On the Transition to Modern Growth

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<th>Authors</th>
<th>B. Ravikumar, and Guillaume Vandenbroucke</th>
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On the Transition to Modern Growth*

B. Ravikumar† G. Vandenbroucke‡

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Abstract

We study a simple model where a single good can be produced using a diminishing-returns technology (Malthus) and a constant-returns technology (Solow). The economy’s output exhibits three stages: (i) stagnation, (ii) transition with increasing growth, and (iii) constant growth in the long run. We map the Malthus technology to agriculture and show that the share of agricultural employment is sufficient to determine the onset of economic transition. Using data on the share, we estimate the onset of transition for the U.S. and Western Europe without using output data. Our model implies that output growth during the transition is a first-order autoregressive process and that the rate of decline in the share of agricultural employment is a sufficient statistic to describe the output growth. Quantitatively, while there is no a priori reason why agricultural employment would pin down output dynamics over two centuries, the autoregressive coefficient on the output growth process is practically the same as the one implied by the rate of decline in the share of agricultural employment.

JEL codes: O10, O13, O40
Keywords: Economic transition, Agricultural employment, Malthus, Solow

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†Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166, USA. Email: b.ravikumar@wustl.edu.

‡Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166, USA. Email: guillaumevdb@gmail.com.
1 Introduction

The standard of living was roughly constant prior to the 19th century despite technological change. Malthus (1798) and Ricardo (1817) accounted for this stagnation with a theory of population and diminishing returns in production. The past two centuries, however, have seen unprecedented growth in living standards, which has spawned a vast literature on transition from stagnation to growth. Our paper belongs to a strand of the literature where a single final good can be produced with two technologies and the economy transitions from one technology to possibly both. Two prominent examples of this strand are Hansen and Prescott (2002), where the transition occurs when physical capital and total factor productivity (TFP) reach a threshold, and Tamura (2002), where endogenous human capital accumulation lowers trade costs and delivers the transition without relying on any exogenous forces.

We develop a simplified version of their models. Our economy has one good that can be produced using two technologies—Malthus and Solow. The Malthus technology is subject to diminishing returns as land is a fixed factor, and the Solow technology has constant returns. Labor is the only flexible factor of production. TFP in the two technologies and population grow exogenously at possibly different rates.

We map the Malthus technology to agriculture and deliver two results. First, the share of agricultural employment determines the onset of economic transition; we do not need information on real gross domestic product per capita (GDP). Specifically, the share declines at a constant rate in the model during the transition, so we can use the rate to project backward and pin down the onset date. Second, during the transition, the share of agricultural employment is a sufficient statistic for GDP dynamics in the model: GDP growth is a first-order autoregressive process with an autoregressive coefficient that maps one for one into the rate of decline of the share of agricultural employment. In sum, we do not need to know the structural parameters, TFPs, or population to estimate either the onset of transition or GDP dynamics.

1 Other strands on transition include Becker, Murphy, and Tamura (1990) and Galor and Weil (2000), where the economy transitions from stagnation and high fertility to growth and lower fertility. In Goodfriend and McDermott (1995), exogenous population growth allows for increasing returns to specialization, and the economy transitions from household production and stagnation to market production and growth. In Jones (2001), the evolution of population and ideas delivers technological progress that helps the economy transition.
Our economy uses only the Malthus technology initially, but it transitions to using both the Malthus and Solow technologies when the two TFPs and population reach a threshold. The onset of economic transition is when employment in the Malthus technology starts declining—labor is employed only in the Malthus technology initially, but it is employed in both technologies during the transition and increasingly more in the Solow technology. Quantitatively, we use data on agricultural employment from Herrendorf, Rogerson, and Valentinyi (2014) to project backward and infer the onset of economic transition. For the United States it is 1840.

In terms of GDP, the economy exhibits three stages: (i) stagnation, (ii) acceleration, where the growth rate increases over time, and (iii) constant growth in the long run. We use GDP data from Delventhal, Fernández-Villaverde, and Guner (2021) and estimate the autoregressive coefficient for the U.S. time series from 1840-2016. The coefficient is almost identical to the one implied by the constant rate of decline in the share of agricultural employment. Note that there is no a priori reason to think the agricultural employment would describe the GDP dynamics accurately, especially given the enormous structural changes in the U.S. during this period.

We also show that our method for estimating the onset of the transition using only agricultural employment data is consistent with GDP growth before and during the transition. That is, we find that the path of U.S. GDP is well described by a constant growth rate before 1840 and an increasing growth rate afterwards.

We repeat the calculations for the United Kingdom, France, the Netherlands, and Sweden. We find the onset of their transitions to be 1783, 1866, 1812, and 1863, respectively. Furthermore, the autoregressive coefficient for each country’s GDP time series is consistent with the one implied by the country’s agricultural employment.

This paper differs from the literature in a few ways. First, previous papers are concerned with the “how”—the channels that led from stagnation to growth. They are not concerned with the “when.” In their quantitative implementations, the onset of economic transition is chosen using narratives or historical evidence related to GDP and the model parameters are calibrated to deliver the chosen onset date. For instance, Hansen and Prescott (2002) use this approach for England and Tamura (2002) uses it for Western Europe + Canada + the U.S. Our paper is concerned with the “when.” We estimate the onset of economic transition without using GDP...
data. We use GDP data for cross validation. Second, our method does not require knowing structural parameters and dynamics of TFPs and population. Cross-country differences in structural parameters and the processes for TFPs and population yield cross-country differences in rates of decline in the share of agricultural employment, which is a sufficient statistic for the onset of economic transition and the dynamics of GDP. Finally, because we do not rely on historical evidence on GDP, our model is useful for studying the economic transition of recent emerging economies.

Two remarks are in order here. First, in models such as Hansen and Prescott (2002) and Tamura (2002) the onset of economic transition is, by definition, when the Malthusian share of employment starts declining, no matter what the details of the model are. Without using any specific model, one could make an ad-hoc assumption that the rate of decline is constant and estimate the onset of transition. Our approach has two advantages: (i) the ad-hoc assumption, without more structure, would have no further implications for GDP dynamics and (ii) we provide a framework where the constant rate of decline is a result, not an assumption.

Second, ours is a model of only economic transition, not demographic transition, as the processes for TFPs and population are exogenous. Endogenizing one or more of these processes could potentially deliver more testable implications, but the fact remains that our simple framework is remarkably consistent with the onset of economic transition and dynamics of GDP, using only agricultural employment. Thus, richer frameworks have to bear the burden of delivering our results.

2 Model

Time is continuous and represented by \( t \geq 0 \). There are two technologies, denoted \( M \) (for Malthus) and \( S \) (for Solow), producing a single consumption good. Technology \( M \) uses land (fixed and normalized to 1) and labor, and exhibits diminishing returns to labor. Technology \( S \) uses only labor and exhibits constant returns to scale. Outputs at date \( t \) from the two technologies are denoted \( Y_t^M \) and \( Y_t^S \):

\[
Y_t^M = (Z_t^M H_t^M)^{1-\alpha}, \\
Y_t^S = Z_t^S H_t^S,
\]
where \( \alpha \in (0, 1) \), \( Z_t^M \) and \( Z_t^S \) are exogenous labor-augmenting productivities, and \( H_t^M \) and \( H_t^S \) are employment in \( M \) and \( S \), respectively. Total gross domestic product is

\[
Y_t = Y_t^M + Y_t^S.
\]

The working population is exogenous and denoted by \( P_t \).

In what follows, we adopt the following two notations. First, we use lowercase letters to denote a variable per unit of population: \( x_t \equiv X_t/P_t \). Second, we use a dot notation to denote the growth rate of a variable:

\[
\dot{X}_t \equiv d\ln(X_t)/dt.
\]

We assume that \( Z_t^S \), \( Z_t^M \), and \( P_t \) grow at constant but potentially different rates \( \dot{Z}^S \), \( \dot{Z}^M \), and \( \dot{P} \), respectively.

\[
Z_t^S = Z_0^S \exp\left(t\dot{Z}^S\right), \quad Z_t^M = Z_0^M \exp\left(t\dot{Z}^M\right), \quad \text{and} \quad P_t = P_0 \exp\left(t\dot{P}\right),
\]

where \( Z_0^S \), \( Z_0^M \), and \( P_0 \) are initial conditions.

The working population is allocated between the two technologies,

\[
H_t^M + H_t^S = P_t.
\]

Labor is perfectly mobile across the two technologies. Thus, the optimal, i.e., output-maximizing, allocation of labor requires:

\[
Z_t^S \leq (1 - \alpha) \left(Z_t^M\right)^{1-\alpha} \left(H_t^M\right)^{-\alpha},
\]

with equality whenever \( H_t^M < P_t \). The left-hand side is the marginal product of labor in technology \( S \), and the right-hand side is the marginal product of labor in technology \( M \). Figure 1 represents the optimal allocation of labor. When \( H_t^M < P_t \), the two marginal products are equalized. If \( Z_t^S \) is low enough (e.g., \( Z_t^S, \text{Low} \)), however, the marginal product of labor in technology \( M \) exceeds that in technology \( S \) even if the entire working population is allocated to technology \( M \). In this case Equation (1) holds with a strict inequality.
If initial conditions are such that

$$Z_0^S < (1 - \alpha) \left( Z_0^M \right)^{1-\alpha} (P_0)^{-\alpha},$$

(2)

then all labor is allocated to technology $M$ at date 0. If inequality (2), evaluated at $t$, continues to hold, then all labor will continue to be allocated to technology $M$ at $t$. Some labor will be allocated to technology $S$ at some date $t^*$ if Equation (1) is satisfied with equality at $t^*$. For this to occur, it must be the case that

$$\dot{Z}^S > (1 - \alpha) \dot{Z}^M - \alpha \dot{P}.$$  

(3)

We assume conditions (2) and (3) are satisfied for the rest of the paper.

The transition date at which technology $S$ starts operating satisfies

$$Z_0^S e^{t^* \dot{Z}^S} = (1 - \alpha) \left( Z_0^M e^{t^* \dot{Z}^M} \right)^{1-\alpha} \left( P_0 e^{t^* \dot{P}} \right)^{-\alpha}. $$
That is,
\[
t^* = \frac{\ln \left[ (1 - \alpha) \left( Z_0^S \right)^{-1} \left( Z_0^M \right)^{1-\alpha} (P_0)^{-\alpha} \right]}{Z^S - (1 - \alpha) \dot{Z}^M + \alpha \dot{P}}.
\] (4)

2.1 Analysis

It is convenient to analyze the economy in two parts: before and after \( t^* \).

**Before the transition** When \( t \leq t^* \), the analysis is straightforward. The entire working population is employed in technology \( M \) and output per capita is that of technology \( M \). Using our notations defined earlier,

\[
\text{Share of agricultural employment : } h_t^M = 1,
\]
\[
\text{GDP : } y_t = y_t^M = \left( Z_t^M \right)^{1-\alpha} (P_t)^{-\alpha},
\]
\[
\text{GDP growth : } \dot{y} = \dot{y}^M = (1 - \alpha) \dot{Z}^M - \alpha \dot{P}.
\]

**During the transition** When both technologies operate, Equation (1) holds with equality and employment in technology \( M \) is

\[
H_t^M = (1 - \alpha)^{1/\alpha} \frac{1}{Z_t^S} \left( \frac{Z_t^M}{Z_t^S} \right)^{(1-\alpha)/\alpha}.
\]

It follows that the employment share of technology \( M \) and its growth rate are

\[
h_t^M = (1 - \alpha)^{1/\alpha} \frac{1}{Z_t^S P_t} \left( \frac{Z_t^M}{Z_t^S} \right)^{(1-\alpha)/\alpha},
\]
\[
\dot{h}_t^M = \frac{1 - \alpha}{\alpha} \dot{Z}^M - \frac{1}{\alpha} \dot{Z}^S - \dot{P}.
\]

A few observations are worth making here. First, condition (3) implies \( \dot{h}_t^M < 0 \). Equation (9) delivers the result that the employment share of technology \( M \) decreases at a constant rate after \( t^* \), as long as the growth rates of the three exogenous variables are constant (see Figure 2). Second, in a world with only population growth, i.e., \( \dot{P} > 0 \) and \( \dot{Z}^M = \dot{Z}^S = 0 \), the employment share of technology \( M \) declines at the rate of population growth. This is because the additional workers are allocated to
technology $S$ where the marginal product of labor is constant. Third, with population growth and TFP growth in technology $S$, i.e., $\dot{P}, \dot{Z}^S > 0$ and $\dot{Z}^M = 0$, the rate of decline of $h^M$ is higher than that with only population growth. This is because the marginal product of labor in technology $S$ is growing, requiring a faster decline of employment in technology $M$ to maintain the equality of marginal products. Finally, if all three exogenous variables are growing, the rate of decline of $h^M$ is less than that with only $\dot{P}, \dot{Z}^S > 0$. This is because the rise in the productivity of technology $M$ allows for a slower decline of employment in $M$ while still maintaining the equality in the marginal products. Note that, even if the two exogenous TFPs grow at the same rate, $h^M$ still declines as long as either $Z^S$ or $P$ is growing.

The economy’s GDP is

$$y_t = Z^S_t + \frac{\alpha}{1 - \alpha} Z^S_t h^M_t.$$  \hfill (10)
The growth rate of $y_t$ is

$$\dot{y}_t = \frac{(1 - \alpha)(1 - h_t^M)}{1 - \alpha + \alpha h_t^M} \dot{Z}_S + \frac{(1 - \alpha)h_t^M}{1 - \alpha + \alpha h_t^M} \dot{Z}_M - \frac{\alpha h_t^M}{1 - \alpha + \alpha h_t^M} \dot{P}. \quad (11)$$

See Appendix A for the derivation of Equations (10) and (11).

Three remarks about Equation (11) are in order. First, if $h_t^M = 1$, Equation (11) is the same as Equation (7): GDP growth before the transition is that of Malthusian output. That is, even though we derived Equation (11) for $t \geq t^*$, this equation describes the growth rate of GDP for all $t$. Second, as $h_t^M \to 0$, Equation (11) implies $\dot{y}_t \to \dot{Z}_S$: Asymptotically, the growth rate of GDP is that of TFP in technology $S$. Third, an implication of (3) and (11) is that $dy/dh^M < 0$, i.e., GDP growth increases monotonically during the transition (see Figure 3). This is true even if the TFPs and population grow at constant rates.
3 Quantitative Analysis

3.1 Determining the onset of modern growth

The onset of “modern” growth in our model is when technology S starts operating at date $t^*$. While $t^*$ is pinned down by Equation (4), the expression is not quantitatively useful since it depends on unobserved variables. In this section, we derive an expression for $t^*$ that is a function of only employment share in the Malthus technology. We then map the Malthus technology to agriculture and use observations on the share of agricultural employment to estimate the onset of the transition.

Under the assumption that $Z^M, Z^S,$ and $P$ grow at constant rates after the onset of modern growth, Equation (9) implies that $\dot{h}^M$ is constant. So, $h^M_t = \exp((t - t^*)\dot{h}^M)$ at any date $t \geq t^*$, or

$$t^* = t - \ln\left(\frac{h^M_t}{\dot{h}^M}\right). \tag{12}$$

Countries could differ in their structural parameters, and levels and growth rates of TFPs and population. In our model, these differences manifest themselves in different $h^M_t$ and $\dot{h}^M_t$ (see Equations 8 and 9). What is remarkable about Equation (12) is that the reasons why different countries have different shares or growth rates of agricultural employment do not matter for estimating the onset of modern growth.

To operationalize Equation (12), consider the specification below for country $i$.

$$\ln(h^M_{t,i}) = \beta_{0,i} + \beta_{1,i}t, \tag{13}$$

which implies $\beta_{1,i} = \dot{h}^M_i$. The transition date is

$$t_{i}^* = -\frac{\beta_{0,i}}{\beta_{1,i}} = -\frac{\ln(h^M_{t,i}) - \beta_{1,i}t}{\beta_{1,i}} = t - \frac{\ln(h^M_{t,i})}{\dot{h}^M_i},$$

which is the same as Equation (12). We run the regression with annual data for the U.S. We find $t^*$ to be 1840. This is illustrated for the U.S. in Figure 4.
3.2 Dynamics of GDP

In our model, the growth rate of GDP increases during the transition and converges to a constant in the long run. In this section, we show that the rate of decline of the share of agricultural employment is a sufficient statistic to characterize the dynamics of GDP during the transition.

From Equation (10) it is easy to see that the long-run path of GDP is that of $Z_t^S$. The relative deviation of GDP from its long-run path, which we denote $\hat{y}_t$, is then

$$\hat{y}_t \equiv \frac{y_t - Z_t^S}{Z_t^S} = \frac{\alpha}{1 - \alpha} h_t^M, \quad \text{for} \quad t \geq t^*.$$  \hspace{1cm} (14)

First, Equation (14) implies that $\hat{y}_t$ grows at rate $h_t^M$. As the share of agricultural employment declines, $\hat{y}_t$ approaches zero and the paths of GDP and $Z_t^S$ converge.
Second, using (14) at two instants $t$ and $t + \omega$, we get

$$\ln(1 + \hat{y}_{t+\omega}) - \ln(1 + \hat{y}_t) + \ln(Z^S_{t+\omega}) - \ln(Z^S_t) = \ln y_{t+\omega} - \ln y_t.$$  

We then approximate $\ln(1 + \hat{y}_{t+\omega}) - \ln(1 + \hat{y}_t) \approx \hat{y}_{t+\omega} - \hat{y}_t$.\(^2\) This yields

$$\hat{y}_{t+\omega} - \hat{y}_t + \ln(Z^S_{t+\omega}) - \ln(Z^S_t) \approx \ln y_{t+\omega} - \ln y_t,$$

which implies

$$\exp(\omega h^M) \hat{y}_t - \hat{y}_t + \omega \hat{Z}^S \approx \ln y_{t+\omega} - \ln y_t.$$  

Similarly, at $t + \omega$,

$$\exp(\omega h^M) \hat{y}_{t+\omega} - \hat{y}_{t+\omega} + \omega \hat{Z}^S \approx \ln y_{t+2\omega} - \ln y_{t+\omega},$$

$$\Rightarrow \exp(\omega h^M) \left( \exp(\omega h^M) \hat{y}_t - \hat{y}_t \right) + \omega \hat{Z}^S \approx \ln y_{t+2\omega} - \ln y_{t+\omega}.$$  

Substituting, rearranging, and evaluating at $\omega = 1$, we get

$$\ln y_{t+2} - \ln y_{t+1} \approx \exp(h^M) (\ln y_{t+1} - \ln y_t) + \hat{Z}^S \left( 1 - \exp(h^M) \right). \quad (15)$$

The rate of decline in the share of agricultural employment is thus a sufficient statistic to describe the dynamics of GDP growth after $t^*$.

Note that (15) is a result, not just an accounting formula. While, as noted earlier, one could estimate $t^*$ by assuming $h^M$ is constant, the ad-hoc assumption would not imply (15). In deriving (15) we have used the model’s optimal allocation of labor across the two technologies and how the model’s parameters map to $h^M$.

**Modern GDP dynamics**  
Equation (15) represents a testable implication: The autoregressive coefficient of the growth rate of GDP, after $t^*$, is the (exponential of the) growth rate of the share of agricultural employment. To test the model, we specify the data-generating process for GDP for country $i$ as

$$\ln y_{t,i} = \gamma_{0,i} + \gamma_{1,i} t + \gamma_{2,i} \exp(\gamma_{3,i} t), \quad \text{for } t \geq t^*_i. \quad (16)$$

---

\(^2\)We are approximating $\ln(1 + \hat{y}_{t+\omega}) - \ln(1 + \hat{y}_t)$ with $\hat{y}_{t+\omega} - \hat{y}_t$, not $\ln(1 + \hat{y}_t)$ with $\hat{y}_t$. 

12
A – GDP and trend

B – Growth rate of GDP

Figure 5: Dynamics of GDP in the United States: 1840-2016

Note: Panel A: The trend of GDP is the fitted values of $\ln y_t$ from Equation (16). Panel B: The growth rate of GDP is the corresponding $\dot{y}_t$.

Source: Delventhal et al. (2021), and authors’ calculations.

Suppressing the country notation, this process has the property that

$$\dot{y}_t \equiv f(t) = \gamma_1 + \gamma_2 \gamma_3 \exp(\gamma_3 t).$$

If $\gamma_2 > 0$ and $\gamma_3 < 0$, then $f(t)$ is increasing at a diminishing rate and $\lim_{t \to \infty} f(t) = \gamma_1$. The autoregressive coefficient on GDP growth is $\exp(\gamma_3)$.

We estimate Equation (16) using data from $t^* = 1840$ to 2016 for the U.S. First, our estimates of $\gamma_2$ and $\gamma_3$ satisfy the conditions above, so GDP growth is increasing at a diminishing rate after $t^*$, as in Figure 3. We illustrate the fit of our specification in Figure 5. Second, we find $\exp(\gamma_3) = 0.991$ and $\exp(\dot{h}^M) = 0.977$. We inferred the latter from the estimation of Equation (13). We conclude that the share of agricultural employment describes the dynamics of U.S. GDP well.

Malthus vs. Solow Is our estimate of the onset of modern growth consistent with a change in the dynamics of GDP? Figure 6 illustrates the U.S. GDP time series before and after $t^*$. Our estimate of 1840 as the year when the U.S. transitioned from Malthus to Solow is consistent with the GDP dynamics in Figure 6.
Figure 6: GDP in the United States: 1650-2016

*Note:* The post-\(t^*\) trend is the same as the one in Figure 5. The pre-\(t^*\) trend is computed assuming a constant rate of growth from the first observation (1650) to the estimated value of GDP at \(t^*\) from Equation (16).

*Source:* Delventhal et al. (2021) and authors’ calculations. There are only three data points for GDP before 1800: in 1650, 1720, and 1775.

### 3.3 Western Europe

The onset of transition and dynamics of GDP for the U.K., France, Netherlands, and Sweden are reported in Table 1. Figures 7-10 illustrate the determination of \(t^*\) and the dynamics of GDP before and after \(t^*\).

As in the case of the U.S., the estimated autoregressive coefficient of GDP dynamics is close to the (exponential of the) growth rate of the share of agricultural employment in each country. Furthermore, it is clear from the right panels of Figures 7-10 that, except for France, the dynamics of GDP changes noticeably at our estimated date \(t^*\). In the case of France, GDP appears to accelerate before 1866.

The onset of transition for France, estimated using only agricultural employment, is too late. The share of agricultural employment in France is an anomaly relative to the
Table 1: Transition and GDP dynamics for Western Europe

<table>
<thead>
<tr>
<th>Country</th>
<th>$t^*$</th>
<th>$\exp(\gamma_3)$</th>
<th>$\exp(h^M)$</th>
<th>GDP sample</th>
<th>$h^M$ sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Kingdom</td>
<td>1783</td>
<td>0.997</td>
<td>0.982</td>
<td>1500–2016</td>
<td>1801–2008</td>
</tr>
<tr>
<td>France</td>
<td>1866</td>
<td>0.984</td>
<td>0.978</td>
<td>1500–2016</td>
<td>1856–2008</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1812</td>
<td>0.997</td>
<td>0.983</td>
<td>1500–2016</td>
<td>1807–2008</td>
</tr>
<tr>
<td>Sweden</td>
<td>1863</td>
<td>0.942</td>
<td>0.978</td>
<td>1500–2016</td>
<td>1850–2008</td>
</tr>
</tbody>
</table>

Note: The estimate of $\gamma_3$ is from regression (16) and that of $h^M$ is from regression (13).
Source: Herrendorf et al. (2014), Delventhal et al. (2021), and authors’ calculations. There are years with missing observations.

U.S. and other European countries. Around 1860, U.S. and France had almost the same share: 53% and 51%, respectively. By the end of World War II, the U.S. share had declined to 12% while France’s share was three times higher at 36%; in other parts of Western Europe the share was 23% in Sweden and 19% in the Netherlands. One reason for the anomaly in France could be the Méline tariffs (see Golob, 1944).
Share of agricultural employment: 1801-2008

GDP: 1500-2016

Figure 7: Onset of transition and GDP dynamics in the United Kingdom

Note: There is only one GDP observation before 1700, in 1500.

Source: Herrendorf et al. (2014), Delventhal et al. (2021), and authors’ calculations.

Share of agricultural employment: 1856-2008

GDP: 1500-2016

Figure 8: Onset of transition and GDP dynamics in France

Source: Herrendorf et al. (2014), Delventhal et al. (2021), and authors’ calculations.
Share of agricultural employment: 1807-2008

Figure 9: Onset of transition and GDP dynamics in the Netherlands

*Note:* There are 3 GDP observations before 1815: in 1500, 1806, and 1807.

*Source:* Herrendorf et al. (2014), Delventhal et al. (2021), and authors’ calculations.

Share of agricultural employment: 1850-2008

Figure 10: Onset of transition and GDP dynamics in Sweden

*Source:* Herrendorf et al. (2014), Delventhal et al. (2021), and authors’ calculations.
4 Conclusion

We develop a simple model where a single good can be produced using two technologies: Malthus (diminishing returns) and Solow (constant returns). TFPs and population are exogenous. The economy’s GDP exhibits three stages: (i) stagnation, (ii) transition with increasing growth, and (iii) constant growth in the long run. We map the Malthus technology to agriculture and show that the share of agricultural employment is sufficient to determine both the onset of economic transition and the dynamics of GDP during the transition.

Quantitatively, we use data on agricultural employment to estimate the onset of transition for the U.S., U.K., France, the Netherlands, and Sweden. Our estimate does not rely on GDP data. We then show that output growth during the transition follows a first-order autoregressive process and the autoregressive coefficient is practically the same as that implied by the rate of decline the share of agricultural employment. There is no a priori reason why agricultural employment would pin down the GDP dynamics over two centuries.

Our method can also be applied to recent emerging economies. For instance, the decline in the share of agricultural employment in China implies that the the onset of transition from Malthus to Solow occurred in 1958. Furthermore, the GDP growth in China since 1958 follows an autoregressive process, and the autoregressive coefficient is identical to the one implied by the share of agricultural employment.
REFERENCES


A Derivation of Equations (10) and (11)

Using the solution for $H_t^M$ for $t \geq t^*$, the output of technology $M$, when both technologies operate, is

$$Y_t^M = (Z_t^M)^{1-\alpha} \left[ (1 - \alpha) \left( \frac{Z_t^M}{Z_t^S} \right)^{(1-\alpha)/\alpha} \right] = (1 - \alpha)^{(1-\alpha)/\alpha} \left( \frac{Z_t^M}{Z_t^S} \right)^{(1-\alpha)/\alpha},$$

implying that output per capita and its growth rate are

$$y_t^M = \frac{1}{Z_t^S h_t^M}, \quad (A.1)$$

$$\dot{y}_t^M = \dot{Z}_t^S + \dot{h}_t^M. \quad (A.2)$$

For technology $S$, output per capita is

$$y_t^S = Z_t^S (1 - h_t^M) = Z_t^S - (1 - \alpha) y_t^M, \quad (A.3)$$

and its rate of growth is

$$\dot{y}_t^S = \frac{d \ln Z_t^S}{dt} \frac{Z_t^S}{y_t^S} - (1 - \alpha) \frac{d \ln y_t^M}{dt} \frac{y_t^M}{y_t^S} = \frac{1}{1 - h_t^M} \dot{Z}_t^S - \frac{h_t^M}{1 - h_t^M} \dot{y}_t^M. \quad (A.4)$$

The economy’s GDP is $y_t = y_t^S + y_t^M$. Using (A.1) and (A.3), this is

$$y_t = Z_t^S + \frac{\alpha}{1 - \alpha} Z_t^S h_t^M. \quad$$

The GDP growth rate is

$$\dot{y}_t = \frac{d \ln y_t^S}{dt} \frac{y_t^S}{y_t} + \frac{d \ln y_t^M}{dt} \frac{y_t^M}{y_t} = \left( 1 - \frac{y_t^M}{y_t} \right) \left( \dot{Z}_t^S \frac{1}{1 - h_t^M} - \dot{y}_t^M \frac{h_t^M}{1 - h_t^M} \right) + \frac{y_t^M}{y_t} \dot{y}_t^M.$$

where

$$\frac{y_t^M}{y_t} = \frac{1}{1-\alpha} \frac{Z_t^S h_t^M}{Z_t^S + \frac{\alpha}{1-\alpha} Z_t^S h_t^M} = \frac{h_t^M}{1 - \alpha + \alpha h_t^M}. \quad$$

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It follows that

\[
\dot{y}_t = \frac{1 - y_t^M/y_t}{1 - h_t^M} \dot{Z}^S + \left( \frac{y_t^M}{y_t} - \frac{h_t^M}{1 - h_t^M} \left( 1 - \frac{y_t^M}{y_t} \right) \right) \dot{y}_M,
\]

\[
= \frac{1 - \alpha}{1 - \alpha + \alpha h_t^M} \dot{Z}^S + \frac{\alpha h_t^M}{1 - \alpha + \alpha h_t^M} \left( \dot{Z}^S + \frac{1 - \alpha}{\alpha} \dot{Z}^M - \frac{1}{\alpha} \dot{Z}^S - \dot{P} \right),
\]

\[
= \frac{(1 - \alpha)(1 - h_t^M)}{1 - \alpha + \alpha h_t^M} \dot{Z}^S + \frac{(1 - \alpha)h_t^M}{1 - \alpha + \alpha h_t^M} \dot{Z}^M - \frac{\alpha h_t^M}{1 - \alpha + \alpha h_t^M} \dot{P}.
\]