ECONOMIC RESEARCH
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Working Paper Number	2023-026G
Revision Date	May 2025
Citable Link	https://doi.org/10.20955/wp.2023.026
Suggested Citation	Ravikumar, B., Vandenbroucke, G., 2025; On the Transition to Sustained Growth: The Importance of Recent Agricultural Employment, Federal Reserve Bank of St. Louis Working Paper 2023-026. URL https://doi.org/10.20955/wp.2023.026

On the Transition to Sustained Growth: The Importance of Recent Agricultural Employment*

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May 2025

Abstract

We study a model where a single good can be produced using a diminishing-returns technology (Malthus) and a constant-returns technology (Solow). We map the former to agriculture and show that the share of agricultural employment declines at a constant rate during the economic transition and that recent observations on the share are sufficient to estimate the onset of transition. Our model implies that (i) output growth is higher and increasing after the onset of transition, (ii) during the transition, it is a first-order autoregressive process, and (iii) the rate of decline in the share of agricultural employment is a sufficient statistic for the autoregressive coefficient. Our quantitative results are consistent with these implications for developed economies over more than a century despite the changes in the sectoral composition of output and for today's developing economies in various stages of development and structural transformation.

JEL codes: O10, O13, O40
Keywords: Malthus, Solow, Agricultural employment, Economic transition

*The views expressed in this article are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System. We thank Simon Alder, Chaoran Chen, Gueorgui Kambourov, Todd Schoellman, three anonymous referees, and participants at the 2023 SAET meeting, 2024 Econometric Society Summer Meeting, 2024 SED meeting, St. Louis Fed brown bag seminar, 2024 ISI conference, and 2025 Vienna Global Macro Workshop for their comments.

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1 Introduction

The standard of living was roughly constant prior to the 19th century despite technological change. Malthus (1798) and Ricardo (1817) accounted for this stagnation with a theory of population and diminishing returns in production. The past two centuries, however, have seen unprecedented growth in living standards, which has spawned a vast literature on the transition from stagnation to growth. Our paper belongs to a strand of the literature where a single final good can be produced with two technologies and the economy transitions from one technology to both. Two prominent examples of this strand are Hansen and Prescott (2002), where the transition occurs when physical capital and total factor productivity (TFP) reach a threshold, and Tamura (2002), where endogenous human capital accumulation lowers trade costs and delivers the transition without relying on any exogenous forces.

We develop a simplified version of their models. Our economy has two technologies—Malthus and Solow. The Malthus technology is subject to diminishing returns, as land is a fixed factor, and the Solow technology has constant returns. Labor is the only flexible factor of production. The final output can be produced using one or both technologies. TFP in the two technologies and population grow exogenously at possibly different rates. Our model is a classical theory of the onset of economic transition and the ensuing dynamics and does not rely on preferences.

Our economy uses only the Malthus technology initially, but it transitions to using both the Malthus and Solow technologies after the two TFPs and population reach a threshold. The onset of economic transition is when employment in the Malthus technology starts declining. Labor is employed in both technologies during the transition and increasingly more in the Solow technology.

We map the Malthus technology to agriculture and deliver two results. First, we determine the onset of economic transition using only the share of agricultural employment; we do not need information on real gross domestic product per capita (GDP, hereafter). Specifically, the share declines at a constant rate in the model during the transition, so we can use a few *recent* observations to estimate the rate of decline and project backward to determine the onset of transition. This result provides a testable implication for countries with historical data on GDP: Does GDP growth change at the agricultural-employment-based onset of transition?

Second, during the transition, the share of agricultural employment is sufficient to determine GDP dynamics in the model: GDP growth is a first-order autoregressive process with a coefficient that is pinned down by the rate of decline in the share of agricultural employment. We do not need to know the structural parameters, TFPs, or population. This result offers another testable implication: Is GDP growth since the onset consistent with the constant rate of decline in the share of agricultural employment estimated from a few recent observations?

Quantitatively, we estimate the rate of decline in the share of agricultural employment for the United States using post-World War II data from Herrendorf, Rogerson, and Valentinyi (2014) and infer the onset of transition. The U.S. economic transition started in 1875.

Annual GDP data for the U.S. is available starting in 1800 (Delventhal, Fernández-Villaverde, and Guner, 2021). When we test our first result, we find that U.S. GDP growth is higher and increasing after 1875.

To test our second result, we estimate the autoregressive coefficient for U.S. GDP growth from 1875 to 2016. We find that the coefficient is almost identical to the one implied by the constant rate of decline in the post-World War II share of agricultural employment. It is surprising that recent agricultural employment would account for the GDP dynamics over 140 years, especially given the structural changes since 1875.

We repeat the calculations for the United Kingdom. Our estimate of the onset of transition is 1812, which is consistent with lower GDP growth before 1812 and higher and increasing growth after. The autoregressive coefficient for GDP growth is almost the same as the one implied by post-World War II agricultural employment. We report similar results for several other middle- and high-income countries.

Next, we examine the economic transition for today's developing economies. Our backward projection based on recent agricultural employment is especially useful for estimating the onset of transition for these economies, which typically do not have a long time series of GDP. For countries with post-World War II data, we consider a sample of countries whose GDP is less than 25% of U.S. GDP in 2016. Using the share of agricultural employment, available only after 1991 for these countries, we estimate the onset of transition. For instance, the onset of transition for India is 1965. The autoregressive coefficient for GDP growth for most of these countries is consistent with the country's rate of decline in the share of agricultural employment.

This paper differs from the transition literature in a few ways. First, in quantitative implementations, the onset of economic transition is chosen using narratives or historical evidence on GDP, and the model parameters are calibrated to deliver the chosen onset of transition. For instance, Hansen and Prescott (2002) use GDP data for England, and Tamura (2002) uses it for Western Europe + Canada + the U.S. In contrast, we estimate the onset of transition without using GDP data. We then use GDP data for cross-validation. Second, our method does not require knowing the structural parameters or growth rates of TFP or population. In our model, cross-country differences in structural parameters and the processes for TFPs and population yield cross-country differences in the share of agricultural employment, which is a sufficient statistic for the onset of transition and GDP growth. Finally, because we do not rely on historical evidence for either GDP or share of agricultural employment, our model is useful for studying the transition of today's developing economies, a set of economies on which the transition literature is usually silent.

Two remarks are in order here. First, in models such as Hansen and Prescott (2002) and Tamura (2002), the onset of economic transition is *by definition* when the Malthusian share of employment starts declining, no matter the details of the model. Without using any specific model, one could make an *ad-hoc assumption* that the rate of decline is constant and estimate the onset of transition. Our approach has two advantages: (i) The ad-hoc assumption, without more structure, would have no further implications for GDP dynamics, and (ii) we provide a framework where the constant rate of decline is a result, not an assumption.

Second, we have left out several forces from our model that have been included in the transition literature. For instance, in the human capital models of Becker, Murphy, and Tamura (1990) and Galor and Weil (2000), the economy transitions from GDP stagnation and high fertility to sustained GDP growth and lower fertility. In Goodfriend and McDermott (1995), exogenous population growth allows for increasing returns to specialization, and the economy transitions from household production and stagnation to market production and growth. In Jones (2001), the evolution of population and ideas delivers technological progress that helps the economy transition. Our analysis, however, raises the question: What do the richer frameworks offer for GDP dynamics? Perhaps they offer deeper theories of GDP by endogenizing the objects we assume are exogenous (population and TFP), by adding state variables such as physical or human capital or by incorporating neoclassical elements such as preferences for food or services. While the deeper theories undoubtedly offer more implications, our simple framework shows that recent agricultural employment is a sufficient statistic for determining GDP dynamics. The burden then for the transition literature is whether the richer frameworks yield similar quantitative implications for the importance of recent employment in agriculture.

In sum, our paper can be viewed in two ways. One is theory ahead of measurement: The model is a measurement device that justifies the empirical specifications in Section 3. The estimates from the specifications help us test the model's implications. This is the approach taken in the rest of the paper. The other equally valid view is a documentation of an empirical regularity and a challenge to the transition literature. The regularity described in Section 3—recent share of agricultural employment is a sufficient statistic for GDP growth—poses a challenge: The transition literature has many elements, none of which seems to be necessary for describing GDP dynamics. While other models of transition might not imply that the share of agricultural employment is a sufficient statistic, as our model does, they have to confront the evidence we present: Recent agricultural employment pins down the GDP dynamics for developed economies over more than a century and for today's developing economies in various stages of development and structural transformation. In the second view, criticisms of our assumptions and details, or lack thereof, do not address our evidence.

2 Model

Time is continuous and represented by $t \geq 0$. There are two technologies, denoted M (for Malthus) and S (for Solow), producing a *single* consumption good. Technology M uses land (fixed and normalized to 1) and labor and exhibits diminishing returns to labor. Technology S uses only labor and exhibits constant returns. Outputs at date t from the two technologies are denoted by Y_t^M and Y_t^S, respectively:

$$Y_t^M = \left(Z_t^M H_t^M\right)^{1-\alpha}, \ \ \alpha \in (0,1), \quad \text{and} \quad Y_t^S = Z_t^S H_t^S,$$

where Z_t^M and Z_t^S are exogenous labor-augmenting productivities and H_t^M and H_t^S are employment in M and S, respectively. Total gross domestic product is

$$Y_t = Y_t^M + Y_t^S.$$

The working population is exogenous and denoted by P_t; we treat population and employment as synonymous. (Since population and TFPs are exogenous, our model is silent on demographic transition or technological change.)

In what follows, we adopt the following two notations. First, we use lowercase letters to denote a variable per unit of population: $x_t \equiv X_t/P_t$. Second, we use a dot notation to denote the *growth rate* of a variable:

$$\text{Growth rate of } X_t \equiv \dot{X}_t \equiv d\ln(X_t)/dt.$$

We assume that Z_t^S, Z_t^M, and P_t grow at constant but potentially different rates $\dot{Z}^S$, $\dot{Z}^M$, and $\dot{P}$, respectively.

$$Z_t^S = Z_0^S \exp\left(t\dot{Z}^S\right),\ Z_t^M = Z_0^M \exp\left(t\dot{Z}^M\right),\ \text{and}\ \ P_t = P_0 \exp\left(t\dot{P}\right),$$

where Z_0^S, Z_0^M, and P_0 are initial conditions.

The working population is allocated between the two technologies,

$$H_t^M + H_t^S = P_t.$$

Labor is perfectly mobile across the two technologies. Thus, the optimal allocation of labor requires that the economy's output of the single good be maximized:

$$Z_t^S \leq (1-\alpha)\left(Z_t^M\right)^{1-\alpha}\left(H_t^M\right)^{-\alpha}, \tag{1}$$

with equality whenever $H_t^M < P_t$. The left-hand side is the marginal product of labor in technology S, and the right-hand side is the marginal product of labor in technology M. Figure 1 represents the optimal allocation of labor. If Z_t^S is low enough (e.g., $Z_{t,\text{Low}}^S$), the marginal product of labor in technology M exceeds that in technology S even if the entire working population is allocated to technology M. In this case, Equation (1) holds with a strict inequality. When Z_t^S is sufficiently high

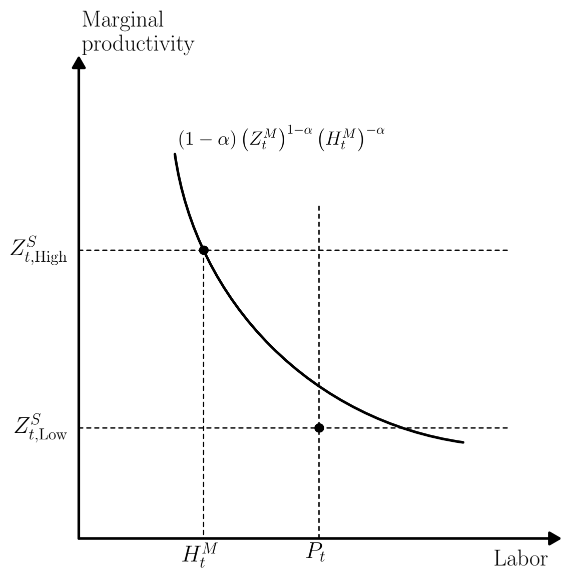


Figure 1: The optimal allocation of labor

(e.g., $Z^S_{t,\text{High}}$), the two marginal products are equalized and $H_t^M < P_t$.

If initial conditions are such that

$$Z_0^S < (1-\alpha)\left(Z_0^M\right)^{1-\alpha}\left(P_0\right)^{-\alpha}, \tag{2}$$

then all labor is allocated to technology M at date 0. If inequality (2) evaluated at t continues to hold, then all labor continues to be allocated to technology M at t. Some labor will be allocated to technology S at some date t^* if Equation (1) is satisfied with equality at t^*. For this to occur, it must be the case that

$$\dot{Z}^S > (1-\alpha)\dot{Z}^M - \alpha\dot{P}. \tag{3}$$

We assume conditions (2) and (3) are satisfied for the rest of the paper.

The date at which the economy starts operating technology S is determined by

$$Z_0^S e^{t^*\dot{Z}^S} = (1-\alpha)\left(Z_0^M e^{t^*\dot{Z}^M}\right)^{1-\alpha}\left(P_0 e^{t^*\dot{P}}\right)^{-\alpha}.$$

That is, the onset of transition from M to S is given by

$$t^* = \frac{\ln\left[(1-\alpha)\left(Z_0^S\right)^{-1}\left(Z_0^M\right)^{1-\alpha}(P_0)^{-\alpha}\right]}{\dot{Z}^S - (1-\alpha)\dot{Z}^M + \alpha\dot{P}}. \tag{4}$$

2.1 Analysis

It is convenient to analyze the economy in two parts: before t^* and during the transition after t^*.

Before the transition When $t \leq t^*$, the analysis is straightforward. The entire working population is employed in technology M, and output per capita is that of technology M. Recall our notation: GDP is real gross domestic product per capita.

$$\begin{aligned} \text{Share of employment in } M &: \; h_t^M = 1, && (5)\\ \text{GDP} &: \; y_t = y_t^M = \left(Z_t^M\right)^{1-\alpha}(P_t)^{-\alpha}, && (6)\\ \text{GDP growth} &: \; \dot{y} = \dot{y}^M = (1-\alpha)\dot{Z}^M - \alpha\dot{P}. && (7) \end{aligned}$$

Note that for the economy to transition from Malthus to Solow, condition (3) has to be satisfied: $\dot{Z}^S$ must exceed the Malthusian GDP growth rate $\dot{y}^M$.

During the transition When both technologies operate, Equation (1) holds with equality and employment in technology M is

$$H_t^M = (1-\alpha)^{1/\alpha}\frac{1}{Z_t^S}\left(\frac{Z_t^M}{Z_t^S}\right)^{(1-\alpha)/\alpha}.$$

It follows that the *employment share* of technology M and its growth rate are

$$\begin{aligned} h_t^M &= (1-\alpha)^{1/\alpha}\frac{1}{Z_t^S P_t}\left(\frac{Z_t^M}{Z_t^S}\right)^{(1-\alpha)/\alpha}, && (8)\\ \dot{h}^M &= \frac{1-\alpha}{\alpha}\dot{Z}^M - \frac{1}{\alpha}\dot{Z}^S - \dot{P}. && (9) \end{aligned}$$

A few comments are worth making here. First, the share of employment in the

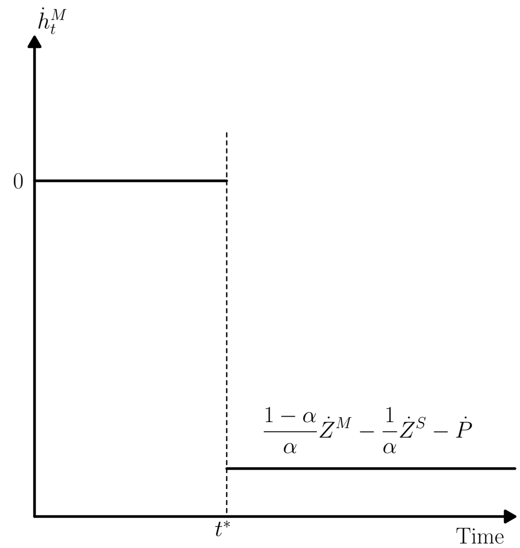


Figure 2: Growth rate of employment share in the Malthus technology

Note: This is an illustrative path of the growth rate of employment share in the Malthus technology under the assumption that the exogenous variables grow at constant, but potentially different, rates.

Malthus technology is the only choice variable in our model. Second, the evolution of the share is the solution to a sequence of static problems. The dynamics in the model are entirely due to the evolution of Z^S, Z^M, and P. Finally, condition (3) implies $\dot{h}^M < 0$ after t^*. Equation (9) delivers the result that the employment share of technology M decreases at a constant rate after t^* (see Figure 2).

The economy's GDP is

$$y_t = Z_t^S + \frac{\alpha}{1-\alpha} Z_t^S h_t^M. \tag{10}$$

The growth rate of y_t is

$$\dot{y}_t = \frac{(1-\alpha)(1-h_t^M)}{1-\alpha+\alpha h_t^M}\dot{Z}^S + \frac{(1-\alpha)h_t^M}{1-\alpha+\alpha h_t^M}\dot{Z}^M - \frac{\alpha h_t^M}{1-\alpha+\alpha h_t^M}\dot{P}. \tag{11}$$

See Appendix A for the derivation of Equations (10) and (11).

Equations (10) and (11) imply that the long-run path of GDP is Z_t^S with growth rate $\dot{Z}^S$. As noted earlier, condition (3) implies that long-run growth rate of GDP is higher than the pre-transition growth rate $\dot{y}^M$.

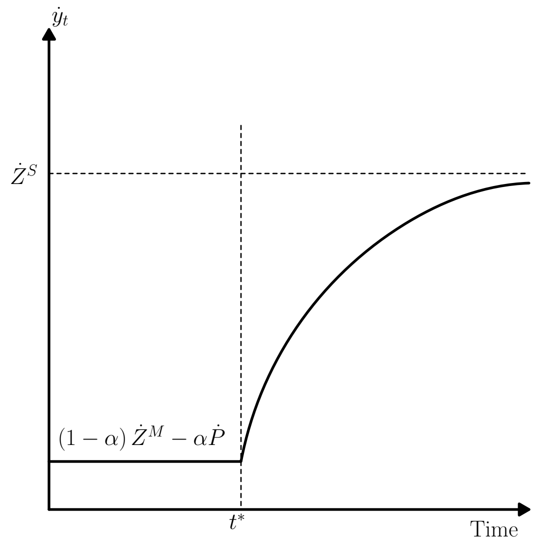


Figure 3: Growth rate of GDP

Note: This is an illustrative path of the growth rate of GDP under the assumption that the exogenous variables grow at constant, but potentially different, rates.

The dynamics of GDP during the transition are described by (11), which implies

$$\frac{d\dot{y}_t}{dt} = \frac{(1-\alpha)\alpha}{\left(1-\alpha+\alpha h_t^M\right)^2}(\dot{h}^M)^2 h_t^M > 0. \tag{12}$$

That is, GDP growth is monotonically increasing during the transition. Figure 3 illustrates an example of a growth path before and after t^*.

Importantly, the dynamics of growth described by (11) and (12) are pinned down entirely by h_t^M—the only endogenous time-varying component on the right-hand sides of (11) and (12). The evolution of h_t^M in (11) describes how rapidly GDP converges to its long-run growth path. To see this, note that

$$\frac{d\dot{y}_t}{dh_t^M} = \frac{(1-\alpha)\,\alpha\dot{h}^M}{[1-\alpha+\alpha h_t^M]^2} < 0.$$

That is, there is a negative association between two endogenous variables $\dot{y}$ and h^M: The economy grows faster when the Malthusian sector is smaller.

The speed at which workers exit the Malthusian sector is given by Equation (9). For instance, higher population growth, ceteris paribus, implies a more rapid exit out of the Malthusian sector. It follows that $d\dot{y}_t/d\dot{P} > 0$. That is, an economy where population grows more rapidly converges faster to its long-run growth path. This does not imply that an economy with a higher $\dot{P}$ exhibits a higher rate of growth at all points in time. First, the economy with the higher $\dot{P}$ will exhibit lower growth in the pre-transition period (Equation 7). Second, the economy with higher $\dot{P}$ will transition earlier since $dt^*/d\dot{P} < 0$ (Equation 4). Similarly, $d\dot{y}_t/d\dot{Z}^S > 0$ and $d\dot{y}_t/d\dot{Z}^M < 0$.

3 Quantitative analysis

The onset of sustained growth in our model is when the Solow technology starts operating at date t^*. First, while Equation (4) pins down t^*, the expression depends on unobserved variables. In this section, we show that the employment share in the Malthus technology is sufficient to determine t^*. Quantitatively, we map the employment share in the Malthus technology in the model to the share of agricultural employment in the data. Second, the level and growth rate of GDP in Equations (10) and (11) also depend on unobserved TFPs and their growth rates. We show that the rate of decline of the share of agricultural employment is a sufficient statistic for the dynamics of GDP growth during the transition after t^*.

3.1 Determining the onset of sustained growth

Under the assumption in Section 2 that Z^M, Z^S, and P grow at constant (but potentially different) rates, Equation (9) implies that $\dot{h}^M$ is constant. So, $h_t^M = \exp\left((t-t^*)\dot{h}^M\right)$ at any date $t \geq t^*$, or

$$t^* = t - \frac{\ln\left(h_t^M\right)}{\dot{h}^M}, \qquad \text{for } t \geq t^*. \tag{13}$$

Equation (13) has several implications. First, the reasons that different countries have different shares or growth rates of agricultural employment do not matter for estimating the onset of sustained growth. Countries could differ in their structural parameters, and in the levels and growth rates of TFPs and population. In our model, these differences manifest themselves in different h_t^M and $\dot{h}_t^M$ (see Equations 8 and 9). Second, both the level and growth rate of the share of agricultural employment are needed to estimate the onset of transition. Countries with the same share of agricultural employment at a point in time could have started their transitions at different times. Third, the onset of economic transition can be determined without using GDP data, which means we can test whether the onset of transition implied by the share of agricultural employment coincides with a change in GDP growth. Finally, since the model implies that the growth rate of the share of agricultural employment is constant, a few recent (presumably more reliable) observations are sufficient to determine the onset of transition. We do not need historical evidence for the share of agricultural employment.

To operationalize Equation (13), consider the specification below for country i:

$$\ln(h_{t,i}^M) = \beta_{0,i} + \beta_{1,i}t, \tag{14}$$

which implies $\beta_{1,i} = \dot{h}_i^M$. At the onset of transition, $h^M = 1$, so

$$\begin{aligned} t_i^* &= -\frac{\beta_{0,i}}{\beta_{1,i}}, \\ &= -\frac{\ln\left(h_{t,i}^M\right) - \beta_{1,i}t}{\beta_{1,i}} = t - \frac{\ln\left(h_{t,i}^M\right)}{\dot{h}_i^M}, \end{aligned} \tag{15}$$

which is the same as Equation (13).

3.2 Dynamics of GDP during the transition

From Equation (10) it is easy to see that the long-run path of GDP is that of Z_t^S. The relative deviation of GDP from its long-run path, which we denote by $\hat{y}_t$, is then

$$\hat{y}_t \equiv \frac{y_t - Z_t^S}{Z_t^S} = \frac{\alpha}{1-\alpha}h_t^M, \quad \text{for} \quad t \geq t^*. \tag{16}$$

First, Equation (16) implies that $\hat{y}_t$ grows at rate $\dot{h}^M$. As the share of agricultural employment declines, $\hat{y}_t$ approaches zero and the paths of GDP and Z^S converge.

Second, using (16) at two instants t and $t+\omega$, we get

$$\ln(1+\hat{y}_{t+\omega}) - \ln(1+\hat{y}_t) + \ln(Z^S_{t+\omega}) - \ln(Z^S_t) = \ln y_{t+\omega} - \ln y_t.$$

We then approximate $\ln(1+\hat{y}_{t+\omega}) - \ln(1+\hat{y}_t) \simeq \hat{y}_{t+\omega} - \hat{y}_t$.[1] This yields

$$\hat{y}_{t+\omega} - \hat{y}_t + \ln(Z^S_{t+\omega}) - \ln(Z^S_t) \simeq \ln y_{t+\omega} - \ln y_t,$$

which implies

$$\exp(\omega\dot{h}^M)\hat{y}_t - \hat{y}_t + \omega\dot{Z}^S \simeq \ln y_{t+\omega} - \ln y_t, \text{ since } \dot{\hat{y}}_t = \dot{h}^M.$$

Similarly, at $t+\omega$,

$$\begin{aligned} \exp(\omega\dot{h}^M)\hat{y}_{t+\omega} - \hat{y}_{t+\omega} + \omega\dot{Z}^S &\simeq \ln y_{t+2\omega} - \ln y_{t+\omega} \\ \Rightarrow \exp(\omega\dot{h}^M)\left(\exp(\omega\dot{h}^M)\hat{y}_t - \hat{y}_t\right) + \omega\dot{Z}^S &\simeq \ln y_{t+2\omega} - \ln y_{t+\omega}. \end{aligned}$$

Substituting, rearranging, and evaluating at $\omega = 1$, we get

$$\ln y_{t+2} - \ln y_{t+1} \simeq \exp(\dot{h}^M)\left(\ln y_{t+1} - \ln y_t\right) + \dot{Z}^S\left(1 - \exp(\dot{h}^M)\right). \quad (17)$$

The rate of decline in the share of agricultural employment is thus a sufficient statistic to describe the dynamics of GDP growth after t^*.

Note that (17) is a result, not just an accounting formula. While, as noted earlier, one could estimate t^* by *assuming* $\dot{h}^M$ *is constant*, the ad-hoc assumption would not imply (17). In deriving (17) we have used the model's optimal allocation of labor in the two technologies.

To estimate the autoregressive coefficient of the growth rate of GDP after t^*, we specify the data-generating process for GDP for country i as

$$\ln y_{t,i} = \gamma_{0,i} + \gamma_{1,i}t + \gamma_{2,i}\exp(\gamma_{3,i}t), \qquad \text{for } t \geq t^*_i. \quad (18)$$

[1] We are approximating $\ln(1+\hat{y}_{t+\omega}) - \ln(1+\hat{y}_t)$ with $\hat{y}_{t+\omega} - \hat{y}_t$, not $\ln(1+\hat{y}_t)$ with $\hat{y}_t$.

Suppressing the country notation, this process has the property that

$$\dot{y}_t \equiv \frac{d \ln y_t}{dt} = \gamma_1 + \gamma_2 \gamma_3 \exp(\gamma_3 t).$$

It is easy to see that

$$\dot{y}_{t+1} - \gamma_1 = (\dot{y}_t - \gamma_1) \exp(\gamma_3),$$

which implies

$$\dot{y}_{t+1} = \exp(\gamma_3) \dot{y}_t + \text{constant}.$$

Hence, the autoregressive coefficient on GDP growth is $\exp(\gamma_3)$.

Equations (17) and (18) thus represent a testable implication: The autoregressive coefficient for GDP growth after t^* is the exponential of the growth rate of the share of agricultural employment. An estimate of the latter is $\exp(\beta_1)$ from (14).

In sum, the share of agricultural employment is sufficient to pin down both the onset of transition and GDP dynamics during the transition.

In the quantitative exercises below, our model's h^M is measured by the share of agricultural employment in the data. So, one interpretation is that agricultural goods are produced using the Malthus technology and that non-agricultural goods are produced using the Solow technology. With such an interpretation, a relative price is involved in the GDP calculation. However, we could easily include the relative price of a non-agricultural good and/or inputs other than labor into one of the two exogenous TFPs in our model. For instance, the evolution of Z^S could capture the dynamics of the relative price. To see this, recall that the optimal evolution of h^M is the solution to a sequence of static problems. Pick an arbitrary period and let q denote the price (or the marginal rate of transformation) of the non-agricultural good relative to that of the agricultural good in that period. Then, the optimal h^M maximizes

$$\left(Z^M H^M\right)^{1-\alpha} + qZ^S(P - H^M).$$

It is easy to see that our analysis goes through just by relabeling qZ^S as the TFP in the Solow technology.

In the next two subsections, we use recent data on the share of agricultural employment and estimate the onset of transition for the United States and the United

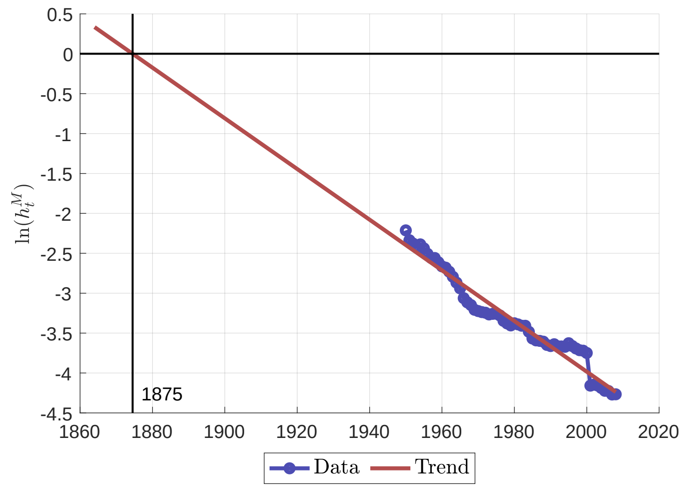


Figure 4: Onset of sustained growth in the United States

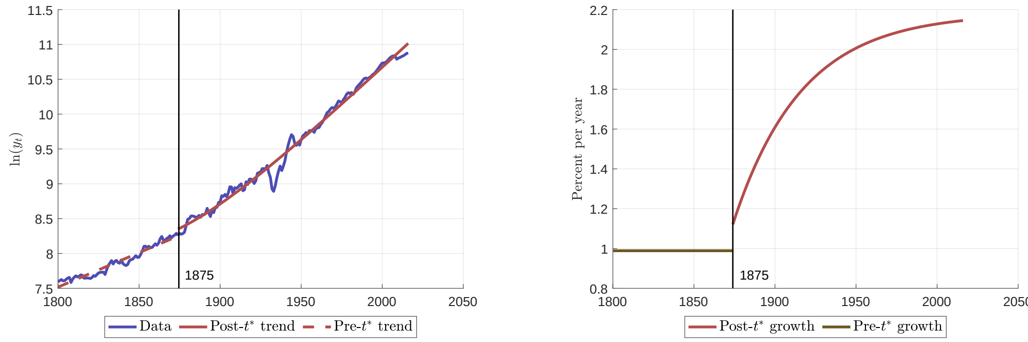
Note: The figure plots the log of the share of agricultural employment from 1949 to 2016.
Source: Herrendorf et al. (2014) and authors' calculations.

Kingdom. Both countries have long time series of annual observations on GDP. We first validate our estimate using GDP data: GDP growth is higher and increasing after the onset of transition relative to before the transition. Second, we estimate the autoregressive coefficient of the growth rate of GDP during the transition and show that the coefficient matches the rate of decline in the recent share of agricultural employment. Finally, we estimate the onset of transition for several countries in Western Europe and show that the evolution of GDP for each country is consistent with its share of agricultural employment.

3.3 The U.S. and the U.K

We estimate β_0 and β_1 in (14) with post-World War II annual data on the share of agricultural employment for the U.S. We find t^* to be 1875 from Equation (15). This is illustrated in Figure 4.

Next, we estimate the coefficients in (18) using post-t^* GDP data for the U.S. We find $\exp(\gamma_3) = 0.977$. The fit is illustrated in Panel A of Figure 5. We check whether



A – Level of GDP

B – Growth rate of GDP

Figure 5: GDP and its growth in the United States: 1800-2016

Note: GDP denotes real per-capita gross domestic product. For GDP, we do not use interpolated observations. Instead, we use only consecutive annual observations, which start in 1800. In Panel A, the pre-1875 trend is based on the best linear fit of the GDP time series from 1800 to 1875. The post-1875 trend is based on the estimated coefficients for the specification in (18).
Source: Delventhal et al. (2021) and authors' calculations.

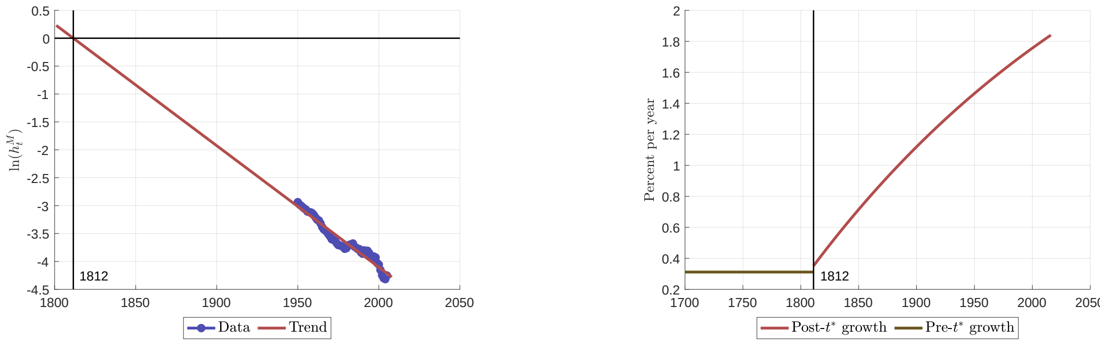
our estimated coefficient is consistent with agricultural employment dynamics—i.e., whether $\exp(\gamma_3)$ approximately equals $\exp(\beta_1)$. We find $\exp(\beta_1) = \exp(\dot{h}^M) = 0.969$.

One way to validate our estimate of t^* is to check whether GDP growth is higher and increasing after t^*. (Recall that we did not use GDP data to estimate the onset of transition.) It is clear in Panel B of Figure 5 that the GDP growth rate is higher and increasing after 1875.

Thus, our estimate of 1875 as the year when the U.S. started the economic transition, based solely on post-World War II agricultural employment, is consistent with (a) the change in GDP growth at 1875 and (b) the autoregressive coefficient for GDP growth after 1875.

As in the case of the U.S., we estimate the coefficients in (14) with post-World War II annual data on the share of agricultural employment for the U.K. From Equation (15), we find t^* to be 1812 for the U.K. This is illustrated in Panel A of Figure 6.

Panel B of the figure illustrates GDP dynamics. Similar to the U.S., the U.K. GDP growth is higher and increasing after 1812. The transition literature pins down the onset of transition by using historical GDP data. For instance, Hansen and Prescott



A – Share of agricultural employment: 1949-2016 B – Growth rate of GDP

Figure 6: Onset of transition and GDP dynamics in the United Kingdom

Note: GDP denotes real per-capita gross domestic product. For GDP, we do not use interpolated observations. Instead, we use only consecutive annual observations, which start in 1700. The pre-1812 trend is based on the best linear fit of the GDP time series from 1700 to 1812. The post-1812 trend is based on the estimated coefficients for the specification in (18).
Source: Herrendorf et al. (2014), Delventhal et al. (2021), and authors' calculations.

(2002) use 1800 as the onset of transition for England. Figure 2 in Leukhina and Turnovsky (2016) indicates the onset of transition to be in the early 1800s.

We estimate the coefficients in (18) using post-1812 GDP data. The autoregressive coefficient of GDP growth $\exp(\gamma_3) = 0.997$. This is almost equal to the (exponential of the) growth rate of the share of agricultural employment: $\exp(\dot{h}^M) = 0.979$.

Speed of convergence Equation (10) implies that

$$y_t - Z_t^S = \frac{\alpha}{1-\alpha} Z_t^S h_t^M.$$

Recall that the long-run growth rate of GDP is $\dot{Z}^S$, the growth rate of the Solow technology. So, the gap between GDP growth at a point in time and its long-run growth rate is $\dot{h}^M + \dot{Z}^S$. In other words, $\dot{h}^M + \dot{Z}^S$ is the speed of convergence to the balanced growth path. Since the model implies $\dot{h}^M = \gamma_3$, the speed of convergence should also be $\gamma_3 + \dot{Z}^S$. Hence, the difference between $\dot{h}^M$ and γ_3 measures the gap in the speed of convergence implied by agricultural employment and that implied directly by GDP dynamics. For the U.S. this gap is 0.008 and for the U.K. it is 0.018.

3.4 Western Europe and other middle- and high-income countries

Panel A of Table 1 reports our results for Western European countries. We do not have data on the share of agricultural employment going back to 1949 for all of these countries as we did for the U.S. and the U.K. We use the share data available from the World Bank, from 1991 to 2022, to estimate the onset of transition.

The fit is remarkable: More than a century's worth of GDP growth in Western Europe is described by the rate of decline in the share of agricultural employment over the past 30 years. Ex-ante, one would not expect the recent observations on agricultural employment to account for GDP dynamics during the 19th and 20th centuries, a period when the sectoral composition of GDP in Western Europe changed dramatically.

For Sweden, our estimate of the onset of transition is 1851. This is consistent with the estimate of 1850s reported by Jörberg (1965). Our estimate of the onset of transition might be late by a few decades for some countries. An example is France, a leading industrial nation in the 19th century: Its shares of agricultural employment for the past 30 years imply the onset of transition was 1889. The share in France, however, is an anomaly. Around 1900, the U.S. and France had almost the same share: 40% and 41%, respectively. In 1954, the U.S. share had declined to 9%, while France's share was almost three times higher at 26%.

Panel B of Table 1 reports our results for middle- and high-income countries. Our estimate of the onset of transition in Japan is 1874, consistent with the Meiji restoration, the transition of Japan to an industrialized economy, which began in 1868 (Tang, 2014). For Canada our estimate is 1881, consistent with an acceleration of economic growth between 1871 and 1891 (Jaworski and Keay, 2022).

3.5 Developing economies

An advantage of our approach, based on recent agricultural employment, is that we can estimate the onset of economic transition for today's developing economies. These economies typically do not have time series of GDP data long enough to determine the onset of transition and are not examined in the transition literature. Table 2 reports the onset of transition and the autoregressive coefficient of GDP growth during the

Table 1: Onset of transition and GDP dynamics

A – Western Europe				
	t^*	$\exp(\dot{h}^M)$	$\exp(\gamma_3)$	$\lvert\dot{h}^M - \gamma_3\rvert$
Austria	1881	0.9766	0.9262	0.0530
Belgium	1900	0.9649	0.9763	0.0118
Denmark	1890	0.9700	0.9080	0.0660
Finland	1905	0.9746	0.9757	0.0011
France	1889	0.9747	0.9604	0.0147
Germany	1908	0.9654	0.8965	0.0741
Greece	1912	0.9815	0.9747	0.0070
Italy	1880	0.9745	0.9766	0.0022
Luxembourg	1916	0.9622	0.9256	0.0388
Netherlands	1861	0.9743	0.9306	0.0458
Norway	1918	0.9693	0.8873	0.0884
Portugal	1925	0.9727	0.9767	0.0041
Spain	1920	0.9673	0.9196	0.0505
Sweden	1851	0.9824	0.9367	0.0476
Switzerland	1865	0.9751	0.8067	0.1896

B – Middle- and high-income countries				
	t^*	$\exp(\dot{h}^M)$	$\exp(\gamma_3)$	$\lvert\dot{h}^M - \gamma_3\rvert$
Argentina	1909	0.9796	0.9433	0.0377
Australia	1891	0.9717	0.8788	0.1005
Brazil	1930	0.9764	0.9643	0.0124
Canada	1881	0.9683	0.9776	0.0095
Chile	1933	0.9650	0.9774	0.0128
Japan	1874	0.9753	0.9762	0.0010
Mexico	1929	0.9771	0.9451	0.0333
New Zealand	1881	0.9815	0.8305	0.1670

Note: GDP denotes real per-capita gross domestic product. For GDP, we do not use interpolated observations. Instead, we use only consecutive annual observations after t^*. For the share of agricultural employment, we use data from 1991 to 2022. The onset of transition is at t^*, the rate of decline in the share of agricultural employment is $\dot{h}^M$, and the autoregressive coefficient on GDP growth is $\exp(\gamma_3)$.
Source: World Bank, Delventhal et al. (2021), and authors' calculations.

transition. The share of agricultural employment is from the World Bank, 1991 to 2022. Using (15) and the coefficients in (14), we estimate the onset of transition for each country. The estimates of the coefficients in (18) yield $\exp(\gamma_3)$.

The sample is the set of countries (i) for which we have annual GDP observations soon after their onset of transition and (ii) whose GDP was below 25% of U.S. GDP in 2016. We exclude several countries based on two rules. First, if t^* predates the onset of contiguous GDP data in Delventhal et al. (2021), we exclude the country. Examples of such countries are Belarus, Bulgaria, Nigeria, Paraguay, Qatar, Russian Federation, Saudi Arabia, and Singapore. Recall that the GDP growth rate is constant in the long-run. Thus, most of the information about γ_3 is in the period that "follows" t^*. We cannot reliably estimate γ_3 when this information is missing. Second, for some countries, the share of agricultural employment increases between 1991 and 2022. This implies t^* is in the future. We exclude such countries, which are Angola, Botswana, Ecuador, Fiji, Macao, Moldova, Suriname, and Uruguay.

Again, our approach fits the data well: $\exp(\gamma_3) \approx \exp(\dot{h}^M)$ for most countries. Note that the countries in Table 2 are in different stages of development. In 2000, China is 8 times as rich as Mozambique. The countries are also in different stages of structural transformation. The share of agricultural employment in Burkina Faso in 2000 is 85%, but in Sri Lanka it is 38%. Despite these differences, GDP dynamics during the transition are pinned down by agricultural employment dynamics.

4 Conclusion

In our model, a single good can be produced using two technologies: Malthus (diminishing returns) and Solow (constant returns). TFPs and population are exogenous. The economy's GDP (real per-capita gross domestic product) exhibits three stages: (i) stagnation, (ii) transition with increasing growth, and (iii) constant growth in the long run. We map the Malthus technology to agriculture and show that agricultural employment is sufficient to determine both the onset of economic transition and the dynamics of GDP during the transition. Specifically, we show that GDP growth during the transition is a first-order autoregressive process and that the rate of decline in the share of agricultural employment pins down the autoregressive coefficient.

Quantitatively, we use recent data on agricultural employment to estimate the onset of transition for the U.S., U.K., and several Western European countries. Our estimate does not rely on GDP data but is consistent with lower growth before the onset of transition and higher and increasing growth after. The autoregressive coefficient of GDP growth during the transition is practically the same as that implied by the rate of decline in the share of agricultural employment. There is no a priori reason that agricultural employment over a recent few years would pin down GDP dynamics over two centuries that were characterized by large structural changes.

Our method is especially useful in the context of developing economies, which do not have historical data and are usually not examined in the transition literature. Again, we find that the share of agricultural employment is sufficient to determine the onset of transition and GDP growth during the transition for economies that are in different stages of development and structural transformation.

Our model is a model of economic transition, not demographic transition, as the processes for TFPs and population are exogenous. Endogenizing one or more of these processes could potentially deliver more testable implications. However, the fact remains that our simple quantitative framework that uses only recent agricultural employment is remarkably consistent with the onset of transition and GDP dynamics.

One might criticize our model for its simplicity (or lack of endogenous fertility, multiple goods, distortions, trade, structural transformation, subsistence agriculture, etc.), but other models that are more elaborate have to confront the quantitative importance of recent agriculture. How is it that a few recent observations on the share of agricultural employment are sufficient to pin down the onset of GDP transition and dynamics of GDP growth during the transition for developed economies over a long period and for developing economies in various stages of development? To answer this question, criticisms of our model will not suffice.

Table 2: Onset of transition and GDP dynamics: Developing economies

	t^*	$\exp(\dot{h}^M)$	$\exp(\gamma_3)$	$\lvert\dot{h}^M - \gamma_3\rvert$		t^*	$\exp(\dot{h}^M)$	$\exp(\gamma_3)$	$\lvert\dot{h}^M - \gamma_3\rvert$
Afghanistan	1969	0.9845	0.9913	0.0069	Liberia	1954	0.9862	0.9949	0.0088
Bangladesh	1971	0.9807	0.9839	0.0033	Malawi	1952	0.9932	0.9301	0.0656
Benin	1970	0.9774	0.971	0.0066	Mongolia	1963	0.9776	0.9929	0.0155
Bolivia	1941	0.984	0.9914	0.0075	Mozambique	1971	0.9929	0.9288	0.0667
Burkina Faso	1975	0.9933	0.9846	0.0088	Myanmar	1963	0.987	0.9887	0.0017
Burundi	1969	0.9969	0.8756	0.1297	Namibia	1959	0.9751	0.9926	0.0178
Cambodia	1987	0.9718	0.7524	0.2559	Nepal	1960	0.9921	0.9767	0.0156
Cameroon	1976	0.9815	0.9908	0.0094	Peru	1943	0.9823	0.8789	0.1112
Chad	1967	0.9931	0.9851	0.0081	Philippines	1955	0.9787	0.8289	0.1661
China	1977	0.9683	0.9834	0.0155	Rwanda	1992	0.9802	0.3045	1.1691
Comoros	1971	0.9791	0.8064	0.1941	Senegal	1974	0.9709	0.9802	0.0095
Congo	1967	0.9897	0.992	0.0023	Sierra Leone	1976	0.9841	0.9889	0.0049
Ethiopia	1959	0.9933	0.9933	0.0000	Sri Lanka	1938	0.9837	0.989	0.0054
Gambia	1956	0.9888	0.8081	0.2018	St. Lucia	1953	0.9649	0.9917	0.0274
Ghana	1964	0.983	0.9872	0.0043	Syria	1951	0.9722	0.9908	0.0190
India	1965	0.9846	0.9857	0.0011	Tanzania	1978	0.9897	0.9801	0.0097
Indonesia	1950	0.9834	0.888	0.1020	Togo	1976	0.9746	0.9849	0.0105
Laos	1980	0.9887	0.9819	0.0069	Yemen	1968	0.9734	0.8709	0.1113
Lesotho	1963	0.9779	0.9909	0.0132	Zambia	1958	0.9916	0.9926	0.0010

Note: GDP denotes real per-capita gross domestic product. For GDP, we do not use interpolated observations. Instead, we use only consecutive annual observations after t^*. For the share of agricultural employment, we use data from 1991 to 2022. The sample is the set of developing economies (i) for which we have annual GDP observations after their onset of transition and (ii) whose GDP was below 25% of U.S. GDP in 2016. The onset of transition is at t^*, the rate of decline in the share of agricultural employment is $\dot{h}^M$, and the autoregressive coefficient on GDP growth is $\exp(\gamma_3)$.

Source: World Bank, Delventhal et al. (2021), and authors' calculations.

References

G. S. Becker, K. M. Murphy, and R. Tamura. Human capital, fertility, and economic growth. *Journal of Political Economy*, 98(5, Part 2):S12–S37, 1990.

M. J. Delventhal, J. Fernández-Villaverde, and N. Guner. Demographic transitions across time and space. *NBER Working Paper Series*, (29480), 2021.

O. Galor and D. N. Weil. Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review*, 90(4):806–828, 2000.

M. Goodfriend and J. McDermott. Early development. *American Economic Review*, 85(1):116–133, 1995.

G. D. Hansen and E. C. Prescott. Malthus to Solow. *American Economic Review*, 92 (4):1205–1217, 2002.

B. Herrendorf, R. Rogerson, and A. Valentinyi. Growth and structural transformation. In P. Aghion and S. N. Durlauf, editors, *Handbook of Economic Growth*, volume 2, chapter 6, pages 855–941. Elsevier, 2014.

T. Jaworski and I. Keay. Globalization and the spread of industrialization in Canada, 1871–1891. *Explorations in Economic History*, 86:101466, 2022.

C. I. Jones. Was an industrial revolution inevitable? Economic growth over the very long run. *The BE Journal of Macroeconomics*, 1(2), 2001.

L. Jörberg. Structural change and economic growth: Sweden in the 19th century. *Economy and History*, 8(1):3–46, 1965.

O. M. Leukhina and S. J. Turnovsky. Population size effects in the structural development of England. *American Economic Journal: Macroeconomics*, 8(3):195–229, 2016.

T. R. Malthus. *An Essay on the Principle of Population.* J Johnson, London, 1798.

D. Ricardo. *On the principles of political economy and taxation.* J Murray, London, 1817.

R. Tamura. Human capital and the switch from agriculture to industry. *Journal of Economic Dynamics and Control*, 27(2):207–242, 2002.

J. P. Tang. Railroad Expansion and Industrialization: Evidence from Meiji Japan. *The Journal of Economic History*, 74(3):863–886, 2014.

A Derivation of Equations (10) and (11)

Using the solution for H_t^M for $t \geq t^*$, the output of technology M when both technologies operate is

$$Y_t^M = \left(Z_t^M\right)^{1-\alpha} \left[(1-\alpha)\frac{\left(Z_t^M\right)^{1-\alpha}}{Z_t^S}\right]^{(1-\alpha)/\alpha} = (1-\alpha)^{(1-\alpha)/\alpha}\left(\frac{Z_t^M}{Z_t^S}\right)^{(1-\alpha)/\alpha},$$

implying that output per capita and its growth rate are

$$y_t^M = = \frac{1}{1-\alpha} Z_t^S h_t^M, \tag{A.1}$$
$$\dot{y}^M = \dot{Z}^S + \dot{h}^M. \tag{A.2}$$

For technology S, output per capita is

$$y_t^S = Z_t^S(1-h_t^M) = Z_t^S - (1-\alpha)y_t^M, \tag{A.3}$$

and its rate of growth is

$$\dot{y}_t^S = \frac{d\ln Z_t^S}{dt}\frac{Z_t^S}{y_t^S} - (1-\alpha)\frac{d\ln y_t^M}{dt}\frac{y_t^M}{y_t^S} = \frac{1}{1-h_t^M}\dot{Z}^S - \frac{h_t^M}{1-h_t^M}\dot{y}^M. \tag{A.4}$$

The economy's GDP (real per-capita gross domestic product) is $y_t = y_t^S + y_t^M$. Using (A.1) and (A.3), this is

$$y_t = Z_t^S + \frac{\alpha}{1-\alpha} Z_t^S h_t^M.$$

The GDP growth rate is

$$\dot{y}_t = \frac{d\ln y_t^S}{dt}\frac{y_t^S}{y_t} + \frac{d\ln y_t^M}{dt}\frac{y_t^M}{y_t} = \left(1 - \frac{y_t^M}{y_t}\right)\left(\dot{Z}^S\frac{1}{1-h_t^M} - \dot{y}^M\frac{h_t^M}{1-h_t^M}\right) + \frac{y_t^M}{y_t}\dot{y}^M,$$

where

$$\frac{y_t^M}{y_t} = \frac{\frac{1}{1-\alpha}Z_t^S h_t^M}{Z_t^S + \frac{\alpha}{1-\alpha}Z_t^S h_t^M} = \frac{h_t^M}{1-\alpha+\alpha h_t^M}.$$

It follows that

$$
\begin{aligned}
\dot{y}_t &= \frac{1 - y_t^M/y_t}{1 - h_t^M}\dot{Z}^S + \left(\frac{y_t^M}{y_t} - \frac{h_t^M}{1 - h_t^M}\left(1 - \frac{y_t^M}{y_t}\right)\right)\dot{y}^M \\
&= \frac{1-\alpha}{1-\alpha+\alpha h_t^M}\dot{Z}^S + \frac{\alpha h_t^M}{1-\alpha+\alpha h_t^M}\left(\dot{Z}^S + \frac{1-\alpha}{\alpha}\dot{Z}^M - \frac{1}{\alpha}\dot{Z}^S - \dot{P}\right) \\
&= \frac{(1-\alpha)(1-h_t^M)}{1-\alpha+\alpha h_t^M}\dot{Z}^S + \frac{(1-\alpha)h_t^M}{1-\alpha+\alpha h_t^M}\dot{Z}^M - \frac{\alpha h_t^M}{1-\alpha+\alpha h_t^M}\dot{P}.
\end{aligned}
$$