ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

A Theoretical Treatment of Foreign Fighters and Terrorism

Authors	Subhayu Bandyopadhyay, and Todd Sandler
Working Paper Number	2024-011A
Creation Date	May 2024
Citable Link	https://doi.org/10.20955/wp.2024.011
Suggested Citation	Bandyopadhyay, S., Sandler, T., 2024; A Theoretical Treatment of Foreign Fighters and Terrorism, Federal Reserve Bank of St. Louis Working Paper 2024-011. URL https://doi.org/10.20955/wp.2024.011

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

A Theoretical Treatment of Foreign Fighters and Terrorism

Subhayu Bandyopadhyay[a]
[a] Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63102 USA
Todd Sandler[b]
[b] Economics Department, School of Economic, Political & Policy Sciences, University of Texas at Dallas, Richardson, TX, 75080 USA

May 2024

Abstract

The paper offers a game-theoretical model that includes three participants – the terrorist organization, its foreign fighters, and the adversarial host government. In stage 1, the terrorist group induces foreign fighters to emigrate through wage incentives, while the host government deters these fighters through proactive border security. Foreign fighters decide whether to emigrate from their source country (extensive margin) in stage 2, after which these fighters determine their level of attacks (intensive margin) in stage 3. Comparative statics to the Nash equilibrium are tied to changes in the employment or opportunity cost in the source country, as well as to changes in radicalization. Our basic model provides a theoretical foundation to recent empirical results. An extension involves a four-stage game with the host government assuming a leadership role prior to the terrorist group choosing its wage incentive.

JEL codes: D74; H56; C72

Keywords: foreign fighters; extensive and intensive margins; three-stage game; selective incentives; proactive border security

---

Corresponding Author: Todd Sandler email: tsandler@utdallas.edu

A Theoretical Treatment of Foreign Fighters and Terrorism

1. Introduction

Foreign fighters are individuals who exit their home or source country to join a terrorist or insurgent group abroad (Hegghammer 2013; Malet 2015). In the current theoretical study, we are particularly interested in foreign fighters who provide their services to non-Western countries especially in the Middle East and Northern Africa, South Asia, and sub-Saharan Africa. American or Western European citizens who traveled to Syria to join the Islamic State of Iraq and Syria (ISIS) are considered foreign fighters as are those citizens who emigrated to Afghanistan to join al-Qaida after its attack on the World Trade Center and other US targets on September 11, 2001 (henceforth, 9/11). Foreign fighters from the United States, Europe, and Australia also journeyed to Somalia, Yemen, and Pakistan to enlist in terrorist and rebel groups (Hegghammer 2013). A surge in foreign fighters can be linked to the rise in religious fundamentalist terrorism after 1990 (Enders and Sandler 2012; Hoffman 2017).

The Rebel Appeals and Incentive Dataset (RAID) documents how 232 militant groups during 1989–2011 applied material (e.g., wages), ideological, or both inducements to recruit their foreign members (Soules 2023). RAID underscores the importance of selective incentives in the form of payments and ideological identity in terms of religion and other common interests when enlisting foreign volunteers to Islamic Jihadist and non-Islamic terrorist and rebel groups. Both considerations can motivate foreign fighters to assume considerable risk associated with membership when providing a public good (e.g., a caliphate) to followers.[1]

When studying foreign fighters, two questions take center stage. First, what factors attract or repel foreign fighters from going abroad given the public good (nonexcludable) nature

[1] On selective incentives, see Olson (1965) and Sandler (1992).

of the benefits from their efforts for the cause? Second, what risks do foreign fighters pose for their home country after their later demobilization (see, e.g., Braithwaite and Chu 2018; Hegghammer 2013; Malet 2015)? Our game-theoretical treatment only pertains to the first question. The motivation for our exercise stems in part from the observation by Humphreys and Weinstein (2008, 436) that "theoretical work has insufficiently explored the interaction of various recruitment strategies" by terrorist and rebel groups to entice foreign recruits to take up arms. The absence of an underlying formal theoretical model means that disparate empirical results cannot be easily explained or attributed to different causes. For example, some empirical studies find that economic considerations such as source-country unemployment can support the decision of fighters to go abroad (e.g., Benmelech and Klor 2020; Verwimp 2016), while other empirical studies do not necessarily link this unemployment to the decision of fighters to emigrate (Brockmeyer et al. 2023; Edgerton 2023).

Our primary purpose is to present a game-theoretical model of foreign fighters that encompasses the determinants of their extensive and intensive margins. Their joining decision corresponds to the former margin, while their effort decision links to the latter margin. To accomplish this task, we initially formulate a three-stage game. In stage 1, the two adversaries – the resident terrorist organization and the domestic (host) government – choose the wage (or selective incentives) for the foreign fighters and the proactive safeguards at the border, respectively, while anticipating how these fighters respond subsequently in terms of their joining decision in stage 2 and their effort levels in stage 3. The foreign fighters' migration decision accounts for their radicalization, group-offered wages, their source-country opportunity cost, and host-government-instituted proactive safeguards at the border. In stage 3, foreign fighters fix their effort by equating the selective wage incentive with their marginal disutility of effort.

After identifying the Nash-equilibrium levels of selective incentives and host-government proactive policy, we perform two comparative-static exercises. First, we allow for a change in employment or economic opportunities in the source country – the so-called opportunity cost of foreign fighters. Second, we permit alternative changes to the potential foreign fighters' radicalization or commitment distribution to the terrorist cause. The comparative statics yield interesting and surprising changes to the equilibrium values of selective wage incentives and host-country proactive measures, and thus to the number and efforts of foreign fighters. In so doing, our analysis underscores how key assumptions are necessary to pin down the influence of such changes on terrorism stemming from the host country. When the source-country employment increases, selective wage incentives for these fighters and proactive measures at the border increase; however, terrorism in the host country may increase or decrease depending on identified opposing influences. We also show that enhanced source-country radicalization decreases terrorist-offered wages but has ambiguous effects on proactive measures and, ultimately, on changes in the number of foreign fighters. Thus, the ambiguity of empirical results in the literature with respect to opportunity costs, selective incentives, and enhanced radicalization can be given a foundation. Even with just three players – the terrorist group, the host government, and the foreign fighters, our model is still rich with interactions and possible outcomes.

In an extension, the host-country government assumes a leadership role when choosing its proactive measures in stage 1, followed by the terrorist group's choice of its wage in stage 2. Stages 3 and 4 now involve the foreign fighters extensive and intensive margins, respectively. In this new four-stage game, there is a reduction in both the government's proactive measures and the terrorist group's wage that improve the welfare of the adversaries compared to the three-stage Nash game. We trace some subtle differences in the comparative statics associated with changes in source-country employment and radicalization for the leadership and non-leadership representations.

The remainder of the paper contains four sections. Section 2 contains the baseline game-theoretical model along with its Nash equilibrium. The comparative-statics analysis follows in Section 3. The government-leadership game is presented in Section 4 with relevant contrasts to the three-stage game. Concluding remarks are contained in Section 5.

2. Baseline model

Terrorist volunteers from a foreign nation (where there is little or no scope for terrorism) join the activities of a terror group abroad and fight against the domestic or host government there. For example, fighters from Western Europe emigrated to Syria and Iraq to enlist in ISIS (Benmelech and Klor 2020; Brockmeyer et al. 2023). So-called Western Jihadists may prefer a foreign venue for a number of reasons – e.g., fewer counterterrorism measures abroad, the ability to gain training, a preference for the foreign venue, greater camaraderie, or attractive inducements (Hegghammer 2013; Humphreys and Weinstein 2008; Kalyvas and Kocher 2007; Soules 2023). Since terrorist recruits may come from abroad, we refer to the foreign nation as the foreign fighters' *source* nation. In the recipient nation, the domestic or host government employs proactive measures (surveillance and capture at the border) to deter foreign volunteers.[2] To address selective incentives for recruits, we characterize the domestic terror group as offering monetary rewards or wages to attract foreign fighters. The key tradeoff faced by the terrorist leaders is to balance benefits of greater volunteering and greater effort by foreign fighters with their compensation expenditures, while accounting for these fighters' radicalization and home

[2] Given our focus on foreign fighters in the context of terrorism, we are abstracting from standard migration issues. In this paper, border protection actions are included in proactive counterterrorism measures to dissuade and capture foreign fighters.

employment opportunities. By contrast, the besieged host government's problem is to minimize terrorist attacks or conflict at home by reducing the inflow of foreign fighters. The benefits from terror or conflict reduction must be weighed against the costs of proactive effort at the border to keep out foreign fighters.

We assume the following terrorism, *T*, production function:

$$T = \phi\left(el^*\right),\ \phi' > 0\,, \tag{1}$$

where *e* is the effort level chosen by a foreign volunteer (the *intensive margin*), where $1 \le e \le \overline{E}$,[3] l^* is the number of foreign volunteers who successfully migrate (the *extensive margin*), and the production function may be linear ($\phi'' = 0$) or concave ($\phi'' < 0$). We assume that the employment rate in the foreign nation is ρ, so that normalizing the foreign wage at unity means that the expected wage (and utility) in the source nation for all radicalized individuals is also ρ.[4] However, if risk-neutral foreign fighters emigrate to join the terrorist organization, their utility is:

$$U^i = \beta^i + w^* e - \delta\left(e\right),\ \delta'\left(e > 1\right) > 0,\ \delta''\left(e \ge 1\right) > 0,\ \delta'\left(e = 1\right) = \delta\left(e = 1\right) = 0\,, \tag{2}$$

where β^i is the utility of fighter *i* from serving in the terror organization; w^* is the wage paid by the terrorist leaders to foreign volunteers; $w^* e$ is the wage receipts of a volunteer who supplies effort *e*; and $\delta\left(e\right)$ is the fighter's disutility from terrorism effort. In the joiners' net gain function, β^i captures the foreign fighters' extent of radicalization or commitment to the cause, viewed by the empirical literature as an important pull factor (see Edgerton 2023; Fox et al. 2023; Gates and Podder 2015; Hegghammer 2013; Malet 2015; Soules 2023). Marginal

---

[3] The default or minimum effort level of a foreign terrorist volunteer is assumed to be unity, while allowing for larger effort levels up to a maximum of $\overline{E}$ in response to appropriate incentives.

[4] The empirical literature on foreign fighters views the employment conditions in the foreign or source country as an important determinant of the extensive margin (see, e.g., Benmelech and Klor 2020; Brockmeyer et al. 2023; Hanson 2021; Verwimp 2016).

disutility of terrorist action is increasing in the effort level.

The domestic government employs proactive effort *g*, which leads to a detection probability of a foreign terrorist of $P(g)$, with $P'(g) > 0$ and $P''(g) < 0$. Thus, the probability of the foreign terrorist successfully joining the terror organization is $p(g) = 1 - P(g)$, with $p'(g) < 0$ and $p''(g) > 0$.

Initially, we envision a three-stage game. In stage 1, the host government chooses its proactive response, *g*, to secure the border from migrating foreign fighters, and the resident terrorist group chooses its rewards, w^*, to attract foreign volunteers. Stage 2 involves radicalized foreigners who choose whether to migrate abroad to join the terror group or to stay put. Finally, in stage 3, successful foreign volunteers determine their terrorism effort, *e*.

2.1 Backward induction

As is standard, we solve the three-stage game employing backward induction starting in the final stage and moving backward. In stage 3, a foreign fighter can only affect *e*, given that choices regarding *w**, *g*, and migration have already been made in earlier stages. Based on Eq. (2), the first-order condition (FOC) associated with maximizing the foreign fighter's utility is:[5]

$$U_e^i = w^* - \delta'(e) = 0 \Rightarrow e = e(w^*),\ e'(w^*) = \frac{1}{\delta''(e)} > 0, \tag{3}$$

where the last equality is obtained from the FOC through the implicit function rule and indicates that greater wages to foreign fighters induce greater effort on their part.[6]

---

[5] We note that $U_e^i(e=1) = w^* > 0$ for any $w^* > 0$ because $\delta'(e=1) = 0$ from Eq. (3). Thus, a corner solution at $e = 1$ is ruled out when $w^* > 0$. We also assume that $\delta'(e = \bar{E})$ is sufficiently large to rule out the possibility of $e = \bar{E}$. The second-order condition is satisfied because $U_{ee}^i = -\delta''(e) < 0$.

[6] For tractability, we assume that $\delta''' = 0$, such that $e'(w^*)$ is a constant.

Moving backward to stage 2 and the choice of the foreign fighters' extensive margin, we assume that the mass of source-country radicalized individuals is $\bar{R}$ with terrorism allegiance parameter β^i distributed uniformly in this radicalized population with a cumulative distribution function $M(\beta)$ and corresponding probability density function m. When foreign terrorist volunteers are caught transiting the border, they lose their time endowment and are simply sent back home. Then, using Eq. (2), remembering that $p(g)$ is the probability of successfully joining the terror group, and recalling ρ as the source nation's expected wage, a radicalized individual *i* will migrate to join the terrorist group abroad if the expected utility from doing so is at least as large as the source nation's expected wage; namely,

$$p(g)\left[\beta^i + w^* e - \delta(e)\right] \geq \rho \Rightarrow \beta^i \geq \frac{\rho}{p(g)} + \delta(e) - w^* e = \beta^c . \tag{4}$$

In Eq. (4), for all βs above a critical threshold β^c, radicalized individuals will attempt migration. Substituting Eq. (3) in Eq. (4) yields:

$$\beta^c \equiv \frac{\rho}{p(g)} + \delta\left[e\left(w^*\right)\right] - e\left(w^*\right)w^* = \beta^c\left(w^*, g; \rho\right), \tag{5}$$

where $\beta^c_{w^*}\left(w^*, g; \rho\right) = -e\left(w^*\right) < 0$, $\beta^c_g\left(w^*, g; \rho\right) = -\frac{\rho p'(g)}{p^2} > 0$, and $\beta^c_\rho\left(w^*, g; \rho\right) = \frac{1}{p} > 0$.

The cutoff $\beta = \beta^c$ is reduced at a higher foreign fighters wage w^*, because the larger reward increases the incentive to migrate and draws in people who may be relatively less motivated (i.e., possess smaller β^i). This concurs with the literature's emphasis on the importance of selective incentives to entice foreign fighters (e.g., Humphreys and Weinstein 2008; Kalyvas and Kocher 2007; Soules 2023). Additionally, greater proactive action *g* at the border reduces the incentive to migrate so that only the more motivated individuals (larger β^i) will choose to become a foreign fighter. A larger ρ raises the opportunity cost of migration and, like a larger g, lifts the threshold motivation for potential migrating fighters.

Given the $M(\beta)$ distribution of β, the fraction of the source-country radicalized population that exceeds the critical β^c is $1-M(\beta^c)$, so that $\left[1-M(\beta^c)\right]\bar{R}$ would-be foreign fighters attempting to migrate, of which a fraction $p(g)$ successfully joins the terror organization. Thus, the number of foreign fighters equals

$$l^* = p(g)\left\{1-M\left[\beta^c\left(w^*, g; \rho\right)\right]\right\}\bar{R} \equiv l^*\left(w^*, g; \rho\right), \tag{6}$$

where, using Eqs. (5) and (6), we show in the appendix that $l^*_{w^*}\left(w^*, g; \rho\right) > 0$, $l^*_g\left(w^*, g; \rho\right) < 0$, and $l^*_\rho\left(w^*, g; \rho\right) < 0$. A larger foreign fighters' wage w^* raises l^* by reducing the critical β, while greater proactive effort g reduces l^* through a direct effect of greater capture of migrants and an increase in the critical β (deterring migration attempts). Finally, a greater opportunity cost ρ reduces l^* by raising the critical β.

In stage 1, the terrorist group and the government choose the wage for foreign fighters and the border's proactive measures, respectively, while looking ahead to the subsequent decisions of the foreign fighters. For the terrorist group, the objective of the terrorist leaders is to maximize terrorism net of payments to foreign fighters, such that given Eq. (1) we can write the terrorist group's objective function as:

$$V = \phi\left(el^*\right) - w^* el^*. \tag{7}$$

Applying Eqs. (3) and (6) we can write this objective as:

$$V = \phi\left[e\left(w^*\right)l^*\left(w^*, g; \rho\right)\right] - w^* e\left(w^*\right)l^*\left(w^*, g; \rho\right) \equiv V\left(w^*, g; \rho\right). \tag{8}$$

The terrorist organization's optimal choice of w^* is defined implicitly by the FOC:[7]

$$V_{w^*}\left(w^*, g; \rho\right) = \left(\phi' - w^*\right)\frac{\partial\left(el^*\right)}{\partial w^*} - el^* = 0, \tag{9}$$

where the first term on the right-hand side measures the terrorist group's gain from the wage-induced rise in effective terrorists' effort (net of the wage paid), while the second term reflects the higher wage bill at the existing effort level. At the optimum, w^* equates the marginal incentive gain to the infra-marginal wage bill cost.

Alternatively, Eq. (9) can be written as $V_{w^*} = \left[\left(\phi' - w^*\right)e' - e\right]l^* + \left(\phi' - w^*\right)el^*_{w^*} = 0$, where the first term on the right-hand side is the marginal benefit of increased effort at a given l^*, which is the marginal benefit of expansion of the *intensive margin*, while the second term measures the marginal benefit of expansion of the *extensive margin* (i.e., larger l^*). Since the second term is strictly positive, the intensive-margin marginal benefit must be negative at the optimum. That is, the terrorist organization trades off an effort distortion against the extensive-margin gain by pushing w^* and $e\left(w^*\right)$ to levels that are greater than what would obtain if the extensive margin were ignored. In the appendix, we show that Eq. (9) reduces to:

$$w^* = \frac{\varepsilon^S \phi'}{1+\varepsilon^S}, \tag{10}$$

where $\varepsilon^S = \dfrac{\partial \ln\left(el^*\right)}{\partial \ln w^*}$ is the wage elasticity of the effective labor supply el^* from the perspective of the terrorist organization. Further use of Eqs. (3) and (6) yields $\dfrac{\partial\left(el^*\right)}{\partial w^*} = l^*e' + mpe^2\overline{R} > 0$,

---

[7] We can verify that $V_{w^*w^*} < 0$ and $V_{w^*}\left(w^* = 0\right) > 0$ if $\delta''\left(e=1\right)$ is sufficiently small. We adopt this assumption to rule out a corner solution at $w^* = 0$.

such that the effective terrorist labor supply curve is positively sloped with respect to w^*, which ensures that $\varepsilon^S > 0$.

Given Eq. (10), w^* is smaller than the marginal product ϕ', consistent with the terrorist organization exploiting its monopsonistic power over its foreign fighters, faced with an upward sloping effective labor supply curve. For a given marginal productivity of labor ϕ', a larger labor supply elasticity ε^S tends to raise w^*, because the terrorist organization experiences a fuller supply response to a higher wage offer. Similarly, for given elasticity ε^S, a larger marginal product raises the incentive for the terrorist organization to expand employment, which it can only do by offering a higher optimal wage rate. We note that Eq. (9) implicitly defines the terrorists' Nash reaction function *vis-à-vis* the government's choice of proactive effort *g*,

$$w^* = w^*\left(g;\rho\right), \quad \frac{\partial w^*}{\partial g} = -\frac{V_{w^*g}}{V_{w^*w^*}} > 0 \Leftrightarrow V_{w^*g} > 0, \tag{11}$$

because $V_{w^*w^*} < 0$. The appendix establishes that $V_{w^*g} > 0$, so that the terrorist organization's reaction path is positively sloped when graphed in $\left(w^*, g\right)$ space.

Next, we turn to the host government's proactive border policy. The government's objective is to minimize its terrorism loss inclusive of the border surveillance costs, which is assumed to be linear, $c\left(g\right) = g$, such that the government's loss function is:

$$G = \phi\left[e\left(w^*\right)l^*\left(w^*, g;\rho\right)\right] + g \equiv G\left(w^*, g;\rho\right). \tag{12}$$

The FOC for the government's loss minimization is:[8]

---

[8] Sufficient diminishing returns in the probability of detection at the border ensures that the second-order condition $G_{gg} > 0$ is satisfied. We also assume that at $g = 0$, $\left|p'\left(g=0\right)\right| = P'\left(g=0\right)$ is sufficiently large to ensure that $G_g\left(g=0\right) < 0$, such that the government is not at a corner solution.

$$G_g = \phi' e l_g^* + 1 = 0\,, \tag{13}$$

where the first term measures the reduction in terrorism due to proaction-induced decline in terrorist migration, while the second term is simply the marginal proaction cost of unity. Optimum proaction equates the government's marginal gain from reduced terrorist migration to proaction's marginal cost. Eq. (13) implicitly defines the government's stage-1 Nash reaction path in $\left(w^*, g\right)$ space as:

$$g = g\left(w^*; \rho\right),\ \frac{\partial g}{\partial w^*} = -\frac{G_{gw^*}}{G_{gg}} > 0 \Leftrightarrow G_{gw^*} < 0\,. \tag{14}$$

Let the elasticity of the marginal product of terror with respect to effective terror effort (i.e., el^*) be $\varepsilon^{\phi'} = -\frac{d\ln\phi'\left(el^*\right)}{d\ln(el^*)} = -\frac{\phi''\left(el^*\right)el^*}{\phi'\left(el^*\right)} \geq 0$. Thus, $\varepsilon^{\phi'}$ is a measure of the concavity of the production function, or alternately a measure of diminishing returns in terror production. In the appendix, we show that

$$G_{gw^*} = \phi'\left(el^*\right)\left[e' l_g^* + me^2 \bar{R} p' - \frac{\varepsilon^{\phi'}\varepsilon^S e l_g^*}{w^*}\right]. \tag{15}$$

Recalling that $p' < 0$ and $l_g^* < 0$, the first two terms inside the square bracket in Eq. (15) are negative. For a linear production function, $\varepsilon^{\phi'} = 0$, such that Eq. (15) implies that $G_{gw^*} < 0$. In that case, Eqs. (14) and (15) yield a positively sloped Nash reaction path of the government in $\left(w^*, g\right)$ space. To reduce the taxonomy of possible cases, we assume that even with diminishing returns ($\phi'' < 0$), $\varepsilon^{\phi'}$ is sufficiently small such that the first two terms inside the bracket in Eq,

(15) dominate ensuring that $G_{gw^*} < 0$.[9]

2.2 Subgame perfect Nash equilibrium

The intersection of the Nash reaction functions in Eqs. (11) and (14) jointly determines the Nash equilibrium:

$$w^{*N} = w^{*N}(\rho) \text{ and } g^N = g^N(\rho). \tag{16}$$

These Nash equilibrium functions can now be plugged into the various equations outlined above to obtain the equilibrium values of the different endogenous variables.

3. Comparative statics

3.1 Changes in wage or unemployment in source country

A major issue in the literature in this area has been to consider how source-country labor market prospects (wage or unemployment rate) may affect the supply of foreign fighters (see, e.g., Benmelech and Klor 2020; Braithwaite and Chu 2018; Brockmeyer et al. 2023; Edgerton 2023; Hanson 2021; Soules 2023; Verwimp 2016). In our framework, ρ is the expected wage in the source nation representing potential foreign fighters' opportunity cost, so that a rise in ρ reflects improved labor market conditions there from enhanced employment (i.e., a fall in source-country unemployment rate). Therefore, we focus now on the comparative-statics effects of changes in ρ on the Nash equilibrium corresponding to the intersection of the Nash reaction paths, captured

[9] For illustration purposes, we consider a simple diminishing returns production function of the form $\phi(L) = L^\sigma$, where $0 < \sigma < 1$. For this production function, $\varepsilon^{\phi'} = -\dfrac{\phi''(L)L}{\phi'(L)} = 1 - \sigma$. For values of σ sufficiently close to unity, $\varepsilon^{\phi'}$ is sufficiently small to ensure $G_{gw^*} < 0$.

by Eq. (16). For that purpose, we provide a graphical treatment using Figure 1, where the mathematical derivations are provided in the appendix.

[Figure 1 near here]

Previously, we established that the reaction paths of the terrorist group and the government are both upward sloping in $\left(w^*, g\right)$ space as displayed in Figure 1 with w^* on the horizontal axis and *g* on the vertical axis. To ensure stability of the Nash equilibrium at *N* where the paths intersect, the terrorist group's reaction path (R^T) must be steeper than the government's reaction path (R^G). At this equilibrium, g^N and w^* denotes the equilibrium proactive measure and equilibrium selective-incentive wage to the terrorist volunteers, respectively. Proposition 1 summarizes the effect of an increase in ρ on the foreign fighters' equilibrium wage rate and the government's equilibrium proactive response.

Proposition 1. An increase in the source nation's wage or employment rate must increase the selective-incentive wage, w^*, and the host government's proactive response, *g*, at the new equilibrium. Although the intensive margin of terror, *e*, must increase as the source-nation wage increases, the effects of that increase on the extensive margin of terror, l^*, and the level of terror are ambiguous.

Proof

Proofs of all propositions are presented in the appendix at the end.

Figure 1 is instructive to see what drives Proposition 1. Using Eqs. (9) and (11) and the inverse function rule, we have:

$$\left(\frac{\partial w^*}{\partial \rho}\right)_{|g} = -\frac{V_{w^*\rho}}{V_{w^*w^*}} > 0 \Leftrightarrow V_{w^*\rho} > 0. \tag{17}$$

In the online appendix, we show that $V_{w^*\rho} > 0$, such that using Eq. (17) we find that at a given *g*, an increase in ρ bolsters w^*. Earlier in Section 2, we have that

$V_{w^*} = \left[\left(\phi' - w^*\right)e' - e\right]l^* + \left(\phi' - w^*\right)el^*_{w^*}$ When ρ rises, l^* falls because of improved source-country employment opportunities. Following Eq. (9), we previously noted that the intensive margin is distorted to allow for more migration. That distortion, measured by the first term on the right-hand side of the equation above, is diminished, permitting the terrorist organization to drive the extensive margin higher to benefit from greater wage-induced migration. That is, the terrorist group responds to partially offset the effect of better labor market opportunities by offering a higher wage incentive, while keeping the extensive margin in mind. Furthermore, if there are diminishing returns to scale, then a ρ-induced reduction in l^* drives up the productivity of terrorists, amplifying the marginal gains for the terrorist organization, which also bolsters w^*.

In Figure 1, a higher ρ shifts the terrorist group's reaction path to the right to, say, dashed path R^{T1} where foreign volunteers require a larger selective incentive to forego a greater opportunity cost at home. Similarly, from Eq. (13) and the inverse function rule, we have:

$$\left(\frac{\partial g}{\partial \rho}\right)_{|w^*} = -\frac{G_{g\rho}}{G_{gg}} \geq 0 \Leftrightarrow G_{g\rho} \leq 0. \tag{18}$$

In the online appendix, we establish that $G_{g\rho} = e^2\phi'' l^*_g l^*_\rho$, which then implies that $G_{g\rho} \leq 0 \Leftrightarrow \phi'' \leq 0$ given the strict negativity of l^*_g and l^*_ρ. Thus, with diminishing returns, a rise in ρ reduces the marginal loss for the government, which raises *g*. The latter effect occurs because a rise in ρ has the direct effect of reducing l^*, which under diminishing returns raises terrorists' productivity, inducing the government to raise its proactive response. Eq. (18) indicates that an increase in ρ shifts up the government's reaction path R^{G1} in Figure 1.[10] Hence, the Nash equilibrium moves from N to N^1, featuring both larger w^* and larger g compared to smaller source-country employment. In other words, an increase in source-country employment rate or wages augments both the foreign terrorists' wage and the government proactive safeguards at the border. Since $e'\left(w^*\right) > 0,$ the effective effort level or intensive margin of each terrorist rises.

Differentiating Eq. (6), we get:

$$\frac{1}{\overline{R}}\frac{dl^*}{d\rho} = mep\frac{dw^*}{d\rho} + \left(1 - M + \frac{m\rho}{p}\right)p'\frac{dg}{d\rho} - m, \tag{19}$$

where the first term on the right-hand side represents terrorists' wage incentives attracting foreign fighters and is migration enhancing, but the other two right-hand side terms capture the deterrent effect of government proactive border efforts and better source-country opportunities, both of which are migration dampening. If the latter two effects dominate, then the extensive margin l^* will fall with an increase in ρ. Finally, notice from Eq. (1) that terror will rise if and only if the effective effort el^* rises. Differentiating el^* with respect to ρ gives:

$$\frac{d\left(el^*\right)}{d\rho} = \left(l^* e' + pme^2\overline{R}\right)\left(\frac{dw^*}{d\rho}\right) + \left(l_g^*\frac{dg}{d\rho} + l_\rho^*\right)e. \tag{20}$$

Given that $l_g^* < 0$, and $l_\rho^* < 0$, the last term in Eq. (20), which includes the deterrent effect of border surveillance and the effect of increased opportunity cost of terrorists, is negative. In

---

[10]Similar qualitative comparative statics are obtained if $\phi'' = 0$. In that case, Eq. (18) suggests that an increase in ρ does not shift the government's reaction path but the terrorist group's reaction path shifts to the right.

contrast, the first term on the right-hand side of Eq. (20), which captures the effects of w^* on the intensive margin $e\left(w^*\right)$ and on the extensive margin l^*, is positive. If the incentive effects of w^* dominate, then, in spite of better labor-market conditions in the source nation, effective terror effort increases and the host nation experiences more terror.

The above ambiguity is in keeping with the foreign fighter literature that come to different empirical conclusions when looking at the impact of higher opportunity cost at home, larger selective incentives to the foreign fighters, different ideological pulls, and different proactive border policies. Even with our stripped-down theoretical model, different background cases will result in different values to terms highlighted above. As such, the comparative-statics outcome of changes in foreign fighters' opportunity costs can result in diverse outcomes in the extensive and intensive margins leading to different findings by Benmelech and Klor (2020), Brockmeyer et al. (2023), Hanson (2021), and others.

3.2 Increased radicalization of foreign fighters

The level of radicalization of a source nation's population can be influenced over time due to internal and external drivers. For example, the rise of religious fundamentalist beliefs in the 1990s and post-9/11 eras raised the number of individuals holding such radical views. As a consequence, religious fundamentalist terrorist organizations gained prominence relative to leftist terrorist groups (see, e.g., Berman 2009; Hoffman 2017; Hou, Gaibulloev, and Sandler 2020; Iannaccone and Berman 2006). With the growth of fundamentalist terrorism, terrorist attacks posed an enhanced risk of carnage as documented by the empirical terrorism literature (e.g., Enders and Sandler 1999, 2000; Gaibulloev and Sandler 2014, 2019). Similarly, individual nations' populations can display altering degrees of radicalization at different points in time brought about by ideological shifts and new grievances associated with domestic and foreign influences (Enders and Sandler 2006). Consequently, after 9/11, terrorist attacks moved from Latin America and Europe and Central Asia to South Asia, sub-Saharan Africa, and North Africa and the Middle East (Gaibulloev and Sandler 2019; Hou, Gaibulloev, and Sandler 2020). Even government counterterrorism tactics can affect the extent of radicalization within and among countries – i.e., harsher proactive measures can foster the radicalization of populations, resulting in more terrorist recruits (i.e., foreign fighters) and attacks (Rosendorff and Sandler 2004).

The question addressed here is: how do changes in source-country radicalization of potential foreign terrorists affect the host government's counterterrorism policy and the host nation's terrorist organization's wage? To answer this question, we must first explain what is meant by the level of radicalization. Recall that β^i measures the utility derived by a radicalized foreign fighter from serving in the terror organization of the domestic/host nation. The more radicalized the individual, the larger the value of that individual's β. Thus, the extent of the rightward shift of the probability density function (pdf), $m(\beta)$, raises the probability of having greater β levels for the radicalized population. That rightward shift captures a measure of increased radicalization.

Following Bandyopadhyay and Sandler (2023), we formalize this idea by introducing a shift parameter k in the pdf of β, so that $m(\beta,k)$. An increase in k causes a rightward shift of the pdf, such that the cumulative probability function, $M(\beta,k)$, of drawing lower β values falls [i.e., $M_k(\beta,k)<0$]. Accordingly, the fraction $1-M(\beta,k)$ of the population characterized by relatively high β values rises.[11] In particular, for a uniform distribution with support

[11] In other words, we conceptualize increased radicalization by first-order stochastic dominance.

$\left[a(k),b(k)\right]$, the pdf is $m(\beta,k)=\dfrac{1}{b(k)-a(k)}$, and the cumulative distribution function is

$M(\beta,k)=\dfrac{\beta-a(k)}{b(k)-a(k)}$, where $b'(k)>0$ and $a'(k)\geq 0$.[12] Taking the partial derivative with

respect to *k* yields $M_k(\beta,k)=-\dfrac{a'(k)(1-M)+Mb'(k)}{b(k)-a(k)}<0$, as required for increases in *k* to represent a more radicalized source-nation population for foreign fighters.

We apply a graphical approach (not drawn here) analogous to Figure 1 to analyze the effect of an increase in *k*. In addition, we assume that $\phi''=0$ for the remainder of the subsection for expositional simplicity.[13] Similar to Eq. (17), the direction of the shift of the terrorist group's reaction function depends on V_{w^*k}, which represents the influence of *k* on the marginal wage benefit V_{w^*} of the terrorist organization. Given Eq. (8), we obtain:

$$V_{w^*k}=\left[\left(\phi'-w^*\right)e'-e\right]l_k^*+e\left(\phi'-w^*\right)l_{w^*k}^*. \tag{21}$$

Using Eq. (6) for the number of foreign fighters and the aforementioned uniform distribution, we get $l_k^*=-p\bar{R}M_k>0$, such that the direct effect of increased radicalization on the supply of foreign fighters is positive. The greater supply of foreign fighters, in turn, reduces the marginal benefit of the terrorist organization of offering higher wages [i.e., the first term on the right-hand side of Eq. (21) is negative as shown in the online appendix]. The last term on the right-hand side of Eq. (21) measures the effect of *k* that works through the wage-incentive term $l_{w^*}^*$. Given

[12] The signs of the derivatives mean that with increased radicalization the pdf of β shifts to the right through an increase in the upper support of the distribution, while allowing for the lower support to remain unchanged or to increase.

[13] Diminishing returns in terror production (i.e., $\phi''<0$) does not substantively change the results. The analysis is available upon request from the authors.

Eq. (6), we get $l^*_{w^*k} = pe\bar{R}m_k$, where $m_k = \frac{a'(k)-b'(k)}{\left[b(k)-a(k)\right]^2}$ so that the sign of $l^*_{w^*k}$ [and hence the sign of the last term in Eq. (21)] is *a priori* ambiguous.

The direction of the shift of the government's reaction function depends on the cross effect G_{gk}, which may be derived from Eq. (13) as:

$$G_{gk} = \phi' e l^*_{gk} . \tag{22}$$

The right-hand side of Eq. (22) captures the impact of increased radicalization on the marginal deterrent effect of proaction and is positive when $l^*_{gk} > 0$. When the latter inequality holds, Eq. (22) implies that the marginal benefit of proactive measures falls with an increase in radicalization, leading the government's reaction path to shift down.

Since $l^*_g < 0$ as proactive measures inhibit foreign fighters supply, if $l^*_{gk} > 0$, then $\frac{\partial\left|l^*_g\right|}{\partial k} < 0$, so that enhanced radicalization reduces the marginal deterrent effect of proaction $\left|l^*_g\right|$. Employing Eq. (6), we derive $l^*_{gk} = \left(\frac{\rho m_k}{p} - M_k\right)\bar{R}p'(g)$. When $m_k < 0$, the pdf is flatter after the rise in *k*, and hence the proaction-induced increase in β^c (recall $\beta^c_g > 0$) has less deterrent effect. In contrast, the second term inside the bracket (i.e., $-M_k$) is always positive since greater radicalization increases the fraction $(1-M)$ of the radicalized population above β^c, leading more fighters to be caught through proactive measures. Hence, if $m_k < 0$, there are opposing effects on the marginal deterrent term l^*_g arising from an increase in radicalization. If $m_k \geq 0$, then $l^*_{gk} < 0$, so that $G_{gk} < 0$ from Eq. (22), implying that the government's reaction path shifts left and up. Comparative statics of radicalization are captured by Proposition 2.

Proposition 2. For a constant-returns-to-scale terror production function, greater radicalization characterized by: (i) an increase in the upper support of the uniform distribution of β reduces the selective-incentive wage w^*, government proaction *g*, and the intensive margin of terror *e*. The impacts on the extensive margin l^* and the level of terror are ambiguous. (ii) A variance-preserving rightward shift of the upper and lower supports of the distribution reduces w^* and e but raises *g* if the shift effects of the reactions functions dominate the strategic complementarity influences. The effects on l^* and terror are again ambiguous.

We now clarify Proposition 2 by considering cases (i) and (ii).

3.2.1 Case (i): $a(k) \equiv 0$, $b(k) > 0$, $b'(k) > 0$

Here, an increase in radicalization allows some individuals to maintain low β values while others gain greater value from their terror participation, thereby adding a greater range of β values. Accordingly, the uniform distribution is now flatter, where $m_k = \dfrac{-b'(k)}{\left[b(k)\right]^2} < 0$ and

$l^*_{w^*k} = pe\bar{R}m_k < 0$. The latter inequality means that the wage-incentive effect related to the extensive margin (i.e., $l^*_{w^*}$) is curtailed by greater radicalization. This follows because more committed followers need less inducement to adhere to the cause. Reduced incentive inducement and greater migration at a given wage rate [see the first term on the right-hand side of Eq. (21)], both limit the marginal benefit of offering a higher w^*. Consequently, the terrorist group's reaction path shifts to the left in (w^*, g) space.

Turning to the government's reaction function, we show in the online appendix that $l^*_{gk} > 0 \Rightarrow \frac{\partial |l^*_g|}{\partial k} < 0$ in case (i). The reduction in $|l^*_g|$ implies a smaller deterrent effect of proaction, resulting in a downward shift of the government's reaction path in response to increased radicalization in the potential foreign fighters' pool. The aforementioned reaction paths' shifts reduce the Nash equilibrium levels of w^* and *g*. Foreign fighters' effort $e(w^*)$ must fall with w^*, but the effect on the number of foreign fighters, $l^* = p(g)\left[1 - M(\beta^c; k)\right]\bar{R}$, is ambiguous because while lower wages deter foreign fighters, reduced host-government proaction and a higher level of radicalization incentivize more migration. Thus, the overall effect of increased radicalization on effective terror effort el^* is ambiguous. The ambiguity is interesting from an empirical viewpoint because enhanced radicalization does not necessarily result in more foreign fighters owing to opposing influences in case (i).

3.2.2 Case (ii): *Variance-preserving shift,* $a'(k) = b'(k) > 0$

The variance of the uniform distribution for the source-country population for foreign fighter is $\frac{\left[a(k) - b(k)\right]^2}{12}$. When *k* increases, the distribution shifts to the right through equal increases in its upper and lower supports [i.e., $a'(k) = b'(k) > 0$]. That shift leaves the variance and the height of the pdf unaffected (i.e., $m_k = 0$) so that $l^*_{w^*k} = pe\bar{R}m_k = 0$, ensuring that V_{w^*k} defined in Eq. (21) is strictly negative. Consequently, the terrorist group's reaction path shifts to the left for which a given proactive level requires smaller foreign fighters' wages.

For the government's reaction path when $m_k = 0$, we have $l^*_{gk} = -M_k\bar{R}p'(g) < 0$, such that $G_{gk} < 0$ from Eq. (22). The associated decline in the marginal loss of proaction (and hence an upward shift of the government's reaction path) stems from greater numbers of potential foreign fighters being caught due to a radicalization-induced influx of such fighters.

The leftward shift of the terrorist group's reaction path is prone to reduce w^*, but the upward shift of the government's reaction path is inclined to raise w^* due to strategic complementarity. Similarly, the upward shift of the government's reaction path tends to raise *g*, but the leftward shift of the terrorists' reaction path tends to pull *g* down due to strategic complementarity. If the shift effects dominate the strategic complementarity effects, then the fall in w^* and the rise in *g* deter foreign fighters. However, the direct effect of increased radicalization $\left(i.e., l_k^*\right)$ is inclined to raise l^*. Without further qualification, radicalization's effect on the effective labor supply of foreign fighters, $e\left(w^*\right)l^*$, is ambiguous.

To summarize the findings of the two special cases of Proposition 2, we find that increased radicalization reduces the foreign fighters' wages but may raise or reduce the host government's proactive measures at the border. If the direct effect of increased radicalization on foreign fighters' supply dominates the sum of the wage and proaction-related effects, then the number of foreign terrorists increases. In addition, when $e'\left(w^*\right)$ is relatively small, el^* and hence terrorism are apt to rise with greater radicalization. Alternatively, greater radicalization may limit terrorist attacks because of a sufficiently large counterterrorism response and a decline in the foreign fighters' wage rate. When the theoretical model captures various considerations, the radicalization's impact on the extent of terrorism is not unambiguous owing to the government's proactive response. As such, our analysis identifies opposing influences or channels of enhanced radicalization that highlights the rich interactions between terrorist efforts to recruit foreign fighters and government efforts to limit their entry at the border.

4. Four-stage policy leadership game

In many contexts, the government may assume a leadership role in setting its policy while anticipating the effect on the impacted adversary's behavior. Leadership would be especially appropriate if it improves the government's well-being in contrast to acting simultaneously with its adversary. Accordingly, this section proposes a four-stage game in which the government moves in stage 1 and anticipates the behavior of the terrorist organization's action in stage 2. Migration and effort decisions of foreign fighters are made in the subsequent stages 3 and 4, respectively. With this alternative sequence of moves, nothing changes in stages 3 and 4 compared with stages 2 and 3, respectively, of the earlier three-stage game. In addition, the terrorist group's optimal wage choice in this game is still represented by Eq. (11) of Section 2.1. Consequently, after suppressing ρ and k from the functional forms, we amend the government's payoff function, previously described in Eq. (12), to:

$$G^L(g) \equiv \phi\left\{e\left[w^*(g)\right]l^*\left[w^*(g), g\right]\right\} + g \tag{23}$$

where the L superscript denotes leadership. The optimal counterterror level, defined by the FOC of the minimization of G^L, equals:

$$G_g^L = \phi'(\cdot)\left[el_g^* + w_g^*(g)\left(l^*e' + el_{w^*}^*\right)\right] + 1 = 0\,. \tag{24}$$

The crucial difference in the choice of optimal proaction described in Eq. (24) compared to Eq. (13) is the added term $\phi'(\cdot)w_g^*(\cdot)\left(l^*e' + el_{w^*}^*\right) > 0$, capturing an additional marginal loss of proaction that works through induced changes in terrorists' wage w^* as follows. An increase in g elicits an increase in w^* because of the positively sloped terrorist reaction path. In turn, the increase in w^* raises terror on the intensive and extensive margins. The government responds to the additional marginal loss in Eq. (24) by reducing its proactive measure compared to the three-stage equilibrium level. The terrorist organization then reacts by lowering its wage, w^*, relative to that for the three-stage equilibrium.

*Proposition 3. W*hen the government acts as a Stackelberg leader in the four-stage game, the government's proactive level and the terrorist organization's wage incentive are both lower compared to the three-stage Nash equilibrium. The four-stage equilibrium yields higher payoffs for both the government and the terrorist group compared to that of the three-stage game.

The paragraph preceding Proposition 3 explains the first sentence of the proposition. Figure 2 uses reaction paths and iso-payoff curves (details of their respective shapes are in the appendix) to depict the comparison of the three-stage Nash policy equilibrium (*N*) and the four-stage leadership policy equilibrium (*L*). Point *L* is where the government chooses its proactive measure to minimize its loss given that the terrorist group is guided by its reaction path R^T. Since the government's loss must decrease with lower w^* [because $G^L_{w^*} = \phi'(\cdot)\left(l^* e' + e l^*_{w^*}\right) > 0$], the government chooses its lowest iso-loss curve G^L given R^T, indicating that *L* is characterized by the tangency of the government's iso-loss curve with the terrorist group's reaction path [this tangency condition is equivalent to Eq. (24) – see the online appendix]. At *L*, both wage w^{*L} and proaction g^L are smaller than their corresponding levels at *N*.

[Figure 2 near here]

In Figure 2, hill-shaped V^L represents the iso-payoff curve for the terrorist organization, whose reaction path, R^T, is the locus of the highest points on the group's iso-payoff curves.

Because the terror group's payoff falls with an increase in proaction, iso-payoff curves above V^L correspond to smaller terrorist payoffs. Consequently, the terrorist iso-payoff curve at *N* (not displayed) denotes a smaller payoff than at V^L. For government leadership, the smaller w^* limits its loss, G^L, compared to that associated with point *N*. That is, the government has a greater payoff (i.e., smaller loss) at *L* compared to *N*. The Pareto improvement for the government *and* the terrorist group in going from the three-stage Nash equilibrium to the government leadership outcome stems from the reduced "arms race" between the two adversaries. Under government leadership, curtailed proaction works as a unilateral de-escalation tool, which then incentivizes the terrorist group to reduce its recruitment efforts. In such adversarial situations, a government may not realize that de-escalation is possible given the complementarity of proaction and terrorist wages behind the positive-sloping reaction paths.

4.1 Changes in wage or unemployment in the source country

For further tractability, we assume that $\phi'' = 0$ for the rest of the analysis. Proposition 4 next outlines sufficient conditions under which a rise in the source-country wage, ρ, leads to an increase in equilibrium levels of government proaction and foreign fighters' wage rate in the policy leadership game.

Proposition 4. For a constant-returns-to-scale terror production function, an increase in the source nation's wage ρ will raise both the foreign fighter's wage rate and the proactive response at the four-stage leadership equilibrium: (a) if the increase in ρ does not affect (or steepens) the terror organization's reaction path; and (b) if the terrorist's intensive margin is more elastic than its extensive margin with respect to the foreign terrorists' wage rate.

When ρ increases, the terrorist group raises w^* at a given proactive response (i.e., its reaction path in Figure 2 shifts rightward). The increase in w^* augments the foreign fighters' incentive to migrate, which encourages government proactive measures. However, there are two other effects related to two sufficiency conditions noted in Proposition 4. First, if the terrorist group's reaction path becomes steeper with an increase in the source-country wage, ρ, then the leadership equilibrium *L* in Figure 2 is, *ceteris paribus*, inclined to move northeast of *L* along the iso-loss curve G^L, thereby raising *g*. Furthermore, there is the effect of a larger ρ on the wage elasticity of effective foreign fighters' labor el^* to consider. When condition (b) in Proposition 4 holds, a larger ρ reduces the marginal effect $\partial el^*/\partial w^*$, thus limiting government's losses from higher terrorist wages coming from a marginal increase in proaction. This effect allows the government to raise *g* to a greater level compared to the initial leadership equilibrium. Thus, when the sufficiency conditions are met, an increase in ρ must raise proactive measures associated with leadership. The equilibrium level of w^* must also increase because of the rightward shift in the terrorist group's reaction path and because of the strategic complementarity effect of an increase in the equilibrium proaction level.

4.2 Increased radicalization of foreign fighters

Here, we maintain all the functional form assumptions of the three-stage model and consider the effect of an increase in radicalization *k* in the leadership model for the two cases outlined in Section 3.2, where increased radicalization tends to raise migration of foreign fighters l^* (i.e., $l_k^* > 0$). The effects of enhanced radicalization on the four-stage equilibrium levels of g and w^* require detailed considerations and are summarized in Proposition 5.

Proposition 5. For a constant-returns-to-scale terror production function (i.e., $\phi'' = 0$), greater radicalization that is characterized by: (i) an increase in the upper support of the uniform distribution of β reduces the selective-incentive wage w^*, government proaction *g*, and the intensive margin of terror *e*, (a) if the increase in *k* does not affect (or flattens) the terror group's reaction path; (b) if the intensive margin (terrorist effort) is more elastic than the extensive margin (number of foreign terrorists) with respect to the foreign terrorists' wage; and (c) if β^c exceeds a critical level $\hat{\beta}$ defined in the online appendix. The effects on the extensive margin l^* and on the level of terrorism are ambiguous. (ii) A variance-preserving rightward shift of the upper and lower supports of the distribution reduces the selective-incentive wage w^*, government proaction *g*, and the intensive margin of terror *e*, (a) if the increase in *k* does not affect (or flattens) the terror organization's reaction function; (b) if the intensive margin of terror is more elastic than its extensive margin with respect to the foreign terrorists' wage; and (c) if the magnitude of the direct effect of radicalization on foreign fighter supply, $\left|l^*_{gk}\right|$, is sufficiently small.

In both cases, as in Section 3.2, w^* is inclined to fall with an increase in *k*. The direct effect of a fall in w^* is a reduced effective terror effort, el^*, and hence a smaller incentive for proaction. Also, in both cases, as the terrorist organization's reaction path becomes flatter with an increase in *k*, the leadership equilibrium drifts, *ceteris paribus*, southwest from point *L* in Figure 2 along the iso-loss curve in the direction of a lower *g*. If conditions (b) and (c) of case (i) of Proposition 5 are met, then a higher *k* raises the marginal effect $\partial el^*/\partial w^*$, amplifying the marginal loss for the government for proaction-induced wage increases. All those effects combine to reduce proaction in case (i). In addition, a flattening of the pdf is associated with $l^*_{gk} > 0$, curtailing the government's incentive to raise proaction. The direction of comparative-static effects in case (i) of Proposition 5 are similar to case (i) of Proposition 2.

In case (ii) of Proposition 5, conditions (a) and (b) both suggest a decrease in proaction along the lines of case (i) above. However, in case (ii), $l^*_{gk} < 0$, which favors proaction. The aforementioned opposing effects of radicalization on proaction motivate condition (c) of case (ii), which requires $\left|l^*_{gk}\right|$ to be small. In this latter situation, the wage-related effects of radicalization dominate the effect coming from $l^*_{gk} < 0$, and hence proaction falls, so that w^* must also fall. Those findings contrast with the direction of proaction change in case (ii) of Proposition 2. If, however, $\left|l^*_{gk}\right|$ is large and dominant, then proaction will rise, similar to case (ii) of Proposition 2.

5. Concluding remarks

Although there is a rich empirical literature on factors affecting foreign fighters' decision to emigrate abroad to bolster terrorist or rebel groups (e.g., Benmelech and Klor 2020; Brockmeyer et al. 2023; Edgerton 2023; Hegghammer 2013; Humphreys and Weinstein 2008; Kalyvas and Kocher 2007), there is no formal theoretical foundation that can identify key parameters that affect the decision of potential foreign fighters to go abroad and exert effort. In particular, we do not know how the source-country labor market or radicalization affects the emigration decisions or would-be foreign fighters. What is needed is a game-theoretic model that accounts for the terrorist organization's selective incentives (i.e., wages) to entice those foreign fighters and the host government's proactive border measures to deter their immigration. Our purpose here is to provide two alternative game frameworks that investigate the interface between the terrorist group and the adversarial government in the host country either when the adversaries move simultaneously or the government assumes a leadership role. In both scenarios, the final two stages of the game allow the potential foreign fighters to decide their joining and effort decisions.

For both frameworks, the comparative statics involve changes in the source-country wages (employment conditions) or changes in the radicalization parameter of the potential pool of foreign fighters. When labor-market conditions in the source country improves, the terrorist group offers a higher wage to induce foreign recruits, the host government raises its proactive measures, and foreign fighters increase their terrorist efforts. However, the influence of improved labor-market conditions in the source country exerts ambiguous effects on the number of foreign fighters and the overall level of terrorism owing to identified opposing influences. Such alternative outcomes in the extensive and intensive margins for foreign fighters are documented in the empirical literature. For instance, Benmelech and Klor (2020) find for ISIS that reduced employment in the source country exerts a positive influence on enticing sought-after recruits to migrate (also see Morris 2023). In contrast, Brockmeyer et al. (2023) show that reduced employment at home may have an ambiguous effect on the extensive margin owing to opposing influence of opportunity cost and travel cost. Edgerton (2023) indicates that there is no necessary influence of economic conditions at home on foreign fighters' migration decision.

For the baseline game where the terrorist group and the host government simultaneously choose wage inducements and proactive border policy in stage 1, respectively, there is a complementarity between terrorist wages and proactive measures resulting in positively sloped reaction paths. When the government assumes a leadership position, we find that the leadership equilibrium Pareto dominates the simultaneous-move Nash equilibrium with both the terrorist group and the adversarial government improving their welfare as leadership reduces both the terrorist wage and the government's proactive response owing to complementarity. Thus, leadership results in de-escalation of adversarial actions that is mutually beneficial. We end by comparing the comparative statics of the three-stage and the four-stage games that involve changes in the source-country's labor market or the radicalization of potential foreign fighters.

Appendix[14]

1. *Supporting the sentence following Eq. (6)*

Using Eqs. (5) and (6) and noting the radicalization parameter *k* in the functional forms, and also recognizing that *m* is independent of β^c for a uniform distribution, we have:

$$l^*_{w^*} = -p(g)m(k)\bar{R}\beta^c_{w^*} = p(g)e(w^*)m(k)\bar{R} > 0\,; \qquad \text{(A1)}$$

$$l^*_g = \left[1 - M(\beta^c;k) + \frac{m(k)\rho}{p(g)}\right]p'(g)\bar{R} < 0\text{; and} \qquad \text{(A2)}$$

$$l^*_\rho = -p(g)m(k)\bar{R}\beta^c_\rho = -m(k)\bar{R} < 0\,. \qquad \text{(A3)}$$

2. *Deriving Eq. (10)*

Utilizing Eq. (9), we get:

$$w^* = \phi' - \frac{el^*}{\dfrac{\partial(el^*)}{\partial w^*}} = \phi' - \frac{w^*}{\left[\dfrac{\partial(el^*)}{\partial w^*}\right]\dfrac{w^*}{el^*}} = \phi' - \frac{w^*}{\varepsilon^S}\,, \qquad \text{(A4)}$$

from which Eq. (10) follows.

3. *Establishing that* $V_{w^*g} > 0$

[14] Many of the details of derivations here can be found in the online appendix.

Since $\partial\left(el^*\right)/\partial w^* = l^* e' + mpe^2\overline{R}$, differentiation of the expression for V_{w^*} in Eq. (9) yields

$$V_{w^*g} = \left(l^* e' + mpe^2\overline{R}\right)\phi'' el_g^* + \left[\left(\phi' - w^*\right)e' - e\right]l_g^* + \left(\phi' - w^*\right)me^2 p'\overline{R} . \tag{A5}$$

From Eq. (A2), we have $l_g^* < 0$, and Eq. (1) specifies that $\phi'' \leq 0$. Thus, the first term on the right-hand side (RHS) of Eq. (A5) is non-negative. In the online appendix, we show that the sum of the last two terms is positive, such that Eq. (A5) establishes that $V_{w^*g} > 0$.

4. *Deriving Eq. (15)*

Differentiating the expression for G_g in Eq. (13) gives:

$$G_{gw^*} = el_g^*\phi'' \frac{\partial\left(el^*\right)}{\partial w^*} + \phi'\left(l_g^* e' + e\frac{\partial l_g^*}{\partial w^*}\right). \tag{A6}$$

Using Eqs. (A2) and (5), we have $\partial l_g^*/\partial w^* = me\overline{R}p'$, such that Eq. (A6) reduces to

$$G_{gw^*} = \phi'\left\{e' l_g^* + me^2\overline{R}p' + \left(\frac{\phi''}{\phi'}\right)\left[\frac{\partial\left(el^*\right)}{\partial w^*}\right]el_g^*\right\}$$

$$= \phi'\left\{e' l_g^* + me^2\overline{R}p' + \left(\frac{\phi'' el^*}{\phi'}\right)\left[\frac{\partial\left(el^*\right)}{\partial w^*}\left(\frac{w^*}{el^*}\right)\right]\frac{el_g^*}{w^*}\right\} . \tag{A7}$$

Given the definitions of ε^S and $\varepsilon^{\phi'}$ provided in Section 2.1, Eq. (A7) reduces to Eq. (15), where $G_{gw^*} < 0$ if $\varepsilon^{\phi'}$ is zero or sufficiently small.

5. *Proof of Proposition 1*

Differentiating Eqs. (9) and (13), and applying Cramer's rule, we get:

$$\frac{dw^*}{d\rho} = \frac{G_{g\rho}V_{w^*g} - G_{gg}V_{w^*\rho}}{D} \text{ and} \tag{A8}$$

$$\frac{dg}{d\rho} = \frac{G_{gw^*}V_{w^*\rho} - V_{w^*w^*}G_{g\rho}}{D}, \tag{A9}$$

where $D = V_{w^*w^*}G_{gg} - V_{w^*g}G_{gw^*}$. We note that $V_{w^*w^*} < 0$, $G_{gg} > 0$, $V_{w^*g} > 0$, and $G_{gw^*} < 0$. In addition, the online appendix shows that the stability of the Nash equilibrium requires that $D < 0$ and that $V_{w^*\rho} > 0$ and $G_{g\rho} \leq 0 \Leftrightarrow \phi'' \leq 0$. Using the signs of the aforementioned cross partials, Eqs. (A8) and (A9) establish that $\frac{dw^*}{d\rho} > 0$ and $\frac{dg}{d\rho} > 0$. These comparative-static results and the discussion in the text following Proposition 1 complete the proof.

6. *Proof of Proposition 2*

Similar to Eqs. (A8) and (A9), the comparative static effects of *k* on w^* and *g* are

$$\frac{dw^*}{dk} = \frac{G_{gk}V_{w^*g} - G_{gg}V_{w^*k}}{D} \text{ and} \tag{A10}$$

$$\frac{dg}{dk} = \frac{G_{gw^*}V_{w^*k} - V_{w^*w^*}G_{gk}}{D}. \tag{A11}$$

To sign V_{w^*k}, recall Eq. (21). Given $l_k^* > 0$, the first term on the RHS of Eq. (21) is negative because $\left(\phi' - w^*\right)e' - e < 0$, as shown in the online appendix. Furthermore, using Eq. (A1), we have $l_{w^*k}^* = pe\bar{R}m_k \leq 0 \Leftrightarrow m_k \leq 0$. Thus, the sum of the two terms in the expression for V_{w^*k} in Eq. (21) is negative, establishing that $V_{w^*k} < 0$ if $m_k \leq 0$. Turning to the cross partial G_{gk}, we note from Eq. (22) that $G_{gk} = \phi' e l_{gk}^* \leq 0 \Leftrightarrow l_{gk}^* \leq 0$, where Eq. (A2) yields

$$l_{gk}^* = \left(\frac{\rho m_k}{p} - M_k\right)\bar{R}p'(g).$$

<u>Case (i)</u>

We know from section 3.2.1 that $m_k < 0$ in case (i). Furthermore, the online appendix shows that $l^*_{gk} > 0$ in case (i). Therefore, based on the analysis presented in the preceding paragraph, $V_{w^*k} < 0$ and $G_{gk} > 0$. Using the signs of these two cross partials and noting that $D < 0$, we get from Eqs. (A10) and (A11) that $dw^*/dk < 0$ and $dg/dk < 0$.

<u>Case (ii)</u>

Recall from Section 3.2.2 that in case (ii), $m_k = 0$, such that $V_{w^*k} < 0$. However, when $m_k = 0$, we have $l^*_{gk} = -M_k \bar{R} p'(g) < 0$, implying that $G_{gk} < 0$. Applying $V_{w^*k} < 0$ and $G_{gk} < 0$ to Eqs. (A10) and (A11), we get some ambiguities in signing the comparative-static effects. The online appendix shows that if the direct-shift effects of radicalization on the two reactions functions dominate the indirect strategic complementarity effects, then $dw^*/dk < 0$ and $dg/dk > 0$.

The discussion above along with the relevant text following the statement of Proposition 2 complete the proof.

7. *Proof of Proposition 3 and shape of iso-payoff curves in Figure 2*

Using Eq. (8) and suppressing ρ from the functional form, an iso-payoff curve for the terrorist group and its slope are defined, respectively, by

$$V\left(w^*, g\right) = \bar{V}\,; \quad \left(\frac{dg}{dw^*}\right)_{V=\bar{V}} = -\frac{V_{w^*}}{V_g}. \qquad \text{(A12)}$$

We note that $V_{w^*} = 0$ at any point on the terrorist group's reaction path, such that Eq. (A12) establishes that at point *L* in Figure 2 the terrorist iso-payoff curve has zero slope. Furthermore, the online appendix establishes that the iso-payoff curve is concave at *L*.

Using Eq. (12), we get a government iso-loss curve and its slope, respectively, as

$$G\left(w^*, g\right) = \bar{G}\,;\ \left(\frac{dw^*}{dg}\right)_{G=\bar{G}} = -\frac{G_g}{G_{w^*}}. \tag{A13}$$

Again, we note that $G_g = 0$ on the government's reaction path R^G (of the three-stage game) in Figure 2, Eq. (A13) implies that the government's iso-loss curve is perfectly vertical when intersecting R^G. The concavity of the iso-loss curve *vis à vis* the vertical axis of Figure 2 is established in the online appendix.

The terrorists' iso-payoff curve V^L in Figure 2 represents a larger payoff compared to the payoff at point *N* because $V_g = \left(\phi' - w^*\right) el_g^* < 0$, implying that the terrorist group's payoff monotonically declines with movement further north of *L*. Similarly, iso-loss curves to the right of point *L* must show greater losses for the government compared to G^L because $G_{w^*} = \phi'(\cdot)\left(e' l^* + e l_{w^*}^*\right) > 0$. Thus, G^L represents a lower loss for the government than at point *N*, which completes the proof of Proposition 3.

8. *Proof of Proposition 4*

Using $w^* = w^*(g;\rho)$, $l^* = l^*\left(w^*, g;\rho\right)$, and $e = e\left(w^*\right)$, we can express Eq. (24) as

$$G_g^L(g;\rho) \equiv \phi'(\cdot)\left\{e(\cdot) l_g^*(\cdot) + w_g^*(\cdot)\left[l^*(\cdot) e' + e(\cdot) l_{w^*}^*(\cdot)\right]\right\} + 1 = 0. \tag{A14}$$

Because $G_{gg} > 0$, Eq. (A14) yields:

$$\frac{dg}{d\rho} = -\frac{G_{g\rho}^L}{G_{gg}} > 0 \Leftrightarrow G_{g\rho}^L < 0. \tag{A15}$$

Differentiation of the expression for $G_g^L(g;\rho)$ in Eq. (A14) and some routine calculations yield:

$$\frac{G_{g\rho}^L(g;\rho)}{\phi'} = \left(e' l_g^* + e l_{gw^*}^*\right) w_\rho^* + \left(l^* e' + e l_{w^*}^*\right)\frac{\partial w_g^*}{\partial \rho} + \left[\frac{\partial\left(l^* e' + e l_{w^*}^*\right)}{\partial \rho}\right] w_g^*, \tag{A16}$$

where $w_\rho^* = \left(\frac{\partial w^*}{\partial \rho}\right)_{|g} > 0$ based on Eq. (17) and the fact that $V_{w^*\rho} > 0$. The online appendix shows that under the sufficiency conditions (a) and (b) of Proposition 4, all the RHS terms in Eq. (A16) are non-positive, while the first and third terms are strictly negative. Thus, given conditions (a) and (b) of Proposition 4, $G_{g\rho}^L < 0 \Rightarrow \frac{dg}{d\rho} > 0$. Recall that $w^* = w^*(g;\rho)$, such that $\frac{dw^*}{d\rho} = w_g^* \frac{dg}{d\rho} + w_\rho^* > 0$ because $\frac{dg}{d\rho} > 0$, $w_g^* > 0$ and $w_\rho^* > 0$, which completes the proof of Proposition 4.

9. *Proof of Proposition 5*

Following the logic of Eqs. (A14) and (A15), we get the effect of increased radicalization on the leadership proactive response as

$$\frac{dg}{dk} = -\frac{G_{gk}^L}{G_{gg}} < 0 \Leftrightarrow G_{gk}^L > 0. \tag{A17}$$

Analogous to Eq. (A16), we can show that

$$\frac{G_{gk}^L(g;k)}{\phi'} = \left(e'l_g^* + el_{gw^*}^*\right)w_k^* + \left(l^*e' + el_{w^*}^*\right)\frac{\partial w_g^*}{\partial k} + \left[\frac{\partial\left(l^*e' + el_{w^*}^*\right)}{\partial k}\right]w_g^* + el_{gk}^*, \tag{A18}$$

where $w_k^* = \left(\frac{\partial w^*}{\partial k}\right)_{|g} = -\frac{V_{w^*k}}{V_{w^*w^*}} < 0$ in both cases (i) and (ii) as shown in the online appendix. In addition, the online appendix establishes that $G_{gk}^L(g;k) > 0$ in cases (i) and (ii) of Proposition 5 if their respective sufficiency conditions are satisfied. Therefore, if the sufficiency conditions are satisfied, then $\frac{dg}{dk} < 0$ implying that $\frac{dw^*}{dk} = w_g^* \frac{dg}{dk} + w_k^* < 0$, because $w_g^* > 0$, $\frac{dg}{dk} < 0$, and

$w_k^* < 0$. Since $e = e\left(w^*\right)$, $e' > 0$, effort must fall because w^* falls, which establishes Proposition 5.

References

Bandyopadhyay, Subhayu, and Todd Sandler. 2023. Voluntary participation in a terror group and counterterrorism policy. *Journal of Economic Behavior and Organization* 215 (C): 500–513.

Benmelech, Efraim, and Esteban F. Klor. 2020. What explains the flow of foreign fighters to ISIS? *Terrorism and Political Violence* 32 (7), 1458–1481.

Berman, Eli. 2009. *Radical, Religious, and Violent: The New Economics of Terrorism*. Cambridge, MA: The MIT Press.

Braithwaite, Alex, and Tiffany S. Chu. 2018. Civil conflicts abroad, foreign fighters, and terrorism at home. *Journal of Conflict Resolution* 62 (3), 1636–1660.

Brockmeyer, Anne, Quy-Toan Do, Clément Joubert, Kartika Bhatia, and Mohamed Abdel Jelil. 2023. Transnational terrorist recruitment: evidence from Daesh personnel records. *The Review of Economics and Statistics* 105 (5), 1092–1109.

Edgerton, Jared F. 2023. Past and present group exclusion and conflict: group marginalization, opportunity, and Islamic state foreign fighter mobilization. *Journal of Conflict Resolution*, Early view. DOI: 10.1177/00220027231190914.

Enders, Walter, and Todd Sandler. 1999. Transnational Terrorism in the Post-Cold War Era. *International Studies Quarterly* 43 (2), 145–167.

Enders, Walter, and Todd Sandler. 2000. Is transnational terrorism becoming more threatening? A time-series investigation." *Journal of Conflict Resolution* 44 (3), 307–332.

Enders, Walter, and Todd Sandler. 2006. Distribution of transnational terrorism among countries by income classes and geography after 9/11. *International Studies Quarterly* 50 (2), 367–393.

Enders, Walter, and Todd Sandler. 2012. *The Political Economy of Terrorism*, 2nd ed. New

York: Cambridge University Press.

Fox, Jonathan, Meirav Mishali-Ram, Ariel Zellman, Matthias Basedau. 2023. Mobilizing for Jihad: how political exclusion and organized protest contribute to foreign fighter outflows. *Terrorism and Political Violence*, Early view DOI: 10.1080/09546553.2023.2217930.

Gaibulloev, Khusrav and Todd Sandler. 2014. An empirical analysis of alternative ways that terrorist groups end. *Public Choice* 160 (1–2), 25–44.

Gaibulloev, Khusrav, and Todd Sandler. 2019. What we have learned about terrorism since 9/11. *Journal of Economic Literature* 57 (2), 275–328.

Gates, Scott, and Sukanya Podder. 2015. Social media, recruitment, allegiance and the Islamic State. *Perspectives on Terrorism* 9 (4), 107–116.

Hanson, Kolby (2021) "Good times and bad apples: rebel recruitment in crackdown and truce." *American Journal of Political Science* 65 (4), 806–825.

Hegghammer, Thomas. 2013. Should I stay or should I go? Explaining variation in Western Jihadists' choice between domestic and foreign fighting, *American Political Science Review* 107 (1), 1–15.

Hoffman, Bruce. 2017. *Inside Terrorism*, 3rd Edition. New York: Columbia University Press.

Hou, Dongfang, Khusrav Gaibulloev, and Todd Sandler. 2020. Introducing extended data on terrorist groups (EDTG), 1970 to 2016. *Journal of Conflict Resolution* 64 (1), 199–225.

Humphreys, Macartan, and Jeremy M. Weinstein. 2008. Who fights? The determinants of participation in civil war. *American Journal of Political Science* 52 (2), 436–455.

Iannaccone, Laurence R. and Eli Berman. 2006. Religious extremism: The good, the bad, and the deadly. *Public Choice* 128 (1–2), 109–129.

Kalyvas, Stathis N., and Matthew A. Kocher. 2007. How "free" is free riding in civil wars?

Violence, insurgency, and the collective action problem. *World Politics* 59 (2), 177–216.

Malet, David. 2015. Foreign fighter mobilization and persistence in a global context." *Terrorism and Political Violence* 27 (3), 454–473.

Morris, Andrea Michelle. 2023. "Who becomes a foreign fighter? Characteristics of the Islamic State's soldiers." *Terrorism and Political Violence*, Early view DOI: 10.1080/09546553.2022.2144730.

Olson, Mancur. 1965. *The Logic of Collective Action*. Cambridge, MA: Harvard University Press.

Rosendorff, B. Peter, and Todd Sandler. 2004. Too much of a good thing? The proactive response dilemma. *Journal of Conflict Resolution* 48 (5), 657–671.

Sandler, Todd. 1992. *Collective Action: Theory and Application*. Ann Arbor, MI: University of Michigan Press.

Soules, Michael J. 2023. Recruiting rebels: Introducing the rebel appeals and incentives dataset." *Journal of Conflict Resolution* 67 (9), 1811–1837.

Verwimp, Philip. 2016. Foreign fighters in Syria and Iraq and the socio-economic environment they faced at home: A comparison of European countries. *Perspectives on Terrorism* 10 (6): 68–81.

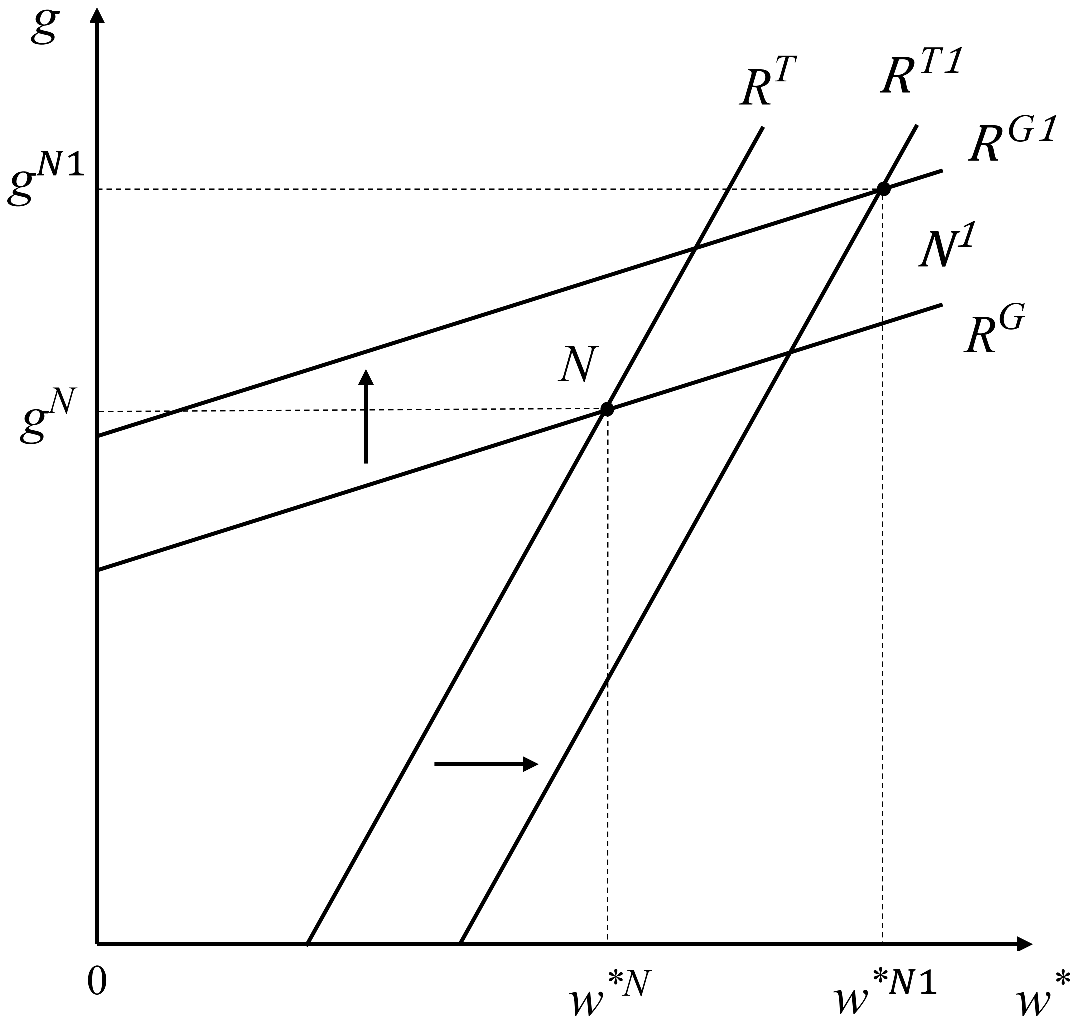

Figure 1. Comparative-static effect of an increase in ρ.

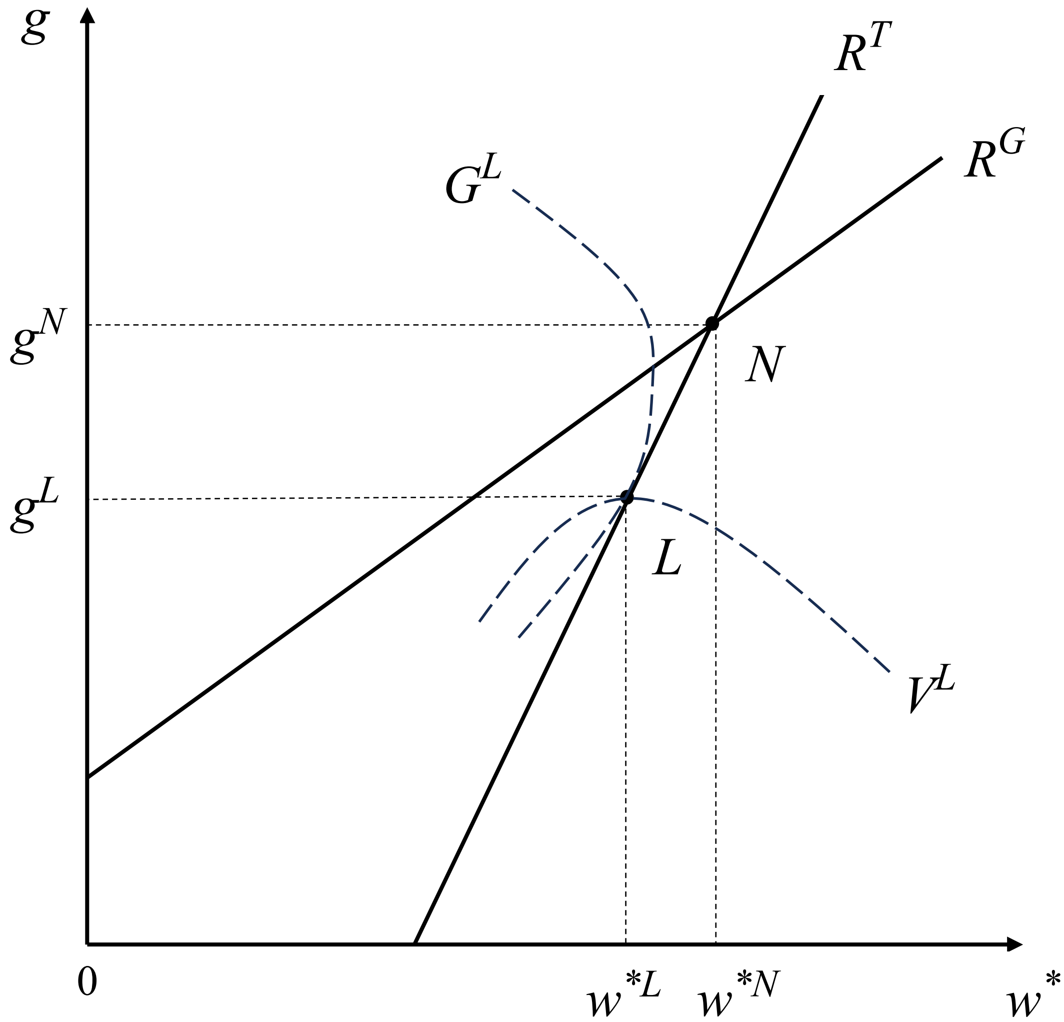

Figure 2. Four-stage and three-stage equilibriums

Online Appendix

(1) *Supporting that* $\left(\phi' - w^*\right)e' - e < 0$ *, and supporting point 3 of the in-paper appendix*

Using Eqs. (9) and (A1), we have:

$$V_{w^*} = 0 \Rightarrow \left(\phi' - w^*\right)e' - e = -\frac{\left(\phi' - w^*\right) pme^2\overline{R}}{l^*} < 0 . \tag{M1}$$

Based on Eq. (M1), the sum of the last two terms of Eq. (A5) is:

$$\left[\left(\phi' - w^*\right)e' - e\right]l_g^* + \left(\phi' - w^*\right)me^2 p'\overline{R} = -\left[\frac{\left(\phi' - w^*\right) pme^2\overline{R}}{l^*}\right]l_g^* + \left(\phi' - w^*\right)me^2 p'\overline{R} . \tag{M2}$$

Given Eqs. (6), (A2), and (M2), we get:

$$\left[\left(\phi' - w^*\right)e' - e\right]l_g^* + \left(\phi' - w^*\right)me^2 p'\overline{R} = -\frac{\left(\phi' - w^*\right) p'm^2e^2\overline{R}\rho}{\left(1 - M\right) p} > 0 , \tag{M3}$$

because $p' < 0$ and $w^* < \phi'$ from Eq. (10), so that Eq. (M3) establishes that the sum of the last two terms of Eq. (A5) is strictly positive.

(2) *Supporting Proposition 1 and point 5 of the in-paper appendix*

To ensure stability, we need the terrorists' reaction path in Figures 1 and 2 to be steeper than the government's reaction path at the three-stage Nash equilibrium, which requires:

$$\frac{1}{\left(\frac{\partial w^*}{\partial g}\right)_{|V_{w^*}=0}} > \left(\frac{\partial g}{\partial w^*}\right)_{|G_g=0} . \tag{M4}$$

Using Eqs. (11) and (14) to substitute for $\left(\frac{\partial w^*}{\partial g}\right)_{|V_{w^*}=0}$ and $\left(\frac{\partial g}{\partial w^*}\right)_{|G_g=0}$, respectively, and simplifying yields $D = V_{w^*w^*}G_{gg} - V_{w^*g}G_{gw^*} < 0$.

Differentiating the expression for V_{w^*} in Eq. (9) and using Eqs. (A1) and (A3), we have:

$$V_{w^*\rho} = \left[\left(\phi' - w^*\right)e' - e + e\phi''\left(l^* e' + pme^2\bar{R}\right)\right] l_\rho^* > 0, \tag{M5}$$

because $\left(\phi' - w^*\right)e' - e < 0$, $\phi'' \leq 0$, and $l_\rho^* < 0$. Similarly, differentiating the expression for G_g in Eq. (13) with respect to ρ gives:

$$G_{g\rho} = \left(e\phi'' l_g^* l_\rho^* + \phi' \frac{\partial l_g^*}{\partial \rho} \right) e. \tag{M6}$$

Differentiating Eq. (A2), and using Eq. (5) to note that $\beta_\rho^c = 1/p$, we have:

$$\frac{\partial l_g^*}{\partial \rho} = \left(-m\beta_\rho^c + \frac{m}{p} \right) \bar{R}p' = 0. \tag{M7}$$

Substituting Eq. (M7) into (M6) yields:

$$G_{g\rho} = e^2 \phi'' l_g^* l_\rho^* \leq 0, \tag{M8}$$

because $\phi'' \leq 0$, $l_g^* < 0$ and $l_\rho^* < 0$. As noted in footnote 10, $\phi'' = 0$, so that $G_{g\rho} = 0$, meaning that the government reaction path does not shift with an increase in ρ.

(3) *Supporting Proposition 2 and point 6 of the in-paper appendix*

Case (i): We show here that $l_{gk}^* > 0$. Based on the functional forms related to the pdf in Sections 3.2 and 3.2.1, we have $m_k = -\frac{b'}{b^2} < 0$, $M = \frac{\beta^c}{b}$, and $M_k = -\frac{Mb'}{b} = -\frac{\beta^c b'}{b^2}$. Recall from the in-paper appendix that $l_{gk}^* = \left(\frac{\rho m_k}{p} - M_k \right) \bar{R}p'(g)$, such that $l_{gk}^* > 0$ if $\frac{\rho m_k}{p} - M_k < 0$. Applying those expressions for m_k, M, and M_k, we get:

$$\frac{\rho m_k}{p} - M_k = \frac{b'\beta^c}{b^2} - \frac{\rho b'}{pb^2} = \frac{b'}{b^2}\left(\beta^c - \frac{\rho}{p} \right) < 0 \text{ if } \beta^c - \frac{\rho}{p} < 0. \tag{M9}$$

Next, we show that $\beta^c - \frac{\rho}{p} < 0$. From Eq. (4), we have $\beta^c - \frac{\rho}{p} = \delta(e) - ew^*$. Given that $e(w^*) > 1$, optimization requires that an individual's utility at $e(w^*) > 1$ exceeds the utility at $e = 1$. Using the latter fact in Eq. (2), we have that, for $w^* > 0$, $U\left[e(w^*) > 1; \beta = \beta^c\right] = \beta^c + w^* e(w^*) - \delta\left[e(w^*)\right] > U(e = 1; \beta = \beta^c) = \beta^c + w^*$, because $\delta(e = 1) = 0$. Note that $\beta^c + w^* e(w^*) - \delta\left[e(w^*)\right] > \beta^c + w^* \Rightarrow w^* e(w^*) - \delta\left[e(w^*)\right] > w^* > 0$. Therefore, $\delta\left[e(w^*)\right] - w^* e(w^*) < 0$. In turn, it must be that $\beta^c - \frac{\rho}{p} = \delta(e) - ew^* < 0$. Thus, applying Eq. (M9), we see that $l_{gk}^* = \left(\frac{\rho m_k}{p} - M_k\right)\bar{R}p'(g) > 0$ in case (i).

Case (ii): Eq. (A10) and $D < 0$ yield $\frac{dw^*}{dk} < 0 \Leftrightarrow G_{gk}V_{w^*g} - G_{gg}V_{w^*k} > 0$. Using $\left(\frac{\partial g}{\partial k}\right)_{|w^*} = -\frac{G_{gk}}{G_{gg}}$ and $G_{gg} > 0$, we get $G_{gk}V_{w^*g} - G_{gg}V_{w^*k} = -G_{gg}\left[V_{w^*k} - \frac{G_{gk}}{G_{gg}}V_{w^*g}\right] = -G_{gg}\left[V_{w^*k} + V_{w^*g}\left(\frac{\partial g}{\partial k}\right)_{|w^*}\right] > 0$, if $V_{w^*k} + V_{w^*g}\left(\frac{\partial g}{\partial k}\right)_{|w^*} < 0$. The latter inequality reduces to $\left|V_{w^*k}\right| > V_{w^*g}\left(\frac{\partial g}{\partial k}\right)_{|w^*}$ because $V_{w^*k} < 0$. Thus, if $\left|V_{w^*k}\right|$ (the direct effect of radicalization on the terrorists' marginal benefit) dominates the strategic complementarity effect, $V_{w^*g}\left(\frac{\partial g}{\partial k}\right)_{|w^*}$, then $\frac{dw^*}{dk} < 0$. Similarly, we can show that $\frac{dg}{dk} > 0$ if the direct effect $\left|G_{gk}\right|$ dominates the strategic complementarity effect, $G_{gw^*}\left(\frac{\partial w^*}{\partial k}\right)_{|g}$.

(4) *Supporting Proposition 3 and point 7 of the in-paper appendix*

Differentiating Eq. (A12) and using $V_{w^*} = 0$, $V_g = \left(\phi' - w^*\right) el_g^* < 0$ and $V_{w^*w^*} < 0$, we get:

$$\left(\frac{d^2 g}{d\left(w^*\right)^2}\right)_{\Big|V=\bar{V}, V_{w^*}=0} = -\frac{V_{w^*w^*}}{V_g} < 0, \tag{M10}$$

establishing concavity of terrorist iso-payoff curve *vis à vis* w^*. Similarly, using Eq. (A13), $G_g = 0$, $G_{gg} > 0$, and $G_{w^*} = \phi'(\cdot)\left(e'l^* + el_{w^*}^*\right) > 0$, we have:

$$\left(\frac{d^2 w^*}{dg^2}\right)_{\big|G=\bar{G}, G_g=0} = -\frac{G_{gg}}{G_{w^*}} < 0, \tag{M11}$$

establishing concavity of the government's iso-loss curves *vis à vis g*.

As asserted in the discussion following Proposition 3, the tangency of G^L with respect to reaction path R^T at point *L* of Figure 2 is equivalent to Eq. (24) because the tangency requires

$$\left(\frac{dw^*}{dg}\right)_{G=\bar{G}} = w_g^*(\cdot), \tag{M12}$$

and Eq. (M12) reduces to Eq. (24) after using Eq. (A13) to substitute $-\frac{G_g}{G_{w^*}}$ for the left-hand side term of Eq. (M12), and noting that $G_g = \phi' el_g^* + 1$ and $G_{w^*} = \phi'(\cdot)\left(e'l^* + el_{w^*}^*\right)$.

(5) *Supporting Proposition 4 and point 8 of the in-paper Appendix*

Differentiating Eq. (A2) and using $\beta_{w^*}^c = -e$, we get $l_{gw^*}^* = me\bar{R}p' < 0$. From Eq. (17), we have $w_\rho^* = -\frac{V_{w^*\rho}}{V_{w^*w^*}} > 0$. Thus, the first term on the right-hand side (RHS) of Eq. (A16) is negative because $l_g^* < 0$. Condition (a) requires the slope $\left(\frac{\partial g}{\partial w^*}\right)_{|V_{w^*}=0}$ of the terrorists' reaction path R^T to increase with ρ, which is the same as requiring the inverse $\left(\frac{\partial w^*}{\partial g}\right)_{|V_{w^*}=0} \equiv w_g^*$, to decrease (or remain unchanged) with an increase in ρ, such that $\frac{\partial w_g^*}{\partial \rho} \leq 0$. Thus, condition (a) ensures that the second term on the RHS of Eq. (A16) is non-positive. Given that $w_g^* > 0$, the third term on the RHS of Eq. (A16) is non-positive if $\frac{\partial\left(l^* e' + e l_{w^*}^*\right)}{\partial \rho} \leq 0$. Notice that

$$\frac{\partial\left(l^* e' + e l_{w^*}^*\right)}{\partial \rho} = e'\left(w^*\right)\left(l_{w^*}^* w_\rho^* + l_\rho^*\right) + e\frac{\partial l_{w^*}^*}{\partial \rho} + l_{w^*}^* e' w_\rho^* . \tag{M13}$$

Noting that $e = e\left(w^*\right)$ and using Eq. (A1), Eq. (M13) reduces to:

$$\frac{\partial\left(l^* e' + e l_{w^*}^*\right)}{\partial \rho} = e'\left(w^*\right)\left(l_\rho^* + 3 l_{w^*}^* w_\rho^*\right). \tag{M14}$$

With $\phi'' = 0$ in Eq. (M5), $V_{w^*\rho} = \left[\left(\phi' - w^*\right)e' - e\right] l_\rho^*$, so that Eq. (17) gives

$w_\rho^* = -\frac{l_\rho^*\left[\left(\phi' - w^*\right)e' - e\right]}{V_{w^*w^*}}$. Substituting the latter expressions for w_ρ^*, the last term in the parentheses in Eq. (M14) yields:

$$l_\rho^* + 3 l_{w^*}^* w_\rho^* = \frac{l_\rho^*}{V_{w^*w^*}}\left\{V_{w^*w^*} + 3 l_{w^*}^*\left[e - \left(\phi' - w^*\right)e'\right]\right\}. \tag{M15}$$

Using Eq. (8) and $\phi'' = 0$, we get $V_{w^*w^*} = 3 l_{w^*}^*\left(\phi' - w^*\right)e' - 2\left(e l_{w^*}^* + l^* e'\right)$. Substituting the latter expression for $V_{w^*w^*}$ in the term inside the curly brackets in Eq. (M15) gives:

$$l^*_\rho + 3l^*_{w^*}w^*_\rho = \frac{l^*_\rho}{V_{w^*w^*}}\left(l^*_{w^*}e - 2l^*e'\right). \tag{M16}$$

Noting that $l^*_\rho < 0$ and $V_{w^*w^*} < 0$, Eq. (M16) implies that $l^*_\rho + 3l^*_{w^*}w^*_\rho \le 0 \Leftrightarrow l^*_{w^*}e - 2l^*e' \le 0$.

Multiplying through the last inequality by $\frac{w^*}{el^*}$, we derive:

$$\frac{l^*_{w^*}w^*}{l^*} - 2\frac{e'w^*}{e} \le 0 \Rightarrow \varepsilon^{l^*} \le 2\varepsilon^e \Rightarrow \varepsilon^e \ge \frac{\varepsilon^{l^*}}{2}, \tag{M17}$$

where $\varepsilon^e = \frac{d\ln e\left(w^*\right)}{d\ln w^*}$ and $\varepsilon^{l^*} = \frac{d\ln l^*}{d\ln w^*}$ are, respectively, the wage elasticity of an individual terrorist's effort and the wage elasticity of the supply of foreign fighters. Thus, in the light of Eqs. (M16) and (M17), $\varepsilon^e \ge \varepsilon^{l^*} > \frac{\varepsilon^{l^*}}{2}$ ensures that $l^*_\rho + 3l^*_{w^*}w^*_\rho < 0$. In turn, noting that $e' > 0$, Eq. (M14) establishes that if $\varepsilon^e \ge \varepsilon^{l^*}$, then $\frac{\partial\left(l^*e' + el^*_{w^*}\right)}{\partial\rho} < 0$. Thus, all three terms on the RHS of Eq. (A16) are non-positive, with the first and last terms strictly negative under conditions (a) and (b) of Proposition 4.

(6) *Supporting Proposition 5 and point 9 of the in-paper appendix*

We have $w^*_k = -\frac{V_{w^*k}}{V_{w^*w^*}} < 0$, because $V_{w^*w^*} < 0$, and using Eq. (21), $V_{w^*k} < 0$ for both cases (i) and (ii). Thus, given that $l^*_g < 0$ and $l^*_{gw^*} < 0$, the first term on the RHS of Eq. (A18) is positive.

Condition (a) for both cases (i) and (ii) of Proposition 5 requires that $\frac{\partial w^*_g}{\partial k} \ge 0$, which means that the slope $\left(\frac{\partial g}{\partial w^*}\right)\Big|_{V_{w^*}=0}$ of R^T in Figure 2 falls (or remains unchanged) when *k* rises. Therefore, when condition (a) is satisfied, the second term on the RHS of Eq. (A18) must be non-negative. Turning to the third term on the RHS of Eq. (A18), we note that

$$\frac{\partial\left(l^{*}e'+el^{*}_{w^{*}}\right)}{\partial k}=e'\left(w^{*}\right)\left(l^{*}_{w^{*}}w^{*}_{k}+l^{*}_{k}\right)+e\frac{\partial l^{*}_{w^{*}}}{\partial k}+l^{*}_{w^{*}}e'w^{*}_{k}\,. \tag{M18}$$

Using Eq. (A1), $\frac{\partial l^{*}_{w^{*}}}{\partial k}=p\bar{R}\left(me'w^{*}_{k}+em_{k}\right)$. With the latter equation and Eq. (A1), Eq. (M18) reduces to:

$$\frac{\partial\left(l^{*}e'+el^{*}_{w^{*}}\right)}{\partial k}=3e'l^{*}_{w^{*}}w^{*}_{k}+e'l^{*}_{k}+pe^{2}\bar{R}m_{k}\,. \tag{M19}$$

Recall that $l^{*}_{k}=-p\bar{R}M_{k}$, such that Eq. (M19) can be written as:

$$\frac{\partial\left(l^{*}e'+el^{*}_{w^{*}}\right)}{\partial k}=3e'l^{*}_{w^{*}}w^{*}_{k}+e'l^{*}_{k}-\frac{e^{2}m_{k}}{M_{k}}l^{*}_{k}\,. \tag{M20}$$

Applying Eq. (A1), we have $\left(\frac{\partial l^{*}_{w^{*}}}{\partial k}\right)_{|w^{*}}=l^{*}_{w^{*}k}=pe\bar{R}m_{k}=-\frac{em_{k}}{M_{k}}l^{*}_{k}$. Given the latter expression for $l^{*}_{w^{*}k}$ and Eq. (21), we get $w^{*}_{k}=\frac{l^{*}_{k}\left[e-\left(\phi'-w^{*}\right)e'+\frac{e^{2}\left(\phi'-w^{*}\right)m_{k}}{M_{k}}\right]}{V_{w^{*}w^{*}}}$, which when substituted in Eq. (M20) yields:

$$\frac{\partial\left(l^{*}e'+el^{*}_{w^{*}}\right)}{\partial k}=l^{*}_{k}\left\{3e'\frac{l^{*}_{w^{*}}}{V_{w^{*}w^{*}}}\left[e-\left(\phi'-w^{*}\right)e'+\frac{e^{2}\left(\phi'-w^{*}\right)m_{k}}{M_{k}}\right]+e'-\frac{e^{2}m_{k}}{M_{k}}\right\}. \tag{M21}$$

Since $l^{*}_{k}>0$, Eq. (M21) establishes that $\frac{\partial\left(l^{*}e'+el^{*}_{w^{*}}\right)}{\partial k}\geq 0$ if the term inside the curly brackets in Eq. (M21) is non-negative. That term can be written as:

$$\{\cdot\} = \frac{e'}{V_{w^*w^*}}\left\{V_{w^*w^*} + 3l^*_{w^*}\left[e - \left(\phi' - w^*\right)e'\right]\right\} + \frac{e^2 m_k}{M_k V_{w^*w^*}}\left[3e'l^*_{w^*}\left(\phi' - w^*\right) - V_{w^*w^*}\right]. \quad \text{(M22)}$$

Using Eqs. (M15) and (M16) and the text between them, we have:

$V_{w^*w^*} + 3l^*_{w^*}\left[e - \left(\phi' - w^*\right)e'\right] = l^*_{w^*}e - 2l^*e'$ and $3e'l^*_{w^*}\left(\phi' - w^*\right) - V_{w^*w^*} = 2\left(el^*_{w^*} + l^*e'\right)$. Making substitutions based on these latter two equations, we reduce Eq. (M22) to:

$$\{\cdot\} = \frac{e'\left(l^*_{w^*}e - 2l^*e'\right)}{V_{w^*w^*}} + \frac{2e^2 m_k\left(l^*_{w^*}e + l^*e'\right)}{M_k V_{w^*w^*}}. \quad \text{(M23)}$$

Eqs. (M21) and (M23) imply that

$$\frac{\partial\left(l^*e' + el^*_{w^*}\right)}{\partial k} \geq 0 \text{ if } \frac{e'\left(l^*_{w^*}e - 2l^*e'\right)}{V_{w^*w^*}} + \frac{2e^2 m_k\left(l^*_{w^*}e + l^*e'\right)}{M_k V_{w^*w^*}} \geq 0. \quad \text{(M24)}$$

Case (i)

Recalling the functional forms related to the pdf from case (i) in point (3) above and evaluating M and M_k at the critical β, we can substitute for $\frac{m_k}{M_k}$ in Eq. (M24) by $\frac{1}{\beta^c}$. Recall that Eqs. (M16) and (M17) establish that if $\varepsilon^e \geq \varepsilon^{l^*}$, then $l^*_{w^*}e - 2l^*e' < 0$. Using the latter fact, $V_{w^*w^*} < 0$, and $\frac{m_k}{M_k} = \frac{1}{\beta^c}$ in Eq. (M24), we have:

$$\frac{\partial\left(l^*e' + el^*_{w^*}\right)}{\partial k} \geq 0 \text{ if } \beta^c \geq \frac{2e^2\left(l^*_{w^*}e + l^*e'\right)}{e'\left(2l^*e' - l^*_{w^*}e\right)} \equiv \hat{\beta}. \quad \text{(M25)}$$

Based on Eq. (M25), we conclude that the third term on the RHS of Eq. (A18) is non-negative if conditions (b) and (c) of case (i) of Proposition 5 are satisfied. Recalling from Section 3.2.1 that $l^*_{gk} > 0$ in case (i), we have that the fourth and last terms on the RHS of Eq. (A18) are positive.

Thus, all terms in Eq. (A18) are non-negative while some are strictly positive when conditions (a) through (c) of case (i) of Proposition 5 are satisfied, thus implying that $G_{gk}^{L}\left(g;k\right)>0$ in case (i).

Case (ii)

Recall from Section 3.2.2 that in case (ii) $m_k = 0$, Eqs. (M21)-(M23) imply that

$$\frac{\partial\left(l^{*}e'+el_{w^{*}}^{*}\right)}{\partial k}=l_{k}^{*}\frac{e'\left(l_{w^{*}}^{*}e-2l^{*}e'\right)}{V_{w^{*}w^{*}}}>0 \text{ if } l_{w^{*}}^{*}e-2l^{*}e'<0, \tag{M26}$$

because $l_k^{*}>0$ and $V_{w^{*}w^{*}}<0$. Earlier, we showed that $l_{w^{*}}^{*}e-2l^{*}e'<0$ when condition (b) of either case of Proposition 5 is satisfied. Thus, the third term on the RHS of Eq. (A18) must be positive. Section 3.2.2 showed that $l_{gk}^{*}<0$ in case (ii). Therefore, the last term on the RHS of Eq. (A18) is strictly negative. If $\left|l_{gk}^{*}\right|$ is sufficiently small, then the positive sum of the other terms of Eq. (A18) dominate. Thus, if conditions (a) through (c) of case (ii) of Proposition 5 hold, then $G_{gk}^{L}\left(g;k\right)>0$.