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Optimal Dynamic Tax-Transfer Policies in Heterogeneous-Agents Economies*

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Abstract

When designing an optimal tax-transfer system, the existing literature identifies two key factors: labor efficiency and debt efficiency. The labor-efficiency approach emphasizes the trade-off between redistribution and distortion in the labor market, while the debt-efficiency approach emphasizes the trade-off between monopoly rent gains and distortion in the asset market. In this paper, we propose a third factor for designing optimal tax-transfer policies: the dynamic-efficiency approach. To demonstrate this, we use an analytically tractable infinite-horizon model incorporating both ex-ante and ex-post heterogeneity. We show that the optimal tax-transfer system is determined at the point where the intertemporal wedge between the market interest rate and the time discount rate is completely eliminated, regardless of the Laffer curve, provided that the government's fiscal space permits an interior Ramsey steady state. Therefore, in line with recent findings by Chien and Wen (2024), incorporating dynamic efficiency could potentially transform the structure of optimal tax-transfer policies within an infinite-horizon model featuring heterogeneous agents.

JEL Classification: E13; E62; H21; H30

Key Words: Ramsey Redistribution, Optimal Tax-Transfer System, Optimal Interest Rate, Laffer Curve, Incomplete Markets, Heterogeneity.

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*The views expressed are those of the individual authors and do not necessarily reflect the official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors. Yili Chien: Federal Reserve Bank of St. Louis; yilichien@gmail.com. Yi Wen: Antai College of Economics and Management, Shanghai Jiaotong University; wenyi001@sjtu.edu.cn.

1 Introduction

When facing an enormous degree of income inequality, how should optimal transfer- or redistribution policies be designed? More specifically, how should a benevolent government choose the level of debt and tax rate to finance transfer programs—such as the Universal Basic Income (UBI)? This is one of the fundamental questions facing macro-economists today.

To sharpen our analysis, imagine a situation where poor agents in society have no labor income or access to saving devices (such as government bonds), while rich agents are able to save through financial instruments and earn stochastic labor income. In such a stark environment, how should a benevolent government with a monopoly power in the debt market choose the interest rate and (possibly progressive) labor taxes to finance lump-sum transfer programs such as UBI?[1]

The existing literature identifies two key factors to address the question. The first relies on the "labor-efficiency" argument based on the adverse effect of a labor tax on labor supply, while the second relies on the "debt-efficiency" argument based on the government's monopoly rents from issuing debt. More specifically, since a labor tax reduces the incentive to work, a benevolent government caring for the income-poor (jobless) agents may have to limit the amount of tax revenues and transfers. This efficiency consideration implies that the planner never set a tax-transfer system that exceeds the peak of the Laffer curve. On the other hand, since reducing the interest rate decreases the burden of debt payment, a benevolent government in the absence of competing private borrowers—the poor cannot borrow—may never issue too much debt to meet rich agents' precautionary demand for financial instruments; instead the government may choose to exercise its monopoly power in the asset market to extract monopoly rents by keeping the interest rate sufficiently low and hence reduce the cost of servicing the debt.

Although the labor-efficiency approach is built originally on static models (for example, Mirrlees (1971)), this argument also remains popular in dynamic models with heterogenous agents and incomplete markets. For example, the works of Heathcote, Storesletten, and Violante (2017), Heathcote and Tsujiyama (2021), Dyrda and Pedroni (2022), and Ferriere, Grubener, Navarro, and Vardishvili (2021) study optimal transfers financed by linear or progressive income taxes in dynamic models with both ex-ante and ex-post heterogeneous agents. Within such a dynamic framework, these authors argue that the main mechanism of designing an optimal tax-transfer

[1]Given that only rich agents pay labor taxes in this environment, the tax system is essentially "progressive" in nature.

policy remains similar to that in static models where balancing the trade-off between redistribution and labor-efficiency is the key consideration of the benevolent government.

On the other hand, in a dynamic model with public debt, the interest payment of debt also imposes a limit on the amount of transfers available to the poor from the government. Therefore, manipulating the interest rate and the debt level could also play an important role in designing the optimal tax-transfer system. In light of this, the works of Yared (2013), Azzimonti and Yared (2017), Bhandari, Evans, Golosov, and Sargent (2017), and Azzimonti and Yared (2019) study optimal redistribution policies financed by government debt and income taxes. These authors show that in order to reduce income inequality through transfers it may be optimal to repress the interest rate on government bonds in order to extract monopoly rents from the wealthy savers.

In this paper, we propose a third factor for consideration in designing optimal tax-transfer policies: the dynamic-efficiency argument. To demonstrate this, we use an analytically tractable infinite-horizon model incorporating both ex-ante and ex-post heterogeneity. We show that the optimal tax-transfer policy entails issuing a sufficient amount of public debt to completely relax the borrowing constraints of rich agents, even if this could imply a skyrocketing amount of public debt in the long run such that the optimal steady-state labor tax approaches 100% (instead of approaching the top of the Laffer curve). That is, incorporating dynamic efficiency could potentially transform the structure of optimal tax-transfer policies within an infinite-horizon model featuring heterogeneous agents.

The reason is as follows. In a typical infinite-horizon Aiyagari-type model, there exists an intertemporal wedge between the market interest rate r and the time discount rate $1/\beta$ such that $r < 1/\beta$ (due to the existence of a liquidity premium in the bond market). Given this intertemporal wedge/distortion, the government has a dominant incentive to frontload consumption at least for the poor by increasing debt supply—i.e., the government opts to borrow aggressively from the future to finance current transfers when the future is discounted more heavily by the time-discounting factor β than by the market discount rate (i.e., the inverse of the interest rate $1/r$). This "arbitrage"opportunity implied by the intertemporal wedge/distortion does not vanish unless the bond supply becomes sufficiently high such that the wedge disappears completely (i.e., until $r = 1/\beta$).

Our result is surprising but also intuitive. In infinite-horizon models with commitment, erasing intertemporal distortion often leads to a first-order welfare gain. In order to get rid of the intertemporal wedge between r and $1/\beta$ (a form of distortion arising from borrowing constraints), the Ramsey planner seeks a tax-transfer policy of lining up the market interest rate and time discount rates if this is feasible within the government's fiscal space. Other concerns such as labor-efficiency and debt-efficiency bring only second-order welfare gains.

The sources of the sharp difference between our result and those obtained by the existing literature are as follows. First, the labor-efficiency argument is based mostly on intuitions from a static model or a finite-period model (including the OLG model) where the intertemporal wedge between the interest rate and the time discount rate—a hallmark feature of heterogeneous-agent models a la Aiyagari (1994)—plays no or a less-critical role in the Ramsey planner's problem. Moreover, a strand of literature such as Heathcote, Storesletten, and Violante (2017) considers only once-and-for-all policy changes, which rely on an unexpected one-time change of a time-invariant tax-transfer system. In such a once-and-for-all policy exercise, the Ramsey planner's ability to frontload consumption is exogenously limited because of the imposed time-invariant restriction on the tax-transfer system. Nonetheless, the key mechanism of our model—consumption frontloading—still operates even in such an environment with once-and-for-all policy changes. For example, in a typical Aiyagari-type model with both ex-ante and ex-post heterogeneity, Ferriere, Grubener, Navarro, and Vardishvili (2021) conduct numerical exercises of a once-and-for-all policy change with non-linear taxes and targeted transfers. Their numerical results of an optimal transition path and the long run steady state do exhibit features that reflect the Ramsey planner's desire of consumption frontloading, such as a declining capital stock and a rising market interest rate during the transition, and a large permanent loss in aggregate output. Hence, the novel finding of our paper—the mechanism behind the intertemporal wedge—provides an additional explanation for their results and takes one critical step forward to fully reveal the dynamic Ramsey plan with endogenously time-varying tax-transfer policies.

Second, for the same reason (i.e., the presence of a wedge between the interest rate and the time discount rate), the debt-efficiency argument based on the trade-off between the gains from monopoly rents and costs of intertemporal distortions is also of second-order importance in the design of a dynamic optimal tax-transfer system. Yet the works of Yared (2013), Azzimonti and Yared (2017), and Bhandari, Evans, Golosov, and Sargent (2017) rely either on a two-period model where the intertemporal wedge between r and $1/\beta$ does not play an important role in the Ramsey allocation, or on a constraint-set analysis not based on solving the Ramsey problem. For example, Bhandari, Evans, Golosov, and Sargent (2017) argue that keeping the interest rate on public debt sufficiently low may be welfare improving and state that "[a]n optimal government debt is determined by a trade-off between gains from exploiting monopoly rents and costs from distorting agents' intertemporal marginal rates of substitution." However, this argument in Bhandari, Evans, Golosov, and Sargent (2017) is not based on the solution to the Ramsey problem. In contrast, we explicitly characterize the Ramsey allocation in our infinite-horizon model—which is similar to their model—and show that the optimal debt level under a Ramsey plan is instead determined by the fiscal space of the government such that in the long run the government will forgo the monopoly rents by setting the interest rate equal to the time discount rate whenever feasible.

On the other hand, Azzimonti and Yared (2017) consider the implications for an optimal fiscal policy when taxes are non-distortionary and households are heterogeneous and borrowing constrained. Their main result is that an optimal redistribution policy always keeps some households borrowing constrained in order to reduce the interest rate on government debt. Their model differs from ours in that they use a two-period model. This means not only is there a terminal period in the private sector's saving decisions, but the critical intertemporal wedge in standard infinite-horizon Aiyagari-type models—the hallmark heterogeneous-agent incomplete-market economies—no longer plays a strategic role in the Ramsey problem.

Our result is in fact anticipated by the seminal work of Albanesi and Armenter (2012), which points out that frontloading consumption (or the intertemporal wedge) leads to a first-order welfare gain in a broad class of second-best economies. However, Albanesi and Armenter (2012) do not provide answers regarding what would happen if the intertemporal wedge never vanishes due to limited fiscal space—such as in the original Aiyagari (1994) model. Our paper not only proves that, if the government fiscal space permits an interior Ramsey steady state, the optimal tax rate and transfers are determined at the point where the interest rate equals the time discount rate; but also shows that if the fiscal space condition is violated the only possible Ramsey steady state is non-interior with aggregate consumption approaching zero and the labor tax rate approaching 100%.

In other words, in neither type of Ramsey steady state—interior or non-interior—do we find the labor-efficiency argument and the debt-efficiency argument critical in the design of an optimal tax-transfer system, precisely because these concerns are strictly dominated by the Ramsey planner's desire to eliminate the intertemporal wedge between the interest rate and the time discount rate.

That is, if the fiscal space permits an interior Ramsey steady state, then the Ramsey planner will issue plenty of debt to equalize the interest rate and the time discount rate in the long run; otherwise, the interior Ramsey steady state does not exist and the Ramsey allocation features zero consumption and a 100% tax rate (even if rich agents may receive substantial Pareto weight in the social welfare function).

We also show that if the Ramsey planner maximizes only the steady-state welfare instead of the entire dynamic path of social welfare, then the intertemporal wedge between the interest rate and the time discount rate no longer matters to the planner; consequently, both the labor-efficiency margin and the debt-efficiency margin become first-order important in determining an optimal tax-transfer system. On the other hand, in the case of no idiosyncratic risk (or no ex-post heterogeneity), since r must equal $1/\beta$, the labor-efficiency concern becomes the only determinant for the Ramsey planner in the design of an optimal tax-transfer system, consistent with the works of Heathcote, Storesletten, and Violante (2017), Heathcote and Tsujiyama (2021), Dyrda and Pedroni (2022), and Ferriere, Grubener, Navarro, and Vardishvili (2021).

An additional contribution of our paper is to provide an analytically tractable model in which the mechanism behind the Ramsey design of the dynamic tax-transfer system is fully transparent both along the transition and in the steady state, yet without the need to assume the existence of a Ramsey steady state—a typical assumption made by a large body of the existing literature (including the Aiyagari (1995) paper). We are able to analytically characterize the necessary and sufficient conditions for the existence of a Ramsey steady state and its properties. In this regard, this paper is also closely related to our previous work in Chien and Wen (2024). The analytical tractability of our model builds on that paper, and the dominant motive for the Ramsey planner to pursue full-self insurance by exploiting the intertemporal wedge between the interest rate and the time discount rate is shown in that paper. However, that paper does not have ex-ante heterogenous agents and does not study the issue of optimal lump-sum transfers—both issues increase the difficulty of model tractability. Hence, our previous work does not contain the new findings in this paper regarding the irrelevance of labor-efficiency and debt-efficiency arguments in the design of a dynamic Ramsey tax-transfer system when both ex-ante and ex-post heterogeneity are present.

The rest of the paper is organized as follows. Section 2 sets up a tractable, infinite-horizon, dynamic model with both ex-ante and ex-post heterogeneity and defines the competitive equilibrium for a given set of government policies. Section 3 solves and characterizes the Ramsey allocation. To help understand the Ramsey allocation and sharpen the intuition of our results, Section 4 considers several special scenarios where the force of the intertemporal wedge no longer matters for the design of the tax-transfer system. Section 5 concludes.

2 The Benchmark Model

2.1 Setup

2.1.1 Firms and Government

Firms. Time is discrete and indexed by $t = 0, 1, 2, ..., \infty$. A representative firm produces output using a linear production technology in labor, $Y_t = N_t$, where Y and N denote aggregate output and labor, respectively.[2] The firm hires labor by paying a competitive real wage, denoted by w_t. The firm's profit maximization in any time t leads to $w_t = 1$.

Government. In each period t, the government can issue bonds B_{t+1}, give lump-sum transfers T_t, and levy a time-varying labor tax τ_t. Let Q_{t+1} denote the price of risk-free bonds in period t, which pays one unit of consumption goods in period $t+1$; then the risk-free interest rate is given by $r_{t+1} \equiv Q_{t+1}^{-1}$. The flow government budget constraint in period t is

$$\tau_t w_t N_t + Q_{t+1} B_{t+1} \geq B_t + T_t, \tag{1}$$

where the initial level of government bonds B_0 is exogenously given. For simplicity without loss of generality, government expenditures are assumed to be zero.

2.1.2 Ex-ante and Ex-post Heterogeneous Agents

There is a unit measure of families exhibiting both ex-ante and ex-post heterogeneity. A constant fraction of the families can participate in both the labor market and the financial market, and the rest does not participate in either market. Therefore, there are two types of families: The first type never participates in market activities and is called "non-participants," while the second type participates in both the labor market and the financial market and is thus called "participants."

[2] Our model can be easily extended to include capital accumulation, but the main result would be similar.

This difference in the status of participation is the source of ex-ante heterogeneity.

On the other hand, the source of ex-post heterogeneity comes from the participant families—each family member is subject to an idiosyncratic employment-status shock in each period. The shock is denoted by $\theta_t \in \{e, u\}$ and follows a first-order Markov process. The shock is identically and independently distributed (iid) across all participants and follows a first-order Markov process. If $\theta_t = e$, then a participant from any family can work and receive labor income; otherwise, if $\theta_t = u$, a participant cannot work in the current period and has no labor income.

This difference between employment and non-employment status within the market participants due to shock θ_t gives rise to ex-post heterogeneity in our model. Let $\pi(e|e)$ and $\pi(e|u)$ denote the transition probability from employed and unemployed states, respectively, to an employed state. Similarly, $\pi(u|e)$ and $\pi(u|u)$ denote the transition probability from employed and unemployed states, respectively, to an unemployed state. In addition, $\pi(e)$ and $\pi(u)$ are the shares of employed or unemployed participants in total population, respectively. For simplicity, we assume that $\pi(e)$ and $\pi(u)$ also represent the initial period's share of employed and unemployed participants.

In addition, the share of non-participants in the total population is exogenously given by $\pi(o)$. Give that the total measure of population is normalized to 1, we have $\pi(o) = 1 - \pi(e) - \pi(u)$. The population share of employed and unemployed participants in each period then must satisfy $\pi(e) = \frac{\pi(e|u)}{\pi(e|u)+\pi(u|e)}(1 - \pi(o))$ and $\pi(u) = \frac{\pi(u|e)}{\pi(e|u)+\pi(u|e)}(1 - \pi(o))$, respectively.

Models with ex-post heterogenous agents are often intractable. To make the model analytically tractable, we introduce a partial risk-sharing technology to the participants, based on the family metaphor of Lucas (1990).[3] That is, each participant family has a unit measure of family members; and in each family there is a family head who maximizes the intertemporal welfare of all family members using a utilitarian welfare criterion (all family members are equally weighted) but faces some limits to the amount of risk sharing. Specifically, in each period after the realization of the idiosyncratic shock to a family member's employment status, the family head can reshuffle wealth among the family members who have the same current status of employment. However, the family head cannot reshuffle resources across family members with different employment status in period t. Hence, the family head is equipped with an incomplete wealth-pooling technology that can only partially hedge the idiosyncratic risk facing the participant-family members.

[3]This partial risk-sharing approach is similar to that in Chien and Wen (2024), Bilbiie (2019), and Bilbiie and Ragot (2021).

Given that there are only two employment statuses, consequently there will be two types of participants in every time period—an employed group and an unemployed group. Namely, in each period t the initial wealth of each participant family member takes only two possible values that depend on the current state of idiosyncratic shock but are independent of the individual's past history of employment status. As a result, the consumption and saving choices of individuals are identical within each group. This wealth-pooling arrangement simplifies the model dramatically and renders the model as well as the Ramsey problem analytically tractable.

Denote the level of consumption, labor supply, and asset holdings (savings) for the period-t employed participants as c_t^e, n_t^e, and a_{t+1}^e, respectively; and similarly let c_t^u and a_{t+1}^u denote consumption and savings for the period-t unemployed participants. Under the partial wealth-pooling technology the total assets available to the employed family members in the beginning of period t are given by $a_t^e \pi(e)\pi(e|e) + a_t^u \pi(u)\pi(e|u)$, and similarly the total initial assets available for the unemployed participants are $a_t^e \pi(e)\pi(u|e) + a_t^u \pi(u)\pi(u|u)$. Hence, for $t \geq 1$, the household budget constraints for the employed and unemployed family members are given, respectively, by

$$\frac{a_t^e \pi(e)\pi(e|e) + a_t^u \pi(u)\pi(e|u)}{\pi(e)} + T_t + \widehat{w}_t n_t^e - c_t^e - Q_{t+1} a_{t+1}^e \geq 0, \tag{2}$$

$$\frac{a_t^e \pi(e)\pi(u|e) + a_t^u \pi(u)\pi(u|u)}{\pi(u)} + T_t - c_t^u - Q_{t+1} a_{t+1}^u \geq 0, \tag{3}$$

where $\widehat{w}_t \equiv (1-\tau_t) w_t$ is the after-tax wage rate and T_t denotes universal lump-sum transfers (or taxes if $T_t < 0$). For the initial period ($t = 0$), the household budget constraints for the employed and unemployed individuals are given, respectively, by

$$a_0^e + \widehat{w}_0 n_0^e + T_0 - c_0^e - Q_1 a_1^e \geq 0, \tag{4}$$

$$a_0^u + T_0 - c_0^u - Q_1 a_1^u \geq 0, \tag{5}$$

where a_0^e and a_0^u are the exogenously-given period-0 initial asset holdings for the employed and unemployed family members, respectively. In addition, the participant family members are subject to borrowing constraints for all $t \geq 0$:

$$a_{t+1}^e \geq 0, \tag{6}$$

$$a_{t+1}^u \geq 0. \tag{7}$$

Since the labor supply of the unemployed members is zero, the utilitarian welfare criterion of the family head is given by

$$\max_{\{c_t^e, c_t^u, n_t^e, a_{t+1}^e, a_{t+1}^u\}} \sum_{t=0}^{\infty} \beta^t \left\{ \left[u(c_t^e) - v\left(n_t^e\right) \right] \pi(e) + u(c_t^u)\pi(u) \right\}, \tag{8}$$

where $\beta \in (0,1)$ is the time discounting factor. We assume that the utility function of the participants takes the standard form:

$$u(c) = \frac{1}{1-\sigma} c^{1-\sigma} \text{ and } v(n) = \frac{1}{1+\gamma} n^{1+\gamma},$$

where $\sigma > 0$ and $\gamma > 0$.

In contrast, the problem of the non-participants is simple and given by:

$$\max_{\{c_t^o\}} \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} (c_t^o)^{1-\sigma} \pi(o)$$

subject to their budget constraints for all $t \geq 0$:

$$T_t - c_t^o \geq 0. \tag{9}$$

Note that in such a set up with respect to the non-participants, a lump-sum tax is no longer feasible and cannot be optimal since a negative T must violate the budget constraint of the non-participants.

The main purpose of including the non-participants in the general setup is to highlight the importance of lump-sum transfers in the presence of ex-ante heterogeneity. For this purpose, we have taken the extreme assumption that the non-participants are unable to work or save and are excluded from any risk-sharing arrangement. As a result, the only way to improve non-participants' welfare is through lump-sum transfers financed by government revenues collected from the participants. This stark set up highlights the exact roles of the labor-efficiency argument and the debt-efficiency argument in the design of an optimal transfer scheme such as UBI.

2.2 Competitive Equilibrium

Definition 1. Given the initial asset holdings of the participants (a_0^e, a_0^u), the initial government bonds B_0, and the sequence of policies $\{\tau_t, B_{t+1}, T_t\}_{t=0}^{\infty}$, a competitive equilibrium is defined as the sequences of prices $\{w_t, Q_{t+1}\}_{t=0}^{\infty}$, aggregate allocations $\{C_t, N_t\}_{t=0}^{\infty}$, and individual allocations $\{c_t^e, c_t^u, c_t^o, n_t^e, a_{t+1}^e, a_{t+1}^u\}_{t=0}^{\infty}$, such that

1. given $\{Q_{t+1}, w_t, \tau_t, T_t\}_{t=0}^{\infty}$, the sequence $\{c_t^e, c_t^u, n_t^e, a_{t+1}^e, a_{t+1}^u\}_{t=0}^{\infty}$ solves the participant family head's problem;

2. given $\{T_t\}_{t=0}^{\infty}$, the sequence $\{c_t^o\}_{t=0}^{\infty}$ solves the non-participant's problem;

3. given $\{w_t\}_{t=0}^{\infty}$, the sequence $\{N_t\}_{t=0}^{\infty}$ solves the representative firm's problem;

4. the government flow budget constraint holds:

$$\tau_t w_t N_t + Q_{t+1} B_{t+1} \geq B_t + T_t;$$

5. all markets clear for $t \geq 0$:

$$\begin{aligned} N_t &= C_t, \\ N_t &= n_t^e \pi(e), \\ C_t &= c_t^e \pi(e) + c_t^u \pi(u) + c_t^o \pi(o), \\ B_t &= a_t^e \pi(e) + a_t^u \pi(u). \end{aligned} \tag{10}$$

To facilitate our analysis, we make the following assumptions throughout the rest of this paper:

Assumption 1. *1. The auto-correlation of the idiosyncratic shock process θ_t is non-negative:* $\pi(e|e) + \pi(u|u) \geq 1$.

2. In period 0, *the employed members have higher initial wealth:* $a_0^e > a_0^u \geq 0$.

Assumption 1 ensures that under the wealth-pooling technology, the wealth of employed family members in each period $t \geq 1$ should be no less than that of unemployed members. This assumption rules out the uninteresting case of a wealth-pooling arrangement where unemployed members could become wealthier than employed members from time to time—this uninteresting possibility arises if the employment shock is negatively autocorrelated (i.e., $\pi(e|e) + \pi(u|u) < 1$).[4]

Proposition 1. *Under Assumption 1 and $B_0 > 0$, the competitive equilibrium has the following properties:*

1. *For all $t \geq 0$, it must be true that $c_t^e \geq c_t^u$ and $a_{t+1}^e > a_{t+1}^u \geq 0$. That is, the borrowing constraints of employed individuals are always slack ($a_{t+1}^e > 0$, implying that the Lagrangian multiplier $\kappa_t^e = 0$). Also, depending on the initial level of debt B_0, the borrowing constraints of unemployed individuals may or may not be binding ($a_{t+1}^u \geq 0$).*

2. *The intertemporal price Q_{t+1} is determined by*

$$Q_{t+1} = \beta \left[u_{c,t+1}^e \pi(e|e) + u_{c,t+1}^u \pi(u|e) \right] / u_{c,t}^e \tag{11}$$

   with the following inequality:

$$Q_{t+1} \geq \beta \left[u_{c,t+1}^e \pi(e|u) + u_{c,t+1}^u \pi(u|u) \right] / u_{c,t}^u.$$

3. *In the steady state, if the asset holdings of unemployed participants are positive such that the corresponding multiplier $\kappa^u = 0$, then the competitive equilibrium features full self-insurance (FSI) with $c^e = c^u$ and $Q = \beta$ (or $r = \beta^{-1}$)*[5]

Proof. See Appendix A.1. □

Proposition 1 states that if asset holdings a_{t+1}^e and a_{t+1}^u are both sufficiently large (because of a sufficiently large B_0) such that all participants' borrowing constraints are slack, then in the steady state the participants can obtain the same level of consumption regardless of their employment status. In this case, the steady-state market interest rate equals the time discount rate: $Q = \beta$. We refer to the allocation where all participants' borrowing constraints are slack as a full self-insurance (FSI) allocation.

A competitive equilibrium with FSI is impossible in a typical Aiyagari model (Aiyagari (1994)) because every participant's marginal utility of consumption follows a supermartingale when $Q = \beta$,

[4]See the proof of Proposition 1 in Appendix A.1 for details.

[5]Throughout this paper, a variable without subscript t is a steady-state value.

which implies that participants' savings (or asset demand) diverge to infinity in the long run, which cannot constitute an equilibrium. In our model, however, the FSI can be achieved even at a finite level of asset holdings because the wealth-pooling technology among family members reduces the need to carry an infinite amount of assets to fully self-insure against highly persistent income risk.[6] This special property renders our model analytically tractable with closed-form solutions. Moreover, this property helps us to understand the key properties of the Ramsey allocation, as should become clear in the next section.

In addition, Proposition 1 also indicates that the Ramsey planner can potentially use fiscal tools (such as government debt and lump-sum transfers) to achieve FSI allocation in this economy when the laissez-faire competitive equilibrium does not feature FSI. For example, by the asset market-clearing condition (10), FSI is feasible if the supply of government bonds is sufficiently high. To make our Ramsey problem interesting, we assume in the rest of the paper that the initial bond supply B_0 as well as the initial distribution of household wealth (a_0^e, a_0^u) are such that the competitive equilibrium (without further policy intervention) does not feature FSI. Namely, B_0 is sufficiently low such that the steady-state competitive equilibrium without government intervention features consumption inequality $c^e > e^u$ and precautionary saving behaviors such that $Q > \beta$.

3 Ramsey Problem

3.1 The Ramsey Problem

We use the primal approach to solve the Ramsey problem. Proposition 6 in Appendix A.2 characterizes the constraint set of the Ramsey planner. Specifically, the constraint set states the conditions for any constructed Ramsey allocation—$\{c_t^e, n_t^e, c_t^u, c_t^o, a_{t+1}^e, a_{t+1}^u, T_t\}_{t=0}^{\infty}$—to satisfy in order to constitute a competitive equilibrium.

The welfare is evaluated in period 0 with Pareto weights ω and $1-\omega$ for participants and

[6] Specifically, in any period t, if the savings of the employed members a_t^e in the last period are sufficiently large, then the period-t initial wealth of the unemployed agents, $\frac{a_t^e \pi(e)\pi(u|e) + a_t^u \pi(u)\pi(u|u)}{\pi(u)}$, can be high enough such that their borrowing constraints are also slack. As a result, household savings (or asset demand) are bounded away from infinity even at the point $r = 1/\beta$.

non-participants, respectively. The welfare function is then assumed to be

$$\sum_{t=0}^{\infty} \beta^t \left\{ \omega([u(c_t^e) - v(n_t^e)] \pi(e) + u(c_t^u)\pi(u)) + (1-\omega)u(c_t^o)\pi(o) \right\}. \tag{12}$$

Armed with Proposition 6, the Ramsey problem is to maximize the social welfare (12) by choosing $\{c_t^e, c_t^u, c_t^o, n_t^e, a_{t+1}^e, a_{t+1}^u, T_t\}_{t=0}^{\infty}$ subject to the constraints (18)-(27) listed in Proposition 6 in the Appendix.

3.2 The First-Best Allocation

Before solving the Ramsey outcome, it is useful to take a look at the first-best allocation as a reference point. The first-best allocation is defined as the optimal allocation chosen by a social planner that maximizes the welfare function (12) subject only to the aggregate resource constraint (18). It is straightforward to show that in the first-best allocation all individual plans are constant over time, denoted by $(c^{e,FB}, c^{u,FB}, c^{o,FB}, n^{e,FB})$, and these plans feature perfect consumption equality:

$$c^{e,FB} = c^{u,FB} = \frac{1-\omega}{\omega} c^{o,FB}.$$

3.3 Definition of Ramsey Steady State

The Ramsey steady state is defined as follows:

Definition 2. Given $\{B_0, a_0^e, a_0^u\}$, a Ramsey steady state is a long-run Ramsey allocation where the variables $\{N_t, C_t, c_t^e, c_t^u, c_t^o, n_t^e\}$ all converge to finite non-negative values. In addition, a Ramsey steady state is called "interior" if none of the variables converge to zero; otherwise, the Ramsey steady state is called "non-interior" if one or more of these variables (such as aggregate consumption C_t) converge to zero.

3.4 Ramsey Outcome

Proposition 2. *Under Assumption 1, there are two possible long-run Ramsey outcomes:*

1. *Under the fiscal-space condition,* $(1-\beta)\frac{\pi(u)}{\pi(u|e)} < 1 - \pi(o)$*, an interior Ramsey steady state exists and has the following properties:*

(a) *The steady-state allocation features FSI with $r = 1/Q = \beta^{-1}$, $T > 0$, $a^e > a^u = 0$, and $c^e = c^u > 0$.*

(b) *Depending on parameter values, the optimal tax rate $\tau \in (0, 1)$ but can be arbitrarily close to 1 depending on how tight the fiscal space condition, $(1-\beta)\frac{\pi(u)}{\pi(u|e)} < 1 - \pi(o)$, is.*

(c) *The interior FSI steady state is the only possible Ramsey steady state.*

2. *If the fiscal-space condition is violated such that $(1-\beta)\frac{\pi(u)}{\pi(u|e)} \geq 1 - \pi(o)$, then an interior Ramsey steady state is impossible, and the only possible Ramsey steady state (if it exists) is non-interior with zero aggregate consumption and a 100% labor-tax rate.*

Proof. See Appendix A.4. □

Notice that this Proposition is robust to the choice of Pareto weight, ω. The message here is that the Ramsey planner would like to implement the FSI interest-rate policy $Q = \beta$ in the steady state whenever feasible. The feasibility depends on whether the optimal tax rate to support FSI exceeds 100%, which depends on the fiscal-space condition expressed analytically in the above proposition. If FSI is infeasible, then a Ramsey steady state (if it exists) must be non-interior.

In a similar setup, Bhandari, Evans, Golosov, and Sargent (2017) show that they can construct a competitive equilibrium without private borrowing that yields higher welfare than a competitive equilibrium with private borrowing. The intuition provided by them is that a tighter borrowing constraint on all types of agents would enable the Ramsey planner to improve welfare by collecting monopoly rents on public debt from bond holders (the participants) and redistributing the gains to the poor (the non-participants) who do not hold bonds. However, they do not solve for the Ramsey problem to show what a Ramsey allocation may look like.

In fact, the Ramsey allocation described in the above Proposition indicates that it is not optimal to repress the interest rate in order to benefit the poor (even if the welfare weight on rich agents is arbitrarily small, $\omega \to 0$). In other words, a steady-state allocation with $r < 1/\beta$ or with monopoly rents from the rich is a feasible competitive equilibrium but not a long-run Ramsey outcome. Our analysis shows that the Ramsey planner has a dominant incentive to pursue FSI allocation even at the cost of an extremely high distortionary tax rate in the long run.

Why must the Ramsey planner pursue an FSI allocation when the marginal benefit of increasing debt may be diminishing while the marginal cost of distortionary future taxes is rising? Our answer provided above is that as long as $\beta < 1$ and $r < \beta^{-1}$, the Ramsey planner has incentives to increase the bond supply to pursue FSI allocation and frontload consumption for both the poor and the rich even if this implies a close to 100% labor tax rate in the long run to finance the sky-rocketing public debt-to-GDP ratio. In other words, as long as $r < 1/\beta$, the marginal benefit of increasing debt dominates the marginal cost of distortionary future taxes.

The FSI Ramsey steady state is consistent with the finding of Albanesi and Armenter (2012), who argue that ruling out intertemporal distortions permanently often induces a first-order welfare gain in a broad class of second-best economies. Hence, if FSI is feasible for the Ramsey planner, it should be the long-run Ramsey outcome. However, Albanesi and Armenter (2012) do not provide analysis on the Ramsey allocation in a typical Aiyagari model where the steady state with $r = 1/\beta$ is infeasible to the Ramsey planner. For this important reason, we modify the Aiyagari model such that FSI is feasible in the long run, and our result indicates that (i) the optimal Ramsey outcome must feature $r = 1/\beta$ if the fiscal-space condition is satisfied and that (ii) it becomes non-interior if the fiscal-space condition is violated.

Clearly, neither the labor-efficiency argument nor the debt-efficiency argument is featured in the Ramsey allocation described in Proposition 2. To understand why these margins do not matter in the Ramsey outcome, we consider several special scenarios in the following section.

4 Understanding the Ramsey Outcome

The first scenario considers the case where there is no ex-ante heterogeneity. We show that in this scenario the Ramsey outcome achieves the first-best allocation where the distortionary labor tax rate is zero and the bond supply is plenty such that all households' borrowing constraints are slack. The bond interest payment is then financed purely by lump-sum taxes. This scenario suggests that a lump-sum transfer is not the best tool to address the inequality issue caused by ex-post heterogeneity.

In the second scenario, we consider the case without ex-post heterogeneity (i.e., in the absence of idiosyncratic risk). We show that in this scenario the competitive equilibrium must feature $r = 1/\beta$; hence, there is no intertemporal distortion and the Ramsey planner's incentive for frontloading consumption is completely shut down. Consequently, the debt-efficiency argument is irrelevant since the Ramsey planner cannot influence the market price Q by changing the bond supply. The Ramsey outcome in this scenario shows that the optimal tax-transfer system is determined solely by the labor-efficiency argument. In particular, if rich agents receive zero Pareto weight in the social welfare function, then the optimal tax rate is indeed implied by the maximal position on the Laffer curve.

The third scenario conducts a different kind of analysis by supposing that the Ramsey planner maximizes only the steady-state welfare of the competitive equilibrium instead of the time-zero present value of the dynamic path of social welfare. In other words, the "future" is no longer "discounted" compared to the "present." In this case, the intertemporal mechanism does not operate anymore. Instead, both the labor-efficiency margin and the debt-efficiency margin are of first-order importance in jointly determining the optimal tax-transfer system. Under the assumption of zero Pareto weight for rich agents, for example, the optimal tax rate is on the top of a "modified" Laffer Curve, which incorporates the consideration of debt-interest payments and reflects the debt-efficiency margin.

4.1 Scenario 1: Without Ex-ante Heterogeneity

Suppose there is no ex-ante heterogeneity ($\pi(o) = 0$) and the fiscal-space condition is met: $(1 - \beta)\frac{\pi(u)}{\pi(u|e)} < 1$. Given the set of fiscal instruments available in our benchmark model, the following Proposition shows that the Ramsey planner is able to achieve the first-best allocation starting in the initial period 0.

Proposition 3. *Given initial value of $B_0 > 0$ and the fiscal-space condition, $(1-\beta)\frac{\pi(u)}{\pi(u|e)} < 1$, the Ramsey outcome achieves the first-best allocation with (i) $Q_{t+1} = \beta$ and (ii) perfect consumption equality $c_t^{e,FB} = c_t^{u,FB}$ for all $t \geq 0$. This allocation can be implemented by the following policy mix:*

1. *The distortionary labor tax rate is zero, $\tau_t = 0$, for all $t \geq 0$.*
2. *The sequence of government debt is determined by the following recursive process for all $t \geq 0$:*
$$B_{t+1} = \frac{1}{\beta}\left(1 - \frac{\pi(u|e)}{\pi(u)}\right) B_t + \frac{1}{\beta}c^{e,FB}.$$
3. *The steady-state lump-sum transfer is strictly negative and given by $T = (\beta - 1)B < 0$.*

Proof. Please refer to Appendix A.5. □

Notice that the fiscal-space condition $(1-\beta)\frac{\pi(u)}{\pi(u|e)} < 1$ implies $\frac{1}{\beta}\left(1 - \frac{\pi(u|e)}{\pi(u)}\right) < 1$, which ensures that the sequence of debt B_{t+1} converges to a finite constant.

This Ramsey outcome suggests that in the absence of non-participants, the government opts to issue plenty of public debt to achieve the first-best allocation where consumption is equalized across the participant family members regardless of their employment status. A lump-sum tax (negative transfer) is used to finance the interest payments of government debt at least in the steady state. The distortionary labor tax is never utilized and always set to zero starting from period 0. This Ramsey outcome highlights the role of government debt in enhancing the self-insurance position of participants as well as the role of lump-sum taxation for interest payments with zero distortion.

This Ramsey outcome may seem intuitive and straightforward. However, there are several subtle issues worth mentioning. First, it indicates that the tool of a lump-sum tax could be very powerful in the environment of ex-post heterogeneous agents. The first-best allocation can be achieved with a complete tax system (at the macroeconomic level) together with an additional aggregate fiscal tool—the unconditional lump-sum tax. In other words, to achieve the first-best allocation, there is no need to have state-contingent fiscal tools based on the status of individual shocks. Second, the government prefers to use bonds instead of lump-sum transfers to achieve the first-best allocation—which features full consumption equality and production efficiency. This result also reflects the fact that the major concern of the Ramsey planner is self-insurance instead of redistribution across employment status. Intuitively, government bonds are more suitable than lump-sum transfers to address the lack-of-insurance problem caused by idiosyncratic risk, and hence the Ramsey planner opts to use debt exclusively instead of lump-sum transfers.

4.2 Scenario 2: Without Ex-post Heterogeneity

Consider the opposite case where there is no ex-post heterogeneity. Specifically, let $\pi(u) = 0$, $\pi(u|e) = 0$, and $\pi(e|e) = 1$. In this case, there is no idiosyncratic risk for rich agents and hence no precautionary saving motives. This economy is similar to that of Werning (2007) without aggregate uncertainty.

Proposition 4. *The Ramsey allocation reaches an interior Ramsey steady state starting in period* 1*. In addition, in the Ramsey steady state the interest rate is equal to the time discount rate:* $r = \beta^{-1}$.

Proof. See Appendix A.6. □

Since there is no precautionary saving motive in the absence of the idiosyncratic shock, all competitive equilibria must feature $Q = \beta$, and hence there is no room for the Ramsey planner to influence the market discount rate Q by adjusting the supply of government bonds. As a result, not only does the "arbitrage" opportunity shown in our benchmark model not exist, but the "debt-efficiency" argument also becomes irrelevant. The optimal tax-transfer system is then determined entirely by the labor-efficiency argument, as in static models.

Figure 1 plots the change of the Ramsey steady-state allocation as the Pareto weight ω on rich agents changes. Specifically, as ω approaches zero such that poor agents receive higher and higher Pareto weight (from left to right in the figure), the tax rate and transfers increase monotonically, and the consumption and labor supply of rich agents decrease monotonically.

We can also show that the conventional Laffer curve argument indeed operates in this scenario. This can be seen most clearly by setting $\omega = 0$. A zero Pareto weight on rich agents indicates that the Ramsey planner does not care about the participants and would like to extract the highest revenue possible from them through labor taxes. According to the Laffer curve wisdom, the optimal tax rate should be determined by the peak of the Laffer curve where the tax revenue is maximized—which is indeed the case, as shown in Figure 2. Hence, the conventional labor-efficiency argument for the design of an optimal tax-transfer system is valid in a heterogeneous-agent environment if it's without ex-post uncertainty (or precautionary saving motives).

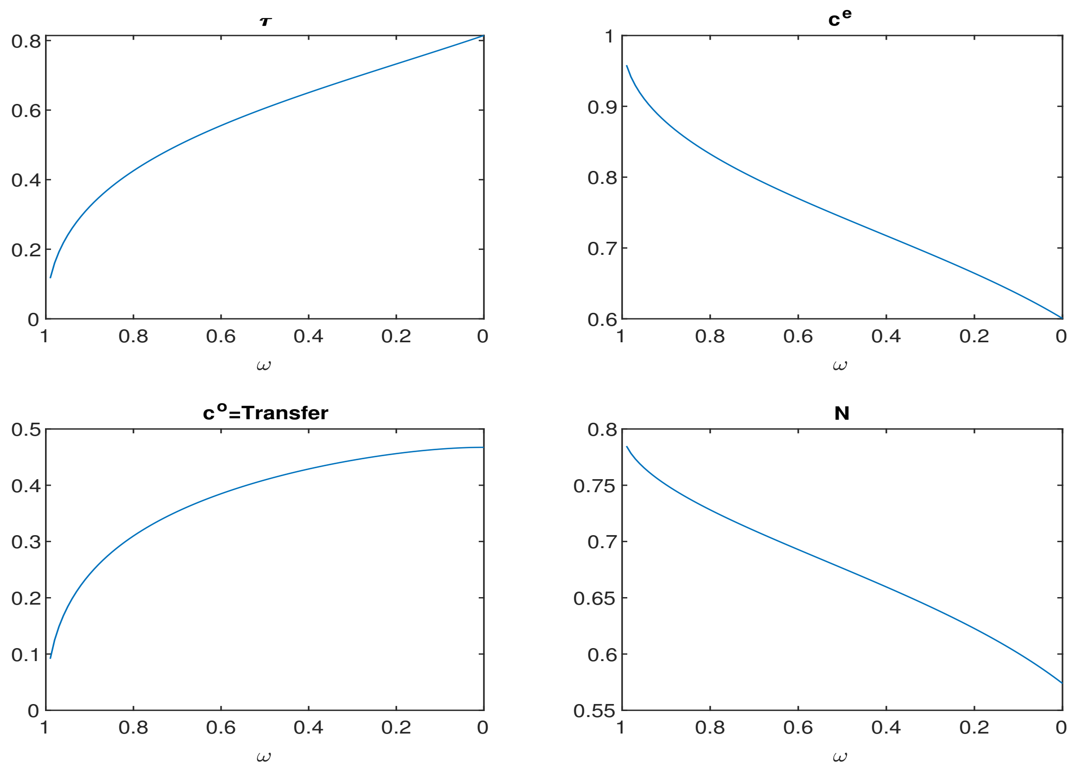
4.3 Scenario 3: Steady-State Welfare Maximization

We now consider the third scenario where the Ramsey planner maximizes only the steady-state welfare of the competitive equilibrium and completely ignores the dynamics. In this case, we prove that the FSI allocation is *not optimal* in the following Proposition. In other words, the optimal Ramsey outcome does indeed feature a repressed interest rate: $r < 1/\beta$, which reflects the debt-efficiency argument of gains from cheap borrowing costs imposed on the rich.

Proposition 5. *If the Ramsey planner's objective is to maximize only the steady-state welfare of the competitive equilibrium, then an FSI allocation is not optimal. Namely, the Ramsey outcome has the following properties:*

1. *The interest rate lies strictly below the time discount rate: $r < 1/\beta$.*

Figure 1: Ramsey Steady State Outcome without Idiosyncratic Risks



Notes: Parameter values are $\gamma = \sigma = 2$, $\pi(o) = 0.2$ and $\pi(e) = 0.8$.

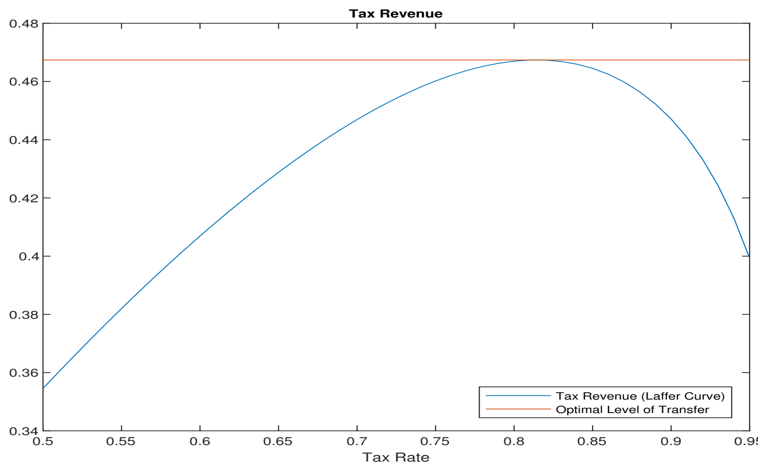
2. *The borrowing constraint of unemployed individuals strictly binds: $a^u = 0$, with $c^e > c^u$.*

Proof. See Appendix A.7. □

The result in Proposition 5 is intuitive. By maximizing only the steady-state welfare of the competitive economy, the Ramsey planner no longer has the incentive to exploit the difference between the interest rate and the time discount rate, because the time discount rate is no longer relevant in maximizing the steady-state welfare. Consequently, the issue of frontloading consumption becomes irrelevant. In this case, the Ramsey planner does not choose FSI allocation by equalizing consumption across the employed and unemployed family members—because the cost of doing so by issuing plenty of debt is too high at the margin when there is no time discounting for future interest payments of debt.

Figure 3 plots the numerical simulations of the Ramsey allocation as the Pareto weight ω varies. It shows that as the value of ω decreases (from left to right), or as the Ramsey planner cares less

Figure 2: Laffer Curve

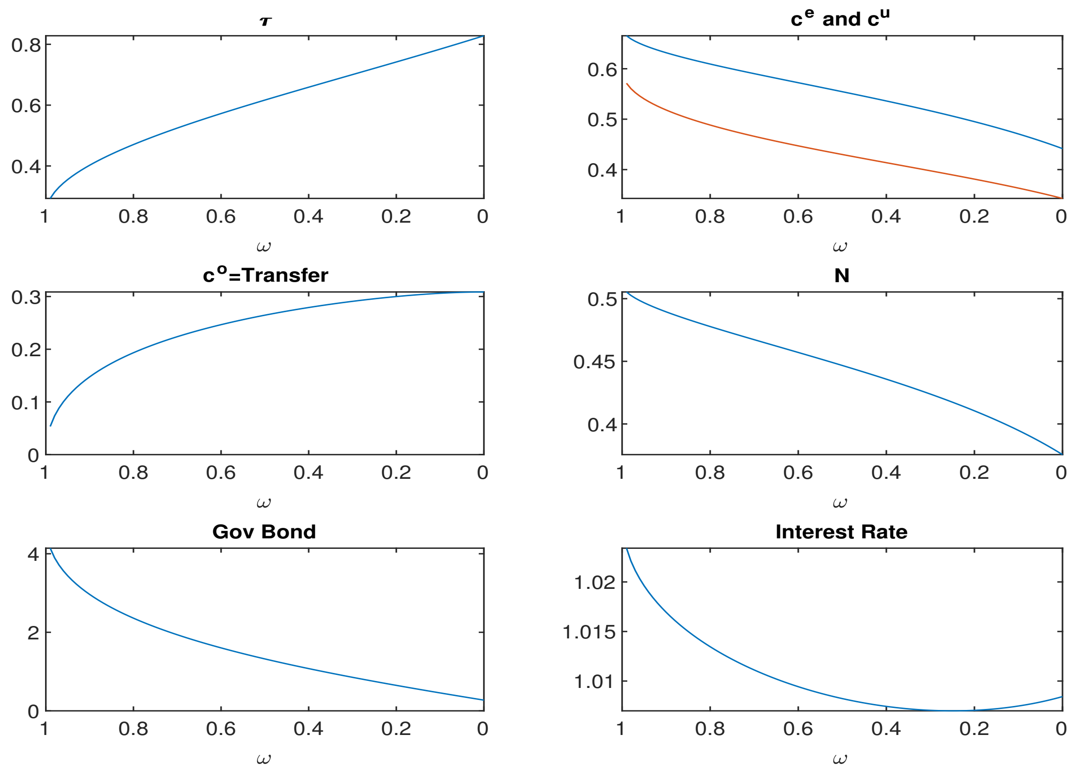


Notes: Parameter values are $\gamma = \sigma = 2$, $\pi(e|e) = \pi(u|u) = 0.95$, $\pi(u|e) = \pi(e|u) = 0.05$, $\pi(o) = \pi(u) = 0.2$ *and* $\pi(e) = \pi(u) = 0.4$.

about rich agents, the implied optimal tax rate τ (top-left panel) rises monotonically; consequently, the consumption of the participants (top-right panel) declines and the consumption of the non-participants (financed by lump-sum transfers) increases. A high tax rate also discourages labor supply and hence reduces output (the middle-right panel). The interest rate lies well below the inverse of the time discount rate, as proved by Proposition 5 (the bottom-right panel). The low interest rate is consistent with the debt-efficiency argument. Interestingly, the interest rate exhibits a non-monotonic behavior when ω is small enough (around $\omega = 0.2$). This is so because as ω declines, the amount of lump-sum transfers received by unemployed participants also rises. As a result, if ω is small enough, the ability of the Ramsey planner to further push down the interest rate (by tightening the borrowing constraint of unemployed participants) becomes limited.

Second, in Figure 4 we demonstrate the labor-efficiency argument based on the Laffer curve: We plot the tax revenue as well as lump-sum transfers (which is the sum of tax revenue plus the bond interest payment) against the tax rate in the case of $\omega = 0$ and hold the supply of government bonds at the optimal level. There are two curves shown in Figure 4; the blue curve is the plot of tax revenue or the standard Laffer curve; the orange curve is the lump-sum transfer that is the sum of tax revenue and bond interest payments, which mimics the Laffer curve but with a maximum lies below and to the left of the Laffer curve. We call the second curve the "modified"

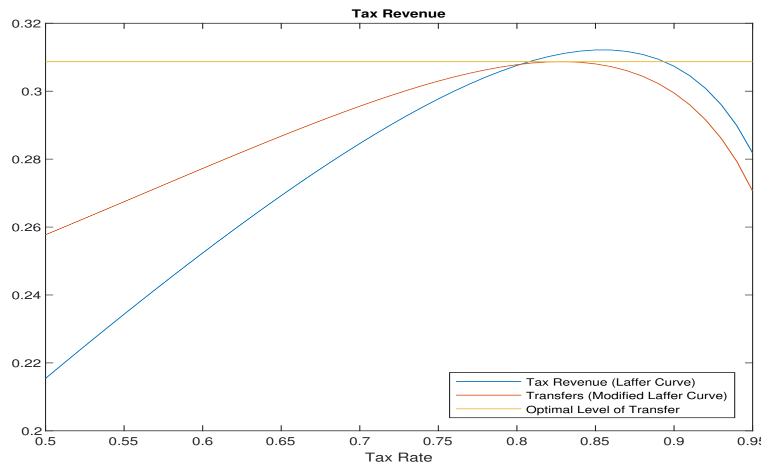
Figure 3: Non-FSI Steady-State Relationship



Notes: Parameter values are $\gamma = \sigma = 2$, $\pi(e|e) = \pi(u|u) = 0.95$, $\pi(u|e) = \pi(e|u) = 0.05$, $\pi(o) = \pi(u) = 0.2$ and $\pi(e) = \pi(u) = 0.4$.

Laffer curve. Since the Ramsey planner would limit its ability to tax labor and hence exercise its monopoly power of borrowing in order to suppress the interest rate or economize on the interest payment, the optimal tax rate lies blow the tax rate implied by the traditional Laffer curve even though the Ramsey planner does not care about the rich at all ($\omega = 0$). As shown in Figure 4, the optimal tax rate is implied by the maximum of the modified Laffer curve instead of the Laffer curve, suggesting that both the labor-efficiency argument and the debt-efficiency argument are relevant in the design of an optimal tax-transfer system.

Figure 4: Laffer Curve



Notes: Parameter values are $\gamma = \sigma = 2$, $\pi(e|e) = \pi(u|u) = 0.95$, $\pi(u|e) = \pi(e|u) = 0.05$, $\pi(o) = \pi(u) = 0.2$ *and* $\pi(e) = \pi(u) = 0.4$.

5 Conclusion

In the beginning of this paper, we asked how an optimal tax-transfer system should be designed to address the enormous degree of income inequality facing both developed and developing countries. This question is even more important in light of the popular support for the UBI policy in the Europe and North America today.

Two conventional wisdoms emerged in the literature to address this question. The first relies on the labor-efficiency argument based on the adverse effect of a labor tax on labor supply, while the second relies on the debt-efficiency argument based on the government's monopoly rents extracted from issuing debt to the wealthy cohort.

We show in this paper that once dynamic efficiency is taken into account, an optimal tax-transfer system entails FSI allocation even if this could imply a skyrocketing amount of public debt in the long run such that the optimal steady-state tax rate approaches 100% (instead of approaching the top of the Laffer curve).

The key reason is that eliminating the intertemporal wedge between the interest rate and the time discount rate—a hallmark feature of Aiyagari-type models—by frontloading consumption leads to a first-order welfare gain in economies with both ex-ante and ex-post heterogeneity. Other concerns such as the "labor-efficiency" and the "debt-efficiency" only bring second-order welfare gains. Hence, neither of the conventional argument holds the key to understanding the design of an optimal tax-transfer program in an infinite-horizon model with both ex-ante and ex-post heterogeneity.

We also showed that the Ramsey allocation follows the labor-efficiency argument if there does not exist ex-post heterogeneity or idiosyncratic risk. In this case, there is no intertemporal wedge since the interest rate and the time discount rate line up perfectly in all competitive equilibria, which also implies that the debt-efficiency argument is irrelevant. On the other hand, if the Ramsey planner cares only about the steady-state welfare, then both the labor-efficiency argument and the debt-efficiency argument become important; in this case the market interest rate lies below the time discount rate and the optimal tax-transfer system is determined both by the tradeoff between redistribution and labor-market efficiency and by the tradeoff between gains from repressing the interest rate and the asset-market efficiency. For example, it is determined at the peak of the modified Laffer curve if a zero Pareto weight is given to the participants. Our analysis thus suggests that the underlying logic of an optimal tax-transfer system is sensitive to models, especially to the presence or absence of the intertemporal wedge between the interest rate and the time discount rate.

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A Appendix

A.1 Proof of Proposition 1

A.1.1 The FOCs of the Family Head's Problem

Let $\beta^t \xi_t^e \pi(e)$, $\beta^t \xi_t^u \pi(u)$, $\xi_0^e \pi(e)$, $\xi_0^u \pi(u)$, $\beta^t \kappa_t^e$ and $\beta^t \kappa_t^u$ be the Lagrangian multipliers associated with constraints (2)-(7), respectively; the FOCs for the participants with respect to c_t^e, c_t^u, a_{t+1}^e, a_{t+1}^u, and n_t^e are given, respectively, by

$$\begin{aligned} u_{c,t}^e &= \xi_t^e, && (13) \\ u_{c,t}^u &= \xi_t^u, && (14) \end{aligned}$$

$$\begin{aligned} Q_{t+1}\xi_t^e &= \beta \left[\xi_{t+1}^e \pi(e|e) + \xi_{t+1}^u \pi(u|e) \right] + \kappa_t^e, && (15) \\ Q_{t+1}\xi_t^u &= \beta \left[\xi_{t+1}^e \pi(e|u) + \xi_{t+1}^u \pi(u|u) \right] + \kappa_t^u, && (16) \end{aligned}$$

$$v_{n,t}^e = \xi_t^e \widehat{w}_t; \tag{17}$$

where $u_{c,t}^e$ and $u_{c,t}^u$ denote the marginal utility of consumption for employed and unemployed individuals in period t; similarly, $v_{n,t}^e$ denotes the marginal disutility of labor for employed individuals in period t.

A.1.2 Proof of $a_{t+1}^e > 0$

First, if $\pi(e|e) + \pi(u|u) \geq 1$ and $a_t^e > a_t^u$, then it must be true that $\frac{a_t^e \pi(e)\pi(e|e) + a_t^u \pi(u)\pi(e|u)}{\pi(e)} \geq \frac{a_t^e \pi(e)\pi(u|e) + a_t^u \pi(u)\pi(u|u)}{\pi(u)}$ and $a_{t+1}^e > a_{t+1}^u$ for all $t \geq 0$; namely, the period t initial wealth of employed family members is no less than that of unemployed members, and the newly accumulated wealth of employed members is greater than that of unemployed members. This can be seen by the following steps:

1. Given $\pi(e|e) + \pi(u|u) \geq 1$, then $a_t^e > a_t^u$ can be rewritten as

$$a_t^e \left[\pi(e|e) + \pi(u|u) - 1 \right] \geq a_t^u \left[\pi(u|u) - 1 + \pi(e|e) \right],$$

where the equality holds if $\pi(e|e)+\pi(u|u)=1$. The equation above together with $\frac{\pi(e)}{\pi(u)}=\frac{\pi(e|u)}{\pi(u|e)}$ give

$$a_t^e\pi(e|e)+a_t^u\frac{\pi(u)}{\pi(e)}\pi(e|u)\geq a_t^u\pi(u|u)+a_t^e\frac{\pi(e)}{\pi(u)}\pi(u|e).$$

By rearranging terms in the equation above, we obtain

$$\frac{a_t^e\pi(e)\pi(e|e)+a_t^u\pi(u)\pi(e|u)}{\pi(e)}\geq\frac{a_t^e\pi(e)\pi(u|e)+a_t^u\pi(u)\pi(u|u)}{\pi(u)}.$$

which means that in every period $t\geq 0$, if $a_t^e>a_t^u$, then it must be true that $\frac{a_t^e\pi(e)\pi(e|e)+a_t^u\pi(u)\pi(e|u)}{\pi(e)}\geq\frac{a_t^e\pi(e)\pi(u|e)+a_t^u\pi(u)\pi(u|u)}{\pi(u)}$,

2. Suppose that employed individuals are no poorer than unemployed individuals in the beginning of each period $t\geq 0$ (namely, $\frac{a_t^e\pi(e)\pi(e|e)+a_t^u\pi(u)\pi(e|u)}{\pi(e)}\geq\frac{a_t^e\pi(e)\pi(u|e)+a_t^u\pi(u)\pi(u|u)}{\pi(u)}$), which together with their higher labor income suggest that employed agents must have higher consumption, $c_t^e\geq c_t^u$. Moreover, in order to smooth consumption, employed individuals have precautionary saving motives to self-insure against the positive possibility of switching to an unemployed state in the future; as a result, employed individuals also have higher savings: $a_{t+1}^e>a_{t+1}^u$.

Given the discussions above, it must be true that $c_t^e\geq c_t^u$ and $a_{t+1}^e>a_{t+1}^u\geq 0$ if (a) $\pi(e|e)+\pi(u|u)\geq 1$, (b) $a_0^e>a_0^u\geq 0$ and (c) $B_t>0$ for all $t\geq 0$. In other words, $\kappa_t^e=0$ and $\kappa_t^u\geq 0$ for all t.

A.1.3 Proof of Equation (11)

Equation (15) together with $\kappa_t^e=0$ gives equation (11).

A.1.4 Proof of Full Self-Insurance

Suppose $\kappa^e=\kappa^u=0$ in the steady state, then equations (15) and (16) imply

$$\frac{\xi^e}{\xi^u}=\frac{\pi(e|e)+\frac{\xi^u}{\xi^e}\pi(u|e)}{\pi(e|u)+\frac{\xi^u}{\xi^e}\pi(u|u)},$$

which can be rewritten as

$$\left(\frac{\xi^e}{\xi^u}\right)^2 \pi(e|u) + \frac{\xi^e}{\xi^u}\left(\pi(u|u) - \pi(e|e)\right) - \pi(u|e) = 0.$$

The equation above has one positive root and one negative root. The positive root implies $\xi^u = \xi^e$, which suggests $c^e = c^u$. The negative root violates the requirement that both $c^u > 0$ and $c^e > 0$. Given that $c^e = c^u$, equation (11) implies $Q = \beta$ in the (interior) steady state.

A.2 Proposition 6 and Its Proof

Proposition 6. *Given the initial asset holdings* (a_0^e, a_0^u), *and the initial government bond,* $B_0 = a_0^e\pi(e) + a_0^u\pi(u)$, *the sequence* $\{c_t^e, n_t^e, c_t^u, c_t^o, a_{t+1}^e, a_{t+1}^u, T_t\}_{t=0}^{\infty}$ *can be supported as a competitive equilibrium if and only if it satisfies the following conditions:*

1. *Resource constraints:*

$$n_t^e\pi(e) - c_t^e\pi(e) - c_t^u\pi(u) - c_t^o\pi(o) \geq 0, \ \forall t \geq 0. \tag{18}$$

2. *Implementability conditions: for* $t = 0$,

$$c_0^e - \frac{v_{n,0}^e}{u_{c,0}^e} n_0^e + Q_1 a_1^e - a_0^e - T_0 = 0, \tag{19}$$

$$c_0^u + Q_1 a_1^u - a_0^u - T_0 = 0, \tag{20}$$

$$c_0^o - T_0 = 0, \tag{21}$$

and, for $t \geq 1$,

$$c_t^e - \frac{v_{n,t}^e}{u_{c,t}^e} n_t^e + Q_{t+1} a_{t+1}^e - \left[\frac{a_t^e\pi(e)\pi(e|e) + a_t^u\pi(u)\pi(e|u)}{\pi(e)}\right] - T_t = 0, \tag{22}$$

$$c_t^u + Q_{t+1} a_{t+1}^u - \left[\frac{a_t^e\pi(e)\pi(u|e) + a_t^u\pi(u)\pi(u|u)}{\pi(u)}\right] - T_t = 0, \tag{23}$$

$$c_t^o - T_t = 0, \tag{24}$$

where

$$Q_{t+1} = \beta \left[\frac{u^e_{c,t+1}\pi(e|e) + u^u_{c,t+1}\pi(u|e)}{u^e_{c,t}} \right].$$

3. *Borrowing constraints and complementary slackness conditions for unemployed participants:* $\forall t \geq 0$,

$$a^u_{t+1} \geq 0, \tag{25}$$

$$g^u_t \geq 0, \tag{26}$$

and

$$g^u_t a^u_{t+1} = 0, \tag{27}$$

where the function g^u_t *is defined as*

$$g^u_t \equiv \frac{u^u_{c,t}}{u^e_{c,t+1}\pi(e|u) + u^u_{c,t+1}\pi(u|u)} - \frac{u^e_{c,t}}{u^e_{c,t+1}\pi(e|e) + u^u_{c,t+1}\pi(u|e)}.$$

A.2.1 Proof of The "If" Part:

Given the initial value of (B_0, a^e_0, a^u_0) as well as the allocation $\{c^e_t, n^e_t, c^u_t, c^o_t, a^e_{t+1}, a^u_{t+1}, T_t\}_{t=0}^{\infty}$, a competitive equilibrium can be constructed by using the conditions in Proposition 6 and by following the steps below that uniquely back up the sequences of the other variables.

1. Aggregate C_t and N_t are chosen to satisfy

$$\begin{aligned} N_t &= n^e_t \pi(e), \\ C_t &= c^e_t \pi(e) + c^u_t \pi(u) + c^o_t \pi(o). \end{aligned}$$

2. $w_t = 1$, which is implied by the firm's problem.

3. $Q_{t+1} = \frac{1}{r_{t+1}}$ is chosen to satisfy the Euler equation of participants:

$$\frac{1}{r_{t+1}} = Q_{t+1} = \beta \frac{(u^e_{c,t+1}\pi(e|e) + u^u_{c,t+1}\pi(u|e))}{u^e_{c,t}} \text{ for all } t > 0 \tag{28}$$

4. τ_t is chosen to satisfy

$$\frac{v^e_{n,t}}{u^e_{c,t}} = \widehat{w}_t = (1 - \tau_t). \tag{29}$$

5. B_{t+1} is pinned down by the asset market-clearing condition

$$B_{t+1} = a^e_{t+1}\pi(e) + a^u_{t+1}\pi(u), \text{ for all } t \geq 0.$$

6. The following constraints are satisfied:

   (a) By plugging equations (29) and (28) into the constraints of participants and non-participants, we can obtain the implementability conditions displayed in equations (19), (20), (21), (22), (23) and (24).

   (b) To satisfy the FOC (16) and borrowing constraints for unemployed participants, we have listed equations (25), (26) and (27) as constraints in Proposition 6.

7. Finally, it is straightforward to verify that the implementability conditions together with the resource constraint imply the government budget constraint.

A.2.2 Proof of The "Only If" Part:

The constraints listed in Proposition 6 are trivially satisfied because they are all part of the competitive-equilibrium conditions.

A.3 Ramsey Problem and Optimality Conditions

A.3.1 Ramsey Problem

The Ramsey planner problem is given by

$$\max_{\{c^e_t, c^u_t, c^o_t, n^e_t, a^e_{t+1}, a^u_{t+1}, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \omega \left[u(c^e_t) - v\left(n^e_t\right) \right] \pi(e) + \omega u(c^u_t)\pi(u) + (1-\omega)u(c^o_t)\pi(o) \right\}$$

subject to

$$\beta^t \mu_t : n^e_t \pi(e) - c^e_t \pi(e) - c^u_t \pi(u) - c^o_t \pi(o) \geq 0, \ \forall t \geq 0.$$

$$\lambda^e_0 \pi(e) : u^e_{c,0} c^e_0 - v^e_{n,0} n^e_0 + Q_1 u^e_{c,0} a^e_1 - u^e_{c,0} a^e_0 - u^e_{c,0} T_0 = 0,$$

$$\lambda^u_0 \pi(u) : u^e_{c,0} c^u_0 + Q_1 u^e_{c,0} a^u_1 - u^e_{c,0} a^u_t - u^e_{c,0} T_0 = 0,$$

$$\lambda^o_0 \pi(o) : u^e_{c,0} c^o_0 - u^e_{c,0} T_0 = 0,$$

$$\beta^t \lambda_t^e \pi(e) : u_{c,t}^e c_t^e - v_{n,t}^e n_t^e + Q_{t+1} u_{c,t}^e a_{t+1}^e - u_{c,t}^e \left[\frac{a_t^e \pi(e)\pi(e|e) + a_t^u \pi(u)\pi(e|u)}{\pi(e)} \right] - u_{c,t}^e T_t = 0,$$

$$\beta^t \lambda_t^u \pi(u) : u_{c,t}^e c_t^u + Q_{t+1} u_{c,t}^e a_{t+1}^u - u_{c,t}^e \left[\frac{a_t^e \pi(e)\pi(u|e) + a_t^u \pi(u)\pi(u|u)}{\pi(u)} \right] - u_{c,t}^e T_t = 0,$$

$$\beta^t \lambda_t^o \pi(o) : u_{c,t}^e c_t^o - u_{c,t}^e T_t = 0,$$

$$\beta^{t+1} \zeta_t^{u,1} : a_{t+1}^u \geq 0,$$

$$\beta^{t+1} \zeta_t^{u,2} : g_t^u \geq 0,$$

$$\beta^{t+1} \zeta_t^{u,3} : g_t^u a_{t+1}^u = 0,$$

where $Q_{t+1} u_{c,t}^e$ are

$$Q_{t+1} u_{c,t}^e = \beta \left[u_{c,t+1}^e \pi(e|e) + u_{c,t+1}^u \pi(u|e) \right]$$

and the function g_t^u is defined as

$$g_t^u(c_t^e, c_t^u, c_{t+1}^e, c_{t+1}^u) \equiv \frac{u_{c,t}^u}{u_{c,t+1}^e \pi(e|u) + u_{c,t+1}^u \pi(u|u)} - \frac{u_{c,t}^e}{u_{c,t+1}^e \pi(e|e) + u_{c,t+1}^u \pi(u|e)}.$$

A.3.2 Ramsey FOCs

We first state the Ramsey FOCs. Denote $\beta^t \mu_t$, λ_0^e, λ_0^u, λ_0^o, $\beta^t \lambda_t^e$, $\beta^t \lambda_t^u$, $\beta^t \lambda_t^o$, $\beta^{t+1} \zeta_t^{u,1}$, $\beta^{t+1} \zeta_t^{u,2}$, $\beta^{t+1} \zeta_t^{u,3}$ as the Lagrangian multipliers for conditions (18)-(27), respectively. For all $t \geq 0$, the FOCs of the Ramsey problem with respect to n_t^e, T_t, a_{t+1}^e and a_{t+1}^u are given, respectively, by

$$v_{n,t}^e + \lambda_t^e (v_{n,t}^e + v_{nn,t}^e n_t^e) = \mu_t, \tag{30}$$

$$\lambda_t^e \pi(e) + \lambda_t^u \pi(u) + \lambda_t^o \pi(o) = 0, \tag{31}$$

$$\lambda_t^e (u_{c,t+1}^e \pi(e|e) + u_{c,t+1}^u \pi(u|e)) = \lambda_{t+1}^e u_{c,t+1}^e \pi(e|e) + \lambda_{t+1}^u u_{c,t+1}^e \pi(u|e), \tag{32}$$

and

$$\begin{aligned} \lambda_t^u (u_{c,t+1}^e \pi(e|e) + u_{c,t+1}^u \pi(u|e)) \quad = \quad & \lambda_{t+1}^u u_{c,t+1}^e \pi(u|u) + \lambda_{t+1}^e u_{c,t+1}^e \pi(e|u) \\ & + \zeta_t^{u,1} + \zeta_t^{u,3} g_t^u(c_t^e, c_t^u, c_{t+1}^e, c_{t+1}^u), \end{aligned} \tag{33}$$

respectively.

For all $t \geq 1$, the FOCs of the Ramsey problem with respect to c_t^e, c_t^u and c_t^o are given, respectively, by

$$\begin{aligned}
& (u_{c,t}^e \omega - \mu_t)\pi(e) + \lambda_t^e(u_{c,t}^e + u_{cc,t}^e c_t^e)\pi(e) - \lambda_t^e u_{cc,t}^e(a_t^e \pi(e)\pi(e|e) + a_t^u \pi(u)\pi(e|u) + T_t \pi(e)) \\
& +\lambda_{t-1}^e a_t^e u_{cc,t}^e \pi(e)\pi(e|e) + \lambda_t^u u_{cc,t}^e c_t^u \pi(u) - \lambda_t^u u_{cc,t}^e(a_t^e \pi(e)\pi(u|e) + a_t^u \pi(u)\pi(u|u) + T_t \pi(u)) \\
& +\lambda_{t-1}^u a_t^u u_{cc,t}^e \pi(u)\pi(e|e) + \lambda_t^o u_{cc,t}^e c_t^o \pi(o) - \lambda_t^o u_{cc,t}^e T_t \pi(o) \\
& +\beta \zeta_t^{u,2} \frac{\partial g_t^u}{\partial c_t^e} + \zeta_{t-1}^{u,2} \frac{\partial g_{t-1}^u}{\partial c_t^e} + \beta a_{t+1}^u \zeta_t^{u,3} \frac{\partial g_t^u}{\partial c_t^e} + a_t^u \zeta_{t-1}^{u,3} \frac{\partial g_{t-1}^u}{\partial c_t^e} \\
= \; & 0,
\end{aligned} \tag{34}$$

$$\begin{aligned}
& (u_{c,t}^u \omega - \mu_t)\pi(u) + \lambda_t^u u_{c,t}^e \pi(u) \\
& +\lambda_{t-1}^u a_t^u u_{cc,t}^u \pi(u)\pi(u|e) + \lambda_{t-1}^e a_t^e u_{cc,t}^u \pi(e)\pi(u|e) \\
& +\beta \zeta_t^{u,2} \frac{\partial g_t^u}{\partial c_t^u} + \zeta_{t-1}^{u,2} \frac{\partial g_{t-1}^u}{\partial c_t^u} + \beta a_{t+1}^u \zeta_t^{u,3} \frac{\partial g_t^u}{\partial c_t^u} + a_t^u \zeta_{t-1}^{u,3} \frac{\partial g_{t-1}^u}{\partial c_t^u} \\
= \; & 0,
\end{aligned} \tag{35}$$

and

$$(u_{c,t}^o(1-\omega) - \mu_t)\pi(o) + \lambda_t^o u_{c,t}^e \pi(o) = 0. \tag{36}$$

For $t = 0$, the FOCs of the Ramsey problem with respect to c_0^e, c_0^u and c_0^o are given, respectively, by

$$\begin{aligned}
& (u_{c,0}^e \omega - \mu_0)\pi(e) + \lambda_0^e(u_{c,0}^e + u_{cc,0}^e c_0^e)\pi(e) - \lambda_0^e u_{cc,0}^e a_0^e \\
& +\lambda_0^u u_{cc,0}^e c_0^u \pi(u) - \lambda_0^u u_{cc,0}^e a_0^u + \beta \zeta_0^{u,2} \frac{\partial g_0^u}{\partial c_0^e} + \beta a_1^u \zeta_0^{u,3} \frac{\partial g_0^u}{\partial c_0^e} = 0,
\end{aligned}$$

$$(u_{c,0}^u \omega - \mu_0)\pi(u) + \lambda_0^u u_{c,0}^e \pi(u) + \beta \zeta_0^{u,2} \frac{\partial g_0^u}{\partial c_0^u} + \beta a_1^u \zeta_0^{u,3} \frac{\partial g_0^u}{\partial c_0^u} = 0,$$

and

$$(u_{c,0}^o(1-\omega) - \mu_0)\pi(o) + \lambda_0^o u_{c,0}^e \pi(o) = 0.$$

Note that

$$\begin{aligned}
\frac{\partial g_t^u}{\partial c_t^u} &= \frac{u_{cc,t}^u}{u_{c,t+1}^e \pi(e|u) + u_{c,t+1}^u \pi(u|u)}, \\
\frac{\partial g_t^u}{\partial c_t^e} &= -\frac{u_{cc,t}^e}{u_{c,t+1}^e \pi(e|e) + u_{c,t+1}^u \pi(u|e)},
\end{aligned}$$

$$\begin{aligned}
\frac{\partial g_{t-1}^u}{\partial c_t^u} &= -\frac{u_{c,t-1}^u u_{cc,t}^u \pi(u|u)}{\left(u_{c,t}^e \pi(e|u) + u_{c,t}^u \pi(u|u)\right)^2} + \frac{u_{c,t-1}^e u_{cc,t}^u \pi(u|e)}{\left(u_{c,t}^e \pi(e|e) + u_{c,t}^u \pi(u|e)\right)^2}, \\
\frac{\partial g_{t-1}^u}{\partial c_t^e} &= -\frac{u_{c,t-1}^u u_{cc,t}^e \pi(e|u)}{\left(u_{c,t}^e \pi(e|u) + u_{c,t}^u \pi(u|u)\right)^2} + \frac{u_{c,t-1}^e u_{cc,t}^e \pi(e|e)}{\left(u_{c,t}^e \pi(e|e) + u_{c,t}^u \pi(u|e)\right)^2}.
\end{aligned}$$

A.4 Proof of Proposition 3

We conjecture and verify that there exists an interior Ramsey steady state featuring FSI and $Q = \beta$.

A.4.1 Existence of FSI Interior Ramsey Steady State

By the following steps, we conjecture and verify that there exists an FSI interior Ramsey steady state featuring (i) $c^e = c^u > 0$, (ii) $a^e > a^u = 0$, and (iii) $\mu < \infty$. Specifically, we show that an interior FSI steady state allocation can satisfy all first order conditions and constraints of the Ramsey planner problem.

1. When $c^e = c^u$ and $\zeta^{u,1} = 0$, the steady-state version of the Ramsey FOCs with respect to a^e and a^u are given by

$$\begin{aligned}
1 &= g_\lambda^e \pi(e|e) + g_\lambda^u \lim_{t\to\infty} \frac{\lambda_t^u}{\lambda_t^e} \pi(u|e) \\
1 &= g_\lambda^u \pi(u|u) + g_\lambda^e \lim_{t\to\infty} \frac{\lambda_t^e}{\lambda_t^u} \pi(e|u),
\end{aligned}$$

where g_λ^e and g_μ denote the steady state growth rate of λ_t^e and λ_t^u, respectively. In addition, the second equation utilizes the fact that $g^u = 0$ if $c^e = c^u$ in steady state. To satisfy these two FOCs, it must be the case that $g_\lambda^e = g_\lambda^u = 1$ and $\lambda^e = \lambda^u$.

2. When $c^e = c^u$, $\zeta^{u,1} = 0$, and $a^u = 0$, the FOC with respect to c_t^u is given by

$$\begin{aligned}
&(u_c^u \omega - \mu)\pi(u) + \lambda^u u_c^e \pi(u) + \lambda^u a^e u_{cc,t}^u \pi(e)\pi(u|e) \\
&+\zeta^{u,2}(\beta \frac{\partial g^u}{\partial c^u} + \frac{\partial g_{-1}^u}{\partial c^u}) = 0,
\end{aligned} \tag{37}$$

the FOC with respect to c_t^e is given by

$$\begin{aligned}
& (u_c^e \omega - \mu)\pi(e) + \lambda^e(u_c^e + u_{cc}^e c^e)\pi(e) - \lambda^e u_{cc}^e a^e \pi(e) + \lambda^e a^e u_{cc}^e \pi(e)\pi(e|e) \\
& -\lambda^e u_{cc}^e T(\pi(e) + \pi(u)) + \lambda^u u_{cc}^e c^u \pi(u) + \lambda^o u_{cc}^e (c^o - T)\pi(o) \\
& +\beta \zeta^{u,2} \frac{\partial g^u}{\partial c^e} + \zeta^{u,2} \frac{\partial g_{-1}^u}{\partial c^e} \\
= \; & 0,
\end{aligned} \tag{38}$$

and the FOC with respect to c_t^o is given by

$$(u_c^o(1-\omega) - \mu)\pi(o) + \lambda^o u_c^e \pi(o) = 0. \tag{39}$$

3. As a result, an interior Ramsey steady state allocation, (c^e, n^e, a^e, c^o, T), together with convergent multipliers $(\mu, \lambda^e = \lambda^u, \lambda^o, \zeta^{u,2})$ can be chosen in order to satisfy the following conditions:

   (a) FOC with respect to n_t^e in steady state

   $$v_n^e(\omega + \lambda^e(1+\gamma)) = \mu \tag{40}$$

   (b) FOC with respect to T_t in steady state

   $$\lambda^e(\pi(e) + \pi(u)) + \lambda^o \pi(o) = 0$$

   (c) The steady state resource constraint

   $$n_t^e \pi(e) - c_t^e(\pi(e) + \pi(u)) - c_t^o \pi(o) = 0$$

(d) The condition (23) is simplified to $a^e\pi(e)\pi(u|e) + T\pi(u) - c^u\pi(u) = 0$, which implies

$$c^u = c^e = a^e \frac{\pi(e)\pi(u|e)}{\pi(u)} + T.$$

(e) The implementability condition for employed participants

$$c^e - \frac{v_n^e}{u_c^e} n^e = a^e(\pi(e|e) - \beta) + T$$

(f) The implementability for o individuals

$$c^o = T$$

(g) The three FOCs with respect to c_t^e, c_t^u and c_t^o in equations (38), (37) and (39)

(h) The constraints from (25) to (27) are trivially satisfied in this FSI Ramsey steady state.

Note that this interior Ramsey steady state depends on the initial conditions of wealth distribution (a_0^e, a_0^u) and Pareto weight parameter ω.

4. We then show that this interior Ramsey steady state is feasible only if $(1-\beta)\frac{\pi(u)}{\pi(u|e)} < (1-\pi(o))$.

(a) Given $Q = \beta$ and $a^u = 0$, the steady-state government budget constraint implies that

$$\tau = \frac{(1-\beta)a^e\pi(e) + T}{n^e\pi(e)},$$

which has to be less than 1. Otherwise, the employed household FOC will be violated.

(b) To ensure that $\tau < 1$, it is required that

$$(1-\beta)a^e\pi(e) + T < n^e\pi(e) = c^u(\pi(e) + \pi(u)) + c^o\pi(o),$$

which together with implementability conditions of unemployed participants and non-participants give

$$(1-\beta)a^e\pi(e) + T < a^e \frac{\pi(u|e)\pi(e)}{\pi(u)}(\pi(e) + \pi(u)) + T.$$

The above equation can be simplified as the following condition:

$$(1-\beta)\frac{\pi(u)}{\pi(u|e)} < (1-\pi(o)) \,.$$

A.4.2 Uniqueness of FSI Interior Ramsey Steady State

We now prove that this FSI interior Ramsey steady state is unique—i.e., there cannot exist an interior Ramsey steady state featuring consumption inequality or partial self-insurance among participants. We prove this by contradiction. Suppose this is not true such that $c^e > c^u > 0$; then given that the borrowing constraint for unemployed individuals must be strictly binding with $a^u = 0$ and $\zeta_t^1 > 0$, we can consider the following arguments:

1. Let g_λ^u, g_λ^e and g_μ denote the steady state growth rate of λ_t^u, λ_t^e and μ, respectively. From the Ramsey FOC with respect to n^e, we know that if λ_t^e diverges, then it must imply $\lambda_t^e \to \infty$. For an interior Ramsey steady state to exist, $\lim_{t\to\infty} \frac{\mu_t}{\lambda_t^e} = v_n^e(1+\gamma) > 0$, a finite positive value.

2. Moreover, we can show that $g_\lambda^e = g_\lambda^\mu$ by the following steps:

   (a) The FOCs with respect to a^e and a^u in steady state are given by

   $$1 < \frac{Q}{\beta} = \pi(e|e) + \frac{u_c^u}{u_c^e}\pi(u|e) = g_\lambda^e \pi(e|e) + g_\lambda^u \frac{\lambda_t^u}{\lambda_t^e}\pi(u|e) < g_\lambda^u \pi(u|u) + g_\lambda^e \frac{\lambda_t^e}{\lambda_t^u}\pi(e|u). \quad (41)$$

   (b) Suppose $g_\lambda^e < g_\lambda^u$, then $\frac{\lambda_t^u}{\lambda_t^e} \to \infty$. Equation (41) becomes $\infty < g_\lambda^u \pi(u|u)$, which is impossible.

   (c) Consider the case that $g_\lambda^e > g_\lambda^u$, which implies $\lim_{t\to\infty} \frac{\lambda_t^u}{\lambda_t^e} = 0$.
   The FOC with respect to c^u (under $\zeta_t^{u,2} = 0$, $a_t^u = 0$) becomes

   $$(u_{c,t}^u \omega - \mu_t)\pi(u) + \lambda_t^u u_{c,t}^e \pi(u) + \lambda_{t-1}^e a_t^e u_{cc,t}^u \pi(u|e) = 0, \quad (42)$$

   which implies

   $$\lim_{t\to\infty} \frac{u_c^u \omega}{\lambda_{t-1}^e} + g_\lambda^u \lim_{t\to\infty} \frac{\lambda_{t-1}^u}{\lambda_{t-1}^e} u_c^e + u_{cc}^u a^e \frac{\pi(u|e)}{\pi(u)} = g_\mu \lim_{t\to\infty} \frac{\mu_{t-1}}{\lambda_{t-1}^e}.$$

As $t \to \infty$, the left-hand side is negative and the right-hand side is positive, which is a contradiction. Then, it must be true that $g^e_\lambda = g^u_\lambda$.

(d) The FOC with respect to T, $-\lambda^o_t \pi(o) = \lambda^e_t \pi(e) + \lambda^u_t \pi(u)$, together with $g^e_\lambda = g^u_\lambda$ imply the growth rate of λ^o, $g^o_\lambda = g^e_\lambda = g^u_\lambda$.

3. The FOC with respect to c^o implies

$$\lim_{t\to\infty} \frac{u^o_{c,t}(1-\omega)}{\lambda^o_t} + u^e_c = \lim_{t\to\infty} \frac{\mu_t}{\lambda^o_t} < 0.$$

Consider the case where $g^o_\lambda = g^e_\lambda = g^u_\lambda > 1$, which implies $\lambda^e_t \to \infty$ and $\lambda^o_t \to -\infty$. Then, the above equation leads to a contradiction that $u^e_c < 0$. Therefore, it must be the case that $\lambda^o_t \to \lambda^o > -\infty$ and $g^e_\lambda = g^u_\lambda = g^o_\lambda = g^\mu = 1$. Given $g^e_\lambda = g^u_\lambda = 1$, equation (41) implies that

$$u^u_c \lambda^e = u^e_c \lambda^u \tag{43}$$

4. Under $\zeta^{u,2} = 0$, $a^u = 0$ and the condition (43), the FOC in the limit with respect to c^e and c^u can be rewritten as

$$u^e_c(\omega + \lambda^e(1-\sigma) + \lambda^e \sigma \frac{T}{c^e}) = \mu, \tag{44}$$

and

$$u^u_c(\omega + \lambda^e(1-\sigma) + \lambda^e \sigma \frac{T}{c^u}) = \mu, \tag{45}$$

respectively. Taking the ratio of the above two equations gives

$$\frac{u^e_c}{u^u_c} \frac{\omega + \lambda^e(1-\sigma) + \lambda^e \sigma \frac{T}{c^e}}{\omega + \lambda^e(1-\sigma) + \lambda^e \sigma \frac{T}{c^u}} = 1,$$

which implies $c^e = c^u$, a contradiction to the $c^e > c^u$ assumption. (We know that $T > 0$)

To summarize, under the condition that $(1-\beta)\frac{\pi(u)}{\pi(u|e)} < 1 - \pi(o)$, the previous subsection A.4.1 proves that there exists an interior steady state featuring FSI. This current subsection A.4.2 further proves that there cannot exist any other interior Ramsey steady states that feature imperfect self-insurance with $Q > \beta$. Hence, the FSI interior Ramsey steady state exists and is the only possible interior steady state in the parameter space that $(1-\beta)\frac{\pi(u)}{\pi(u|e)} < 1 - \pi(o)$. However, there may exist Ramsey steady states that are non-interior; for this we now turn.

A.4.3 Non-Interior Ramsey Steady State

The proof in the subsection A.4.1 and A.4.2 also imply that if $(1-\beta)\frac{\pi(u)}{\pi(u|e)} \geq 1-\pi(o)$, there is no interior Ramsey steady state. In this subsection, we further prove that there exists a non-interior Ramsey steady state. The proof proceeds by considering all possible cases of a non-interior Ramsey steady state.

Case 1 Consider a non-interior steady-state allocation where (i) $\lim_{t\to\infty} c_t^e = c^e > 0$ and (ii) $\lim_{t\to\infty} c_t^u = 0$. The resource constraint then implies that $0 < c^e = \lim_{t\to\infty} c_t^e \leq \lim_{t\to\infty} n_t^e = n^e$. We show by the following steps that this allocation is not optimal or infeasible and hence must be ruled out.

1. Given that $\lim_{t\to\infty} c_t^e > 0$ and $\lim_{t\to\infty} c_t^u = 0$, we can assume $a_t^u = 0$ without loss of generality.
2. This non-interior allocation implies $Q_{t+1} > \beta$ and $Q_{t+1} \to \infty$
$$Q_{t+1} = \beta \left[\frac{u_{c,t+1}^e}{u_{c,t}^e}\pi(e|e) + \frac{u_{c,t+1}^u}{u_{c,t}^e}\pi(u|e) \right]$$
3. Given $Q_{t+1} \to \infty$, in order to satisfy the long-run government budget constraint that
$$0 < \tau n^e \pi(e) + \lim_{t\to\infty} Q_{t+1}B_{t+1} = \lim_{t\to\infty} B_t + \lim_{t\to\infty} T_t,$$
it has to be the case that $B_{t+1} \to 0$, $Q_{t+1}B_{t+1}$ converges to a finite non-negative value, and $\lim T_t > 0$.
4. By the asset market-clearing condition, $B_{t+1} \to 0$ implies $a_t^e \to 0$.
5. Taking the limit of unemployed participants' implementability condition gives
$$0 = \lim_{t\to\infty} c_t^u = \lim_{t\to\infty} a_t^e \frac{\pi(u|e)}{\pi(u)} + \lim_{t\to\infty} T_t = \lim_{t\to\infty} T_t > 0,$$
which leads to a contradiction.

Case 2 Consider the allocation where (i) $c_t^e = c_t^u > 0$ in transition for a finite period of time $t > 0$ but (ii) $\lim_{t\to\infty} c_t^e = \lim_{t\to\infty} c_t^u = 0$. Now, we show that this allocation violates the Ramsey FOCs and hence cannot be an optimal allocation.

1. We first show that $\lambda_t^e \to \lambda^e < \infty$. Assume otherwise, $\lambda_t^e \to \infty$. Given that $c_t^e = c_t^u > 0$ for a certain period of time $t > 0$, we know that $\zeta_t^{u,1} = 0$ and $g_t^u(c_t^e, c_t^u, c_{t+1}^e, c_{t+1}^u) = 0$ starting from a finite time t. Beyond period t, the Ramsey FOCs with respect to a_{t+1}^u can be rewritten as

$$1 = \frac{\lambda_{t+1}^u}{\lambda_t^u}\pi(u|u) + \frac{\lambda_{t+1}^e}{\lambda_t^u}\pi(e|u),$$

which together with $\lambda_t^e \to \infty$ imply $\lambda_t^u \to \infty$. In addition, the equation above can be rewritten as

$$\left(\frac{\lambda_{t+1}^u}{\lambda_t^u}\right)\left(\frac{\lambda_{t+1}^e}{\lambda_{t+1}^u}\right)\pi(e|u) = 1 - \frac{\lambda_{t+1}^u}{\lambda_t^u}\pi(u|u) < \pi(e|u),$$

where the last inequality utilizes the fact that $\frac{\lambda_{t+1}^u}{\lambda_t^u} > 1$. Similarly, by using the fact that $\frac{\lambda_{t+1}^e}{\lambda_t^e} > 1$, the Ramsey FOCs with respect to a_{t+1}^e can be rewritten as

$$\left(\frac{\lambda_{t+1}^e}{\lambda_t^e}\right)\left(\frac{\lambda_{t+1}^u}{\lambda_{t+1}^e}\right)\pi(u|e) = 1 - \frac{\lambda_{t+1}^e}{\lambda_t^e}\pi(e|e) < \pi(u|e).$$

Multiplying the above two equations gives

$$\frac{\lambda_{t+1}^u}{\lambda_t^u}\frac{\lambda_{t+1}^e}{\lambda_t^e} < 1,$$

which leads to a contradiction since $\frac{\lambda_{t+1}^u}{\lambda_t^u} > 1$ and $\frac{\lambda_{t+1}^e}{\lambda_t^e} > 1$. Hence, it must be the case that $\lambda_t^e \to \lambda^e < \infty$ and $\lambda_t^u \to \lambda^u = \lambda^e$. In addition, $\lambda_t^o \to \lambda^o > -\infty$, as indicated by the FOC with respect to T.

2. Given $c_t^e = c_t^u > 0$ in transition for a finite period of time $t > 0$, then it must be the case that $Q_{t+1} \to \beta$.

3. The Ramsey FOC with respect to n_t^e is given by

$$\frac{v_{n,t}^e}{\lambda_t^e} + v_{n,t}^e(1+\gamma) = \frac{\mu_t}{\lambda_t^e}.$$

Given $\lambda_t^e \to \lambda^e > 0$, it must be the case that $v_{n,t}^e \to v_n^e > 0$ and $\mu_t \to \mu > 0$. Otherwise, the above FOC is violated. As $c_t^e \to 0$ and $n_t^e \to n^e > 0$, $\tau_t \to 1$. The resource constraint then implies

$$c_t^o \to c^o = n^e \frac{\pi(e)}{\pi(o)} > 0. \tag{46}$$

4. The sum of implementability conditions of employed and unemployed participants together with $c_t^e = c_t^u \to 0$ and with $\tau_t \to 1$ imply

$$0 \geq (\beta - 1)(\lim_{t\to\infty} a_t^e \pi(e) + \lim_{t\to\infty} a_t^u \pi(u)) = \lim_{t\to\infty} T_t(\pi(e) + \pi(u)),$$

and hence $\lim_{t\to\infty} T_t \leq 0$. The implementability condition of o agent

$$c^o = \lim_{t\to\infty} T_t.$$

As a result, we get

$$c^o = \lim_{t\to\infty} T_t < 0,$$

which is a contradiction to equation (46).

We then know that case 2 cannot be a long-run Ramsey outcome.

Case 3 Consider the allocation where (i) $c_t^e > c_t^u > 0$ for all $t < \infty$ and (ii) $\lim_{t\to\infty} c_t^e = \lim_{t\to\infty} c_t^u = 0$. We show through the following steps that this allocation is feasible and could satisfy all the optimal conditions provided $\sigma \geq 1$. Hence, there is a possible non-interior Ramsey steady state.

1. This allocation implies the following:

   (a) Given that $c_t^e > c_t^u > 0$ and $a_t^u = 0$ for $t < \infty$ as well as $\lim_{t\to\infty} c_t^u = 0$, the implementability condition of unemployed participants is given by

   $$c_t^u = a_t^e \frac{\pi(e)\pi(u|e)}{\pi(u)} + T_t,$$

   which can be satisfied if $\lim_{t\to\infty} a_t^e = \lim_{t\to\infty} T_t = 0$. Hence, $B_t \to 0$ according to the asset market-clearing condition. The implementability condition of nonparticipants is given by

   $$c_t^o = T_t,$$

which can be satisfied if $\lim_{t\to\infty} c_t^o = 0$. The implementability condition of employed participants is reduced to

$$c_t^e - \frac{v_{n,t}^e}{u_{c,t}^e} n_t^e + Q_{t+1} a_{t+1}^e - a_t^e \pi(e|e) - T_t = 0,$$

which is satisfied in the limit given that $Q_{t+1} a_{t+1}^e \to 0$ and $\lim_{t\to\infty} c_t^e = \lim_{t\to\infty} \frac{v_{n,t}^e}{u_{c,t}^e} n_t^e = 0$. Finally, by these implementability conditions, the convergent rates of c_t^e, c_t^o, c_t^u, a_t^e and T_t have to be the same.

(b) In order to induce $\lim_{t\to\infty} c_t^e = \lim_{t\to\infty} n_t^e = 0$, the tax rate τ_t must converge to 1. Otherwise, it violates the households' FOCs.

(c) The borrowing constraints and complementary slackness conditions for unemployed participants in the Ramsey problem are all trivially satisfied.

2. So far we have shown that this non-interior steady-state allocation can satisfy all constraints of the Ramsey planner problem. Now, we further show that this allocation satisfies all of the Ramsey FOCs by properly choosing convergent properties of the Ramsey multipliers:

   (a) Given that $c_t^e > c_t^u > 0$ for $t < \infty$ and that $a_t^u = 0$, it must be true that $\zeta_t^{u,1} > 0$ and $\zeta_t^{u,2} = 0$ for all $t < \infty$.

   (b) Let $\mu_t \to \infty$, $\lambda_t^e \to \infty$, and $\frac{\mu_t}{\lambda_t^e} \to 0$; the Ramsey FOC with respect to n_t^e in equation (30) is satisfied in the limit as $t \to \infty$:

$$\lim_{t\to\infty} \frac{\mu_t}{\lambda_t^e} = (1+\gamma) \lim_{t\to\infty} v_{n,t}^e = 0.$$

   (c) The FOCs of a_{t+1}^e and a_{t+1}^u in equations (32) and (33) can be rewritten, respectively, as

$$\pi(e|e) + \frac{u_{c,t+1}^u}{u_{c,t+1}^e} \pi(u|e) = \frac{\lambda_{t+1}^e}{\lambda_t^e} \pi(e|e) + \frac{\lambda_{t+1}^u}{\lambda_t^e} \pi(u|e), \tag{47}$$

$$\pi(e|e) + \frac{u_{c,t+1}^u}{u_{c,t+1}^e} \pi(u|e) = \frac{\lambda_{t+1}^u}{\lambda_t^u} \pi(u|u) + \frac{\lambda_{t+1}^e}{\lambda_t^u} \pi(e|u) + \frac{\zeta_t^{u,1} + \zeta_t^{u,3} g_t^u}{\lambda_t^u u_{c,t+1}^e}. \tag{48}$$

Let $\frac{u_{c,t}^u}{u_{c,t}^e}$, $\frac{\lambda_{t+1}^e}{\lambda_t^e}$, $\frac{\lambda_{t+1}^u}{\lambda_t^u}$, $\frac{\lambda_{t+1}^e}{\lambda_t^u}$, and $\frac{\lambda_{t+1}^u}{\lambda_t^e}$ all converge to finite positive constants. Hence, equation (47) is satisfied. In addition, let g_t^u also converge to a finite non-negative constant according to its definition, so equation (48) can be satisfied if $\lim_{t\to\infty}\frac{\zeta_t^{u,1}+\zeta_t^{u,3}g_t^u}{\lambda_t^u u_{c,t+1}^e}$ is chosen to be a finite constant, which is possible and does not lead to contradictions.

(d) The first order condition with respect to T_t can be satisfied by letting

$$\frac{\lambda_t^o}{\lambda_t^e} = -\frac{1}{\pi(o)}(\pi(e) + \frac{\lambda_t^u}{\lambda_t^e}\pi(u)),$$

which implies the growth rate of λ_t^e, λ_t^u and λ_t^o are the same.

(e) The first order condition with respect to c_t^o can be rewritten as

$$\frac{u_{c,t}^o}{\lambda_t^o u_{c,t}^e}(1-\omega) + 1 = \frac{\mu_t}{u_{c,t}^e \lambda_t^o}. \tag{49}$$

Taking the limit of the equation above gives

$$\lim_{t\to\infty}\frac{u_{c,t}^o}{\lambda_t^o u_{c,t}^e} = -\frac{1}{1-\omega},$$

which can be satisfied only if $\frac{u_{c,t}^o}{u_{c,t}^e} \to \infty$ and hence $\frac{c_t^o}{c_t^e} \to 0$.

(f) Given $\zeta_t^1 > 0$, $\zeta_t^2 = 0$, $a_t^u = 0$, $c_t^o = T_t$ and $c_t^u = a_t^e \frac{\pi(e)\pi(u|e)}{\pi(u)}$, the Ramsey FOC with respect to c_t^e can be rewritten as

$$u_{c,t}^e\left(\omega + \lambda_t^e(1-\sigma)\right) + \left(\lambda_{t-1}^e - \lambda_t^e\right) a_t^e u_{cc,t}^e \pi(e|e) - u_{cc,t}^e c_t^o = \mu_t,$$

which can be further transformed to

$$\frac{\omega}{\sigma\lambda_t^e} + \left(1 - \frac{\lambda_{t-1}^e}{\lambda_t^e}\right)\frac{\pi(u)\pi(e|e)}{\pi(e)\pi(u|e)}\frac{c_t^u}{c_t^e} + \frac{1}{\lambda_t^e}\frac{c_t^o}{c_t^e} = \frac{\mu_t}{\sigma\lambda_t^e u_{c,t}^e} + 1 - \frac{1}{\sigma}.$$

As $t \to \infty$, the equation above becomes

$$0 < \left(1 - \lim_{t\to\infty}\frac{\lambda_{t-1}^e}{\lambda_t^e}\right)\lim_{t\to\infty}\frac{c_t^u}{c_t^e} = \frac{\pi(u|e)\pi(e)}{\pi(e|e)\pi(u)}(1 - \frac{1}{\sigma}). \tag{50}$$

Hence, given $\sigma \geq 1$ (a necessary condition), the above FOC can be satisfied and does not lead to contradictions.

(g) Under the conditions that $\zeta_t^1 > 0$, $\zeta_t^2 = 0$, $a_t^u = 0$, and $c_t^u = a_t^e \frac{\pi(e)\pi(u|e)}{\pi(u)}$, the Ramsey

FOC with respect to c_t^u can be simplified to

$$u_{c,t}^u \omega + \lambda_t^u u_{c,t}^e + \lambda_{t-1}^e c_t^u u_{cc,t}^u = \mu_t,$$

which can be rewritten as

$$\frac{\omega}{\lambda_{t-1}^e} + \frac{\lambda_t^u}{\lambda_{t-1}^e} \frac{u_{c,t}^e}{u_{c,t}^u} - \sigma = \frac{\mu_t}{\lambda_t^e} \frac{\lambda_t^e}{\lambda_{t-1}^e} \frac{1}{u_{c,t}^u}.$$

Since $\lim_{t\to\infty} \frac{\mu_t}{\lambda_t^e} \frac{\lambda_t^e}{\lambda_{t-1}^e} \frac{1}{u_{c,t}^u} = 0$, in the limit the equation above becomes

$$\lim_{t\to\infty} \frac{\lambda_t^u}{\lambda_{t-1}^e} \frac{u_{c,t}^e}{u_{c,t}^u} = \sigma. \tag{51}$$

We then have two sub-cases to consider:

i. $\sigma > 1$. Equation (51) can be satisfied if $\frac{\lambda_t^u}{\lambda_{t-1}^e}$ converges to a finite positive constant. From 2(c), we know that $\frac{u_{c,t}^e}{u_{c,t}^u}$ also converges to a finite positive constant given the convergence of $\frac{c_t^u}{c_t^e}$.

ii. $\sigma = 1$. Equation (51) can be satisfied if both $\frac{\lambda_t^u}{\lambda_{t-1}^e}$ and $\frac{c_t^u}{c_t^e}$ converge to finite positive values.

Hence, both sub-cases above are possible and do not lead to contradictions.

A.5 Proof of Proposition 3

The proof is done by construction. Conjecture that the Ramsey planner can achieve the first best allocation starting from period 0. In other words, all constraints except the resource constraint do not bind: $\lambda_t^e = \lambda_t^u = 0$ and $\zeta_t^{u,1} = \zeta_t^{u,2} = \zeta_t^{u,3} = 0$ for all t. It is then straightforward to see that the Ramsey planner problem becomes the social planner problem.

Given the first-best allocation, the following steps show that the Ramsey planner can choose the corresponding policy in order to achieve the first best allocation.

1. Given that $v_n^{e,FB} = u_c^{e,FB}$, the optimal labor tax is set to zero for all $t \geq 0$:

$$\tau_t = 1 - \frac{v_{n,t}^{e,FB}}{u_{c,t}^{e,FB}} = 0$$

2. Given $u_{c,t}^{e,FB} = u_{c,t+1}^{e,FB} = u_c^{e,FB}$, $Q_{t+1} = \beta \frac{u_{c,\,t+1}^{FB}}{u_{c,\,t}^{FB}} = \beta$. As a result, the government budget constraint is then reduced into

$$\beta B_{t+1} - B_t = T_t, \tag{52}$$

which implies that a steady state lump sum tax, $T = (\beta - 1)B < 0$.

3. Consider the case where $a_{t+1}^u = 0$ for all $t \geq 0$. We show the sequences of T_t and a_{t+1}^e can be chosen to satisfy the implementability conditions in the following:

   (a) The T_0 is chosen to satisfies the implementability condition of unemployed participants at period 0:

   $$T_0 = c_0^{FB} - a_0^u$$

   (b) a_1^e is chosen such that the implementability condition of employed family members is satisfied:

   $$\beta \frac{a_1^e}{\pi(e)} = \frac{a_0^e}{\pi(e)} + T_0 + n_0^{e,FB} - c_0^{e,FB},$$

   which is strictly positive given $\frac{a_0^e}{\pi(e)} > \frac{a_0^u}{\pi(u)}$.

   (c) Given a_1^e, for all $t \geq 1$, the T_t and a_{t+1}^e are chosen such that

   $$T_t = c^{u,FB} - \frac{a_t^e \pi(e)\pi(u|e)}{\pi(u)},$$

   and

   $$\beta a_{t+1}^e = a_t^e \pi(e|e) + T_t + n_t^{e,FB} - c_t^{e,FB},$$

   which are the implementability condition of unemployed and employed individuals, respectively.

   (d) The government budget constraint (52) together with the implementability condition of employed imply

   $$\beta B_{t+1} - B_t = T_t = c^{e,FB} - \frac{B_t \pi(u|e)}{\pi(u)},$$

   which can be further expressed as

   $$B_{t+1} = \frac{1}{\beta}\left(1 - \frac{\pi(u|e)}{\pi(u)}\right) B_t + \frac{1}{\beta} c^{e,FB}.$$

Hence, for B_t to converge to a finite positive number, it requires

$$1 - \frac{\pi(u|e)}{\pi(u)} < \beta,$$

which is exactly the fiscal-space condition.

Therefore, as shown by the steps above, the first-best allocation can be implemented by the Ramsey planner.

A.6 Proof of Proposition 4

A.6.1 Ramsey Problem

Under the assumption that $\pi(u) = 0$, $\pi(u|e) = 0$ and $\pi(e|e) = 1$, the Ramsey planner problem is simplified into

$$\max_{\{c_t^e, c_t^o, n_t^e, a_{t+1}^e\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{\omega \left[u(c_t^e) - v\left(n_t^e\right)\right] \pi(e) + (1-\omega) u(c_t^o)\pi(o)\right\}$$

subject to

$$\beta^t \mu_t : n_t^e \pi(e) - c_t^e \pi(e) - c_t^o \pi(o) \geq 0, \ \forall t \geq 0.$$

$$\beta^t \lambda_t^e \pi(e) : u_{c,t}^e c_t^e - v_{n,t}^e n_t^e + \beta u_{c,t+1}^e a_{t+1}^e - u_{c,t}^e a_t^e - u_{c,t}^e c_t^o = 0, \ \forall t \geq 0$$

where $Q_{t+1} u_{c,t}^e$ are

$$Q_{t+1} u_{c,t}^e = \beta u_{c,t+1}^e.$$

A.6.2 Ramsey FOCs

For all $t \geq 0$, the FOCs of the Ramsey problem with respect to a_{t+1}^e, and n_t^e are given, respectively, by

$$\lambda_t^e = \lambda_{t+1}^e, \tag{53}$$

and

$$\omega v_{n,t}^e + \lambda_t^e v_{n,t}^e (1+\gamma) = \mu_t. \tag{54}$$

The FOCs of the Ramsey problem with respect to c_t^e and c_t^o are given, respectively, by

$$u^e_{c,t}\omega + \lambda^e_t u^e_{c,t}(1-\sigma) - \lambda^e_t u^e_{cc,t}(a^e_t + c^o_t) + \lambda^e_{t-1} a^e_t u^e_{cc,t} = \mu_t \tag{55}$$

$$u^o_{c,t}(1-\omega)\pi(o) = \mu_t \pi(o) + \lambda^e_t u^e_{c,t}\pi(e). \tag{56}$$

A.6.3 Ramsey Outcome

The FOCs imply that $\lambda^e_t = \lambda^e_{t+1} = \lambda^e$ for all t. The constant λ^e together with Ramsey FOCs imply a static Ramsey outcome starting at period $t = 1$. We then conjecture and verify that the Ramsey outcome has the following properties:

1. The Ramsey outcome jumps to its steady state at period 1.
2. $Q_{t+1} = \beta$ for all $t \geq 1$
3. $a^e_t = a^e$ for all $t \geq 1$

As a result, given a^e, the Ramsey allocation, $(c^e_t, c^o_t, n^e_t, \lambda^e_t, \mu_t)$, for each period $t \geq 1$, can be solved by following five equations:

1. The three FOCs with respect to n^e_t, c^e_t and c^o_t :

$$v^e_{n,t}(1 + \lambda^e(1+\gamma_n)) = \mu_t,$$

$$u^e_{c,t}\omega + \lambda^e u^e_{c,t}(1-\sigma) - \lambda^e u^e_{cc,t} c^o_t = \mu_t,$$

   and

$$u^o_{c,t}(1-\omega)\pi(o) = \mu_t \pi(o) + \lambda^e u^e_{c,t}\pi(e)$$

2. The resource constraint and implementability condition for e individuals:

$$n^e_t \pi(e) - c^e_t \pi(e) - c^o_t \pi(o) \geq 0,$$

   and

$$c^e_t - \frac{v^e_{n,t}}{u^e_{c,t}} n^e_t + (\beta - 1)a^e - c^o_t = 0$$

3. Note that there is no dynamic link among these five equations listed above and hence the Ramsey allocation reaches its steady state starting at period 1. The optimal labor tax rate is constant given that $\tau_t = 1 - \frac{v^e_{n,t}}{u^e_{c,t}} = 1 - \frac{v^e_n}{u^e_c} = \tau$.

4. The steady state a^e is pinned down by the set of FOCs and constraints at period 0 as well as the initial a^e_0.

A.7 Proof of Proposition 5

To maximize the steady-state welfare of the economy, the Ramsey problem under the primal approach becomes:

$$\max_{\{c^e,c^u,c^o,n^e,a^e,a^u\}} \omega\left[u(c^e) - v\left(n^e\right)\right]\pi(e) + \omega u(c^u)\pi(u) + (1-\omega)u(c^o)\pi(o)$$

subject to

$$\mu : n^e\pi(e) - c^e\pi(e) - c^u\pi(u) - c^o\pi(o) \geq 0, \tag{57}$$

$$\lambda^e : u^e_c c^e\pi(e) - v^e_n n^e\pi(e) + a^e\beta(u^e_c\pi(e|e) + u^u_c\pi(u|e))\pi(e) - u^e_c(a^e\pi(e)\pi(e|e) + a^u\pi(u)\pi(e|u)) - u^e_c c^o\pi(e) = 0, \tag{58}$$

$$\lambda^u : u^e_c c^u\pi(u) + a^u\beta(u^e_c\pi(e|e) + u^u_c\pi(u|e))\pi(u) - u^e_c(a^e\pi(e)\pi(u|e) + a^u\pi(u)\pi(u|u)) - u^e_c c^o\pi(u) = 0, \tag{59}$$

$$\zeta^{u,1} : a^u \geq 0, \tag{60}$$

$$\zeta^{u,2} : g^u(c^e, c^u) \equiv \pi(e|e) + \frac{u^u_c}{u^e_c}\pi(u|e) - \frac{u^e_c}{u^u_c}\pi(e|u) - \pi(u|u) \geq 0, \tag{61}$$

$$\zeta^{u,3} : a^u g^u(c^e, c^u) = 0. \tag{62}$$

A.7.1 Ramsey FOCs

Denote $\mu, \lambda^e, \lambda^u, \lambda^o$, $\zeta^{u,1}$, $\zeta^{u,2}$ and $\zeta^{u,3}$ as the Lagrangian multipliers for constraints (57) to (62), respectively. The FOCs of the Ramsey problem with respect to n^e, a^e and a^u are given, respectively, by

$$v^e_n(\omega + \lambda^e(1+\gamma)) = \mu, \tag{63}$$

$$\lambda^e\beta(u^e_c\pi(e|e) + u^u_c\pi(u|e)) = \lambda^e u^e_c\pi(e|e) + \lambda^u u^e_c\pi(u|e), \tag{64}$$

and

$$\lambda^u \beta(u_c^e \pi(e|e) + u_c^u \pi(u|e)) = \lambda^u u_c^e \pi(u|u) + \lambda^e u_c^e \pi(e|u) + \zeta^{u,1} + \zeta^{u,3} g(c^e, c^u). \tag{65}$$

The FOCs of the Ramsey problem with respect to c^e, c^u and c^o are given, respectively, by

$$\begin{aligned}
& (u_c^e \omega - \mu)\pi(e) + \lambda^e u_c^e (1-\sigma)\pi(e) - \lambda^e u_{cc}^e (a^e \pi(e)\pi(e|e) + a^u \pi(u)\pi(e|u) + c^o \pi(e)) \\
& + \lambda^e \beta a^e u_{cc}^e \pi(e)\pi(e|e) + \lambda^u u_{cc}^e c^u \pi(u) - \lambda^u u_{cc}^e (a^e \pi(e)\pi(u|e) + a^u \pi(u)\pi(u|u) + c^o \pi(u)) \\
& + \lambda^u \beta a^u u_{cc}^e \pi(u)\pi(e|e) + \beta \zeta^{u,2} \frac{\partial g^u}{\partial c^e} + a^u \zeta^{u,3} \frac{\partial g^u}{\partial c^e} \\
= \; & 0,
\end{aligned} \tag{66}$$

$$\begin{aligned}
& (u_c^u \omega - \mu)\pi(u) + \lambda^u u_c^e \pi(u) + \lambda^u \beta a^u u_{cc}^u \pi(u)\pi(u|e) + \lambda^e \beta a^e u_{cc}^u \pi(e)\pi(u|e) \\
& + \zeta^{u,2} \frac{\partial g^u}{\partial c^u} + a^u \zeta^{u,3} \frac{\partial g^u}{\partial c^u} = 0,
\end{aligned} \tag{67}$$

and

$$(u_c^o(1-\omega) - \mu)\pi(o) - \lambda^e u_c^e \pi(e) - \lambda^u u_c^e \pi(u) = 0. \tag{68}$$

A.7.2 No FSI

We then show that the Ramsey outcome cannot feature FSI. Namely, it must be the case that $a^u = 0$ or $\zeta^{u,1} > 0$, $c^e > c^u$ and $g^u(c^e, c^u) > 0$. The proof is done by contradiction. Consider a Ramsey steady state featuring full self-insurance; that is, $a^u \geq 0$ or $\zeta^{u,1} = 0$, $c^e = c^u$ and $g^u(c^e, c^u) = 0$. Then, Ramsey FOC, equation (64), is simplified as

$$\lambda^e(\beta - \pi(e|e)) = \lambda^u \pi(u|e),$$

which implies

$$\frac{\lambda^u}{\lambda^e} = \frac{\beta - \pi(e|e)}{1 - \pi(e|e)} < 1. \tag{69}$$

However, the Ramsey FOC with respect to a^u, equation (65), becomes

$$\lambda^u(\beta - \pi(u|u)) = \lambda^e \pi(e|u),$$

which can be written as

$$\frac{\lambda^u}{\lambda^e} = \frac{1 - \pi(u|u)}{\beta - \pi(u|u)} > 1,$$

thus contradicting with equation (69). Hence, $c^e = c^u$ cannot be the outcome of this Ramsey problem of maximizing the steady-state welfare of the competitive equilibrium.