ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

Trade and Inequality in an Overlapping Generations Model with Capital Accumulation

Authors	Jun Nie, B. Ravikumar, and Michael J. Sposi
Working Paper Number	2024-018B
Revision Date	September 2024
Citable Link	https://doi.org/10.20955/wp.2024.018
Suggested Citation	Nie, J., Ravikumar, B., Sposi, M.J., 2024; Trade and Inequality in an Overlapping Generations Model with Capital Accumulation, Federal Reserve Bank of St. Louis Working Paper 2024-018. URL https://doi.org/10.20955/wp.2024.018

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Trade and Inequality in an Overlapping Generations Model with Capital Accumulation*

Jun Nie† B. Ravikumar‡ Michael Sposi§

September 2024

Abstract

We study the lifecycle aspect of within-country inequality that stems from capital and labor services supplied by individuals. Our environment is a combination of a multicountry trade model and an overlapping generations model with production and capital accumulation. Trade liberalization increases the measured total factor productivity in each country, which increases the marginal product of capital and incentivizes capital accumulation. Higher capital stock and higher measured productivity raise the marginal product of labor and, hence, wages. Inequality, measured by the ratio of old agents' income to young agents' income, evolves over time due to capital accumulation during the transition from autarky to an open-economy world. Immediately after liberalization, inequality increases. Over time, capital accumulates at a diminishing rate and inequality declines.

JEL codes: F11, D31, E22

Keywords: Trade, Inequality, Lifecycle, Capital accumulation

*The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

†Southern Methodist University, 3100 Dyer St. Dyer TX, 75275. jnie@smu.edu

‡Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166. b.ravikumar@wustl.edu

§Southern Methodist University, 3100 Dyer St. Dyer TX, 75275. msposi@smu.edu

1 Introduction

“Free trade” is not free. Trade liberalization creates winners and losers. As noted by Rodrik (2021), “Redistribution is the flip side of the gains from trade [...] As an economy opens up to trade, domestic relative prices change, producing income redistribution alongside gains from trade (p.1).”

The notion that trade liberalization creates inequality is an old one (Stolper and Samuelson, 1941). Recent literature has focused on implications of trade liberalization for inequality along several channels: skill (Wood, 1995), sector/occupation (Autor, Dorn, Hanson, and Song, 2014), class (Adao, Carrillo, Costinot, Donaldson, and Pomeranz, 2022), region (Autor, Dorn, and Hanson, 2013), consumption basket (Fajgelbaum and Khandelwal, 2016), or fragmentation (Beladi, Marjit, Oladi, and Raei, forthcoming). For instance, as trade liberalization induces countries to specialize, workers in some sectors lose their jobs; inequality emerges between workers in the sectors that the country chooses to specialize in and workers in the sectors that the country abandons.[1] Labor economists, however, have documented that one major “source” of income inequality among workers is age. In the U.S., median earnings of a 50-year-old is fifty percent higher than that of a 25-year-old. Furthermore, the factor composition of income varies with age. Figure 1 illustrates the share of capital income over the lifecycle: The income is predominantly from labor when young and from capital when old.[2]

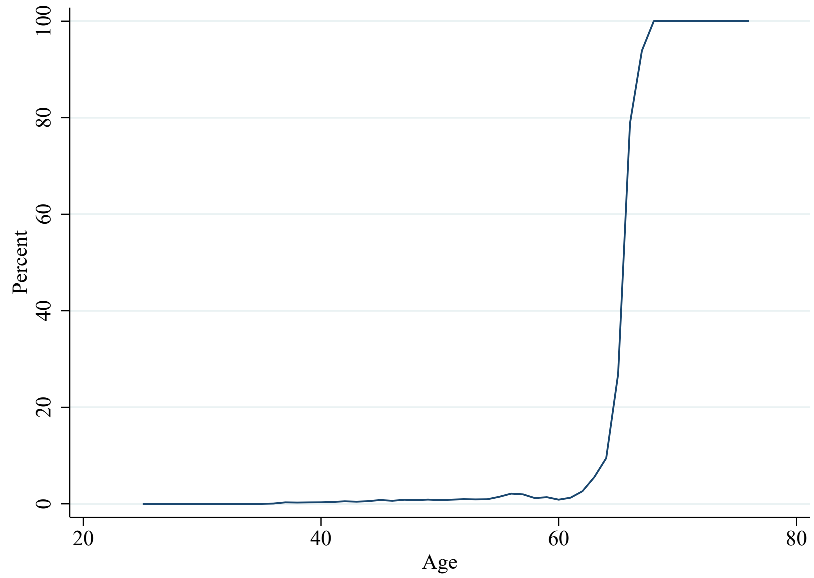
Our focus is on the lifecycle aspect of within-country inequality that stems from capital and labor services supplied by individuals. We accomplish two tasks in this paper. First, we study the effect of trade liberalization on inequality between different age groups at a point in time. Second, we study the evolution of the inequality over time. Our environment is a combination of an overlapping generations model and a multicountry trade model. The overlapping generations model is that of Diamond (1965) that has production and capital accumulation. The trade model is that of Armington (1969).

The economy lives forever. Each agent in the economy lives for two periods, denoted as “young” and “old.” Each young agent is endowed with 1 unit of labor. Agents work while young, but consume in both periods of their lives. Each country produces a unique

[1] For the recent shift from globalization to protectionism, see Goldberg (2023) for arguments related to fairness and inequality and Ravikumar, Santacreu, and Sposi (2024) for how losses from protectionism are smaller than the gains from liberalization.

[2] Our measure includes income from salary (labor) and capital (interest payments, dividends, and rents) and excludes net sales of assets.

Figure 1: Capital Income as a Share of Household Income in the United States



Source: Current Population Survey (CPS), 1969-2019, U.S. Bureau of Labor Statistics.
Notes: The figure above plots capital income as a share of household income for a synthetic cohort, ages 25 to 76 years. To construct the income of the synthetic cohort, we take the median income for 25-year-olds in 1968, 26-year-olds in 1969, and so on, up to 2019. Total income is the sum of salary income and capital income; the latter includes dividends, rents, and interest payments. It does not include wealth or potential value of capital.

variety by combining capital and labor in a Cobb-Douglas fashion, augmented by country-specific fundamental productivity. International trade is subject to bilateral iceberg costs. In each destination, all varieties are combined to form a composite good. The composite good is allocated to consumption and investment in the destination. Trade is balanced in each period. Saving by the young generation in a period equals the beginning-of-period stock of capital in the next period.

In our environment, the income is from only labor while young, but from only capital when old. Measured total factor productivity in autarky in each country is its fundamental productivity. In an open-economy world, measured total factor productivity in each country depends on the volume of trade. Trade lowers the aggregate price index, thereby heightening the real value of aggregate output, which manifests as an increase in measured productivity. The increase in measured productivity amplifies the marginal product of capital, which incentivizes capital accumulation. Higher capital stock and higher measured productivity raise the marginal product of labor and, hence, wages. We measure inequality at a point in time by the ratio of old agents' income (from capital) to that of the young (from labor). The dynamics of inequality stem from capital accumulation during transition from autarky to an open-economy world.

Our results are as follows. The steady-state inequality in an open-economy world is the same as that in autarky. This is because of the Cobb-Douglas production function and one final good. That is, we do not have Heckscher-Ohlin forces that affect the factor composition of income based on heterogeneous factor intensities.

The inequality, however, varies along the transition path as the world moves from autarky to frictionless trade. When countries are symmetric, the home trade share in each country falls and measured productivity increases. Upon impact, the capital-labor ratio is unchanged, so the inequality initially increases. Over time, capital accumulates at a diminishing rate and inequality declines back to its autarky level. Symmetry also implies that the inequality is identical across countries at every point in time. When countries are asymmetric, we resort to numerical simulations to study the transition paths. For instance, consider the case when the asymmetry is in the fundamental productivity. Upon liberalization, less-productive countries realize a larger increase in measured productivity and inequality. Along the transition, less-productive countries also accumulate capital at a faster rate and inequality stays higher relative to more-productive countries. As capital deepens, the inequality declines back to the autarky level for each country.

While inequality is "worse" along the transition path from a closed-economy world to an open-economy world, lifetime utility is not. As capital accumulates, returns to labor continue to increase, while returns to capital diminish, although the latter remain above the autarky steady state level. So, each cohort experiences greater lifetime utility than the preceding cohort. Using the change in real income per capita as a measure of gain, a measure traditionally used in the trade literature, all countries experience dynamic gains from trade liberalization. Less-productive and/or small countries experience more gains even though they experience higher inequality relative to more-productive and/or large countries.

Our paper investigates some of the factors highlighted in the literature that inform the debate on whether trade or technology has had a greater influence on rising inequality in advanced economies. The main aspect of inequality in that literature is the skill premium and the framework is Heckscher-Ohlin. On the one hand, to the extent that modern technologies (e.g., information and communications) complement skilled workers, technological change widens the income gap between high- and low-skilled workers; Card and DiNardo (2002) provide a comprehensive summary of that literature. Berman and Machin (2000) argue that technological change diffuses from one country to another and affects the skill premium in other countries.

On the other hand, trade liberalization could increase the returns to skilled labor in developed countries where skilled labor is relatively more abundant; see, for instance, Krugman (2008), Slaughter (2001), Wood (1995), and Wood (1998). The focus of our paper is on the lifecycle aspect of inequality, i.e., the age-profile of income and factor composition of income, in a Ricardian framework. Trade liberalization affects the returns to capital differently from the returns to labor. Since capital income is a larger share of older individuals' income, the differential impact on the factor returns affects the age-profile of income.

In the next section, we describe the model and the equilibrium dynamics of inequality. In Section 3 we provide a numerical analysis of the inequality for the case of asymmetric countries. Concluding remarks are in Section 4.

2 Model

Our framework is a combination of a two-period overlapping generations model (Diamond, 1965) and a multi-country trade model (Armington, 1969). There are N countries indexed by $n = 1, ..., N$. Time is discrete and time is indexed by $t = 0, 1, ..., \infty$.

In each period there is a representative household that lives for two periods – young and old. The population of young is L_n^y, and the population of old is L_n^o, so the total population in country n is $L_n^y + L_n^o$. Young agents inelastically supply one unit of labor and divide their earnings between consumption and saving. Old agents consume their saving plus the returns on saving. There is a representative firm that rents capital and labor from households to produce output. All markets are competitive and agents have perfect foresight.

We first describe the equilibrium of a world in autarky. We then introduce international trade and describe the corresponding equilibrium.

Households The household born in period t has time-separable preferences given by

$$\left((c_{n,t}^y)^{\frac{\sigma-1}{\sigma}} + \beta(c_{n,t+1}^o)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where c denotes consumption; subscript t denotes the time period and superscripts "y" and "o" refer to young and old, respectively. The discount factor is $\beta \in (0, 1)$, and the intertemporal elasticity of substitution is $\sigma \geq 0$.

The household faces budget constraints in each period of life:

$$\begin{aligned} P_{n,t}c^{y}_{n,t} + P_{n,t}s_{n,t} &\leq W_{n,t} \\ P_{n,t+1}c^{o}_{n,t+1} &\leq P_{n,t+1}(1 + r_{n,t+1})s_{n,t}, \end{aligned}$$

where $s_{n,t}$ denotes saving by the young in period t, $W_{n,t}$ is the nominal wage rate in period t, $r_{n,t+1}$ is the real interest rate between periods t and $t+1$, and $P_{n,t}$ is the price of goods in period t. When young, the household allocates the wage income to consumption and saving. When old, it collects principal plus interest on its savings and finances its consumption.

The household's intertemporal Euler equation is

$$\frac{c^{o}_{n,t+1}}{c^{y}_{n,t}} = \beta^{\sigma}(1 + r_{n,t+1})^{\sigma}.$$

Thus, the household's optimal saving is described by

$$s_{n,t} = \left(\frac{\beta^{\sigma}(1 + r_{n,t+1})^{\sigma-1}}{1 + \beta^{\sigma}(1 + r_{n,t+1})^{\sigma-1}} \right) \left(\frac{W_{n,t}}{P_{n,t}} \right). \tag{2}$$

Firms There is a representative firm that rents capital and labor to produce output according to a Cobb-Douglas production function:

$$Y_{n,t} = A_n(K_{n,t})^{\alpha}(L_n)^{1-\alpha}, \tag{3}$$

where A_n is country n's time-invariant fundamental productivity, $K_{n,t}$ and L_n are the capital and labor used in production, and α is capital's share in value added.

Investment In each period a fraction $\delta \in [0, 1]$ of the capital stock depreciates during production. Investment, $I_{n,t}$, adds to the future capital stock according to

$$K_{n,t+1} = (1 - \delta)K_{n,t} + I_{n,t}. \tag{4}$$

2.1 Autarky

Competitive factor markets imply that the real rental rate and real wage rate are equal to the marginal products of capital and labor, respectively, and the price of output equals the marginal cost of production:

$$\frac{R_{n,t}}{P_{n,t}} = \alpha A_n \left(\frac{K_{n,t}}{L_n}\right)^{\alpha-1}, \tag{5}$$

$$\frac{W_{n,t}}{P_{n,t}} = (1-\alpha) A_n \left(\frac{K_{n,t}}{L_n}\right)^{\alpha}, \text{ and} \tag{6}$$

$$P_{n,t} = \frac{\left(\frac{R_{n,t}}{\alpha}\right)^{\alpha} \left(\frac{W_{n,t}}{1-\alpha}\right)^{1-\alpha}}{A_n}.$$

Equilibrium In the absence of arbitrage, the capital accumulation technology (4) implies that the real interest rate on household savings must equal the marginal product of capital, net of depreciation:

$$r_{n,t} = \frac{R_{n,t}}{P_{n,t}} - \delta. \tag{7}$$

In equilibrium, the capital stock available in the next period equals the savings by the young in the current period and the labor employed equals the size of the young population:

$$K_{n,t+1} = s_{n,t} L_n^y \tag{8}$$

$$L_n = L_n^y. \tag{9}$$

Equation (8), using conditions (2), (5), (6), and (7), yields the *equilibrium* law of motion for capital:

$$K_{n,t+1} = \left(\frac{\beta^{\sigma} \left(\alpha A_n \left(\frac{K_{n,t+1}}{L_n}\right)^{\alpha-1} + 1 - \delta \right)^{\sigma-1}}{1 + \beta^{\sigma} \left(\alpha A_n \left(\frac{K_{n,t+1}}{L_n}\right)^{\alpha-1} + 1 - \delta \right)^{\sigma-1}} \right) (1-\alpha) A_n K_{n,t}^{\alpha} L_n^{1-\alpha}. \tag{10}$$

Equilibrium Inequality in Autarky The focus of this paper is the income inequality across age groups and how this inequality varies over time. We define inequality as the ratio of old-to-young earnings, in line with our empirical metric in Figure 1:

$$\rho_{n,t} \equiv \frac{P_{n,t} r_{n,t} K_{n,t}}{W_{n,t} L_n}. \tag{11}$$

That is, the numerator captures the interest income flow received by old agents, and the denominator captures labor income flow received by young agents. As in Figure 1, the numerator does not include wealth, i.e., the value of the capital stock. We study the properties of $\rho_{n,t}$ both in the steady state and along the transition.

Lemma 1. *In an autarky steady state, each country's capital-labor ratio is proportional to its "scaled" fundamental productivity level:* $\frac{K_n}{L_n} \propto A_n^{\frac{1}{1-\alpha}}$.

Proof. Denote the steady-state capital stock by $K_n^\star$. In steady state, the law of motion (10) implies

$$1 = \left(1 + \beta^{-\sigma}\left(\alpha A_n \left(\frac{K_n^*}{L_n}\right)^{\alpha-1} + 1 - \delta\right)^{1-\sigma}\right)^{-1} (1-\alpha) A_n \left(\frac{K_n^*}{L_n}\right)^{\alpha-1}.$$

Define $x \equiv A_n \left(\frac{K_n^*}{L_n}\right)^{\alpha-1}$. Clearly, the solution for x depends only on the parameters $(\alpha, \beta, \sigma, \delta)$. Hence, the steady-state capital-labor ratio, $K_n^\star/L_n$, in country n is proportional to its "scaled" fundamental productivity level $A_n^{\frac{1}{1-\alpha}}$. □

Countries with higher productivity will have higher steady-state capital-labor ratios. The scaling factor, $\frac{1}{1-\alpha}$, captures the fact that higher productivity increases resources available for saving, which in turn induces more production capacity in the future through higher capital stock, the impact of which is governed by capital's share, α.

Theorem 1. *In an autarky steady state, the inequality does not depend on fundamental productivity or the population size.*

Proof. To see this, apply equation (11) to the steady state:

$$\begin{aligned}
\rho_n^\star \equiv \frac{P_n^\star r_n^\star K_n^\star}{W_n^\star L_n} &= \frac{P_n^\star (R_n^\star / P_n^\star - \delta) K_n^\star}{W_n^\star L_n} \\
&= \frac{R_n^\star K_n^\star - P_n^\star \delta K_n^\star}{W_n^\star L_n} \\
&= \frac{\alpha}{1-\alpha} - \delta \frac{(K_n^\star/L_n)^{1-\alpha}}{(1-\alpha) A_n}.
\end{aligned}$$

Using Lemma 1, $\frac{(K_n^\star/L_n)^{1-\alpha}}{A_n}$ is a constant that depends only on $(\alpha, \beta, \sigma, \delta)$. □

Transitional Dynamics Consider a world not in a steady state. For instance, suppose that the initial capital stock is not equal to the steady-state capital stock. Along the transition path, if the current capital stock exceeds the steady-state level of capital, then the return to new investment will be lower than its steady-state level. As a result aggregate investment slows down and so does the growth rate of the capital stock. Eventually, the economy settles to a steady state where new investment exactly replaces the depreciated capital stock. Along this transition path, the returns to labor change at a different rate from the returns to capital, resulting in inequality dynamics over time.

Theorem 2. *In autarky, along the transition path, the change in inequality depends only on change in the capital-labor ratio.*

Proof. To see this, apply equation (11):

$$\begin{aligned}\rho_{n,t} \equiv \frac{P_{n,t} r_{n,t} K_{n,t}}{W_{n,t} L} &= \frac{P_{n,t}(R_{n,t}/P_{n,t} - \delta) K_{n,t}}{W_{n,t} L_n} \\ &= \frac{R_{n,t} K_{n,t} - P_{n,t} \delta K_{n,t}}{W_{n,t} L_n} \\ &= \frac{\alpha}{1-\alpha} - \delta \frac{(K_{n,t}/L_n)^{1-\alpha}}{(1-\alpha) A_n}.\end{aligned}$$

□

As capital becomes more abundant relative to labor, its marginal product declines, whereas the marginal product of labor increases. So, earnings accruing to capital owners (the old) decline relative to those of labor owners (the young), which decreases inequality over time.

Since the production function is Cobb Douglas, the share of GDP that accrues to capital is equal to α, so why is $\rho_{n,t}$ not equal to $\frac{\alpha}{1-\alpha}$? The reason is that the interest income received by the old agents is not the capital share of GDP. In the absence of arbitrage, the interest rate on savings accounts for not only the rental rate received by the old when they rent the capital to the representative firm but also the rate at which capital depreciates after production. To see this, consider the accounting for aggregate demand in period t, consisting of consumption by the current young and current old, along with gross saving of the young:

$$L_n^y P_{n,t} c_{n,t}^y + L_n^o P_{n,t} c_{n,t}^o + L_n^y P_{n,t} s_{n,t} = P_{n,t}(1 + r_{n,t}) K_{n,t} + W_{n,t} L_n.$$

Using $r_{n,t} = R_{n,t}/P_{n,t} - \delta$ yields

$$L_n^y P_{n,t} c_{n,t}^y + L_n^o P_{n,t} c_{n,t}^o + L_n^y P_{n,t} s_{n,t} = P_{n,t} \left(1 + \frac{R_{n,t}}{P_{n,t}} - \delta\right) K_{n,t} + W_{n,t} L_n$$

Dividing through by $P_{n,t}$ and using the facts that $K_{n,t+1} = L_n^y s_{n,t}$ and $K_{n,t+1} = (1 - \delta)K_{n,t} + I_{n,t}$ gives rise to the GDP accounting identity:

$$\underbrace{L_n^y c_{n,t}^y + L_n^o c_{n,t}^o}_{C_{n,t}} + \underbrace{K_{n,t+1} - (1-\delta)K_{n,t}}_{I_{n,t}} = \underbrace{\frac{R_{n,t}}{P_{n,t}} K_{n,t} + \frac{W_{n,t}}{P_{n,t}} L_n}_{Y_{n,t}} .$$

In other words, aggregate investment in the economy equals aggregate saving, which in turn is the gross saving by the young minus the gross dis-saving by the old. The latter embeds the depreciated capital stock.

2.2 Open-economy World

In this section we study a world with international trade. Household and firm behavior are the same as in the previous section. We now introduce a "retail firm" and modify the market-clearing conditions.

Production and International Trade Each country produces a differentiated good according to the Cobb-Douglas production function (3).

Denote $q_{ni,t}$ as quantity of goods that country n sources from country i at time t. There is a retail firm that aggregates goods from all sources to construct a composite good as follows:

$$Q_{n,t} = \left[\sum_{i=1}^{N} (q_{ni,t})^{\frac{\theta}{1+\theta}} \right]^{\frac{1+\theta}{\theta}},$$

where θ is the elasticity of substitution of goods among source countries (trade elasticity) and $Q_{n,t}$ is the aggregate composite good. The quantity of the composite good determines the resources available for consumption and investment in country n. International trade is subject to iceberg costs denoted by $d_{ni} \geq 1$ for $\forall i \neq n$. We normalize $d_{nn} = 1$.

Prices in the Open-economy World The price that country n faces to purchase one unit from country i is given by

$$p_{ni,t} = \frac{u_{i,t}}{A_i} d_{ni},$$

where $u_{i,t} = \left(\frac{R_{i,t}}{\alpha}\right)^{\alpha} \left(\frac{W_{i,t}}{1-\alpha}\right)^{1-\alpha}$.

Thus, the optimal quantity of goods that country n sources from country i is

$$q_{ni,t} = \left(\frac{p_{ni,t}}{P_{n,t}}\right)^{-\theta} Q_{n,t},$$

and the ideal price index is given by

$$P_{n,t} = \left(\sum_{i=1}^{N}\left(\frac{u_{i,t}d_{ni}}{A_i}\right)^{-\theta}\right)^{-\frac{1}{\theta}}.$$

The share of country n's spending on goods originating from country i is given by

$$\pi_{ni,t} = \frac{A_i^{\theta} u_{i,t}^{-\theta} d_{ni}^{-\theta}}{\sum_{i'=1}^{N} A_{i'} u_{i',t}^{-\theta} d_{ni'}^{-\theta}}. \tag{12}$$

We define country n's measured productivity as $Z_{n,t} = A_n(\pi_{nn,t})^{-1/\theta}$. The equilibrium prices in the open-economy world are analogous to those in autarky after replacing the fundamental productivity with the measured productivity. That is, the equilibrium prices satisfy

$$\frac{R_{n,t}}{P_{n,t}} = \alpha Z_{n,t}\left(\frac{K_{n,t}}{L_n}\right)^{\alpha-1}$$
$$\frac{W_{n,t}}{P_{n,t}} = (1-\alpha) Z_{n,t}\left(\frac{K_{n,t}}{L_n}\right)^{\alpha}$$
$$P_{n,t} = \frac{\left(\frac{R_{n,t}}{\alpha}\right)^{\alpha}\left(\frac{W_{n,t}}{1-\alpha}\right)^{1-\alpha}}{Z_{n,t}}$$

Equilibrium in the Open-economy World In addition to the conditions of (i) no arbitrage and (ii) factor markets clearing—equations (7), (8), and (9)—we impose balanced trade to complete the characterization of the open economy:

$$\sum_{i=1}^{N} P_{n,t}Q_{n,t}\pi_{ni,t} = \sum_{i=1}^{N} P_{i,t}Q_{i,t}\pi_{in,t}. \tag{13}$$

This condition gives rise to the familiar accounting identity that output equals the sum of consumption, investment, and net exports. To see why, note that the composite good purchased by country n, $Q_{n,t}$, constitutes the sum of country n's aggregate consumption and investment:

$$Q_{n,t} = C_{n,t} + I_{n,t}.$$

The left-hand side of equation (13), which is country n's total spending on goods from all countries including itself, is equal to country n's total spending on the composite good since $\sum_{i=1}^{n} \pi_{ni,t} = 1$. The right-hand side of equation (13) is the sum of all destination countries' spending on goods produced by country n and is thus country n's output and total income. By deflating the aggregate income by the aggregate price level $P_{n,t}$, it follows that

$$Y_{n,t} = Q_{n,t}.$$

Similar to autarky, the equilibrium law of motion for capital is given by

$$K_{n,t+1} = \left(\frac{\beta^{\sigma} \left(\alpha Z_{n,t+1} \left(\frac{K_{n,t+1}}{L_n} \right)^{\alpha-1} + 1 - \delta \right)^{\sigma-1}}{1 + \beta^{\sigma} \left(\alpha Z_{n,t} \left(\frac{K_{n,t+1}}{L_n} \right)^{\alpha-1} + 1 - \delta \right)^{\sigma-1}} \right) (1-\alpha) Z_{n,t} K_{n,t}^{\alpha} L_n^{1-\alpha}.$$

Finally, inequality in the open-economy world is given by

$$\rho_{n,t} \equiv \frac{P_{n,t} r_{n,t} K_{n,t}}{W_{n,t} L_n} = \frac{\alpha}{1-\alpha} - \delta \frac{(K_{n,t}/L_{n,t})^{1-\alpha}}{(1-\alpha) Z_{n,t}}.$$

In the open-economy world, the two equilibrium relationships above are analogous to those in autarky, after replacing the fundamental productivity A_n with the measured productivity Z_n. Table 2 in Appendix A provides the full set of equilibrium conditions. The following sequence of claims (Lemma 2, Theorem 3, and Theorem 4) leverages this observation. In particular, Lemma 1 can now be generalized. The proofs of these claims are omitted since they are similar to those in Section 2.1.

Lemma 2. *In an open-economy world, each country's steady-state capital-labor ratio is proportional to its "scaled" measured productivity level:* $\frac{K_n}{L_n} \propto Z_n^{\frac{1}{1-\alpha}}$.

Since the effect of bilateral trade costs is summarized by each country's home trade share, trade integration influences each country through the measured productivity.

The next result is the counterpart of Theorem 1 for the open-economy world.

Theorem 3. *In an open-economy world, the steady-state inequality does not depend on fundamental productivity or the population size.*

The next result is the counterpart of Theorem 2 and yields one notable difference relative to autarky along the transition path.

As in the case of autarky, the transitional dynamics are due to capital accumulation. Trade liberalization decreases the home trade share, which increases the measured TFP. The increase in measured TFP increases the marginal product of capital, which stimulates capital accumulation. While capital stock does not change on impact, the higher rate of return on capital induces capital to increase gradually. The rate of accumulation depends on the investment rate, which is governed by the intertemporal Euler equation. Trade also implies that the dynamics of capital in country i depend on the evolution of capital stocks in all other countries.

Theorem 4. *In an open-economy world, along the transition path, change in inequality depends on the change in the capital-labor ratio and change in the measured productivity.*

Unlike autarky, the measured productivity $Z_{n,t}$ in the open economy need not be constant along a transition path since $Z_{n,t} = A_n \pi_{nn,t}^{-1/\theta}$. That is, the home trade share adjusts when capital stocks grow asymmetrically across countries. The home trade share increases in locations whose capital stocks grow relatively faster, reflecting relatively growing endowments and therefore occupying an increasing share of world trade.

Theorem 4 demonstrates that the transitional dynamics in an open-economy world are more complicated than in autarky. However, when countries are symmetric, the dynamics are straightforward, as we describe next.

Symmetric Countries In the special case where all of the countries are identical, equation (12) implies that the home trade share must be equal to

$$\pi_{nn,t} = \left(1 + (N-1)d^{-\theta}\right)^{-1},$$

where d is the symmetric bilateral trade cost. Consider a scenario in which the world is initially in an autarkic steady state, then unexpectedly moves to a world with lower trade costs. The home trade share immediately declines from 1 to some value $\pi^\star \in [1/N, 1)$ and remains constant thereafter. In turn, the measured productivity Z_n immediately increases and remains permanently higher thereafter.

Upon impact, the capital-labor ratio is unchanged, so the inequality $\rho_{n,t}$ initially increases. Over time, capital gradually accumulates at a diminishing rate and inequality steadily declines back to its initial steady-state level. Moreover, the ratio of young to old earnings is identical across countries at every point in time.

We turn to numerical analysis in the next section to evaluate an open-economy world with heterogeneous countries.

3 Numerical Analysis

In this section we numerically evaluate the implications of moving from a relatively closed world ($d_{ni} = 100$) to a frictionless-trade world ($d_{ni} = 1$). Our algorithm for computing the exact transitional dynamics of the open-economy world is based on one from Ravikumar, Santacreu, and Sposi (2024), and is described in Appendix A. We assume the world is initially closed and in steady state. We then study the transition paths to the frictionless-trade world when the countries differ along one of the following dimensions: fundamental productivity or population size.

To compute the transitions numerically, we assign plausible values for key parameters. Some values are established estimates in the literature and some are disciplined so that the model is consistent with empirical evidence. The parameter values are reported in Table 1. We interpret one period in the model to correspond to about thirty years. We set the discount factor to $\beta = 0.29$, corresponding to an annual discount factor of 0.96, or an annual real interest rate of about 4 percent. We set the intertemporal elasticity of substitution to $\sigma = 0.75$ so that consumption when young and old are complements in the agent's preferences. We assign $\alpha = 0.33$ based on Gollin (2002) and set $\delta = 0.85$, corresponding to an annual depreciation rate of roughly 6 percent. Finally, we set trade elasticity to $\theta = 4$ following Simonovska and Waugh (2014).

Table 1: Common Parameters

Discount factor	β	0.29
Intertemporal elasticity	σ	0.75
Capital's share in value added	α	0.33
Depreciation rate	δ	0.85
Trade elasticity	θ	4

We set the number of countries $N = 5$. Albeit somewhat arbitrary, this choice facilitates "third-country effects" that would be inadmissible in a two-country setting.[3] Additionally,

[3]Third-country effects are quantitatively important when considering changes in trade costs between heterogeneous countries (see, for instance, Anderson and vanWincoop, 2003; Ghironi, 2024).

it allows us to span a reasonable degree of heterogeneity while maintaining parsimony.

3.1 Asymmetric Fundamental Productivity

In this case we assume that countries differ in their fundamental productivity: $A_n = n$. That is, country 5 is five times as productive as country 1. The countries have identical population size and age shares: $L_n^y + L_n^o = 1, L_n^y = L_n^o = 1/2$. The other parameters are as in Table 1 for all countries.

Figure 2 plots the transitional dynamics for inequality (top), the capital-labor ratio (middle), and measured productivity (bottom). The dynamics are relative to the autarky state: Each country's autarky steady state is different, but the focus of the figure is on how each country transitions from its autarky steady state to the new frictionless-trade steady state. In period 1, we remove bilateral trade barriers. On impact, the inequality in all countries increases. The initial boost in the first period is driven by measured productivity because capital-labor ratio is unchanged relative to the initial steady state. Moreover, the increase is particularly pronounced in low-productivity countries. That is, opening up to trade allows less-productive countries to access better technologies from more-productive countries and realize a larger boost in measured productivity.

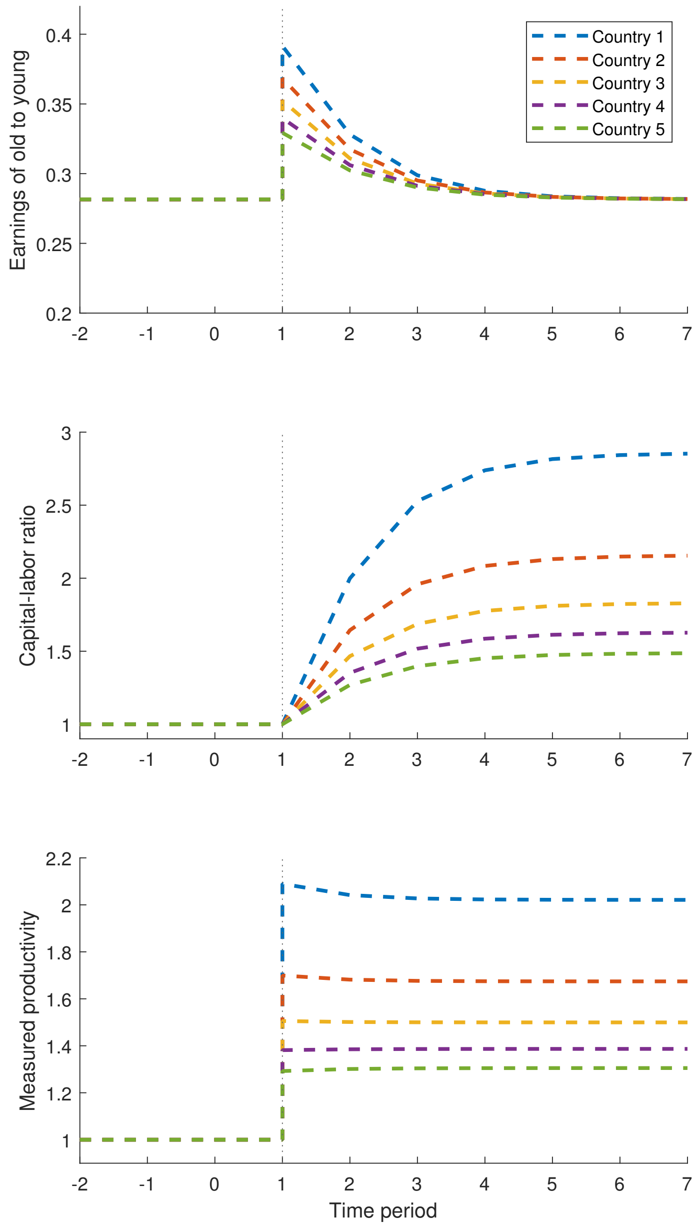
From period 2 onward, countries start to accumulate capital. Given that less-productive countries have a larger increase in measured productivity, they also experience a higher return to capital. Optimal saving decisions then imply that they accumulate capital faster than more-productive countries. As capital deepens, the ratio of old agents' earnings to young agents' earnings declines to its initial steady-state level.

3.2 Asymmetric Population

In this case we assume that countries differ in their total population size but have identical fundamental productivity and age shares: $A_n = 1, L_n^y = L_n^o = n/2$. That is, countries with a higher index n have a larger population.

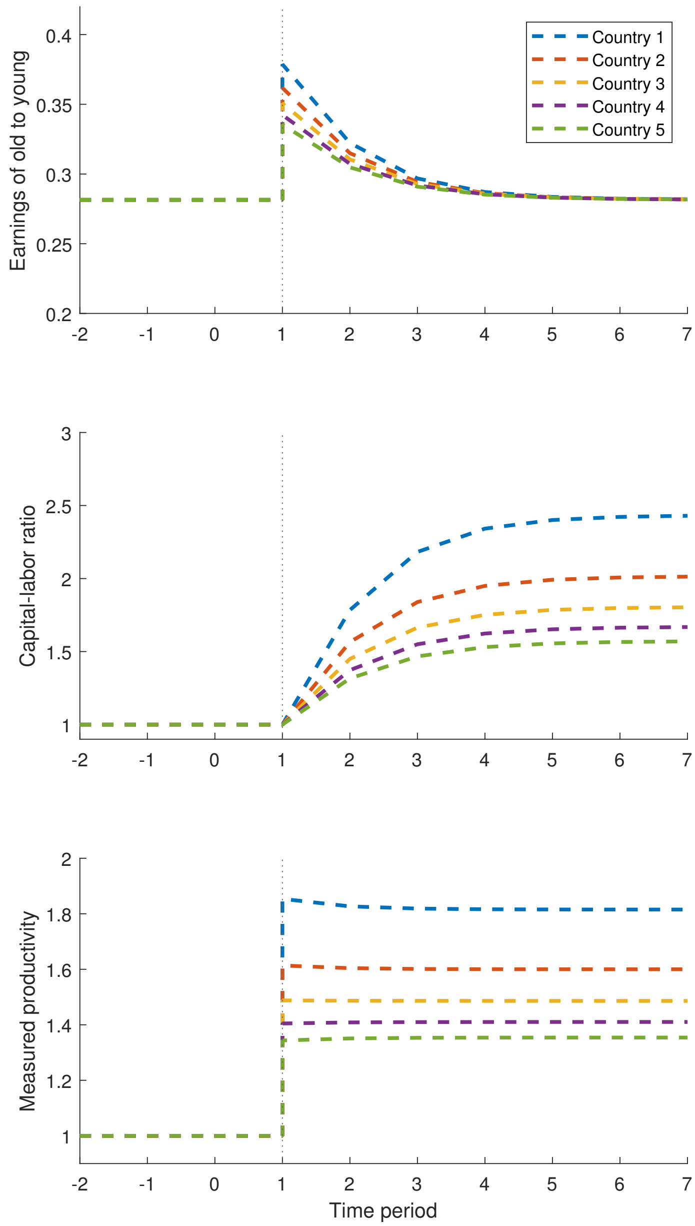
The transitional dynamics for the inequality and its decomposition are similar to the case of asymmetric productivity; see Figure 3. Smaller countries experience a sharper rise in inequality initially compared with larger countries. The inequality then declines back to its autarky steady-state level.

Figure 2: Transition Paths with Heterogeneity in Country Productivity



Notes: Countries are asymmetric in fundamental productivity $A_n = n$. The trade liberalization shock occurs in period 1. The top panel plots the transitional dynamics for earnings ratio of old to young $\frac{P_{n,t} r_{n,t} K_{n,t}}{W_{n,t} L_n}$ from initial steady state (autarky) to final steady state (frictionless trade). The middle and bottom panels plot the transition paths for the capital-labor ratio $\frac{K_{n,t}}{L_n}$ and measured productivity $Z_{n,t}$ relative to initial steady state.

Figure 3: Transition Paths with Heterogeneity in Country Population



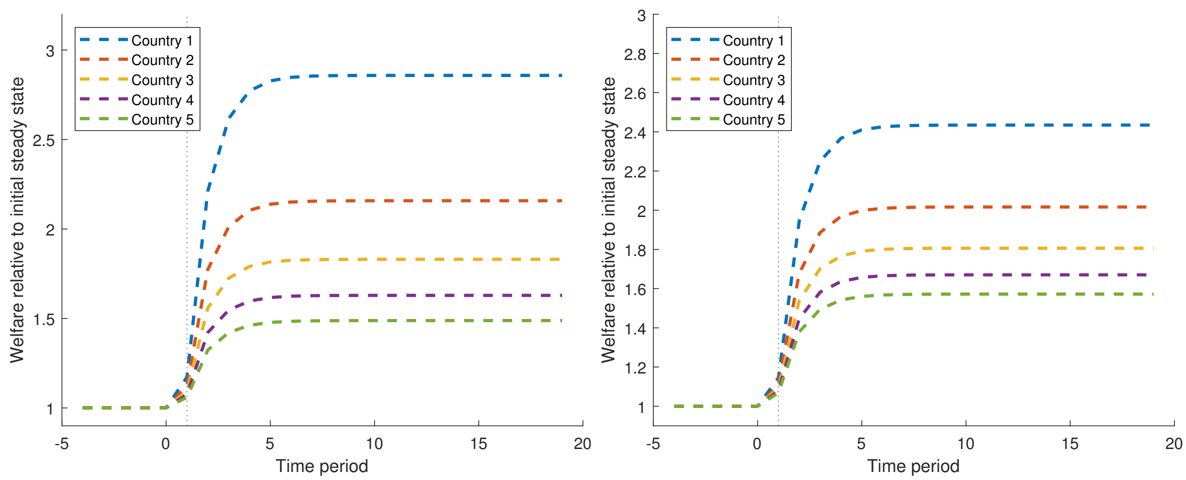
Notes: Countries are asymmetric in population size $L_n^y = L_n^o = n/2$. The trade liberalization occurs in period 1. The top panel plots the transitional dynamics for earnings ratio of old to young $\frac{P_{n,t} r_{n,t} K_{n,t}}{W_{n,t} L_n}$ from initial steady state (autarky) to final steady state (frictionless trade). The middle and bottom panels plot the transition paths for the capital-labor ratio $\frac{K_{n,t}}{L_n}$ and measured productivity $Z_{n,t}$ relative to initial steady state.

3.3 Welfare Dynamics

The previous analysis focused on income differences across individuals within a time period and how these differences evolved over time following trade integration. We now examine cross-cohort differences in welfare as measured by realized lifetime utility in equation (1).

Figure 4 illustrates the evolution of welfare following a one-time permanent trade liberalization that occurs in period 1. The left panel depicts the case where countries differ in their fundamental productivity ($A_n = n$), while the right panel depicts the case where countries differ in their population size ($L_n^y = L_n^o = n/2$).

Figure 4: Evolution of Household Welfare

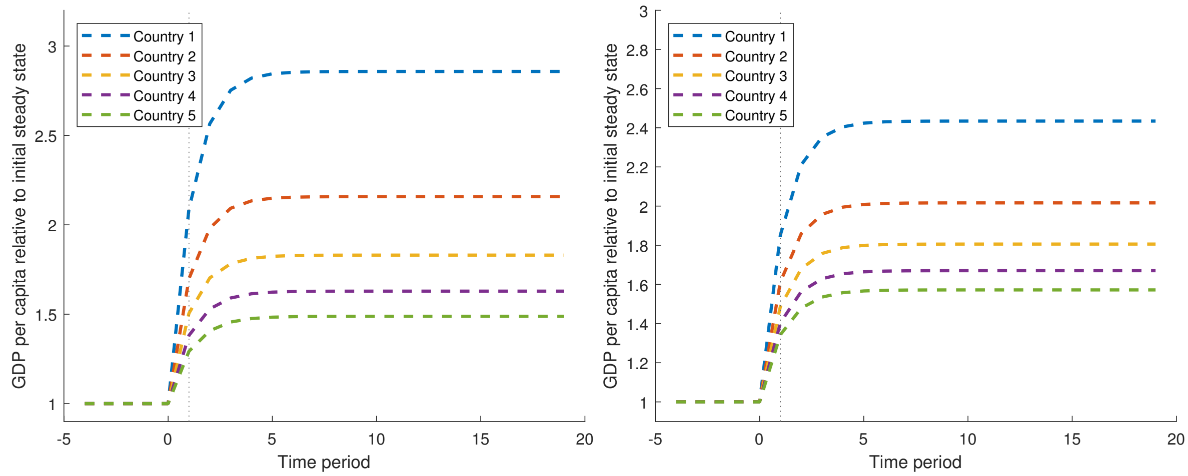


Notes: This figure plots the transition for each cohort's welfare (lifetime utility) along the path from the autarky steady state to the frictionless-trade steady state. Welfare is plotted relative to its autarky steady-state level. The left panel depicts the case when countries differ in fundamental productivity, $A_n = n$. The right panel depicts the case when countries differ in population size, $L_n^y = L_n^o = n/2$. The trade liberalization shock occurs in period 1.

Clearly, in the steady state, each cohort experiences the same lifetime utility as the preceding cohort. However, at the time of the trade liberalization, period 1, the current old realize an unexpectedly higher return to their saving and enjoy more consumption than the old from the preceding cohort. Simultaneously, the young in period 1 realize higher labor returns, have higher lifetime earnings, and higher welfare than the previous cohort. Over time, as capital gradually accumulates, returns to labor continue rising, while returns to capital diminish, although they remain above their autarky steady-state level. Thus, each successive cohort experiences greater lifetime utility than their immediate predecessor.

In this setting, neither the young nor the old are worse off from trade integration, although the inequality between the young and old is higher along the transition path than in the steady state. This is because each generation experiences both sides of the inequality: the upside when old and the downside when young. That is, faster expansion of resources coincides with an unequal distribution of those resources along the transition path. Figure 5 plots the evolution of real GDP per capita, a common metric used in trade models when evaluating gains from trade. Similar to models with an infinitely-lived representative household, smaller and less-productive countries realize a higher growth rate following trade liberalization.

Figure 5: Evolution of GDP per Capita



Notes: This figure plots the transition for real GDP per capita from the autarky steady state to the frictionless-trade steady state. GDP per capita is plotted relative to its autarky steady-state level. The left panel depicts the case when countries differ in fundamental productivity, $A_n = n$. The right panel depicts the case when countries differ in population size, $L_n^y + L_n^o = n$. The trade liberalization shock occurs in period 1.

4 Conclusion

We study the lifecycle aspect of within-country inequality. Our framework is a multicountry trade model embedded in an overlapping generations model with production and capital accumulation. We measure inequality by the ratio of old agents' income, which is payment to their capital services, to that of the young, which is payment to their labor services. The dynamics of inequality in our model stem from the effect of trade liberalization on the marginal products of capital and labor. Trade liberalization increases the measured total factor productivity in each country, which increases the marginal product of capital and incentivizes capital accumulation. Higher capital stock and higher measured productivity raise the marginal product of labor and, hence, wages. Inequality evolves over time due to capital accumulation during the transition from autarky to a frictionless-trade world. Immediately after liberalization, inequality increases since the capital-labor ratio does not adjust instantaneously. Over time, capital accumulates at a diminishing rate and inequality declines.

We show that, when countries differ in their fundamental productivities, trade liberalization increases inequality more in less-productive countries. Furthermore, the less-productive countries accumulate capital faster and the inequality in less-productive countries is persistently above that in more-productive countries.

While inequality in each country increases on impact from trade liberalization and declines only gradually, there are overall gains from trade. Measured by welfare, each cohort fares better than the preceding cohort along the transition path from autarky to frictionless trade.[4]

Even though our results on inequality over the lifecycle are in the context of an Armington trade model, the underlying effects of trade liberalization on measured productivity and capital accumulation are also valid in the Eaton and Kortum (2002) multicountry Ricardian framework. For instance, Ravikumar, Santacreu, and Sposi (2023) show that trade liberalization induces countries to specialize in production in the direction of their comparative advantage, which in turn boosts measured productivity and capital accumulation.

[4]The effect of trade liberalization on cross-country income inequality has been explored by Waugh (2010) and Mutreja, Ravikumar, and Sposi (2018).

References

Adao, Rodrigo, Paul Carrillo, Arnaud Costinot, Dave Donaldson, and Dina Pomeranz. 2022. "Imports, Exports, and Earnings Inequality: Measures of Exposure and Estimates of Incidence." *The Quarterly Journal of Economics* 137 (3):1553–1614.

Anderson, James E. and Eric vanWincoop. 2003. "Gravity with gravitas: A solution to the border puzzle." *American Economic Review* 93 (1):170–192.

Armington, Paul S. 1969. "A Theory of Demand for Products Distinguished by Place of Production." *Staff Papers-International Monetary Fund* :159–178.

Autor, David H., David Dorn, and Gordon H. Hanson. 2013. "The China Syndrome: Local Labor Market Effects of Import Competition in the United States." *American Economic Review* 103 (6):2121–68.

Autor, David H., David Dorn, Gordon H. Hanson, and Jae Song. 2014. " Trade Adjustment: Worker-Level Evidence." *The Quarterly Journal of Economics* 129 (4):1799–1860.

Beladi, Hamid, Sugata Marjit, Reza Oladi, and Sepideh Raei. forthcoming. "Vertical Specialization, Global Expansion of Supply Chain, and Convergence." *Macroeconomic Dynamics* .

Berman, Eli and Stephen Machin. 2000. "Skill-Biased Technology Transfer Around the World." *Oxford Review of Economic Policy* 16 (3):12–22.

Card, David and John E. DiNardo. 2002. "Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles." *Journal of Labor Economics* 20 (4):733–783.

Diamond, Peter. 1965. "National Debt in a Neoclassical Growth Model." *American Economic Review* 55 (5):1126–1150.

Eaton, Jonathan and Samuel Kortum. 2002. "Technology, Geography, and Trade." *Econometrica* 70 (5):1741–1779.

Fajgelbaum, Pablo D. and Amit K. Khandelwal. 2016. "Measuring the Unequal Gains from Trade." *The Quarterly Journal of Economics* 131 (3):1113–1180.

Ghironi, Fabio. 2024. "International Economic Sanctions and Third-Country Effects." *IMF Economic Review* 72:611–652.

Goldberg, Pinelopi Koujianou. 2023. *The unequal effects of globalization*. MIT Press.

Gollin, Douglas. 2002. "Getting Income Shares Right." *Journal of Political Economy* 110 (2):458–474.

Krugman, Paul R. 2008. "Trade and Wages, Reconsidered." *Brookings Papers on Economic Activity* 2008:103–137.

Mutreja, Piyusha, B. Ravikumar, and Michael Sposi. 2018. "Capital Goods Trade, Relative Prices, and Economic Development." *Review of Economic Dynamics* 27:101–122.

Ravikumar, B., Ana Maria Santacreu, and Michael Sposi. 2023. "TFP, Capital Deepening, and Gains from Trade." *Federal Reserve Bank of St. Louis Review* Third Quarter:150–76.

———. 2024. "Trade liberalization versus protectionism: Dynamic welfare asymmetries." *European Economic Review* 163:104692.

Rodrik, Dani. 2021. "A Primer of Trade and Inequality." Working Paper 29507, National Bureau of Economic Research.

Simonovska, Ina and Michael E. Waugh. 2014. "The Elasticity of Trade: Estimates and Evidence." *Journal of International Economics* 92 (1):34–50.

Slaughter, Matthew. 2001. "International Trade and Labour-market Outcomes: Results, Questions, and Policy Options." *The Economic Journal* 108 (450):1452–1462.

Stolper, Wolfgang F. and Paul A. Samuelson. 1941. "Protection and Real Wages." *The Review of Economic Studies* 9 (1):58–73.

Waugh, Michael E. 2010. "International Trade and Income Differences." *American Economic Review* 100 (5):2093–2124.

Wood, Adrian. 1995. "How Trade Hurt Unskilled Workers." *Journal of Economic Perspectives* 9 (3):57–80.

———. 1998. "Globalisation and the Rise in Labour Market Inequalities." *The Economic Journal* 108 (450):1463–1482.

Appendix

A Dynamic Equilibrium and Computational Algorithm

This section of the Appendix provides equilibrium conditions for the frictionless-trade world and an algorithm to compute the exact transitional dynamics. The conditions leverage the fact that, under frictionless trade, $d_{ni} = 1$ and $\pi_{ni} = \pi_{ii}$ for all (n, i).

Table 2: Dynamic Equilibrium Conditions

1	$p_{n,t} = \left(\sum_{n'=1}^{N} (u_{n',t})^{-\theta} (A_{n'})^{\theta}\right)^{-1/\theta}$	$\forall(n,t)$
2	$\pi_{nn,t} = (u_{n,t})^{-\theta} (A_n)^{\theta} \Big/ \left(\sum_{n'=1}^{N} (u_{n',t})^{-\theta} (A_{n'})^{\theta}\right)$	$\forall(n,i,t)$
3a	$r_{n,t} = \frac{R_{n,t}}{P_{n,t}} - \delta$	$\forall(n,t)$
3b	$r_{n,T+1} = r_{n,T}$	$\forall(n,t)$
4	$s_{n,t} = \left(1 + \beta^{-\sigma} (1 + r_{n,t+1})^{-\sigma}\right)^{-1} (W_{n,t}/P_{n,t})$	$\forall(n,t)$
5	$c^{y}_{n,t} = \frac{W_{n,t}}{P_{n,t}} - s_{n,t}$	$\forall(n,t)$
6a	$c^{o}_{n,t+1} = (1 + r_{n,t+1})\, s_{n,t}$	$\forall(n,t)$
6b	$c^{o}_{n,1} = (1 + r_{n,1}) K_{n,1}/L^{o}_{n}$	$\forall(n,t)$
7	$C_{n,t} = L^{y}_{n,t} c^{y}_{n,t} + L^{o}_{n,t} c^{o}_{n,t}$	$\forall(n,t)$
8	$K_{n,t+1} = L^{y}_{n,t} s_{n,t}$	$\forall(n,t)$
9	$I_{n,t} = K_{n,t+1} - (1-\delta) K_{n,t}$	$\forall(n,t)$
10	$Q_{n,t} = C_{n,t} + I_{n,t}$	$\forall(n,t)$
11	$W_{n,t} = (1-\alpha) \sum_{i=1}^{N} P_{i,t} Q_{i,t} \pi_{nn,t} \Big/ (L^{y}_{n,t} + L^{o}_{n,t})$	$\forall(n,t)$
12	$R_{n,t} = \alpha \sum_{i=1}^{N} P_{i,t} Q_{i,t} \pi_{nn,t} \Big/ K_{n,t}$	$\forall(n,t)$
13	$P_{n,t} Q_{n,t} = \sum_{i=1}^{N} P_{i,t} Q_{i,t} \pi_{nn,t}$	$\forall(n,t)$

Note: Units costs $u_{n,t} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} (r_{n,t})^{\alpha} (W_{n,t})^{(1-\alpha)}$.

1. Choose T sufficiently large to ensure the economy settles into a steady state by that period. Guess a sequence of factor price vectors, $\{\mathbf{w}_t, \mathbf{R}_t\}_{t=1}^{T}$. Normalize the wage vectors in each period to the $(N-1)$-simplex: $\Delta \equiv \left\{ \left(\{\mathbf{w}_t\}_{t=1}^{T}\right) \in \mathbb{R}^{NT} : \sum_{n=1}^{N} W_{n,t} L^{y}_{n} = 1, \forall(t) \right\}$.

2. Compute country-level prices, $P_{n,t}$, and bilateral trade shares, $\pi_{ni,t}$, using conditions 1 and 2 in Table 2, respectively.

3. Compute real interest rates, $r_{n,t}$, using condition 3.

4. Compute demand for saving by young, $s_{n,t}$, using condition 4.

5. Compute demand for consumption by young, $c^y_{n,t}$, and demand for consumption by old, $c^o_{n,t}$, using conditions 5, and 6, respectively. Then compute aggregate consumption, $C_{n,t}$, based on condition 7.

6. Compute the sequence of capital stocks, $K_{n,t}$, using condition 8 and recover aggregate investment using condition 9.

7. Compute the composite goods demand, $Q_{n,t}$, using condition 10.

8. Update the sequence of factor price vectors $\{\mathbf{w}_t, \mathbf{R}_t\}_{t=1}^{T}$ using conditions 11 and 12, respectively.

9. If the updated factor prices are close to the previous guess, stop. Otherwise, go back to step 2. Once factor prices converge, condition 13 will hold by Walras' law.