Growth-at-Risk is Investment-at-Risk

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Growth-at-Risk is Investment-at-Risk*

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Abstract

We investigate the role financial conditions play in the composition of U.S. growth-at-risk. We document that, by a wide margin, growth-at-risk is investment-at-risk. That is, if financial conditions indicate U.S. real GDP growth will be in the lower tail of its conditional distribution, we know that the main contributor is a decline in investment. Consumption contributes under extreme financial stress. Government spending and net exports do not play a role.

JEL Nos.: C12, C32, C38, C52
Keywords: growth-at-risk, real-time data, quantiles, expected shortfall

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1 Introduction

There is now a substantial empirical literature on downside risks to real GDP growth – often described as growth-at-risk. Much of the work focuses on the role of financial conditions. For example, Adrian et al. (2019; ABG) use quantile regressions to examine the link between the Chicago Fed’s National Financial Conditions Index (NFCI) and downside risks to U.S. real GDP growth. Adrian et al. (2022) provide comparable evidence for other advanced economies. Building on this work, alternative sources of tail risk have also been considered, including climate change (Kiley, 2021), natural disasters (Bayoumi et al., 2021), and uncertainty (Hengge, 2019). In addition, alternative methods have been developed for modeling tail risk to GDP, including Bayesian additive regression trees (Clark et al., 2021), GARCH-based location-scale models (Brownlees and Souza, 2020), and generalized autoregressive score models (Delle Monache et al., 2021).

One of the main empirical results in ABG is that tighter current financial conditions ($z$) is associated with greater downside risk to U.S. real GDP growth ($y$). One way they do so is by analyzing its expected shortfall. Succinctly, for some level of tail risk $\alpha$ (e.g., 5%), forecast origin $t$, forecast horizon $\tau$, and conditional quantile for real GDP growth $Q_\alpha(y_{t+\tau}|y_t, z_t)$, expected shortfall is $E(y_{t+\tau}|y_{t+\tau} < Q_\alpha(y_{t+\tau}|y_t, z_t))$. An example of this is provided in Figure 1, which plots the NFCI and expected shortfall at the 1- and 4-quarter horizons across our sample.\(^1\) Significant declines in the expected shortfall are almost always associated with large increases in the NFCI and thus drastically tightening financial conditions. In contrast, NFCI values below or near zero do not appear to have any clear effect on growth-at-risk at either horizon. These findings broadly support much of the literature on macroeconomic models with financial frictions, which highlight the nonlinearities in the response of output to shocks of varying magnitude and sign. Notably, through a model where experts face a risk-sharing constraint, Brunnermeier and Sannikov (2014) evaluate the effect of macro shocks during normal times (i.e., near the steady state) and find that only large negative shocks matter. More recently, Forni et al. (2022) utilize a vector moving average approach to reach the same conclusions with a specific focus on financial shocks.

\(^1\)Note that Figure 1 provides in-sample estimates of expected shortfall. Instead, for the remainder of the paper we focus on results that emphasize the real-time importance of financial conditions for real GDP growth. Section 3 provides methodological details.
In this paper, we revisit the original work of ABG, but from a different perspective. We investigate the role financial conditions have on the composition of U.S. growth-at-risk. That is, if current financial conditions indicate that U.S. real GDP growth is going to be in the lower tail of its conditional distribution next quarter, what does this imply for growth of consumption, investment, government spending, and net exports next quarter? In this paper, we provide a collection of stylized facts that give a more nuanced view of the economy than when we say (aggregate) growth is at risk. While we do not do so here, we have in mind that these facts may prove useful for calibrating or validating macroeconomic models of household and firm behavior in the presence of financial frictions (e.g., Brunnermeier and Sannikov, 2014) or even disaster risk (e.g., Gourio, 2012).

Our approach to answering this question is related to Acharya et al.’s (2017) marginal expected shortfall but is closer to Banulescu and Dumitrescu’s (2015) component expected shortfall. Let \( w \in \{c, i, g, nx\} \) denote the growth rate of consumption, investment, government spending, and net exports. Using copula-based methods to link the conditional distributions of \( y_{t+\tau} \) and \( w_{t+\tau} \), we form estimates of \( E(w_{t+\tau} \mid y_{t+\tau} < Q_\alpha(y_{t+\tau} \mid y_t, z_t)) \), which we refer to as an approximate marginal expected shortfall (aMES). Each of these is then scaled by the contribution of the level of the component to the level of GDP to form what we refer to as an approximate component expected shortfall (aCES). Methodological details are provided in Section 3.

Figure 2 provides our main result. At the 1- and 4-quarter horizons we plot real-time estimates of each aCES. By a wide margin, growth-at-risk is driven by investment, as its aCES is almost always the only notable negative contributor. Consumption-at-risk only contributes to downside risks in cases of the most extreme financial distress, as was the case at the height of the Great Recession. Unsurprisingly for U.S. data, contributions from government spending and net exports are negligible.

Inspired by Chari et al.’s (2007) treatment of durables consumption as a form of invest-

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2 In many ways, our perspective is analogous to that of Adrian and Brunnermeier (2016), who instead measure contributions to systemic risk of individual banks as a function of their various characteristics (e.g., leverage and size). A key difference is that they evaluate what one component predicts about the aggregate, whereas we do the inverse.

3 It is approximate because even after weighting by their contribution to the level of GDP, the scaled sum of the growth rates is not equal to the growth rate of the sum of the components. The accuracy of the approximation is discussed later in section 3.
ment in their “Business Cycle Accounting,” we also conduct a comparable exercise using the subcomponents of consumption and investment - the details of which we provide later. As expected, durables account for the bulk of consumption’s limited negative contribution to growth-at-risk. Non-durables contribute a little, while services have no impact. In contrast, both residential and non-residential investment always contribute negatively.

The remainder of the paper proceeds as follows. Section 2 describes the data and evaluates the usefulness of the NFCI for forecasting tails risks associated with the growth of each component of GDP. Section 3 delineates our methodology for forming the composition of GDP growth-at-risk and presents our main empirical results as well as the detailed results on investment, consumption, and net exports. Section 4 concludes.

2 Financial Conditions and the Components of Growth-at-Risk

Our decomposition of growth-at-risk builds on quantile forecasts. As in much of the literature, these forecasts are conditional on one lag of GDP growth and one lag of the quarterly average of the NFCI. The usefulness of financial conditions indices for predicting downside risk to growth in GDP is well established (e.g., Adrian, et al., 2022) despite being the subject of some criticism (e.g., Reichlin, Ricco, and Hasenzagl, 2020). Even so, their usefulness for forecasting the individual components of GDP growth has not been established, and hence we begin our analysis by doing so in the following.

2.1 Quantile Regression Framework

Our evidence is based on fully real-time out-of-sample exercises. These entail sequences of $\tau$-step-ahead quantile forecasts formed sequentially across quarterly forecast origins $t = R, ..., T - \tau$. In each instance they are formed using current vintage data $\{w_s(t), x'_s(t)\}_{s=1}^t$. These data consist of a scalar predictand $w_s(t)$ and vector of predictors $x_s(t)$ associated with observations $s = 1, ..., t$. As an example, when $\tau = 1$, ABG set $w_s(t)$ to current quarter GDP growth (i.e., $y_s(t)$) and set $x_s(t)$ to $(1, y_{s-1}(t), z_{s-1}(t))'$ where $z$ denotes the NFCI. Later we will see other cases in which the quantile regressions treat $w_s(t)$ as current quarter consumption growth (or some other component) while maintaining the same definition of $x_s(t)$ so that lagged GDP growth remains a predictor. We also consider cases in which
τ = 4, and in these cases $w_s(t)$ is defined as average quarterly growth over four consecutive quarters.

For a chosen $\alpha \in (0, 1)$, the $\tau$-step-ahead conditional $\alpha$—quantile forecasts are based on linear models $x'_s(t)\beta^{(\alpha)}$. The parameter vector $\beta^{(\alpha)}$ is re-estimated as we move across forecast origins by minimizing the relevant tick loss function and takes the form

$$\hat{\beta}_t^{(\alpha)} = \arg \min_\beta \sum_{s=1}^{t-\tau} (\alpha - 1(w_{s+\tau}(t) \leq x'_s(t)\beta))(w_{s+\tau}(t) - x'_s(t)\beta).$$

The quantile forecasts $x'_t(t)\hat{\beta}_t^{(\alpha)}$ are evaluated against the future realization $w_{t+\tau}(t')$ for some vintage $t'$ such that $t' \geq t+\tau$. This yields a sequence of $P$ forecast errors, denoted $\hat{u}_t^{(\alpha)}(t') = w_{t+\tau}(t') - x'_t(t)\hat{\beta}_t^{(\alpha)}$ and subsequent loss $L(\hat{u}_t^{(\alpha)}(t')) = (\alpha - 1(\hat{u}_t^{(\alpha)}(t') \leq 0))\hat{u}_t^{(\alpha)}(t')$.

2.2 Data

We obtain GNP and GDP (including components) data from the Federal Reserve Bank of Philadelphia’s real-time data set (RTDS). We obtain NFCI data from the FRED and ALFRED databases. For NFCI vintages before May 25, 2011, we use the unofficial NFCI vintages constructed by Amburgey and McCracken (2023). The data date back to 1973Q1, and we obtain vintages from January 1988 through April 2022. Throughout, we generalize by using the term GDP despite using GNP in vintages before December 1991.

Forecast origins take place at the start of the second month of a given quarter, and we evaluate forecasts for the horizons $\tau = 1, 4$. All forecasts are constructed using a recursive estimation scheme with an initial estimation sample of fifteen years. As a result, forecast origins do not necessarily start at the first available vintage, due to an insufficient number of observations for the direct multi-step estimation. Put together, when $\tau = 1(4)$ the first forecast origins are May 1, 1988 (February 1, 1989), with target dates 1988Q2 (1989Q4). This procedure continues up to 2019Q4. In the main text, we abstract from evaluating any forecasts with horizons after 2019Q4 to avoid the atypical macroeconomic data releases during the COVID-19 pandemic. For the NFCI, we use non-calendar 3-month averages—that is, averages that use the most recent 3 months of observations at a given origin. To evaluate tick loss, we use the initial release of each series.

4The only exception is aggregate Private Domestic Investment for which we obtain data from the ALFRED database. For the other series we opt for the RTDS due to its convenience and deeper availability in the case of exports and imports.

5See Online Appendix A for an analysis of the COVID-19 period.
2.3 Evaluating Forecast Accuracy

As noted in section 2.1, we are interested in evaluating the usefulness of the NFCI for forecasting quantiles of each GDP component. To do so, we compare a NFCI-containing model to a baseline model with the NFCI omitted. Formally, the unrestricted model takes the form \( \hat{w}_{t,\tau}^{(\alpha)} = x_t(t)\beta_t^{(\alpha)} \) for \( x_t(t) = (1, y_t(t), z_t(t))' \), where \( \hat{w}_{t,\tau}^{(\alpha)} \) is the \( \tau \)-quarter-ahead component forecast. The restricted model omits \( z_t(t) \) as a predictor. To evaluate whether the NFCI is a useful predictor, we use the OOS-t test for nested quantile forecasting models developed in Amburgey and McCracken (2023), which takes the form

\[
OOS - t = \frac{P^{-1/2} \sum_{t=1}^{T-1} L(\hat{u}_{1,t+\tau}^{(\alpha)}) - L(\hat{u}_{2,t+\tau}^{(\alpha)})}{\sqrt{\Omega}},
\]

where \( L(\hat{u}_{1,t+\tau}^{(\alpha)}) \) and \( L(\hat{u}_{2,t+\tau}^{(\alpha)}) \) represent the tick loss of the restricted and unrestricted models for the forecast target date, \( t + \tau \), and the conditional quantile, \( \alpha \).

Table 1 displays the results from this exercise. The first column gives the predicted series, and the second row gives the \( \alpha \)-quantile, with \( \alpha \in (0.05, 0.1) \) representing the lower tail and \( \alpha \in (0.9, 0.95) \) representing the upper tail. For each exercise we display the percentage change in average tick loss associated with including lagged NFCI as a predictor (e.g., a value of -0.457 means the model including the NFCI had 45.7% lower tick loss on average compared to the baseline model). In parenthesis, we report the p-values associated with a two-sided test of the null using the OOS-t test delineated above. For ease of viewing, we split results for the 1-quarter- and 4-quarter-ahead forecasts into two panels.

The results for GDP largely mirror the existing literature. In the lower tail, the model that includes lagged NFCI performs significantly better than the baseline model. Conversely, in the upper tail the baseline model performs better, albeit not at a significant level. The consumption results mimic those of GDP. Namely, the NFCI model performs better in the lower tail and worse in the upper tail. Perhaps more noteworthy are the investment results. Here we observe a good example of how financial conditions have distinct effects on the components. In contrast to GDP and consumption, the NFCI model performs better at forecasting investment growth across all quantiles (upper and lower) at the shortest

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6The test is an adaptation of West (1996) that permits non-differentiability of the tick loss function as well as the presence of data revisions. See Amburgey and McCracken (2023) for details on estimation of the asymptotic variance \( \Omega \).
horizon and significantly so for all but the largest quantile. In addition, at the longer horizon, the NFCI model does better in the lower tail as well as the median, while doing worse in the upper tail. The government spending results are a mixed bag, with the NFCI model performing worse (better) than the baseline model for $\tau = 1(4)$. Finally, in all cases except for one, the net export forecasts are improved by the NFCI, usually significantly so. Amalgamating the results in Table 1, it is generally clear that the inclusion of NFCI is helpful for forecasting growth-at-risk, as well as the components of growth-at-risk – absent government spending. It’s also clear that the degree of predictive content for the components differs from that for GDP – especially for investment, government spending, and net exports.

3 Decomposing Growth-at-Risk

In the previous section we established that the NFCI is a useful predictor of downside risk to growth in consumption, investment, and net exports. In this section we build on these observations and decompose growth-at-risk using an approximate component-wise expected shortfall. This requires a method for linking the downside risks between real GDP growth and each of its components, and here we opt for a parametric copula to do so. Other applications of copula-based methods in the growth-at-risk literature include Coe and Vahey (2020), who study the relationship between financial conditions and the distribution of future U.S. real GDP growth across a long historical sample dating back to 1875, and Chernis et al. (2023), who revisit the results in ABG using a copula-based quantile regression framework designed to limit the influence of outliers like those associated with the COVID-19 period.

3.1 Our Methodology

We begin by using real-time conditional quantile forecasts to construct marginal conditional distributions of GDP growth as well as growth of the components. At each forecast origin and for each $\alpha = 0.05, 0.1, \ldots, 0.95$, we use one lag of both the NFCI and GDP growth as predictors when forming these quantile forecasts. Within a comparable framework, ABG use these forecasts to inform the parameters of an assumed skewed-t marginal conditional distribution. Instead, we use a semi-parametric method developed in Mitchell, Poon, and Zhu (2022, MPZ), which builds on Koenker and Zhao (1996). By doing so we impose fewer
restrictions on the shapes of the marginal conditional distributions.\textsuperscript{7}

In all cases we follow Algorithm 1 of MPZ. At each forecast origin, their approach treats the forecasted quantiles as the actual quantiles of the marginal distribution. Values between these quantiles are simulated using 1000 uniform draws. Beyond the lowest (highest) quantile, the 0.05 and 0.10 (0.90 and 0.95) quantile forecasts are used to estimate the parameters of a normal distribution, and then 1000 values are drawn from the implied tail of this normal distribution. This collection of quantiles and simulated values are then used to form a kernel-based estimate of the marginal distribution.\textsuperscript{8} From here we can easily estimate $E(y_{t+\tau}|y_{t+\tau} < Q_{0.05}(y_{t+\tau}|y_t, z_t))$, i.e., the expected shortfall displayed in Figure 1.\textsuperscript{9}

Given the sequence of marginal distributions, we now construct a sequence of bivariate joint conditional distributions between GDP growth and growth in each of the individual components. To do so, we link the corresponding marginal distributions using a Student’s-t copula, rather than a Gaussian copula, as it permits greater flexibility of the bivariate tail risks. The hyperparameters associated with this copula are estimated using maximum likelihood over the full sample.\textsuperscript{10}

The need for joint distributions follows immediately from the aMES, $E(w_{t+\tau}|y_{t+\tau} < Q_{0.05}(y_{t+\tau}|x_t))$. To estimate this object we need the ability to generate draws from the joint distribution $F(y_{t+\tau}, w_{t+\tau}|x_t)$ for which $y_{t+\tau} < Q_{0.05}(y_{t+\tau}|x_t)$. But having constructed the joint distributions, drawing from them is straightforward and the average of these draws forms our estimate.

Figure 3 plots the 1- and 4-quarter-ahead aMES forecasts for each component. By a large margin, growth in investment exhibits the greatest and most consistent risk. Across both horizons, its aMES is nearly always negative, and when financial conditions are tight, it experiences substantial declines. For the 1-quarter-ahead forecasts, leading into the 1990-91, 2001, and 2008 recessions, investment growth is predicted to decline roughly $-40\%$, $-25\%$, and $-80\%$ (!), respectively. At the 4-quarter horizon the predicted declines are lower but are still substantial at $-20\%$, $-15\%$, and $-35\%$, respectively. In contrast, the forecasts for

\textsuperscript{7}In unreported results we follow the methodology of ABG and obtain similar findings.

\textsuperscript{8}We use a normal kernel with the theoretically optimal bandwidth from Bowman and Azzalini (1997).

\textsuperscript{9}In Figure 1, rather than the out-of-sample forecasts, we plot the in-sample estimates obtained at the last horizon, i.e., 2019Q4. The in-sample estimates are obtained by simply following the same set of steps outlined here, but using the fitted quantiles rather than the out-of-sample quantile forecasts.

\textsuperscript{10}There are two hyperparameters: the degrees of freedom and the Pearson correlation coefficient.
consumption are nearly always positive. The exception occurs when financial conditions are at their tightest leading into the Great Recession period. Here we observe a substantial decline of roughly $-7\%$ at both horizons.

The third row of panels indicates that, unsurprisingly, government spending is largely unaffected by financial conditions. At both horizons there appears to be a clear, but insignificant, downward trend in the expected shortfall since the 1990s. Finally, the last row of panels shows that the downside risk to net exports tends to move in the positive direction during economic downturns — or rather the risk to imports is greater than the risk to exports. Outside of recessions the expected shortfall of net exports remains near 0, however.

### 3.2 The Composition of GDP Growth-at-Risk

Now that we have the individual aMES components, we can finalize our decomposition of growth-at-risk, adjusting for the fact that these components contribute to GDP by very different magnitudes. On average over our sample, consumption is roughly 70% of GDP while the remaining pieces are roughly 15% (noting that net exports is negative). To account for these features we rescale the results in Figure 3 relative to each component’s total proportion of GDP. We denote a component’s rescaled aMES as its aCES. Formally, a component’s aCES is given by

$$\frac{W_t}{Y_t} E(w_{t+\tau}|y_{t+\tau} < Q_\alpha(y_{t+\tau}|x_t)),$$

where $Y_t$ and $W_t$ represent the levels of GDP and one of its components, respectively. With this adjustment, we return to our main result in Figure 2. First consider the sum of all four components’ aCES (i.e., the total area of the plot) as an approximation of the expected shortfall. The patterns displayed by this approximation largely mimic those in Figure 1. Importantly, periods associated with tight financial conditions tend to coincide with markedly negative expected shortfalls.

As we already noted, despite making up only approximately 15% of GDP, investment is the sole consistent contributor to the downside risk to growth, as it is the only element that is consistently negative. Specifically, at the 1- and 4-quarter horizons, investment makes up an astounding 92.5% and 90% of total negative contributions in our sample, respectively. On the other hand, consumption contributes only in the presence of extreme financial distress,
as we saw during the Great Recession. Across our sample, consumption makes up a mere
6% and 10% of total negative contributions at the 1- and 4-quarter horizons, respectively,
with the majority of these contributions coming from the Great Recession period at both
horizons. Downside risks from government spending and net exports are negligible.

While we find our decomposition instructive, as we’ve noted earlier, it is still an ap-
proximation. Nevertheless, on average across all forecast origins in Figure 2, and for both
horizons, the average deviation between the weighted sum of the aCES values and the actual
ES for GDP growth is roughly 10 bps, albeit with a standard deviation of 1%.

3.3 A Detailed Composition of GDP Growth-at-Risk

In our main analysis we made a choice about the granularity of GDP’s components. Here
we revisit this choice with an eye toward the robustness of our findings and any insights
 gained from a more thorough breakdown of GDP. For example, while growth-at-risk from
consumption and investment make up the bulk of our findings, it may be that these cat-
egories remain too broad to understand their underlying contributions to growth-at-risk.
As an example, Mankiw (1985) emphasizes that consumption of durables is more sensi-
tive to interest rates than consumption of nondurables or services. In addition, Leamer
(2007) argues that residential investment plays a dominant role in GDP weakness with
non-residential investment playing a lesser role.

In that spirit, our more detailed analysis of the composition of growth-at-risk applies the
same modeling choices outlined in Sections 3.1 and 3.2, but with a different set of series, w.
Now we analyze how consumption of nondurable goods (CNDUR), consumption of durable
goods (CDUR), consumption of services (CSER), nonresidential investment (INRES), res-
idential investment (IRES), exports (X), and imports (M) contribute to growth-at-risk.11
12

Figure 4 plots the detailed composition of growth-at-risk at the 1-quarter- and 4-quarter
ahead horizons. The top panel focuses on consumption, which typically did not contribute
negatively to growth-at-risk in Figure 2. Even so, our detailed decomposition reveals that
consumer durables is a regular downside risk, as it displays significant negative contributions

11 For imports we multiply equation (2) by -1 since imports contribute negatively to the level of GDP.
12 We choose to omit change in inventories since it makes up less than 1% of investment. We also omit
government spending due to its insignificance in our initial decomposition.
that tend to arise leading into, and during, recessions. This aligns with our expectation given Chari et al.’s (2007) treatment of durables consumption as a form of investment. Nondurables provide substantial contributions to growth-at-risk only during the height of the financial crisis, reinforcing Houthakker and Taylor’s (1970) assertion that certain non-durables can behave as durables in the right economic environment. Across the sample, the contribution from services consumption remains positive and is generally invariant to financial conditions, with the exception that during the height of the Great Recession it dips sharply to zero.

The middle panel gives us a more detailed understanding of investment-at-risk. In contrast to the components of consumption, both nonresidential and residential investment contribute to growth-at-risk as they remain below zero for most of the sample. The balanced contributions to growth-at-risk contrast with Leamer’s (2007) assessment above. It’s important to note however, that Leamer (2007) is emphasizing the timing of contributions to GDP relative to business cycle peaks, while we’re considering risks to GDP at any stage of the business cycle.

Finally, the bottom panel indicates that exports and imports contribute to growth-at-risk more than Figure 2 reveals. During expansions both exports and imports contribute little to growth-at-risk, but during recessions they deliver significant negative and positive contributions, respectively. Throughout the entire sample there is a clear trend of changes to exports and imports canceling each other out. In other words, trade tends to decrease in both directions during tail events rather than only exports or only imports.

4 Conclusion

In this paper we address a previously unexplored aspect of growth-at-risk. Using the concept of component-wise expected shortfall, we provide evidence on its composition. By a wide margin we find that U.S growth-at-risk is dominated by investment-at-risk, be it residential or nonresidential. Consumption, and especially consumption of durables, contributes to growth-at-risk but only in the most extreme cases of financial stress. Almost no contribution comes from net exports or government spending.

It is worth noting that while we expect investment to be a primary contributor to GDP
growth-at-risk, the relative importance of each component may vary significantly across countries. For example, in contrast to exports’ moderate role in the U.S. economy, they make up approximately half of Germany’s GDP. Thus, in Germany’s case, one might expect exports-at-risk to make up a more substantial proportion of growth-at-risk. While beyond the scope of this paper, investigating the composition of growth-at-risk in other countries seems a sensible avenue for future research.
References


Delle Monache, Davide, De Polis, Andrea, and Ivan Petrella (2021), “Modeling and Fore-


### Table 1: CM Tests of Equal Tick Loss

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<td>(0.00)</td>
<td>(0.43)</td>
<td>0.18</td>
<td>0.00</td>
<td>0.09</td>
<td>(0.00)</td>
<td>(0.16)</td>
<td>0.14</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: This table gives the percentage change in the average tick loss when lagged NFCI is added to a baseline model that only includes an intercept and lagged real GDP growth as a predictor (e.g., -0.457 denotes roughly a 45% lower average tick loss in the unrestricted model than in the restricted model). $P$-values associated with two-sided OOS-t tests are provided in parentheses. Asterisks indicate statistical significance at the 5% level. Results are reported for the horizons $\tau = 1$ and $\tau = 4$, and the quantiles $\alpha \in \{0.05, 0.1, 0.5, 0.9, 0.95\}$. All models are estimated with a recursive scheme.
Figure 1: NFCI and Expected Shortfall

Notes: This figure plots the 1-quarter- and 4-quarter-ahead NFCI and in-sample expected shortfall of GDP.
Figure 2: Real-time composition of growth-at-risk

Notes: This figure plots the 1-quarter- and 4-quarter-ahead aCES of each GDP component. Forecasts are out-of-sample and use a recursive estimation scheme.
Figure 3: Marginal Component Expected Shortfall

Notes: This Figure plots the 1-quarter- and 4-quarter-ahead aMES of each GDP component. Forecasts are out-of-sample and use a recursive estimation scheme.
Notes: This figure plots the 1-quarter- and 4-quarter-ahead aCES of each GDP subcomponent. The top, middle, and bottom panels plot the detailed aCES of consumption, investment, and net exports, respectively. Each component is broken down into its subcomponents. For brevity, we exclude government spending. Forecasts are out-of-sample and use a recursive estimation scheme.
Online Appendix

A: COVID-19

In the main analysis we omitted forecasts with horizons after 2019Q4. As noted in Amburgey and McCracken (2023), the enormous swings in GDP growth led to wildly different parameter estimates from quantile regressions when using that period for estimation. These in turn led to highly inaccurate tail forecasts that were not in-line with the accuracy seen in earlier periods. Nonetheless, COVID-19 caused an unprecedented shock to the economy, and there is value to identifying how the components-at-risk evolved throughout that period.

To do so, we apply the methodology described in section 3.1 when constructing the conditional marginal and joint distributions – with one modification. Rather than re-estimate the quantile regression parameters recursively across the COVID-19 period, for each forecast origin after 2019Q4, we fix the parameter estimates at that value (e.g., Lenza and Primiceri (2022). By doing so we maintain most of the “real-time” nature of our exercises (i.e., by using vintage data), albeit with a look-ahead bias due to how we manage the parameter estimates.

With that in mind, Figure A1 provides the composition of growth-at-risk from 2020Q1 through 2022Q2. Aside from the forecasts associated with origins 2020Q3 and 2020Q4, the composition remains by and large the same as that in Figure 2. Investment-at-risk is omnipresent with consumption-at-risk remaining positive throughout. On the other hand, the forecasts associated with origins 2020Q3 and 2020Q4 (i.e, 2020Q3 and 2020Q4 for 1-quarter ahead, and 2021Q2 and 2021Q3 for 4-quarter ahead) display wild swings due to the unstable nature of GDP growth during these periods. Specifically, in 2020Q2 and 2020Q3, GDP growth was -35% and 30%, respectively.

Focusing on the 1-quarter horizon, the approximate expected shortfall (i.e., the total area of the aCES) drops to a record low of -16% in 2020Q3 before jumping to a record high of 6% in 2020Q4. Perhaps most notably, for the first and only time across our sample,

\footnote{Lenza and Primiceri (2022) and Chernis et al. (2022) note comparable issues in the context of VARs and quantile regressions, respectively.}
investment-at-risk contributes positively to downside-risk in 2020Q4. Patterns in the 4-quarter horizon forecasts mirror those outlined above.

Figure A2 provides the detailed composition of growth-at-risk over the same period. At the peak of the recession, for the first time in our sample, services meaningfully contribute negative values to growth-at-risk. This reflects the large drop in service-related activity caused by government-mandated closures of businesses in the industry. Consumer durables and nondurables display similar patterns to that of the Great Recession, albeit with nondurables making up a more substantial portion of downside risk at the height of the downturn. The middle panel of Figure A2 tells us that non-residential investment was the driver of investment’s swing from negative to positive contributions during this period. For example, at the 1-quarter horizon, nonresidential investment’s aCES swung from -9% to 4.5% between 2020Q3 and 2020Q4. Even more so than our main results, this stands in stark contrast to Leamer (2007), who argues that residential investment plays a more prominent role in recessions.

References


Figure A1: Real-time composition of growth-at-risk, COVID-19

Notes: See Figure 2 for details.
Figure A2: Real-time composition of growth-at-risk: detailed, COVID-19

Notes: See Figure 4 for details.