Trade and Inequality in an Overlapping Generations Model with Capital Accumulation

Authors: Jun Nie, B. Ravikumar, and Michael J. Sposi

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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

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Abstract

We study the lifecycle aspect of within-country inequality that stems from capital and labor services supplied by individuals. Our environment is a combination of a multicountry trade model and an overlapping generations model with production and capital accumulation. Trade liberalization increases the measured total factor productivity in each country, which increases the marginal product of capital and incentivizes capital accumulation. Higher capital stock and higher measured productivity raise the marginal product of labor and, hence, wages. Inequality, measured by the ratio of old agents’ income to that of the young, evolves over time due to capital accumulation during the transition from autarky to an open-economy world. Immediately after liberalization, inequality increases. Over time, capital accumulates at a diminishing rate and inequality declines.

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*The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.
†Southern Methodist University, 3100 Dyer St. Dyer TX, 75275. jnie@smu.edu
‡Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63106. b.ravikumar@wustl.edu
§Southern Methodist University, 3100 Dyer St. Dyer TX, 75275. msposi@smu.edu
1 Introduction

“Free trade” is not free. Trade liberalization creates winners and losers. As noted by Rodrik (2021), “Redistribution is the flip side of the gains from trade [...] As an economy opens up to trade, domestic relative prices change, producing income redistribution alongside gains from trade (p.1).”

The notion that trade liberalization creates inequality is an old one (Stolper and Samuelson, 1941). Recent literature has focused on implications of trade liberalization for inequality along several channels: skill (Wood, 1995), sector/occupation (Autor, Dorn, Hanson, and Song, 2014), class (Adao, Carrillo, Costinot, Donaldson, and Pomeranz, 2022), region (Autor, Dorn, and Hanson, 2013), consumption basket (Fajgelbaum and Khandelwal, 2016), or fragmentation (Beladi, Marjit, Oladi, and Raci forthcoming). For instance, as trade liberalization induces countries to specialize, workers in some sectors lose their jobs; inequality emerges between workers in the sectors that the country chooses to specialize in and workers in the sectors that the country abandons. Labor economists, however, have documented that one major “source” of income inequality among workers is age. In the U.S., median earnings of a 50-year-old is fifty percent higher than that of a 25-year-old. Furthermore, the factor composition of income varies with age. Figure illustrates the share of capital income over the lifecycle: The income is predominantly from labor when young and from capital when old.

Our focus is on the lifecycle aspect of within-country inequality that stems from capital and labor services supplied by individuals. We accomplish two tasks in this paper. First, we study the effect of trade liberalization on inequality between different age groups at a point in time. Second, we study the evolution of the inequality over time. Our environment is a combination of an overlapping generations model and a multicountry trade model. The overlapping generations model is that of Diamond (1965) that has production and capital accumulation. The trade model is that of Armington (1969).

The economy lives forever. Each agent in the economy lives for two periods, denoted as “young” and “old.” Each young agent is endowed with 1 unit of labor. Agents work while young, but consume in both periods of their lives. Each country produces a unique

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1For the recent shift from globalization to protectionism, see Goldberg (2023) for arguments related to fairness and inequality and Ravikumar, Santacreu, and Sposi (2024) for how losses from protectionism are smaller than the gains from liberalization.

2Our measure includes income from salary (labor) and capital (interest payments, dividends, and rents) and excludes net sales of assets.
variety by combining capital and labor in a Cobb-Douglas fashion, augmented by country-specific fundamental productivity. International trade is subject to bilateral iceberg costs. In each destination, all varieties are combined to form a composite good. The composite good is allocated to consumption and investment in the destination. Trade is balanced in each period. Saving by the young generation in a period equals the beginning-of-period stock of capital in the next period.

In our environment, the income is from only labor while young, but from only capital when old. Measured total factor productivity in autarky in each country is its fundamental productivity. In an open-economy world, measured total factor productivity in each country depends on the volume of trade. Trade lowers the aggregate price index, thereby heightening the real value of aggregate output, which manifests as an increase in the measured productivity. The increase in measured productivity amplifies the marginal product of capital, which incentivizes capital accumulation. Higher capital stock and higher measured productivity raise the marginal product of labor and, hence, wages. We measure inequality at a point
in time by the ratio of old agents’ income (from capital) to that of the young (from labor). The dynamics of inequality stem from capital accumulation during transition from autarky to an open-economy world.

Our results are as follows. The steady-state inequality in an open-economy world is the same as that in autarky. This is because of the Cobb-Douglas production function and one final good. That is, we do not have Heckscher-Ohlin forces that affect the factor composition of income based on heterogeneous factor intensities.

The inequality, however, varies along the transition path as the world moves from autarky to frictionless trade. When countries are symmetric, the home trade share in each country falls and measured productivity increases. Upon impact, the capital-labor ratio is unchanged, so the inequality initially increases. Over time, capital accumulates at a diminishing rate and inequality declines back to its autarky level. Symmetry also implies that the inequality is identical across countries at every point in time. When countries are asymmetric, we resort to numerical simulations to study the transition paths. For instance, consider the case when the asymmetry is in the fundamental productivity. Upon liberalization, less-productive countries realize a larger increase in measured productivity and inequality. Along the transition, less-productive countries also accumulate capital at a faster rate and inequality stays higher relative to more-productive countries. As capital deepens, the inequality declines back to the autarky level for each country.

While inequality is “worse” along the transition path from a closed-economy world to an open-economy world, lifetime utility is not. As capital accumulates, returns to labor continue to increase, while returns to capital diminish, although the latter remain above the autarky steady state level. So, each cohort experiences greater lifetime utility than the preceding cohort. Using the change in real income per capita as a measure of gain, a measure traditionally used in the trade literature, all countries experience dynamic gains from trade liberalization. Less-productive and/or small countries experience more gains even though they experience higher inequality relative to more-productive and/or large countries.

In the next section, we describe the model and the equilibrium dynamics of inequality. In Section 3 we provide a numerical analysis of the inequality for the case of asymmetric countries. Concluding remarks are in Section 4.
2 Model

Our framework is a combination of a two-period overlapping generations model (Diamond, 1965) and a multi-country trade model (Armington, 1969). There are \( N \) countries indexed by \( n = 1, ..., N \). Time is discrete and time is indexed by \( t = 0, 1, ..., \infty \).

In each period there is a representative household that lives for two periods – young and old. The population of young is \( L_n^y \), and the population of old is \( L_n^o \), so the total population in country \( n \) is \( L_n^y + L_n^o \). Young agents inelastically supply one unit of labor and divide their earnings between consumption and saving. Old agents consume their saving plus the returns on saving. There is a representative firm that rents capital and labor from households to produce output. All markets are competitive and agents have perfect foresight.

We first describe the equilibrium of a world in autarky. We then introduce international trade and describe the corresponding equilibrium.

**Households** The household born in period \( t \) has time-separable preferences given by

\[
\left( \frac{c_{y,n,t}^{\sigma-1}}{\sigma} + \beta \left( \frac{c_{o,n,t+1}^{\sigma-1}}{\sigma} \right) \right)^{\frac{1}{\sigma-1}}, \tag{1}
\]

Consumption is denoted using a subscript \( t \) for the time period, and superscripts “\( y \)” and “\( o \)” refer to young and old, respectively. The discount factor is \( \beta \in (0, 1) \), and the intertemporal elasticity of substitution is \( \sigma \geq 0 \).

The household faces budget constraints in each period of life:

\[
P_{n,t}c_{y,n,t} + P_{n,t}s_{n,t} \leq W_{n,t} \tag{2}
\]

\[
P_{n,t+1}c_{o,n,t+1} \leq P_{n,t+1}(1 + r_{n,t+1})s_{n,t}, \tag{3}
\]

where \( s_{n,t} \) denotes saving by the young in period \( t \), \( W_{n,t} \) is the nominal wage rate in period \( t \), \( r_{n,t+1} \) is the real interest rate between periods \( t \) and \( t+1 \), and \( P_{n,t} \) is the price of goods in period \( t \). When young, the household allocates the wage income to consumption and saving. When old, it collects principal plus interest on its savings and finances its consumption.

The household’s intertemporal Euler equation is

\[
\frac{c_{o,n,t+1}^{\sigma}}{c_{y,n,t}^{\sigma}} = \beta^{\sigma} (1 + r_{n,t+1})^\sigma. \tag{4}
\]
Thus, the household’s optimal saving is described by

\[ s_{n,t} = \left( \frac{\beta \sigma (1 + r_{n,t+1})^{\sigma - 1}}{1 + \beta \sigma (1 + r_{n,t+1})^{\sigma - 1}} \right) \left( \frac{W_{n,t}}{P_{n,t}} \right). \] (4)

**Firms** There is a representative firm that rents capital and labor to produce output according to a Cobb-Douglas production function:

\[ Y_{n,t} = A_n (K_{n,t})^\alpha (L_n)^{1-\alpha}, \] (5)

where \( A_n \) is country \( n \)’s time-invariant fundamental productivity, \( K_{n,t} \) and \( L_n \) are the capital and labor used in production, and \( \alpha \) is capital’s share in value added. In each period a fraction \( \delta \in [0, 1] \) of the capital stock depreciates during production.

### 2.1 Autarky

Competitive factor markets imply that the real rental rate and real wage rate are equal to the marginal products of capital and labor, respectively, and the price of output equals the marginal cost of production:

\[ \frac{R_{n,t}}{P_{n,t}} = \alpha A_n \left( \frac{K_{n,t}}{L_n} \right)^{\alpha - 1}, \] (6)

\[ \frac{W_{n,t}}{P_{n,t}} = (1 - \alpha) A_n \left( \frac{K_{n,t}}{L_n} \right)^\alpha, \] and (7)

\[ P_{n,t} = \frac{(\frac{R_{n,t}}{\alpha})^\alpha (\frac{W_{n,t}}{1-\alpha})^{1-\alpha}}{A_n}. \] (8)

**Equilibrium** In the absence of arbitrage, the real interest rate on the household’s savings must equal the marginal product of capital, net of depreciation:

\[ r_{n,t} = \frac{R_{n,t}}{P_{n,t}} - \delta. \] (9)

In equilibrium, the capital stock available in the next period equals the savings by the young in the current period and the labor employed equals the size of the young population:
The above equilibrium conditions imply that the law of motion for capital is

\[ K_{n,t+1} = s_{n,t}L_n^y \]
\[ L_n = L_n^y. \]

(10)

(11)

Equilibrium Inequality in Autarky

The focus of this paper is the income inequality across age groups and how this inequality varies over time. We define inequality as the ratio of old-to-young earnings, in line with our empirical metric in Figure 1:

\[ \rho_{n,t} \equiv \frac{P_{n,t}r_{n,t}K_{n,t}w_{n,t}}{K_{n,t}}. \]

(13)

That is, the numerator captures the interest income flow received by old agents, and the denominator captures labor income flow received by young agents. As in Figure 1, the numerator does not include wealth, i.e., the value of the capital stock. We study the properties of \( \rho_{n,t} \) both in the steady state and along the transition.

**Lemma 1.** In an autarky steady state, each country’s capital-labor ratio is proportional to its “scaled” fundamental productivity level: \( \frac{K_n}{L_n} \propto A_n^{1-\alpha}. \)

**Proof.** Denote the steady-state capital stock by \( K_n^* \). In steady state, the law of motion (12) implies

\[ 1 = \left( 1 + \beta^{-\sigma} \left( \alpha A_n \left( \frac{K_n^*}{L_n} \right)^{\alpha-1} + 1 - \delta \right)^{1-\sigma} \right)^{1-\sigma} (1 - \alpha) A_n \left( \frac{K_n^*}{L_n} \right)^{\alpha-1}. \]

Define \( x \equiv A_n \left( \frac{K_n^*}{L_n} \right)^{\alpha-1} \). Clearly, the solution for \( x \) depends only on the parameters \((\alpha, \beta, \sigma, \delta)\). Hence, the steady-state capital-labor ratio, \( K_n^*/L_n \), in country \( n \) is proportional to its “scaled” fundamental productivity level \( A_n^{1-\alpha} \). \( \square \)
Countries with higher productivity will have higher steady-state capital-labor ratios. The scaling factor, \( \frac{1}{1-\alpha} \), captures the fact that higher productivity increases resources available for saving, which in turn induces more production capacity in the future through higher capital stock, the impact of which is governed by capital’s share, \( \alpha \).

**Theorem 1.** In an autarky steady state, the inequality does not depend on fundamental productivity or the population size.

**Proof.** To see this, apply equation (13) to the steady state:

\[
\rho_n^* \equiv \frac{P_n^* r_n^* K_n^*}{w_n^* L_n} = \frac{P_n^* (R_n^*/P_n^* - \delta) K_n^*}{w_n^* L_n} = \frac{R_n^* K_n^* - P_n^* \delta K_n^*}{w_n^* L_n} = \frac{\alpha}{1 - \alpha} - \delta \left(\frac{K_n^*}{L_n}\right)^{1-\alpha} \frac{A_n}{(1 - \alpha)A_n}.
\]

Using Lemma 1, \( \left(\frac{K_n^*}{L_n}\right)^{1-\alpha} \) is a constant that depends only on \((\alpha, \beta, \sigma, \delta)\). \(\square\)

**Transitional Dynamics**  Consider a world not in a steady state. For instance, the initial capital stock is not equal to the steady-state capital stock.

**Theorem 2.** In autarky, along the transition path, the change in inequality depends only on change in the capital-labor ratio.

**Proof.** To see this, apply equation (13):

\[
\rho_{n,t} \equiv \frac{P_{n,t} r_{n,t} K_{n,t}}{W_{n,t} L_n} = \frac{P_{n,t} (R_{n,t}/P_{n,t} - \delta) K_{n,t}}{W_{n,t} L_n} = \frac{R_{n,t} K_{n,t} - P_{n,t} \delta K_{n,t}}{W_{n,t} L_n} = \frac{\alpha}{1 - \alpha} - \delta \left(\frac{K_{n,t}}{L_n}\right)^{1-\alpha} \frac{A_n}{(1 - \alpha)A_n}.
\]

\(\square\)

As capital becomes more abundant relative to labor, its marginal product declines, whereas the marginal product of labor increases. So, earnings accruing to capital owners (the old) decline relative to those of labor owners (the young), which decreases inequality over time.
Since the production function is Cobb Douglas, the share of GDP that accrues to capital is equal to \( \alpha \), so why is \( \rho_{n,t} \) not equal to \( \frac{\alpha}{1-\alpha} \)? The reason is that the interest income received by the old agents is not the capital share of GDP. In the absence of arbitrage, the interest rate on savings accounts for not only the rental rate received by the old when they rent the capital to the representative firm but also the rate at which capital depreciates after production. To see this, consider the accounting for aggregate demand in period \( t \), consisting of consumption by the current young and current old, along with gross saving of the young:

\[
L_n^y P_n t c_{n,t}^y + L_n^o P_n t c_{n,t}^o + L_n^y P_n t s_{n,t} = P_n t (1 + r_{n,t}) K_{n,t} + W_{n,t} L_n.
\]

Using \( r_{n,t} = R_{n,t}/P_{n,t} - \delta \) yields

\[
L_n^y P_n t c_{n,t}^y + L_n^o P_n t c_{n,t}^o + L_n^y P_n t s_{n,t} = P_n t \left( 1 + \frac{R_{n,t}}{P_{n,t}} - \delta \right) K_{n,t} + W_{n,t} L_n
\]

Dividing through by \( P_{n,t} \) and using the facts that \( K_{n,t+1} = L_n^y s_{n,t} \) and \( K_{n,t+1} = (1 - \delta) K_{n,t} + I_{n,t} \) gives rise to the GDP accounting identity:

\[
\left( \frac{L_n^y c_{n,t}^y}{C_{n,t}} \right) + \left( \frac{L_n^o c_{n,t}^o}{I_{n,t}} \right) + \left( 1 - \delta \right) K_{n,t} = \frac{R_{n,t}}{P_{n,t}} K_{n,t} + \frac{W_{n,t}}{P_{n,t}} L_n.
\]

In other words, aggregate investment in the economy equals aggregate saving, which in turn is the gross saving by the young minus the gross dis-saving by the old. The latter embeds the depreciated capital stock.

### 2.2 Open-economy World

In this section we study a world with frictionless international trade. Household and firm behavior are the same as in the previous section. We now introduce a “retail firm” and modify the market-clearing conditions.

**Production and International Trade** Each country produces a differentiated good according to the Cobb-Douglas production function \([5]\).

Denote \( q_{n,i,t} \) as quantity of goods that country \( n \) sources from country \( i \) at time \( t \). There is
a retail firm that aggregates goods from all sources to construct a composite good as follows:

\[ Q_{n,t} = \left[ \sum_{i=1}^{N} (q_{ni,t})^{\theta \frac{1+\theta}{1+\theta}} \right]^{\frac{1}{\theta}}, \]  

(14)

where \( \theta \) is the elasticity of substitution of goods among source countries (trade elasticity) and \( Q_{n,t} \) is the aggregate composite good. The quantity of the composite good determines the resources available for consumption and investment in country \( n \). International trade is subject to iceberg costs denoted by \( d_{ni,t} \geq 1 \) for \( \forall i \neq n \). We normalize \( d_{nn,t} = 1 \).

**Prices in the Open-economy World**  
The price that country \( n \) faces to purchase one unit from country \( i \) is given by

\[ p_{ni,t} = \frac{u_{i,t}}{A_i} d_{ni,t}, \]

where \( u_{i,t} = \left( \frac{r_{i,t}}{A_i} \right)^{\alpha} \left( \frac{w_{i,t}}{1-\alpha} \right)^{1-\alpha}. \)

Thus, the optimal quantity of goods that country \( n \) sources from country \( i \) is

\[ q_{ni,t} = \left( \frac{p_{ni,t}}{P_{n,t}} \right)^{-\theta} Q_{n,t}, \]

(15)

and the ideal price index is given by

\[ P_{n,t} = \left( \sum_{i=1}^{N} \left( \frac{u_{i,t}d_{ni,t}}{A_i} \right)^{-\theta} \right)^{-\frac{1}{\theta}}. \]

(16)

The share of country \( n \)'s spending on goods originating from country \( i \) is given by

\[ \pi_{ni,t} = \frac{A_i^\theta u_{i,t}^{-\theta} d_{ni}^{-\theta}}{\sum_{l=1}^{N} A_l u_{l,t}^{-\theta} d_{nl}^{-\theta}}. \]

(17)

We define country \( n \)'s measured productivity as \( Z_{n,t} = A_n (\pi_{nn,t})^{-1/\theta} \). The equilibrium prices in the open-economy world are analogous to those in autarky after replacing the fundamental
productivity with the measured productivity. That is, the equilibrium prices satisfy

\[
\frac{R_{n,t}}{P_{n,t}} = \alpha Z_{n,t} \left( \frac{K_{n,t}}{L_n} \right)^{\alpha-1} \tag{18}
\]

\[
\frac{W_{n,t}}{P_{n,t}} = (1 - \alpha) Z_{n,t} \left( \frac{K_{n,t}}{L_n} \right)^\alpha \tag{19}
\]

\[
P_{n,t} = \frac{\left( \frac{R_{n,t}}{\alpha} \right)^\alpha \left( \frac{W_{n,t}}{1-\alpha} \right)^{1-\alpha}}{Z_{n,t}} \tag{20}
\]

**Equilibrium in the Open-economy World** In addition to the conditions of (i) no arbitrage and (ii) factor market clearing—equations (9), (10), and (11)—we impose balanced trade to complete the characterization of the open economy:

\[
\sum_{i=1}^{N} P_{n,t} Q_{n,t} \pi_{n_i,t} = \sum_{i=1}^{N} P_{i,t} Q_{i,t} \pi_{in,t}. \tag{21}
\]

This condition gives rise to the familiar accounting identity that output equals the sum of consumption, investment, and net exports. To see why, note that the composite good purchased by country \( n \), \( Q_{n,t} \), constitutes the sum of country \( n \)'s aggregate consumption and investment:

\[
Q_{n,t} = C_{n,t} + I_{n,t}.
\]

The left-hand side of equation (21), which is country \( n \)'s total spending on goods from all countries including itself, is equal to country \( n \)'s total spending on the composite good since \( \sum_{i=1}^{n} \pi_{n_i,t} = 1 \). The right-hand side of equation (21) is the sum of all destination countries’ spending on goods produced by country \( n \) and is thus country \( n \)'s output and total income. By deflating the aggregate income by the aggregate price level \( P_{n,t} \), it follows that

\[
Y_{n,t} = Q_{n,t}.
\]

Similar to autarky, the equilibrium law of motion for capital is given by

\[
K_{n,t+1} = \frac{\beta^\sigma \left( \alpha Z_{n,t+1} \left( \frac{K_{n,t+1}}{L_{n,t+1}} \right)^{\alpha-1} + 1 - \delta \right)^{\sigma-1}}{1 + \beta^\sigma \left( \alpha Z_{n,t} \left( \frac{K_{n,t+1}}{L_{n,t+1}} \right)^{\alpha-1} + 1 - \delta \right)^{\sigma-1}} \left( 1 - \alpha \right) Z_{n,t} K_{n,t}^{\alpha} L_{n}^{1-\alpha}. \tag{22}
\]
Finally, inequality in the open-economy world is given by

\[ \rho_{n,t} \equiv \frac{P_{n,t} r_{n,t} K_{n,t}}{W_{n,t} L_n} = \frac{\alpha}{1 - \alpha} - \delta \frac{(K_{n,t}/L_{n,t})^{1-\alpha}}{(1 - \alpha) Z_{n,t}}. \]  

(23)

In the open-economy world, the two equilibrium relationships above are analogous to those in autarky, after replacing the fundamental productivity \( A_n \) with the measured productivity \( Z_n \). Table 2 in Appendix A provides the full set of equilibrium conditions. The following sequence of claims leverages this observation. The proofs are omitted since they are analogous to those in the closed-economy section. In particular, Lemma 1 can now be generalized.

**Lemma 2.** In an open-economy world, each country’s steady-state capital-labor ratio is proportional to its “scaled” measured productivity level:

\[ \frac{K_n}{L_n} \propto Z_n^{\frac{1}{1-\alpha}}. \]

Since the effect of bilateral trade costs is summarized by each country’s home trade share, trade integration influences each country through the measured productivity.

The next result is the counterpart of Theorem 1 for the open-economy world.

**Theorem 3.** In an open-economy world, the steady-state inequality does not depend on fundamental productivity or the population size.

The next result is the counterpart of Theorem 2 and yields one notable difference relative to autarky along the transition path.

**Theorem 4.** In an open-economy world, along the transition path, change in inequality depends on the change in the capital-labor ratio and change in the measured productivity.

Unlike autarky, the measured productivity \( Z_{n,t} \) in the open economy need not be constant along a transition path since \( Z_{n,t} = A_n \pi^{n\theta}_{mn,t} \). That is, the home trade share adjusts when capital stocks grow asymmetrically across countries. The home trade share increases in locations whose capital stocks grow relatively faster, reflecting relatively growing endowments and therefore occupying an increasing share of world trade.

Theorem 4 demonstrates that the transitional dynamics in an open-economy world are more complicated than in autarky. However, when countries are symmetric, then the dynamics are straightforward, as we describe next.
Symmetric Countries In the special case where all of the countries are identical, equation (17) implies that the home trade share must be equal to
\[ \pi_{nn,t} = \left( 1 + (N - 1)d^{-\theta} \right)^{-1}, \]
where \( d \) is the symmetric bilateral trade cost. Consider a scenario in which the world is initially in an autarkic steady state, then unexpectedly moves to a world with lower trade costs. The home trade share immediately declines from 1 to some value \( \pi^* \in [1/N, 1) \) and remains constant thereafter. In turn, the measured productivity \( Z_n \) immediately increases and remains permanently higher thereafter.

Upon impact, the capital-labor ratio is unchanged, so the inequality \( \rho_{nn,t} \) initially increases. Over time, capital gradually accumulates at a diminishing rate and inequality steadily declines back to its initial steady-state level. Moreover, the ratio of young to old earnings is identical across countries at every point in time.

We turn to numerical analysis to evaluate an open-economy world with heterogeneous countries.

3 Numerical Analysis

In this section we numerically evaluate the implications of moving from a relatively closed world \( (d_{ni} = 100) \) to a frictionless-trade world \( (d_{ni} = 1) \). Our algorithm for computing the exact transitional dynamics of the open-economy world is based on one from Ravikumar, Santacreu, and Sposi (2024), and is described in Appendix A. We assume the world is initially closed and in steady state. We then study the transition paths to the frictionless-trade world when the countries differ along one of the following dimensions: fundamental productivity or population size.

In each case we assign values for key parameters as reported in Table 1. We interpret one period in the model to correspond to about thirty years. We set the discount factor to \( \beta = 0.29 \), corresponding to an annual discount factor of 0.96, or an annual real interest rate of about 4 percent. We set the intertemporal elasticity of substitution to \( \sigma = 0.75 \) so that consumption when young and old are complements in the agent’s preferences. We assign \( \alpha = 0.33 \) based on Gollin (2002) and set \( \delta = 0.85 \), corresponding to an annual depreciation rate of roughly 6 percent. Finally, we set trade elasticity to \( \theta = 4 \) following Simonovska and Waugh (2014).

We set the number of countries \( N = 5 \). Albeit somewhat arbitrary, this choice facilitates
“third-country effects” that would be inadmissible in a two-country setting. Additionally, it allows us to span a reasonable degree of heterogeneity while maintaining parsimony.

### 3.1 Asymmetric Fundamental Productivity

In this case we assume that countries differ in their fundamental productivity: $A_n = n$. That is, country 5 is five times as productive as country 1. The countries have identical population size and age shares: $L^y_n + L^o_n = 1, L^y_n = L^o_n = 1/2$. The other parameters are as in Table 1 for all countries.

Figure 2 plots the transitional dynamics for inequality (top), the capital-labor ratio (middle), and measured productivity (bottom). In period 1, we remove bilateral trade barriers. On impact, the inequality in all countries increases. The initial boost in first period is driven by measured productivity because capital-labor ratio is unchanged relative to the initial steady state. Moreover, the increase is particularly pronounced in low-productivity countries. That is, opening up to trade allows less-productive countries to access better technologies from more-productive countries and realize a larger boost in measured productivity.

From period 2 onward, countries start to accumulate capital. Given that less-productive countries have a larger increase in measured productivity, they also accumulate capital faster than more-productive countries. As capital deepens, the ratio of old to young earnings declines back toward its initial steady-state level.

### 3.2 Asymmetric Population

In this case we assume that countries differ in their total population size but have identical fundamental productivity and age shares: $A_n = 1, L^y_n + L^o_n = n, L^y_n = L^o_n = n/2$. That is, countries with a higher index $n$ have a larger population.

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Third-country effects are quantitatively important when considering changes in trade costs between heterogeneous countries (see, e.g., Anderson and van Wincoop 2003, Ghironi 2024).
Figure 2: Transition Paths with Heterogeneity in Country Productivity

Notes: Countries are asymmetric in fundamental productivity $A_n = n$. The trade liberalization shock occurs in period 1. The top panel plots the transitional dynamics for earnings ratio of old to young $\frac{P_{n,t}W_{n,t}K_{n,t}}{W_{n,t}L_{n,t}}$ from initial steady state (autarky) to final steady state (frictionless trade). The middle and bottom panels plot the transition paths for the capital-labor ratio $\frac{K_{n,t}}{L_{n,t}}$ and measured productivity $Z_{n,t}$ relative to initial steady state.
Notes: Countries are asymmetric in population size $L_n^y + L_n^o = n$. The trade liberalization occurs in period 1. The top panel plots the transitional dynamics for earnings ratio of old to young $\frac{P_{n,t} r_{n,t} K_{n,t}}{W_{n,t} L_n}$ from initial steady state (autarky) to final steady state (frictionless trade). The middle and bottom panels plot the transition paths for the capital-labor ratio $\frac{K_{n,t}}{L_n}$ and measured productivity $Z_{n,t}$ relative to initial steady state.
The transitional dynamics for the inequality and its decomposition are similar to the case of asymmetric productivity; see Figure 3. Smaller countries experience a sharper rise in inequality initially compared with larger countries. The inequality then declines back to its autarky steady-state level.

### 3.3 Welfare Dynamics

The previous analysis focused on income differences across individuals within a time period and how these differences evolved over time following trade integration. We now examine cross-cohort differences in welfare as measured by realized lifetime utility in equation (1).

Figure 4 illustrates the evolution of welfare following a one-time permanent trade liberalization that occurs in period 1. The left panel depicts the case where countries differ in their fundamental productivity \( A_n = n \), while the right panel depicts the case where countries differ in their population size \( L^n_y + L^n_o = n \).

**Figure 4: Evolution of Household Welfare**

Notes: This figure plots the transition for each cohort’s welfare (lifetime utility) along the path from the autarky steady state to the frictionless-trade steady state. Welfare is plotted relative to its autarky steady-state level. The left panel depicts the case when countries differ in fundamental productivity, \( A_n = n \). The right panel depicts the case when countries differ in population size, \( L^n_y + L^n_o = n \). The trade liberalization shock occurs in period 1.

Clearly, in the steady state, each cohort experiences the same lifetime utility as the preceding cohort. However, at the time of the trade liberalization, period 1, the current old realize an unexpectedly higher return to their saving and enjoy more consumption than the old from the preceding cohort. Simultaneously, the young in period 1 realize higher labor
returns, have higher lifetime earnings, and higher welfare than the previous cohort. Over time, as capital gradually accumulates, returns to labor continue rising, while returns to capital diminish, although they remain above their autarky steady-state level. Thus, each successive cohort experiences greater lifetime utility than their immediate predecessor.

In this setting, neither the young nor the old are worse off from trade integration, although the inequality between the young and old is higher along the transition path than in the steady state. This is because each generation experiences both sides of the inequality: the upside when old and the downside when young. That is, faster expansion of resources coincides with an unequal distribution of those resources along the transition path. Figure 5 plots the evolution of real GDP per capita, a common metric used in trade models when evaluating gains from trade. Similar to models with an infinitely-lived representative household, smaller and less-productive countries realize a higher growth rate following trade liberalization.

Figure 5: Evolution of GDP per Capita

Notes: This figure plots the transition for real GDP per capita from the autarky steady state to the frictionless-trade steady state. GDP per capita is plotted relative to its autarky steady-state level. The left panel depicts the case when countries differ in fundamental productivity, $A_n = n$. The right panel depicts the case when countries differ in population size, $L_n^h + L_n^o = n$. The trade liberalization shock occurs in period 1.

4 Conclusion

We study the lifecycle aspect of within-country inequality. Our framework is a multicountry trade model embedded in an overlapping generations model with production and capital
accumulation. We measure inequality by the ratio of old agents’ income, which is payment to their capital services, to that of the young, which is payment to their labor services. The dynamics of inequality in our model stem from the effect of trade liberalization on the marginal products of capital and labor. Trade liberalization increases the measured total factor productivity in each country, which increases the marginal product of capital and incentivizes capital accumulation. Higher capital stock and higher measured productivity raise the marginal product of labor and, hence, wages. Inequality evolves over time due to capital accumulation during the transition from autarky to a frictionless-trade world. Immediately after liberalization, inequality increases since the capital-labor ratio does not adjust instantaneously. Over time, capital accumulates at a diminishing rate and inequality declines.

When countries differ in their fundamental productivities, we show that trade liberalization increases inequality more in less-productive countries. Furthermore, the less-productive countries accumulate capital faster and the inequality is persistently above those in more-productive countries.

While inequality increases on impact from trade liberalization and declines only gradually, there are overall gains from trade. Measured by welfare, each cohort fares better than the preceding cohort along the transition path from autarky to frictionless trade.

Even though our results on inequality over the lifecycle are in the context of an Armington trade model, the underlying effects of trade liberalization on measured productivity and capital accumulation are also valid in the Eaton and Kortum (2002) multicountry Ricardian framework. For instance, Ravikumar, Santacreu, and Sposi (2023) show that trade liberalization induces countries to specialize in production in the direction of their comparative advantage, which in turn boosts measured productivity and capital accumulation.
References


Appendix

A Dynamic Equilibrium and Computational Algorithm

This section of the Appendix provides equilibrium conditions for the frictionless-trade world and an algorithm to compute the exact transitional dynamics. The conditions leverage the fact that, under frictionless trade, $d_{ni} = 1$ and $\pi_{ni} = \pi_{ii}$ for all $(n, i)$.

Table 2: Dynamic Equilibrium Conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_{n,t} = \left( \sum_{n'=1}^{N} (u_{n',t})^{-\theta} (A_{n'})^{\theta} \right)^{-1/\theta}$</td>
</tr>
<tr>
<td>2</td>
<td>$\pi_{nn,t} = (u_{n,t})^{-\theta} (A_{n})^{\theta} \left( \sum_{n'=1}^{N} (u_{n',t})^{-\theta} (A_{n'})^{\theta} \right)$</td>
</tr>
<tr>
<td>3a</td>
<td>$r_{n,t} = \frac{R_{n,t}}{p_{n,t}} - \delta$</td>
</tr>
<tr>
<td>3b</td>
<td>$r_{n,T+1} = r_{n,T}$</td>
</tr>
<tr>
<td>4</td>
<td>$s_{n,t} = \left( 1 + \beta^{-\sigma} (1 + r_{n,t+1})^{-\sigma} \right)^{-1} (w_{n,t}/P_{n,t})$</td>
</tr>
<tr>
<td>5</td>
<td>$c_{y,n,t} = \frac{w_{n,t}}{P_{n,t}} - s_{n,t}$</td>
</tr>
<tr>
<td>6a</td>
<td>$c_{o,n,t+1} = (1 + r_{n,t+1}) s_{n,t}$</td>
</tr>
<tr>
<td>6b</td>
<td>$c_{o,n,1} = (1 + r_{n,1}) K_{n,1}/L_{n}$</td>
</tr>
<tr>
<td>7</td>
<td>$C_{n,t} = L_{n,t}^{y} c_{y,n,t} + L_{n,t}^{o} c_{o,n,t}$</td>
</tr>
<tr>
<td>8</td>
<td>$K_{n,t+1} = L_{n,t}^{y} s_{n,t}$</td>
</tr>
<tr>
<td>9</td>
<td>$I_{n,t} = K_{n,t+1} - (1 - \delta) K_{n,t}$</td>
</tr>
<tr>
<td>10</td>
<td>$Q_{n,t} = C_{n,t} + I_{n,t}$</td>
</tr>
<tr>
<td>11</td>
<td>$w_{n,t} = (1 - \alpha) \sum_{i=1}^{N} P_{i,t} Q_{i,t} \pi_{nn,t} / (L_{n,t}^{y} + L_{n,t}^{o})$</td>
</tr>
<tr>
<td>12</td>
<td>$R_{n,t} = \alpha \sum_{i=1}^{N} P_{i,t} Q_{i,t} \pi_{nn,t} / K_{n,t}$</td>
</tr>
<tr>
<td>13</td>
<td>$P_{n,t} Q_{n,t} = \sum_{i=1}^{N} P_{i,t} Q_{i,t} \pi_{nn,t}$</td>
</tr>
</tbody>
</table>

Note: Units costs $u_{n,t} = \alpha^{-\alpha} (1 - \alpha)^{-1-\alpha} (r_{n,t})^{\alpha} (w_{n,t})^{1-\alpha}$.

1. Choose $T$ sufficiently large to ensure the economy settles into a steady state by that period. Guess a sequence of factor price vectors, $\{w_{t}, R_{t}\}_{t=1}^{T}$. Normalize the wage vectors in each period to the $(N-1)$-simplex: $\Delta \equiv \left\{ \{w_{t}\}_{t=1}^{T} \in \mathbb{R}^{NT} : \sum_{n=1}^{N} w_{n,t} L_{n}^{y} = 1, \forall (t) \right\}$.

2. Compute country-level prices, $P_{n,t}$, and bilateral trade shares, $\pi_{ni,t}$, using conditions 1 and 2, respectively.

3. Compute real interest rates, $r_{n,t}$, using condition 3.
4. Compute demand for saving by young, \( s_{n,t} \), using condition 4.

5. Compute demand for consumption by young, \( c_{y,n,t} \), and demand for consumption by old, \( c_{o,n,t} \), using conditions 5, and 6, respectively. Then compute aggregate consumption, \( C_{n,t} \), based on condition 7.

6. Compute the sequence of capital stocks, \( K_{n,t} \), using condition 8 and recover aggregate investment using condition 9.

7. Compute the composite goods demand, \( Q_{n,t} \), using condition 10.

8. Update the sequence of factor price vectors \( \{ w_t, R_t \}^T_{t=1} \) using conditions 11 and 12, respectively.

9. If the updated factor prices are close to the previous guess, stop. Otherwise, go back to step 2. Once factor prices converge, condition 13 will hold by Walras’ law.