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On the Economic Mechanics of Warfare*

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Abstract

How do war-related expenditures affect economically-relevant outcomes at a war's conclusion (e.g., prevailing side, duration, and casualties)? I present a model of attrition and characterize the effects of resources at a military conclusion (one side cannot fight anymore) and a political conclusion (one side quits). I analyze the Pacific War through the lenses of the model both theoretically and empirically. I find that a parsimonious parameterization reproduces well the aggregate patterns of destruction, measured in ship tonnage, for both belligerents.

JEL: E6, H56, N4
Keywords: War; Attrition; Military spending.

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1 Introduction

War is a frequent and costly activity of mankind. In this paper, I examine the role of aggregate resources in determining how wars evolve and conclude. More specifically, I analyze how the resources allocated to wars by belligerents affect economically-relevant outcomes such as the duration, the destruction inflicted on each side, the nature of the conclusion, and the prevailing side at the time of conclusion.

My analysis is inspired by two strands of literature. First, military historians (e.g. O'Brien, 2015, Nolan, 2017) have argued against a battle-centric view of wars and in favor of analyzing wars as processes of mutual attrition.[1] Second, combat models developed in Operations Research since Lanchester (1916)'s seminal work are systems of differential equations describing the attrition of opposing forces during battles. My approach is to use a Lanchester-type model and interpret it as a representation of war as a whole.

I assume a state of war to exist between two countries. The countries are endowed with initial "weapons" stocks and technologies that use weapons to destroy their opponent's weapons. The countries also receive per-period exogenous flows of weapons ("reinforcements"). I interpret the initial stocks as resulting from prewar investments, and reinforcements as resulting from wartime expenditures. These are the resources the effects of which I analyze.

The destruction experienced by countries is mitigated by their reinforcements and the destruction they inflict on their opponents. The model's outcome is thus the joint dynamics of weapons accumulation and destruction. I discuss two possible conclusions: In the first, which I label "military," the war concludes when a country's weapons stock reaches an exogenously-determined low threshold and the country cannot fight anymore. In the second, which I label "political," the war concludes when a country's casualties reach an

[1] O'Brien's first sentence, for example, is "*There were no decisive battles in World War II*" and Nolan, for his part, insists that attrition is key to understanding the outcomes of conflict from the Punic wars to the World Wars of the 20th century.

exogenously-determined high threshold and political forces request that the fighting ends.

Literature

Some questions economists have addressed about wars deal with its financing (e.g., Hall and Sargent, 2021), its causes studied via game theoretical methods (e.g., Kimbrough et al., 2020) or via empirical methods (e.g., Blattman and Miguel, 2010), and its consequences for an array of issues such as fertility (e.g., Caldwell, 2004), the marriage market (e.g., Abramitzky et al., 2011), the formation of states (e.g., Fernández-Villaverde et al., 2023), etc. I do not address these questions.

Instead, my work contributes to the defense economics literature. An often cited result of Lanchester's theory is a set of "laws", such as the square law, the linear law, etc. (see Epstein, 1985). These have been used for two distinct, albeit related, purposes in defense economics. First, they provide a theory of how to measure military effectiveness as a function of the number of military units and the quality of an average unit.[2] Hildebrandt (1999), for instance, argues that the Lanchester laws are akin to production functions, i.e., mappings from military inputs to military effectiveness, which are useful for cost-benefit analysis in defense related matters. Pugh (1993) and Kirkpatrick (1995) use the Lanchester laws to analyze how countries maximize equipment effectiveness under a budget. Second, the Lanchester laws provide a theory of the prevailing side in a military conclusion: The side with the highest combat effectiveness prevails. Hirshleifer (1991, 2000) notes that, as such, the Lanchester laws yield success functions akin to those from the contest literature, i.e., mappings from resources to the probability that one side prevails. Anderton (1990) and Anderton and Carter (2007), for instance, use Lanchester laws to assess the conditions under which a country would prevail after attacking another country if the attacker always had an advantage.

[2] Under the square law, effectiveness is the product of quality and the square of quantity.

I complement this literature to the extent that my objective is the same as that of Hirshleifer (2000): It is to address the “macrotechnology of conflict.” For armed conflicts, thus, it amounts, as I indicated earlier, to analyzing the mapping from aggregate resources to the outcomes of wars or, for short, the aggregate technology of destruction. Unlike Hirshleifer, however, I interpret Lanchester’s equations as an aggregate technology and exploit their entire dynamics, not just the condition for one side to prevail, i.e., not just the square law.[3] Thus, I discuss how aggregate resources determine the paths of casualties, the duration of war, which side losses in a military conclusion, which side sues for peace first in a political conclusion, and which type of conclusion is likely to prevail.[4]

My work also relates to the Operations Research (OR) literature where the Lanchester equations are often used. Taylor (1980a,b) and Caldwell et al. (2000) provides comprehensive presentations. But OR is an applied discipline aiming to improve decision making (see Morse and Kimball, 1950). Hence, in military matters, it is to a large extent concerned with producing tactical insights. The tactical insight from the Lanchester equations I use is well known to OR scholars: the principle of force concentration emanating from the square law. I offer no tactical insights because my focus is on aggregate destruction.

Insofar as OR scholars have confronted the Lanchester model with data, however, I complement their work by proposing another confrontation of the model with data. In line with my focus on aggregate destruction, I use data for an entire war, namely the Pacific War, instead of a single battle. So, my approach differs from the OR literature, whose interest in tactical questions is illustrated by attempts at validating the Lanchester equations mostly from individual battle data: The battle of Gettysburg (Pennsylvania, Jul. 1863) was studied by Armstrong and Sodergren (2015). The battle of Iwo Jima (Bonin Islands, Feb.-Mar. 1945) was studied by Engel (1954), Samz (1972),

[3] Hirshleifer (2000, p. 791) states explicitly his interpretation of Lanchester’s equations as describing the “microtechnology” of combat.

[4] These considerations are not ignored by Hirshleifer, but they are mentioned as limitations of the Lanchester model (Hirshleifer, 2000, p. 784 and fn. 16).

and Stymfal (2022). The Ardennes campaign (Belgium, Dec. 1944-Jan. 1945) was studied by Bracken (1995), Fricker (1998), Chen and Chu (2001), and Lucas and Turkes (2004). The battle of Kursk (Russia, Jul.-Aug. 1943) was studied by Lucas and Turkes (2004) and Dinges (2001). The battle of the Atlantic was studied by Morse and Kimball (1950).

Roadmap

I analyze the model in Section 2. In 2.1 and 2.2, I present the setup, the solution, and the dynamics of weapons stocks and casualties. In 2.3, I define a military conclusion. I explain the determination of the prevailing side, the duration of war, and how final casualties depend on resources. In 2.4, I define a political conclusion and explain how resources matter for the duration of war and casualties at a political conclusion.

In Sections 3 and 4 I use the model to discuss the Pacific War, which opposed Japan to the U.S. and its allies, from December 1941 to August 1945, in the larger context of World War 2. Section 3 is theoretical. I discuss the conditions under which a political conclusion prevails over a military conclusion, or vice versa. I frame the discussion in a context where the belligerents differ in ways that are stylized representation of the differences between the U.S. and Japan during the Pacific War. Section 4 is empirical. I discuss data and present an application of the model to the war as a whole.

2 The model

2.1 Setup

The model is a version of Lanchester (1916)'s model. Time is continuous and there is no uncertainty. There are two countries, Red and Blue, with weapons stocks denoted K_t^R and K_t^B, respectively. A country's weapons stock is an input used to inflict a flow of destruction on the opposite side's weapons stock. Let $\theta^R > 0$ denote Red's attrition coefficient, which is the flow of Blue weapons destroyed per Red weapon at each point in time. Blue's attrition coefficient, $\theta^B > 0$, has a similar interpretation. Let $X^R \geq 0$ and $X^B \geq 0$ denote constant reinforcement flows at each point in time. The laws of motion for K_t^R and K_t^B are

$$
\begin{aligned}
dK_t^R/dt &= -\theta^B K_t^B + X^R, &(1)\\
dK_t^B/dt &= -\theta^R K_t^R + X^B, &(2)
\end{aligned}
$$

where the initial conditions K_0^R and K_0^B are given. I use the term "resources" to refer to initial weapons stocks and/or reinforcements.

A weapon is a combination of human and physical capital, e.g., soldier & rifle or aircraft & crew. I assume complementarity between human and physical capital and, hence, interpret "destruction" as both casualties and material destruction. (I use these terms interchangeably). I do not distinguish between lethal and non-lethal casualties. It is conceivable that one aircraft & crew effects as much destruction as many soldiers & rifles, suggesting substitutability between different weapon types. I consider an aggregate weapon for simplicity.

Note the assumption that the flow of Blue casualties, $\theta^R K_t^R$, is independent of the stock of Blue weapons (and vice versa). Thus, additional Blue weapons are not destroyed by Red when the Red stock is constant. This assumption, referred to as "aimed fire" in the Operations Research literature (e.g., Taylor, 1980a), can be opposed to an "area fire" assumption, where the flow of Blue casualties increases with Blue's stock. I assume aimed fire for simplicity.

I assume no destruction other than that of weapons and, thus, abstract from some form of destruction found in war: First, there is no destruction of the productive capacities of a country. Second, there are no civilians casualties. I present, in Appendix E, a version of the model with civilian casualties and show that the dynamics are isomorphic to that of the model without civilians.

There are no decisions. The model represents the mechanics of attrition in a manner similar to the Solow model's representation of the mechanics of capital accumulation. Adding decisions would be a natural extension. Yet, absent a theory of why there is a war, it is difficult to assign objective functions to the belligerents. My goal is to describe tradeoffs that decision makers would have to internalize regardless of their objectives.

The steady state $(\bar{K}^R, \bar{K}^B)$ of system (1)-(2) is a stalemate where each country's reinforcements are destroyed by its opponent: $\bar{K}^R = X^B/\theta^R$ and $\bar{K}^B = X^R/\theta^B$. Define $\tilde{K}_t^R = K_t^R - \bar{K}^R$ and $\tilde{K}_t^B = K_t^B - \bar{K}^B$. Then, (1)-(2) become

$$\begin{pmatrix} d\tilde{K}_t^R/dt \\ d\tilde{K}_t^B/dt \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -\theta^B \\ -\theta^R & 0 \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} \tilde{K}_t^R \\ \tilde{K}_t^B \end{pmatrix}. \tag{3}$$

Let $\pm\lambda$ be the eigenvalues of $\mathcal{M}$ with corresponding eigenvectors $[1, \pm v]'$, where $\lambda = -\sqrt{\theta^R\theta^B}$ and $v = \sqrt{\theta^R/\theta^B}$.

2.2 Dynamics

Weapons stocks Standard methods (Appendices A) yield the solution

$$\tilde{K}_t^R = \frac{1}{2}\left[e^{t\lambda}\mathcal{A} - e^{-t\lambda}\mathcal{B}\right]\frac{1}{v}, \tag{4}$$

$$\tilde{K}_t^B = \frac{1}{2}\left[e^{t\lambda}\mathcal{A} + e^{-t\lambda}\mathcal{B}\right], \tag{5}$$

where the constant $\mathcal{A}$ and $\mathcal{B}$ depend on initial conditions:

$$\mathcal{A} = \tilde{K}_0^B + v\tilde{K}_0^R \quad \text{and} \quad \mathcal{B} = \tilde{K}_0^B - v\tilde{K}_0^R.$$

Recall that $\lambda < 0$, so the stalemate is a saddle-point. Note also, from Equation (3), that $dK_t^B/dt > 0$ whenever $K_t^R < \bar{K}^R$ because Red does not offset Blue reinforcements and, thus, K_t^B increases. Conversely, if K_t^R is above its stalemate, Blue reinforcements do not offset the destruction caused by Red

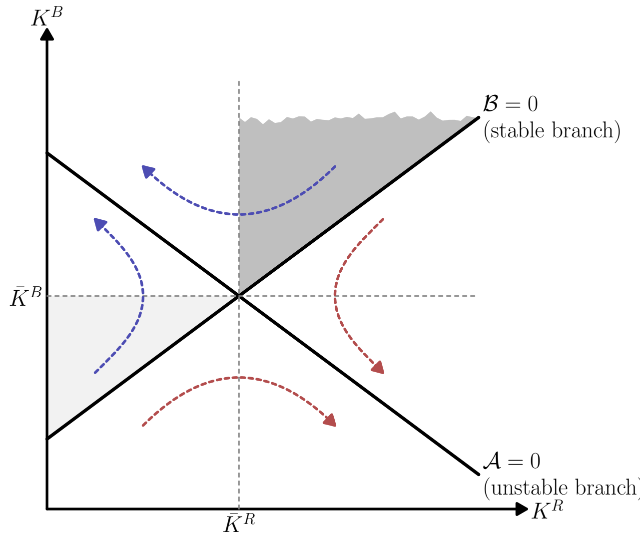


Figure 1: Weapons stocks dynamics

Note: The figure shows the phase diagram of system (1)-(2). The stalemate is at $(\bar{K}^R, \bar{K}^B)$. The dotted-blue (dotted-red) arrows indicate trajectories leading to a Blue (Red) military victory. Above the stable branch: (i) The light-gray (dark-gray) area indicates initial conditions such that Red's (Blue's) weapons stock's trajectory is $\cap$-shaped ($\cup$-shaped). (ii) The non-shaded area indicates initial conditions such that all trajectories are monotonic.

and, thus, K_t^B decreases. The same logic applies to the Red weapons stock. Figure 1 summarizes these observations in a phase diagram.

Weapons stocks' trajectories need not be monotonic. The blue arrow starting off in the light-shaded area of Figure 1 represents a case where both weapons stocks are initially below their stalemate: The Blue stock increases monotonically; the Red stock increases until $K_t^B = \bar{K}^B$, when $t = \ln(-\mathcal{B}/\mathcal{A})/(2\lambda)$, and then decreases. It is the reverse with the blue arrow starting off in the dark-shaded area: The Red stock decreases monotonically; the Blue stock decreases until $K_t^R = \bar{K}^R$, when $t = \ln(\mathcal{B}/\mathcal{A})/(2\lambda)$, and then increases. With initial conditions in the northwest quadrant, stocks evolve monotonically.

Casualties The flows of casualties are $d_t^B = \theta^R K_t^R$ for Blue and $d_t^R = \theta^B K_t^B$ for Red. Their dynamics mimic the weapons stocks dynamics, so they need not be monotonic. I show (Appendix A) that, at any time t, the Red stock is decreasing with the initial Blue stock and with Blue reinforcements. This follows from the "aimed fire" assumption (Section 2.1): Given the Red stock, additional Blue weapons are not exposed to destruction, but they destroy Red weapons. This, in turn, implies fewer Blue destruction. It follows that d_t^B is decreasing in K_0^B and X^B:

$$\frac{\partial K_t^R}{\partial X^B} < 0 \Rightarrow \frac{\partial d_t^B}{\partial X^B} < 0 \quad \text{and} \quad \frac{\partial K_t^R}{\partial K_0^B} < 0 \Rightarrow \frac{\partial d_t^B}{\partial K_0^B} < 0. \tag{6}$$

I also show (Appendix A) that the Blue stock is increasing with K_0^B and X^B, implying that d_t^R is increasing as well:

$$\frac{\partial K_t^B}{\partial X^B} > 0 \Rightarrow \frac{\partial d_t^R}{\partial X^B} > 0 \quad \text{and} \quad \frac{\partial K_t^B}{\partial K_0^B} > 0 \Rightarrow \frac{\partial d_t^R}{\partial K_0^B} > 0. \tag{7}$$

Symmetric results hold for the effect of Red resources on d_t^B and d_t^R. Thus a country's flow of casualties is decreasing in the resources the country commits to the war and increasing in the resources committed by its opponent.

Let D_t^R and D_t^B denote total casualties at t for Red and Blue, respectively:

$$D_t^R = \int_0^t d_u^R du \quad \text{and} \quad D_t^B = \int_0^t d_u^B du.$$

In the remainder of the paper I refer to d_t^B and d_t^R as "flow-casualties" and to D_t^B and D_t^R as "casualties." I show (Appendix A) that

$$D_t^R = tX^R + K_0^R - K_t^R, \tag{8}$$
$$D_t^B = tX^B + K_0^B - K_t^B. \tag{9}$$

A country's casualties at t are the initial stock plus the reinforcements committed until t, net of the remaining stock. Casualties at the war's conclusion depend on how and when the war concludes.

2.3 Military conclusion

I adopt the following definition: A "military" conclusion is when the war ends because a belligerent's weapons stock reaches a critically low, exogenously determined, threshold. I assume the threshold is zero. Not all wars end with a military conclusion, but World War 2 is an example. The fighting ability of both Germany and Japan was close to nil by the end of the war.

The prevailing side Figure 1 illustrates the condition under which a military conclusion is a victory for Blue or for Red, or is a stalemate. Initial conditions above the stable branch imply that the Red stock eventually reaches 0, while the Blue stock reaches a positive value. This can be seen from Equations (4) and (5) since $e^{t\lambda}$ converges to 0 while $e^{-t\lambda}$ diverges. It follows that, when $\mathcal{B} > 0$, $\tilde{K}_t^B$ eventually increases while $\tilde{K}_t^R$ eventually decreases. Thus,

$$\begin{aligned} \mathcal{B} > 0 &\Rightarrow \text{Blue victory,} \\ \mathcal{B} < 0 &\Rightarrow \text{Red victory,} \\ \mathcal{B} = 0 &\Rightarrow \text{stalemate.} \end{aligned}$$

The condition for a Blue military victory can be expressed as

$$\sqrt{\theta^B} K_0^B + X^B / \sqrt{\theta^R} > \sqrt{\theta^R} K_0^R + X^R / \sqrt{\theta^B}. \tag{10}$$

Following the Operations Research literature, I refer to the units in Equation (10) as "fighting strength." The general form for fighting strength is

$$\sqrt{\text{attrition coefficient}} \times \text{quantity of weapon.}$$

The Blue fighting strength (left-hand side of 10), is the sum of that arising from the initial weapons stock, $\sqrt{\theta^B} K_0^B$, and that arising from reinforcements,

$X^B/\sqrt{\theta^R}$. Note that the latter is in the same units as it can be written $\sqrt{\theta^R}\bar{K}^R$. Red's fighting strength is defined in the same manner.[5]

Equation (10) is a modified version of the so-called Lanchester Square Law and deserves some comments. Consider the case where $X^B = X^R = 0$. The Blue fighting strength is then $\sqrt{\theta^B}K_0^B$, which increases faster with the weapons stocks than with the attrition coefficient. That is because additional Blue weapons destroy Red weapons and dilute Red's ability to attrit Blue. A higher Blue attrition coefficient serves the first purpose but not the second. (The same logic applies for Red). This argument is an implication of "aimed fire" technology (Equations 1 and 2): Since the destruction experienced by one side is independent of its weapons stock, fielding more weapons has no adverse effect. The property that the size of the weapons stocks matters more than the quality (measured by the attrition coefficient) is the main tactical insight of the Lanchester model and has often been viewed as a rationalization of the practice of concentrating forces.

In the general case, when $X^R, X^B \geq 0$, there is an additional benefit from a higher attrition coefficient: It reduces the opposing force's fighting strength arising from reinforcements. A higher Blue attrition coefficient, for instance, reduces the contribution of Red reinforcements to Red fighting strength. That is because Blue destroys Red reinforcements with a lower stalemate stock, and, thus, it is easier for Blue to exceed its stalemate and attrit Red.

Duration of war I assume, for the remainder of this section, that $\mathcal{B} > 0$, so that Blue is poised to obtain a military victory at τ^M:

$$K^R_{\tau^M} = 0. \tag{11}$$

[5] In the defense economics literature, fighting strength is sometime called "effectiveness" (e.g. Kirkpatrick, 1995).

I show (Appendix B.1) that

$$\frac{\partial \tau^M}{\partial X^B} < 0, \qquad \frac{\partial \tau^M}{\partial X^R} > 0, \qquad \frac{\partial \tau^M}{\partial K_0^B} < 0, \qquad \frac{\partial \tau^M}{\partial K_0^R} > 0. \tag{12}$$

The duration of war before a military conclusion, τ^M, is decreasing in the resources of the country obtaining the military victory, and increasing in the resources of its opponent.

When Blue allocates more resources to the war, via K_0^B and/or X^B, Blue's ability to attrit Red is heightened and the Red stock depletes faster. Hence, the war is shorter. Recall that the additional Blue resources are not causing more Blue casualties because of the aimed fire technology. If Red allocates more resources to the war while $\mathcal{B}$ remains positive, Blue suffers additional flow-casualties. This slows down Blue's ability to attrit Red, and, therefore, the war takes longer before Blue prevails militarily.

The results in (12) imply that the level curves of τ^M are straight lines,

$$\left.\frac{dK_0^B}{dK_0^R}\right|_{d\tau^M=0} = -\frac{v}{\tanh(\tau^M \lambda)} \tag{13}$$

where tanh is the hyperbolic tangent. It is immediate, from the properties of the hyperbolic tangent, that the level curves (i) are increasing, (ii) are steeper than the stable branch whose slope is v, and (iii) have a slope that decreases with τ^M. Point (i) indicates that Blue needs a larger initial stock to offset an increase in Red's initial stock and maintain the duration of war constant. Point (ii) indicates that such increase in K_0^B cannot "just" maintain the condition for a military victory, i.e., $d\mathcal{B} = 0$, as this would require extra time to destroy the additional Red weapons. Point (iii) indicates that, as the war gets shorter, each additional Red weapon must be destroyed faster for the duration of war to be constant. The level curves in the (X^R, X^B) plane have similar properties.

Casualties Casualties at τ^M follow from Equations (8) and (9):

$$D^R_{\tau^M} = \tau^M X^R + K^R_0 \quad \text{and} \quad D^B_{\tau^M} = \tau^M X^B + K^B_0 - K^B_{\tau^M}. \tag{14}$$

Red loses all the resources it commits to the war. Blue casualties are mitigated by the end-of-war Blue weapons stock. I show (Appendix B.2) that

$$\frac{\partial D^R_{\tau^M}}{\partial X^B} < 0, \qquad \frac{\partial D^R_{\tau^M}}{\partial X^R} > 0, \qquad \frac{\partial D^R_{\tau^M}}{\partial K^B_0} < 0, \qquad \frac{\partial D^R_{\tau^M}}{\partial K^R_0} > 0, \tag{15}$$

and

$$\frac{\partial D^B_{\tau^M}}{\partial X^B} < 0, \qquad \frac{\partial D^B_{\tau^M}}{\partial X^R} > 0, \qquad \frac{\partial D^B_{\tau^M}}{\partial K^B_0} < 0, \qquad \frac{\partial D^B_{\tau^M}}{\partial K^R_0} > 0. \tag{16}$$

That is, in a Blue military victory, Blue casualties are reduced by Blue resources committed to the war and increased by Red resources. Red casualties behave in the **same** (not symmetric) manner. That is, Red casualties are also reduced by Blue resources and increased by Red resources.

To understand these results, consider the effects of an increase in the initial stock of Blue weapons, from K^B_0 to $K^B_{0,\text{new}}$, illustrated in Figure 2. Panel A represents Blue flow-casualties (the solid and dashed lines) and Blue casualties (the areas under the lines). Blue flow-casualties converge to zero because the stock of Red weapons converges to zero in a Blue military victory. Recall that Blue flow-casualties are lower at each point in time when K^B_0 is higher (Equation 6), implying that the dashed line is below the solid line. Thus, the light-shaded area in Panel A indicates a reduction in Blue casualties. This reduction combines two effects acting in the same direction: the lower flow-casualties at each point in time and the shorter war (Equation 12).

Panel B represents Red flow-casualties and Red casualties. Two effects operate in opposite directions. First, Red flow-casualties are higher at each point in time (Equation 7). Second, the war is shorter. The dark-shaded area represents the increase in Red casualties due to the first effect. The light-shaded area represents the decrease due to the second effect. In the end, the second effect dominates, as indicated in Equation (15). Thus, in a Blue

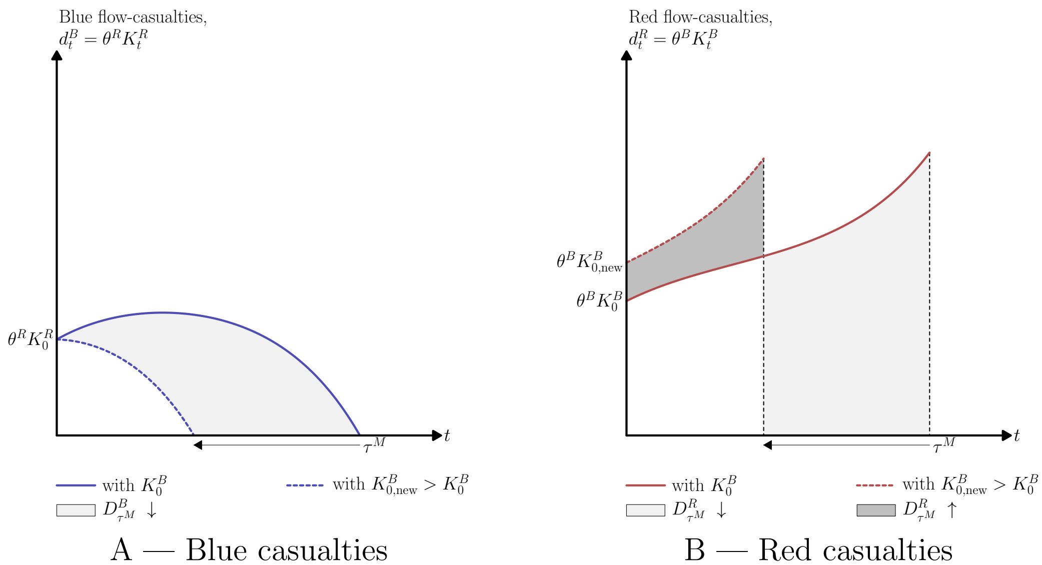


Figure 2: The effect of K_0^B on casualties when $\mathcal{B} > 0$

Note: In panel A, the vertical axis measures Blue flow-casualties, d_t^B. Thus, the area under a line and up to time t represents Blue casualties, $D_t^B = \int_0^t d_u^B du$. The left-pointing arrow under the horizontal axis indicates the shorter duration of war. Panel B reads similarly.

military victory, Red casualties are reduced when the initial stock of Blue weapons is larger. The effect of an increase in Blue reinforcements, X^B can be understood in a similar way.

Figure 3 shows the effect of an increase in Red resources, namely X^R. In Panel A, Blue flow-casualties are higher at each point in time because X^R is higher—this is the symmetric effect of that described in Equation (7) for d_t^R. Thus, the dark shaded area indicates an increase in Blue casualties resulting from two effects acting in the same direction: the higher flow-casualties and the longer war. Panel B shows Red flow-casualties, which are lower at each point in time—this is the symmetric effect of that described in Equation (6) for d_t^B. The effect on Red casualties combines two effects acting in opposite directions: The light-shaded area represents a reduction of Red casualties due to fewer flow-casualties. The dark-shaded area represents an increase due to the longer

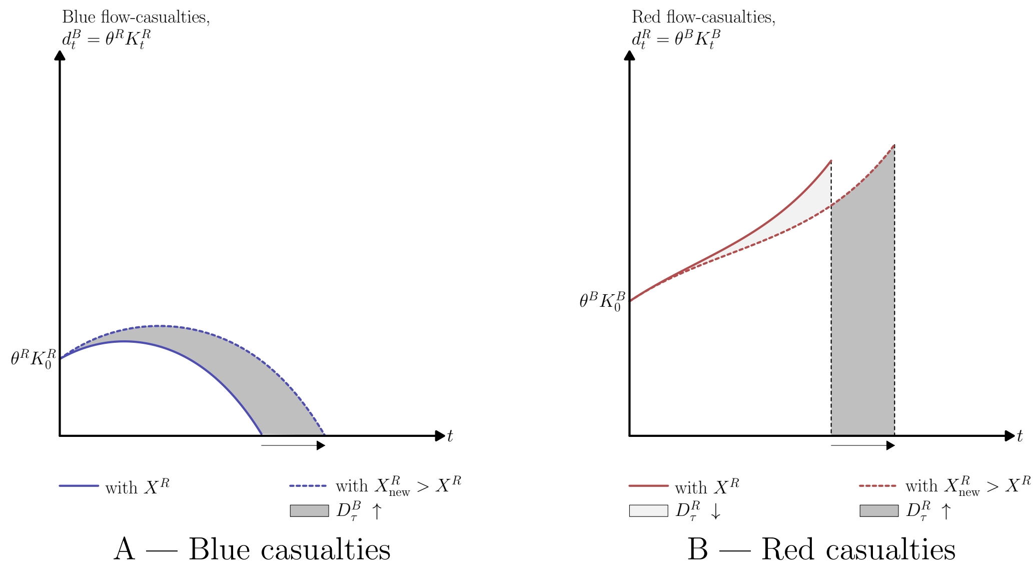


Figure 3: The effect of X^R on casualties when $\mathcal{B} > 0$

Note: In panel A, the vertical axis measures Blue flow-casualties, d_t^B. Thus, the area under a line and up to time t represents Blue casualties, $D_t^B = \int_0^t d_u^B du$. The right-pointing arrow under the horizontal axis indicates the longer duration of war. Panel B reads similarly.

war. In the end, as indicated in Equation (15), the second effect dominates and Red casualties increase as a result of increased Red reinforcements. The effect of the initial Red weapons stock can be understood similarly.

The results in Equations (15) and (16) can be stated more generally as: At a military conclusion, **both** sides' casualties decrease with the resources of the victorious belligerent and increase with the resources of the defeated.

The role of GDP Let Y^R and Y^B denote the (constant) GDP of Red and Blue, respectively. I assume that, before the war, Red and Blue are at steady states where they allocate constant fractions of GDP to their military. Thus, at the start of war their initial weapons stocks are proportional to their GDP:

$$K_0^B = s^B Y^B \quad \text{and} \quad K_0^R = s^R Y^R.$$

I refer to s^R and s^B as prewar (military) saving rates for the sake of exposition.[6] I further assume that, in wartime, Red and Blue allocate constant fractions (possibly different than in peacetime) of their GDP to reinforcements

$$X^B = \sigma^B Y^B \quad \text{and} \quad X^R = \sigma^R Y^R,$$

where σ^B and σ^R are wartime investment rates in weapons.

The condition for a Blue military victory (Equation 10) becomes

$$s^B + \frac{\sigma^B}{\sqrt{\theta^R \theta^B}} > \underbrace{\frac{\sqrt{\theta^R} Y^R}{\sqrt{\theta^B} Y^B}}_{\mathcal{Y}} \left(s^R + \frac{\sigma^R}{\sqrt{\theta^R \theta^B}} \right). \tag{17}$$

The ratio $\mathcal{Y}$ indicates the role of each country's GDP in determining the prevailing side in a military conclusion. Red may allocate larger fractions of its GDP to the military in peacetime and wartime ($s^R > s^B$ and $\sigma^R > \sigma^B$), but that is not enough for a military victory if Blue's GDP is high. This, per se, is not a "finding," however, since the preceding paragraphs have already established that larger resources are conducive to a military victory (e.g., Equation 10). The lesson to draw from Equation (17) is **how** exactly does GDP matter? And **how much**? The answer is through $\mathcal{Y}$.

As with condition (10), the role of military technology, via the attrition coefficients, is small relative to the role of GDP: The elasticity of $\mathcal{Y}$ with respect to relative attrition is 1/2, while its elasticity with respect to relative GDP is 1. As Red GDP becomes smaller relative to Blue's, the relative attrition in favor of Red, necessary to maintain $\mathcal{Y}$ constant, increases quadratically. A poor country is thus at a disadvantage relative to a rich country not only because it has fewer resources, but also because the resource gap is increasingly difficult to offset with better technology the poorer the country is.

I made no reference to GDP per capita. In the model, GDP per capita does

[6] With a constant saving rate, as in the Solow model for instance, the capital-to-output ratio is proportional to the saving rate, albeit not equal to it.

not play a role. It is conceivable that countries with high GDP per capita innovate more and have access to better military technologies, i.e., more favorable relative attrition, than countries with low GDP per capita. This effect is trumped by the size effect I described.

The role of GDP for the duration of war can be gauged by rewriting the level curves of τ^M from Equation (13),

$$\left.\frac{ds^B}{ds^R}\right|_{d\tau^M=0} = -\frac{\mathcal{Y}}{\tanh(\tau^M\lambda)}.$$

Consider increases in Red's prewar investment rate, s^R. Since Blue is poised to obtain a military victory, these increases lengthen the war (Equation 12). Blue can offset the increased duration by raising s^B. The lower $\mathcal{Y}$, that is the richer Blue relative to Red, the cheaper it is for Blue to do so. Remark that I use the term "cheaper" in reference to the *good* cost that Blue must face. I am not referring to the possibility that the marginal utility of consumption of a representative Blue individual could be lower than for a representative Red individual. The relationship above is technological in this sense.

2.4 Political conclusion

I assume there are exogenously-determined casualties thresholds at which countries sue for peace. Let $\bar{D}^R$ and $\bar{D}^B$ denote these thresholds for Red and Blue, respectively. I label such conclusions "political" because the fighting strength of the country suing for peace need not be lower than its opponent's. For example, Blue can sue for peace when $\mathcal{B} > 0$.

The Vietnam war is an example of a conflict without a military conclusion. No belligerent was incapable of fighting when it concluded. Instead, political forces compelled decision makers to reduce the U.S. involvement in the war: Even though the war lasted from 1964 to 1975, the peak of U.S. troops in

Vietnam was in April 1969 (Anderson, 2002, p. 187).[7] Evidence of political discontent with the war is numerous, e.g., the anti-war movement or the repeal of the Tonkin Gulf Resolution in 1970.

World War 1 is another example. Allied forces were not in Germany when the war ended, and Douglas Haig, the commander of the British Expeditionary Force, said of the November 1918 armistice: "*Germany is not broken in a military sense*" (Liddell Hart, 2012, ch. 13).

Mueller (1973) is a classic study of how cumulative casualties in war negatively affect public opinion. Gartner and Segura (1998) argue that flow casualties also matter, not just the cumulative casualties, because flow casualties better capture the effects of "pivotal events and shocks." Given the deterministic nature of the model, however, it seems reasonable to follow Mueller's approach, and define political thresholds in term of cumulative casualties.

Duration of war Blue sues for peace at τ^B such that $D^B_{\tau^B} = \bar{D}^B$ if the war has not yet reached a different conclusion. Using Equation (9),

$$K^B_{\tau^B} = \tau^B X^B + K^B_0 - \bar{D}^B. \tag{18}$$

Similarly, Red sues for peace at τ^R such that:

$$K^R_{\tau^R} = \tau^R X^R + K^R_0 - \bar{D}^R. \tag{19}$$

Figure 4 illustrates the determination of τ^B and τ^M in a case where $\mathcal{B} > 0$ and Blue's political threshold is reached first. Thus, Blue sues for peace at date τ^B even though it was on a path to a military victory and Red would have sued for peace before being militarily defeated ($\tau^R < \tau^M$). Note that if τ^B and/or τ^R exist, they are unique because the left-hand side of Equations (18) and (19) are every where steeper (with respect to time) than K^B_t and K^R_t. Existence, however, is not guaranteed: It is clear, from Figure 4 that (18) and (19) may

[7] The Tonkin Gulf Resolution was passed in 1964. Saigon fell in 1975.

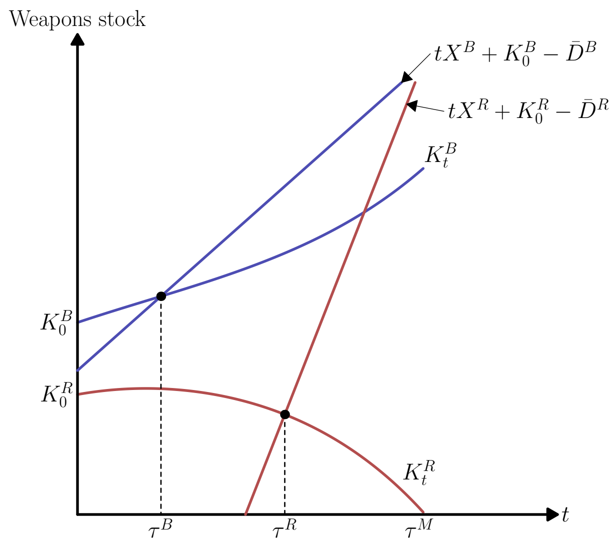


Figure 4: Weapons stock trajectories and political thresholds when $\mathcal{B} > 0$

not have solutions for large enough values of $\bar{D}^B$ and $\bar{D}^R$.

How do resources affect the date at which belligerents reach their political thresholds? I show (Appendix C.1) that

$$\frac{\partial \tau^B}{\partial X^B} > 0, \qquad \frac{\partial \tau^B}{\partial X^R} < 0, \qquad \frac{\partial \tau^B}{\partial K_0^B} > 0, \qquad \frac{\partial \tau^B}{\partial K_0^R} < 0, \tag{20}$$

and that

$$\frac{\partial \tau^R}{\partial X^B} < 0, \qquad \frac{\partial \tau^R}{\partial X^R} > 0, \qquad \frac{\partial \tau^R}{\partial K_0^B} < 0, \qquad \frac{\partial \tau^R}{\partial K_0^R} > 0. \tag{21}$$

The date at which a belligerent reach its political threshold is increasing with the resources it commits and decreasing with the resources committed by its opponent. It is worth noting that this result is independent of the sign of $\mathcal{B}$. It holds regardless of which country is poised to obtain a military victory.

Suppose Blue commits additional resources to the war, either via K_0^B or via X^B. This reduces Blue flow-casualties at each point in time (Equation 6) and, thus, lengthens the time necessary for Blue to reach its political threshold: τ^B increases. Again, this is because additional Blue weapons imply higher Red flow-casualties (Equation 7) and, thus, impair Red's ability to destroy Blue weapons. Higher Red flow-casualties shorten the time necessary for Red to reach its political threshold: τ^R decreases. The effects of Red resources on τ^B and τ^R have similar explanations.

One lesson from Equations (20) and (21) may appear counter-intuitive: Suppose Blue faces an "unpopular" war and that such unpopularity is reflected in a "low" threshold $\bar{D}^B$. To reduce the need to sue for peace, Blue decision makers have an incentive to allocate more instead of fewer resources to the war. By allocating more resources to the war Blue would be pushing τ^B into the future and lowering τ^M, the time required before a military victory. If Blue obtained a military victory it would then, by definition, have suffered fewer casualties than $\bar{D}^B$. The opposite "policy", that is the policy of allocating fewer resources to the unpopular war, does not economize casualties under the aimed-fire assumption. On the contrary, allocating fewer resource causes casualties to accumulate faster because it lowers Blue's ability to attrit Red.

Casualties The casualties of the country suing for peace are given by its political threshold. How are its opponent's casualties affected by resources? I show (Appendix C.2) that

$$\frac{\partial D^R_{\tau^B}}{\partial X^B} > 0, \qquad \frac{\partial D^R_{\tau^B}}{\partial X^R} < 0, \qquad \frac{\partial D^R_{\tau^B}}{\partial K_0^B} > 0, \qquad \frac{\partial D^R_{\tau^B}}{\partial K_0^R} < 0,$$

and

$$\frac{\partial D^B_{\tau^R}}{\partial X^B} < 0, \qquad \frac{\partial D^B_{\tau^R}}{\partial X^R} > 0, \qquad \frac{\partial D^B_{\tau^R}}{\partial K_0^B} < 0, \qquad \frac{\partial D^B_{\tau^R}}{\partial K_0^R} > 0.$$

Consider a political conclusion initiated by Red at τ^R, i.e., a Blue political victory. Blue casualties increase with Red resources and decrease with Blue resources. Red resources have two effects: First more Red resources imply higher Blue flow-casualties—this is the symmetric effect of that described in Equation (7) for d_t^R. Second the war is longer (Equation 21). Symmetrically, more Blue resources lower Blue flow-casualties and lead to a shorter war.

3 Dictatorship v. democracy

In this section, I discuss the conditions under which a political conclusion prevails over a military conclusion, or vice versa. I frame the discussion in a specific context: One belligerent is a relatively rich democracy (Blue) and the other is a relatively poor dictatorship (Red). I first define the sense in which, in the model, Blue is richer than Red and the (arguably narrow) sense in which Blue is a democracy and Red a dictatorship. Second, I characterize the conditions under which each belligerent may prevail and in what type (military or political) of conclusion. Finally, I argue that the scenario of a democracy at war with a poorer dictatorship is representative of the Pacific War which opposed the U.S. to Japan, from December 1941 to August 1945, in the larger context of World War 2. I further argue that the analysis suggests an interpretation of the outcome of the Pacific War that, to the best of my knowledge, has not been emphasized in the historical literature.

Setup

To represent Blue being richer than Red I assume $\mathcal{B} > 0$ because, in the context of the model, this assumption indicates that Blue's resources are large enough, compared with Red's resources, to permit a military victory (Equations 10 or 17). Therefore, if the war was just a contest of resources, Blue would prevail militarily at τ^M. Note that the restriction $\mathcal{B} > 0$ allows for Red's initial weapons stock to be higher than Blue's. This would the case if Red had acquired a large army via peace time investments while Blue had not. The assumption $\mathcal{B} > 0$ when $K_0^R > K_0^B$ implies that Blue's ability to reinforce must be high enough relative to Red's.[8] In other words, the dictatorship may have a large army at the onset of war while the rich democracy may not and, yet, the dictatorship's gross domestic product may not be enough to allow for sufficient reinforcements. Hence, in a contest of resources, the rich democracy would prevail militarily despite its initially small army.

A war is not just a contest or resources, however. It implies an element of politics and, hence, I introduce a political distinction between Blue and Red. To represent Blue as a democracy I assume that its military leaders are subject to a civilian oversight which will not accept to continue the war beyond the finite casualty limit $\bar{D}^B$. In contrast, to represent Red as a dictatorship I assume that its military leaders are not subject to civilian oversight as are Blue's. Nevertheless, it is plausible that despite the absence of civilian oversight, Red's population could revolt, overthrow its leaders, and put an end to the war when casualties become excessive. In such case there would also be a finite casualty threshold for Red. My assumption, then, is that the distinction between Blue's democratic regime and Red's dictatorial regime lies in the Red threshold being higher than the Blue threshold. That is, in the Blue democracy the push for peace due to casualties initiates sooner than in the Red dictatorship. For instance, elections in a democracy allow the public to orchestrate a transfer of power at regular intervals, whereas coordination to overthrow a dictatorial regime is more difficult. To simplify the following discussion, I carry this distinction to the limit and assume $\bar{D}^R$ to be infinite.

In sum, I assume

$$\mathcal{B} > 0 \quad \text{and} \quad 0 < \bar{D}^B < +\infty = \bar{D}^R,$$

and seek to discuss questions such as: Who prevails when a rich country, whose participation to the war is subject to a political constraint, faces a poor country whose participation is not subject to a political constraint? How does the willingness to "accept" casualties mitigate the economic disadvantage of

[8]Suppose $K_0^R = aK_0^B$ with $a > 1$, then $\mathcal{B} > 0 \Leftrightarrow X^B > vX^R - v(1 - av)\theta^B K_0^B$.

the poor dictatorship, if at all? Similarly, does a low tolerance for casualties offset the economic advantage of the rich democracy?

Analysis

Since $\bar{D}^R = +\infty$ Red never sues for peace. The war's conclusion is either military with Red's forces annihilated at τ^M, or political with Blue suing for peace at τ^B. Define a "frontier" of initial conditions such that Red's weapons stock reaches zero precisely when Blue's casualties reach their threshold:

$$BM = \left\{ \left(K_0^R, K_0^B \right) : \tau^M = \tau^B \right\}. \tag{22}$$

Note that BM is not defined everywhere: For low enough values of K_0^R there may not be a K_0^B such that $\mathcal{B} > 0$ and $\tau^M = \tau^B$. That is because τ^B is not guaranteed to exists, as indicated in Section 2.4. To understand this, consider an initial condition immediately above the stable branch, i.e., $\mathcal{B} > 0$. First, suppose that K_0^R is "low." Then, Red does not inflict "high" casualties on Blue and, hence, Blue does not reach its political threshold before the war's military conclusion. Furthermore, any initial condition with a higher value of K_0^B would imply an even earlier military conclusion and fewer Blue casualties. Thus, for such "low" value of K_0^R, a military conclusion favorable to Blue is the only possible conclusion of war. Second, suppose that K_0^R is "high." Then, Red inflicts "high" casualties on Blue during the war and Blue may reach its threshold before obtaining a military victory.

I show (Appendix D) that the BM frontier, where it exists, is increasing and steeper than the stable branch:

$$\left. \frac{dK_0^B}{dK_0^R} \right|_{\tau^M = \tau^B} > \left. \frac{dK_0^B}{dK_0^R} \right|_{\mathcal{B}=0} = v > 0.$$

The BM frontier is increasing because it is an iso-casualty line for Blue: Consider a point on BM. An increase in K_0^R implies an increase in Blue's casualties

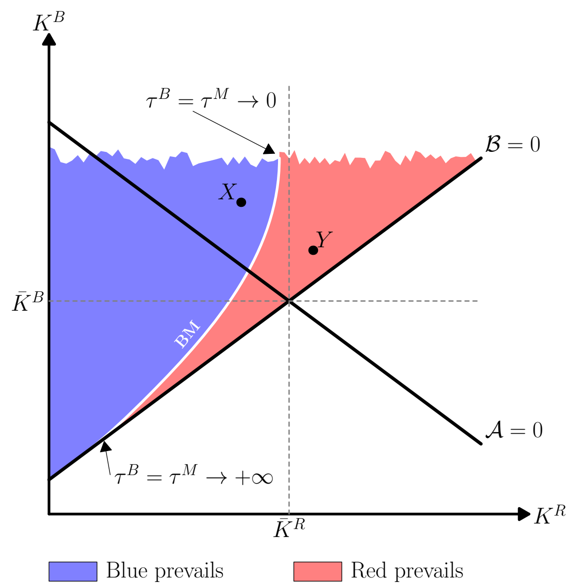


Figure 5: War between a rich democracy (Blue) and a poor dictatorship (Red)

Note: I show in Appendix D that (i) BM is increasing and convex; (ii) BM's slope approaches v (the slope of the stable branch, $\mathcal{B} = 0$) as the duration of war approaches infinity, i.e., near $\mathcal{B} = 0$; (iii) BM's slope approaches infinity as K_0^R increases; and (iv) the duration of war decreases as K_0^R increases along BM.

(Equation 16). But Blue's casualties are fixed at $\bar{D}^B$ on BM and, hence, K_0^B must increase to offset the effect of K_0^R (Equation 16). I also show that the BM frontier is convex.

Consider now a point on the BM frontier and contemplate an increase in K_0^B holding K_0^R constant: τ^M decreases (Equation 12) while τ^B increases (Equation 20). It follows that initial conditions above the BM frontier imply $\tau^M < \tau^B$ and, hence, such initial conditions lead to a military conclusion where Blue prevails and Red's military is annihilated. By the same argument, initial conditions between BM and the stable branch lead to a conclusion where Blue sues for peace and Red prevails, even though Blue is on a path toward a military victory. Figure 5 illustrates this discussion.

It is evident from Figure 5 that both Blue and Red resources remain determinant for the war's outcome. Consider point X, for instance, which is such that Blue is poised to obtain a military victory. Since the BM frontier is increasing, the area to the southeast of X intersect with the red area. In other words, lower Blue resources and higher Red resources are conducive to Red prevailing. Similarly, points to the northwest of Y intersect with the blue area, indicating that higher Blue resources and lower Red resources are conducive to Blue prevailing. It is important to note that, even though these conclusions resemble the properties of the military victory in Section 2.3, i.e., the belligerent with the most resources tends to prevail, they are different. Here, the belligerent with the most resources is more likely to prevail but not necessarily in a military sense. The role of resources via their effects on the accumulation of casualties is critical. In one case, when Red is poor enough relative to Blue, Red cannot inflicts casualties at a fast enough pace to bring Blue to its political threshold before Blue effects Red's destruction. In the opposite case, if Red is rich enough relative to Blue, then Red can inflict casualties at a pace that is fast enough to bring Blue to sue for peace before Red is destroyed.

It is interesting to note an implication of the convexity of the BM frontier. Suppose Red and Blue are not at war but perceive war to be likely. Suppose also that they have enough information to know that their military spending and weapon stocks place them on the BM frontier. A war would lead to the destruction of Red and intolerable casualties for Blue and, so, each has an incentive to accumulate additional weapons whenever the other does the same. Such simultaneous accumulation of weapons, in an attempt to avoid war, resembles an arms race.[9] Remark that an arms race can, similarly, take place along the stable branch. This, however, only occurs if neither Blue nor Red face a casualty threshold and, hence, the only possible outcome of war is a military conclusion. The interesting aspect of the arms race along the BM frontier arises from the convexity of BM while the stable branch is linear: As Red accumulates a larger weapons stock, the increase in Blue's

[9]This discussion is about the payoff functions in the arms race game, not about the solution of the game. The literature on arms races is vast (e.g. Smith, 2020).

weapons stock required to dissuade Red from war becomes larger. Along the stable branch, however, linearity implies that the increase in Blue's weapons stock necessary to dissuade an attack is constant. In other words, Blue's finite casualty threshold implies that an arms race with the Red dictatorship is increasingly costly.

The Pacific War

When Japan started a war with the United States on December 7, 1941, the latter was a larger economy: The GDP difference was 5-fold and the population difference was almost 2-fold, both in favor of the U.S.[10]. I briefly describe the decades-long march to war before the Pearl Harbor attack in Appendix F.1.

In the United States, the armed forces were part of the Roosevelt and Truman administrations who answered to the American people. Concerns for casualties among American leaders was paramount. In June 1945, for example, as the decision whether to invade or to continue bombing-and-blockading Japan drew closer, President Truman called a meeting of the military and civilian leaders of the war "and stated unequivocally that 'it is his intention to make his decision on the campaign with the purpose of economizing to the maximum extent possible on the loss of American lives. Economy in the use of time and in money cost is comparatively unimportant.'" (From Giangreco, 2017, p. 77.) I report, in Appendix F.2, additional evidence that civilian and military leaders perceived a limit to the casualties the American people would tolerate.

In contrast to the U.S., in Japan the war was run by the military. I report, in Appendix F.2, evidence indicating that, even though Japan was not a dictatorship in the strictest sense, civilian control of the military was almost non-existent. Furthermore, the Japanese Army had "popularized the concept [that] death in battle or suicide was preferable to capture" as early as the Russo-Japanese war of 1904-05. (Drea, 2016, p. 119.) This attitude was later exacerbated. I report, in Appendix F.2, evidence that, during the Pacific war,

[10] Maddison Project Database 2020.

Japanese leaders did not seek minimal casualties.

Given these observations, it is natural to map the American-Japanese confrontation in the Pacific to the framework developed above: The U.S. was richer and with an explicit concern about casualties while Japan was poorer and exhibited noticeably less concern for casualties. Thus, in the language of the model, the former can be represented as the Blue democracy and the latter as the Red dictatorship.

Historical evidence indicates that the Japanese military leaders (i) wanted a short war because they knew that Japanese GDP could not sustain a protracted competition against U.S. GDP; and (ii) expected to obtain a rapid political victory because they hoped the U.S. would quickly sue for peace. Historian C. J. Nolan writes

> "The short naval war delusion took hold in Japan at all levels, not just in spite but *because* of its known scientific, economic, industrial and military weakness against [America]. Japanese admirals did not plan a long war deluded that they could win. They planned what they desperately hope would be a short naval war, deluded that they *might* win, while knowing that was the only kind of war in which they stood even a gambler's chance" (Nolan, 2017, p 514.)

I report, in Appendix F.3, further evidence of Japanese knowledge that they could not defeat the U.S. in a military sense and that their plan for victory was for the U.S. to quickly sue for peace.

It is often mentioned, in the historical literature, that Japanese leaders misjudged the U.S. tolerance for casualties (see Appendix F.3). Under such narrative, Japan lost because the U.S. casualty threshold, $\bar{D}^B$, was higher than it (Japan) assessed. So, Japan was not in a position to obtain a political victory because of the U.S. "will to fight." The analysis in this section suggests another interpretation of Japan's defeat: The failure to note that the larger U.S. GDP, which was conducive to a Japanese military defeat in the long-run, was not conducive to a Japanese political victory in the short run. In other words, the **same** reason that made a "long" war unwinnable for Japan also implied that the war would be "long" before the U.S. reached its political threshold as Japanese leaders expected. The large U.S. GDP implied that Japan's ability to wage war would be destroyed before the U.S. reached its political threshold.

In sum, the large U.S. GDP, relative to that of Japan, played two roles. First, it ensured that Equation (17) held and that the U.S. could eventually obtain a military victory—a point emphasized by many authors. Second, it ensured that the date at which the U.S. would sue for peace, τ^B, could be pushed into the future—a point not emphasized, to the best of my knowledge, in the historical literature.

Note that this second point remains true regardless of the U.S. political threshold $\bar{D}^B$: No matter how high or low $\bar{D}^B$ is, the higher the U.S. GDP the farther into the future will $\bar{D}^B$ be reached. Formally,

$$\frac{\partial \mathcal{B}}{\partial Y^B} > 0, \quad \text{and} \quad \frac{\partial \tau^B}{\partial Y^B} > 0.$$

The first inequality (derived from 17) is the first effect: a large U.S. GDP is conducive to a U.S. military victory. The second inequality (derived from 20) is the second effect: a large U.S. GDP is not conducive to an early Japanase political victory.[11]

4 The Pacific War

In this section, I consider a discrete-time version of the model, with time measured in months, and confront it to Pacific War data. I first discuss the data and, then, show that a parsimonious parameterization of the model replicates the patterns of casualties well.

[11] I am not arguing that the U.S. did not suffer high casualties during the Pacific war.

4.1 Data

The Pacific War was fought in sea, air, and land battles from the Aleutian in the north to the Coral Sea in the south, and from Hawaii in the Central Pacific to the Philippines and Japan in the west. A variety of weapons type were used: Troops from different services with different training and experience, bomber planes, fighter planes, tanks and other armored vehicles, ships of all sizes and functions, etc. One could model each weapons type separately, each with their type-specific initial stocks, reinforcements, and vectors of attrition coefficients (e.g., troops destroy troops at a different rate than they destroy airplanes). Indeed, modeling various weapons type is of first-order importance in Operations Research, particularly in the planning of battles. Data availability limits one's ability to apply such granular model to the entirety of the war, however. Furthermore, such modeling is not necessary to show that the model is an empirically adequate description of the mechanics of aggregate destruction in war. Just as the aggregate production function has proven useful in studying macroeconomics, I suggest that the process of aggregate destruction is useful to study the economic mechanics of warfare.

So, I continue to use the notion of an aggregate weapon as in Section 2. There remains the question of measurement. I use displacement of naval ships, measured in tons. My reasons are: First, there exists data because tonnage is a long-established metric for measuring and comparing ships. Second, tonnage permits an obvious aggregation in which an aircraft carrier displacing 25,000 tons counts more than a destroyer displacing 2,500 tons. Third, using tonnage to measure weapons amounts to assuming that the larger the ship the more weapons it represents. This is reasonable to the extent that larger ship carry more and/or bigger guns, more personnel and materiel, and, for some, more airplanes. For instance, the large aircraft carrier Yorktown ($25,500$ tons) carried up to 90 airplanes while the smaller Princeton ($11,500$ tons) carried up to 45 airplanes. Thus, to a first-order approximation, a ship's tonnage aggregates its ability to bring different weapons type to bear on ones' opponent. Since, in the land battles of the war, personnel and materiel were transported by ships and since many air battles were waged from aircraft carriers, it seems appropriate to use tonnage as a unit.[12]

I use data from Willmott (2010). Japanese losses are reported in tons per month in Table 12.3 (p. 460). I combine warships and military shipping losses to construct Japanese flow-casualties, d_t^R.

For the U.S., I assemble monthly time series of both reinforcements and losses. I proceed as follows. First, Willmott's Appendix 10.4 (p. 418-429) reports American ships lost by date, location, cause, name, and tonnage. I use this data to construct American flow-casualties, d_t^B. I exclude the ships lost in the Atlantic and/or reported lost to German forces. I include the ships lost to weather or that ran aground, provided it was in the Pacific. It is not transparent how the Japanese data account for weather-related losses.

Second, Willmott's Appendix 10.1.2 (p. 379-381) reports American ships by type, name, and month of commissioning, i.e., when a ship enters active service. I count a ship as reinforcement, i.e., X_t^B, in the month when it is commissioned. The exact tonnage of each ship is not reported, however. Yet, there are twelve types of ship in the data set and, hence, I convert the number of ships to tonnage by imputing the same tonnage to each ship of a given type. For instance, 8 destroyers were commissioned in January 1943, which I count as $8 \times 2,200$ tons of reinforcement in that month. I use $2,200$ tons per destroyer even though this masks some heterogeneity: The Aaron Ward (sunk Apr. 7, 1943) displaced $2,200$ tons, while the Henley (sunk Oct. 3, 1943) displaced $1,850$ tons, and the Perkins (sunk Nov. 29, 1943) displaced $2,300$ tons. Table 1 shows displacement by type of ship and an example of each type with such displacement.

Third, I count ships that were sunk and later returned to service as loss when sunk and reinforcement when returned. For instance, losses on Dec. 7, 1941, include the battleship California. But she was raised, fixed, and returned to

[12] Even in land battles ship tonnage was key: During the battle of Iwo Jima (Bonin Islands, Feb.-Mar. 1945), for instance, the U.S. navy was able to bring flows of reinforcement to the island while the Japanese navy did not.

Table 1: Tonnage of American Warship

Type	Displacement	Example	
Fleet carrier	25,500	Yorktown	sunk Jun. 6, 1942
Light carrier	11,500	Princeton	sunk Oct. 24, 1944
Escort carrier	10,400	Gambier Bay	sunk Oct. 25, 1944
Battleship	33,190	California	sunk Dec. 7, 1941
Heavy cruiser	11,420	Chicago	sunk Jan. 30, 1943
Light cruiser	8,340	Juneau	sunk Nov. 13, 1942
Destroyer	2,200	Aaron Ward	sunk Apr. 7, 1943
Submarine	2,424	Cisco	sunk Sep. 29, 1943
Destroyer escort	1,350	Eversole	sunk Oct. 10, 1944
Minesweeper	1,190	Hovey	sunk Jan. 6, 1945
Transport	9,600	McCawley	sunk Jun. 30, 1943
Oiler	7,470	Mississinewa	sunk Nov. 20, 1944

Source: Willmott (2010, p. 418-429).

service in May 1944. I apply similar treatments to the battleships Maryland, Nevada, Tennessee, and West Virginia, and to the destroyer escorts Cassin, Downes, and Shaw.

Willmott (2010, p. 379) reports fleets' sizes in the Pacific on Dec. 7, 1941. For Japan it was 10 fleet carriers, 10 battleships, 18 heavy cruisers, 20 light cruisers, 112 destroyers, and 65 submarines. For the U.S. it was 3 fleet carrier, 8 battleships, 13 heavy cruisers, 11 light cruisers, 80 destroyers, and 56 submarines. I convert these figures to date-0 tonnages, K_0^R and K_0^B, using Table 1. One caveat, here, is that ships such as transports or oilers are not included in this list. Yet, they existed in both navies: The U.S. Navy, for instance, had two oilers in Pearl Harbor on that day.[13] I use the data from Willmott, however, because it provides information on a homogeneous set of ships for both sides. I assume it is preferable to not count oilers for any side rather

[13]https://www.history.navy.mil/research/library/online-reading-room/title-list-alphabetically/s/ships-present-at-pearl-harbor.html

than counting them for one side only.[14] These calculations yield $K_0^R = 1,363k$ tons and $K_0^B = 894k$ tons. Thus, the Japanese fleet was larger than the U.S. Pacific fleet at the start of the war. This should not be surprising given the large advantage of the Japanese in aircraft carriers. I already discussed, In Section 3, the situation were $\mathcal{B} > 0$ and yet $K_0^R > K_0^B$.

Figure 6 shows the time series of cumulative losses for Japan (Panel A) and the U.S. (Panel B), and the time series of U.S. reinforcements (Panel C). The scale of Japanese losses are large compared to American losses throughout most of the war. Panel C reveals a pattern of increasing reinforcements brought into the conflicts by the U.S. The peak in the last months of 1944 is worth noting. It is due to the commissioning of a large number of transport ships. This is illustrated by the dashed line, which represents a counterfactual time series of U.S. reinforcement excluding transport ships. It is also worth noting that this increase started in September 1944 and, hence, it is not preparatory to the invasion of Europe, which started in June 1944.

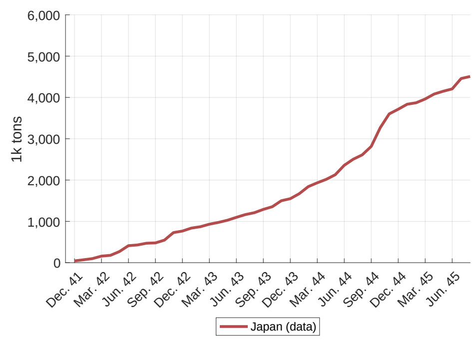
4.2 Empirical analysis

The discrete version of Equations (1)-(2), allowing for time-varying reinforcement for Blue, is

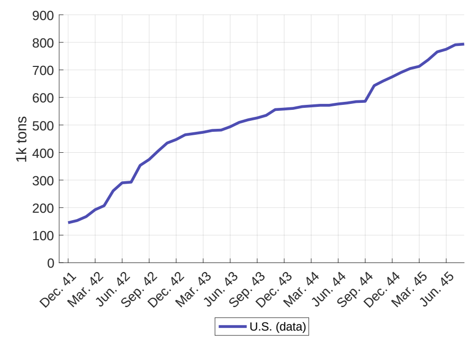
$$\begin{aligned} K_{t+1}^R &= K_t^R - \theta^B K_t^B + X^R, \\ K_{t+1}^B &= K_t^B - \theta^R K_t^R + X_t^B. \end{aligned}$$

The unknown parameters are the attrition coefficients, θ^B and θ^R, and Japan's reinforcements, X^R. I adopt a two-step approach to choosing these parameters. In the first, I allow the attrition coefficients to be time-specific. In the

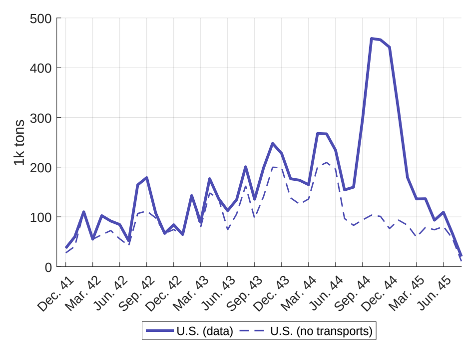
[14] There are additional caveats with the figures I constructed. First, I do not have a list of Japanese reinforcement per tonnage and months. Second, the list of American reinforcements does not indicate whether a ship was slatted to operate in the Pacific or elsewhere. Third, not all ship types are represented: The list of American losses includes "gun boats" which are not reported in the list of ships commissioned; conversely, the list of ships commissioned includes "frigates" which do not appear in losses.



A – Japan losses, D_t^R



B – U.S. losses, D_t^B



C – U.S. reinforcements, X_t^B

Figure 6: Cumulative losses for Japan and the U.S. and U.S. reinforcements

Note: D_t^R is the discrete time version of casualties: $D_t^R = \sum_{j=0}^{t} d_t^R$. Casualties for the U.S. are computed similarly.
Source: Willmott (2010) and author's calculations.

second, I allow the attrition coefficients to assume a restricted set of values changing at dates informed by the first step. In both, I assume time-invariant reinforcements for Japan and impose the following discipline: The parameters must be such that (i) the model-implied casualties are close to the observed casualties in Panels A and B of Figure 6 and (ii) the model-implied stock of Japanese weapons reaches 0 in the last (45th) period of the war. I impose such terminal condition because the Japanese Navy was virtually destroyed by the end of the Pacific War. Willmott (2010, p. 364) reports that whole classes of destroyers were wiped out, that there remain only one Japanese fleet carrier but that it was ineffective for lack of airplanes, etc.[15]

It is convenient to denote the set of unknown coefficient by ω, the size of which will differ in the first and the second step. Let $\hat{K}_t^R(\omega)$ and $\hat{D}_t^R(\omega)$ denote the model-implied stock of weapons and casualties for Red conditional on ω, respectively. Let $\hat{K}_t^B(\omega)$ and $\hat{D}_t^B(\omega)$ be defined similarly. Initial conditions are given by data and, so,

$$\hat{K}_0^R(\omega) = 1,363 \quad \text{and} \quad \hat{K}_0^B(\omega) = 894. \tag{23}$$

Step 1 — Matching casualties with time-specific attrition

With time-specific attrition coefficients it is possible to exactly match the observed casualties. To see that, let

$$\omega = \{X^R\}$$

[15] In assuming that the war concludes in its 45th month with the annihilation of the stock of Japanese weapons, the model is silent about the atom bomb. The debate surrounding the atom bomb's contribution to the end of the Pacific War is not new and the model has nothing to contribute. However, it is a matter of fact, not interpretation, that, by August 1945, the Japanese Navy was destroyed.

and consider the following version of the model

$$\begin{aligned}\hat{K}^R_{t+1}(\omega) &= \hat{K}^R_t(\omega) - d^R_t + X^R,\\ \hat{K}^B_{t+1}(\omega) &= \hat{K}^B_t(\omega) - d^B_t + X^B_t,\end{aligned}$$

with initial conditions given by Equation (23). Note that flow-casualties, d^R_t and d^B_t, are the observed data. So, the model matches casualties by construction. It remains to determine ω such that Japan's weapons stock reaches 0 in the last period of the war. That is, solve

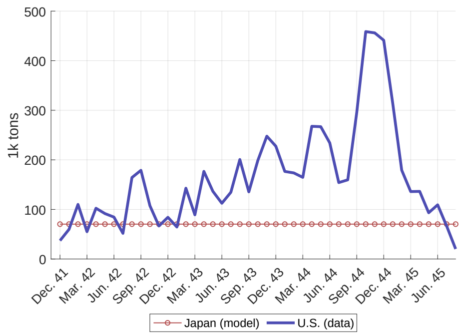
$$\hat{K}^R_{44}(\omega) = 0.$$

The discipline on ω (equivalently X^R) follows the logic from Equation (12): A larger values of X^R leads to a longer war while a smaller value leads to a shorter war. The solution to the equation above is $X^R \simeq 70$. Given X^R, the attrition coefficients can be immediately computed via

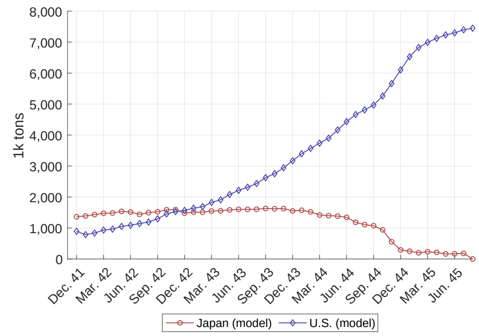
$$\theta^R_t = d^R_t/\hat{K}^R_t(\omega) \quad \text{and} \quad \theta^B_t = d^B_t/\hat{K}^B_t(\omega).$$

Figure 7 shows the model's implications. Panel A compares the model-implied Japanese reinforcements to the observed U.S. reinforcements. Japanese reinforcement are significantly lower. Panel B shows the implied stock of weapons for each belligerent.

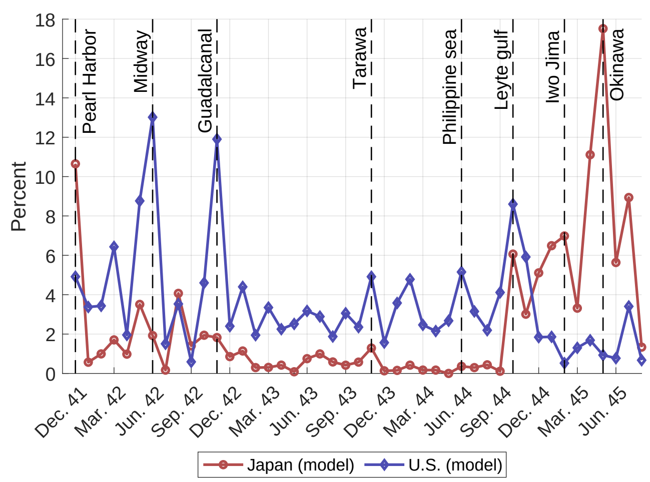
Panel C of Figure 7 is the most informative, and deserves a detailed discussion. First, it indicates the dates (month) of some major battles of the war. These dates correspond to peaks in attrition coefficients for one or both belligerent, indicating that the time series of casualties do match the actual evolution of the war. Second, during the Pearl Harbor attack, Japan's attrition coefficient is both higher than the U.S.'s coefficient and higher than its own subsequent values until September 1944. This is likely the surprise effect of the Pearl Harbor attack. Third, there is a noticeable increase in Japan's attrition coefficient in the last months of 1944, starting with the battle of Leyte Gulf in the



A – Reinforcements, X^R and X_t^B



B – Weapons stocks, $\hat{K}_t^R(\omega)$ and $\hat{K}_t^B(\omega)$



C – Attrition coefficients, θ_t^R and θ_t^B

Figure 7: The quantitative model with time-specific attrition coefficients

Note: In this calibration the attrition coefficients are such that the model exactly replicates the observed casualties of both sides, in each month of the war.
Source: Willmott (2010) and author's calculations.

Philippines (October 1944). This maybe attributable to the increased reliance of Japan on suicide tactics (the Kamikaze) which became more prevalent starting with the battle of Leyte Gulf (e.g. Craig, 1997, p. 1-4). Furthermore, as the war approached the large islands of the Philippines (and a few months later Okinawa), land-based Japanese airplanes played an increasing role in the fighting, mechanically implying a higher attrition of U.S. tonnage per Japanese tonnage. The first coordinated wave of Kamikaze planes at Leyte Gulf was, indeed, land-based. Finally, there is a noticeable decrease in the U.S.'s attrition coefficient after the battle of Leyte Gulf. This may reflect a composition effect in the U.S. fleet, more than a consequence of the battle itself. As noted earlier there was a large influx of U.S. reinforcements, at the end of 1944, mostly from transport ships—see panel C of Figure 6. Transport ships are likely less effective at destroying enemy weapons than the average warship and hence, would have lowered the average attrition coefficient of the U.S. fleet.

Step 2 — A parsimonious model of the Pacific War

The lesson from the exercise above is an interpretation of what may have caused variations in attrition coefficients of both the U.S. and Japanese fleet. There are three points to retain. First, The Pearl Harbor attack justifies an abnormally high attrition coefficient for Japan, due to the surprise effect. This coefficient can be computed, without solving the model, from U.S. flow-casualties in December 1941 ($145k$ tons) and the initial weapons stock of Japan:

$$\theta_0^R = \frac{145}{1,363} = 10.6\%.$$

Note that this is the figure that transpires from Panel C of Figure 7. Second, the post-Leyte Gulf attrition coefficient for Japan exhibits an increase due to changing tactics. Third, the post-Leyte Gulf attrition coefficient for the U.S. exhibits a decline due to a large influx of transport ships. These observations suggest a parsimonious number of parameters: two attrition coefficients per country (before and after Leyte Gulf) and Japanese reinforcements:

$$\omega = \left(\theta^R_{\text{low}}, \theta^R_{\text{high}}, \theta^B_{\text{low}}, \theta^B_{\text{high}}, X^R\right).$$

These parameters must then solve

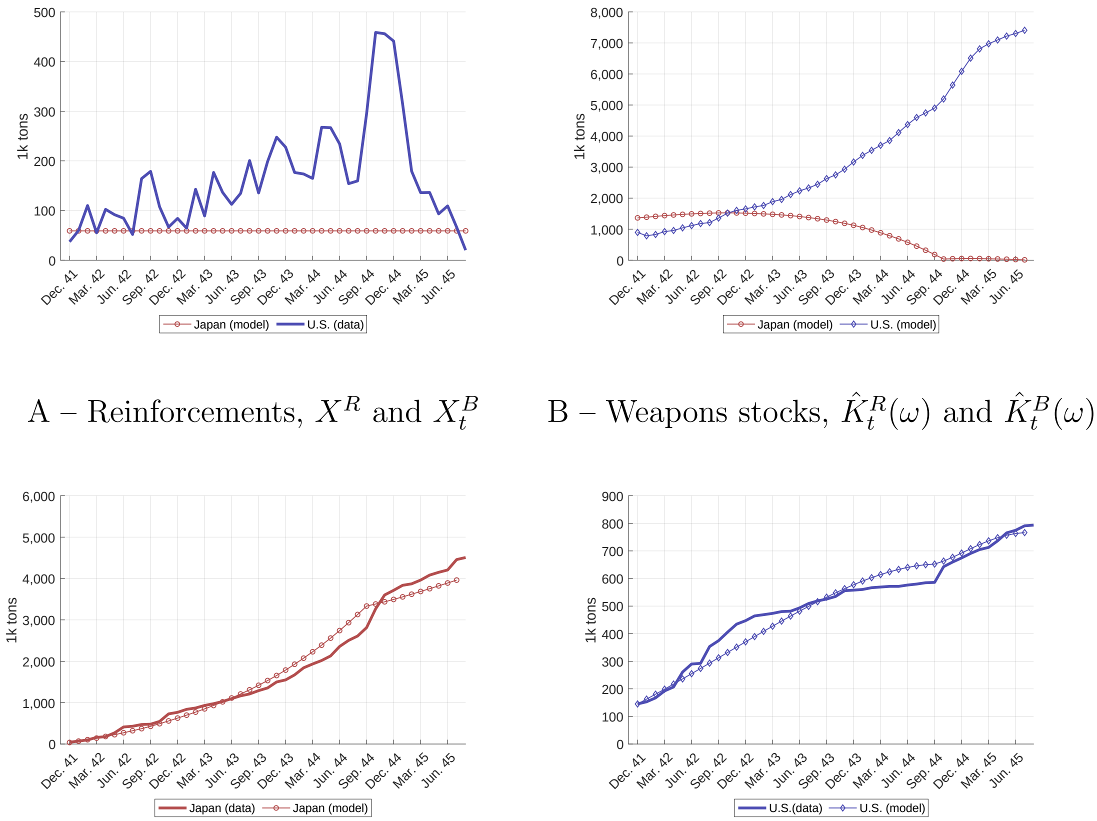
$$\begin{aligned}
\min_{\omega} \quad & \sum_{t=0}^{44} \left(\hat{D}^R_t(\omega)/D^R_t - 1\right)^2 + \sum_{t=0}^{44} \left(\hat{D}^B_t(\omega)/D^B_t - 1\right)^2, \\
\text{s.t.} \quad & \hat{K}^R_0(\omega) = 1,363 \quad \text{and} \quad \hat{K}^B_0(\omega) = 894, \\
& \hat{K}^R_{44}(\omega) = 0, \\
& \theta^R = 0.106 \times \mathbb{I}_{t=0} + \theta^R_{\text{low}} \times \mathbb{I}_{0<t<35} + \theta^R_{\text{high}} \times \mathbb{I}_{t\geq 35}, \\
& \theta^B = \theta^B_{\text{high}} \times \mathbb{I}_{t<35} + \theta^B_{\text{low}} \times \mathbb{I}_{t\geq 35}.
\end{aligned}$$

where the first constraint represents initial conditions, the second represents the terminal condition, and the last two constraints restrict the path of attrition coefficients. There are 90 data points and 5 parameters. The solution for this problem is displayed in Table 2. Figure 8 shows the model's implication under this calibration. Estimated Japanese reinforcements remain low compared to observed U.S. reinforcement and, importantly, the model-implied casualties mimic the data well.

Table 2: Coefficients of the parsimonious model

θ^R_{low}	θ^R_{high}	θ^B_{low}	θ^B_{high}	X^R
1.3%	29.3%	0.9%	4.1%	59.0

So, a parsimonious (5 parameters) model is a quantitatively accurate description of attrition on both side. Furthermore, the changing attrition coefficients can be rationalized as exogenous technological changes, e.g, changes in tactics and/or composition of the weapons stocks. In what follows, I put the model to use by asking: Could the U.S. have won the Pacific war at a cheaper



A – Reinforcements, X^R and X_t^B

B – Weapons stocks, $\hat{K}_t^R(\omega)$ and $\hat{K}_t^B(\omega)$

C – Japan's losses, D_t^R

D – U.S.'s losses, D_t^B

Figure 8: The parsimonious quantitative model

Note: In this calibration the Japanese attrition coefficients assume three values: In period 0 (Pearl Harbor), before Leyte Gulf, and after Leyte Gulf. The U.S. attrition coefficients assume two values, before and after Leyte Gulf.
Source: Willmott (2010) and author's calculations.

cost? The theory in Section 2 indicated a trade-off: As per period spending increases duration decreases and, hence, the total cost of war may increase or decrease. To answer the question I construct counterfactual time series of U.S reinforcements as

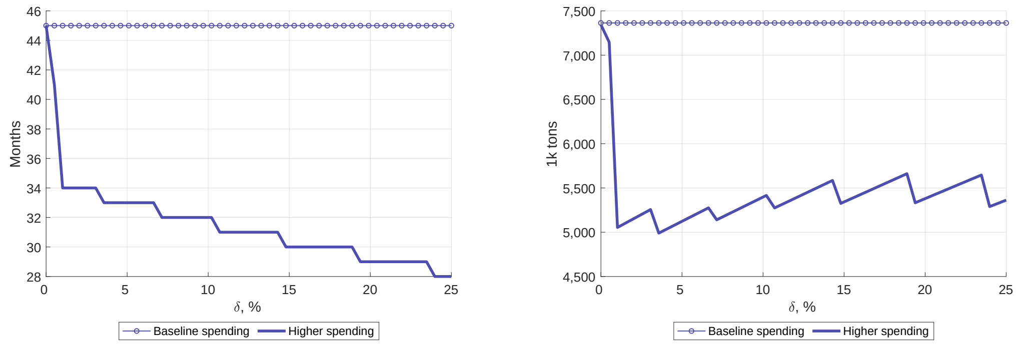
$$\tilde{X}_t^B = (1+\delta)X_t^B$$

and compute the model's solution holding all other parameters constant. The total cost of war is the (non-discounted) sum of reinforcements up to the end of war, that is as long as the implied stock of Japanese weapons is positive:

$$\mathcal{C}(\delta) = \sum_{\{t:\hat{K}_t^R(\omega)>0\}} \tilde{X}_t^B.$$

Note the following: First, I only consider $\delta > 0$. With $\delta < 0$, that is with less spending, the war is longer which would require making assumptions about U.S. reinforcements beyond August 1945. Second, with $\delta > 0$ the per-period spending increases but the "timing" of spending does not. For instance, the surge in late 1944 remains. The question is whether the war will last long enough for the late-1944 surge to be even counted in the total cost. Finally, Japan could react to different U.S. reinforcements in ways that the model cannot account for by construction. Thus, as I indicated earlier, this exercise should be interpreted as describing the properties of a payoff function, not as the solution of a game.

Figure 9 displays the results of these calculations for δ ranging from 0% (baseline) to 25%. On Panel A, the duration of war is decreasing in δ, consistent with the results in Equation (12). The sharp initial decrease can be understood from an inspection of Panel B of Figure 8: In the last year of the war the stock of Japanese tonnage is relatively low and flat and, so, it can be brought to zero all at once via a small increase in δ. Beyond the initial decrease, duration decreases more gradually. On Panel B, the total cost of the war decreases with duration at low values of δ, but increases as δ rises. The conclusion, therefore, is that additional spending by the U.S. would have lowered the cost of war. The minimum cost would have been achieved with an increase in spending of



A – Duration B – Total cost, $\mathcal{C}(\delta)$

Figure 9: Duration and the total cost of war for the U.S.

Source: Author's calculations.

about two and a half percent.

5 Conclusion

I used a combat model of attrition, à la Lanchester (1916), to represent war as a whole. I analyzed how aggregate resources determine the casualties, the duration of war, which side losses in a military conclusion, which side first sues for peace in a political conclusion, and which conclusion prevails.

I presented a theoretical analysis of how resources matter when a country has fewer resources but higher tolerance for casualties than its opponents. I confronted the model to Pacific War data and argued that it fits the data well. I showed, with an example, how to use the model for questions such as: could the U.S. have won the Pacific War at a cheaper cost? The answer was yes: With approximately 2.5% additional monthly spending, the duration of war may have been shortened enough to reduce the overall cost of the war.

Resources are exogenous in the model. Endogenizing the production and destruction of productive capacities seems a natural extension.[16] Modeling decisions, such as the allocation of resources toward the production of consumption goods versus the production of military equipment also seems an important extension. I leave these interesting extensions for future work.

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[16] There exists work on this issue such as Findlay (1996).

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A Weapons stocks and casualties

Let $\mathcal{P}$ represent the matrix of eigenvectors of $\mathcal{M}$ in Equation (3). Standard methods imply that the solution of (3) can be expressed as

$$\begin{aligned}\begin{pmatrix}\tilde{K}_t^R \\ \tilde{K}_t^B\end{pmatrix} &= \mathcal{P}\begin{pmatrix}e^{\lambda t} & 0 \\ 0 & e^{-\lambda t}\end{pmatrix}\mathcal{P}^{-1}\begin{pmatrix}\tilde{K}_0^R \\ \tilde{K}_0^B\end{pmatrix}, \\ &= \frac{1}{2v}\begin{pmatrix}e^{\lambda t} & e^{-\lambda t} \\ ve^{\lambda t} & -ve^{-\lambda t}\end{pmatrix}\begin{pmatrix}\mathcal{A} \\ -\mathcal{B}\end{pmatrix}.\end{aligned}$$

where $\mathcal{A} = \tilde{K}_0^B + v\tilde{K}_0^R$ and $\mathcal{B} = \tilde{K}_0^B - v\tilde{K}_0^R$. So,

$$\begin{aligned}\tilde{K}_t^R &= \frac{1}{2}\left[e^{\lambda t}\mathcal{A} - e^{-\lambda t}\mathcal{B}\right]\frac{1}{v}, \qquad \text{(A.1)} \\ &= \sinh(\lambda t)\tilde{K}_0^B/v + \cosh(\lambda t)\tilde{K}_0^R, \\ \tilde{K}_t^B &= \frac{1}{2}\left[e^{\lambda t}\mathcal{A} + e^{-\lambda t}\mathcal{B}\right], \qquad \text{(A.2)} \\ &= \cosh(\lambda t)\tilde{K}_0^B + \sinh(\lambda t)v\tilde{K}_0^R,\end{aligned}$$

where sinh and cosh are the hyperbolic sine and cosine functions, respectively. It it immediate that

$$\begin{array}{rclrcl}
\partial K_t^B/\partial X^B &=& \sinh(\lambda t)/\lambda, & \partial K_t^R/\partial X^B &=& [1-\cosh(\lambda t)]/\theta^R, \\
\partial K_t^B/\partial X^R &=& [1-\cosh(\lambda t)]/\theta^B, & \partial K_t^R/\partial X^R &=& \sinh(\lambda t)/\lambda, \\
\partial K_t^B/\partial K_0^B &=& \cosh(\lambda t), & \partial K_t^R/\partial K_0^B &=& \sinh(\lambda t)/v, \\
\partial K_t^B/\partial K_0^R &=& \sinh(\lambda t)v, & \partial K_t^R/\partial K_0^R &=& \cosh(\lambda t).
\end{array}$$

It follows, from the properties of the hyperbolic functions, that

$$\frac{\partial K_t^B}{\partial X^B} > 0\ ,\ \frac{\partial K_t^B}{\partial X^R} < 0\ ,\ \frac{\partial K_t^B}{\partial K_0^B} > 0\ ,\ \frac{\partial K_t^B}{\partial K_0^R} < 0,$$

and

$$\frac{\partial K_t^R}{\partial X^B} < 0\ ,\ \frac{\partial K_t^R}{\partial X^R} > 0\ ,\ \frac{\partial K_t^R}{\partial K_0^B} < 0\ ,\ \frac{\partial K_t^R}{\partial K_0^R} > 0.$$

The following relationships are useful: $\mathcal{A} - \mathcal{B} = 2v_1 \tilde{K}_0^R$ and $\mathcal{A} + \mathcal{B} = 2\tilde{K}_0^B$. Red casualties are $D_t^R = \int_0^t \theta^B K_u^B du = \theta^B \int_0^t [\tilde{K}_u^B + \bar{K}^B] du$:

$$D_t^R = \theta^B \frac{1}{2}\left[\mathcal{A}\int_0^t e^{\lambda u} du + \mathcal{B}\int_0^t e^{-\lambda u} du\right] + tX^R = tX^R + K_0^R - K_t^R.$$

Blue casualties are $D_t^B = \int_0^t \theta^R K_u^R du = \theta^R \int_0^t [\tilde{K}_u^R + \bar{K}^R] du$:

$$D_t^B = -\frac{\lambda}{2}\left[\mathcal{A}\int_0^t e^{\lambda u} du - \mathcal{B}\int_0^t e^{-\lambda u} du\right] + tX^B = tX^B + K_0^B - K_t^B.$$

B Military conclusion

Assume that $\mathcal{B} > 0$ and $K_{\tau^M}^R = 0$.

B.1 Duration

Note, from Equations (A.1)-(A.2), that

$$\begin{aligned} 4\left(v\tilde{K}_t^R\right)^2 &= e^{2\lambda t}\mathcal{A}^2 + e^{-2\lambda t}\mathcal{B}^2 - 2\mathcal{A}\mathcal{B}, \\ 4\left(\tilde{K}_t^B\right)^2 &= e^{2\lambda t}\mathcal{A}^2 + e^{-2\lambda t}\mathcal{B}^2 + 2\mathcal{A}\mathcal{B}. \end{aligned}$$

It follows that

$$\left(\tilde{K}_t^B\right)^2 - \left(v\tilde{K}_t^R\right)^2 = \mathcal{A}\mathcal{B} \Rightarrow \tilde{K}_{\tau^M}^B = \bar{K}^B + \sqrt{\left(v\bar{K}^R\right)^2 + \mathcal{A}\mathcal{B}} > 0.$$

Implicitly differentiating $K_{\tau^M}^R = 0$ yields

$$\begin{aligned} \frac{\partial \tau^M}{\partial X^B} &= -\frac{\partial K_{\tau^M}^R/\partial X^B}{\partial K_{\tau^M}^R/\partial \tau} = \frac{[1-\cosh(\lambda\tau^M)]/\theta^R}{\theta^B \tilde{K}_{\tau^M}^B} < 0, \\ \frac{\partial \tau^M}{\partial X^R} &= -\frac{\partial K_{\tau^M}^R/\partial X^R}{\partial K_{\tau^M}^R/\partial \tau} = \frac{\sinh(\lambda\tau^M)/\lambda}{\theta^B \tilde{K}_{\tau^M}^B} > 0, \\ \frac{\partial \tau^M}{\partial K_0^B} &= -\frac{\partial K_{\tau^M}^R/\partial K_0^B}{\partial K_{\tau^M}^R/\partial \tau} = \frac{\sinh(\lambda\tau^M)/v}{\theta^B \tilde{K}_{\tau^M}^B} < 0, \\ \frac{\partial \tau^M}{\partial K_0^R} &= -\frac{\partial K_{\tau^M}^R/\partial K_0^R}{\partial K_{\tau^M}^R/\partial \tau} = \frac{\cosh(\lambda\tau^M)}{\theta^B \tilde{K}_{\tau^M}^B} > 0. \end{aligned}$$

B.2 Casualties

Recall that $D_t^R = \theta^B \int_0^t K_u^B du$. Then,

$$\begin{aligned} \frac{\partial D_{\tau^M}^R}{\partial X^B} &= \theta^B K_{\tau^M}^B \frac{\partial \tau^M}{\partial X^B} + \theta^B \int_0^{\tau^M} \frac{\partial K_u^B}{\partial X^B} du = \frac{1-\cosh\left(\lambda\tau^M\right)}{\theta^R}\mathcal{Z} < 0, \\ \frac{\partial D_{\tau^M}^R}{\partial X^R} &= \theta^B K_{\tau^M}^B \frac{\partial \tau^M}{\partial X^R} + \theta^B \int_0^{\tau^M} \frac{\partial K_u^B}{\partial X^R} du = \tau + \frac{\sinh\left(\lambda\tau^M\right)}{\lambda}\mathcal{Z} > 0, \\ \frac{\partial D_{\tau^M}^R}{\partial K_0^B} &= \theta^B K_{\tau^M}^B \frac{\partial \tau^M}{\partial K_0^B} + \theta^B \int_0^{\tau^M} \frac{\partial K_u^B}{\partial K_0^B} du = \frac{\sinh\left(\lambda\tau^M\right)}{v}\mathcal{Z} < 0, \\ \frac{\partial D_{\tau^M}^R}{\partial K_0^R} &= \theta^B K_{\tau^M}^B \frac{\partial \tau^M}{\partial K_0^R} + \theta^B \int_0^{\tau^M} \frac{\partial K_u^B}{\partial K_0^R} du = 1 + \cosh\left(\lambda\tau^M\right)\mathcal{Z} > 0, \end{aligned}$$

where $\mathcal{Z} \equiv \bar{K}^B/\tilde{K}^B_{\tau^M}$. Similarly, recall that $D^B_t = \theta^R \int_0^t K^R_u du$. Then,

$$
\begin{aligned}
\frac{\partial D^B_{\tau^M}}{\partial X^B} &= \theta^R \int_0^{\tau^M} \frac{\partial K^R_u}{\partial X^B} du = \tau - \frac{\sinh\left(\lambda\tau^M\right)}{\lambda} < 0, \\
\frac{\partial D^B_{\tau^M}}{\partial X^R} &= \theta^R \int_0^{\tau^M} \frac{\partial K^R_u}{\partial X^R} du = \frac{\cosh\left(\lambda\tau^M\right) - 1}{\theta^B} > 0, \\
\frac{\partial D^B_{\tau^M}}{\partial K^B_0} &= \theta^R \int_0^{\tau^M} \frac{\partial K^R_u}{\partial K^B_0} du = 1 - \cosh\left(\lambda\tau^M\right) < 0, \\
\frac{\partial D^B_{\tau^M}}{\partial K^R_0} &= \theta^R \int_0^{\tau^M} \frac{\partial K^R_u}{\partial K^R_0} du = -\upsilon \sinh\left(\lambda\tau^M\right) > 0.
\end{aligned}
$$

C Political conclusion

Assume that Blue reaches $\bar{D}^B$ at τ^B and Red reaches $\bar{D}^R$ at τ^R:

$$\tau^B X^B + K^B_0 - K^B_{\tau^B} = \bar{D}^B, \tag{C.1}$$

$$\tau^R X^R + K^R_0 - K^R_{\tau^R} = \bar{D}^R. \tag{C.2}$$

C.1 Duration

Implicitly differentiating Equation (C.1) yields

$$
\begin{aligned}
\frac{\partial \tau^B}{\partial X^B} &= \frac{\tau^B - \partial K^B_{\tau^B}/\partial X^B}{\partial K^B_{\tau^B}/\partial \tau^B - X^B} = \frac{1}{\theta^R K^R_{\tau^B}} \left(\frac{\sinh\left(\lambda\tau^B\right)}{\lambda} - \tau^B \right) > 0, \\
\frac{\partial \tau^B}{\partial X^R} &= -\frac{\partial K^B_{\tau^B}/\partial X^R}{\partial K^B_{\tau^B}/\partial \tau^B - X^B} = \frac{1}{\theta^R K^R_{\tau^B}} \left(\frac{1 - \cosh\left(\lambda\tau^B\right)}{\theta^B} \right) < 0, \\
\frac{\partial \tau^B}{\partial K^B_0} &= \frac{1 - \partial K^B_{\tau^B}/\partial K^B_0}{\partial K^B_{\tau^B}/\partial \tau^B - X^B} = \frac{1}{\theta^R K^R_{\tau^B}} \left(\cosh\left(\lambda\tau^B\right) - 1 \right) > 0, \\
\frac{\partial \tau^B}{\partial K^R_0} &= -\frac{\partial K^B_{\tau^B}/\partial K^R_0}{\partial K^B_{\tau^B}/\partial \tau^B - X^B} = \frac{1}{\theta^R K^R_{\tau^B}} \upsilon \sinh\left(\lambda\tau^B\right) < 0.
\end{aligned}
$$

Implicitly differentiating Equation (C.2) yields

$$
\begin{aligned}
\frac{\partial \tau^R}{\partial X^B} &= -\frac{\partial K^R_{\tau^R}/\partial X^B}{\partial K^R_{\tau^R}/\partial \tau^R - X^R} = \frac{1}{\theta^B K^B_{\tau^R}}\left(\frac{1-\cosh\left(\lambda\tau^R\right)}{\theta^R}\right) < 0,\\
\frac{\partial \tau^R}{\partial X^R} &= \frac{\tau^R - \partial K^R_{\tau^R}/\partial X^R}{\partial K^R_{\tau^R}/\partial \tau^R - X^R} = \frac{1}{\theta^B K^B_{\tau^R}}\left(\frac{\sinh\left(\lambda\tau^R\right)}{\lambda} - \tau^R\right) > 0,\\
\frac{\partial \tau^R}{\partial K^B_0} &= -\frac{\partial K^R_{\tau^R}/\partial K^B_0}{\partial K^R_{\tau^R}/\partial \tau^R - X^R} = \frac{1}{\theta^B K^B_{\tau^R}}\left(\frac{\sinh\left(\lambda\tau^R\right)}{v}\right) < 0,\\
\frac{\partial \tau^R}{\partial K^R_0} &= \frac{1-\partial K^R_{\tau^R}/\partial K^R_0}{\partial K^R_{\tau^R}/\partial \tau^R - X^R} = \frac{1}{\theta^B K^B_{\tau^R}}\left(\cosh\left(\lambda\tau^R\right)-1\right) > 0.
\end{aligned}
$$

C.2 Casualties

Red casualties at τ^R are $D^R_{\tau^R} = \bar{D}^R$ and Blue casualties at τ^B are $D^B_{\tau^B} = \bar{D}^B$. Using results from Sections A and C.1, Blue casualties at τ^R satisfy

$$
\frac{\partial D^B_{\tau^R}}{\partial X^B} = \frac{\partial}{\partial X^B}\left(\theta^R \int_0^{\tau^R} K^R_u du\right) = \underbrace{\frac{\theta^R K^R_{\tau^R}}{\theta^B K^B_{\tau^R}}\frac{1-\cosh\left(\lambda\tau^R\right)}{\theta^R}}_{<0} + \underbrace{\tau^R - \frac{\sinh\left(\lambda\tau^R\right)}{\lambda}}_{<0} < 0,
$$

$$
\frac{\partial D^B_{\tau^R}}{\partial X^R} = \frac{\partial}{\partial X^R}\left(\theta^R \int_0^{\tau^R} K^R_u du\right) = \underbrace{\frac{\theta^R K^R_{\tau^R}}{\theta^B K^B_{\tau^R}}\left(\frac{\sinh\left(\lambda\tau^R\right)}{\lambda} - \tau^R\right)}_{>0} + \underbrace{\frac{1}{\theta^B}\left(\cosh\left(\lambda\tau^R\right)-1\right)}_{>0} > 0,
$$

$$
\frac{\partial D^B_{\tau^R}}{\partial K^B_0} = \frac{\partial}{\partial K^B_0}\left(\theta^R \int_0^{\tau^R} K^R_u du\right) = \underbrace{\frac{\theta^R K^R_{\tau^R}}{\theta^B K^B_{\tau^R}}\frac{\sinh\left(\lambda\tau^R\right)}{v}}_{<0} + \underbrace{1-\cosh\left(\lambda\tau^R\right)}_{<0} < 0,
$$

$$
\frac{\partial D^B_{\tau^R}}{\partial K^R_0} = \frac{\partial}{\partial K^R_0}\left(\theta^R \int_0^{\tau^R} K^R_u du\right) = \underbrace{\frac{\theta^R K^R_{\tau^R}}{\theta^B K^B_{\tau^R}}\left(\cosh\left(\lambda\tau^R\right)-1\right)}_{>0} + \underbrace{\frac{\theta^R}{\lambda}\sinh\left(\lambda\tau^R\right)}_{>0} > 0,
$$

Red casualties at τ^B are derived similarly.

D The BM frontier

Let τ denote the duration of war along the BM frontier. So, $\tau X^B + K_0^B - K_\tau^B = \bar{D}^B$. Differentiating and using results from Appendix A and Appendix B.1 yields

$$\left.\frac{dK_0^B}{dK_0^R}\right|_{BM} = -\frac{\cosh(\tau\lambda)b - \sinh(\tau\lambda)v}{\sinh(\tau\lambda)b/v + 1 - \cosh(\tau\lambda)} > 0 \tag{D.1}$$

where $b \equiv X^B/(\theta^B \tilde{K}_\tau^B) > 0$ since $\tilde{K}_\tau^B > 0$ at the military conclusion when $K_\tau^R = 0$. Furthermore, the numerator in Equation (D.1) is monotonically increasing and the denominator is monotonically decreasing. Thus, the slope of BM is monotonically decreasing with τ:

$$\frac{\partial}{\partial\tau}\left.\frac{dK_0^B}{dK_0^R}\right|_{BM} < 0. \tag{D.2}$$

It is convenient to write

$$\left.\frac{dK_0^B}{dK_0^R}\right|_{BM} = v\frac{\tanh(\tau\lambda) - b/v}{\frac{1}{\cosh(\tau\lambda)} + \tanh(\tau\lambda)\, b/v - 1},$$

where $\tanh(x) \equiv \sinh(x)/\cosh(x)$ is the hyperbolic tangent Then, from the properties of the hyperbolic cosine and tangent, it follows that

$$\lim_{\tau\to\infty}\left.\frac{dK_0^B}{dK_0^R}\right|_{BM} = v \quad \text{and} \quad \lim_{\tau\to 0}\left.\frac{dK_0^B}{dK_0^R}\right|_{BM} = +\infty. \tag{D.3}$$

It follows, from (D.2) and (D.3), that the BM frontier is steeper than the stable branch ($\mathcal{B} = 0$). It also follows that, as τ approaches infinity, i.e., near the stable branch, the slope of BM approaches that of the stable branch.

Finally, note that the slope of BM is monotonically decreasing in b and that

$$\begin{aligned}\lim_{b\to\infty}\left.\frac{dK_0^B}{dK_0^R}\right|_{BM} &= \lim_{b\to\infty}\frac{v}{\tanh(\tau\lambda)}\frac{\tanh(\tau\lambda)/b - 1/v}{\frac{1}{b\sinh(\tau\lambda)} + 1/v - \frac{1}{b\tanh(\tau\lambda)}} \\ &= -\frac{v}{\tanh(\tau\lambda)} = \left.\frac{dK_0^B}{dK_0^R}\right|_{d\tau^M=0},\end{aligned}$$

where the last equality is Equation (13). It follows that, for finite values of b,

$$\left.\frac{dK_0^B}{dK_0^R}\right|_{BM} > \left.\frac{dK_0^B}{dK_0^R}\right|_{d\tau^M=0}. \tag{D.4}$$

As K_0^R increases along BM the corresponding increase in K_0^B is more than the increase needed to maintain τ^M constant. Hence, the duration of war decreases and the slope of BM increases. In sum, BM is increasing and convex.

E Civilians

Let Blue allocates a fraction $\alpha^B \in (0,1)$ of its weapons stock to destroying Red weapons and the rest to destroying Red civilian resources. I interpret civilian resources as combinations of human and material capital, and I assume a rate of transformation, η^R, from civilian resources to weapons. Let $\alpha^R \in (0,1)$ and η^B have symmetric interpretations. Equations (1) and (2) become

$$dK_t^R/dt = -\theta^B\alpha^B K_t^B + X^R, \quad \text{and} \quad dK_t^B/dt = -\theta^R\alpha^R K_t^R + X^B.$$

The flow of Red civilian resources destroyed by Blue weapons at t, expressed in Red weapons, is then $\eta^R\theta^B(1-\alpha^B)K_t^B$. Red casualties become

$$\begin{aligned} D_t^R &= \alpha^B\theta^B\int_0^t K_u^B du + \eta^R\theta^B(1-\alpha^B)\int_0^t K_u^B du, \\ &= \left(1+\eta^R\frac{1-\alpha^B}{\alpha^B}\right)\left(tX^R + K_0^R - K_t^R\right). \end{aligned}$$

A symmetric result holds for Blue casualties. There are two differences between this model and that in the main body of the paper. First, the laws of motion of weapons stock are like in Equations (1) and (2) with modified attrition coefficients, that is $\theta^R\alpha^R$ instead of θ^R and $\theta^B\alpha^B$ instead of θ^B. Second, casualties are scaled versions of that in Equations (8) and (9). Thus, this model is isomorphic to that in the main body of the paper, and the analysis remains valid with this representation of civilian casualties.

F The Pacific War

F.1 Before Pearl Harbor

After its victory in the 1904-05 Russo-Japanese War, Japan obtained control of the Liadong Peninsula in South Manchuria and, to protect its assets in the peninsula, created the Kwantung Army in 1906.[17] During World War 1, Japan joined the Entente and seized German possessions in China and the Pacific.[18]

World War 1 changed the strategic outlook of the Japanese armed forces, particularly the Army. Europeans expected a short war in 1914. Instead, the conflict lasted over four years and mobilized national resources on a scale never observed before. Some Japanese Army leaders concluded that future conflicts would be similarly long and demanding of resources Japan did not have. Another lesson that Japanese Army leaders drew from World War 1 was defiance toward alliances, which they believed led Europe to war in 1914. Thus, autarky and the control of Manchuria's resources seemed better options to protect Japan than alliances and trade.[19] Some politicians in Tokyo held similar views. For instance, Matsuoka Yosuke, a member of Parliament and future Foreign Minister, referred to Manchuria as "Japan's lifeline."[20]

During the 1930s, Japan extended its hold on Manchuria and, in 1937, it invaded the rest of China, starting the second Sino-Japanese war. In the words of a Japanese general, the war turned into "an endless bog." (From Barnhart, 1987, p. 89.) By 1940, China had not yielded and, instead of self-sufficiency, Japan was relying on foreign resources, particularly oil from the United States, to fight. Thus, in 1940, Japan signed the Tripartite Pact and planned an expansion southward to seize the resources of French Indochina and the oil fields of the Dutch East Indies, then under German control.

A southward move, it was perceived, would be opposed by the United States. Because of the war in China, the American public was increasingly hostile to the shipping of oil and scrap iron to Japan. Furthermore, American officials were likely to defend the "Open Door" policy in China.[21] The U.S. fleet in Hawaii and the U.S. military presence in the Philippines both posed threats to

[17] Chen (2015), Clements (2022, p. 105).
[18] Louis (1966), Morison (2001b, p. 7), and Hotta (2014, p. 43).
[19] Barnhart (1987, ch. 1), Drea (2016, p. 137-138).
[20] Drea (2016, p. 168).
[21] Barnhart (1987, ch. 6)

the southward move and so, in 1941, Japan decided to preemptively eliminate these threats and to attack the United States.

F.2 Attitudes toward casualties

The United States

Historian D. M. Giangreco writes "Americans' concern over casualties was very real and hard to miss." (Giangreco, 2017, p. 11.) Elections held during the war, including a presidential election in 1944 and congressional elections in 1942 and 1944, provide ample evidence that the American public's view of casualties was a concern for civilian and military leaders.

In mid-1944, with the invasions of Normandy in Europe and of the Mariana Islands in the Pacific, the U.S. Army experienced what it termed the "casualty surge": An average of 65,000 monthly casualties from June 1944 to May 1945, with a peak of 88,000 in December 1944. These figures, however, underestimate the casualty weight of the war since they exclude losses from diseases and psychiatric breakdowns and losses for the Navy and Marines in the Pacific.[22]

The publication of casualties was politically sensitive. On the one hand, casualties demonstrated America's commitment to its allies as well as to its enemies, or the lack thereof. On the other hand, casualties could affect the resolve of the American public. The Roosevelt administration, looking for an optimal strategy between these considerations, adopted a variety of measures, such as changing the calculations of casualties in the second half of 1944; substituting "reinforcements" for "replacements" in its communications; and instituting a point system to bring home longer-serving veterans.[23]

On several occasions war leaders indicated their concern for the effect of casualties on the resolve of the American public. In December 1943, for instance, Secretary of the Navy F. Knox wrote to a friend that "we must be prepared for casualties on a scale we have never known since the Civil War—and we are not at all prepared for them." In another letter Knox indicated that when the American public "finds out that the war with Japan will take a couple of years more, I just wonder whether they will have the guts to stay with it to the finish." (From Giangreco, 2017, p. 9.)

[22] Giangreco (2017, p. 3).
[23] Giangreco (2004), Giangreco (2017, ch.2).

In 1944, Gen. G. C. Marshall, the Army Chief of Staff, said in a public address that

> "We are daily confronted with the bitter human cost of this great struggle. We do not have the destroyed homes of England or daily casualties among our peaceful population as they do; but because of our expanding battlefront our military casualties are steadily increasing (...) The great battle now in progress must be kept going, every front must be kept blazing until we break the control of the German Army and people (...) [It is] far better to accept heavy casualties for a brief period than the much greater total which inevitably accumulates from the daily attrition of prolonged periods of inactivity on the battlefield." (From Giangreco, 2017, p. 19)

In March 1945, Marshall wrote to U.S. representative W. H. Hess: "I and others in responsible places in the War Department, are keenly sensitive to the daily casualties we are suffering." Such correspondence illustrates the essence of civilian oversight of the military.

Gen. D. MacArthur, who briefly contemplated running for president in 1944, claimed throughout the war that his strategies were more concerned with the lives of soldiers than were the strategies of the Navy or the Marines. For instance, after the Buna campaign (New Guinea, Jan. 1943) MacArthur announced in a communiqué to the press that "[The] utmost care was taken for the conservation of our forces with the result that probably no campaign in history against a thoroughly prepared and trained army produced such complete and decisive results with a lower expenditure of life and resources." (From McManus, 2019, p. 341) This, and other similar communiqués of MacArthur, have raised many objections.[24] Regardless of their truthfulness, however, they reveal U.S. military leaders' concern for the public's perception of casualties.

After the battle of Tarawa (Gilbert Islands, Nov. 1943) a controversy erupted in the American press about the need for the high casualties—nearly 3,000 in three days, including 1,000 killed.[25] Roosevelt himself hesitated before allowing the release of a film showing dead American soldiers on Tarawa's beaches.[26] The controversy also affected cooperation between the armed services: The

[24] Jones (1989, p. 46).
[25] Toll (2015, p. 365)
[26] Boomhower (2017, p. 136).

Army favored a careful approach to fighting while the Marines favored frontal assaults. Which approach was the most economical in lives was central to the controversy.[27] The same issue resurfaced during the battle for Saipan (Marianna archipelago, June 1944) and severely affected Army-Marines relations.

Japan

Japan in 1941 was not a dictatorship in the strictest sense. Yet, its parliamentary system had ceased to function in the fall of 1940 and national policies were increasingly influenced by the armed forces without effective constraints from civilian authorities.[28] The official historian of the U.S. Navy in World War 2, S. E. Morisson writes that "it was the army that made Japanese foreign policy before 1920 and after 1931." (Morison, 2001b, p. 6.) Historians D. M. Goldstein and K. V. Dillon, make a similar point:

> "Where, in this picture was the civilian government? It was firmly under the thumb of the military (...) Thus one may well form the opinion—correctly, we believe—that for most if not all of 1941 Japanese-U.S. diplomacy was irrelevant. Any agreement reached between the Foreign ministry and the State Deparment would be meaningless unless the Japanese military supported it. And the Japanese military—particularly the Army—was not interested in peace." (Goldstein and Dillon, eds, 1993, p. 2)

In 1928, Kwantung army officers took it upon themselves to assassinate Zhang Zuolin, the then-warlord of Manchuria. Army officials in Tokyo recommended that the matter be dealt with internally, against the wishes of the prime minister. Eventually, the emperor acceded to the Army's demands and asked the prime minister to step down. Historian E. J. Drea concludes "The army had placed its prestige above the law (...) Generals had condoned a criminal conspiracy and assassination (...) and threatened to bring down the cabinet if the army did not get its way. (...) Open contempt for the civilian cabinets and politicians became a familiar pattern in the army's continuing illegal attempts to achieve its domestic and international ends." (Drea, 2016, p. 166). Indeed, during the 1930s, attempted coups and assassinations befell the opponents of militarism: Prime minister Hamaguchi was shot in 1930 and prime minister

[27] McManus (2019, p. 528).
[28] Hotta (2014, p. 19)

Inukai was assassinated in 1932. In 1936 a coup attempt by junior Army officers resulted in the assassination of two former prime ministers (Takahashi and Saito) and other high-ranking government officials. It was the Kwantung Army that, in 1931, staged the Mukden incident and invaded Manchuria without a mandate from the civilian government. In 1937 it was again the Army that escalated the Marco Polo bridge incident, which marked the beginning of the second Sino-Japanese war and the Pacific war.

There are numerous indications that war casualties imposed less of a constraint on Japanese leaders than on U.S. leaders. In January 1941, Gen. Tojo, then minister of the Army and soon-to-be prime minister, issued a *Field Service Code* for the soldiers of the Japanese Army. Its most notable quote is "Never accept alive the shame of capture (...) Die so as not to live the disgrace of such an offense." (From Clements, 2022, p. 224.) The influence of this document and whether such exhortation should be taken literally has been debated. Yet, Clements reports that many contemporaries and historians believe it should. Regardless of the reason, however, it remains the case that Japanese did indeed seek to avoid casualties less than the Americans did. There are the well-known cases of Japanese soldiers, sailors and airmen committing suicide in order to destroy U.S. targets, e.g., the Kamikaze pilots. It must be noted that, even if such tactics became prevalent in the last year of the war, they existed at the onset: During the attack on Pearl Harbor, midget submarines participated in what was essentially a suicide mission.[29]

Suicide, instead of surrender, was often the outcome of battles in which the Japanese found themselves on the loosing end. A suicide charge on Attu (Aleutians Islands, May 1943) left 27 survivors from a garrison of 2,600 soldiers. Before the charge the division's medical officer had executed patients too weak to participate in the charge.[30] At Tarawa, after the suicide charges that concluded the battle, only 17 survivors remained from a garrison of 3,600 soldiers. At Kwajalein (Marshall Islands, Feb. 1944) another suicide charge left 51 survivors from a garrison of 3,500.[31] Finally, at Saipan (Marianna Island, June 1944) a suicidal charge of nearly 3,000 fighters concluded the battle in which 27,000 Japanese died.[32]

Self-inflicted casualties were not confined to the suicide of those fighting the Americans. On Okinawa (Ryukyu Islands, Apr.-Jun. 1944), where a large

[29] Goldstein and Dillon, eds (1993, p. 146), Hotta (2014, p. 257).
[30] Drea (2016, p. 231).
[31] Clements (2022, p. 260-261).
[32] Morison (2001a, p. 336).

civilian population lived, the Japanese Army commander had declared that the Army would confiscate all food for its benefit once the Americans landed. After the landings, there were numerous murders and summary executions of civilians by the Japanese Army.[33] Furthermore, the Army encouraged mass suicides of civilians. Kinjo Shigeaki, an Okinawan who was sixteen during the war, recalls:

> "[During the war] there was *'gyokusai'* (...) meaning people giving up their lives joyfully for their country (...) A week prior to the group suicide a sergeant gave out two hand grenades each to the members of the village youth organization (...) They were directed to throw one of them at the enemy and use the other to engage in *gyokusai* (...) In short, what was planned was a *gyokusai* of everyone on the island. It's often been argued that the military never actually issued orders to commit suicide, but that's beside the point."(From Cook and Cook, 1992, p. 364).

On Saipan, where about 22,000 civilian lived, several hundred committed suicide. E. J. Drea writes that "(...) the army had imposed its standard of no surrender onto the civilian population to legitimize the notion of death before dishonor and collective suicide for all Japanese." (Drea, 2016, p. 240.)

F.3 Japan's plan

In 1940 the Japanese government set up an "Institute of Total War Studies" comprising thirty members drawn from the armed forces, different branches of government, businesses, and the press. The institute was to study a hypothetical war with the U.S. and its allies while the war on China continued. Researchers from the institute presented their work to the cabinet, including Prime Minister Konoye and Minister of the Army Tojo (who was to become prime minister), in August 1941. Historian H. P. Willmott reports that

> "[the cabinet] was subjected to the conclusion that the Japanese economy and manpower resources could not sustain the burden of the China war should that conflict continue for another five to ten years; that Japan could never win a war with the United States; and that in a war with the United States Japan's position, in terms

[33] Drea (2016, p. 247).

> of shipping, would become extremely difficult after late 1943; and that by the end of 1944 Japan would have reached the point at which it would no longer be able to wage war effectively (...) It is very rare that a state is ever given so detailed an analysis of its own mortality."(Willmott, 2010, p. 452).

A detailed analysis of the decision to go to war despite such warnings is beyond the scope of this paper. I summarize a few aspects of that decision below, in support of my discussion in Section 3. Interested readers should consult the references I quote.

Adm. Yamamoto, the commander of the combined fleet in 1941, did not favor war with the U.S.[34] He was in agreement with the conclusion of the Institute of Total War Studies. In September 1941 he indicated to his superior Adm. Nagano, the Navy Chief of Staff, "it is obvious that a Japanese-American war will become a protracted one. As long as tides of war are in our favor, the United States will never stop fighting. As a consequence the war will continue for several years, during which matériel will be exhausted, vessels and arms will be damaged, and they can be replaced only with great difficulties (...) We must not start a war with so little a chance of success." (From Asada, 2006, p. 277.) As commander of the combined fleet, however, Yamamoto's duty was to conceive the attack. He explained the thinking behind his Pearl Harbor plan in a letter to Minister of the Navy Adm. Oikawa in January 1941:

> "The most important thing we have to do first of all in a war with the U.S., I firmly believe, is to fiercely attack and destroy the U.S. main fleet at the outset of the war, so that the morale of the U.S. Navy and her people goes down to such an extent that it cannot be recovered (...) [A] surprise attack on the enemy air forces in the Philippines and Singapore should definitely be made almost at the same time of launching attacks on Hawaii (...) when the U.S. main forces is destroyed, I think those [southern forces] will loose morale to such an extent that they could hardly be of much use in actual bitter fighting." (From Goldstein and Dillon, eds, 1993, p. 116-117.)

Clearly, Yamamoto's view on America's will to fight seems different in September than in January. Did he exaggerate the weakness of America's will to fight

[34]The combined fleet comprised battleships, aircraft carriers, and supporting ships. It was the main fighting instrument of the Japanese Navy.

to convince his superiors who were not keen on his plan? Historian S. Asada reports that "Japanese historians have generally concluded that Yamamoto indeed shared with his countrymen a general underestimation of the American people." (Asada, 2006, p. 280.) Regardless of Yamamoto's own (maybe changing) views, however, he was aware of a pervasively low opinion of Americans in his country. In the summer of 1941, he told Konoye "There are some who say that if we give a terrible blow to the enemy at the outset of the war, warships cannot be replenished quickly, so Americans will raise their hands and surrender." (From Asada, 2006, p. 280.) Indeed, the view that Americans would not stand war for long was pervasive in the Navy. S. Asada writes,

> "The Japanese Navy as a whole suffered from grievous misperceptions of American national character. Chihaya Masataka, a lieutenant commander at the time of the Pearl Harbor attack, later wrote 'We thought that we could easily tackle them [Americans]; a race so steeped in material comfort and absorbed in the pursuit of pleasure was spiritually degenerate.' Rear Admiral Nakazawa, chief of the Operations Section in 1939-41, believed that the American people, 'a composite nation of immigrants, lacked unity, could not withstand adversity and privations, and regarded war a form of sport, so that if we deal a severe blow at the outset of hostilities they would loose the will to fight.' Ambassador Grew, who knew the Japanese as well as any American, testified 'They regarded us as a decadent nation [in which] pacifism and isolationism practically ruled the policiy of our Government.'" (Asada, 2006, p. 292.)

Because of such low esteem for the Americans, Japanese leaders started a war they expected to be short. There was no plan for a protracted war. On October 7, 1941, Gen. Tojo asked Adm. Oikawa if he had confidence in victory in a war against the U.S. Oikawa replied "That, I am afraid, I do not have... If the war continues for a few years, we do not know what the outcome would be." (From Hotta, 2014, p. 199.) The same day, Adm. Nagano told Gen. Sugyiama, the Army chief of Staff, "there is a chance of winning for now. As far as the future is concerned, the question of victory or defeat will depend on the total combination of material and psychological strength."(From Hotta, 2014, p. 200.) Adm. Sadatoshi, chief of the Operations Section in 1944, wrote after the war "I saw this as a limited war (...) Our plan was to cause the enemy a great damage and thus to win a balance of force in our favor and terminate hostilities on the basis of a compromise settlement while Japan still had a margin of strength left." (From Asada, 2006, p. 287.)

Three weeks before the Pearl Harbor attack, on November 15, 1941, the *"Plan for the Facilitation of the Conclusion of War with the United States, Britain, and the Netherlands"* was approved by the Japanese high command. It stated

> "We aim to demolish the Far Eastern bases belonging to the United States, Britain and the Netherlands quickly so as to ensure our survival and defense, while actively seeking the surrender of the Chiang Kai-shek regime, cooperating with Germany and Italy to prompt British surrender first, and trying to deprive the United States of its will to continue the war." (From Hotta, 2014, p 277.)

Note that Japan planned on its enemies—the Chinese, the British, and the Americans—to surrender, not necessarily to be military defeated.