ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

Battles and Diseases in the U.S. Civil War

Authors	Guillaume Vandenbroucke
Working Paper Number	2024-032A
Creation Date	September 2024
Citable Link	https://doi.org/10.20955/wp.2024.032
Suggested Citation	Vandenbroucke, G., 2024; Battles and Diseases in the U.S. Civil War, Federal Reserve Bank of St. Louis Working Paper 2024-032. URL https://doi.org/10.20955/wp.2024.032

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Battles and Diseases in the U.S. Civil War*

Guillaume Vandenbroucke†

September 2024

Abstract

Wars create short-term fluctuations in mortality. Belligerents might mitigate their own casualties with larger armies that hinder their opponent's fighting ability. But diseases are frequent in wars and, thus, may reduce the benefits of larger armies. First, I analyze these competing mechanisms in a dynamic model of wartime attrition. Second, I calibrate the model using U.S. Civil War data and find that if the Union had fielded a 50%-larger army in 1861, Union casualties would have been marginally lower. The theory provides the insight for this quantitative result.

JEL: E6, H56, N4
Keywords: War; Attrition; Diseases.

*The views expressed in this article are mine and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

†Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166, USA. Email: guillaumevdb@gmail.com.

1 Introduction

Population flows (births, deaths, and migration) are the subject of a vast literature in Economics. On mortality, economists have studied, among other topics, the secular declines accompanying demographic transitions and the short-term fluctuations of epidemics and famines.[1] But, short-term mortality fluctuations also result from wars and, historically, wars and epidemics are often simultaneous. Furthermore, a war's outcomes (duration, casualties, prevailing side, etc) depend on prewar and wartime expenditure of resources by belligerents. In this paper, I investigate the role of resources in the dynamics of battle- and disease-related casualties in wars.

I use a version of Lanchester (1916)'s model to represent military attrition. Two belligerents are assumed to enter a state of war at date 0 with their respective weapon stocks and a technology to destroy their opponent's stock. The belligerents receive reinforcement flows during the war and face non-combat attrition, i.e., diseases. The war ends when a belligerent's weapon stock reaches an arbitrary low threshold or when its casualties reach an arbitrary high threshold. When, all else equal, a belligerent increases its weapon stock date 0, as a result of additional prewar spending, two opposing mechanisms must be considered. First, the larger stock hinders its opponent's ability to inflict casualties and, hence, may hasten the conclusion of war. Thus, the larger stock is conducive to a reduction in the belligerent's own battle-related casualties. Second, the larger stock is subjected to diseases and, hence, raises the belligerent's own disease-related casualties.

In Section 2, I present the model and use it to analyze the respective strength of these mechanisms. I show that if a belligerent, say Blue, increases its date-0 weapon stock, all else equal its casualties increase for a period of time due to diseases. Beyond that period of time, its casualties fall below what they would have been absent the increase in the initial stock. The final effect on Blue casualties depends, therefore, on the duration of the war. If Blue's opponent sues for peace quickly, the war maybe short enough that Blue casualties are larger as a result of having fielded a larger force. In the opposite case, Blue casualties are lower. It is also possible that the final effect on Blue casualties is zero.

[1] For an example of a discussion of famines and crisis mortality, see Fogel (1989), for an example of a discussion of long-term trends in mortality, see Hejkal et al. (forthcoming).

In Section 3, I apply the model to the case of the U.S. Civil War during which both battle- and disease-related casualties were large. I calibrate the model and show that it reproduces the dynamics of observed weekly casualties for both sides. I then ask: Was there an incentive for the Union to field a larger (say 50% larger) army in 1861? Put differently: Given the Confederacy's army, what would have been the benefit for the Union to start the war with a larger army? The calibrated model implies that the effect of Union casualties would have been marginal. The theory of Section 2 provides the insight for this finding.

Operation Research scholars often use the Lanchester model to generate tactical insights or analyze specific battles (e.g., Taylor, 1980a,b, Lucas and Turkes, 2004, Armstrong and Sodergren, 2015, Stymfal, 2022). I depart from this tradition: I use the model to represent war as whole and focus on the dynamics of casualties more than on the determination of the prevailing side. The Lanchester model is also used in Defense Economics as a theory of measurement of military effectiveness (e.g., Hildebrandt, 1999, Pugh, 1993, Kirkpatrick, 1995). Finally, my work is similar to that prompted by the recent Covid-19 pandemic, which builds on epidemiological models to discuss how economic activity interacts with the dynamics of infections, deaths, and recoveries. The interest reader can consult, for example, the Covid-19 special issues of the *Journal of Economic Dynamics and Control* (July 2022) and of the *Journal of Public Economics* (May 2021) and the references therein. Similarly to the authors in this literature, I build on an Operation Research's model of wartime attrition, to ask how resources matter for the dynamics of wartime casualties.

2 Model

There are two sides, Blue (Union) and Grey (Confederacy), entering a state of war at date 0. Each side is endowed with an initial stock of the aggregate weapon, denoted k_0^i where $i \in \{B, G\}$, and receives an exogenous flow of reinforcement, x^i, at each point in time. I assume constant reinforcements for simplicity.

I model disease-related attrition via depreciation at the time-invariant rates δ^i. I indicate, in Section 3.1, the quantitative importance of diseases and the differences between the Union and the Confederacy. Hence, I allow depreciation to be belligerent-specific. I restrict it to be time-invariant for simplicity, but it should be noted that attrition from diseases could be seasonal.[2]

Blue's combat casualties at a point in time are $\theta^G k_t^G$ where θ^G is Grey's attrition coefficient: each unit of Grey weapon "destroys" θ^G units of Blue weapon. Grey's combat casualties are defined similarly. Two points are worth noting. First, Blue's combat casualties depend solely on Grey's weapon stock and vice versa. This specification, referred to as "aimed fire," can be opposed to an "area fire" specification where a belligerent's combat casualties also increase in its own weapons stock. Second, the technology is linear, allowing for analytical solutions and ease of analysis.

The laws of motions for the aggregate weapon stocks are

$$\begin{aligned} dk_t^B/dt &= -\delta^B k_t^B - \theta^G k_t^G + x^B, \quad (1) \\ dk_t^G/dt &= -\delta^G k_t^G - \theta^B k_t^B + x^G. \quad (2) \end{aligned}$$

Many variations of the Lanchester equations have been studied in the Operations Research literature (e.g., Taylor, 1980a,b). In the present version the war concludes when either (i) a belligerent's weapon stock reaches an exogenously-determined low threshold, e.g., zero, and that belligerent is assumed to be militarily defeated; or (ii) a belligerent's casualties reach an exogenously-determined high threshold and that belligerent is assumed to ask for terms. I define the first type of conclusion as a "military conclusion" and the second type as a "political conclusion." In Vandenbroucke (forthcoming), I discuss the conditions under which one belligerent prevails and in which of these two conclusions.

The Lanchester model is similar to the Solow model in two dimensions. First, there are no decisions: The model describes the dynamics of attrition in a way that resembles the Solow model's description of the dynamics of capital accumulation. Its value is pointing out mechanisms that would be internalized by decision makers whatever their objectives might be. Second, the model features aggregate weapon stocks, just as the Solow model features an aggregate capital stock. Distinguishing between different types of weapon (e.g., infantry, cavalry, or artillery) is often empirically impractical due to data limitations. Furthermore, even though such distinctions are critical for tactical insights when contemplating a particular battle, it is not obvi-

[2] Note that δ^i may be interpreted as other forms of non-combat attrition, such as desertions.

ous that they imply more or better insights when contemplating war as a whole. I interpret the aggregate weapon as follows. Different types of weapons may require different combinations of physical and human capital. Within a type, however, the composition may be rigid: In the infantry a man and a rifle are complementary; in the artillery a few (approximately six) men are needed to operate a gun, etc. I assume the aggregate weapon to inherit such fixed-proportion characteristic: It is a Leontieff composite of physical and human capital. Hence, when a unit becomes a casualty, whether from disease or from combat, both the physical and human capital are withdrawn from the fight. The model is silent on whether this means death/destruction, wounding/incapacitation, or capture of the unit.

2.1 Dynamics

Let $(\bar{k}^B, \bar{k}^G)$ denote the steady state of system (1)-(2), i.e., a stalemate where reinforcements offset attrition, and define $\tilde{k}_t^i \equiv k_t^i - \bar{k}^i$. System (1)-(2) becomes

$$\begin{bmatrix} d\tilde{k}_t^B/dt \\ d\tilde{k}_t^G/dt \end{bmatrix} = \underbrace{\begin{bmatrix} -\delta^B & -\theta^G \\ -\theta^B & -\delta^G \end{bmatrix}}_{A} \begin{bmatrix} \tilde{k}_t^B \\ \tilde{k}_t^G \end{bmatrix}. \tag{3}$$

Let λ_1 and λ_2 denote the eigenvalues of A with associated eigenvectors $[1; v_1]'$ and $[1; v_2]'$. I show in Appendix A that if mean combat-related attrition exceeds mean disease-related attrition, that is, if

$$\sqrt{\theta^G \theta^B} > \sqrt{\delta^G \delta^B} \tag{4}$$

then

$$\lambda_1 < 0, \ \lambda_2 > 0, \ v_1 > 0, \ \text{and } v_2 < 0. \tag{5}$$

Thus, under condition (4), the system is a saddle.[3] I assume, in what follows, that condition (4) is satisfied

[3]Along the saddle one stock converges to zero while the other diverges from almost everywhere in the (k^G, k^B)-plane. If (4) is not satisfied, both stocks converge toward zero.

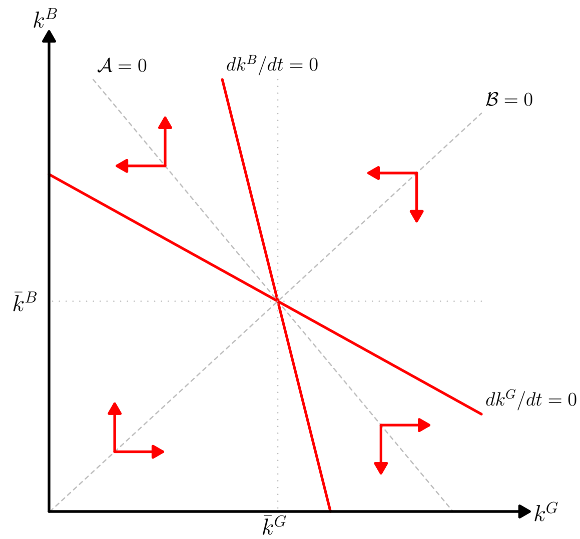


Figure 1: Weapon' stocks dynamics

Note: See Appendix A for the derivation of $\mathcal{A}$ and $\mathcal{B}$. The loci $dk^B/dt = 0$ and $dk^G/dt = 0$ can be deduced from (1)-(2) and condition (4) ensures that $dk^B/dt = 0$ is steeper than $dk^G/dt = 0$.

Standard methods (see Appendix A) yield the solution

$$k_t^G = \bar{k}^G + \frac{1}{v_2 - v_1}\left[\mathcal{A}e^{\lambda_1 t} + \mathcal{B}e^{\lambda_2 t}\right], \tag{6}$$

$$k_t^B = \bar{k}^B + \frac{1}{v_2 - v_1}\left[\mathcal{A}v_1 e^{\lambda_1 t} + \mathcal{B}v_2 e^{\lambda_2 t}\right]. \tag{7}$$

where $\mathcal{A} \equiv \tilde{k}_0^G v_2 - \tilde{k}_0^B$ and $\mathcal{B} \equiv \tilde{k}_0^B - v_1 \tilde{k}_0^G$. The equation $\mathcal{B} = 0$ defines the stable branch of the system while $\mathcal{A} = 0$ defines the unstable branch. Figure 1 illustrates weapon stocks dynamics. Initial conditions above the stable branch, i.e., $\mathcal{B} > 0$, imply trajectories leading to $k^G = 0$, i.e., the annihilation of Grey's forces and, hence, a military victory for Blue. Initial conditions below the stable branch have the opposite implications. Along any trajectories casualties accumulate for each side.

2.2 Casualties

I now analyze the casualties of each side and focus on the effect of Blue's initial weapon stock. The theoretical results below form the basis for interpreting the quantitative discussion of the U.S. Civil War in Section 3. There are, of course, other questions of interest beside the effect of Blue's initial weapon stock. For the sake of this paper, however, I restrict myself to one question: What could have been the benefit to the Union of having been better prepared (i.e., a higher k_0^B) at the onset of the war?

The cumulative casualties of Blue and Grey, up to date, t are

$$D_t^B = \int_0^t (\theta^G k_u^G + \delta^B k_u^B) du \qquad \text{and} \qquad D_t^G = \int_0^t (\theta^B k_u^B + \delta^G k_u^G) du. \tag{8}$$

I show in Appendix A that

$$\partial D_t^G / \partial k_0^B > 0.$$

That is, if Blue started the war with more resources, i.e., a larger weapon stock, then Grey casualties would accumulate faster and, hence be larger at all points in the war. I also show that

$$\begin{array}{ll} \text{If } \delta^B = 0 & : \partial D_t^B / \partial k_0^B < 0 \forall t > 0 \\ \text{If } \delta^B > 0 & : \exists\ \tau > 0 \text{ such that } \partial D_t^B / \partial k_0^B \gtreqless 0 \text{ whenever } t \lesseqgtr \tau. \end{array}$$

This result deserves a discussion. If, on the one hand, Blue does not experience disease-related attrition ($\delta^B = 0$), then starting the war with a larger weapon stock implies fewer Blue casualties at all points during the war. That is because Blue's larger stock implies more Grey combat casualties and, hence, impedes Grey's ability to inflict casualties on Blue. If, on the other hand, Blue experiences disease-related attrition ($\delta^B > 0$), then starting the war with a larger weapon stock may increase or decrease Blue casualties depending upon the duration of the war. That is because initially, when $t < \tau$, Blue casualties are larger than if Blue had started with a lower stock due to an excess of disease-related casualties. But disease-driven casualties are only a fraction of Blue's stock and, so, Blue still inflicts more casualties on Grey than without the augmented initial stock. Thus, Blue's stock is unambiguously higher throughout the war while Grey's stock is unambiguously lower (Appendix A). Hence, eventually, when $t > \tau$, Grey's ability to inflict combat-related casualties on Blue becomes so limited that Blue's casualties are reduced beyond what Blue would have experienced with a lower initial stock.

I assume δ^B is invariant to changes in Blue's initial stock. If a larger initial stock brings about an increase in the transmission of communicable diseases within Blue's army, however, one might want to consider the joint effect of an increase in k_0^B and in δ^B. Such effect would raise Blue's casualties from diseases at each point in time and, hence, increase the time needed for Blue casualties to be reduced by a larger initial stock, that is τ.

It follows from the discussion above that, under some conditions, Blue could hasten victory by committing additional resources to the war while not affecting its final casualties. Figure 2 illustrates such special case. The continuous line on the top panel represents Grey's casualties and the continuous line on the bottom panel represents Blue's casualties. I assume the war ends at date T^* not because Grey is militarily defeated but, instead, because Grey casualties reach the threshold $\bar{D}^G$. I interpret $\bar{D}^G$ as a political threshold because it represents Grey's willingness to accept casualties as expressed, for instance, via votes, protests, or civil unrest. In short, the scenario in Figure 2 is one where Blue prevails at T^* because Grey gives up the fight. It is also possible to interpret $\bar{D}^G$ as representing an assessment of the military situation by Grey leaders, e.g., when casualties have reached $\bar{D}^G$, military leaders may assess they do not have the remaining resources to prevail.

The dashed lines in the top and bottom panels of Figure 2 represent casualties if, all else equal, Blue had started with a larger weapon stock. As per the discussion above: (i) Grey casualties are everywhere above their initial trajectories and (ii) Blue casualties are initially higher than their initial trajectories but, eventually, lower. Under this new scenario, the war ends earlier, at $T^{**} < T^*$, because Grey casualties reach the threshold $\bar{D}^G$ earlier than before. The figure illustrates the special case where, at T^{**}, Blue casualties are equal to what they were in the initial situation. So, in this case, even though Blue managed to obtain an earlier conclusion in its favor, its casualty cost is unchanged. This need not happen since the figure illustrates a special case: Blue casualties could be higher or lower when the initial Blue stock is higher. This depends on parameters. In sum, Blue casualties (i) can be the same or close as illustrated by the figure, (ii) can only be equal or higher when $\delta^B > 0$, and (iii) must be lower when $\delta^B = 0$.

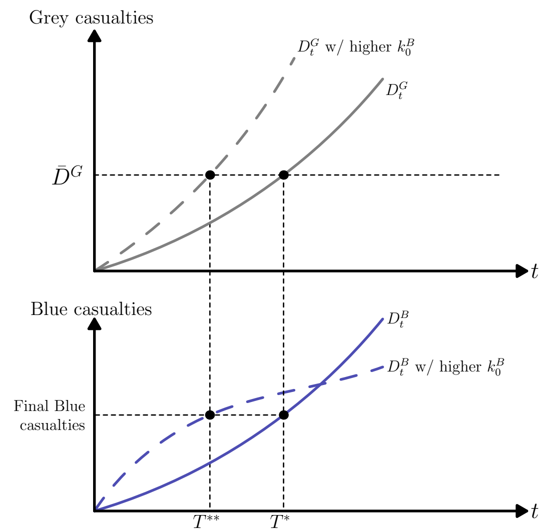


Figure 2: The case when Blue's larger initial weapon stock does not change Blue casualties

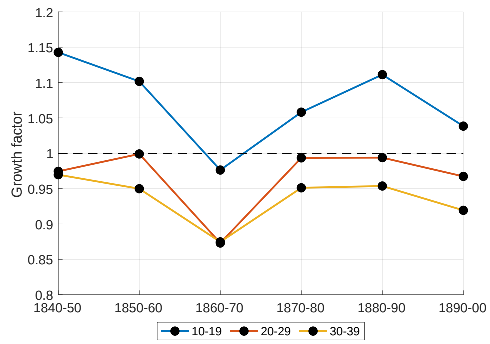
It is worth emphasizing that when Blue prevails with at least as much casualties after having committed more resources to the war, it is entirely because of disease-related casualties. More specifically, it is **not** because Blue forces are more "exposed" to Grey's fire. Such mechanism does not exist under the aimed-fire assumption.

3 The U.S. Civil War

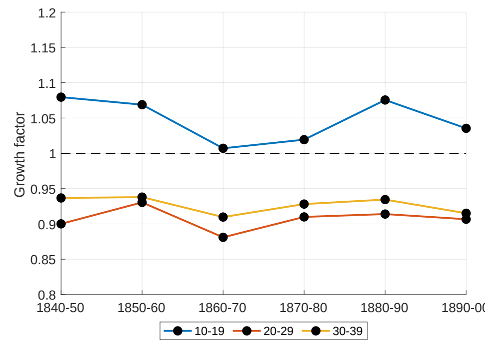
3.1 Facts

The Civil war lasted 220 weeks, from April 1861 to April 1865.[4] Even though the estimates of deaths due to the war vary significantly in the literature, they are all large: An often cited figure is in the neighborhood of $620,000$ (e.g. McPherson, 1988) while a more recent estimate is about $750,000$ (Hacker, 2011). Since the U.S. population in the 1860 census was approximately 27 millions, the low estimate implies that the Civil War would have caused the deaths of more than 2% of the population and

[4] General Lee surrendered to General Grant on April 9, 1865 at Appomattox, VA. Yet, there was one more battle in Palmito Ranch, TX, in May 1865, with little consequences.



A – Male



B – Female

Figure 3: Population growth by cohorts

Note: A point indicates the growth (factor) over a particular decade for a cohort of a given age at the start of the decade. For example, 1.1 for the 10-19 male cohort over the period 1850-60 indicates that the cohort who was 10-19 in 1850 was 10% larger when it reached the age 20-29 (in 1860).
Source: Carter et al. (2006), series Aa292-Aa345 and author's calculations.

approximately 10% of the 6 millions men between the age of 20 and 50 in 1860.[5]

Figure 3 illustrates, albeit imperfectly, the effect of the Civil War on the U.S. population. The figure reports the growth factor, over each of the last six decades of the 19th century, for cohorts of given ages at the start of the decades. For example, the figure 1.1 for the 10-19 male cohort over the period 1850-60 (Panel A) indicates that the cohort who was 10-19 in 1850 was 10% larger when it reached the age 20-29 in 1860. A few lessons can be drawn: First, young cohorts had a tendency to increase in size in the decades before and after the war. That is because immigration in the 10-19 age group must have offset mortality. Second, older cohorts had a tendency to decrease, in the decades before and after the war, because immigration did not offset mortality for these age groups. Finally, and most importantly, during the 1860s, the male cohorts declined significantly while the female cohorts were less affected. This is likely the result of excess mortality due to the Civil War. Remark that the decline in the size of the 10-19 cohort in 1860 is moderate relative to the decline in the older cohorts in 1860. That is likely because men in their 20s and 30s were more exposed to the risk of dying because of the war than were men in their teens.

[5]Population figures from IPUMS USA, University of Minnesota, www.ipums.org.

Greer (2005) assembled times series of weekly casualties for the Union and the Confederacy. Casualty figures count those killed in battle, wounded, or taken prisoners. The data also include, in a separate count, an estimate of weekly attrition due to diseases. The main source used by Greer is Kennedy, ed (1998) adjusted after interviews with historians of specific battles. The choice to include prisoners of war in the casualty count was dictated, in part, by a concern for consistency: Prisoners are not reported separately from the dead and wounded for each battle. Hence, excluding prisoners from the casualty count could not be done consistently across battles. Another consideration for including prisoners is that they often did not return to the fighting. A prisoner exchange program, the Dix-Hill cartel, existed, but only for a brief period of time: It was signed in July 1862 and suspended in July 1863 (Pierce, 1993). Similarly, wounded men are counted as casualties even though they sometimes returned to the fighting. The data does not permit to distinguish between those who did return and the others.

Figure 4 shows the time series of cumulative weekly battle casualties for the Union and the Confederacy. Adding battle casualties and casualties from diseases estimated by Greer (2005) yields

$$\text{Total Union casualties} = \underbrace{454,000}_{\text{battle cas.}} + \underbrace{248,000}_{\text{disease cas.}} = 702,000 \tag{9}$$

and

$$\text{Total Confederate casualties} = \underbrace{454,000}_{\text{battle cas.}} + \underbrace{167,000}_{\text{disease cas.}} = 621,000. \tag{10}$$

It is worth noting the quantitative importance of diseases and the disparity between the Union and the Confederacy. For the Union, 35% of casualties are disease-related while for the Confederacy the figure is 27%. Another point of note is the difference between casualties and deaths. Using Hacker (2011)'s $750,000$ figure, total deaths amount to 57% of casualties.

Long (1971, p. 706) gives estimates of the size of the Union and Confederate armed forces. These estimates have been the subject of extensive research in the historical literature. Long's figures are for 9 dates for the Union and 7 for the Confederacy. Long does not offer an estimate of the size of the Confederate armed forces at the start of April 1861. I assumed it to be the same as that of the Union. Goldin and

Lewis (1975, fn. g, p. 305) also report armed forces size at various points during the war. The data is represented on Figure 5 with linear interpolation between Long's missing data points.

3.2 Confronting model and data

Attrition

I start with a measure of the attrition coefficients. Given Blue's weapons stock, k_t^B, and Grey's battle casualties, d_t^G, an estimate of Blue's attrition coefficient can be constructed by imposing that, if applied to the observed stock of Blue weapons, the attrition coefficient yields total Grey casualties:

$$\sum_t \theta^B k_t^B = \sum_t d_t^G,$$

which yields

$$\theta^B = \frac{\sum_t d_t^G}{\sum_t k_t^B} = 0.0028,$$

where the numerator, i.e., $454,000$ is indicated in Equation (10) and the denominator can be deduced from the data in Figure 5. Similarly,

$$\theta^G = \frac{\sum_t d_t^B}{\sum_t k_t^G} = 0.0060.$$

Note that $\theta^G > \theta^B$ because total battle casualties are similar for both sides (Figure 4) while the size of the Grey armed forces is noticeably smaller than that of the Blue (Figure 5).

The depreciation rates can be constructed in a similar fashion. Given Blue's weapons stock and non-combat casualties, an estimate of Blue's depreciation rate must be such that, if applied to the observed stock of Blue weapons, the depreciation rate yields the total of Blue's non-combat casualties:

$$\sum_t \delta^B k_t^B = 248,000,$$

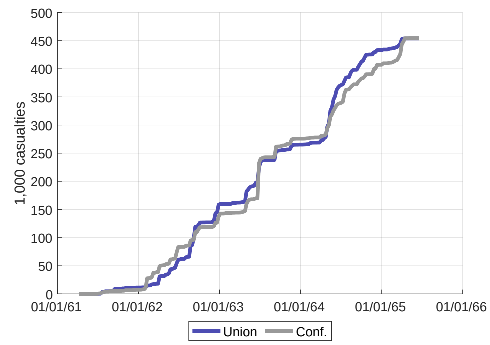


Figure 4: Cumulative battle casualties, 1861-1865

Note: The figure shows weekly cumulative battle casualties (killed, wounded, and prisoners). It excludes casualties from diseases.
Source: Greer (2005).

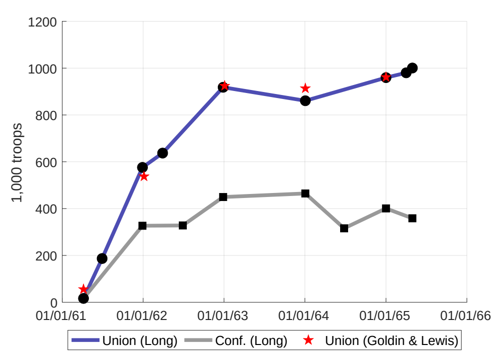


Figure 5: Armed force sizes, 1861-1865

Note: The figure shows the estimated size of armed forces. Black markers indicate the dates for which Long provides estimates. I linearly interpolated between the estimate. The red pentagrams indicate estimates from Goldin and Lewis.
Source: Long (1971, p. 706) and Goldin and Lewis (1975, fn. g, p. 305).

which yields

$$\delta^B = \frac{248,000}{\sum_t k_t^B} = 0.0015.$$

Similarly,

$$\delta^G = \frac{167,000}{\sum_t k_t^G} = 0.0022.$$

Reinforcements

Given the attrition and depreciation rates, it remains to estimate reinforcement flows which are not readily observed. My strategy is to use draft dates for the Union and the Confederacy as critical dates between which reinforcement flows are constants to be estimated. These constants may be different for the Union and the Confederacy. The argument for resorting to this approach is that many soldiers, on each side, were volunteers but that the flow of volunteers was not steady. With the realization that the war would last longer than previously anticipated and that volunteering was not reliable, drafts were put into effects. Thus, draft dates are rough indications of changes in reinforcement flows.

I now present this argument in more details. When the war started soldiers were mostly volunteers. In the Union, for instance, the public's interest for serving is exemplified by an article published in the New York Tribune in May 1861:

> "Had the President called—as we fervently wish he had—for Five Hundred Thousand men... they would all have been promptly furnished by the Free States alone and every man a glad volunteer"
>
> Quoted in Shannon (1928, p. 38)

The recruiting of volunteers, however, quickly collapsed for a hosts of reasons. For instance, the Union's Secretary of War, E. Stanton, briefly suspended recruiting in April 1862—possibly believing that the confederacy was on the verge of defeat—only to reinstate it in June—when realizing the confederacy was not about to be defeated (Shannon, 1928, pp. 265-68). Furthermore, potential volunteers increasingly required enticement to join, as their opportunity cost rose with the demand for industrial and farm labor (Shannon, 1928, p. 260). Finally, the military outcomes of 1862 convinced Union leaders that the war was going to last longer than first anticipated. A contemporary historian wrote:

> "We have sedulously deceived ourselves as to the magnitude and duration of the struggle in which we are engaged... Deluded with the idea that the rebellion was constantly near its end, we have habitually resorted to temporary expedient when a permanent system was indispensable"
> Quoted in Shannon (1928, p. 267)

A number of drafts were decided to remedy this situation. Levine (1981) records four drafts in the Union: first in July 1863, and then in March, July, and December 1864. The drafts were not popular (e.g. the New York City draft riots of July 1863) and their effectiveness have been debated in the historical literature. Some men simply failed to report, some received exemptions for physical or other reasons, some legally paid substitute to serve instead of them, etc. For example, just 3% of those called in the draft of July 1863 served in person (Levine, 1981, Table 1, p. 819). In the Confederacy, drafts were decided in April and September 1862 as well as in December 1864. As in the Union, the drafts provided exemptions that depended on age or wealth (e.g. the so-called "twenty slave law").

I interpret the drafts dates as indications that decision makers acknowledged the need for adjusting the reinforcements they provided their armed forces. The timing of the drafts, however, do not necessary correspond to the actual change in the flow of reinforcements. For instance, after the Union's volunteer recruiting efforts restarted in June 1862, almost one year elapsed before Lincoln signed the Enrollment Act of March 1863, which paved the way for the draft of July 1863. Anticipation of the draft could have either discouraged or encouraged volunteers to sign up before July 1863. Furthermore, raising volunteers was difficult in the first place: When Lincoln called for 100,000 volunteers (June 1863) it took seven weeks to assemble 12,000 and, then, the call was rescinded (Shannon, 1928, p. 296). If assembling volunteers took time, unpopular drafts may have been equally lengthy. Because of these considerations, my approach is to interpret the drafts dates as *indications that Union leaders changed their reinforcement policies in 1863 and 1864, and confederate leaders changed theirs in 1862 and 1864.* I then use two critical dates for the Union: the 1st week of July 1863 and the 1st week of July 1864. I "merge" the three drafts of 1864 into one single date to reduce the number of parameters and because, for the reasons I explained, the draft dates cannot be interpreted as the exact moments when the reinforcement flows changed. I proceed similarly for the Confederacy and use the first week of July 1862 and the first week of July 1864 as two critical dates.

With two critical dates for each side, there remains 2×3 reinforcement flows to estimate. Let the list of these unknown reinforcement be

$$\omega = \{x_i^B, x_i^G\}_{i=1}^3,$$

and define

$$x_t^B = \begin{cases} x_1^B: & t < \text{Jul. } 1863, \\ x_2^B: & \text{Jul. } 1863 < t < \text{Jul. } 1864, \\ x_3^B: & t > \text{Jul. } 1864, \end{cases} \qquad x_t^G = \begin{cases} x_1^G: & t < \text{Jul. } 1862 \\ x_2^G: & \text{Jul. } 1862 < t < \text{Jul. } 1864, \\ x_3^G: & t > \text{Jul. } 1864. \end{cases}$$

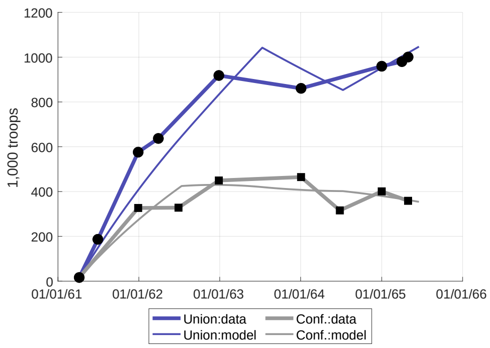
I determine these coefficients by fitting the model to the time series of army sizes represented in Figure 5:

$$\begin{aligned} \min_{\omega} \quad & \sum_t [k_t^B - k_t^B(\text{data})]^2 + \sum_t [k_t^G - k_t^G(\text{data})]^2, \\ \text{s.t.} \quad & k_0^B = k_0^B(\text{data}) \text{ and } k_0^G = k_0^G(\text{data}), \\ & k_{t+1}^B = (1-\delta^B)k_t^B + x_t^B - \theta^G k_t^G, \\ & k_{t+1}^G = (1-\delta^G)k_t^G + x_t^G - \theta^B k_t^B. \end{aligned}$$

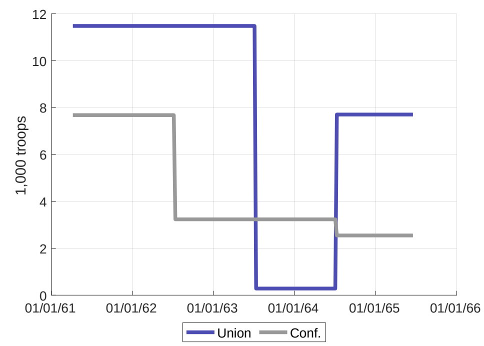
Panel A of Figure 6 shows the model's fit to the army sizes data and Panel B shows the estimated paths of reinforcements. Panels C and D show the implied casualties for Blue and Grey, respectively. It is worth noting the model's overall fit to the data despite its parsimonious parameterization: six parameters.

3.3 Discussion

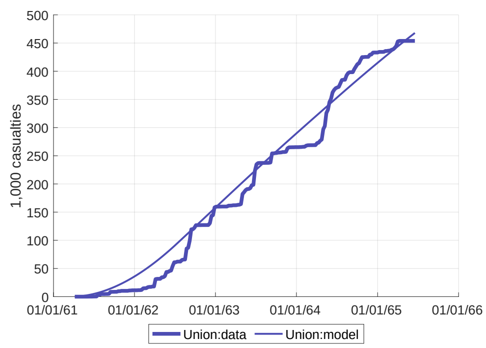
The conclusion of the previous exercise is that the Lanchester model can be used as an empirically relevant description of the dynamics of casualties in war. Thus, in turn, it can also be used for counterfactual experiments. I propose one such experiment now: What would have casualties been if the Union had been better prepared to fight the



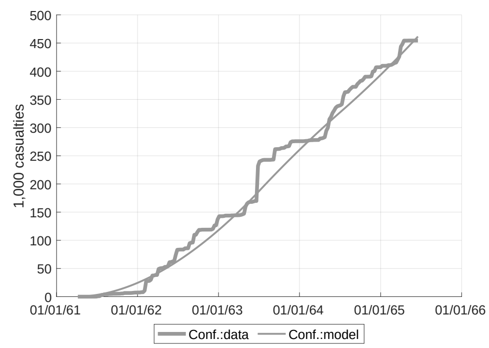
A – Army sizes (targeted)



B – Reinforcements



C – Union casualties



D – Confederate casualties

Figure 6: The model's fit

Note: The figure shows the model's fit to the army sizes (Panel A) and to the cumulative casualties (Panels B and C).
Source: Long (1971), Greer (2005), and author's calculations.

war in the sense of having had a larger stock of weapon in April 1861

In conducting the experiment, I hold reinforcement flows as they are described in Panel B of Figure 6. I also hold the initial Confederate weapon stock constant. Finally, I use final confederate casualties (633,000 in the model v. 621,000 in the data) as the threshold level at which the war concludes. My argument for this approach is that the war did not conclude with the annihilation of the confederate army (Panel A of Figure 6). It is reasonable, therefore, to consider that General Lee's surrender was motivated, among other reasons, by a desire to put an end to the accumulation of casualties.[6]

It is important to understand that, because the model does not offer a theory of behavior, the counterfactual experiment I propose cannot be interpreted as a "policy" experiment. Instead, it should be interpreted as an exploration of the incentives that the technology of warfare afforded decision makers at the time: If the Union had been able to field an army 50% larger in April 1861, and if the confederacy had not behaved differently, would the outcome have been noticeably different for the Union? The answer, from the model, is no. In the counterfactual experiment, the casualties for the Union are 716,000 versus 719,000 in the baseline (702,000 in the data). The war is shorter by 1-2 weeks in the counterfactual. Thus, the case of the U.S. Civil war resembles that illustrated in Figure 2: Because of diseases, there is a mechanism whereby the effect of fielding a larger army does not reduce one's casualties significantly.

4 Conclusion

I presented a version of the Lanchester model of attrition. I allowed for non-combat attrition which I interpreted as disease-related attrition. I show that if a belligerent fields a larger army at the start of the war, two opposite effects come into play: The larger army tends to reduce casualties from combat because the war may be shorter; It also tends to increase casualties from diseases because it is larger. Theoretically, I show that it is possible for the final effect on casualties to be zero, or arbitrarily

[6] Lee's actual motivations probably accounted for an assessment of the military situation and a diminished likelihood of success if the war continued. The model, however, is not equipped to address such issues.

small.

I calibrated the model to data from the U.S. Civil War. The model offers an empirically relevant description of the dynamics of wartime casualties for both sides. Using the calibrated model, I asked whether the outcome for the Union would have been significantly different if its army had been larger in 1861—under the assumption that the Confederacy would not have reacted to such larger Union army. The answer is no: Union Casualties would have been only marginally smaller than in the baseline calibration.

References

Armstrong, Michael J and Steven E Sodergren, "Refighting Pickett's Charge: Mathematical Modeling of the Civil War Battlefield," *Social Science Quarterly*, 2015, *96* (4), 1153–1168.

Carter, Susan B, Scott S Gartner, Michael R Haines, Alan L Olmstead, Richard Sutch, and Gavin Wright, *Historical statistics of the United States: millennial edition*, Vol. 1: Population, Cambridge University Press New York, 2006.

Fogel, Robert W, "Second thoughts on the European escape from hunger: famines, price elasticities, entitlements, chronic malnutrition, and mortality rates," *NBER Working Paper Series On Historical Factors in Long-Run Growth*, 1989, (1).

Goldin, Claudia D and Frank D Lewis, "The economic cost of the American Civil War: Estimates and implications," *The Journal of Economic History*, 1975, *35* (2), 299–326.

Greer, Darroch, "Counting Civil War Casualties, Week-By-Week, For The Abraham Lincoln Presidential Library and Museum," https://www.mathscinotes.com/wp-content/uploads/2021/11/ALPLM_CountingCasualties_080519_lo.pdf 2005. [Online; accessed 15-August-2024].

Hacker, J David, "A census-based count of the Civil War dead," *Civil war history*, 2011, *57* (4), 307–348.

Hejkal, John, B. Ravikumar, and Guillaume Vandenbroucke, "Technology Adoption, Mortality, and Population Dynamics," *Economic Journal*, forthcoming.

Hildebrandt, Gregory G, "The military production function," *Defence and Peace Economics*, 1999, *10* (3), 247–272.

Kennedy, Frances H, ed., *The Civil War Battlefield Guide*, Houghton Mifflin Harcourt, 1998.

Kirkpatrick, David L, "The rising unit cost of defence equipment—The reasons and the results," *Defence and Peace Economics*, 1995, *6* (4), 263–288.

Lanchester, Frederick W, *Aircraft in Warfare: The Dawn of the Fourth Arm*, Constable limited, 1916.

Levine, Peter, "Draft Evasion in the North during the Civil War, 1863-1865," *The Journal of American History*, March 1981, *4* (67).

Long, E. B., *The Civil War Day by Day: An Almanac, 1861-1865*, Doubleday, 1971.

Lucas, Thomas W and Turker Turkes, "Fitting Lanchester Equations to the Battles of Kursk and Ardennes," *Naval Research Logistics*, 2004, *51* (1), 95–116.

McPherson, James M, *Battle cry of freedom: The Civil War era*, Oxford University Press, 1988.

Pierce, Donald R, *The effects of the cessation of exchange of prisoners during the Civil War*, Master thesis, U.S. Army Command and General Staff College, 1993.

Pugh, Philip G, "The procurement nexus," *Defence and peace economics*, 1993, *4* (2), 179–194.

Shannon, Fred A, *The organization and administration of the Union army, 1861-1865*, Vol. 1, The Arthus H Clark Company: Cleveland, 1928.

Stymfal, Matthew G, *Revisiting Engel's verification of Lanchester's square law using battle of Iwo Jima data*, Master thesis, Naval Postgraduate School, 2022.

Taylor, James G, *Lanchester-Type Models of Warfare (vol. I)*, Naval Postgraduate School, 1980.

— , *Lanchester-Type Models of Warfare (vol. II)*, Naval Postgraduate School, 1980.

Vandenbroucke, Guillaume, "On the Economic Mechanics of Warfare," *International Economic Review*, forthcoming.

A Model solution and analysis

The eigenvalues, λ_i, and corresponding eigenvectors, $[1; v_i]'$, for A in (3) are

$$\lambda_1 = -\frac{\delta^G+\delta^B}{2} - \frac{\sqrt{\left(\delta^B-\delta^G\right)^2+4\theta^G\theta^B}}{2}, \quad v_1 = \frac{\delta^B-\delta^G}{2\theta^B} + \frac{\sqrt{\left(\delta^B-\delta^G\right)^2+4\theta^G\theta^B}}{2\theta^B},$$

$$\lambda_2 = -\frac{\delta^G+\delta^B}{2} + \frac{\sqrt{\left(\delta^B-\delta^G\right)^2+4\theta^G\theta^B}}{2}, \quad v_2 = \frac{\delta^B-\delta^G}{2\theta^B} - \frac{\sqrt{\left(\delta^B-\delta^G\right)^2+4\theta^G\theta^B}}{2\theta^B}.$$

Note that $\lambda_1 < 0$, $v_1 > 0$, and $v_2 < 0$. The condition for a saddle is $\lambda_2 > 0$, or

$$\sqrt{\left(\delta^B-\delta^G\right)^2+4\theta^G\theta^B} > \delta^G+\delta^B \;\Leftrightarrow\; \theta^G\theta^B > \delta^G\delta^B.$$

It is useful to note that, by definition, $A \cdot [1; v_i]' = [\lambda_i; \lambda_i, v_i]'$, implying $v_i = -(\lambda_i + \delta^G)/\theta^B = -\theta^G/(\lambda_i + \delta^B)$.

The solution of (3) is of the form

$$\tilde{k}_t^G = r_1 e^{\lambda_1 t} + r_2 e^{\lambda_2 t} \quad \text{and} \quad \tilde{k}_t^B = b_1 e^{\lambda_1 t} + b_2 e^{\lambda_2 t}.$$

where r_1, r_2, b_1, and b_2 are coefficients to be determined. The coefficients must satisfy the initial condition, hence $\tilde{k}_0^G = r_1 + r_2$ and $\tilde{k}_0^B = b_1 + b_2$. Rewrite the solution of (3) as

$$\tilde{k}_t^G = r_1 e^{\lambda_1 t} + \left(\tilde{k}_0^G - r_1\right) e^{\lambda_2 t}, \tag{A.1}$$

$$\tilde{k}_t^B = b_1 e^{\lambda_1 t} + \left(\tilde{k}_0^B - b_1\right) e^{\lambda_2 t}. \tag{A.2}$$

The coefficients r_1 and b_1 must also satisfy the laws of motions in (3),

$$r_1\lambda_1 e^{\lambda_1 t} + \left(\tilde{k}_0^G - r_1\right)\lambda_2 e^{\lambda_2 t} = -\theta^B\left[b_1 e^{\lambda_1 t} + \left(\tilde{k}_0^B - b_1\right)e^{\lambda_2 t}\right] - \delta^G\left[r_1 e^{\lambda_1 t} + \left(\tilde{k}_0^G - r_1\right)e^{\lambda_2 t}\right],$$

and

$$b_1\lambda_1 e^{\lambda_1 t} + \left(\tilde{k}_0^B - b_1\right)\lambda_2 e^{\lambda_2 t} = -\theta^G\left[r_1 e^{\lambda_1 t} + \left(\tilde{k}_0^G - r_1\right)e^{\lambda_2 t}\right] - \delta^B\left[b_1 e^{\lambda_1 t} + \left(\tilde{k}_0^B - b_1\right)e^{\lambda_2 t}\right].$$

Identifying the coefficients on $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ in these equations yields a system of two equations:

$$r_1 v_1 = b_1 \qquad \text{and} \qquad \left(\tilde{k}_0^G - r_1\right) v_2 = \left(\tilde{k}_0^B - b_1\right)$$

with solution

$$r_1 = \frac{\tilde{k}_0^G v_2 - \tilde{k}_0^B}{v_2 - v_1} \qquad \text{and} \qquad b_1 = r_1 v_1. \tag{A.3}$$

Remark that $\tilde{k}_0^G - r_1 = \left(\tilde{k}_0^B - v_1\tilde{k}_0^G\right)/(v_2 - v_1)$, so that the solution of system (3) becomes

$$\begin{aligned}
\tilde{k}_t^G &= \frac{\tilde{k}_0^G v_2 - \tilde{k}_0^B}{v_2 - v_1} e^{\lambda_1 t} + \frac{\tilde{k}_0^B - v_1\tilde{k}_0^G}{v_2 - v_1} e^{\lambda_2 t}, \\
&= \frac{1}{v_2 - v_1}\left[\mathcal{A}e^{\lambda_1 t} + \mathcal{B}e^{\lambda_2 t}\right], \\
\tilde{k}_t^B &= \frac{\tilde{k}_0^G v_2 - \tilde{k}_0^B}{v_2 - v_1} v_1 e^{\lambda_1 t} + \frac{\tilde{k}_0^B - v_1\tilde{k}_0^G}{v_2 - v_1} v_2 e^{\lambda_2 t}, \\
&= \frac{1}{v_2 - v_1}\left[\mathcal{A}v_1 e^{\lambda_1 t} + \mathcal{B}v_2 e^{\lambda_2 t}\right].
\end{aligned}$$

where $\mathcal{A} = \tilde{k}_0^G v_2 - \tilde{k}_0^B$ and $\mathcal{B} = \tilde{k}_0^B - v_1\tilde{k}_0^G$. Recall that $\lambda_1 < 0$, and $\lambda_2 > 0$. Thus, the system's stable branch is $\mathcal{B} = 0$ and the system unstable branch is $\mathcal{A} = 0$.

It is immediate that

$$\frac{\partial k_t^G}{\partial k_0^B} = \frac{1}{v_2 - v_1}\left(-e^{\lambda_1 t} + e^{\lambda_2 t}\right) < 0 \qquad \text{and} \qquad \frac{\partial k_t^B}{\partial k_0^B} = \frac{1}{v_2 - v_1}\left(-v_1 e^{\lambda_1 t} + v_2 e^{\lambda_2 t}\right) > 0,$$

where the inequalities follow from (5). Consider now the effect of k_0^B on D_t^B, the Blue casualties defined in Equation (8):

$$\begin{aligned}
\frac{\partial D_t^B}{\partial k_0^B} &= \theta^G \int_0^t \frac{\partial}{\partial k_0^B} k_u^G du + \delta^B \int_0^t \frac{\partial}{\partial k_0^B} k_u^B du, \\
(v_2 - v_1)\frac{\partial D_t^B}{\partial k_0^B} &= -\theta^G \frac{e^{\lambda_1 t} - 1}{\lambda_1} + \theta^G \frac{e^{\lambda_2 t} - 1}{\lambda_2} - \delta^B v_1 \frac{e^{\lambda_1 t} - 1}{\lambda_1} + \delta^B v_2 \frac{e^{\lambda_2 t} - 1}{\lambda_2},
\end{aligned}$$

which is also

$$\begin{aligned}
(v_2 - v_1)\frac{\partial D_t^B}{\partial k_0^B} &= \left(-\theta^G - \delta^B v_1\right)\frac{e^{\lambda_1 t} - 1}{\lambda_1} + \left(\theta^G + \delta^B v_2\right)\frac{e^{\lambda_2 t} - 1}{\lambda_2}, \\
&= v_1\left(e^{\lambda_1 t} - 1\right) - v_2\left(e^{\lambda_2 t} - 1\right),
\end{aligned}$$

where the last equality uses the definition of the eigenvalues and eigenvectors of A. Finally,

$$\frac{\partial D_t^B}{\partial k_0^B} = 1 + \frac{v_1}{v_2 - v_1} e^{\lambda_1 t} - \frac{v_2}{v_2 - v_1} e^{\lambda_2 t}.$$

A similar calculation leads to

$$\frac{\partial D_t^G}{\partial k_0^B} = \frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{v_2 - v_1}.$$

It follows from (5) that $\partial D_t^G/\partial k_0^B > 0$. To assess the sign of $\partial D_t^B/\partial k_0^B$ define

$$f(t) \equiv \frac{\partial D_t^B}{\partial k_0^B} = 1 + \frac{v_1}{v_2 - v_1} e^{\lambda_1 t} - \frac{v_2}{v_2 - v_1} e^{\lambda_2 t}.$$

Then, $f(t) < 1$ for all t, $f(0) = 0$ and $\lim_{t\to\infty} f(t) = -\infty$. Furthermore, the slope of f is

$$f'(t) = \frac{\lambda_1 v_1 e^{\lambda_1 t} - \lambda_2 v_2 e^{\lambda_2 t}}{v_2 - v_1}.$$

It is useful to note the following result:

$$\lambda_1 v_1 - \lambda_2 v_2 = -\frac{\delta^B}{\theta^B}\sqrt{\left(\delta^B - \delta^R\right)^2 + 4\theta^R\theta^B}. \tag{A.4}$$

Proposition 1: $\partial D_t^B / \partial k_0^B < 0$ for all t whenever $\delta^B = 0$.

Proof of Proposition 1: Equation (A.4) implies $v_1\lambda_1 = v_2\lambda_2$ whenever $\delta^B = 0$. So,

$$f'(t)\bigg|_{\delta^B=0} = \frac{v_1\lambda_1}{v_2 - v_1}\left(e^{\lambda_1 t} - e^{\lambda_2 t}\right) < 0.$$

The inequality follows (5). Thus, $\partial D_t^B / \partial k_0^B$ is decreasing. Its maximum attains at $t = 0$ where it is 0. The result follows. ■

Proposition 2: If $\delta^B > 0$, there exists one date $\tau > 0$ such that $\partial D_t^B / \partial k_0^B > 0$ for $t < \tau$ and $\partial D_t^B / \partial k_0^B < 0$ for $t > \tau$.

Proof of Proposition 2: Equation (A.4) implies $v_1\lambda_1 - v_2\lambda_2 < 0$ whenever $\delta^B > 0$. The slope of f at $t = 0$ is then

$$f'(0)\bigg|_{\delta^B>0} = \frac{\lambda_1 v_1 - \lambda_2 v_2}{v_2 - v_1} > 0.$$

Thus, f is increasing at 0 where it is equal to 0. By continuity f is then positive in a neighborhood above $t = 0$. Since $\lim_{t\to\infty} f(t) = -\infty$ and since f is continuous, there exists a date $\tau > 0$ at which f intersects with the horizontal axis. Uniqueness of τ follows from $f''(t) < 0$ for all t. ■