–983.

Desmet, K. and Parro, F. (2025). Spatial dynamics. Technical report, National Bureau of Economic Research.

Diamond, R. and Gaubert, C. (2022). Spatial sorting and inequality. *Annual Review of Economics*, 14:795–819.

Dix-Carneiro, R. (2014). Trade liberalization and labor market dynamics. *Econometrica*, 82(3):825–885.

Donald, E., Fukui, M., and Miyauchi, Y. (2025a). Optimal dynamic spatial policy. *Working Paper*.

Donald, E., Fukui, M., and Miyauchi, Y. (2025b). Unpacking aggregate welfare in a spatial economy. Technical report, National Bureau of Economic Research.

Dvorkin, M. (2023). Heterogeneous agents dynamic spatial general equilibrium. *Federal Reserve Bank of St. Louis, Working Paper*, (2023-005).

Fajgelbaum, P. D. and Gaubert, C. (2025). Optimal spatial policies. Technical report, National Bureau of Economic Research.

Floden, M. and Lindé, J. (2001). Idiosyncratic risk in the United States and Sweden: Is there a role for government insurance? *Review of Economic Dynamics*, 4(2):406–437.

Ganong, P. and Shoag, D. (2017). Why has regional income convergence in the us declined? *Journal of Urban Economics*, 102:76–90.

Gaubert, C., Kline, P., Vergara, D., and Yagan, D. (2021). Trends in US spatial inequality: concentrating affluence and a democratization of poverty. *AEA Papers and Proceedings*, 111:520–525.

Giannone, E. (2017). Skilled-biased technical change and regional convergence. *CREI. Unpublished manuscript.*

Giannone, E., Li, Q., Paixao, N., and Pang, X. (2024). Unpacking moving. *Working Paper.*

Greaney, B., Parkhomenko, A., and Van Nieuwerburgh, S. (2024). Dynamic urban economics. *Available at SSRN 4897077*.

Heathcote, J., Perri, F., and Violante, G. L. (2010). Unequal we stand: An empirical analysis of economic inequality in the United States, 1967–2006. *Review of Economic dynamics*, 13(1):15–51.

Hotz, V. J. and Miller, R. A. (1993). Conditional choice probabilities and the estimation of dynamic models. *The Review of Economic Studies*, 60(3):497–529.

Karahan, F. and Ozkan, S. (2013). On the persistence of income shocks over the life cycle: Evidence, theory, and implications. *Review of Economic Dynamics*, 16(3):452–476.

Kleinman, B., Liu, E., and Redding, S. J. (2023). Dynamic spatial general equilibrium. *Econometrica*, 91(2):385–424.

Lamadon, T., Mogstad, M., and Setzler, B. (2019). Income volatility, taxation and the functioning of the US labor market. *IRS - SOI Working Paper*.

Lillard, L. A. and Weiss, Y. (1979). Components of variation in panel earnings data: American scientists 1960-70. *Econometrica*, pages 437–454.

Lillard, L. A. and Willis, R. J. (1978). Dynamic aspects of earning mobility. *Econometrica*, pages 985–1012.

Luccioletti, C. (2023). Should governments subsidize homeownership? Working Paper.

MaCurdy, T. E. (1982). The use of time series processes to model the error structure of earnings in a longitudinal data analysis. *Journal of Econometrics*, 18(1):83–114.

Meghir, C. and Pistaferri, L. (2011). Earnings, consumption and life cycle choices. In *Handbook of Labor Economics*, volume 4, pages 773–854. Elsevier.

Moffitt, R., Abowd, J., Bollinger, C., Carr, M., Hokayem, C., McKinney, K., Wiemers, E., Zhang, S., and Ziliak, J. (2022). Reconciling trends in us male earnings volatility: Results from survey and administrative data. *Journal of Business & Economic Statistics*, 41(1):1–11.

Mongey, S. and Waugh, M. (2024). Discrete choice, complete markets, and equilibrium. *Working Paper*.

Ruggles, S., Flood, S., Sobek, M., Backma, D., Chen, A., Cooper, G., Richards, S., Rodgers, R., and Schouweiler, M. (2024). IPUMS USA: version 15.0 [dataset]. *Minneapolis, Mn: Ipums*, 15.

Sun, J. E. (2024). The distributional consequences of climate change: The role of housing wealth, expectations, and uncertainty. Working Paper.

Suss, J., Kemeny, T., and Connor, D. S. (2024). Geowealth-US: Spatial wealth inequality data for the United States, 1960–2020. *Scientific Data*, 11(1):253.

Traiberman, S. (2019). Occupations and import competition: Evidence from Denmark. *American Economic Review*, 109(12):4260–4301.

Vanhapelto, T. (2022). House prices and rents in a dynamic spatial equilibrium. *Working Paper*.

Wang, N. (2003). Caballero meets Bewley: The permanent-income hypothesis in general equilibrium. *American Economic Review*, 93(3):927–936.

Appendix A Data

A.1 Wealth data

The traditional source of micro-level wealth data for the United States is the Survey of Consumer Finances, produced by the Board of Governors of the Federal Reserve System. Unfortunately, the publicly available sample does not contain information on individuals' location, such as state or city. In our empirical analysis, we use wealth from the Survey of Income and Program Participation (SIPP), which has individual-level data on wealth and many other characteristics.

In this appendix, we compare these two surveys and find that they provide similar information on wealth for the overall US economy, which gives us confidence about our state-level analysis. Then, we describe the Geowealth-US (Suss et al., 2024) data and compare it to the SIPP.

A.1.1 Survey of Income and Program Participation

The Survey of Income and Program Participation is a household-based survey designed as a series of nationally representative panels. Each panel generally features a large sample of households that are interviewed multiple times over a three or four-year period, depending on the year. From 1996 to 2017, the survey is structured as non-overlapping panels, with surveys every 4 months. Beginning in 2018, the survey switched to having overlapping panels interviewed annually. The SIPP provides the most comprehensive information available on how the nations economic well-being changes over time, which has been the SIPP's defining characteristic since 1983. The wealth data is part of the "Assets and Liabilities" topical module available in different waves across the SIPP's panels over time. There is a module with information about financial assets in every year from 1994 to 2021, excluding 2006, 2007, 2008, and 2012. The different asset and liability categories available in the SIPP are not as comprehensive and detailed as those in the Survey of Consumer Finances, as we discuss below.

Table 8: State-Level Net Worth to Household Income Ratios

State	Total net worth over total household income		Non-housing net worth over total household income	
	Mean	Median	Mean	Median
Alabama	2.16	0.77	1.33	0.21
Alaska	2.15	0.61	1.29	0.27
Arizona	2.22	0.61	1.27	0.20
Arkansas	1.97	0.42	1.24	0.12
California	3.12	0.95	1.66	0.35
Colorado	3.21	1.35	1.93	0.44
Connecticut	2.98	1.07	1.80	0.37
Delaware	2.41	0.84	1.24	0.18
District of Columbia	2.63	0.36	1.73	0.27
Florida	2.43	0.56	1.47	0.19
Georgia	2.50	0.65	1.50	0.19
Hawaii	4.20	2.22	2.54	0.70
Idaho	2.70	0.81	1.47	0.27
Illinois	2.60	0.95	1.67	0.35
Indiana	2.42	0.78	1.65	0.27
Iowa	2.83	1.10	2.04	0.52
Kansas	2.55	0.71	1.74	0.36
Kentucky	2.09	0.85	1.34	0.29
Louisiana	2.13	0.75	1.27	0.20
Maine	2.59	0.91	1.54	0.59
Maryland	2.83	1.15	1.89	0.50
Massachusetts	3.28	1.22	1.97	0.49
Michigan	2.49	0.73	1.61	0.22
Minnesota	3.06	1.60	2.17	0.83
Mississippi	1.71	0.52	0.95	0.14
Missouri	2.38	0.71	1.59	0.25
Montana	3.10	1.50	1.75	0.43
Nebraska	2.22	1.09	1.44	0.48
Nevada	2.25	0.82	1.50	0.22
New Hampshire	2.63	1.00	1.76	0.56
New Jersey	2.86	1.16	1.84	0.39
New Mexico	2.42	0.64	1.39	0.18
New York	2.85	0.85	1.69	0.38
North Carolina	2.51	0.90	1.66	0.27
North Dakota	2.66	0.78	1.71	0.42
Ohio	2.41	0.90	1.61	0.30
Oklahoma	2.40	0.77	1.56	0.24
Oregon	2.96	0.90	1.74	0.26
Pennsylvania	2.65	1.06	1.77	0.38
Rhode Island	2.63	0.52	1.69	0.24
South Carolina	2.26	0.86	1.44	0.24
South Dakota	3.47	1.81	2.08	0.82
Tennessee	2.35	0.83	1.41	0.24
Texas	2.27	0.78	1.42	0.23
Utah	2.91	1.24	1.68	0.45
Vermont	3.32	1.59	2.48	0.89
Virginia	2.73	1.14	1.80	0.44
Washington	3.00	1.09	1.76	0.42
West Virginia	2.03	0.78	1.15	0.15
Wisconsin	2.84	0.93	2.00	0.47
Wyoming	3.75	1.76	2.11	1.17
Summary Statistics				
Mean	2.65	0.91	1.64	0.33
Standard deviation	0.37	0.25	0.23	0.14
Median	2.60	0.90	1.66	0.35

Moments computed using SIPP data, 2014-2019. Wealth and income are the sum for the household. Sample restricted to individuals between 25 and 60 years old. Top and bottom 1% of the income and wealth distribution are trimmed.

A.1.2 Survey of Consumer Finances

The Survey of Consumer Finances (SCF) is administered by the Board of Governors and measures the financial characteristics of the US population at the household level. There are two samples in the SCF - the first of which includes about 75% of the observations and is intended to provide good coverage of broad population characteristics such as home ownership. The second sample is made up of relatively wealthy households to help capture the top of the wealth distribution. Sampling is generally done at the level of a "Primary Economic Unit" (PEU) made up of an economically dominant individual or couple and the individuals who are financially dependent on them. Wealth data in the SCF is available every 3 years starting in 1989.

Since not all assets and liabilities in the SCF are included in the SIPP we construct our measures of net-worth and net-worth excluding housing using the following: Business assets; checking and savings, which includes bonds, CDs, money market accounts, and other cash; stock; private IRA; vehicle net worth; house value; residential debt; nonresidential assets; other assets.

A.1.3 Comparing the SIPP and SCF

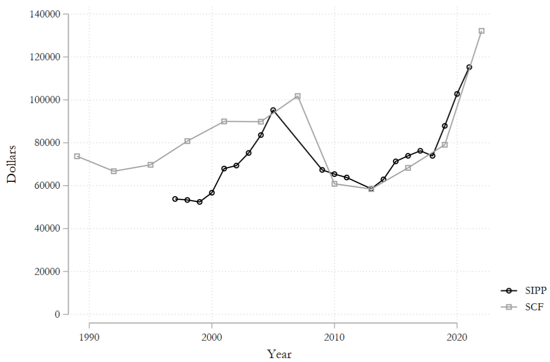
The two main differences between the SIPP and the SCF are: (1) oversampling of the top wealth individuals in the SCF, allowing for a better representation of top wealth distribution, while there is oversampling of lower income individuals in the SIPP, to better capture individuals using government programs; and (2) more detailed asset coverage with larger asset and liabilities categories listed in the SCF. These differences across the surveys translate into differences in the estimated wealth moments. In an effort to better compare moments of wealth across the two surveys, we construct comparable measures of net-worth across surveys using the assets and liabilities that are listed in both. In addition, we focus our attention on moments of the wealth distribution up the 90th percentile, as the SIPP may not capture well the upper end of the wealth distribution.

The figures below show the evolution over time of moments of the median and percentiles 75 and 90 of the wealth distribution across surveys. We deflate wealth by the CPI for that year.

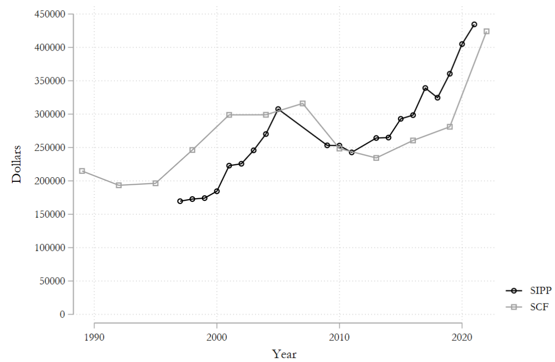
As Figure 9 shows, the moments of net-worth across surveys behave similarly over time and the levels are very close to each other, except for the earlier periods shown in the SIPP.

Figure 9: Moments of the Wealth Distribution in the United States

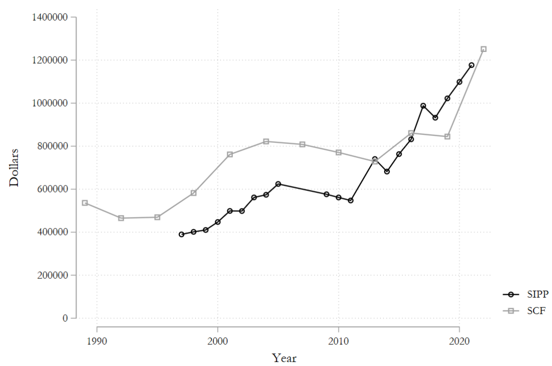
(a) 50th percentile of net worth



(b) 75th percentile of net worth



(c) 90th percentile of net worth



Note: 2007 Dollars. Deflated by CPI. Sources: SIPP, SCF, and authors' calculations.

A.2 Geowealth-US

The Geowealth-US dataset (Suss et al., 2024) provides moments of the wealth distribution and measures of wealth inequality at subnational levels in the United States from 1960 to 2020. It fills a significant gap in the available data for wealth in the United States by offering estimates of key moments of the wealth distribution at various geographic levels.

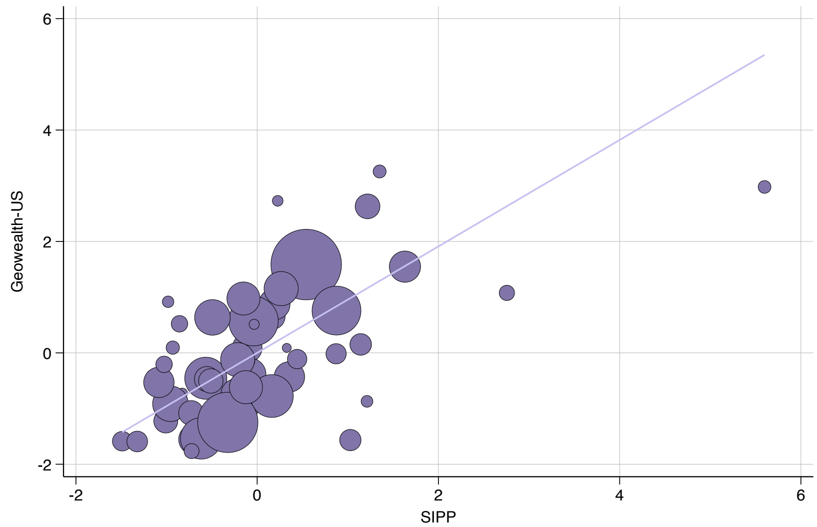
The Geowealth-US dataset is constructed using a sophisticated machine-learning-based imputation framework that links the Survey of Consumer Finances from the Federal Reserve, which contains detailed information on wealth but no region or location information, to microdata from the US Census Bureau, such as public use microdata from different decennial Census of Population and the American Community Survey, which contain detailed information of the location of individuals but no information on wealth. The process consists of four main steps: (1) Modeling Household Wealth: A stacked ensemble of predictive models, including generalized linear regression, elastic net regression, random forest, and gradient boosting machines, is built using data from the SCF to accurately predict household wealth using observable characteristics of individuals. The key requirement is that observables used to predict wealth are present in both types of surveys. (2) Predicting Wealth: The trained models are then applied to impute wealth for each household in the Decennial and American Community Survey (ACS) microdata for different periods of time. (3) Handling Top Wealth Holders: To address the issue of top wealth holders, the dataset employs Pareto tail estimation, a statistical technique designed to model the extreme upper end of the wealth distribution more accurately. (4) Estimating Wealth and Inequality: The imputed data is aggregated to estimate wealth and inequality for widely-used spatial units, such as commuting zones, allowing for detailed geographical analysis.

While the use of imputed data is not ideal, there are ample examples in the macro-labor literature that proceed in a similar way. For example, Blundell et al. (2008) produce imputed measures of non-durable consumption in the Panel Study of Income Dynamics using information and estimates from the Consumer Expenditure Survey. Our main results for the cross-section use information from the SIPP, and we only use Geowealth-US to compute the evolution over time of wealth moments across states and calculate convergence measures.

A.2.1 Correlation with the SIPP

Suss et al. (2024) validate their measures of wealth through comparisons with other established sources of wealth data, showing strong correlations at both the national and state levels. The dataset's estimates for top wealth shares are highly consistent with other national measures, including the raw SCF data and SCF data augmented with the Forbes 400 list. However, for the very wealthiest households, the Geowealth-US model's estimates may differ.

Figure 10: Comparison of mean total net-worth to mean household income ratio



Ratio of mean total net-worth to mean income by state in deviations from the national average. Moments computed using Geowealth-US for the year 2020 and the SIPP, pooling years 2014-2019. In the SIPP, wealth and income are the sum for the household and the sample restricted to individuals between 25 and 60 years old.

At the state level and for mean state wealth, the Geowealth-US data shows a strong correlation with measures from the US Census Bureau's Survey of Income and Program Participation (SIPP). For 2020, the correlation coefficient with the SIPP data is very high for both mean and median wealth, providing robust validation for the subnational estimates.

Since our sample selection criteria in the SIPP can affect this correlation, we compare the correlation of the wealth to income ratio across states in the SIPP and Geowealth-US. Figure 10 shows the scatterplot of the ratio of the mean total net worth to mean income by state across the two datasets. The SIPP cannot capture well the upper end of the wealth distribution, and this generates differences in the level of mean wealth across the two surveys for the whole economy. To control for this, the figure shows the ratios by state in deviation from the ratio for the nation. As the figure shows, the correlation is quite high, around 0.63. Moreover, a simple regression of the Geowealth-US measure and the SIPP measure has a regression coefficient of 0.96, which indicates that a change in the ratio computed using the SIPP predicts almost an identical change in the ratio in the Geowealth-US.

Appendix B Proofs

B.1 Proof of Proposition 1

Equations (2) and (3) describe the problem of the worker.

$$V_{j,t}(a,z,\varepsilon) = \max_{\ell}\left\{\varepsilon_\ell e^{\psi_{j\ell}} E_{\eta',z'|z,\ell}\left[\max_{c_{j\ell,t},a_{j\ell,t+1}}\left\{\left(-\frac{1}{\gamma}\right)e^{-\gamma\, c_{j\ell,t}-A_\ell} + \beta\left[E_{\varepsilon'}\left[V_{\ell,t+1}(a_{j\ell,t+1},z',\varepsilon')\right]\right]\right\}\right]\right\}$$

$$\text{s.t.:}\qquad c_{j\ell,t} + \frac{1}{(1+r_t)}\,a_{j\ell,t+1} + q_{\ell,t} = w_{\ell,t}(1-\tau)\,(1+z'+\eta') + a$$

Begin with the optimal choice of consumption and savings after the reallocation decision has occurred. We can write the first-order condition for consumption and savings as,

$$e^{-\gamma\, c_{j\ell,t}(a,z',\eta')-A_\ell} = \beta(1+r_t)\,\frac{\partial E_{\varepsilon'}\left[V_{\ell,t+1}(a_{j\ell,t+1}(a,z',\eta'),z',\varepsilon')\right]}{\partial a_{j\ell,t+1}(a,z',\eta')}. \tag{24}$$

Since the realization of preference shock ε_ℓ for the chosen labor market multiplies the period and continuation utility, it does not affect differentially consumption and savings and does not show up in the first-order condition.

We now use a guess-and-verify strategy. Assume that optimal consumption, savings and ex-ante value function, $v_{j,t}(a,z) = E_\varepsilon\left[V_{j,t}(a,z,\varepsilon)\right]$, take the following form,

$$\begin{aligned}
c_{\ell,t}(a,z',\eta') &= \Xi_{\ell,t}(z') + \kappa_t\,(a + \tilde{y}_{\ell,t} + \tilde{w}_{\ell,t}\,\eta') \\
a_{\ell,t+1}(a,z',\eta') &= (1+r_t)\,(\tilde{w}_{\ell,t}\,z' - \Xi_{\ell,t}(z')) + (1+r_t)(1-\kappa_t)\,(a + \tilde{y}_{\ell,t} + \tilde{w}_{\ell,t}\,\eta') \\
v_{j,t}(a,z) &= -\frac{1}{\gamma}\frac{1}{\kappa_t}\,e^{-\gamma\,\kappa_t\,a}\,\Omega_{j,t}(z)
\end{aligned}$$

where, given the guess for consumption, next period assets are obtained using the budget constraint. Also, given the guess for the ex-ante value function, we have

$$\frac{\partial E_{\varepsilon'}\left[V_{\ell,t+1}(a_{\ell,t+1}(a,z',\eta'),z',\varepsilon')\right]}{\partial a_{\ell,t+1}(a,z',\eta')} = \frac{\partial v_{\ell,t+1}(a_{\ell,t+1}(a,z'\eta'),z')}{\partial a_{\ell,t+1}(a,z',\eta')} = -\gamma\,\kappa_{t+1} v_{\ell,t+1}(a_{\ell,t+1}(a,z',\eta'),z'). \tag{25}$$

The unknown functions $\Xi_{\ell,t}(z')$ and κ_t can be obtained by matching coefficients after taking the logarithm in the first-order condition (24),

$$\begin{aligned}-\gamma\left[\Xi_{\ell,t}(z')+\kappa_t\left(a+\tilde{y}_{\ell,t}+\tilde{w}_{\ell,t}\,\eta'\right)\right]-A_\ell &= \log\left(\beta(1+r_t)\right)+\log\left(\Omega_{\ell,t+1}(z')\right)\\ &\quad -\gamma\kappa_{t+1}\,\left[(1+r_t)\left(\tilde{w}_{\ell,t}\,z'-\,\Xi_{\ell,t}(z')\right)+(1-\kappa_t)(1+r_t)\left(a+\tilde{y}_{\ell,t}+\tilde{w}_{\ell,t}\,\eta'\right)\right].\end{aligned}$$

Then,

$$\begin{aligned}\kappa_t &= \frac{\kappa_{t+1}(1+r_t)}{1+\kappa_{t+1}(1+r_t)}\\ \Xi_{\ell,t}(z') &= -\frac{1}{\gamma}\left(\frac{1}{1+\kappa_{t+1}(1+r_t)}\right)\left[\log\left(\beta(1+r_t)\right)+\log\left(\Omega_{\ell,t+1}(z')\right)+A_\ell-\gamma\kappa_{t+1}(1+r_t)\tilde{w}_{\ell,t}\,z'\right].\end{aligned} \tag{26}$$

Using (24) and (25), we get that,

$$e^{-\gamma\,c_{\ell,t}(a,z',\eta')-A_\ell}=\beta(1+r_t)\left(-\gamma\kappa_{t+1}\right)\,E_{\varepsilon'}\left[V_{\ell,t+1}(a_{\ell,t+1}(a,z',\eta'),z',\varepsilon')\right]. \tag{27}$$

Substituting into the value function of the worker under optimal consumption/savings choices, and taking expectations over ε, we have,

$$v_{j,t}(a,z)=E_\varepsilon\left[\max_\ell\left\{\varepsilon_\ell e^{\psi_{j\ell}}E_{\eta',z'|z,\ell}\left[\left(-\frac{1}{\gamma}\right)\frac{1}{\kappa_t}e^{-\gamma c_{\ell,t}(a,z',\eta')-A_\ell}\right]\right\}\right]. \tag{28}$$

Then, replacing consumption for the guess,

$$v_{j,t}(a,z)=\left(\frac{1}{\gamma}\right)\frac{1}{\kappa_t}\,e^{-\gamma\kappa_t\,a}\,E_\varepsilon\left[\max_\ell\left\{-\varepsilon_\ell\,e^{\psi_{j\ell}-A_\ell}E_{\eta',z'|z,\ell}\left[e^{-\gamma\left(\Xi_{\ell,t}(z')+\kappa_t\left(\tilde{y}_{\ell,t}+\tilde{w}_{\ell,t}\,\eta'\right)\right)}\right]\right\}\right]. \tag{29}$$

Finally, given the guess for the ex-ante value function,

$$\begin{aligned}\Omega_{j,t}(z) &= -E_\varepsilon\left[\max_\ell\left\{-\varepsilon_\ell e^{\psi_{j\ell}-A_\ell}E_{\eta',z'|z,\ell}\left[e^{-\gamma\left(\Xi_{\ell,t}(z')+\kappa_t\left(\tilde{y}_{\ell,t}+\tilde{w}_{\ell,t}\,\eta'\right)\right)}\right]\right\}\right]\\ &= \bar{\lambda}\left[\frac{1}{J}\sum_{\ell=1}^{J}e^{\nu A_\ell-\nu\psi_{j\ell}+\nu\,\gamma\kappa_t\tilde{y}_{\ell,t}}\left(E_{\eta',z'|z,\ell}\left[e^{-\gamma\,\Xi_{\ell,t}(z')-\gamma\kappa_t\tilde{w}_{\ell,t}\,\eta'}\right]\right)^{-\nu}\right]^{-1/\nu},\end{aligned} \tag{30}$$

where the last line uses properties of the Weibull distribution. In this way, the guess for consumption, savings and ex-ante value function are verified. Moreover, replacing this last expression one period forward in (26), defines the recursion for $\Xi_{\ell,t}(z')$ in the proposition.

Spatial Euler equation. Note that we can write (24) using (25) and (28) as,

$$e^{-\gamma\, c_{\ell,t}(a,z',\eta')-A_\ell} = \beta(1+r_t)\left(-E_\varepsilon\left[\max_m\left\{-\varepsilon_\ell e^{\psi_{\ell m}}\, E_{\eta'',z''|z',m}\left[e^{-\gamma c_{m,t+1}(a',z'',\eta'')-A_m}\right]\right\}\right]\right),$$

and using properties of the Weibull distribution, we get,

$$e^{-\gamma\, c_{\ell,t}(a,z',\eta')-A_\ell} = \beta(1+r_t)\bar{\lambda}\left[\frac{1}{J}\sum_{\ell=1}^{J} e^{-\nu\psi_{\ell m}}\left(E_{\eta'',z''|z',m}\left[e^{-\gamma c_{m,t+1}(a',z'',\eta'')-A_m}\right]\right)^{-\nu}\right]^{-1/\nu},$$

We already used a property of the Weibull distribution to compute the expectation of the maximum over ε. We also show that properties of the distribution allows us characterize the share of movers. For this, we simplify the notation using the following lemma.

Lemma 1. *Let ε be a vector of size J of i.i.d. random variables distributed Weibull with identical shape parameter $\nu > 0$ and scale parameter $\lambda = \bar{\lambda}\,\frac{J^{1/\nu}}{\Gamma\left(1+\frac{1}{\nu}\right)}$, and let $\mathbb{X} \in \mathbb{R}^J_{(-)}$ be a vector of strictly negative scalars that may vary by labor market ℓ, then the expectation of the maximum of $\varepsilon\,\mathbb{X}_t$ and the share of draws for which element ℓ is maximum are,*

$$\begin{aligned} E_\varepsilon\left[\max_\ell \varepsilon_\ell\,\mathbb{X}_\ell\right] &= -\bar{\lambda}\left(\frac{1}{J}\sum_{\ell=1}^{J}(-\mathbb{X}_\ell)^{-\nu}\right)^{-1/\nu}, \\ \mu_{j\ell,t} = Pr\left(\varepsilon_\ell\,\mathbb{X}_\ell \geq \varepsilon_k\,\mathbb{X}_k;\,\forall k\right) &= \frac{(-\mathbb{X}_\ell)^{-\nu}}{\sum_{\ell=1}^{J}(-\mathbb{X}_\ell)^{-\nu}}. \end{aligned}$$

First, note that $E_\varepsilon\left[\max_\ell \varepsilon_\ell\,\mathbb{X}_\ell\right] = -E_\varepsilon\left[\min_\ell \varepsilon_\ell\,(-\mathbb{X}_\ell)\right]$. Since all elements $\mathbb{X}_\ell$ are negative, the terms $(-\mathbb{X}_\ell)$ are positive. Let $F(x)$ and $f(x)$ be the distribution and density functions of a

Weibull random variable, respectively, then,

$$
\begin{aligned}
E_\varepsilon\left[\min_\ell \varepsilon_\ell\left(-\mathbb{X}_\ell\right)\right] &= \sum_{\ell=1}^{J}\int_0^\infty \varepsilon_\ell\left(-\mathbb{X}_\ell\right)\prod_{k\neq\ell} Pr_{\varepsilon_k}\left(\varepsilon_k(-\mathbb{X}_k)\geq\varepsilon_\ell(-\mathbb{X}_\ell)\right) f(\varepsilon_\ell)\mathrm{d}\varepsilon_\ell \\
&= \sum_{\ell=1}^{J}\int_0^\infty \varepsilon_\ell\left(-\mathbb{X}_\ell\right)\prod_{k\neq\ell}\left[1-F(\varepsilon_\ell(-\mathbb{X}_\ell)/(-\mathbb{X}_k))\right] f(\varepsilon_\ell)\mathrm{d}\varepsilon_\ell \\
&= \sum_{\ell=1}^{J}\int_0^\infty \varepsilon_\ell\left(-\mathbb{X}_\ell\right)\prod_{k\neq\ell} e^{-(\varepsilon_\ell(-\mathbb{X}_\ell)/(-\lambda\mathbb{X}_k))^\nu}\frac{\nu}{\lambda}\left(\frac{\varepsilon_\ell}{\lambda}\right)^{\nu-1} e^{-(\varepsilon_\ell/\lambda)^\nu}\mathrm{d}\varepsilon_\ell \\
&= \sum_{\ell=1}^{J}\int_0^\infty (-\mathbb{X}_\ell)\nu e^{-\left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu \sum_{k=1}^{J}\left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu}\left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu \mathrm{d}\varepsilon_\ell
\end{aligned}
$$

define $z=\left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu\sum_{k=1}^{J}\left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu$, then using a change of variables,

$$
\begin{aligned}
E_\varepsilon\left[\min_\ell \varepsilon_\ell\left(-\mathbb{X}_\ell\right)\right] &= \sum_{\ell=1}^{J}(-\mathbb{X}_\ell)\lambda\left(\sum_{k=1}^{J}\left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu\right)^{-1-\frac{1}{\nu}}\left[\int_0^\infty z^{1/\nu}e^{-z}\mathrm{d}z\right] \\
&= \lambda\left(\sum_{\ell=1}^{J}(-\mathbb{X}_\ell)^{-\nu}\right)^{-1/\nu}\Gamma\left(1+\frac{1}{\nu}\right).
\end{aligned}
$$

Then, given the assumption for λ,

$$
E_\varepsilon\left[\max_\ell \varepsilon_\ell\,\mathbb{X}_\ell\right] = -E_\varepsilon\left[\min_\ell \varepsilon_\ell\left(-\mathbb{X}_\ell\right)\right] = -\bar{\lambda}\left(\frac{1}{J}\sum_{\ell=1}^{J}(-\mathbb{X}_\ell)^{-\nu}\right)^{-1/\nu}.
$$

Finally, letting $(-\mathbb{X}_\ell)=e^{\psi_{j\ell}-A_\ell}E_{z'|z}\left[e^{-\gamma\left(\Xi_{\ell,t}(z')+\kappa_t\left(\tilde{y}_{j\ell,t}+\tilde{w}_{\ell,t}\,(z'+\eta')\right)\right)}\right]$, then Lemma 1 shows that (30) holds.

To find the mobility probabilities, we follow similar steps.

$$\begin{aligned}
Pr\left[\varepsilon_\ell\, \mathbb{X}_\ell \geq \varepsilon_k\, \mathbb{X}_k; \quad \forall k\right] &= Pr\left[\varepsilon_\ell\,(-\mathbb{X}_\ell) \leq \varepsilon_k\,(-\mathbb{X}_k); \quad \forall k\right] \\
&= \int_0^\infty \prod_{k\neq\ell} Pr_{\varepsilon_k}\left(\varepsilon_k(-\mathbb{X}_k) \geq \varepsilon_\ell(-\mathbb{X}_\ell)\right) f(\varepsilon_\ell)\mathrm{d}\varepsilon_\ell \\
&= \int_0^\infty \prod_{k\neq\ell} \left[1 - F(\varepsilon_\ell(-\mathbb{X}_\ell)/(-\mathbb{X}_k))\right] f(\varepsilon_\ell)\mathrm{d}\varepsilon_\ell \\
&= \int_0^\infty \prod_{k\neq\ell} e^{-(\varepsilon_\ell(-\mathbb{X}_\ell)/(-\lambda\mathbb{X}_k))^\nu}\, \frac{\nu}{\lambda}\left(\frac{\varepsilon_\ell}{\lambda}\right)^{\nu-1} e^{-(\varepsilon_\ell/\lambda)^\nu}\mathrm{d}\varepsilon_\ell \\
&= \int_0^\infty e^{-\left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu \sum_{k=1}^J \left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu}\, \frac{\nu}{\lambda}\left(\frac{\varepsilon_\ell}{\lambda}\right)^{\nu-1}\mathrm{d}\varepsilon_\ell
\end{aligned}$$

define $z = \left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu \sum_{k=1}^J \left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu$, then using a change of variables,

$$\begin{aligned}
Pr\left[\varepsilon_\ell\, \mathbb{X}_\ell \geq \varepsilon_k\, \mathbb{X}_k; \quad \forall k\right] &= \left(\sum_{k=1}^J \left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu\right)^{-1} \left[\int_0^\infty e^{-z}\mathrm{d}z\right] \\
&= \frac{(-\mathbb{X}_\ell)^{-\nu}}{\sum_{k=1}^J (-\mathbb{X}_k)^{-\nu}}.
\end{aligned}$$

Letting $(-\mathbb{X}_\ell) = e^{\psi_{j\ell}} E_{z'|z}\left[e^{-\gamma\,\left(\Xi_{\ell,t}(z') + \kappa_t\left(\tilde{y}_{j\ell,t} + \tilde{w}_{\ell,t}\,(z'+\eta')\right)\right)}\right]$, we have the expression (7) defining mobility.

B.2 Proposition 2

Let $\bar{\Lambda}_{j,t-1}(z)$ be the distribution of workers that at the end of period $t-1$ were located in labor market j and had a labor supply shock z. We take this distribution to be conditional on the period, so that integration over labor market and z adds to one. Then, the density of workers that at the end of period t located in labor market ℓ and had a labor supply of $1 + z' + \eta'$ is $\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)\, f_{\epsilon',\ell}(z' - \rho\, z)\, f_{\eta',\ell}$, where $\mu_{j\ell,t}(z)$ is the conditional probability that a worker in j with a past shock z moves to ℓ in period t, $f_{\epsilon',\ell}(z' - \rho\, z)$ is the conditional density that a worker in labor market ℓ with past shock z gets a labor supply shock of z', and $f_{\eta',\ell}$ is the density that the worker gets a shock η'.

A fraction δ of workers leave the economy and are replaced by workers in the same labor market ℓ, but with $z = 0$. Then, equation (14) is the result of integrating the density over j, z and η', taking into account the exit probability and the new workers arriving in the economy,

$$\bar{\Lambda}_{\ell,t}(z') = (1-\delta)\sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}(\eta')\,\mathrm{d}z\,\mathrm{d}\eta' + \delta\sum_{j=1}^{J}\mu_{j\ell,t}(0)f_{\epsilon',\ell}(z')\left(\int \bar{\Lambda}_{j,t-1}(z)\mathrm{d}z\right).$$

Note that the realization for η' is observed after reallocation, and thus there is no selection on this shock. Simplifying this expression for the integral over η' leads to equation (14)

Define the total consumption on the labor market ℓ at the end of period t by workers with labor supply z' as $\bar{c}_{\ell,t}(z')\,\bar{\Lambda}_{\ell,t}(z')$. Similarly, denote $\bar{a}_{\ell,t+1}(z')\,\bar{\Lambda}_{\ell,t}(z')$ as the total assets of the workers, implicitly defining the average asset holdings.

Total consumption and assets are the result of the aggregation of consumption and savings decisions of workers who moved from j to ℓ and face a new realization of z'. Using the optimal decisions from Proposition 1, we have,

$$\begin{aligned}
\bar{c}_{\ell,t}(z')\,\bar{\Lambda}_{\ell,t}(z') &= \sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_{\epsilon',\ell}(z'-\rho\, z)\,(1-\delta)\left(\Xi_{\ell,t}(z') + \kappa_t\tilde{y}_{j\ell,t} + \kappa_t\tilde{w}_{\ell,t}\,\eta' + \bar{a}_{j,t}(z)\right)\mathrm{d}z\mathrm{d}\eta' + \\
&\quad \sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(0)f_{\epsilon',\ell}(z')\,\delta\left(\Xi_{\ell,t}(z') + \kappa_t\tilde{y}_{j\ell,t} + \kappa_t\tilde{w}_{\ell,t}\,\eta' + \bar{a}^0\right)\mathrm{d}z\mathrm{d}\eta' \qquad (31)
\end{aligned}$$

$$\begin{aligned}
\bar{a}_{\ell,t+1}(z')\,\bar{\Lambda}_{\ell,t}(z') &= \sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}(\eta')(1-\delta)\Big[(1+r_t)(1-\kappa_t)\left(\tilde{y}_{j\ell,t} + \tilde{w}_{\ell,t}\,\eta' + \bar{a}_{j,t}(z)\right) \\
&\qquad + (1+r_t)(\tilde{w}_{\ell,t}z' - \Xi_{\ell,t}(z'))\Big]\mathrm{d}z\,\mathrm{d}\eta' + \qquad (32) \\
&+\sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(0)f_{\epsilon',\ell}(z')\, f_{\eta',\ell}(\eta')\delta\Big[(1+r_t)(1-\kappa_t)\left(\tilde{y}_{j\ell,t} + \tilde{w}_{\ell,t}\,\eta' + \bar{a}^0\right) + (1+r_t)(\tilde{w}_{\ell,t}z' - \Xi_{\ell,t}(z'))\Big]\mathrm{d}z\,\mathrm{d}\eta'.
\end{aligned}$$

Where the last line in each of these expressions uses the fact that newborns start with an identical value for assets equal to $\bar{a}^0$. Note that these expressions are equal to (12) and (13) after simplifying the integrals over η' as the density integrates to one and the mean of this variable is zero.

In addition, aggregation over past values of assets is simply the average value of assets at the end of period $t-1$ times the number of individuals in that labor market.

Appendix C Estimation of income process: more details

We provide additional details on how we construct moments of earnings dynamics needed for the estimation of the income process and details on how we estimate mobility matrix $\mu_{j\ell}(z)$.

C.1 Minimum distance and moments

As we mentioned in the main text, a few sets of moments allow us to estimate the parameters that characterize the income process. First, mean log-earnings in the model is, Compute the expectation of log-earnings for the cross-section of workers conditional on labor market,

$$E\left[\log(earnings_{i\ell t})|\ell, t\right] \quad = \quad \log(\tilde{w}_{\ell,t}) + \underbrace{E\left[z'_{i\ell t}|\ell, t\right]}_{f(\rho,\sigma^2_{\epsilon,\ell,t},\mu_{j\ell}(z))},$$

where the expectation for the persistent shock conditional on labor market and workers' characteristics would, in general, not be zero due to selection, and is a function of parameters ρ, $\sigma^2_{\epsilon,\ell,t}$, and the distribution of z in a location (which also depends on mobility μ, see equation 18). At the same time, mean log earnings can easily be computed in the data. Since real world earnings may depend on a richer set of characteristics, such as education and experience, we use the state fixed effect from a Mincer regression that includes a rich set of demographic controls, along with state-time and industry-time fixed effects. We interpret the state fixed effect as the data counterpart to mean log-earnings by state in the model. Thus, conditional on values for ρ, $\sigma^2_{\epsilon,\ell,t}$ and μ, we can pin down a value for $E\left[z'_{i\ell t}|\ell, t\right]$, and obtain $\log(\tilde{w}_{\ell,t})$ as the difference between the data moment and the expectation in the model.

Next, define residual log-earnings in the model as log-earnings minus the conditional expectation. Then, the variance of residual log earnings conditional on labor market, time, is

$$Var\left[\left(\log(earnings_{i\ell t}) - E\left[\log(earnings_{i\ell t})\right)|\ell, t\right]|\ell, t\right] \quad = \quad \underbrace{Var\left[z'_{i\ell t}|\ell, t\right]}_{f(\rho,\sigma^2_{\epsilon,\ell,t},\mu_{j\ell}(z))} + \underbrace{Var\left[\eta'_{i\ell t}\right]}_{\sigma^2_{\eta,\ell,t}},$$

where the variance of the persistent shock is a function of parameters ρ, $\sigma^2_{\epsilon,\ell,t}$, and μ, which characterize the distribution of z over locations (see equation 19).

Similarly, for workers that stay in the same state over two consecutive periods, we can compute the covariance of their earnings in period t and period $t-1$,

$$E\left[\left(\log(earnings_{i\ell t}) - E\left[\log(earnings_{i\ell t})\right]|\ell, t\right)\left(\log(earnings_{i\ell t-1}) - E\left[\log(earnings_{i\ell t-1})\right]|\ell, t-1\right)|\ell, t, t-1\right] = Cov((z_{i\ell t-1} + \eta_{i\ell t-1}), (z'_{i\ell t} + \eta'_{i\ell t})|j, \ell, t-1, t),$$

where the covariance in the model depends on parameters ρ, $\sigma^2_{\epsilon,\ell,t}$, and the distribution of z in a location, as detailed in equation (20).

Then, our estimation algorithm takes a value of ρ and estimates of $\mu_{j\ell}(z)$, and searches for values of $\sigma^2_{\epsilon,\ell,t}$ and $\sigma^2_{\eta,\ell,t}$, to minimize the distance between moments in the data and those from the model. For each candidate value of $\sigma^2_{\epsilon,\ell,t}$ and $\sigma^2_{\eta,\ell,t}$, we estimate the distribution of z across locations over two consecutive periods in a stationary economy, and generate moments in the model. We do this for a large grid of values of ρ from 0.8 to 0.98. Once the minimum distance estimator has optimal values of $\sigma^2_{\epsilon,\ell,t}$ and $\sigma^2_{\eta,\ell,t}$ conditional on a ρ, we compute the covariance of residual earnings of period t and $t-2$ for the whole economy and compare this moment with that of the data. We select the value of ρ for which the moment in the model matches the data counterpart. Since the CPS-ORG only has information about wages of individuals one year apart, we use the PSID to compute this second-order covariance of earnings across all individuals, irrespective of their location. For the variance and covariance of order one, we use data from the CPS-ORG.

C.2 Estimation of mobility matrix $\mu_{j\ell}(z)$

The main difficulty in estimating the mobility matrix $\mu_{j\ell}(z)$ is that the persistent component of residual earnings, z, is unobserved. Residual earnings according to our model, $z + \eta$, on the other hand, have a data counterpart, and estimating a mobility matrix conditional on residual earnings can be done in the usual way. We now describe how we estimate the mobility matrix conditional on z.

Taking logs on both sides of (7) and assuming a stationary equilibrium, we can approximate the difference in log-mobility in the model as

$$\log(\mu_{j\ell}(z)) - \log(\mu_{jj}(z)) = \mathbb{C}_{1,j\ell} + \mathbb{C}_{2,j\ell}\, z + \mathcal{O}_{\ell j}$$

where the expression for log-mobility in the model is not exactly linear in z due to Jensens' inequality and potential nonlinearities in $\Xi(z)$ in the last term of (7) in the numerator. Thus, $\mathcal{O}_{\ell j}$ denotes the approximation error. Parameters $\mathbb{C}_{1,j\ell}$ and $\mathbb{C}_{2,j\ell}$ for all j and ℓ depend on the underlying model parameters and, in a stationary equilibrium, the stationary values of the endogenous variables. Note that the approximation error can be smaller if we use a higher order polynomial in z to approximate the expression.[50]

To obtain a data counterpart to this expression we proceed as follows. In the data, we can estimate mobility matrices across labor markets conditional on observed residual log-earnings $\zeta = z + \eta$. We convey our ideas in an easier way if we assume we can estimate parameters of a multinomial logit using grouped data, such that we have many observations on mobility by origin, destination, and residual log-earnings (for example, by having estimates of mobility in different periods of time, all from the same stationary equilibrium). The estimated empirical multinomial Logit satisfies,

$$\log(\tilde{\mu}_{j\ell,t}(\zeta)) - \log(\tilde{\mu}_{jj,t}(\zeta) = \tilde{\mathbb{C}}_{1,j\ell} + \tilde{\mathbb{C}}_{2,j\ell}\, \zeta + e_{j\ell,t},$$

where constants, $\tilde{\mathbb{C}}_{1,j\ell}$ and $\tilde{\mathbb{C}}_{2,j\ell}$, are origin-destination specific and the tilde denotes that these are estimates from the data. $e_{j\ell,t}$ is a regression error, which may arise due to classical measurement error. As mentioned before, in the data we do not observe z but rather residual earnings, ζ.

Using the definition of residual log-earnings in the expression for mobility in the model, we have,

$$\log(\mu_{j\ell,t}(\zeta)) - \log(\mu_{jj,t}(\zeta)) = \mathbb{C}_{1,j\ell} + \mathbb{C}_{2,j\ell}\, \zeta + u_{j\ell,t},$$

where, $u_{j\ell,t} = -\mathbb{C}_{2,j\ell}\, \eta + e_{j\ell,t}$. This expression links log-mobility differences in the model and

[50]Under different parameter combinations, we computed this expression in our model and found that it is mostly linear around the regions of the state space with the highest density.

the data. It is clear that the estimated parameters using residual earnings may not consistently estimate model parameters $\mathbb{C}_{1,j\ell}$ and $\mathbb{C}_{2,j\ell}$, as the error $u_{j\ell,t}$ is correlated with the regressor ζ.

Given that $\zeta = z + \eta$, and that η is assumed i.i.d., we can map our problem to a OLS estimation under measurement error in the regressor, where the regressor we would ideally use is z but we only have available ζ from the data, which has additional i.i.d. error. In this case, we know that $\tilde{\mathbb{C}}^m_{1,j\ell}$ is a consistent estimator of $\mathbb{C}_{1,j\ell}$, and $\tilde{\mathbb{C}}_{2,j\ell} = \frac{\sigma^2_{z,j}}{\sigma^2_{z,j}+\sigma^2_{\eta,j}} \mathbb{C}_{2,j\ell}$, where $\sigma^2_{\eta,j}$ and $\sigma^2_{z,j}$ are defined in the model.

Thus, if we know the values of $\sigma^2_{z,j}$ and $\sigma^2_{\eta,j}$, and using the empirical estimates $\tilde{\mathbb{C}}_{2,j\ell}$, we can recover a consistent estimate of $\mathbb{C}_{2,j\ell}$. After this, using $\mathbb{C}_{1,j\ell}$ and $\mathbb{C}_{2,j\ell}$, we can obtain model-consistent estimates of mobility, $\mu_{j\ell}(z)$.

Then, in our estimation procedure for the variances of the income process, for each candidate value of $\sigma^2_{z,j}$ and $\sigma^2_{\eta,j}$ evaluated, and using the estimates of $\tilde{\mathbb{C}}_{2,j\ell}$ from the data, we recover $\mathbb{C}_{2,j\ell}$ and construct $\mu_{j\ell}(z)$, and the model counterparts to the variance and covariance of residual log-earnings.[51]

C.3 Estimation results

Figures 5 and 6 show the results of estimation in a map of US states. The following table shows the detailed information, together with summary statistics. $\sigma^2_{\epsilon,j}$, $\sigma^2_{\eta,j}$ are the estimates of the variance of persistent and transitory income shocks, respectively. $E[z|j]$ shows the mean level of the persistent shock in the stationary equilibrium. While the mean value of z is zero in the aggregate, by assumption, due to selection, the mean may differ by location. w_j shows the estimate of unit wages by location.

[51]In practice, we do not estimate a multinomial Logit using grouped data. For each origin state j, we estimate a multinomial logit using individual data for destination states and include residual earnings as a regressor. For this we use data from the American Community Survey (ACS), pooling several years, as described in the text.

Table 9: Income process estimation results

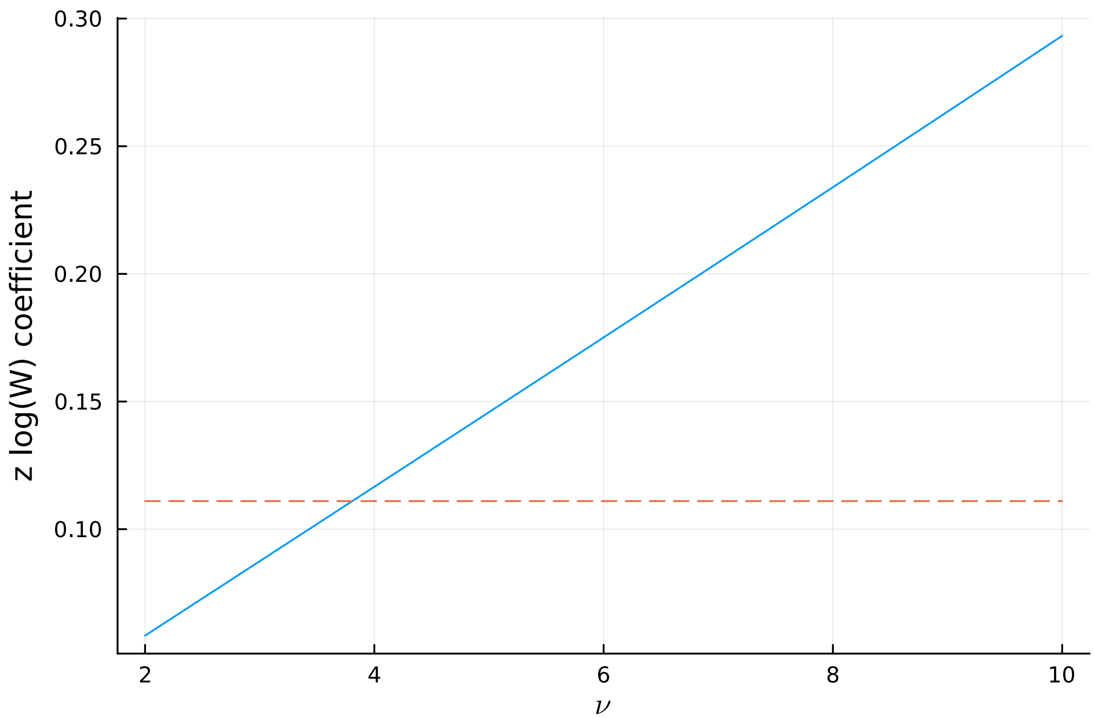
state	$\sigma^2_{\epsilon,j}$	$\sigma^2_{\eta,j}$	$E[z\|j]$	w_j
Alabama	0.011	0.098	-0.007	0.914
Alaska	0.014	0.108	0.015	1.114
Arizona	0.015	0.092	-0.006	0.977
Arkansas	0.013	0.094	-0.010	0.890
California	0.021	0.100	0.011	1.094
Colorado	0.018	0.094	0.006	1.027
Connecticut	0.017	0.093	0.011	1.125
Delaware	0.013	0.091	0.005	1.005
District of Columbia	0.023	0.089	0.073	1.209
Florida	0.015	0.111	-0.011	0.942
Georgia	0.015	0.109	0.002	0.952
Hawaii	0.016	0.109	0.005	1.047
Idaho	0.017	0.073	-0.011	0.922
Illinois	0.020	0.090	0.003	1.021
Indiana	0.015	0.075	-0.008	0.941
Iowa	0.015	0.077	-0.004	0.929
Kansas	0.018	0.076	-0.011	0.931
Kentucky	0.015	0.083	-0.011	0.910
Louisiana	0.016	0.100	-0.002	0.935
Maine	0.017	0.062	-0.008	0.934
Maryland	0.020	0.099	0.013	1.125
Massachusetts	0.017	0.107	0.017	1.122
Michigan	0.014	0.086	-0.008	0.973
Minnesota	0.017	0.066	0.001	1.051
Mississippi	0.014	0.094	-0.018	0.894
Missouri	0.016	0.078	-0.009	0.956
Montana	0.015	0.080	-0.019	0.902
Nebraska	0.015	0.066	-0.006	0.934
Nevada	0.014	0.112	0.002	1.006
New Hampshire	0.019	0.070	0.010	1.033
New Jersey	0.023	0.108	0.016	1.124
New Mexico	0.017	0.102	-0.015	0.949
New York	0.019	0.112	0.012	1.055
North Carolina	0.015	0.106	-0.011	0.947
North Dakota	0.013	0.073	-0.015	0.951
Ohio	0.016	0.064	-0.007	0.957
Oklahoma	0.014	0.103	-0.008	0.907
Oregon	0.017	0.075	-0.008	1.022
Pennsylvania	0.018	0.073	-0.005	0.990
Rhode Island	0.015	0.077	-0.006	1.037
South Carolina	0.014	0.087	-0.004	0.915
South Dakota	0.013	0.067	-0.010	0.894
Tennessee	0.017	0.083	-0.004	0.896
Texas	0.018	0.093	0.004	0.952
Utah	0.017	0.079	-0.007	0.974
Vermont	0.014	0.075	-0.005	0.947
Virginia	0.021	0.091	0.006	1.056
Washington	0.021	0.070	0.012	1.106
West Virginia	0.013	0.099	-0.013	0.887
Wisconsin	0.016	0.052	-0.007	0.970
Wyoming	0.011	0.075	-0.028	0.964
Summary Statistics				
Mean	0.018	0.091	0.001	1.002
Standard deviation	0.003	0.016	0.010	0.072
Median	0.017	0.093	0.002	0.974

The estimation uses pooled CPS-ORG data for years 2010-2019. Sample restricted to non-movers that can be matched over two consecutive years in the survey. Individuals must be employed at the times of the survey, be between 25 and 60 years old, and have non-imputed earnings information. Mobility measures estimated using ACS data, years 2011 to 2019. See the text for details about the estimation procedure. Estimation conditional on a value for $\rho = 0.934$.

Appendix D Additional Tables and Figures

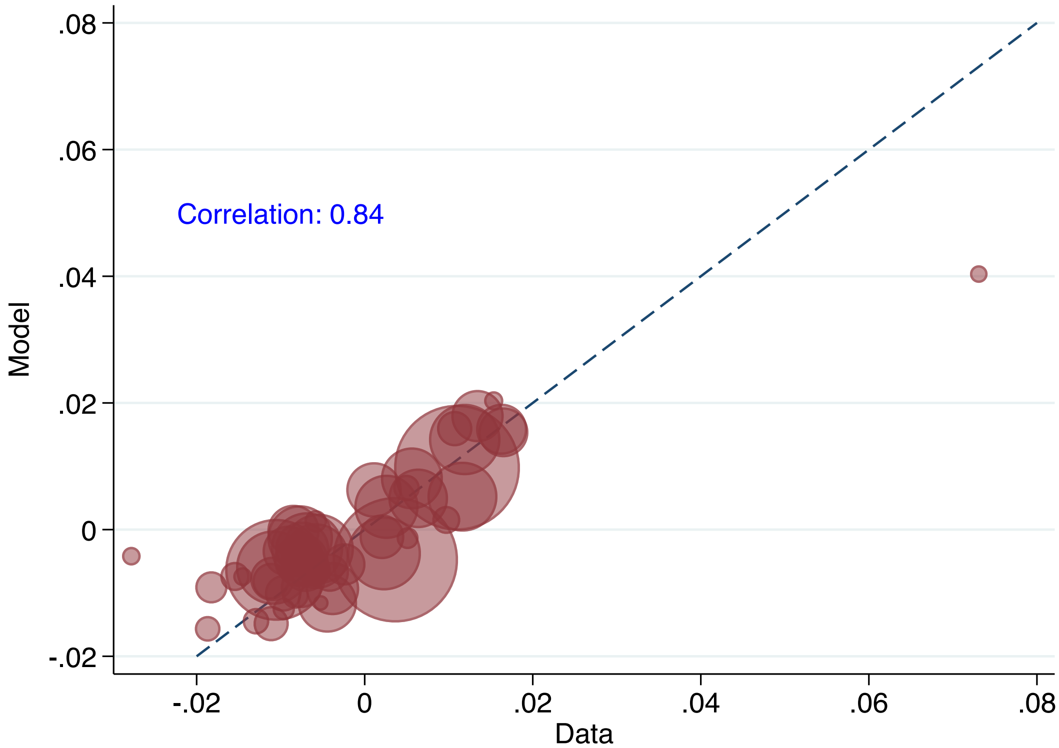
In this appendix, we present additional information about identification of the parameter ν and the patterns of sorting in the model and the data.

Figure 11: Identification of Weibull Shape Parameter ν



Note: This figure shows how the coefficient of a regression of z_j on $\log(W_j)$ varies with the Weibull shape parameter ν. We target this coefficient to estimate ν. z_j is average individual productivity in j, and W_j is wage. For this exercise, we hold $\overline{\lambda}$ fixed at its estimated value (0.37), and re-estimate moving costs Ψ as described in Section 4. The dashed horizontal line shows the empirically estimated coefficient that we target. The estimated value of ν is 3.95.

Figure 12: Average Productivity z_j in Model vs. Data



Note: This figure shows average individual productivity z_j by state in the data versus the estimated model. The empirical values of z_j are estimated in our first-stage estimation as described in Section 4. The dashed line represents the 45 degree line of perfect fit.

Appendix E Extension: Life cycle economy

We now develop a life cycle version of the model. We assume that individuals live for a maximum of S periods, starting their life at age s equal to one. In this version, we abstract from probabilistic death. Earnings have a component that varies deterministically with age Υ_s, so that earnings are equal to $w_{\ell,t}(1-\tau)\Upsilon_s(1+z'+\eta')$. For example, if workers gain experience as they transit through the labor market, then Υ_s increases with age. Newborn workers enter the economy with an amount of assets $\bar{a}_{j,1}$, which can vary by region. In addition, we assume that individuals derive utility from leaving bequest at the end of their lives (warm glow), with a functional form that is also CARA, but with a different parameter.

Let $\tilde{y}_{\ell,s,t} = w_{\ell,t}(1-\tau)\Upsilon_s - q_{\ell,t}$ and $\tilde{w}_{\ell,s,t} = w_{\ell,t}(1-\tau)\Upsilon_s$. Then, we can write the problem of the worker for $s < S$ as,

$$\begin{aligned} V_{j,s,t}(a,z,\varepsilon) &= \max_{\ell} \left\{ \varepsilon_\ell \, e^{\psi_{j\ell,s}} E_{\eta',z'|z,\ell} \left[\max_{c_{j\ell,s,t}, a_{j\ell,s+1,t+1}} \left\{ \left(-\frac{1}{\gamma}\right) e^{-\gamma\, c_{j\ell,s,t} - A_\ell} + \beta \left[E_{\varepsilon'} \left[V_{\ell,s+1,t+1}(a_{j\ell,s+1,t+1}, z', \varepsilon') \right] \right] \right\} \right] \right\} \\ \text{s.t.:} \quad & c_{j\ell,s,t} + \frac{1}{(1+r_t)}\, a_{j\ell,s+1,t+1} = \tilde{y}_{\ell,s,t} + \tilde{w}_{\ell,s,t}(z'+\eta') + a, \end{aligned}$$

and for $s = S$,

$$\begin{aligned} V_{j,S,t}(a,z,\varepsilon) &= \max_{\ell} \left\{ \varepsilon_\ell \, e^{\psi_{j\ell,s}} E_{\eta',z'|z,\ell} \left[\max_{c_{j\ell,S,t}, b_{j\ell,S,t+1}} \left\{ \left(-\frac{1}{\gamma}\right) e^{-\gamma\, c_{j\ell,S,t} - A_\ell} + \bar{\beta} \left(-\frac{1}{\phi}\right) e^{-\phi\, b_{j\ell,S,t+1}} \right\} \right] \right\}, \\ \text{s.t.:} \quad & c_{j\ell,S,t} + \frac{1}{(1+r_t)}\, b_{j\ell,S,t+1} = \tilde{y}_{\ell,S,t} + \tilde{w}_{\ell,S,t}(z'+\eta') + a. \end{aligned}$$

where $b_{j\ell,S,t+1}$ are bequests and $\bar{\beta}$ is a parameter that affects the intensity of the bequest motive.

Proposition 3. *The optimal consumption-savings decisions of the worker, after choosing the location, are characterized by,*

$$\begin{aligned} c_{j\ell,s,t}(a,z',\eta') &= \Xi_{\ell,s,t}(z') + \kappa_{s,t} \left[\tilde{y}_{\ell,s,t} + \tilde{w}_{\ell,s,t}\, \eta' + a \right], \\ a_{j\ell,s+1,t+1}(a,z',\eta') &= (1+r_t)(1-\kappa_{s,t}) \left[\tilde{y}_{\ell,s,t} + \tilde{w}_{\ell,s,t}\eta' + a \right] + (1+r_t) \left[\tilde{w}_{\ell,s,t} z' - \Xi_{\ell,s,t}(z') \right], \qquad \textit{for } s < S, \\ b_{j\ell,S,t+1}(a,z',\eta') &= (1+r_t)(1-\kappa_{S,t}) \left[\tilde{y}_{\ell,S,t} + \tilde{w}_{\ell,S,t}\eta' + a \right] + (1+r_t) \left[\tilde{w}_{\ell,S,t} z' - \Xi_{\ell,S,t}(z') \right]. \end{aligned}$$

Parameter $\kappa_{s,t}$ varies by age s and time, and $\Xi_{\ell,s,t}(z')$ depends on the realization of the shock z', the selected labor market, age, and time. Moreover, the ex-ante lifetime utility and the share of individuals moving from labor market j to ℓ, $\mu_{j\ell,s,t}(z)$ are,

$$v_{j,s,t}(a,z) = \left(-\frac{1}{\gamma}\right) \frac{1}{\kappa_{s,t}} e^{-\gamma\kappa_{s,t}\, a}\, \bar{\lambda} \left(\frac{1}{J} \sum_{\ell=1}^{J} e^{\nu(\mathbb{B}_{1,j\ell,s,t})} \left(E_{z'|z,\ell} \left[e^{-\gamma \Xi_{\ell,s,t}(z')} \right] \right)^{-\nu} \right)^{-(1/\nu)}, \tag{33}$$

$$\mu_{j\ell,s,t}(z) = \frac{e^{\nu(\mathbb{B}_{1,j\ell,s,t})} \left(E_{z'|z,\ell} \left[e^{-\gamma \Xi_{\ell,s,t}(z')} \right] \right)^{-\nu}}{\sum_{m=1}^{J} e^{\nu(\mathbb{B}_{1,jm,s,t})} \left(E_{z'|z,m} \left[e^{-\gamma \Xi_{m,s,t}(z')} \right] \right)^{-\nu}} \tag{34}$$

where $\mathbb{B}_{1,j\ell,s,t} = -\psi_{j\ell,s} + A_\ell + \gamma\kappa_{s,t}\tilde{y}_{\ell,s,t} - (\kappa_{s,t}\gamma\tilde{w}_{\ell,s,t})^2 \sigma^2_{\eta,\ell}/2$ and $\Xi_{\ell,s,t}(z')$ and $\kappa_{s,t}$ are defined *recursively as,*

$$\kappa_{s,t} = \frac{\kappa_{s+1,t+1}(1+r_t)}{1+\kappa_{s+1,t+1}(1+r_t)}, \qquad \textit{for } s < S \tag{35}$$

$$\kappa_{S,t} = \frac{\phi(1+r_t)}{\gamma+\phi(1+r_t)} \tag{36}$$

$$\begin{aligned}\Xi_{j,s,t}(z) &= \left(-\frac{1}{\gamma}\right)\left(\frac{1}{1+\kappa_{s+1,t+1}(1+r_t)}\right)\Bigg(\log\left(\beta(1+r_t)\right) - \gamma\kappa_{s+1,t+1}(1+r_t)\tilde{w}_{j,s,t}z + \log(\bar{\lambda}) + \\ &\quad + A_\ell - \frac{1}{\nu}\log\left[\frac{1}{J}\sum_{m=1}^{J} e^{\nu(\mathbb{B}_{1,jm,s+1,t+1})}\left(E_{z'|z,m}\left[e^{-\gamma\Xi_{m,s+1,t+1}(z')}\right]\right)^{-\nu}\right]\Bigg), \qquad \textit{for } s < S\end{aligned} \tag{37}$$

$$\Xi_{j,S,t}(z) = -\frac{\log\left(\bar{\beta}(1+r_t)\right)+A_\ell}{\gamma+\phi(1+r_t)} + \kappa_{S,t}\tilde{w}_{\ell,S,t}z. \tag{38}$$

Proof. The proof uses backward induction. We begin with the last period of life, S, where the worker chooses a location and then decides how to split available resources between consumption and bequests. Conditional on the location and the realization of the shocks, the first order condition of the problem is,

$$e^{-\gamma c_{j\ell,S,t}-A_\ell} = \bar{\beta}(1+r_t)e^{-\phi b_{j\ell,S,t+1}} \tag{39}$$

Taking logs and using the budget constraint, we have that,

$$c_{j\ell,S,t}(a,z',\eta') = -\frac{\log\left(\bar{\beta}(1+r_t)\right)+A_\ell}{\gamma+\phi(1+r_t)} + \frac{\phi(1+r_t)}{\gamma+\phi(1+r_t)}\tilde{w}_{\ell,S,t}z' + \frac{\phi(1+r_t)}{\gamma+\phi(1+r_t)}\left[\tilde{y}_{\ell,s,t} + \tilde{w}_{\ell,s,t}\,\eta' + a\right],$$

which defines $\Xi_{\ell,S,t}(z)$ and $\kappa_{S,t}$. Optimal bequests are obtained using the expression for consumption and the budget constraint. Using the first order condition and substituting into the expression for the value function, we have that, evaluated at the optimum,

$$\begin{aligned} v_{j,S,t}(a,z) &= E_\varepsilon\left[\max_\ell\left\{\varepsilon_\ell\, e^{\psi_{j\ell,s}-A_\ell}E_{\eta',z'|z,\ell}\left[-\frac{1}{\gamma}e^{-\gamma\, c_{j\ell,S,t}}\left(\frac{\phi(1+r_t)+\gamma}{\phi(1+r_t)}\right)\right]\right\}\right] \\ &= E_\varepsilon\left[\max_\ell\left\{\varepsilon_\ell\, e^{\psi_{j\ell,s}-A_\ell}E_{\eta',z'|z,\ell}\left[-\frac{1}{\gamma\kappa_{S,t}}e^{-\gamma\, c_{j\ell,S,t}}\right]\right\}\right]. \end{aligned}$$

Replacing the optimal consumption and taking expectations, we get,

$$\begin{aligned} v_{j,S,t}(a,z) &= E_\varepsilon\left[\max_\ell\left\{\varepsilon_\ell\, e^{\psi_{j\ell,s}-A_\ell} E_{\eta',z'|z,\ell}\left[-\frac{1}{\gamma\kappa_{S,t}}e^{-\gamma\Xi_{\ell,S,t}(z')-\gamma\kappa_{S,t}\left[\tilde{y}_{\ell,S,t}+\tilde{w}_{\ell,S,t}\,\eta'+a\right]}\right]\right\}\right] \\ &= \frac{1}{\gamma\kappa_{S,t}}e^{\gamma\kappa_{S,t}a}E_\varepsilon\left[\max_\ell\left\{-\varepsilon_\ell\, e^{\psi_{j\ell,s}-A_\ell}e^{-\gamma\kappa_{S,t}\tilde{y}_{\ell,S,t}+(\gamma\kappa_{S,t}\tilde{w}_{\ell,S,t})^2\sigma_\eta^2}\,E_{\eta',z'|z,\ell}\left[e^{-\gamma\Xi_{\ell,S,t}(z')}\right]\right\}\right] \end{aligned}$$

Using the results from Lemma 1,

$$v_{j,S,t}(a,z) = -\frac{1}{\gamma\kappa_{S,t}}e^{\gamma\kappa_{S,t}a}\,\bar{\lambda}\left(\frac{1}{J}\sum_{\ell=1}^{J}e^{-\nu\psi_{j\ell,s}+\nu A_\ell+\nu\gamma\kappa_{S,t}\tilde{y}_{\ell,S,t}-\nu(\gamma\kappa_{S,t}\tilde{w}_{\ell,S,t})^2\sigma_\eta^2/2}\left(E_{\eta',z'|z,\ell}\left[e^{-\gamma\Xi_{\ell,S,t}(z')}\right]\right)^{-\nu}\right)^{-1/\nu}$$

Therefore, the value function for $s=S$ has the form $v_{j,S,t}(a,z) = -\frac{1}{\gamma\kappa_{S,t}}e^{\gamma\kappa_{S,t}a}\,\Omega_{j,S,t}(z)$. In addition, mobility rates are,

$$\mu_{j\ell,S,t}(z) = \frac{e^{\nu\mathbb{B}_{1,j\ell,S,t}}\left(E_{\eta',z'|z,\ell}\left[e^{-\gamma\Xi_{\ell,S,t}(z')}\right]\right)^{-\nu}}{\sum_{m=1}^{J}e^{\nu\mathbb{B}_{1,jm,S,t}}\left(E_{\eta',z'|z,\ell}\left[e^{\gamma\Xi_{m,S,t}(z')}\right]\right)^{-\nu}}.$$

with $\mathbb{B}_{1,j\ell,S,t} = -\psi_{j\ell,s}+A_\ell+\gamma\,\kappa_{S,t}\,\tilde{y}_{\ell,S,t}-(\gamma\,\kappa_{S,t}\,\tilde{w}_{\ell,S,t})^2\sigma_{\eta,\ell}^2/2$.

For any other period s such that $1 \le s \le S-1$, we guess that the value function one period ahead has the form $v_{j,s+1,t+1}(a',z') = -\frac{1}{\gamma\kappa_{s+1,t+1}}e^{\gamma\kappa_{s+1,t+1}a'}\,\Omega_{j,s+1,t+1}(z')$, which is verified for $s=S$. Then, we show that the optimal consumption and savings decision for any other $s<S$ and the resulting value function follow the same structure as the ones for $s=S$, confirming the guess.

Begin with the optimal choice of consumption and savings for a worker of age s at time t after the reallocation decision has occurred. The first-order condition for consumption and savings is,

$$e^{-\gamma\,c_{j\ell,s,t}(a,z',\eta')-A_\ell} = \beta(1+r_t)\,\frac{\partial v_{\ell,s+1,t+1}(a_{j\ell,s+1,t+1}(a,z',\eta'),z')}{\partial a_{j\ell,s+1,t+1}(a,z',\eta')}. \tag{40}$$

Assume that optimal consumption, savings and ex-ante value function next period take the following form,

$$\begin{aligned}
c_{j\ell,s,t}(a,z',\eta') &= \Xi_{\ell,s,t}(z') + \kappa_{s,t}\,(a+\tilde{y}_{j\ell,s,t}+\tilde{w}_{\ell,s,t}\,\eta') \\
a_{j\ell,s+1,t+1}(a,z',\eta') &= (1+r_t)\,[\tilde{w}_{\ell,s,t}\,z' - \Xi_{\ell,s,t}(z')] + (1+r_t)(1-\kappa_{s,t})\,(a+\tilde{y}_{j\ell,s,t}+\tilde{w}_{\ell,s,t}\,\eta') \\
v_{\ell,s+1,t+1}(a',z') &= -\frac{1}{\gamma}\frac{1}{\kappa_{s+1,t+1}}\,e^{-\gamma\,\kappa_{s+1,t+1}\,a'}\,\Omega_{\ell,s+1,t+1}(z')
\end{aligned}$$

Given the guess for the ex-ante value function, we have

$$\frac{\partial v_{\ell,s+1,t+1}(a_{j\ell,s+1,t+1}(a,z'\eta'),z')}{\partial a_{j\ell,s+1,t+1}(a,z',\eta')} = -\gamma\,\kappa_{s+1,t+1}v_{\ell,s+1,t+1}(a_{j\ell,s+1,t+1}(a,z',\eta'),z'). \tag{41}$$

The unknown functions $\Xi_{\ell,s,t}(z')$ and $\kappa_{s,t}$ can be obtained by matching coefficients after taking logarithm in the first order condition ,

$$\begin{aligned}
-\gamma\left[\Xi_{\ell,s,t}(z') + \kappa_{s,t}\left(a+\tilde{y}_{j\ell,s,t}+\tilde{w}_{\ell,s,t}\,\eta'\right)\right] &= \log\left(\beta(1+r_t)\right) + A_\ell + \log\left(\Omega_{\ell,s+1,t+1}(z')\right) \\
&\quad -\gamma\kappa_{s+1,t+1}\,\left[(1+r_t)\,\left(\tilde{w}_{\ell,s,t}\,z' - \Xi_{\ell,s,t}(z')\right) + (1-\kappa_{s,t})(1+r_t)\left(a+\tilde{y}_{j\ell,s,t}+\tilde{w}_{\ell,s,t}\,\eta'\right)\right].
\end{aligned}$$

Then,

$$\begin{aligned}
\kappa_{s,t} &= \frac{\kappa_{s+1,t+1}(1+r_t)}{1+\kappa_{s+1,t+1}(1+r_t)} \\
\Xi_{\ell,s,t}(z') &= -\frac{1}{\gamma}\left(\frac{1}{1+\kappa_{s+1,t+1}(1+r_t)}\right)\left[\log\left(\beta(1+r_t)\right) + A_\ell\log\left(\Omega_{\ell,s+1,t+1}(z')\right) - \gamma\kappa_{s+1,t+1}\,(1+r_t)\,\tilde{w}_{\ell,s,t}\,z'\right].
\end{aligned}$$

Using the first order condition and the derivative of the value function, we get that,

$$e^{-\gamma\,c_{j\ell,s,t}(a,z',\eta')-A_\ell} = \beta(1+r_t)\,(-\gamma\kappa_{s+1,t+1})\;v_{\ell,s+1,t+1}(a_{j\ell,s+1,t+1}(a,z',\eta'),z'). \tag{42}$$

Substituting into the value function of the worker under optimal consumption/savings choices, and taking expectations over ε, we have,

$$v_{j,s,t}(a,z) = E_\varepsilon\left[\max_\ell\left\{\varepsilon_\ell\,e^{\psi_{j\ell,s}-A_\ell}E_{\eta',z'|z,\ell}\left[\left(-\frac{1}{\gamma}\right)\frac{1}{\kappa_{s,t}}e^{-\gamma c_{j\ell,s,t}(a,z',\eta')}\right]\right\}\right]. \tag{43}$$

Then, replacing for the functional form of consumption ,

$$v_{j,s,t}(a,z) = \left(\frac{1}{\gamma}\right) \frac{1}{\kappa_{s,t}}\, e^{-\gamma \kappa_{s,t}\, a}\, E_{\varepsilon}\left[\max_{\ell} \left\{-\varepsilon_{\ell}\, e^{\psi_{j\ell,s} - A_{\ell}} E_{\eta',z'|z,\ell}\left[e^{-\gamma\left(\Xi_{\ell,s,t}(z') + \kappa_{s,t}\left(\tilde{y}_{j\ell,s,t} + \tilde{w}_{\ell,s,t}\, \eta'\right)\right)}\right]\right\}\right]. \tag{44}$$

Using the results in Lemma 1,

$$v_{j,s,t}(a,z) \quad = \quad -\frac{1}{\gamma \kappa_{s,t}} e^{\gamma \kappa_{s,t} a}\, \bar{\lambda} \left(\frac{1}{J}\sum_{\ell=1}^{J} e^{\nu \mathbb{B}_{1,j\ell,s,t}} \, \left(E_{\eta',z'|z,\ell}\left[e^{-\gamma \Xi_{\ell,s,t}(z')}\right]\right)^{-\nu}\right)^{-1/\nu},$$

where $\mathbb{B}_{1,j\ell,s,t} = -\psi_{j\ell,s} + A_{\ell} + \gamma\, \kappa_{s,t}\, \tilde{y}_{\ell,s,t} - (\gamma\, \kappa_{s,t}\, \tilde{w}_{\ell,s,t})^2 \sigma_{\eta,\ell}^2/2$, and the value function for s has the form $v_{j,s,t}(a,z) = -\frac{1}{\gamma \kappa_{s,t}} e^{\gamma \kappa_{s,t} a}\, \Omega_{j,s,t}(z)$. Mobility rates are,

$$\mu_{j\ell,s,t}(z) \quad = \quad \frac{e^{\nu \mathbb{B}_{1,j\ell,s,t}} \, \left(E_{\eta',z'|z,\ell}\left[e^{-\gamma \Xi_{\ell,s,t}(z')}\right]\right)^{-\nu}}{\sum_{m=1}^{J} e^{\nu \mathbb{B}_{1,jm,s,t}} \, \left(E_{\eta',z'|z,\ell}\left[e^{-\gamma \Xi_{m,s,t}(z')}\right]\right)^{-\nu}}.$$

Finally, using the definition of $\Omega_{\ell,s+1,t+1}(z')$, we have,

$$\begin{aligned} \Xi_{\ell,s,t}(z') \quad = \quad & -\frac{1}{\gamma}\left(\frac{1}{1+\kappa_{s+1,t+1}(1+r_t)}\right)\Bigg[\log\left(\beta(1+r_t)\right) + A_{\ell} + \gamma \kappa_{s+1,t+1}\, (1+r_t)\, \tilde{w}_{\ell,s,t}\, z' + \log(\bar{\lambda}) - \\ & \frac{1}{\nu}\log\left(\frac{1}{J}\sum_{m=1}^{J} e^{\nu \mathbb{B}_{1,\ell m,s+1,t+1}} \, \left(E_{\eta',z''|z',m}\left[e^{-\gamma \Xi_{m,s+1,t+1}(z'')}\right]\right)^{-\nu}\right)\Bigg]. \end{aligned}$$

The evolution of the distribution of individuals over labor markets can be computed as follows. Let $\bar{\Lambda}_{j,s-1,t-1}(z)$ be the distribution of workers at the end of period $t-1$, with age $s-1$ in labor market j and shock z. We assume that individuals who leave the economy are replaced by newborns in the same location with $z = 0$. Newborns start with a level of assets that is $\bar{a}_{j,1,t}$ and depends on bequests of location j. Integrating the distribution over labor markets, age, and z adds to one.

The distribution of individuals evolves according to,

$$\bar{\Lambda}_{\ell,s,t}(z') = \sum_{j=1}^{J} \int\int \bar{\Lambda}_{j,s-1,t-1}(z)\mu_{j\ell,s,t}(z) f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}(\eta')\,\mathrm{d}z\,\mathrm{d}\eta', \qquad \text{for } 1 < s \leq S,$$

$$\bar{\Lambda}_{\ell,1,t}(z') = \sum_{j=1}^{J} \int\int \bar{\Lambda}_{j,S,t-1}(z)\mu_{j\ell,1,t}(0) f_{\epsilon',\ell}(z')\, f_{\eta',\ell}(\eta')\,\mathrm{d}z\,\mathrm{d}\eta', \qquad \text{for } s = 1.$$

Note that the realization for η' is observed after reallocation, thus there is no selection on this shock.

Define total consumption in labor market ℓ at the end of period t by workers with labor supply z' and age s, to be $\bar{c}_{\ell,s,t}(z')\,\bar{\Lambda}_{\ell,s,t}(z')$. Similarly, denote $\bar{a}_{\ell,s,t+1}(z')\,\bar{\Lambda}_{\ell,s,t}(z')$ as the total asset holdings of the workers, implicitly defining the average asset holdings.

Total consumption and assets are the result of the aggregation of consumption and savings decisions of workers who moved from j to ℓ and face a new realization of z'. Using the optimal decisions, we have,

$$\begin{aligned}\bar{c}_{\ell,s,t}(z')\,\bar{\Lambda}_{\ell,s,t}(z') &= \sum_{j=1}^{J} \int \bar{\Lambda}_{j,s-1,t-1}(z)\mu_{j\ell,s,t}(z) f_{\epsilon',\ell}(z'-\rho\, z) \times \\ &\qquad \big(\Xi_{\ell,s,t}(z') + \kappa_{s,t}\tilde{y}_{j\ell,s,t} + \kappa_{s,t}\bar{a}_{j,s,t}(z)\big)\,\mathrm{d}z \qquad \text{for } 1 < s \leq S, \\ \bar{c}_{\ell,1,t}(z')\,\bar{\Lambda}_{\ell,1,t}(z') &= \sum_{j=1}^{J} \int \bar{\Lambda}_{j,S,t-1}(z)\mu_{j\ell,1,t}(0) f_{\epsilon',\ell}(z') \times \\ &\qquad \big(\Xi_{\ell,1,t}(z') + \kappa_{1,t}\tilde{y}_{j\ell,1,t} + \bar{a}_{j,1,t}\big)\,\mathrm{d}z \qquad \text{for } s = 1\end{aligned}$$

$$\begin{aligned}\bar{a}_{\ell,s+1,t+1}(z')\,\bar{\Lambda}_{\ell,s,t}(z') &= \sum_{j=1}^{J} \int \bar{\Lambda}_{j,s-1,t-1}(z)\mu_{j\ell,s,t}(z) f_{\epsilon',\ell}(z'-\rho\, z) \times \\ &\big[(1+r_t)\left(\tilde{w}_{\ell,s,t}z' - \Xi_{\ell,s,t}(z')\right) + (1+r_t)(1-\kappa_{s,t})\left(\tilde{y}_{\ell,s,t} + \bar{a}_{j,s,t}(z)\right)\big]\,\mathrm{d}z \qquad \text{for } 1 < s < S, \\ \bar{a}_{\ell,2,t}(z')\,\bar{\Lambda}_{\ell,1,t}(z') &= \sum_{j=1}^{J} \int \bar{\Lambda}_{j,S,t-1}(z)\mu_{j\ell,1,t}(0) f_{\epsilon',\ell}(z') \times \\ &\big[(1+r_t)\left(\tilde{w}_{\ell,1,t}z' - \Xi_{\ell,1,t}(z')\right) + (1+r_t)(1-\kappa_{1,t})\left(\tilde{y}_{\ell,1,t} + \bar{a}_{j,1,t}\right)\big]\,\mathrm{d}z \qquad \text{for } s = 1.\end{aligned}$$

Bequests are

$$\bar{b}_{\ell,S,t}(z')\,\bar{\Lambda}_{\ell,S,t}(z') \;=\; \sum_{j=1}^{J}\int \bar{\Lambda}_{j,S-1,t-1}(z)\mu_{j\ell,S,t}(z)f_{\epsilon',\ell}(z'-\rho\,z)\times$$
$$\big((1+r_t)(1-\kappa_{S,t})\,[\tilde{y}_{\ell,S,t}+\bar{a}_{j,S,t}(z)]+(1+r_t)\ \big[\tilde{w}_{\ell,S,t}z'-\Xi_{\ell,S,t}(z')\big]\big)\,\mathrm{d}z.$$