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From Population Growth to TFP Growth*

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Abstract

A slowdown in population growth reduces business dynamism by increasing the share of older firms. We explore how this affects productivity growth using a business dynamics model with endogenous productivity. The growth rate of older firms is a key factor in determining the impact of population growth on productivity. Quantitatively, this effect is substantial for both the U.S. and Japan. In the U.S., slowing population growth reduces TFP growth by 0.3 percentage points from 1970 to 2060, with an even larger effect in Japan. However, TFP growth reacts slowly due to short-run counterbalancing factors.

Keywords: population growth, economic growth, firm dynamics, demographics, productivity, innovation, TFP, Japan.

JEL Codes: E20, J11, O33, O41.

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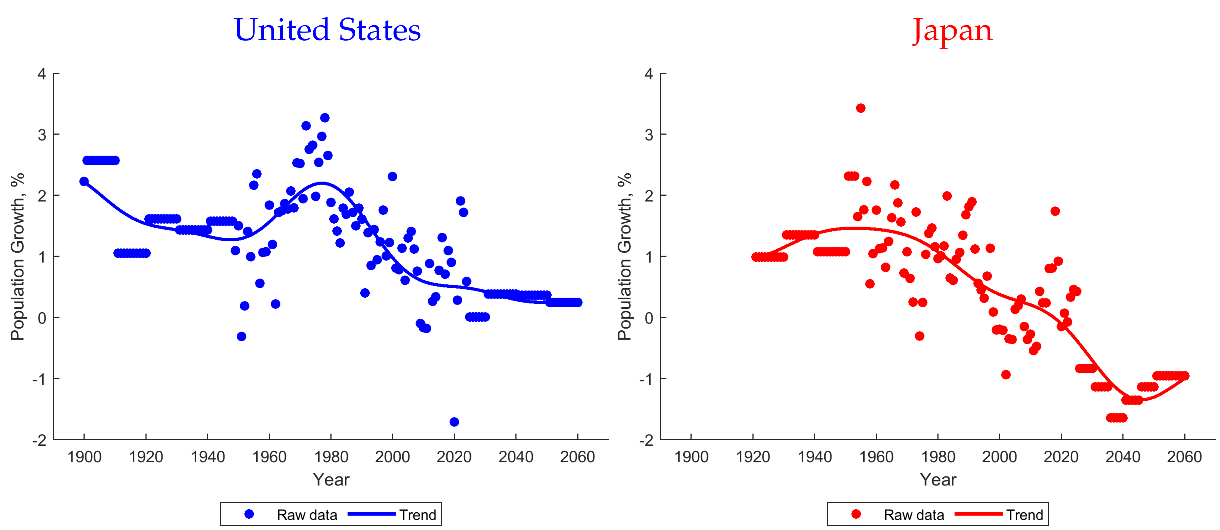
1 Introduction

In developed countries, there is an increasing concern that slowing population growth may lead to a decline in economic growth.[1] Figure 1 shows the estimated trends for the United States and Japan, the countries that we will consider in our quantitative exercises. At the beginning of our data in 1900, or in the 1970s when the baby boomers entered the job market, the trend in US labor force growth was approximately 2.2%. In contrast, current forecasts estimate that in 2060, it will be less than 0.3%. The fall is more dramatic in Japan, and it started earlier. While the trend labor force growth was close to 1.5% in 1950, the forecast for the 2040s is below -1.3%.

Figure 1: Labor force growth rate



Source: See appendix 11.7.

Several recent studies (Karahan, Pugsley and Şahin, 2024; Peters and Walsh, 2022; Hopenhayn, Neira and Singhania, 2022) have demonstrated that a slowing in population growth in the United States has led to a decline in business dynamism, as it increases the share of old businesses. But how does this shift in population and business demographics affect productivity growth? We aim to provide an answer to this specific question.

This paper incorporates population growth and endogenous productivity growth into a business dynamics model to achieve this goal. As in Hopenhayn (1992), businesses typ-

[1]Although we usually refer to it as population growth, the driving force in our study is labor force growth.

ically start with low productivity and gradually increase their productivity throughout their life cycle. However, the intensity of innovation within the business, which is endogenous, determines the productivity of younger businesses. A business innovation improves on previous innovations, as in Romer (1990) and Aghion and Howitt (1992). This innovation is the first driver of growth in our model. The productivity of older businesses also increases with age. This productivity growth represents the second growth engine in our model, capturing the technology advancements made by mature, leading businesses. Therefore, for our model to display balanced growth, one of these two forces—innovation by young businesses or technological advancements by established leading businesses—must be present. Additionally, every period, some incumbent businesses exit, while new businesses enter.[2]

The main theoretical result is that in comparing BGPs, the shape of the businesses' life-cycle profile determines the sign and magnitude of the impact of population growth on productivity growth. In particular, we identify a "sufficient statistic"— the growth rate of surviving old businesses. If the growth rate of the size of surviving old businesses is negative, a fall in the population growth rate will result in a decrease in the rate of aggregate productivity growth. Two components make up the mechanism for this result. First, as population growth declines, so does the growth rate in the number of businesses; otherwise, the average firm size will diverge. As a result, an economy with a lower population growth rate will have a lower proportion of young businesses, as a small number of new businesses relative to existing businesses implies that most businesses are old. The second element of the mechanism underlying the result above relies on the productivity growth of old businesses relative to overall productivity growth. The "sufficient statistic" is precisely a measure of these two growth rates. If the size of surviving old businesses decreases over the life cycle, their productivity is expanding at a slower rate than the average productivity of the economy. Thus, combining these two factors, as the labor force expands more slowly and the proportion of old businesses increases, if the productivity growth of old businesses is lower than the average, the total productivity growth will also

[2]We assume exit is exogenous for the theoretical characterization of the model, but we relax this assumption in a subsequent section.

be lower.

These theoretical results are feasible because there is a fully elastic supply of startups, entrants' innovation is constant, and we assume that survival rates and the growth rate of productivity of leading businesses are exogenous. Subsequently, we examine numerical versions of the model that relax these assumptions. The comparison of BGPs with different population growth rates provides the main quantitative findings. For these exercises, we consider not only the model analyzed in the theoretical section of the paper, but also our benchmark extended model that incorporates congestion at entry and spillovers from new to existing businesses' productivity growth. We conclude that population growth has a significant impact on productivity growth. In the benchmark case, a drop in population growth, as projected for the US from 1970 to 2060, implies a long-run decline in productivity growth of approximately 0.3 percentage points. Similarly, for Japan, the predicted decline in population growth from 1950 to 2060 implies, in the long run, a 0.6 percentage point reduction in productivity growth.

We also analyze the robustness of the impact of population growth on productivity growth on the BGP by modifying the model in different ways. First, we consider a process for productivity that is closer to the more standard AR(1) process. We show that, depending on the details of the specification, it yields results that are slightly larger or slightly smaller. We also argue that these differences are due to the inability of that specification to accurately replicate moments about the life cycle of businesses, which, as we should in theory, are key to our results. Second, we consider extending the model to incorporate endogenous exit. While there is a force that would amplify the results (the exit rate declines even more), there is a counterveiling force (higher innovation) that implies very little change in the results. Finally, we consider the role of adding endogenous innovation by leading businesses and show that it would amplify the results, but only for very low population growth rates.

Next, we compute transitional dynamics for the economies calibrated to the US and Japan. Since computing a transition is computationally challenging, the transitional analysis utilizes a benchmark quantitative model that incorporates congestion at entry and spillovers from new to mature businesses' productivity growth. The main experiment involves providing the model with time series data for the trend in labor force growth and simulating the evolution of total factor productivity (TFP) growth. A salient feature of the impact of population growth on TFP growth is that it takes a considerable amount of time to occur. We investigate why the response of TFP growth is sluggish and discover two significant factors: a labor reallocation effect and a level-vs-growth effect. Both effects fade in the long term, leading to our main result comparing BGPs; however, in the meantime, they partially offset the drop in TFP growth.

The paper's final section validates the mechanism proposed in this work. The dynamic correlation between labor force growth and productivity growth produced by the model is very similar to the correlation found in data for US states. This result, obtained using local projections, supports the proposed mechanism and its quantitative significance.[3]

A recent paper, Alon et al. (2018), is closely related, as it documents how declining entry and the aging of incumbent businesses reduce aggregate productivity growth. Their analysis is empirical, using rich Census data to quantify the impact of shifts in business age composition. Our paper complements theirs by studying the same channel within a structural equilibrium model that endogenizes innovation, entry, and exit, and allows for general equilibrium forces. Taken together, the two approaches provide complementary evidence on the importance of firm dynamics for aggregate productivity.

The modeling choices make our work more closely related to two recent papers that study the relationship between population growth and business dynamism using similar firm dynamics models. According to Karahan, Pugsley and Şahin (2024) and Hopenhayn, Neira and Singhania (2022), the slowdown in labor force growth has resulted in a startup deficit, which can explain the widely recognized decline in business dynamism.[4] These papers share several characteristics of our framework. As a validation exercise, we show that our model can also reproduce the reduction in business dynamism in the US. However, we focus on the impact of the same driving force on TFP growth rather than on

[3]Furthermore, instrument variable regressions suggest a causal effect of labor force growth on productivity growth in section 11.4 in the online appendix. In addition, Section 11.5 in the online appendix demonstrates that the model reproduces the recent slowdown in business dynamism in the United States, a phenomenon examined in several recent papers.

[4]Related, Engbom (2018) focuses on the age of workers and the dynamism of businesses.

business dynamism.[5]

The paper most closely related to ours is Peters and Walsh (2022), which also studies the link between population growth and productivity. Their framework builds on Klette and Kortum (2004) and emphasizes scale effects, market concentration, and markups, while abstracting from firm life-cycle dynamics. In contrast, our approach builds on Hopenhayn (1992) and focuses on how innovation intensity varies with firm age, yielding a novel mechanism that connects slower population growth to weaker TFP growth through the growth of older businesses. The two approaches are complementary: their emphasis is on variety and competition, ours on life-cycle dynamics, and together they highlight distinct channels through which demographics affect aggregate growth.

Several other papers have been written about the importance of population growth to economic prosperity. Jones (2020) investigates the extreme case of long-run negative population growth in the context of models of ideas, including various endogenous and semi-endogenous growth models. He discovers that negative population growth leads to stagnant living standards as the population vanishes. Recently, Kalyani (2022) finds a negative association between inventors' creativity and age and argues that a larger proportion of older workers in the labor force will result in lower productivity growth because inventors are, on average, less creative. Finally, Cooley, Henriksen and Nusbaum (2019) examines the impact of this demographic change on output growth through capital accumulation and labor productivity, while Vandenbroucke (2021) investigates the slowdown in output per worker growth in the 1960s and 1970s.

Our empirical findings relate to those of Karahan, Pugsley and Şahin (2024) and Crump et al. (2019) due to the use of an instrumental variable strategy and the shared focus on the slowdown in labor force growth as the driving force. Karahan, Pugsley and Şahin (2024) shows that the decline in firm entry originates from the slowdown in labor supply growth. Crump et al. (2019) demonstrates that the aging of firms—attributed by Karahan, Pugsley and Şahin (2024) to the decline in the growth rate of the labor force—significantly impacts job destruction, and consequently, the inflow into unemployment. There are two

[5] Although it is not the focus of Engbom (2018)'s analysis, the transitional dynamics shown in that paper's figure 10 reveal a slight decline in growth between 1970 and 2050.

key differences between their work and ours. First, while they focus on the start-up rate, we examine productivity growth. Second, we use local projections to capture the dynamics of the response.

2 Model

The economy is made up of businesses and households. Households own businesses and make decisions about consumption and investment. Businesses are the most important part of the framework, as they innovate, hire workers, and rent capital. In equilibrium, slower labor-force growth reduces the number of startups, changing business demographics, which is critical for determining the relationship between population growth and productivity growth.

2.1 Household

A representative household populates the economy and solves

$$\max_{\{c_t\},\{\mathrm{k}_t\}} \sum_{t=1}^{\infty} \frac{(\beta g_{M_t})^{t-1} c_t^{1-\epsilon}}{1-\epsilon} \tag{1}$$

subject to

$$c_t + g_{M,t+1}\mathrm{k}_{t+1} = w_t + s_t - e_t + r_t\mathrm{k}_t + (1-\delta)\mathrm{k}_t,$$

where $\mathrm{k}_t \equiv \mathrm{K}_t/M_t$ is capital per person, $s_t \equiv \sum_i S_{i,t}/M_t$ is business surplus per person, $e_t \equiv E_t/M_t$ is the initial cost of starting businesses per person, δ is depreciation rate, β is discount factor, and $g_{M,t+1}$ is population growth rate. Note that this representative household also owns the businesses.

2.2 Businesses

To simplify the stochastic process for productivity while preserving flexibility to match key calibration facts, we classify businesses into two types: laggard and leading. Laggard businesses operate at low, flat productivity but can, with some success probability, transition to the leading group, which entails a one-time productivity jump. Leading businesses not only start from a higher productivity level but also experience steady growth thereafter. Exit rates differ across types: laggards face a higher likelihood of exit than leaders.

The total number of businesses is N_t. They have decreasing returns to scale and solve

$$S_{i,t}(x_{i,t}, w_t, r_t) = \max_{k_{i,t}, l_{i,t}} \{x_{i,t}^{\zeta} k_{i,t}^{\alpha} l_{i,t}^{1-\alpha-\zeta} - w_t l_{i,t} - r_t k_{i,t}\}, \tag{2}$$

taking as given wages w_t and capital rental rate r_t. $x_{i,t}$ denotes the productivity of business i at time t, and the solutions for labor $l_{i,t}$, capital $k_{i,t}$, and output $y_{i,t}$ are linear in productivity $x_{i,t}$,

$$l_i = x_i \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha} \right]^{\frac{1}{\zeta}},\ k_i = x_i \left[\left(\frac{\alpha}{r}\right)^{\alpha+\zeta} \left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha-\zeta} \right]^{\frac{1}{\zeta}},$$

$$y_i = x_i \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha-\zeta} \right]^{\frac{1}{\zeta}}.$$

Also, the average productivity in the economy is $X \equiv \frac{1}{N} \sum_i x_i$. Combining average productivity with the expressions above, we get useful expressions for aggregate variables that we will use later to define the economy's equilibrium. Therefore, output, labor, and capital are

$$L = \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha} \right]^{\frac{1}{\zeta}} NX,\ K = \left[\left(\frac{\alpha}{r}\right)^{\alpha+\zeta} \left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha-\zeta} \right]^{\frac{1}{\zeta}} NX,$$

$$Y = \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha-\zeta} \right]^{\frac{1}{\zeta}} NX.$$

Innovators draw inspiration from the ideas of leading businesses to generate their own innovative ideas. A new technology takes one period to start production. Let χ be the average productivity of leading businesses, i.e., $\chi \equiv \frac{1}{N_S} \sum_{i \in \text{leading}} x_i$, where N_S is the number of leading businesses. An innovator will then choose an innovation step size, g, which measures the difference between the innovator's potential productivity, $\hat{x}$, and the reference productivity, χ. Thus, the cost of research for generating $\hat{x}$ is proportional to how far ahead of the pack the project is, $R(\hat{x}/\chi) = \frac{1}{z_R} \left(\frac{\hat{x}}{\chi}\right)^{\iota}$, with $\iota > 2$. Although we assume that this cost is paid entirely at the innovation stage, one period before the firm's birth, the growth caused by R&D realizes stochastically over time. Since there are no financing frictions, this cost could be the expected discounted investment costs paid over time with no significant change in the model.[6] After the research stage, innovators develop ideas to start their businesses. The probability of entering the market (σ) hinges on the amount of money spent on developing the project, $D(\sigma) = \sigma^2/(2z_D)$.[7]

The value of a project started with potential productivity $\hat{x}$ is

$$I(\hat{x};\{w_t\},\{r_t\}) = \sum_{t=1}^{\infty} \hat{\beta}_t \mathbb{E}_{\hat{x}}[S(x_t; w_t, r_t)|\hat{x}],$$

where $\hat{\beta}_t$ is the market discount factor.[8] At the time of innovation, an innovator chooses σ and $\hat{x}$ to maximize its payoff,

$$V(\{w_t\},\{r_t\},\chi_t) = \max_{\sigma_t,\hat{x}_t} \sigma_t \underbrace{I(\hat{x}_t;\{w_t\},\{r_t\})}_{\text{Revenue from project}} - \underbrace{w_t R(\hat{x}_t/\chi_t)}_{\text{Research cost}} - \underbrace{w_t D(\sigma_t)}_{\text{Development cost}} . \tag{3}$$

In partial equilibrium, solving this problem yields the innovator's chosen step size of innovation, g^*, the probability of starting the project, σ^*, and the maximized payoff, V^*. The value V^* is essential because the household is willing to start a business if this value covers the initial fixed cost. As a result, in equilibrium with entry, the following free-entry condition must be met:

$$V_t \leq w_t c_E. \tag{4}$$

The assumption that the entry cost increases one-to-one with wages makes the model tractable and is common in growth models (e.g. Klette and Kortum, 2004). The assumption is also supported by the data presented in Klenow and Li (2022).

Specifically, we consider businesses that start with potential productivity χg. If the project succeeds in becoming a leading business at age=1, its productivity will equal χg right away; if it does not, it will become a laggard with productivity $\theta \chi g$, where $\theta < 1$. Each period, a fraction λ of laggard projects becomes leading businesses, and their productivity increases from $\theta \chi g$ to χg. While being a laggard, the business's productivity remains constant. On the other hand, the productivity of leading ventures grows at a

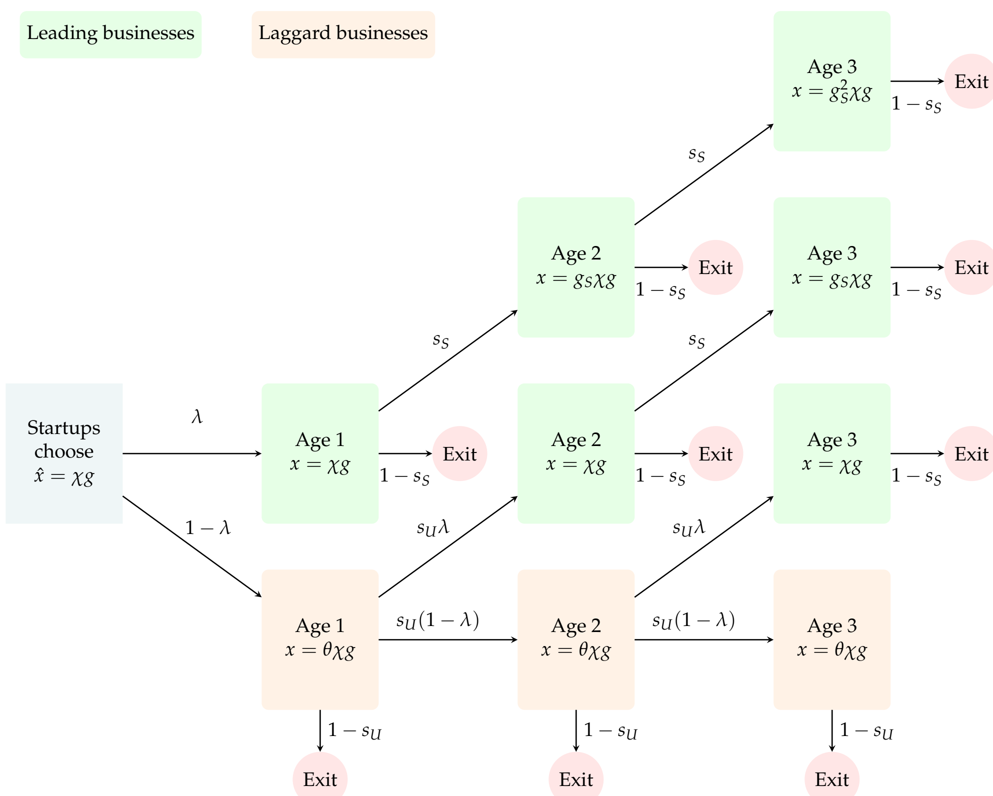
[6] Similarly, since there are no financing frictions, we could alternately interpret that a share of emerging businesses that exit at each age a are emerging businesses that choose again R&D at each age a as they would choose the same innovation step size as new businesses.

[7] The option choice of σ by innovators, which resembles Greenwood, Han and Sanchez (2022), is unimportant for our results, but it simplifies some of the expressions.

[8] Specifically, $\hat{\beta}_t = \prod_{j=1}^{t} \frac{1}{(1+r_j-\delta)}$.

constant pace g_S. The survival rates, s_S and s_U, for leading and laggard businesses differ. Specifically, we assume that $s_S > s_U$ represents the larger exit rate of laggard businesses compared to leading businesses. Although we do not impose this condition in the calibration section, we find that leading businesses are more likely to survive than laggard businesses, which allows us to capture the growth over the life cycle of average business size and the size of surviving businesses. Figure 2 depicts a three-period example of the business lifecycle.

Figure 2: A business's life-cycle (example up to age 3)



The reason for this simplified structure for productivity is that it allows us to construct some useful expressions for a business life-cycle. The businesses born a years ago (i.e., those age a today) can be divided into businesses that today are (i) out of business, (ii) laggards, and (iii) leaders. The following two expressions represent the contribution of the last two groups to the average productivity of age a businesses relative to their potential productivity. Of course, the contribution of businesses that are currently out of business is zero.

For laggard businesses, the expression is simply

$$\Lambda_{U,a} \equiv \theta(1-\lambda)^a (s_U)^{a-1},$$

where the first part, θ, is included because laggard businesses' productivity is θ of the potential productivity, and this term is written relative to potential productivity. The second term is the probability that businesses remain laggards until age a, and the last term is the probability that the business does not exit before age a.

The same expression for leading businesses is more involved,

$$\Lambda_{S,a}(g_S) \equiv \sum_{j=1}^{a} \left[{g_S}^{j-1} (s_U)^{a-j} (1-\lambda)^{a-j} \lambda (s_S)^{j-1} \right].$$

But also, in this case, the first part, g_S^{j-1}, is in place to adjust the productivity relative to potential productivity. Thus, for businesses that became leading j years ago, the adjustment factor is g_S^{j-1}, to account for the productivity growth rate since they became leading. The following four terms together make the probability that a business survived and remained a laggard until age $a-j$, it became leading at age $a-j$, and survived as a leading business from age $a-j$ until a.

Why is this notation useful? We can use it to compute the average productivity based on potential productivity, $\hat{x}_t$, and the number of new businesses, n_t. In particular, for age 1 businesses, average productivity is simply

$$X_{1,t} = \frac{\text{sum prod of age-1 businesses}}{\text{number of age-1 businesses}} = \frac{\hat{x}_t \left(\Lambda_{S,1}(g_S) + \Lambda_{U,1} \right) n_t}{\left(\Lambda_{S,1}(1) + \Lambda_{U,1}/\theta \right) n_t} = \hat{x}_t \left(\lambda + (1-\lambda)\theta \right).$$

Similarly, for age 2 businesses, average productivity is simply

$$\begin{aligned} X_{2,t} &= \frac{\hat{x}_{t-1} \left(\Lambda_{S,2}(g_S) + \Lambda_{U,2} \right) n_{t-1}}{\left(\Lambda_{S,2}(1) + \Lambda_{U,2}/\theta \right) n_{t-1}} \\ &= \frac{\hat{x}_{t-1} \left(s_{U,1}(1-\lambda)\lambda + g_S s_{S,1}\lambda + \theta s_{U,1}(1-\lambda)(1-\lambda) \right)}{\left(s_{U,1}(1-\lambda)\lambda + s_{S,1}\lambda + s_{U,1}(1-\lambda)(1-\lambda) \right)}. \end{aligned}$$

Note that we can also compute the average productivity of the pool of age-1 and age-2 businesses as

$$X_{1-2,t} = \frac{\text{sum prod of age-1 and age-2 businesses}}{\text{number of age-1 and age-2 businesses}}$$
$$= \frac{\hat{x}_t\left(\Lambda_{S,1}(g_S)+\Lambda_{U,1}\right)n_t + \hat{x}_{t-1}\left(\Lambda_{S,2}(g_S)+\Lambda_{U,2}\right)n_{t-1}}{\left(\Lambda_{S,1}(1)+\Lambda_{U,1}/\theta\right)n_t + \left(\Lambda_{S,2}(1)+\Lambda_{U,2}/\theta\right)n_{t-1}}.$$

Following this logic, the average productivity of all businesses in the economy is

$$X_t = \frac{\sum_{a=1}^{\infty}\hat{x}_{t-a+1}\left(\Lambda_{S,a}(g_S)+\Lambda_{U,a}\right)n_{t-a+1}}{\sum_{a=1}^{\infty}\left(\Lambda_{S,a}(1)+\Lambda_{U,a}/\theta\right)n_{t-a+1}}. \quad (5)$$

This equation is crucial to solving the model since it depends on two key equilibrium variables: potential productivity $\hat{x}_t$ and the number of entrants n_t.

Similarly, the expected productivity at age a of entrants whose potential productivity is $\hat{x}$ is

$$\mathbb{E}[x_a|\hat{x}] = (\Lambda_{S,a}(g_S)+\Lambda_{U,a})\hat{x}, \quad (6)$$

and the survival probability up to age a is

$$\Lambda_{S,a}(1)+\Lambda_{U,a}/\theta. \quad (7)$$

Finally, we can use this notation to write the law of motion for the number of projects (by type and total) given the number of entrants n_t at a given time t as

$$N_{U,t} = \sum_a n_{t-a}\Lambda_{U,a}/\theta, \quad (8)$$

$$N_{S,t} = \sum_a n_{t-a}\Lambda_{S,a}(1). \quad (9)$$

$$N_t = N_{U,t} + N_{S,t}. \quad (10)$$

2.3 Equilibrium

To close the model, three market-clearing conditions must be met. The labor-market-clearing condition implies that population equals the sum of labor for production, entry, research, and development, which is

$$M_t = L_t + \frac{n_t}{\sigma}\left(c_E + \frac{1}{z_R}g_t^{\iota} + \frac{1}{2z_D}\sigma_t^2\right). \quad (11)$$

Likewise, the capital-market-clearing condition is

$$\mathrm{K}_t = \left[\left(\frac{\alpha}{r_t} \right)^{\alpha+\zeta} \left(\frac{1-\alpha-\zeta}{w_t} \right)^{1-\alpha-\zeta} \right]^{\frac{1}{\zeta}} N_t \times X_t \tag{12}$$

where $\mathrm{K}_t = \mathrm{k}_t M_t$. Finally, the goods-market-clearing condition is

$$Y_t = C_t + I_t, \tag{13}$$

where $C_t = c_t M_t$ and $I_t = \mathrm{K}_{t+1} - (1-\delta)\mathrm{K}_t$.

We now define the notion of equilibrium in the economy.

Definition 1. Given a sequence for labor supply $\{M_t\}$, an equilibrium is a sequence of prices $\{w_t, r_t\}$, business choices $\{l_{i,t}, k_{i,t}, g_t, \sigma_t\}$, household choices $\{c_t, \mathrm{k}_t\}$, a measure of entrants $\{n_t\}$, and the number of projects, $\{N_{t,S}, N_{t,U}, N_t\}$, such that: (a) c_t and k_t solve the optimization problem of a household (1), (b) $l_{i,t}$ and $k_{i,t}$ solve the business's static problem (2), (c) σ_t and g_t are the innovation choices that results from problem (3), (d) The free entry condition (4) is satisfied, (e) $N_{t,S}$, $N_{t,U}$, and N_t are in accordance with the laws of motion (8), (9), and (10), (f) The clearing conditions for the labor market (11), the capital market (12) and the goods market (13) are satisfied.

The optimal step size of innovation is obtained by solving (3). When we incorporate the free entry condition (4) into the solution for the step size of innovation, we find that the step size of innovation is constant in equilibrium. The next lemma presents this result. *All proofs are in appendix 10.1.*

Lemma 1 (Step size of innovation)**.** *The equilibrium step size of innovation is constant,*

$$g^* = \left(\frac{2 c_E z_R}{\iota - 2} \right)^{\frac{1}{\iota}}. \tag{14}$$

This lemma implies that the step size of innovation is determined by only three parameters: the slope of the innovation cost, the entry cost, and research efficiency. The free entry condition is crucial to this result. Many important aspects of the economy influence income levels, but not the size of innovation, as in Atkeson and Burstein (2010).[9] This result simplifies the analysis because it implies that the productivity growth rate of

[9]Technically, having a constant step size requires that expected profit be a monomial function of potential productivity $\hat{x}$.

young businesses, determined by g^*, and that of old businesses, determined by g_S, will be constant.

One may think that a model with constant g^* and g_S cannot capture what happens in reality. However, it is worth highlighting that Alon et al. (2018), Karahan, Pugsley and Şahin (2024), and Hopenhayn, Neira and Singhania (2022) found that there are minimal changes in the life-cycle profile of business dynamics statistics like the exit rate and average size in the US since there is available data, which is consistent with minor changes in g^* and g_S over time in our model. For robustness, our quantitative exercises include cases in which g^* and g_S are not constant.

3 Balanced growth path

In this section, we characterize a balanced growth path for this economy and investigate how it is impacted by changes in the constant population growth rate g_M. The following lemma characterizes the economy's BGP equilibrium.

Lemma 2 (Characterization of the balanced growth path)**.** *Given a constant growth rate of the labor supply greater than the old businesses' survival rate, $g_M > s_{S,\infty}$, there is a unique BGP equilibrium in which the following occurs: (a) Aggregate variables Y, K, and C grow at constant rates, (b) Wages grow at the same rate, $g_w = (g_X)^{(1-\alpha)/\zeta}$, (c) The interest rate is fixed at $r = \frac{(g_w)^\epsilon}{\beta} - (1-\delta)$, (d) The step size g and the probability of starting business σ are constant, (e) Business size is constant because the number of businesses grows at the same rate as the labor in production and population, $g_N = g_L = g_M$, and (f) Average productivity and average leading business productivity grow at the same rate, $g_X = g_\chi$.*

In the BGP described above, the average productivity of all projects, X, is a function of the potential productivity of new projects today, $\hat{x}_1$, and other parameters:

$$X = \frac{\hat{x}_1 \sum_{a=1}^{\infty} \left(\frac{1}{g_X g_N}\right)^{a-1} \left(\Lambda_{S,a}(g_S) + \Lambda_{U,a}\right)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_N}\right)^{a-1} \left(\Lambda_{S,a}(1) + \Lambda_{U,a}/\theta\right)}, \tag{15}$$

where the growth rate of the number of businesses, g_N, is used to account for the increase in the number of businesses over time. Similarly, χ, which is today's reference productivity for innovators, is

$$\chi = \frac{\hat{x}_1 \sum_{a=1}^{\infty} \left(\frac{1}{g_X}\right)^{a-1} \left(\frac{1}{g_N}\right)^{a-1} \Lambda_{S,a}(g_S)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_N}\right)^{a-1} \Lambda_{S,a}(1)}. \tag{16}$$

Because potential productivity today $\hat{x}_0 = g_X \hat{x}_1$ equals the step size of innovation multiplied by the average productivity of leading businesses; i.e., $\hat{x}_0 = \chi g$, we can derive an equation that defines the relationship between the step size g, productivity growth rate g_X, and the number of businesses growth rate g_N. This equation implies that g_X solves

$$g = \frac{\sum_{a=1}^{\infty} \left(\frac{1}{g_M}\right)^{a} \Lambda_{S,a}(1)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_X g_M}\right)^{a} \Lambda_{S,a}(g_S)}. \tag{17}$$

We can immediately see from equation (17) the two sources of growth that determine g_X: g_S and g. We can also see the potential role of population growth g_M, which will be the focus of the following subsection.

To gain more intuition on the workings of the model, consider for a moment a case in which all new businesses become leading at age 1 ($\lambda = 1$). The cost of this simplification is that in this simpler case, we can distinguish only between entrants and incumbents (not between young and old businesses), and all incumbents' productivity growth and exit rates will be the same.

In this case, however, we can find a closed-form solution for g_X as a function of g, g_S, and the share of incumbent businesses. In particular, we find that

$$\text{share of incumbent} = \frac{(s_S/g_M) \times n + (s_S/g_M)^2 \times n + ...}{n + (s_S/g_M) \times n + (s_S/g_M)^2 \times n + ...} = \frac{s_S}{g_M},$$

and total productivity growth is simply

$$g_X = g_S \times (s_S/g_M) + g^* \times (1 - s_S/g_M). \tag{18}$$

This equation makes it immediately clear that there will be positive total productivity growth ($g_X > 1$) even if only leading businesses have positive productivity growth ($g_S > 1, g = 1$) or if only new businesses have positive innovation ($g > 1, g_S = 1$). In addition, note that this result holds regardless of the value of population growth (as long as $g_M > s_S$).

3.1 The "sufficient statistic"

Before studying the impact of population growth on productivity growth, we show that the employment-size growth rate of surviving old businesses converges to the simple ratio of productivity growth rates, g_S/g_X.

Lemma 3 (Growth rate of the size of surviving old businesses)**.** *In a balanced growth equilibrium, the employment growth rate of surviving businesses converges monotonically to g_S/g_X as the age$\to \infty$.*

As we will show below, this variable will serve as a "sufficient statistic" for characterizing the influence of population growth on productivity growth.

3.2 The impact of population growth on TFP growth

TFP is measured as in the data by

$$TFP \equiv \frac{Y}{K^{\tilde{\alpha}} M^{1-\tilde{\alpha}}},$$

where the share of capital $\tilde{\alpha}$ is simply

$$\tilde{\alpha} \equiv 1 - \text{labor share of income} = 1 - \frac{wM}{Y}.$$

Note that $\tilde{\alpha}$ is different from α, and it is constant in a BGP but may vary along a transition and across BGPs.

Starting from the definition of TFP, we can obtain

$$\begin{aligned} TFP &= \left(\frac{L}{M}\right)^{1-\tilde{\alpha}} \left(\frac{Y}{L}\right)^{\frac{\tilde{\alpha}}{\alpha}-1} \left(\frac{NX}{L}\right)^{\frac{\zeta\tilde{\alpha}}{\alpha}} \\ &= \left(\frac{L}{M}\right)^{1-\tilde{\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha-\tilde{\alpha}}{1-\alpha}} \left(\frac{NX}{L}\right)^{\frac{\zeta(1-\tilde{\alpha})}{1-\alpha}}. \end{aligned}$$

Given the equality among total labor force growth, labor force for production, and the growth in the number of businesses ($g_M = g_L = g_N$) and constancy of interest rate ($g_r = 0$) along a BGP, growth in TFP along a BGP can be written as

$$g_{TFP} = g_X^{\frac{\zeta(1-\tilde{\alpha})}{1-\alpha}}.$$

Therefore, the main question for understanding the impact of population growth on TFP is how population growth affects average productivity growth, or $\frac{dg_X}{dg_M}$. The following lemma, which is the paper's main theoretical result, employs equation (17) to characterize the impact of g_M on g_X.[10]

Lemma 4 (The sign of the impact of population growth on productivity growth)**.** *In a balanced growth equilibrium, if the growth rate of the size of surviving old businesses is negative, then an increase in the labor force growth rate* g_M *raises average productivity* g_X*; i.e., if* $g_S/g_X < 1 \Rightarrow dg_X/dg_M > 0$.

We explain this result after presenting the following result, which describes how the same 'sufficient statistic' determines the size of the impact of population growth on productivity growth.

Lemma 5 (Magnitude of the impact of population growth on productivity growth)**.** *Suppose the growth rate of the size of surviving old businesses is negative. Suppose there are two economies with the same average productivity growth* (g_X) *and the same labor force growth* (g_M)*, but the growth rate in the size of old businesses decreases faster in one than in the other.*[11] *Then, the impact of population growth on productivity growth,* dg_X/dg_M*, is larger in the economy where the growth rate in the size of old businesses decreases more rapidly.*

These findings are better understood by considering that there are two mechanisms. First, recall that the growth in the number of businesses must be equal to the growth rate in the number of new businesses. Therefore, an increase in g_M (and consequently in g_N) reduces the share of old businesses. Second, total productivity growth equals the weighted average of productivity growth of businesses of various ages. As a result, if the productivity growth of old businesses is lower than the average, a decline in g_M (and an increase in the share of old businesses) will harm total productivity growth. Because the ratio of the average productivity growth rate to the productivity growth rate of old businesses equals the growth rate of the size of surviving old businesses, we referred to this variable as a "sufficient statistic," as it is the only information required to identify the sign of the impact of population growth on productivity growth.

[10] As the share of capital $\tilde{\alpha}$ varies across BGPs, $d\tilde{\alpha}/dg_M$ is a non-zero value. Specifically, it is most likely negative, thereby amplifying the effect of population growth on TFP growth. Further discussion on this topic can be found in online appendix 11.3.

[11] Although productivity growth is endogenously determined in this model, we can set productivity growth arbitrarily by adjusting z_R.

To see this logic more clearly in an equation, recall the case in which all new businesses become leading businesses at age 1 ($\lambda = 1$). There, total productivity growth is given by equation (18). Clearly, productivity growth (g_X) is the weighted average of the productivity growth of incumbent businesses (g_S) and the step size of innovation by new businesses (g). Decreasing population growth increases the weight on incumbents (s/g_M), and it hurts total productivity growth (g_X) as long as the size of incumbent businesses declines over their life-cycle (what happens if g_S/g_X). These findings also imply that the calibration of the productivity life-cycle profile is crucial for the quantitative results presented in the following sections of this paper.

The results in this section are achievable because there is a perfectly elastic supply of new businesses, entrants' innovation is constant, and we assume that survival rates and the growth rate of productivity of leading businesses are exogenous. In the following sections, we examine versions of the model that allow for congestion at entry, non-constant innovation by entrants, and endogenous exit and innovation by leading businesses. Our quantitative analysis reveals that these forces amplify the effects discussed in this section.

4 Quantitative model

4.1 Entry congestion and innovation spillovers

Before proceeding to the quantitative analysis of the model, we add two realistic features to the model presented in the previous section: congestion of entering businesses and spillovers to older businesses. These extensions will depend on two key parameters: ϕ and γ. By setting $\phi = \gamma = 0$, these two extensions can be removed. This exercise will evaluate the significance of these features in relation to our quantitative results.

First, we modify the free entry condition (4) to account for potential "congestion." According to Hopenhayn (1992), the working assumption in the model described above is that as long as the free entry condition is satisfied, the number of entrants is perfectly elastic, so n_t can be chosen to scale up or down the number of businesses and clear the labor market. We modify the free entry as in Karahan, Pugsley and Şahin (2024) to add a more realistic response of the number of businesses to economic conditions. In particular, we replace c_E with $c_E(\tilde{n}_t/\tilde{M}_t)^{\phi}$, where $\tilde{n}_t = n_t/\bar{n}_t$ and $\bar{n}_t$ represents the number of entrants in normal times (i.e., our reference period 1980-1999), and $\tilde{M}_t$ is defined analogously. Now, the modified free entry condition is $V_t \leq w_t c_E(\tilde{n}_t/\tilde{M}_t)^{\phi}$, which means that to increase the number of entrants into the economy, the value of entry must also increase. The key parameter is ϕ, which we will calibrate based on previous estimates.

The primary implication of this feature is that the step size of innovation is no longer independent of the population growth rate. Because of congestion, equation (14) is replaced by $g^* = \left(\frac{2c_E(\tilde{n}/\tilde{M})^{\phi} z_R}{\iota-2}\right)^{\frac{1}{\iota}}$. Now, the share of entry in the population, n/M, affects the innovation intensity, g^*. Therefore, there is another channel through which the population growth rate, g_M, affects the economy's productivity growth g_X.[12]

Second, we consider that the productivity growth of leading businesses, g_S, may be a function of their previous productivity growth. This equation captures the idea that leading businesses may benefit (with some delay) from younger businesses' innovation. Thus, the productivity growth rate of already-leading businesses is

$$g_{S_t} = \bar{g_S} + \gamma(g_{\chi_{t-1}} - \bar{g_\chi}), \tag{19}$$

where $\bar{g_S}$ is a constant representing the productivity growth rate of leading businesses in normal times and $\bar{g_\chi}$ is a constant growth rate for reference productivity leading businesses' productivity in normal times. The key parameter is γ, which we will estimate using data on the relationship between employment growth by mature businesses and overall productivity growth (more on this in the calibration section).

Since the productivity growth of leading projects g_S depends on g_χ, which is equal to g_X along a balanced growth path, expected productivity for leading projects $\Lambda_{S,a}(g_S)$ will depend on g_X; i.e., $\Lambda_{S,a}(g_S(g_X))$. Thus, equation (17), which determines the growth rate of productivity in the economy, is replaced by

$$\left(\frac{2c_E z_R}{\iota - 2}\right)^{\frac{1}{\iota}} = \frac{\sum_{a=2}^{\infty}\left(\frac{1}{g_N}\right)^{a-1}\Lambda_{S,a}(1)}{\sum_{a=2}^{\infty}\left(\frac{1}{g_X g_N}\right)^{a-1}\Lambda_{S,a}(g_S(g_X))}.$$

If spillover is positive, this extension will amplify the effect of g_M on g_X.

Although this will serve as our benchmark quantitative model for the remainder of

[12] The full expression for n/M is included in the online appendix (11.8).

the paper, we add two additional features to the BGP analysis at the end of this section: endogenous exit and endogenous choice of innovation by leading businesses. Incorporating these features substantially complicates the model, but their effect is limited, and if anything, they amplify the results implied by our benchmark model.

4.2 Calibration

We calibrate the model to aggregate statistics and business dynamics data for the United States and Japan.[13] The model can reproduce key stylized facts with relatively few parameters. What is essential for our results is not the micro-level information about firm dynamics but aggregate moments about business dynamics by age.[14] In particular, the model should reproduce the average size of all age-a businesses (defined as total employment over total number of age-a businesses) and the transition rates from age a to age $a+1$, which will be given by the percent of age-a businesses that exit (i.e., the exit rate) and the change in the average size of businesses between age a and $a+1$ restricted to businesses that are in operation at both ages a and $a+1$ (i.e., the growth rate of the size of surviving businesses).

We calibrate the model to reproduce the average from 1980 to 1999. The calibrated parameters based on previous papers or obtained directly from data are shown in the top panel of Table 1. We assign values to the remaining parameters, as shown in the bottom panel of Table 1, to reproduce key stylized facts, including establishment size, life-cycle profiles, and exit rates.[15]

Thus, the value of g_M in the US, 1.0167, and in Japan, 1.0072, are the averages for the years 1980-1999. The exponent of the research cost function, ι, is set to the same

[13] Note that in our calibration strategy, we allow the US and Japan to differ not only in labor force growth, but also in parameters governing productivity dynamics and the cost of innovation. We recognize that the two countries differ in many other dimensions. These differences are absorbed in our calibration through productivity and innovation parameters. Importantly, we do not interpret the model as explaining why the US and Japan differ overall. Our objective is to assess how the sharp decline in population growth has shaped Japan's productivity trajectory. This interpretation is valid as long as other differences are not themselves driven by demographic change.

[14] We could add more heterogeneity by adding iid productivity shocks, and it would not change the results as long as we reproduce business dynamics by age.

[15] We use establishment-level data to capture product or project-level activity, which is more in line with the process of innovation that we model. Establishment-level data are frequently used as a proxy of project-level analysis, assuming that one establishment produces one product (e.g., Klenow and Li (2020) and Garcia-Macia, Hsieh and Klenow (2019)).

value as in Greenwood, Han and Sanchez (2022). They estimated it to be equivalent to the impact of innovation expenditures on a firm's stock market value. Similarly, the parameter ϕ, which determines the degree of congestion, is set as in Karahan, Pugsley and Şahin (2024). The parameter γ, which influences the diffusion of innovation from new to old businesses, is calibrated using the value estimated in appendix 11.1. There, we compare various specifications for regressing current old-establishment productivity growth on the economy's past productivity growth.

Table 1: Parameters' values and targets of calibration

Parameter	Value	Basis
Entry cost, c_E	1	Normalization
Decreasing returns, ζ	0.2	Standard
Capital share, α	0.32	Standard
Depreciation rate, δ	0.07	Standard
Risk aversion, ϵ	2	Standard
Discount factor, β	0.96	Standard
Labor force growth rate, g_M	(1.0167, 1.0072)	Average g_M 1980-1999
Research cost exponent, ι	2.56	GHS
Convexity of aggregate entry cost, ϕ	0.55	KPS
Elasticity of g_S to g_χ, γ	0.342	See appendix 11.1.
Research cost slope, z_R	(0.954, 1.914)	Average prod. growth
Development cost slope, z_D	(0.00868 0.0176)	Average estab. size
Jump of prod. when becoming a leader, $1/\theta$	(18.6, 52.5)	Average size by age
Productivity growth of leading businesses, $\bar{g_S}$	(1.060, 1.030)	Growth of old estab.
Survival of leading businesses, s_S	(0.965, 0.972)	Exit rate of old estab.
Survival of laggard businesses, $s_{U,a}$	See Figure 8	Life-cycle profile of exit rate
Success probability, λ_a	See Figure 8	Growth of estab.

Note: Figure 8 is placed in appendix 11.2. The parameters with different values for the United States and Japan are shown in parentheses, with the United States representing the first number and Japan representing the second. GHS is an abbreviation for Greenwood, Han and Sanchez (2022), and KPS is an abbreviation for Karahan, Pugsley and Şahin (2024).

We now discuss the joint calibration of parameters to match specific targets. To begin, it is worth noting that since all parameter values are determined during the process of matching moments, there is no one-to-one correspondence between parameters and targets. However, some parameters have a greater influence on some moments, as shown in the Table 8 in online appendix 11.9.

First, because the value of productivity for research, z_R, affects the growth rate of average productivity in the economy, we calibrate the BGP to replicate the average productivity growth in the United States and Japan from 1980 to 1999.

Second, note that the jump in productivity when businesses become leaders, governed by θ, is a normalization.[16] We choose θ so that leading businesses employ, on average, around 200 workers, whereas laggard businesses employ, on average, about 10 workers. Then, we calibrate the success probability λ_a for ages $a \geq 2$ by assuming it changes exponentially with age to minimize the number of parameters to search on. Thus, all that is required are the starting probability and the decay constant. Figure 8 in appendix 11.2 shows the resulting life-cycle profile of success probability. Both the United States and Japan have extremely low success probabilities. As a result, leading businesses are uncommon. In addition, note that we find that λ_a is decreasing in age a, which is consistent with Greenwood, Han and Sanchez (2022)'s finding that the odds of success by venture capital funding round decrease with the age of the project.

Third, old businesses' employment growth is g_S/g_X as discussed in Lemma 3. As a result, we calibrate g_S so that the model accurately reproduces the employment growth for old establishments in the data.

Finally, we calibrate the parameters that determine the life-cycle profile of the survival probability. We assume that the probability of leading businesses surviving is constant. The resulting survival probability is very similar to 0.97 for the United States and Japan, reflecting the fact that the exit rate of old establishments is quite low in both countries' data. Because leading businesses are rare in the model, the resulting life-cycle profile of survival probabilities for laggard businesses is closely related to the survival probability for establishments in the data.[17] The resulting profiles for s_S and s_U for the United States

[16] The success probability would decrease if we increase the value of θ. In other words, if leading businesses are larger, the calibration would indicate that there are fewer of them to match the growth rate of surviving businesses. Case 5 in Table 3 shows that the results do not change in an alternative calibration with a lower value of θ.

[17] In the same manner as the choice of success probability, we calibrate the survival probability $s_{U,a}$ using an exponential decay function while allowing it to converge to a non-zero value. Hence, the initial

and Japan are also shown in Figure 8 in appendix 11.2.[18]

As a result, we calibrate 10 parameters using 32 moments for the United States (31 bars in Figure 3 plus average productivity growth) and 20 moments for Japan (19 bars in Figure 3 plus average productivity growth). Each moment receives equal weight in a calibration routine that selects parameter values to minimize the distance between the model-implied moments and the targets.

Figure 3 depicts the fit of calibration targets.[19] The left panel shows how well the model matches the exit rate life-cycle patterns for the US and Japan. Exit rates in both countries decline with age, and the rates in the US are higher than those in Japan, particularly for newer establishments. The model accurately predicts these trends, which is critical for determining the relative relevance of young and old businesses.

The middle panel of Figure 3 depicts the fit of the life-cycle profile of establishment size as measured in employment. For the plot, we normalize employment by the employment of age-one establishments such that the plot starts at one. Two facts are important. First, the model accurately reproduces the profiles for the US and Japan. Second, Japan's profile is much lower than that of the US. While establishments 27 years or older in the US are approximately 3.5 times larger than those one year old, the same ratio for establishments 29 years or older in Japan is 1.5.

Finally, Figure 3's right panel shows the growth of surviving businesses. Contrary to the average establishment size by age, these profiles are unaffected by business selection into exit. Thus, the differences between the middle panel and the right panel help identify the differences in survival rates of leading and laggard businesses.[20] As our theoretical

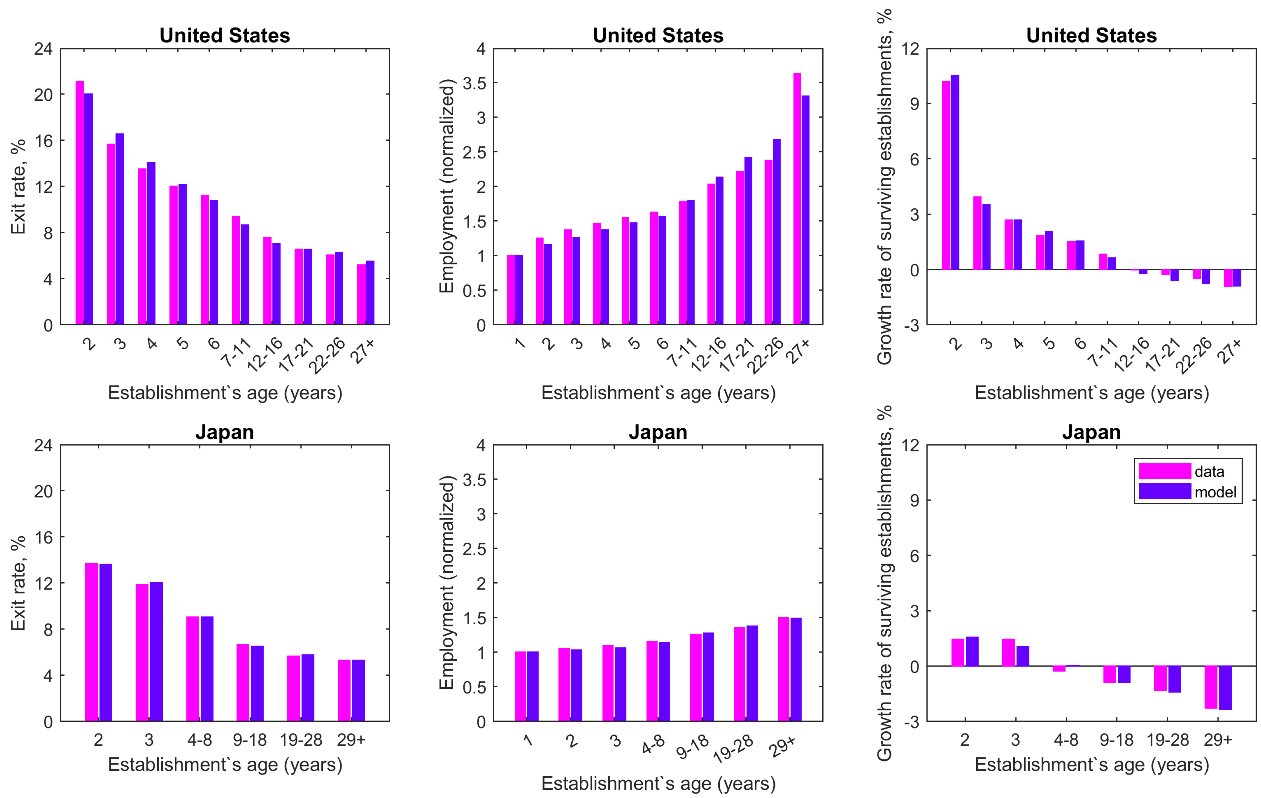
probability, long-run probability, and decay constant are calibrated.

[18]We show in Figure 9 that the model also reproduces well the share of young businesses, which could be expected given the fit of the exit rates.

[19]The source of data for the United States is the Business Dynamics Statistics (BDS), which the United States Census Bureau produces. In Figure 3, the average of 1980-2019 is taken for all three measures. The data source for Japan is the Economic Census and Establishment and Enterprise Census conducted by the Statistics Bureau. The exit rate and growth of surviving establishments by age are based on data from 2004, as these measures are only available for that year, compared with the data from 2001. For the employment size, we have extracted the year-of-birth fixed effect as the life-cycle profile for Japan, which, unlike the one for the US, is influenced by the year of birth. More on data sources in the online appendix 11.7.

[20]In particular, if the growth rate of average employment size is equal to the growth rate of average employment size of surviving businesses, then the exit rate of leading and laggard businesses should be equal.

Figure 3: Fit of life-cycle profiles



analysis showed, it is critical to reproduce the growth of surviving businesses. Notably, there is a diminishing growth profile with age, and older businesses tend to experience an average decline in size. Recall this was our "sufficient statistic" described in Lemma 3. The patterns in the data suggest that a decline in population growth will result in a decrease in aggregate productivity growth along the BGP, as stated in Lemma 4. Additionally, the decline in the size of surviving old businesses is more rapid in Japan than in the US. Given the finding in our Lemma 5, we are likely to find a larger impact of labor force growth on productivity growth in Japan than in the US. In terms of the calibration, the growth rate of the size of surviving old businesses is the prime target for calibrating g_S. Consequently, we obtain $g_S = 1.06$ for the US and $g_S = 1.03$ for Japan. Figure 12 in the appendix displays the implied average productivity growth of surviving businesses.

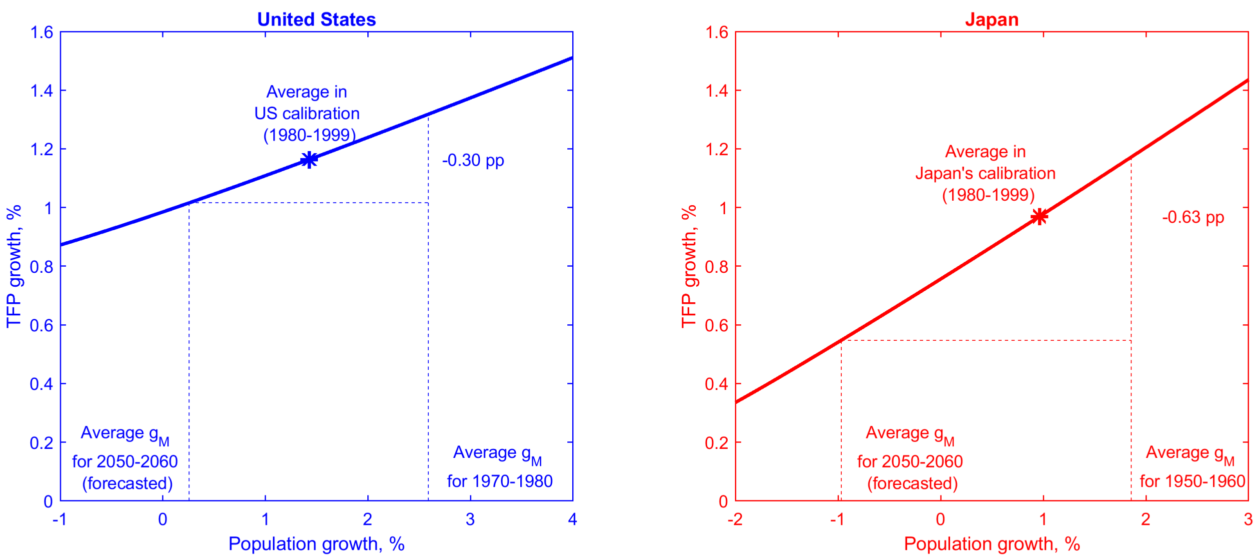
5 Balanced growth path analysis

This section shows how variations in the population growth rate impact the productivity growth rate along the BGP.

5.1 Results using the benchmark model

We first present the benchmark findings, followed by an explanation of how spillovers and congestion impact the results. The exercise is straightforward. We take the model's BGP calibrations for the US and Japan from 1980 to 1999, modify g_M, and find the new value of g_X, which is proportional to g_{TFP} in the BGP. This exercise is repeated for various values of g_M. Figure 4 shows the results for the US (left panel) and Japan (right panel).

Figure 4: Impact of population growth on TFP (comparison across BGPs)



The calibrated point is indicated by the stars in Figure 4, and the lines depict how g_{TFP} changes as g_M increases. Additionally, vertical lines represent historical periods of high labor force growth and the projected labor force growth for the years 2050 to 2060.[21] Using these time frames as examples, we illustrate how drops in labor force growth imply major shifts in the pace of productivity growth of the economy. Our model predicts a 0.3-percentage-point drop in productivity growth for the US as a result. For Japan, population

[21]The labor force growth projections are taken from the Bureau of Labor Statistics (BLS) for the US and the Cabinet Office (CAO) for Japan.

growth declined by more than three percentage points between the indicated 100 years (1950-1960 to 2050-2060), which implies a 0.6 percentage point reduction in productivity growth.

5.2 Role of key model's features

What role do the key characteristics of the model play in the outcomes shown in Figure 4? To address this, we compare our benchmark model (column A) results with those for three alternative models in Table 2. The findings of a model with no congestion ($\phi = 0$), a model with no spillovers ($\gamma = 0$), and the simplest model with neither congestion nor spillovers ($\phi = \gamma = 0$) are shown in columns B, C, and D, respectively.

Table 2: The role of key features on the impact of population growth on TFP along the BGP

Periods	Data's growth in labor force, %	Model's implied growth in TFP in the BGP, % (A) Benchmark	(B) No congestion	(C) No spillover	(D) Simplest
United States					
1970-1980	2.59	1.32	1.29	1.30	1.27
1980-1999	1.43	1.20	1.20	1.20	1.20
2050-2060	0.26	1.02	1.06	1.05	1.08
Difference in pp	-2.33	-0.30	-0.23	-0.24	-0.19
Japan					
1950-1960	1.86	1.17	1.14	1.11	1.09
1980-1999	0.96	0.91	0.91	0.91	0.91
2050-2060	-0.97	0.55	0.59	0.64	0.67
Difference in pp	-2.83	-0.63	-0.54	-0.47	-0.41

The results in Table 2 show that congestion and spillovers are important, although around 65% of the impact would still be seen absent them. The overall impact for the US decreases from 0.30 percentage points to 0.19 percentage points when both factors are removed. Japan's decrease falls from 0.63 to 0.41 percentage points. As a consequence, we conclude that the impact is a reduction of approximately 0.08-0.13 percentage points in productivity growth for every percentage point reduction in the US population growth, and between 0.15 and 0.22 percentage points in productivity growth for every percentage point reduction in Japan's population growth.

Why do these features amplify the impact on TFP growth? First, congestion reduces the entry cost when the population growth rate decreases because there are fewer entrants. This declining entry cost discourages innovation efforts, as entrants can compensate for the entry cost with lower productivity. Consequently, congestion increases the effect through smaller innovation. Second, the impact of spillovers is more straightforward: the decline in population growth leads to lower productivity growth, resulting in smaller spillovers to the productivity growth of existing businesses, which is also part of overall productivity growth.

In addition, Table 8 in online appendix 11.9 presents the results of a sensitivity analysis for the size of the impact of g_M on g_{TFP}. We find that the most critical parameters affecting the size of the impact are the survival of leading businesses, s_S, and their productivity growth, g_S. We find that an increase in s_S or a decrease in g_S would significantly increase the size of the impact of g_M on g_{TFP}. Note that the last result is in line with Lemma 5.

6 Robustness of BGP results

In this section, we modify and extend the "simplest" model to demonstrate the robustness of our BGP results to specific assumptions.

6.1 Robustness to alternative productivity process

We examine the sensitivity of our results to variations in several key features of our benchmark productivity process. We proceed in two steps. First, we provide alternative specifications. Second, we turn to a quantitative evaluation of alternative specifications.

6.1.1 Alternative productivity processes

Case 1: Standard AR(1) Process In the standard business dynamics framework, productivity x_t follows an autoregressive process:

$$\ln(x_{t+1}) = (1-\rho)\mu + \rho \ln(x_t) + \epsilon_{t+1}.$$

To simplify the analysis while maintaining the key dynamics, we consider a discrete two-state version where x starts at $\{x_L = \theta \hat{x}_t,\ x_H = \hat{x}_t\}$, and where $\mu = v\hat{x}_t$. This specification ensures that the values of initial productivity grow at the same rate as $\hat{x}_t$ on the balanced growth path, and thus there is a balanced growth in which different birth cohorts of businesses have the same life-cycle profile of size.

Note that when the shock $\epsilon = 0$, productivity evolves deterministically throughout the life cycle. To capture the stochastic transitions between the two productivity states, we specify the shock ϵ_{t+1} as:

$$\epsilon_{i,t+1} = \begin{pmatrix} 0 \\ \\ \rho\left(\ln(x_{-i,t}) - \ln(x_{i,t})\right) \end{pmatrix},$$

where the top row represents persistence within the same state, and the bottom row induces transitions between states. Transitions follow a Markov process with probabilities $P(i|i) = 1 - \lambda$ and $P(i|-i) = \lambda$.

Case 2: AR(1) with two states allowing λ to Depend on Age This case presents a straightforward *generalization* of Case 1. Specifically, we relax the assumption of a constant transition probability and instead allow the transition rate to vary with age: $P(i|i) = 1 - \lambda_a,\ P(i|-i) = \lambda_a$. This feature is also incorporated into our benchmark calibration, enhancing the model's ability to accurately match the observed life cycle profile of businesses. In particular, younger businesses—more likely to transition out of low productivity—can be captured with higher values of λ_a. At the same time, older firms exhibit higher persistence, consistent with the empirical decline in transition rates over time.

Case 3: Deterministic Productivity Growth Depending on Level This case brings us a step closer to the productivity process used in our benchmark model. As before, we assume that productivity can take two initial values, $\{x_L = \theta \hat{x}_t,\ x_H = \hat{x}_t\}$, but we now depart from the AR(1) framework used in Cases 1 and 2. Instead, we introduce a different mechanism for firm growth: deterministic productivity growth rates for each type of productivity. Specifically, we assume that low- and high-productivity firms grow deterministically at rates g_{x_L} and g_{x_H}, respectively.

Cases 1-2 and Case 3 are closely related. Since $g_{x_{H_t}} = x_{H_t}/x_{H_{t-1}}$ and $g_{x_{L_t}} = x_{L_t}/x_{L_{t-1}}$, in cases 1 and 2 can be rewritten as

$$g_{x_{Ht}} = \left(\frac{\nu \hat{x}}{x_{Ht}}\right)^{1-\rho}, \quad g_{x_{Lt}} = \left(\frac{\nu \hat{x}}{x_{Lt}}\right)^{1-\rho},$$

which allows us to define $g_{x_{Ht}}$ and $g_{x_{Lt}}$ in those cases as well. There are two key differences. First, in Case 3, $g_{x_{Ht}}$ and $g_{x_{Lt}}$ are constant, whereas in Cases 1-2, they are time-varying due to the mean-reverting nature of the process. Second, while g_{x_H} will exceed g_{x_L} in Case 3, this is not possible in Cases 1-2 because x_H is, by definition, larger than x_L. The specification in Case 3 provides greater flexibility in the productivity growth path. In particular, productivity growth does not converge to 1, and high-productivity firms can grow faster than low-productivity firms, unlike in Cases 1- 2. For simplicity, we set $g_{x_L} = 1$, meaning that low-productivity firms that do not transition remain at a constant productivity level.

The transition between low and high productivity continues to be age-dependent, governed by λ_a, with young businesses facing a high transition probability early in their life cycle, as in Case 2, allowing this setup to replicate the steep early growth observed in the data despite having $g_{x_L} = 1$.

As in the previous cases, we retain the simplifying assumption that a firm's productivity path depends solely on its current age, not on the timing of its transition to high productivity. In other words, once a firm transitions, it follows the same high-productivity trajectory as firms that were always in the high state. This abstraction makes the model more tractable while still capturing the essential dynamics.

The feature that makes us favor this specification over the AR(1) is that leading businesses, which are more likely to survive to old age, continue to grow even at older ages. The parameter determining this growth is g_{x_H}. As we show in the theoretical results, this parameter plays a central role in shaping the model's predictions for aggregate productivity dynamics.

Case 4: Our Benchmark This case introduces two key modifications relative to Case 3, bringing us to our benchmark specification.

First, we relax the assumption of symmetric transitions. Unlike in the previous cases, the transition probability λ is now asymmetric: we allow firms to move from low to high productivity, but we assume that firms cannot transition back. In other words, the high-productivity state becomes absorbing. This assumption greatly simplifies our analytical expressions.

Second, we allow the high-productivity level to depend on a firm's history—specifically, on the age at which it transitions into the high state. This assumption reflects a key feature of our benchmark model: productivity grows with tenure in the high-productivity state. For example, a firm born with high productivity reaches $\hat{x}g_H^9$ by age 10, while a firm that transitions to high productivity at age 9 reaches only $\hat{x}g_H$ by age 10.

Case 5: Lower Jump between States (smaller θ) The productivity gap between low- and high-productivity states is governed by the parameter θ in all the previous cases. For robustness, in this case, we consider setting θ to half its benchmark value (as in the previous case). Note that in this case, the calibration procedure will return transition probabilities from low to high productivity that are higher, to allow the model to reproduce the same observed growth in business size with age.

6.1.2 Evaluation

To assess the empirical performance of each specification, we calibrate all five cases to the same set of data moments using the same distance-minimization routine. The results of this exercise are shown in the Figure 11 in the appendix. Reassuringly, all five specifications provide a reasonably good fit to the data—including Case 1, which most closely resembles a standard AR(1) process adapted to a two-state setting. However, some key moments—particularly the size of older businesses and the rapid growth of the youngest businesses—are not well matched in Case 1 and, to a lesser extent, in Case 2.

Ultimately, the central question is whether our main result is sensitive to alternative specifications. To address this, Table 3 reports the effect of changes in population growth g_M on total factor productivity growth g_{TFP} along a balanced growth path in the simplest

Table 3: Robustness of the effect of a change in population growth on TFP growth

	1970-1980, %	1980-1999, %	2050-2060	Difference in pp
g_M, data	2.59	1.43	0.26	-2.33
g_{TFP}, case 1	1.26	1.20	1.10	-0.16
g_{TFP}, case 2	1.28	1.20	1.06	-0.22
g_{TFP}, case 3	1.26	1.20	1.10	-0.15
g_{TFP}, case 4*	1.27	1.20	1.08	-0.19
g_{TFP}, case 5	1.28	1.20	1.08	-0.20

Note: (*) benchmark specification.

model with the alternative cases for the productivity process. The key takeaway is that all five specifications deliver similar qualitative results, with estimates ranging from -0.15 to -0.22, compared to -0.19 in our benchmark case.

Quantitatively, however, Case 1 delivers one of the smallest effects (-0.16). The reason is straightforward. In Case 1, the largest miss in the calibration is that young businesses grow less in the model than in the data, which dampens the effect of population growth—operating through business aging—on total productivity growth. By allowing λ to depend on age, Case 2 addresses this issue. The remaining shortcoming is that the productivity of old businesses does not grow with the AR(1) specification. In this case, the productivity of old businesses grows too slowly relative to the data, resulting in an exaggerated impact of population growth—again through business aging—on total productivity growth.

We conclude that allowing the productivity process to directly capture the growth dynamics of both young and old businesses is crucial. While an AR(1) specification performs reasonably well, modifying the process to match these moments better enables the model to reproduce the magnitude of the impact of population growth on TFP growth more accurately.

6.2 Robustness to incorporating endogenous exit

In this section, we consider an extension of the model incorporating endogenous exit. To give businesses a reason to exit, we incorporate a fixed cost shock $c_a w\varepsilon$ to the profits of laggard businesses that depend on the age of the business.[22] If the pre-fixed costs expected discounted profits I_a is larger than the fixed cost, they pay the fixed costs and continue their businesses. If not, they exit the market.

We also keep an exogenous exit probability at the exit rate level for very old laggard businesses in the original calibration, $s_{U,\infty}$. Thus, the survival probability for laggard businesses is now given by $s_{U,a} = s_{U,\infty} \Pr(c_a w\varepsilon \leq I_a) = s_{U,\infty} \times F(I_a/(wc_a))$, where F is the distribution of ε. For tractability, we assume that F is Type III extreme value (or Weibull) distribution; i.e., $\varepsilon \sim Weibull(1, \vartheta)$. With this assumption, the survival probability for laggard businesses is given by

$$s_{U,a} = s_{U,\infty} \left[1 - \exp\left(- \left(\frac{I_a}{wc_a} \right)^{\vartheta} \right) \right],$$

where ϑ is a parameter of the distribution. In addition, we incorporate the expected fixed cost in expected profits given survival. We calibrated c_a to get the same age profile of the exit rate as the exogenous exit case. Also, we choose $\vartheta = 0.43$ such that the effect of population growth on the economy's exit rate across BGPs coincides with the effect in Hopenhayn, Neira and Singhania (2022).[23]

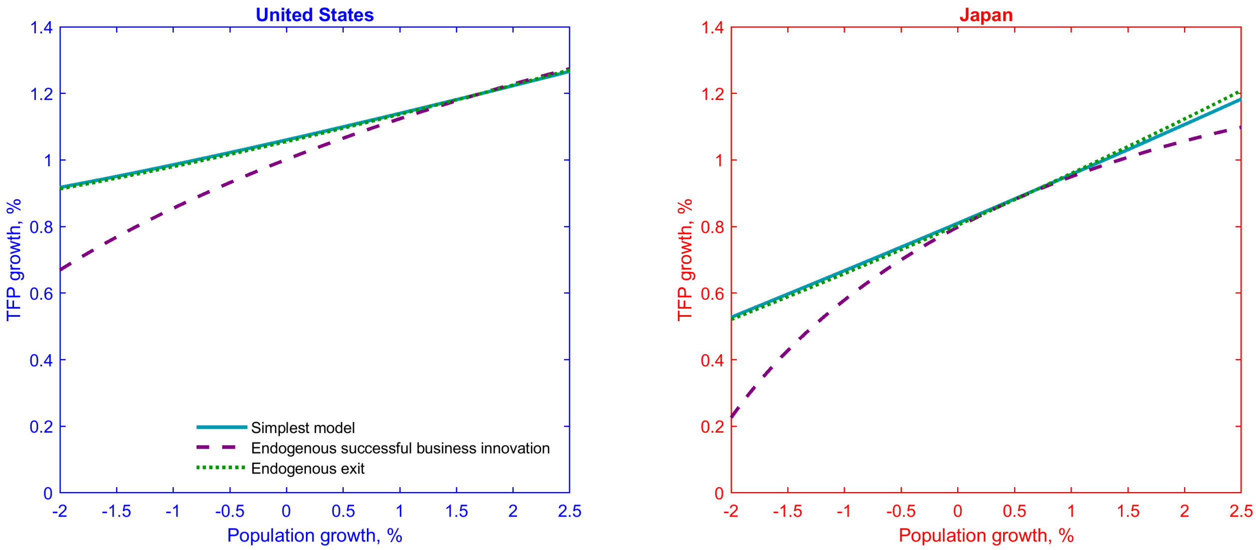
Figure 5 shows how the effect of the population growth rate on TFP growth compares between the model introduced in section 2 and the model with endogenous exit. The effect is very small for the US and Japan's calibrations. Why is the amplification so small? Because there are two counterbalancing forces. First, a decline in population growth produces an increase in survival rate for each age because wages grow slower. This effect on the survival rate for every age increases the share of old businesses, which magnifies the effect on productivity growth due to the growth composition effect. Second, there

[22] Leading businesses could also have to pay a fixed cost, but we assume it is negligible compared to their revenue, so they would not exit for this reason (they are still subject to the exogenous probability of survival).

[23] In particular, we match that when population growth declines from 2.66% to 0.78%, the exit rate declines by 0.88 percentage points. As a reference, with exogenous exit, the composition effect due to firms getting older would imply a decline of 0.80 percentage points.

is also a change in the step size of innovation, g. Given that the survival rates increase endogenously for each age, businesses find it beneficial to innovate more.

Figure 5: Endogenous exit and leading business innovation



6.3 Robustness to incorporating endogenous innovation

In this section, we incorporate endogenous productivity growth by leading businesses, g_S, into the model.

Denote $I_a(x)$ to be the value of a leading business at age a. It satisfies the following Bellman equation:

$$I_a(x, X, w) = S(x, X, w) + \frac{s_S}{1 + r - \delta} \max_{g_S} \left[I_{a+1}(g_S x, g_X X, g_w w) - C_S(g_S) w \right], \tag{20}$$

where $C_S(g_S)$ is the cost function of achieving productivity growth of g_S. We consider a general case for $C_S(g_S)$ that allows for the cost of productivity growth to depend on the distance of the business productivity to the frontier (i.e., the average of leading businesses, χ) and to the average productivity of the economy (i.e., X). In particular, we assume

$$C_S(g_S) = c_S g_S^{\iota} \left(\frac{x}{X}\right)^{\xi} \left(\frac{x}{\chi}\right)^{1-\xi}, \tag{21}$$

and let ξ determine which distance is more relevant. In this case, we find by solving the

Bellman equation that

$$g_S = \left(\frac{A g_w}{c_S \iota g_X} \right)^{\frac{1}{\iota - 1}}, \tag{22}$$

where A is a function of L/N, χ/X, and g_w/g_X. This equation, in conjunction with the free entry condition, determines g_S.[24] To obtain quantitative results, we calibrate two parameters: c_S and ξ. We calibrate c_S to have the same g_S in the reference period (1980-1999) as in our benchmark quantitative model, and ξ to have the elasticity of g_S to g_X consistent with the result in Table 6 when comparing between the averages for 1980-1999 and 2000-2019. We find that innovation by leading businesses is hump-shaped in population growth; consequently, the effect of population growth on TFP growth is larger for low-population growth states and smaller for high-population growth states compared to the simplest case. This implies that the effect on TFP growth can be larger in the future than in the main analysis, as the population growth is slowing down.

7 Transitional Dynamics

The concern that declining population growth will have an influence on TFP growth is clearly long-term. However, we address several interesting questions by computing transitional dynamics. In this section, we compute transitional dynamics and use the results to answer three questions. The first question is: How important has population growth been for the slowdown in TFP in the US and Japan in the last 40 years? We show that the share accounted for population growth is significant, but it is a fraction of the

[24] In particular, A satisfies

$$A = \frac{\zeta}{1-\alpha-\zeta} \frac{L}{N} \left(\frac{\chi}{X} \right)^{1-\xi} + \frac{s_S}{1-r-\delta} \left(A \frac{g_w}{g_X} \right)^{\frac{\iota}{\iota-1}} (c_S \iota)^{-\frac{1}{\iota-1}} \left(1 - \frac{1}{\iota} \right),$$

and a free entry condition implies that

$$\left(\frac{c_E}{D} \right)^{\frac{\iota-2}{2\iota}} = \left[A \left(\sum_{a=1}^{\infty} \hat{\beta}_a \left(\frac{g_w}{g_X} \right)^a \Lambda_{u,a} \frac{\lambda_a}{1-\lambda_a} \right) + \theta \frac{L}{N} \left(\frac{\chi}{X} \right)^{1-\xi} \frac{\zeta}{1-\alpha-\zeta} \left(\sum_{a=1}^{\infty} \hat{\beta}_a \left(\frac{g_w}{g_X} \right)^a \Lambda_{u,a} \right) \right] \left(\frac{\chi}{X} \right)^{\xi}$$

where

$$D = z_D^{\frac{\iota}{\iota-2}} \left(\frac{z_R}{\iota} \right)^{\frac{2}{\iota-2}} \left(\frac{\iota-2}{2\iota} \right)$$

is a constant value.

differences among BGPs. This result suggests that the transition is slow, and a significant part of the impact of the decline in population growth on TFP growth will occur in the future. Thus, a natural question is: How much of this effect will be observed in the future? Finally, we ask: What are the causes of the TFP growth's sluggish reaction to population growth changes? We show that the interplay of level-vs-growth and labor-reallocation effects is partially responsible for this.

7.1 Impact on TFP growth in the last 40 years

We use transitions between BGPs to compare the drop caused by the decline in population growth with data for the United States and Japan.

We select as the starting BGPs those that correspond to labor force growth in the first years that we observe a significant decline in the trend of the growth rate of the labor force (1950 for Japan and 1970 for the US).[25] We utilize the BGPs corresponding to the value of g_M in 2020 for the final BGP in both countries.[26] The entire transitions for the main variables are shown in Figure 10 in the online appendix.

Table 4 describes the changes in the last 40 years, focusing on two sub-periods.[27], respectively. The numbers for the "slowdown" in TFP growth are the difference in TFP growth (in percentage points) between the average trend growth in 1980-99 and 2000-19. For instance, TFP trend growth in the US was 1.195% in 1980-99 and 1.010% in 2000-19, so the difference is 0.184 pp. The decline in TFP growth was more severe in Japan: 0.451 pp.

How much of the drop in TFP growth can be attributed to the slowdown in population growth? To answer this question, we take the simulated transitions given the evolution of the trend in population growth and compute the average growth in TFP in the period

[25] The Christiano and Fitzgerald (2003) filter is used to extract the slow-moving trend of labor force growth. We chose this filter because it allows us to choose the parameters to capture the long-run movement in the labor force. We set the parameters at 2 and 40 and also consider the changes using other values.

[26] In a later section, we show the amplification obtained by including forecasts for g_M until 2060.

[27] We begin the analysis in 1980 since other circumstances, such as World War II and the high economic growth period in Japan in the post-war period would likely be influential in previous decades. Recall that in the previous section, we calibrated the model to have a balanced growth rate of TFP equal to the average growth of TFP between 1980 and 1999. To facilitate the comparison with data, for the transitions, we recalibrated z_R for the US and Japan to make sure that the average growth in the period 1980-1999 is equal in the model in the data. Only very small changes were necessary; the new values are 0.886 and 1.622 for the US and Japan

Table 4: TFP growth slowdown since 1980-99 to 2000-19, data-model comparison

	United States		Japan	
	Change in g_{TFP} 1980-99 − 2000-19	Share accounted for	Change in g_{TFP} 1980-99 − 2000-19	Share accounted for
Data	0.184	–	0.451	–
Benchmark	0.091	49.7%	0.109	24.2%
No congestion	0.082	44.5%	0.101	22.3%
No spillover	0.076	41.2%	0.088	19.6%
Simplest	0.070	38.3%	0.082	18.2%

1980-1999 and 2000-2019 as we did in the data. Using the benchmark model, we find that the drop in population growth accounts for 49.7% of the decline in TFP growth in the US. If we abstract away from congestion, this share is reduced by about one-tenth, while removing spillovers decreases it by less than one-fifth. In the simplest scenario, the drop in population growth explains 38.3% of the decline in TFP growth in the US over that time period. The analysis is similar in Japan. The benchmark model accounts for roughly 24.2% of the drop in TFP growth, whereas the simplest model (without congestion and spillovers) accounts for around 18.2%. Although the model generates a slowdown in Japan, which is larger than the one in the US, the smaller share accounted for population growth in Japan reflects its weaker recent economic performance (the slowdown in Japan during this period is much larger than in the US).

7.2 Future decline in productivity

The results in Table 4 together with BGP analysis shown in Figure 4 suggest that the transition between BGPs is slow and, as a consequence, part of the effect of population growth on productivity growth will occur in the future. The top panel of Table 5 shows the expected decline in TFP growth between 2020 and 2050 and between 2020 and 2100. The expected changes for these subperiods in the US are -0.05 and -0.06 percentage points, respectively. Note that this implies that the impact of population growth on productivity growth will increase by almost 50% in the future (it was -0.091 between the average of 1980-1999 and 2000-2019). The expected change in productivity in Japan is much larger: -0.14 and -0.17 percentage points between 2020 and 2050 and between 2020 and 2100, respectively. The faster labor force growth decline in Japan than in the US between 2005 and 2020 causes this result.

To complete the analysis of expected TFP growth, we re-computed the transitions, incorporating the forecast for the decline in labor force growth until 2060. The bottom panel of Table 5 shows the results. Since labor force growth is expected to continue decreasing, the impact is larger, particularly between 2020 and 2100. For that period, the expected change in productivity growth is -0.08 for the US and -0.34 for Japan.

Table 5: Expected change in TFP growth as a consequence of the decline in the growth of the labor force (percentage points)

Country	United States	Japan
Benchmark		
Between 2020 and 2050	-0.05	-0.14
Between 2020 and 2100	-0.06	-0.17
Including forecast for g_M		
Between 2020 and 2050	-0.07	-0.24
Between 2020 and 2100	-0.08	-0.34

7.3 Why is the response so slow? Two counterbalancing factors

Why is there a slow reaction of g_{TFP} to g_M, as mentioned in the preceding subsection? To answer this question, we evaluate the economy's response to a one-time change in population growth. We conduct this experiment using the model calibrated for the US.

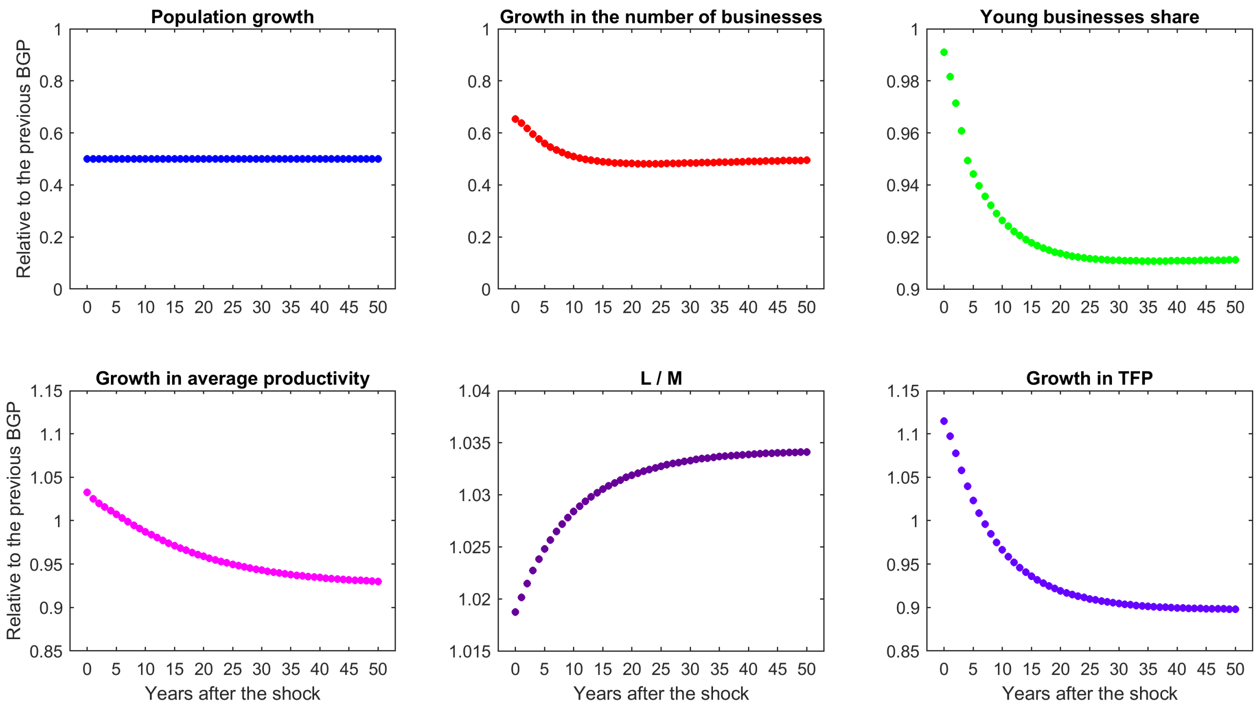
Recall that TFP is given by

$$TFP = \left(\frac{L}{M}\right)^{1-\tilde{\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha-\tilde{\alpha}}{1-\alpha}} \left(\frac{NX}{L}\right)^{\frac{\zeta(1-\tilde{\alpha})}{1-\alpha}}.$$

Therefore, in a transition, TFP growth is not simply equal to growth in average productivity, X. As long as the share of workers in the production sector, L/M, the interest rate, r, and the average firm size, N/L, are changing, they will cause changes in TFP growth. These three drivers of TFP growth in the transition are absent in the BGP because they are constant. It turned out that the most important of these three components is L/M, which should not be surprising, given that it has the largest exponent.

Figure 6 depicts the economy's response to the change in population growth rates that we feed into the model. The shock is a permanent fall in population growth from 2% to 1% in the year designated as zero.[28] The values displayed are relative to the values in the BGP with population growth equal to 2%.

Figure 6: Response to a permanent decline in population growth



First, note that during the period when the shock is realized, population growth (top left panel) is half of its original value, and the growth in the number of businesses (top middle panel) also declines quickly, although at a slightly slower rate. The decline in the growth rate of the number of businesses generates a slow fall in the share of young businesses (top right panel), which takes more than 20 years to reach the value of the new BGP. As previously discussed, in the long run, this results in a decline in average productivity growth (bottom left panel). However, during the shock period and for a few years afterward, average productivity growth is larger than in the original BGP. This is

[28]For simplicity, we assume agents learn about the shock only one period before it occurs. This assumption allows for the growth in a number of businesses to coincide with population growth in the same period, as the decision to enter is made one period in advance.

referred to as the level-vs-growth effect. The long-run decline in average productivity growth is due to a rise in the proportion of older businesses in the economy, which have lower productivity growth than the average. However, on impact, a higher proportion of older businesses positively influences average productivity growth because younger businesses are less productive than older businesses. Thus, in the short run, younger businesses' lower productivity level outweighs their greater productivity growth, and average productivity growth rises. The decline in the entry rate captured by the drop in the growth rate of the number of businesses (shown in the top middle panel) generates a decline in employment in the innovation sector of the economy, so a larger share of workers are employed in the production sector (L/M increases in the bottom middle panel). This effect, which we refer to as the labor-reallocation effect, is the second driver of the short-run increase in TFP growth after a decline in the growth rate of the population.

The labor-reallocation and level-vs-growth effects operate together to drive TFP growth (panel F) in the short run in the opposite direction that it moves in the long run as population growth changes. As a result, these forces partially counterbalance the short-run effect on TFP growth and create its sluggish response to changes in population growth.

We also investigate the sensitivity of the speed of convergence to the model's parameters, as shown in Table 8 of the online appendix 11.9. The most important parameters are s_S, g_S, and β. We find that decreases in s_S and β, or increases in g_S, materially increase the speed of convergence.

8 Validation using US state-level data

As mentioned in the introduction, there is vast empirical literature documenting the impact of population growth on productivity growth.[29] In this section, we offer new evidence using state-level US data.[30] The purpose is to validate the proposed mechanism as closely as possible.

We study the impact of population growth on productivity growth throughout the US

[29]See Peters (2022) and references therein.

[30]A concern about using state-level data is that there may be knowledge spillover across states. Note that if the spillover is positive, meaning that faster growth in one state generates higher growth in other states, then the coefficient in our regression will be downward-biased.

states using local projections. The focus is on examining whether the dynamics following a change in population growth, as described in the previous section, are present in the data.

For the analysis in this section, we would ideally need a lengthy time series of state-level TFPs, which are not available in the US. As a result, we use real GDP per worker, which is referred to as labor productivity. Our real GDP per worker and labor force data range from 1977 to 2019. To keep the analysis comparable, we also use labor productivity from the model, which is calculated using the following expression: $\log(g_{prod,t}) = 1/(1 - \tilde{\alpha}) \times \log(g_{TFP,t}) - \tilde{\alpha}/(1 - \tilde{\alpha}) \times \log(g_{r,t})$. We run an anticipated transition from 1900 to 2060 for the 50 US states to generate simulated time series using the model.

A difficulty with analyzing the relationship between changes in productivity growth and labor force growth is that, as shown in the previous section, it is not monotonic. Productivity growth rises at first and then falls due to a slowdown in labor force growth, as shown in Figure 6. This pattern suggests that we should estimate a dynamic model. As a result, we follow Jordà (2005) and employ local projections.

Our left-hand-side variable is the change in labor productivity growth rate between i years after the shock and the year before the shock, for each state s,

$$\Delta(g_{prod})^s_{t+i,t-1} = g^s_{prod,t+i} - g^s_{prod,t-1}.$$

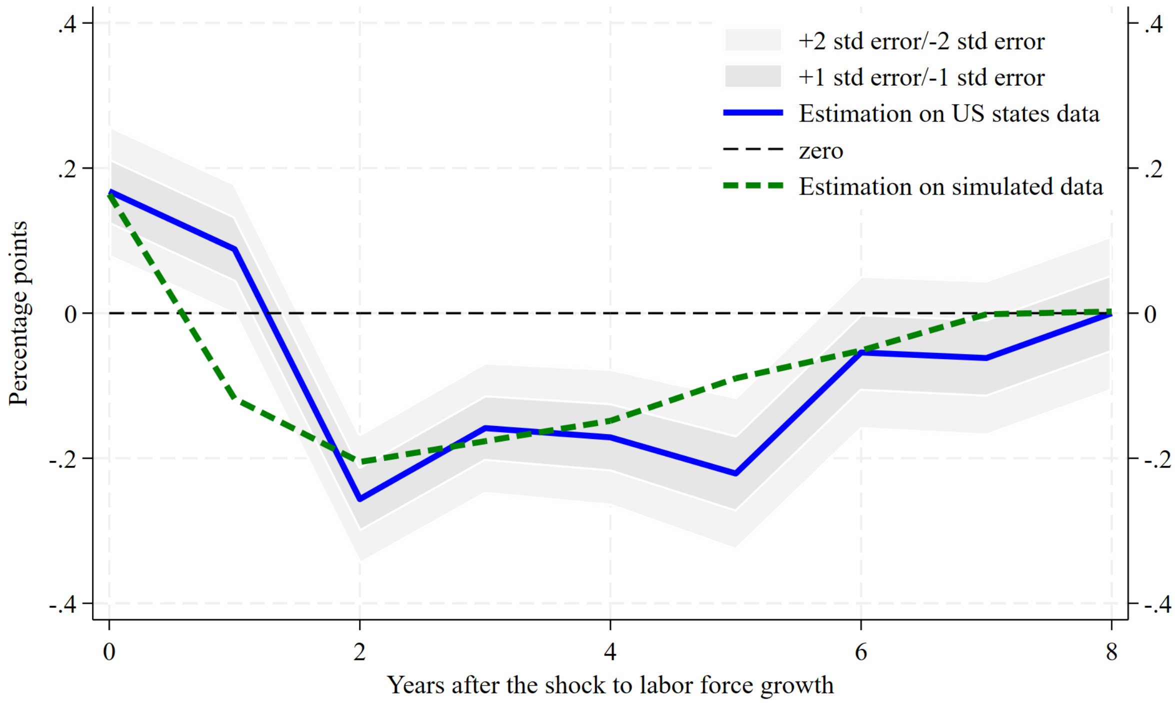
We regress this variable on the change in the growth rate of the labor force,

$$\Delta(g_{prod})^s_{t+i,t-1} = \beta^i_0 + \beta^i_1 \times \Delta(g_M)^s_{t,t-1} + \text{controls},$$

for $i = 0, 1, ..., 8$. Note that these are nine different regressions, one for each value of i. The controls included are seven lags of $\Delta(g_M)^s_{t,t-1}$, seven lags of $\Delta(g_{prod})^s_{t,t-1}$, and a quadratic polynomial in year.

Figure 7 depicts the outcome of estimating local projections in data for US states and in the model-simulated data for US states. A reduction in labor force growth has a positive and significant effect on labor productivity growth. Then it decreases and becomes negative and significant two years after the shock, and it remains negative and significant three, four, and five years after the shock.

Figure 7: Change in labor productivity growth after a 1pp *decline* in labor force growth



Note: The shaded areas represent one (darker) and two (lighter) standard error bands.

9 Conclusions

At least since Solow (1957), the persistent improvement in living standards around the globe has been largely attributed to TFP growth. However, the trend growth in TFP has recently slowed in industrialized economies (Cette, Fernald and Mojon, 2016; Fernald et al., 2017). On the other hand, population growth has declined in most developed economies, and this trend is projected to continue in the next decades. In fact, the latest United Nations projections suggest that the world's population could reach zero growth during the 2080s (UN, 2022). Therefore, determining the potential impact of population growth on TFP growth is crucial. We propose a theory that explains the connection between these two trends. According to our theory, the slowdown in population growth has been, and will likely continue to be, a drag on TFP growth in the coming decades due to the projected slowing of population growth.

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10 Appendix

10.1 Proofs

While we assume in section 2 that the success probability λ and survival rate of laggard businesses s_U are constant, all the lemmas are proven under a more generalized case here. Specifically, we assume that they are weakly decreasing in age, which is consistent with the calibration in section 4.2.

10.1.1 Proof of Lemma 1

First, the value of a project with potential productivity $\hat{x}_t$ is

$$I_t(\hat{x}_t; \{w_t\}, \{r_t\}) = \sum_{j=t+1}^{\infty} \hat{\beta}_j \mathbb{E}_{\hat{x}_t}[S(x_j; w_j, r_j)|\hat{x}_t].$$

Note that using equation (2), we have that

$$S_t = \zeta x_t \left[\left(\frac{\alpha}{r_t} \right)^{\alpha} \left(\frac{1-\alpha-\zeta}{w_t} \right)^{1-\alpha-\zeta} \right]^{\frac{1}{\zeta}},$$

so we can rewrite I_t as

$$I_t(\hat{x}_t; \{w_t\}, \{r_t\}) = \sum_{j=t+1}^{\infty} \hat{\beta}_j \zeta \left[\left(\frac{\alpha}{r_j} \right)^{\alpha} \left(\frac{1-\alpha-\zeta}{w_j} \right)^{1-\alpha-\zeta} \right]^{\frac{1}{\zeta}} \mathbb{E}[x_j|\hat{x}_t] = \Gamma(w_t, r_t)\hat{x}_t,$$

where

$$\Gamma(w_t, r_t) = \zeta \alpha^{\frac{\alpha}{\zeta}} (1-\alpha-\zeta)^{\frac{1-\alpha-\zeta}{\zeta}} \sum_{j=t+1}^{\infty} \hat{\beta}_j (\Lambda_{S,j-t}(g_S) + \Lambda_{U,j-t})(r_j)^{-\frac{\alpha}{\zeta}} (w_j)^{-\frac{1-\alpha-\zeta}{\zeta}}.$$

using equation (6) to substitute for $E[x_j|\hat{x}_t]$.

Then, we can solve equation (3). This equation is altered to

$$V(\{w_t\}, \{r_t\}, \chi_t) = \max_{\sigma_t, g_t} \sigma_t \Gamma(w_t, r_t) \chi_t g_t - \frac{1}{z_R}(g_t)^{\iota} w_t - \frac{\sigma_t^2}{2z_D} w_t, \tag{23}$$

where $g_t \equiv \hat{x}_t/\chi_t$ is the step size of innovation. Note that the FOCs with respect to σ_t and g_t are

$$\frac{\partial V_t}{\partial \sigma_t} = \Gamma(w_t, r_t)\chi_t g_t - \frac{w_t}{z_D}\sigma_t = 0, \quad \frac{\partial V_t}{\partial g_t} = \Gamma(w_t, r_t)\sigma_t \chi_t - \frac{\iota w_t}{z_R} g_t^{\iota-1} = 0.$$

The solutions are

$$g_t^* = \left(\frac{z_R z_D}{\iota} \Gamma(w_t, r_t)^2 \left(\frac{\chi_t}{w_t} \right)^2 \right)^{\frac{1}{\iota - 2}}, \tag{24}$$

$$\sigma_t^* = \frac{z_D \Gamma(w_t, r_t)}{w_t} \chi_t g_t^*. \tag{25}$$

Substituting equations (24) and (25) into (23), we obtain

$$V_t = \frac{\iota - 2}{2 z_R} \left(\frac{z_R z_D}{\iota} \Gamma(w_t, r_t)^2 \left(\frac{\chi_t}{w_t} \right)^2 \right)^{\frac{\iota}{\iota - 2}} w_t = \frac{\iota - 2}{2 z_R} (g_t^*)^\iota w_t. \tag{26}$$

Finally, we replace equation (26) into the free entry condition (equation (4)),

$$V_t = \frac{\iota - 2}{2 z_R} (g_t^*)^\iota w_t = c_E w_t,$$

which determines the step size g^*,

$$g^* = \left(\frac{2 c_E z_R}{\iota - 2} \right)^{\frac{1}{\iota}}.$$

Since this solution is equal to equation (14), this concludes the proof of Lemma 1.

10.1.2 Proof of Lemma 2

Let $\cdot'$ denote values in the next period. Suppose that $g_M > s_{S,\infty}$ and $\{\sigma, g, c, k, N\}$ solves the old problem. The existence of a balanced growth path will be shown when $\{\sigma, g, g_w c, g_w k, g_M N\}$ solves the new one for $M' = g_M M$.

First, the Euler equation derived from equation (1), $(g_w)^\epsilon = \beta(1 + r - \delta)$, shows r is constant, and so $\hat{\beta}_t' = \hat{\beta}_t$.

Second, when we observe $N' = g_M N$, we will get a certain value of g_X from equation (17). Note that the right-hand side of equation (17) is a monotonic (increasing) function of g_X and $\lim_{g_X \to 0} RHS \to 0$ and $\lim_{g_X \to \infty} RHS \to \infty$ for any $g_M \in (0, \infty)$. Given the LHS is positive, we have a unique g_X for every g_M.

Third, the low of motion of the number of businesses, equations (8)-(10), shows $n' = g_N n$.

In addition, equations (15) and (16) give $\chi' = g_X \chi$ and $\hat{x}' = g_X \hat{x}$. Equation (24) gives $w' = (g_X)^{\frac{\zeta}{1-\alpha}} w$, and it gives $L' = g_M L$, from the equation for labor demand.

To see the firm side, from equation (2), $S(\hat{x}', w', r') = g_w S(\hat{x}, w, r)$, and so $I(\hat{x}', w', r') =$

$g_w I(\hat{x}, w, r)$. Therefore, the objective function for the value of projects in equation (3) inflates by the factor g_w. Note that $R(\hat{x}'/\chi') = R(\hat{x}/\chi)$ and $D(\sigma') = D(\sigma)$. Also, both sides of equation (4) inflates by g_w, so the new solution still satisfies the free entry condition.

Regarding market clearing conditions, all labor, capital, and goods markets hold in the new problem at the conjectured solution: both sides of equation (11) inflate by g_M, equation (12) by $g_w g_M$, and equation (13) by $g_w g_M$.

Lastly, consider the budget constraint in equation (1). At the conjectured solution, both sides inflate by the factor g_w. As such, the budget constraint also holds with the new solution.

Thus, by canceling out this factor of proportionality, the new problem reverts back to the old one, and we have shown that it is a BGP.

Finally, we show that population growth must be higher than the old businesses' survival rate, $g_M > s_{S,\infty}$ to have a BGP. For this, note that the lower bound for the growth in the number of businesses is $s_{S,\infty}$. This is the case because (i) in that lower bound, the number of entrants is at its lower bound (zero), and (ii) we assumed that leading businesses live longer than laggard ones, $s_{S,\infty} \geq s_{U,\infty}$. Hence, if $g_M < s_{S,\infty}$, then the population growth rate is lower than the growth rate of the number of businesses, $g_M < g_N$, which contradicts the condition $g_M = g_N$ that should hold in any balanced growth path.

10.1.3 Proof of Lemma 3

This proof consists of two parts: (i) Prove that $g_X > g_S$ if and only if the employment growth rate of surviving leading businesses is negative, and (ii) prove that the employment growth rate of surviving *old* businesses is asymptotically equivalent to that of surviving *leading* businesses.

First, the employment of a business i at time t is

$$l_{i,t} = x_{i,t} \left[\left(\frac{\alpha}{r_t} \right)^{\alpha} \left(\frac{1 - \alpha - \zeta}{w_t} \right)^{1-\alpha} \right]^{\frac{1}{\zeta}}.$$

Given that the business is leading at time t and surviving at time $t+1$, the business's employment at time $t+1$ is written as

$$l_{i,t+1} = g_S x_{i,t} \left[\left(\frac{\alpha}{r_{t+1}} \right)^{\alpha} \left(\frac{1-\alpha-\zeta}{w_{t+1}} \right)^{1-\alpha} \right]^{\frac{1}{\zeta}}.$$

Since $r_{t+1} = r_t$ and $w_{t+1} = (g_X)^{\frac{\zeta}{1-\alpha}} w_t$ in a BGP, the employment growth surviving of leading businesses $l_{i,t+1}/l_{i,t} = g_S/g_X$.

Next, to show the asymptotic equivalence between the employment growth rate of surviving *old* businesses and that of surviving *leading* businesses, recall that we assume that old leading businesses are more likely to survive than old laggard businesses, $s_{S,\infty} \geq s_{U,\infty}$. It implies that the share of laggard businesses converges to zero, so the employment growth rate of old businesses becomes equivalent to that of surviving businesses. This concludes the proof of Lemma 3.

We can see this step algebraically by considering the expression for the employment growth rate of surviving businesses of all ages:

$$\textit{surviving growth} = \frac{g_S}{g_X}(1-\Delta_a) + \Delta_a \left(\frac{(1-\lambda_a) + \frac{\lambda_a}{\theta}}{g_X} \right),$$

where Δ_a is the employment share of laggard businesses. Note that it depends on the employment share, not the business share. Since leading businesses are larger than laggard businesses, the employment share converges faster than the business share.

10.1.4 Proof of Lemma 4

Combining equations (14) and (17) and arranging it gives an equilibrium expression for the relationship between g_M and g_X:

$$\left(\frac{2c_E z_R}{\iota - 2} \right)^{-\frac{1}{\iota}} = \frac{\sum_{a=1}^{\infty} \left(\frac{1}{g_M} \right)^a \left(\frac{g_S}{g_X} \right)^a \frac{\Lambda_{S,a}(g_S)}{(g_S)^a \Lambda_{S,a}(1)} \Lambda_{S,a}(1)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_M} \right)^a \Lambda_{S,a}(1)}.$$

The right-hand side can be interpreted as the weighted average of

$$\tilde{x}_a \equiv \left(\frac{g_S}{g_X} \right)^a \frac{\Lambda_{S,a}(g_S)}{(g_S)^a \Lambda_{S,a}(1)}$$

with weights $(1/g_M)^{a-1} \Lambda_{S,a}(1)$; i.e.,

$$\left(\frac{2c_E z_R}{\iota - 2} \right)^{-\frac{1}{\iota}} = \sum_{a=1}^{\infty} (\tilde{x}_a) \times \frac{((1/g_M)^a \Lambda_{S,a}(1))}{\sum_{a=1}^{\infty} (1/g_M)^a \Lambda_{S,a}(1)}. \tag{27}$$

Now, $\tilde{x}_a$ is decreasing in g_X for all $a \geq 1$. As such, *ceteris paribus*, the increase in g_X decreases the right-hand side of equation (27). Also, if $g_S < g_X$, $\tilde{x}_a$ is decreasing in age a since $\Lambda_{S,a+1}(g_S)/\Lambda_{S,a}(g_S) \leq g_S \Lambda_{S,a+1}(1)/\Lambda_{S,a}(1)$. Therefore, we need larger weights on young businesses to increase the right-hand side of equation (27) if $g_S < g_X$, which means that we need to increase g_M.

To put them together, if $g_S < g_X$, an increase in g_M must increase g_X to keep the right-hand side of equation (27) constant, which concludes the proof of Lemma 4.

10.1.5 Proof of Lemma 5

Start with equation (17). Totally differentiating it by g_M and reorganizing it gives

$$\frac{1}{1+\frac{g_M}{g_X}\frac{dg_X}{dg_M}} \frac{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \Lambda_{S,a}(1)a}{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \Lambda_{S,a}(1)} = \frac{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \left(\frac{1}{g_X}\right)^a \Lambda_{S,a}(g_S)a}{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \left(\frac{1}{g_X}\right)^a \Lambda_{S,a}(g_S)}. \tag{28}$$

Therefore, for given g_M and g_X, dg_X/dg_M is larger if the right-hand side of equation (28) is smaller. Note that only the right-hand side depends on g_S. Since the growth rate in the size of old businesses decreases faster in the economy with smaller g_S, we need to show

$$\frac{d}{dg_S} \frac{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \left(\frac{1}{g_X}\right)^a \Lambda_{S,a}(g_S)a}{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \left(\frac{1}{g_X}\right)^a \Lambda_{S,a}(g_S)} > 0.$$

Arranging it gives

$$\frac{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \left(\frac{1}{g_X}\right)^a \Lambda_{S,a}(g_S)a\frac{\Lambda'_{S,a}(g_S)}{\Lambda_{S,a}(g_S)}}{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \left(\frac{1}{g_X}\right)^a \Lambda_{S,a}(g_S)a} > \frac{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \left(\frac{1}{g_X}\right)^a \Lambda_{S,a}(g_S)\frac{\Lambda'_{S,a}(g_S)}{\Lambda_{S,a}(g_S)}}{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \left(\frac{1}{g_X}\right)^a \Lambda_{S,a}(g_S)}, \tag{29}$$

where $\Lambda'_{S,a}(g_S) \equiv d\Lambda_{S,a}(g_S)/dg_S$. Note that both sides of equation (29) can be interpreted as weighted averages of $\Lambda'_{S,a}(g_S)/\Lambda_{S,a}(g_S)$. Since LHS of equation (29) has more weights on older ages, a sufficient condition for this equation to hold is $\Lambda'_{S,a}(g_S)/\Lambda_{S,a}(g_S)$ is increasing in age a.

Now, using the definition of $\Lambda_{S,a}(g_S)$, we can write $\Lambda'_{S,a}(g_S)/\Lambda_{S,a}(g_S)$ as

$$\frac{\Lambda'_{S,a}(g_S)}{\Lambda_{S,a}(g_S)} = \frac{\sum_{j=1}^{a}(j-1)\omega_{a,j}}{\sum_{j=1}^{a}\omega_{a,j}},$$

where $\omega_{a,j} \equiv g_S{}^{j-1}\left(\prod_{k=0}^{a-j-1}(1-\lambda_k)\right)\left(\prod_{k=1}^{a-j} s_{U,k}\right)\left(\prod_{k=a-j+1}^{a-1} s_{S,k}\right)\lambda_{a-j}$. Notice that the subscript j represents how many periods have passed since each business becomes leading, so $w_{a,j}$ is the aggregate productivity of leading businesses at age a that succeeded at age $a-j+1$. Therefore, what we want to show is

$$\frac{\sum_{j=1}^{a+1}(j-1)\omega_{a+1,j}}{\sum_{j=1}^{a+1}\omega_{a+1,j}} > \frac{\sum_{j=1}^{a}(j-1)\omega_{a,j}}{\sum_{j=1}^{a}\omega_{a,j}} \tag{30}$$

for all ages a. Using a property $\omega_{a+1,j+1} = g_S s_{S,a}\omega_{a,j}$, we can transform equation (30) into

$$\frac{\sum_{j=1}^{a}\omega_{a,j}}{\omega_{a+1,1}/g_S s_{S,a}} > \frac{\sum_{j=1}^{a}(j-1)\omega_{a,j}}{\sum_{j=1}^{a}\omega_{a,j}}. \tag{31}$$

Since we assume that the survival probability for leading businesses is higher than that for laggard $s_{S,a} > s_{U,a}$ for all ages and the success probability λ_a decreases in age a, $\omega_{a,j+1} > \omega_{a,j}$ because

$$\frac{\omega_{a,j+1}}{\omega_{a,j}} = \frac{g_S s_{S,a-j}\lambda_{a-j-1}}{(1-\lambda_{a-j-1})s_{U,a-j}\lambda_{a-j}}.$$

Under these assumptions, we can prove equation (31):

$$\text{(LHS)} = \frac{\sum_{j=1}^{a}\omega_{a,j}}{\omega_{a+1,1}/g_S s_{S,a}} > \frac{\sum_{j=1}^{a}\omega_{a,j}}{\omega_{a,1}} > \frac{\sum_{j=1}^{a}\omega_{a,j}}{\sum_{j=1}^{a}\omega_{a,j}/a} = a > \frac{\sum_{j=1}^{a}(j-1)\omega_{a,j}}{\sum_{j=1}^{a}\omega_{a,j}} = \text{(RHS)}.$$

Hence, dgX/dgM is larger in the economy in which the growth rate in the size of old businesses decreases faster.

11 Online Appendix

11.1 Spillovers calibration

In this section, we estimate the value of γ for equation (19). To obtain a measure of g_S, we use BDS data and the following procedure. Use the data to construct the share of old establishments (N_{old}/N) and the share of workers in old establishments (L_{old}/L), where we consider an establishment as old if it is 16 years old or older. Then, note that from the equation for aggregate labor in production L, we can construct data on $\log([X_{old}/X]_t)$ as it is equal to $\log([L_{old}/L]_t) - \log([N_{old}/N]_t)$. Finally, we obtain g_S as $g_S = \Delta \log([X_{old}/X]_t) + \Delta \log(X_t) = \Delta \log([X_{old}/X]_t) + g_X$.

The OLS estimation of equation (19) is in the first column of Table 6. That is the coefficient we use in our benchmark model. The second column is the same regression but adds a linear trend. It yields similar results.

Table 6: Estimates for calibration of spillovers

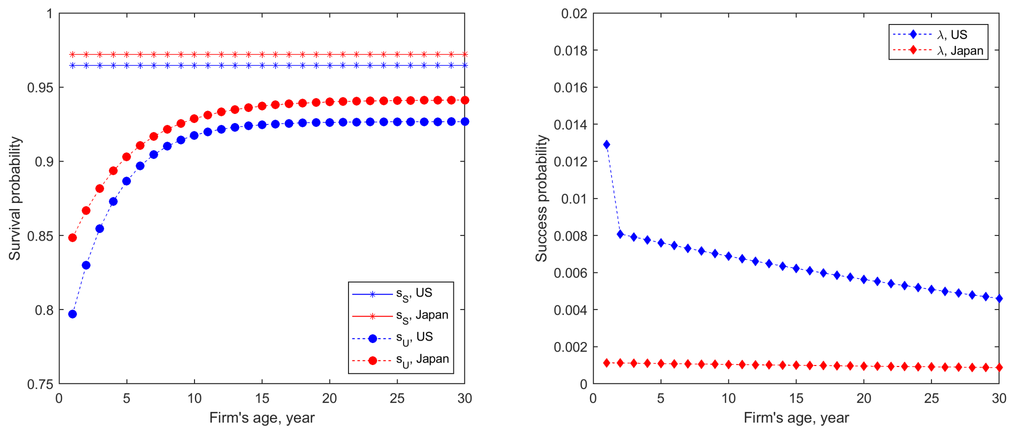
Regression for g_s	OLS		Instrumental Variables			
$g_{x,t-1}$	0.342*	0.304	0.385*	0.389*	0.460**	0.468**
	(0.186)	(0.199)	(0.207)	(0.200)	(0.205)	(0.196)
Trend	no	yes	no	yes	no	yes
R squared	0.124	0.137	0.108	0.091	0.115	0.091
First stage statistic F	-	-	14.549	12.659	25.452	22.187
Hansen's χ^2, p value	-	-	0.126	0.142	0.166	0.195
Instruments	-	-	VC	VC	VC & entry	VC & entry
Observations	26	26	23	23	23	23

Note: "VC" stands for lag growth rate of (i) VC total investment, (ii) early stage investment, (iii) seed investment, and (iv) expansion stage investment. Similarly, "Entry" stands for lag growth rate in the entry rate.

One may be concerned that overall productivity growth in $t-1$ may be affected by the productivity of the old businesses in period t. Ideally, we want the variation in g_X that is independent of the productivity growth of already leading businesses. We chose it as an instrument of g_X venture capital (VC) investment because it is expected to affect g_X through innovation by new firms. And VC investment should not directly affect the productivity growth of already leading businesses. Using this IV approach, we find slightly larger estimates.

11.2 Profiles of survival and success probabilities

Figure 8: Probability of survival and success over the life-cycle



11.3 Effect of population growth on the share of capital $\tilde{\alpha}$

Starting since the definition of $\tilde{\alpha}$, note that since $wL/Y = 1 - \alpha - \zeta$ from the definition of technology, we can obtain

$$\tilde{\alpha} = 1 - (1 - \alpha - \zeta)\frac{M}{L},$$

On the business side, the following equation holds:

$$Y = rK + wL + S, \tag{32}$$

Note that $rK = \alpha Y, wL = (1 - \alpha - \zeta)Y$, and $S = \zeta Y$.

Conversely, on the household side, we have

$$Y = rK + wM + S - E. \tag{33}$$

Note that $w(M - L) = E$ because entry, research, and development require only labor.

Combining equations (32) and (33), we derive

$$\frac{M}{L} = 1 + \frac{\zeta}{1 - \alpha - \zeta}\frac{E}{S},$$

so the ratio of E to S affects the share of workers $\tilde{\alpha}$ through the ratio of M to L.

Additionally, in the context of the free entry condition, the initial cost of starting a business E equals the expected future profit that entrants will earn until they exit. Therefore, the ratio E/S represents the ratio of expected future profit to current profit. Along a BGP, this ratio transforms into:

$$\frac{E}{S} = \frac{\sum_{t=1}^{\infty} \left(\frac{g_w}{(1+r-\delta)g_X}\right)^t (\Lambda_{S,t}(g_S) + \Lambda_{U,t})}{\sum_{t=1}^{\infty} \left(\frac{1}{g_M g_X}\right)^t (\Lambda_{S,t}(g_S) + \Lambda_{U,t})} = \frac{\sum_{t=1}^{\infty} \left(\frac{\beta}{g_X^{\frac{\zeta(\epsilon-1)}{1-\alpha}+1}}\right)^t (\Lambda_{S,t}(g_S) + \Lambda_{U,t})}{\sum_{t=1}^{\infty} \left(\frac{1}{g_M g_X}\right)^t (\Lambda_{S,t}(g_S) + \Lambda_{U,t})}.$$

Given that dg_X/dg_M is substantially smaller than one both from model and empirical results, an increase in g_M decreases the denominator more than the numerator, as long as $\frac{\zeta(\epsilon-1)}{1-\alpha}$ is not large. Consequently, $d\tilde{\alpha}/dg_M$ is likely negative, amplifying the effect of population growth on TFP growth, although extreme parameter values can alter this relationship.

11.4 Cross-sectional IV regressions

Although the local projection analysis of the data and model reveals similar correlations between labor force growth and labor productivity growth, this analysis does not enable us to determine the *causal* impact of labor force growth on labor productivity growth. For example, one possible explanation for our result is that workers relocate to states with greater expected labor productivity growth. Nonetheless, our mechanism for the effect of labor force growth on labor productivity growth is quite specific, as it is based on a decrease in the number of new businesses. Karahan, Pugsley and Şahin (2024) validates this mechanism by demonstrating a causal relationship between labor force growth and the number of startups or the startup rate. In particular, they identify that a 1-percentage-point decrease in the working-age population growth rate roughly translates to a nearly 1-percentage-point decrease in the startup rate. Because our model was constructed using a firm-dynamics model similar to Karahan, Pugsley and Şahin (2024)'s framework, it is not surprising that we attain the same kind of relationship, as shown in panels for population growth and growth in average productivity in Figure 6.

Karahan, Pugsley and Şahin (2024), inspired by Shimer (2001), used a past birth rate as an instrumental variable for labor force growth. This variable is a powerful instrument because, as previous research has shown, there is a close connection between current labor force growth and the birth rate some 20 years ago. Furthermore, in our scenario, the birth rate many years ago is unlikely to have a direct impact on current labor productivity growth. Unfortunately, we find that this instrument is too weak to be used for the yearly dynamics across the states examined above using local projections. Lagged state-level birth rate, on the other hand, is a significant predictor of differences in labor force growth after averaging state data over 20 years. This fact enables us to use cross-sectional regressions in an attempt to identify the causal effect of labor force growth on labor productivity growth.

For the 20 years from 2004 to 2024, we average labor productivity growth, g_{prod}, and labor force growth, g_M. We use the birth rate pushed back 20 years as an instrument for g_M, so the average is from 2000 to 2004. We control by two potentially important variables. First, we control by the state's initial GDP per capita (average from 2004 to 2024) because state convergence would suggest a negative link between the initial level of development and future growth. Second, we include the state's population (average from 1990 to 2019), as many growth theories indicate that scale effects may exist.

The findings of four specifications are presented in Table 7. The first two columns show the results of OLS regressions, while the following two columns show the results of Instrumental Variable (IV) regressions. For each method, we perform the analysis using the average of labor force growth for the same period, as well as for 1999 and 2019, to avoid period overlap in the averages. The results are pretty similar.

Table 7, regardless of specification, demonstrates that labor force growth has an effect on labor productivity of around a 0.2 percentage point change in labor productivity growth for every 1-percentage-point change in labor force growth. It should be noted that these estimates are comparable to the effect estimated in the local projections for the years following the shock. Note also that the magnitudes are comparable, although slightly smaller, than those estimated by Peters (2022) using forced population expulsions in post-war Germany.

Table 7: Impact of labor force growth on labor productivity growth, cross-sectional regressions for US states

Dependent variable Average g_{prod}	OLS year-lag on g_M no lag	OLS year-lag on g_M 5 years lag	IV, lagged birth rate year-lag on g_M no lag	IV, lagged birth rate year-lag on g_M 5 years lag
Average g_M	0.195*	0.183	0.106**	0.157***
	(0.070)	(0.100)	(0.042)	(0.059)
log(Initial gdp pc)	-0.004**	-0.005**	-0.004***	-0.005***
	(0.001)	(0.001)	(0.001)	(0.001)
log(Population)	0.000	0.000	0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
R-squared	0.122	0.110	0.107	0.108
First-stage reg F stat	–	–	59.263	46.034
Observations	50	50	50	50

Note: There is also a constant in each regression, and the values in parentheses are robust standard errors. The states include all US states except the District of Columbia.

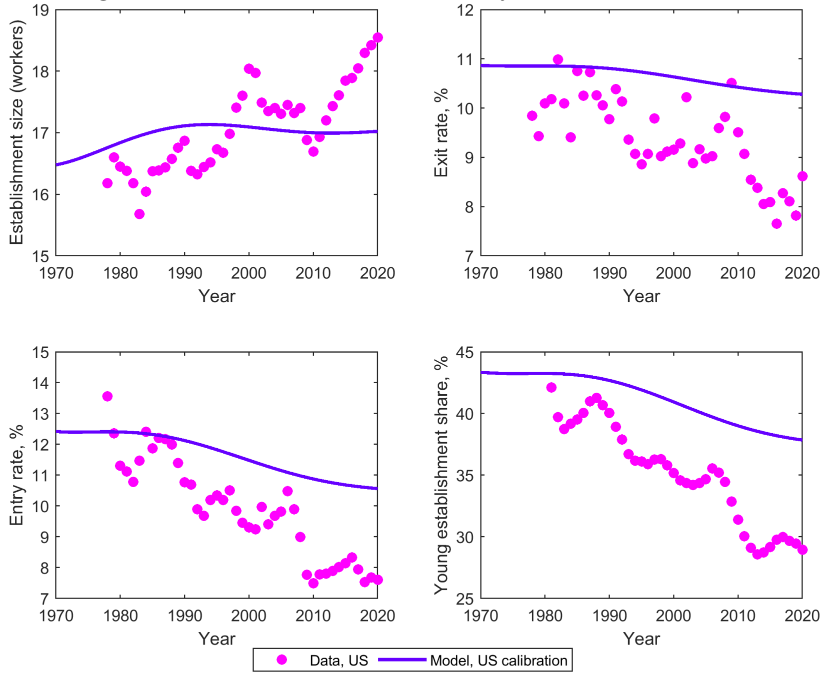
11.5 The decline in US business dynamism

In this section, we examine how this model predicts the decline in business dynamism in the United States. Since we do not use trend data for the variables we compare, it may be challenging for the model to reproduce the US decline in business dynamism once it is fed the labor force trend. In that sense, the exercise in this section serves as a model validation exercise.

We use the calibration in the previous section to compute the model's transitional dynamics. To begin, we compute a new BGP based on the 1970 population growth. Then, we compute a transition in which the agents are aware of the change in g_M exactly 100 years in advance. Thus, agents can predict the change—this appears appealing because low-frequency population size changes are predictable. Since we have g_M data up to 2020, we assume that g_M remains constant at the 2020 level thereafter.

Because the time series for these variables in the US begins around 1980, the three plots in Figure 9 begin with the model's prediction in 1970. Overall, Figure 9 indicates that the model, to some extent, captures the decline in business dynamism in the US. The model predicts the decline in the exit rate, the entry rate, and the proportion of young

Figure 9: Decline of business dynamism in the US



establishments.[31] It also predicts a slight increase in the average business size. It is reassuring that the model performs reasonably well in this dimension. The mechanism is straightforward and similar to that described in Karahan, Pugsley and Şahin (2024) and Hopenhayn, Neira and Singhania (2022). As fewer establishments enter the market (bottom left panel) due to slowing population growth, the share of young establishments (bottom right panel) decreases. Then, since older establishments are less likely to exit and larger, the exit rate falls (top right panel), and the average business size increases (top left panel). Previous research has shown a link between population growth and business dynamism; therefore, our interest in business dynamism is primarily a validation exercise.

11.6 Dicussion: Our mechanism vis-à-vis scale effects

We have abstracted away from scale effects on growth up to this point by extending the firm-dynamics model of Hopenhayn (1992). However, those models have a long history, as elegantly explained by Jones (2022). In this section, we briefly discuss a particular type of model with scale effects, specifically a model in which each business produces a distinct type of good or variety.

[31] We define young establishments as age 5 or younger.

Assume the technology for a project of productivity x_i, capital k_i and labor l_i is $y_i = x_i k_i^{\alpha} l_i^{1-\alpha}$, and the final consumption good is a CES combination of goods or varieties according to

$$Y = \left[\sum_{i=1}^{N} y_i^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}}\right]^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}},$$

where N is the number of firms, each producing a different variety as in Peters and Walsh (2022). These expressions give the following formula for total output,

$$Y = N^{\frac{1}{(\tilde{\sigma}-1)}} \tilde{X} K^{\alpha} M^{1-\alpha},$$

where $\tilde{X} \equiv \left(\sum_i \frac{1}{N} x_i^{\tilde{\sigma}-1}\right)^{\frac{1}{\tilde{\sigma}-1}}$ is the CES aggregation of productivity and N is the number of goods or varieties (K and M are total capital and labor force, as before). Therefore, the growth in TFP is

$$g_{TFP} = g_{\tilde{X}} + \frac{1}{\tilde{\sigma}-1} g_N.$$

This equation implies that TFP growth is proportional to the growth of average productivity across firms and the growth in the number of businesses/varieties g_N. Recall that for comparisons of balanced-growth paths in our benchmark model, g_{TFP} is proportional to g_X; therefore, the last term is the key difference when considering new varieties.

In a BGP, the growth rate in the number of varieties, g_N, must equal the growth rate of the population, g_M. Thus, this equation implies that, unlike in our model, there is a direct effect of population growth on TFP growth across BGPs. The magnitude of this effect is given by the parameter $\tilde{\sigma}$. Calibrating $\tilde{\sigma} = 4$ such that it is consistent with the "degree of diminishing returns" calibrated in Jones (2022), this equation says that for each 1-percentage-point decline in population growth, there would be a 0.33 percentage point decline in productivity growth. This number is larger but comparable with the numbers we found for the change in g_X: 0.13 for the US's calibration and 0.22 for Japan's calibration. Thus, the new mechanism introduced in this paper would increase the impact of population growth on productivity growth between 30-60% (from 0.33 to 0.46-0.55 percentage points). Similarly, Peters and Walsh (2022) finds that in the long run, for each point of decline in population growth, productivity growth declines by about 0.23 percentage points. They also report that almost all of this is due to the decline in the number of varieties, which is the term highlighted in this section. Thus, the impact we find for our new mechanism would increase its effect on productivity growth in the US by 56% (from 0.23 to 0.36 percentage points).

11.7 Data sources

11.7.1 US data

Civilian labor force. Civilian labor force data come from the Bureau of Labor Statistics (BLS) Current Population Survey from 1949 to 2019 and from Lebergott (1966) from 1900 to 1948. The civilian labor force definition in BLS includes the population 16 years of age and over, while the definition in Lebergott (1966) includes the population 10 years of age and over. The labor force growth projections are based on the Bureau of Labor Statistics' (BLS) "A look at the future of the U.S. labor force to 2060," published in September 2016.

Establishment data. Establishment data come from the U.S. Census Bureau's Business Dynamics Statistics (BDS). It provides annual measures of establishment openings and closings, and job creation and destruction by age group, which allows computing life-cycle patterns of establishment that we use as targets in Figure 3 and other dynamism in Figure 9. The data is available since 1978. An establishment is a fixed physical location where economic activity occurs. A firm may have one establishment or many establishments.

Total factor productivity. Total factor productivity (TFP) data are calculated using Penn World Table (PWT) 10.0. While the value of TFP is directly available in PWT, we calculate TFP from real GDP, the number of persons engaged, and capital stock by assuming a Cobb-Douglas production function. It ignores the effect of the change in human capital, which makes the data more consistent with our model. The data have been available since 1950.

Venture capital investment. Venture capital investment data are from the PwC/CB Insights MoneyTree™ Report.

11.7.2 Japan data

Labor force. Labor force data are sourced from the Statistics Bureau of Japan's (SBJ) Labor Force Survey from 1953 to 2019 and the National Institute of Population and Social Security Research (IPSS) from 1920 to 1952. The definition includes the population aged 15 years and over. The labor force growth projections are based on the Ministry of Internal Affairs and Communications (MIC) report, "Information and Communications in Japan 2022," published in July 2022. It offers only a projection of the working-age population, so we assume the ratio of the labor force to the working-age population has remained constant since 2020.

Establishment data. Establishment data come from SBJ's Establishment and Enterprise Census from 1981 to 2006 and from the Economics Census from 2009 to 2021. They provide the number of establishments and employment by age group, but these data are not annual (available only for the years 1981, 1986, 1991, 1996, 2001, 2004, 2006, 2009, 2012, 2014, 2016, and 2021). The number of establishments and employment by both age groups and three kinds of status (existing, newly organized, and closed) are only available in 2004, so the exit rate and growth of surviving establishments by age group, which are the targets, are calculated based on the data in 2004 by comparing with the data in 2001. Accurately, the calculation will not provide the annual exit rate and growth, but rather a three-year average. Therefore, the fittings for these targets in Figure 3 are based on this three-year average at an annual rate.

We have extracted the birth-year fixed effect for employment size by age, another target value. The life-cycle profile for Japan is primarily influenced by the birth year, while it remains very stable over time in the United States. In particular, we first assume that all ages in the same age group grow their establishment size at the same rate over the year. (For example, the establishment size for the age group between 3 and 7 grew by 1.2% annually from 2001 to 2004; we assume age 3, 4, 5, 6, and 7 establishments in 2001 increased their size by 1.2% every year between 2001 and 2004.) Then, we regress establishment size growth on fixed effects of age a and year t: Establishment size growth$_{a,t} = u_a + v_t + \epsilon_{a,t}$, and extract u_a to obtain the average growth in establishment size by age. Please note that with age a and year t, the born year is identically defined by $t - a + 1$.

Total factor productivity. TFP data are calculated using PWT 10.0, as in the US case.

11.7.3 US data (state-level)

Civilian labor force. Civilian labor force data come from the BLS Current Population Survey for the years 1976 to 2019 and from *Historical Statistics of the United States, Colonial Times to 1970* issued by the Census Bureau for the years 1900 to 1950. We interpolate the data between 1950 and 1976. The projections for 2030 are based on the Projections Managing Partnership (PMP), which is grounded in employment data. We use only data between 1976 and 2019 to estimate empirical data, where yearly data are available. For the simulated data, we utilize the whole dataset to reproduce TFP growth.

Real GDP and total nonfarm employees. Real GDP data come from the Bureau of Economic Analysis (BEA) Gross Domestic Product by State and Personal Income by State. Total nonfarm employees for the corresponding years are from the BLS. We use these statistics to derive labor productivity.

Population. Population data are from the Census Bureau. This is one of the control variables in the cross-sectional regressions for US states.

11.8 More on BGP for the benchmark case

Section 3 illustrates the balanced growth path without two extensions and subsequently introduces two additional features (congestion and spillover). However, these two features somewhat change the property of the balanced growth path.

First, since the productivity growth of leading businesses, g_S, depends on g_χ, which is equal to g_X along a balanced growth path due to spillovers, the expected productivity for leading projects, $\Lambda_{S,a}(g_S)$, will also depend on g_X. Let it denote as $\Lambda_{S,a}(g_S(g_X))$. As such, the RHS of the equation for the step size g^* (17) is written in this form:

$$g^* = \frac{\sum_{a=1}^{\infty} \left(\frac{1}{g_M}\right)^a \Lambda_{S,a}(1)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_X g_M}\right)^a \Lambda_{S,a}(g_S(g_X))}. \tag{34}$$

Next, because of congestion, the LHS of equation (17) (or equation (14)) is altered to

$$g^* = \left(\frac{2c_E(\tilde{n}/\tilde{M})^{\phi} z_R}{\iota - 2}\right)^{\frac{1}{\iota}}, \tag{35}$$

so we need to specify n/M. Using the equation for labor demand, we can derive

$$\frac{N}{L} = \frac{w^{\frac{1-\alpha}{\zeta}}}{X}\left(\frac{\alpha}{r}\right)^{-\frac{\alpha}{\zeta}}(1-\alpha-\zeta)^{-\frac{1-\alpha}{\zeta}}. \tag{36}$$

Also, from equation (24),

$$\begin{aligned} w^{\frac{1-\alpha}{\zeta}} =& \chi(g^*)^{-\frac{\iota-2}{2}}\left(\frac{z_R z_D}{\iota}\right)^{\frac{1}{2}}\zeta\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{\zeta}}(1-\alpha-\zeta)^{\frac{1-\alpha-\zeta}{\zeta}}\times \\ & \sum_{t=1}^{\infty}\hat{\beta}_t(\Lambda_{S,t}(g_S)+\Lambda_{U,t})\left(\frac{1}{g_w}\right)^{\frac{1-\alpha-\zeta}{\zeta}(t-1)}. \end{aligned} \tag{37}$$

Note that

$$\Gamma(w,r) = \zeta\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{\zeta}}\left(\frac{1-\alpha-\zeta}{w}\right)^{\frac{1-\alpha-\zeta}{\zeta}}\sum_{t=1}^{\infty}\hat{\beta}_t(\Lambda_{S,t}(g_S)+\Lambda_{U,t})\left(\frac{1}{g_w}\right)^{\frac{1-\alpha-\zeta}{\zeta}t}$$

along a balanced growth path. Therefore, from equations (36) and (37),

$$\frac{N}{L} = \frac{\chi}{X}(g^*)^{-\frac{\iota-2}{2}}\left(\frac{z_R z_D}{\iota}\right)^{\frac{1}{2}}\frac{\zeta}{1-\alpha-\zeta}\sum_{t=1}^{\infty}\hat{\beta}_t(\Lambda_{S,t}(g_S)+\Lambda_{U,t})\left(\frac{1}{g_w}\right)^{\frac{1-\alpha-\zeta}{\zeta}t}. \tag{38}$$

Since

$$N = n\left(\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^{a}(\Lambda_{S,a}(1)+\Lambda_{U,a}/\theta)\right)$$

from equations (8) and (9), and

$$\frac{\chi}{X} = \frac{\left(\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^{a-1}(\Lambda_{S,a}(1)+\Lambda_{U,a}/\theta)\right)\left(\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^{a-1}\Lambda_{S,a}(g_S(g_X))\right)}{\left(\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^{a-1}\Lambda_{S,a}(1)\right)\left(\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^{a-1}(\Lambda_{S,a}(g_S(g_X))+\Lambda_{U,a})\right)}$$

from equations (15) and (16), equation (38) gives this equation:

$$\begin{aligned} \frac{n}{L} =& (g^*)^{-\frac{\iota-2}{2}}\frac{\left(\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^{a}\Lambda_{S,a}(g_S)\right)}{\left(\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^{a}\Lambda_{S,a}(1)\right)\left(\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^{a}(\Lambda_{S,a}(g_S)+\Lambda_{U,a})\right)}\times \\ & \left[\left(\frac{z_R z_D}{\iota}\right)^{\frac{1}{2}}\left(\frac{\zeta}{1-\alpha-\zeta}\right)\left(\sum_{t=1}^{\infty}\left(\frac{\beta}{g_w^{\frac{1-\alpha-\zeta}{\zeta}}}\right)^{t}(\Lambda_{S,t}(g_S)+\Lambda_{U,t})\right)\right]. \end{aligned}$$

Substituting g^* for the RHS of equation (17) and g_w for $(g_X)^{\frac{\zeta}{1-\alpha}}$, we can derive the following expression for n/L:

$$\frac{n}{L} = \left(\frac{z_R z_D}{\iota}\right)^{\frac{1}{2}} \left(\frac{\zeta}{1-\alpha-\zeta}\right) \times \left(\frac{\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^a \Lambda_{S,a}(g_S(g_X))}{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \Lambda_{S,a}(1)}\right)^{\frac{\iota}{2}} \times \tag{39}$$

$$\left(\frac{\sum_{t=1}^{\infty}\left(\frac{\beta}{g_X^{1+\frac{\zeta(\epsilon-1)}{1-\alpha}}}\right)^t (\Lambda_{S,t}(g_S(g_X)) + \Lambda_{U,t})}{\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^a (\Lambda_{S,a}(g_S(g_X)) + \Lambda_{U,a})}\right). \tag{40}$$

Also, from equations (4), (11), (24), (25), and (26),

$$\frac{M}{L} = 1 + \frac{\zeta}{1-\alpha-\zeta}\left(\frac{\sum_{t=1}^{\infty}\left(\frac{\beta}{g_X^{1+\frac{\zeta(\epsilon-1)}{1-\alpha}}}\right)^t (\Lambda_{S,t}(g_S(g_X)) + \Lambda_{U,t})}{\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^a (\Lambda_{S,a}(g_S(g_X)) + \Lambda_{U,a})}\right). \tag{41}$$

Therefore, from equations (40) and (41), the step size g^* is influenced by g_M and g_X through the number of entrants per capita. These equations and the equation for the step size

$$\left(\frac{2c_E(n/M)^{\phi} z_R}{\iota - 2}\right)^{\frac{1}{\iota}} = \frac{\sum_{a=2}^{\infty}\left(\frac{1}{g_M}\right)^{a-1} \Lambda_{S,a}(1)}{\sum_{a=2}^{\infty}\left(\frac{1}{g_X g_M}\right)^{a-1} \Lambda_{S,a}(g_S(g_X))}$$

define the equilibrium g_X for each g_M.

11.9 Sensitivity analysis

We simulate a shock of g_M from 1.02 to 1.01 to g_{TFP} using the US calibration as a benchmark case. The impact size is the change in g_{TFP} that persists in the long run as a result of that change. The elasticity of "Impact Size" represents the elasticity of the impact of g_M on g_{TFP} to a change in a parameter value. Specifically, an "Impact Size" elasticity equal to X means that a 1% change in the parameter results in an X% increase in the response of g_M to g_{TFP}.

"Convergence Speed" is the share of the change in g_{TFP} that occurred 20 periods after the g_M shock (relative to the long-run impact). The “Convergence Speed” elasticity equal to Z means that a 1% change in the parameter results in a Z% increase in the explained share of g_{TFP} change 20 periods after the shock.

Finally, note that for the parameters that vary over age, the exact change to λ and s_U is applied to all ages.

Table 8: Role of parameters for the impact size and the convergence speed

	Elasticity	
	Impact	Convergence
Parameter	Size	Speed
Survival rate of leading businesses, s_S	8.184	-9.827
Survival rate of laggard businesses, s_U	1.704	0.267
Decreasing returns, ζ	1.049	0.092
Capital share, α	0.017	0.187
Spillover elasticity, γ	0.236	0.031
Entry cost exponent, ϕ	0.169	-0.092
Success probabilities, λ	0.028	0.071
Depreciation rate, δ	-0.002	-0.119
Risk aversion, ϵ	-0.190	0.256
Inverse of Jump in prod at success, θ	-0.049	-0.079
Research cost exponent, ι	-0.059	-0.179
Discount factor, β	2.412	-4.212
Productivity growth old businesses, g_S	-28.165	6.856

11.10 Computational Details

In this section, we describe the method used to compute the model.

11.10.1 Balanced Growth Path

We first solve for a BGP using the 1980-1999 population and productivity growth averages as targets. Substituting these target values for g_M and g_X in equation (34) gives the step size g^* for the reference periods. We then derive the initial productivity growth to define the BGP state before the population growth shock. Given the step size and the discussion in the online appendix 11.8, we can find an equation for g_X as a function of the exogenous value g_M. Since an analytical solution does not exist, we use Newton's method. We can also derive the other variables using the properties of the BGP as described in Lemma 2.

11.10.2 Transitional Periods

After computing the BGP, we compute the transitional dynamics in two steps. First, we guess the growth rate of the number of entrants $\{g_{n_t}\}$ and solve for the equilibrium set of prices $\{g_{w_t}, r_t\}$ using the capital, goods, and labor market clearing conditions. We then derive $\{g_{n_t}\}$ using the free entry condition. Each step requires finding the roots numerically, so our code has a nested structure of two Newton's method computations. Importantly, we solve all periods simultaneously, not sequentially; innovators stand on the shoulders of previous innovators, which requires solving the model from the past to the future. However, they must also choose the step size of productivity, considering expected profits, which requires solving the model from the future to the past.

In the first step, we derive the number of businesses by age given the conjectured $\{g_n\}$ and the values in the initial BGP. We can also determine productivity by age, as the step size satisfies equation (35) even during transitional periods. Next, we guess the interest rate $\{r_t\}$. As the number of businesses N_t and average productivity X_t are already known, $\{g_{w_t}\}$ can be derived easily by the labor market clearing condition (11) given $\{r_t\}$. With this set of prices $\{g_{w_t}, r_t\}$, we solve the household and business optimization problems. Lastly, we fix $\{r_t\}$ using Newton's method, incorporating capital and goods market clearing conditions.

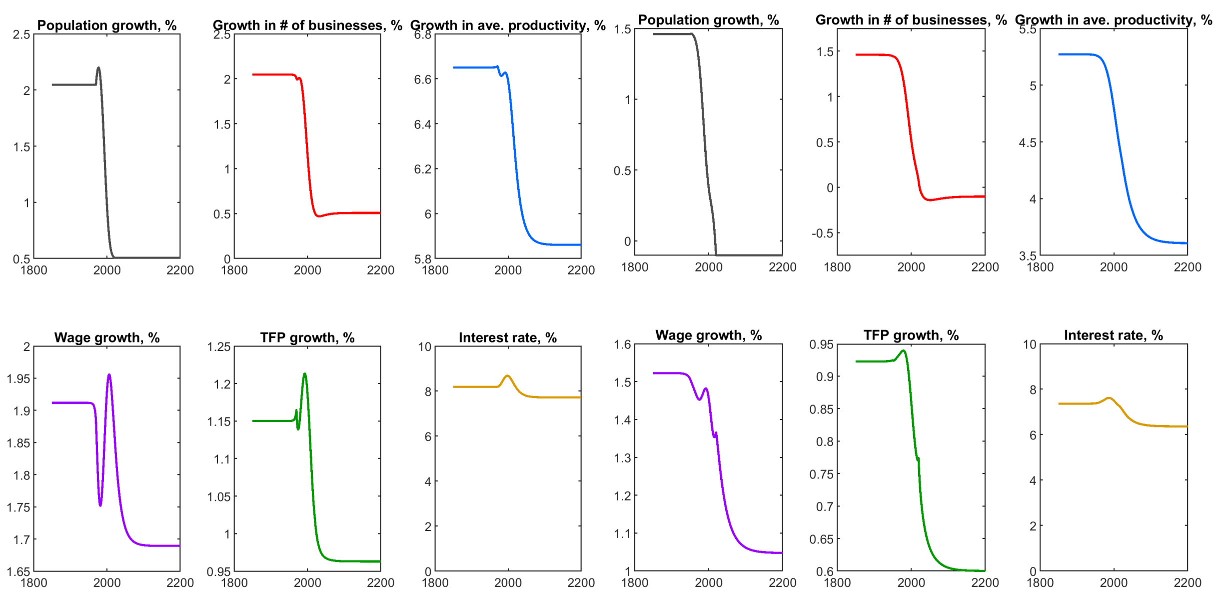
In the second step, we compute the expected profits for entrants and, therefore, the value of businesses. Since it should equal the entry cost, we adjust $\{g_{n_t}\}$ using Newton's method. Since changes in $\{g_{n_t}\}$ affect $\{r_t\}$, we return to the first step for every repetition. When the value of businesses minus the entry cost converges to zero sufficiently ($< 10^{-6}$), the computation is finished.

11.11 Full transitions

This section shows the full transitions computed for the US and Japan. As mentioned in the main text, the transition's input is the labor force growth trend, which is shown with a black line in Figure 10.

Figure 10: Full Transitions

United States | Japan



11.12 Calibration of alternative productivity processes

Figure 11: Fit of targets of alternative specifications

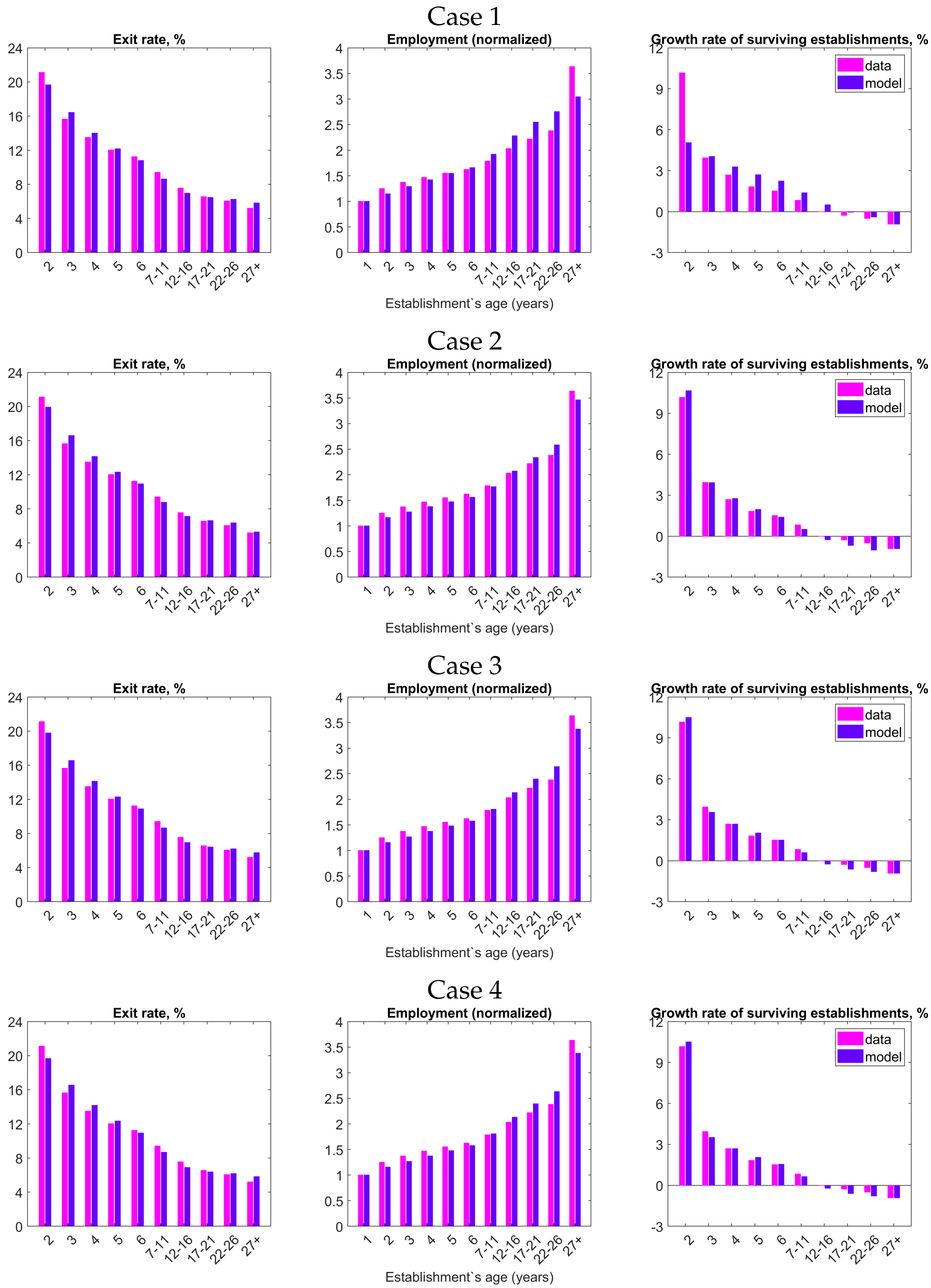


Figure 12: Productivity growth by establishment age