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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Heterogeneous Agents Dynamic Spatial General Equilibrium*

Maximiliano Dvorkin†

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Abstract

I develop a dynamic model of migration and labor market choice with incomplete markets and uninsurable income risk to quantify the effects of international trade on workers' employment reallocation, earnings, and wealth. Macroeconomic conditions in different labor markets and idiosyncratic shocks shape agents' labor market choices, consumption, earnings, and asset accumulation over time. Despite the rich heterogeneity, the model is highly tractable as the optimal consumption, labor supply, capital accumulation, and migration and reallocation decisions of individual workers across different markets have closed-form expressions and can be aggregated. I study the asymmetric impact of international trade on the evolution of employment, earnings, and wealth, and decompose the frictions workers face to reallocate across U.S. sectors and regions into those with a transitory effect and those with long-lasting consequences.

Keywords: International trade, migration, spatial equilibrium, dynamic Roy models, human capital, wealth, inequality.

JEL Classification: F16, F66, R23, E21, E24, J24, J61.

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1 Introduction

In theory, globalization and international trade are good, as they promote a more efficient allocation of resources across countries and across different factors of production, expanding production possibilities and income. In practice, however, considerable debate exists on whether globalization has adverse effects, particularly on the labor market. Workers with different characteristics and skills may be asymmetrically exposed to the large swings in labor demand caused by these forces, and while on aggregate there could be important benefits, these may not be evenly distributed across individuals.

In recent years, an important body of literature analyzed the effects of import competition, highlighting the displacement of workers from the manufacturing sector and from production occupations. For individual workers, depressed labor market conditions in some industries and regions translated into job displacement, protracted periods of unemployment, earnings losses and sectoral and occupational change.[1]

The very influential works of Autor, Dorn, and Hanson (2013) and Autor, Dorn, Hanson, and Song (2014) on the effects of the surge of Chinese imports into the U.S. economy show that workers more exposed to trade with China endure long-lasting earnings loses. In this paper I complement these facts and show that the wealth of more exposed workers is also negatively affected by the increased trade with China.

A few economic mechanisms may explain these facts. On the one hand, trade exposure may lead to job displacement, which destroys industry and/or occupation specific human capital. Thus, compared to non-displaced workers, those affected by trade will find reemployment in positions for which they are poorly matched and with a low labor productivity. On the other hand, if workers seek reemployment in similar industries and occupations to avoid human capital loses, labor demand and wages will remain depressed, affecting earnings. In terms of wealth, exposed workers can use their savings or reduce their investments in human capital or assets as a way to smooth consumption over time. In addition, regions with more exposed industries will see a relative decline in the value of real estate and revenues of local businesses as economic conditions remain weak, affecting individuals' wealth. As argued by Xu, Ma, and Feenstra (2019), the "China shock" operated in part through the housing market and find evidence that commuting zones that experienced larger increases in import exposure also had smaller increases in housing prices. Thus, the margins of adjustment to a trade shock extend beyond labor reallocation across industries or regions, and include also changes in

[1] For a discussion of labor market effects of international trade, see Artuç, Chaudhuri, and McLaren (2010); Autor, Dorn, and Hanson (2013); Caliendo, Dvorkin, and Parro (2019); Dix-Carneiro (2014); Helpman, Itskhoki, and Redding (2010); Pierce and Schott (2016); Traiberman (2019).

consumption, savings, and investments in human capital and wealth.

To rationalize the empirical evidence, I develop a heterogeneous agents dynamic spatial general equilibrium model of labor reallocation and regional migration where workers invest in their human capital and save in assets. The model extends the setup developed in Caliendo, Dvorkin, and Parro (2019) and Dvorkin and Monge-Naranjo (2019) to an economy with heterogeneous agents facing idiosyncratic shocks affecting their human capital, or efficient units of labor, and the returns on their assets.

Every period workers make a set of optimal discrete and continuous choices to maximize lifetime utility. The model combines elements of the dynamic discrete choice literature, such as labor reallocation over industries or occupations and migration over regions, with optimal decisions on continuous variables, such as human capital investments and assets accumulation. In my setting, the evolution of a worker's human capital and assets is driven by his labor market choices, idiosyncratic labor-market-specific shocks and the costs of switching industries/occupations and migrating.

In this economy, agents are heterogeneous in the level of human capital and the assets they bring from the previous period and are subject to idiosyncratic shocks affecting their labor supply and the effective return of their assets. The effect of these shocks on the evolution of individual human capital and assets depend on investments and on the reallocation and migration decision of workers. I characterize the worker's human capital and asset investment choices and the equilibrium assignment of workers to labor markets. I show how individual consumption/investment decisions can be written as an optimal portfolio problem conditional on labor market choices, leading to decision rules that are homogeneous in wealth and a portfolio allocation that is similar across individuals but differs by labor market. Exploiting properties of extreme value distributions and the optimal consumption and savings policies, I show how the ensuing discrete problem leads to a tractable characterization.

At the market level, the total supply of human capital and assets in a labor market depends on the aggregation across heterogeneous workers arriving from all locations and sectors. This requires aggregating across different sources of heterogeneity, namely, individual shocks, stocks of assets and human capital, conditional on individuals' optimal selection into labor market via migration and reallocation. A key result of the paper is that individual investment decisions and the stocks of human capital and assets can be aggregated, and the equilibrium evolution of the aggregate supplies of human capital and assets across labor markets can be represented in closed-form. At the market level, the total supply of human capital and assets in a labor market depends on the aggregation across heterogeneous workers arriving from all locations and sectors. This requires aggregating across different sources of heterogeneity,

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I then use the model to study how worker's individual choices change with the exposure to import competition from China, and in turn how this choices shape the equilibrium allocation of workers to different labor markets, the dynamics of aggregate human capital and assets, the behavior of earnings inequality, and the welfare of the different workers in the economy in general equilibrium.

An important recent literature on dynamic labor reallocation and migration following Artuç, Chaudhuri, and McLaren (2010) explores some of these frictions, but for the most part they are modeled as switching costs in terms of utility, disconnected from human capital and earnings. The recent works by Caliendo, Dvorkin, and Parro (2019); Dix-Carneiro (2014); Traiberman (2019) are some important examples, but they cannot account for the heterogeneous and persistent effects of trade (for observationally equivalent workers) on earnings as documented by Autor, Dorn, Hanson, and Song (2014). A recent exception, although applied to a different context, is the work by Coen-Pirani (2021), who develops a model of migration with human capital (or labor productivity) accumulation that interacts with worker's reallocation/migration decision in an economy with perfectly symmetric regions.

It is important to highlight that individuals in my model face idiosyncratic earnings risk and capital-income risk. These sources of risk are not new to the literature but a much larger body of work in heterogeneous agents models with incomplete markets models focuses on earnings risks. Some notable exceptions are the works by Angeletos (2007), Moll (2014) and Guvenen, Kambourov, Kuruscu, and Ocampo (2022), who develop incomplete markets models with idiosyncratic capital-income risk. Empirically, fluctuations in capital-income over time for the same individual and large differences in the level of capital income across individuals are a prevalent feature of the data. Moreover, while in reality some risks on assets' return can be diversified or hedged, a very large share of the capital stock in the United States is in the form of private businesses and private equity, and real estate, and individuals tend to have a poorly diversified portfolio. Benhabib, Bisin, and Zhu (2011) develops an incomplete markets model with both stochastic labor and capital income processes. My paper contributes to this literature by adding geographic and sectoral components, with workers optimally reallocating across industries and occupations and migrating across regions, where the reallocation decision influences the future evolution of capital, earnings and reallocation.

By and large, the recent literature on dynamic worker reallocation and migration has abstracted from capital accumulation decisions. However, in a recent influential paper Kleinman, Liu, and Redding (2023) incorporate investment and capital dynamics in different sectors and regions by immobile rentiers. In their setup workers cannot save, thus frictions affecting the evolution of financial assets are not taken into account by workers. Relative to Kleinman, Liu, and Redding (2023), this paper studies how the ownership of capital is distributed in the economy and how it is affected by individuals' migration and reallocation decisions and by international trade. Ferriere, Navarro, and Reyes-Heroles (2021), Giannone, Li, Paixao, and Pang (2020), and Greaney (2020) develop models with asset accumulation and labor reallocation/migration decisions, but the setting rapidly becomes intractable as the number of labor markets increase. My model remains tractable and can be easily extended to saving in many different types of assets. Carroll and Hur (2020) develop a heterogeneous agent model with earnings risk and incomplete markets but abstracts from labor reallocation and mobility. Lyon and Waugh (2019) allow for labor reallocation across industries, but a labor market is defined at the level of each individual good variety in a trade model.

Bilal and Rossi-Hansberg (2021) argued that workers sort over industries and regions to exploit their comparative advantage, but also trade-off static gains in terms of amenities and wages for future earnings potential and capital accumulation. The evolution of workers' human and assets in my model also depend on workers comparative advantage and their reallocation decisions, taking into account the dynamic gains and losses on both forms of capital.

This paper is organized as follows. Section 2 presents evidence on the long-lasting effects of international trade on earnings and wealth. Section 3 develops a dynamic model of migration and labor market choice with incomplete markets and uninsurable income risk. Section 4, presents the quantitative exercise, comparing the effects of trade at the micro and macro levels between the calibrated version of model and the data. This section also decomposes the effects of trade and discusses the role of main frictions. Section 5 concludes.

2 Imports, earnings and wealth: empirical evidence

In this section I follow closely Autor, Dorn, Hanson, and Song (2014) and study how exposure to increased competition from China affects the evolution of earnings and wealth of U.S. workers. For this I use data from the National Longitudinal Survey of Youth 1979 (NLSY79), which is a nationally representative survey of a cohort of over 12,500 young men and women living in the United States in 1979. Individuals in this cohort were ages 14 to 22 when first interviewed in 1979 and the U.S. Bureau of Labor Statistics interviewed these individuals yearly up to 1994 and every two years after that. This survey contains information on demographic characteristics, education and employment choices, earnings and wealth, among other things.[2]

As argued in Autor, Dorn, and Hanson (2013) and Autor, Dorn, Hanson, and Song (2014), the expansion in U.S.-China international trade since the reestablishment of diplomatic relations in the early 1970's was gradual and did not really take off until the early 1990's. For example, in 1987 U.S. imports from and exports to China represented around 1.5% of all U.S. imports or exports, while trade with Canada or Europe was over ten times larger. However, by the year 2000, U.S. imports from China represented over 8% of all imports, and by 2017 this figure peaked at around 22%. Thus, following these works, I take 1991 to be the year in which the "China shock" starts and use workers' industry of employment in the year 1991 to isolate the dynamic effects of workers' exposure to import competition.[3] Workers reallocation to other industries after 1991, is part of the dynamic response to the shock, thus focusing on the industry in 1991 abstracts from selection into other industries after the shock.

In the empirical analysis, the trade exposure of a U.S. worker to Chinese imports is measured as,

$$\Delta IP_j = \frac{\Delta M_j^{U-C}}{Y_j + M_j - X_j} \tag{1}$$

where ΔM_j^{U-C} is the change in U.S. imports from China between 1991 and 2007 for industry j, Y_j is total U.S. production of industry j in 1991, M_j and X_j are total U.S. imports and exports, of industry j in 1991. The subindex j denotes the industry of employment of the worker in 1991. In this way, ΔIP_j represents the change in import penetration of industry j between 1991 and 2007 as a share of initial absorption.

I study the evolution of real earnings and wealth post 1991 and how it changes due to the exposure of Chines imports. In particular, let $\tilde{E}_{ij\tau}$ and $\tilde{W}_{ij\tau}$, be defined as,

$$\begin{aligned} \tilde{E}_{ij\tau} &= \sum_{t=1992}^{\tau} \frac{E_{ijt}}{\bar{E}_{ij0}} \times 100, \\ \tilde{W}_{ij\tau} &= \frac{W_{ij\tau} - W_{ij0}}{|\bar{W}_{ij0}|} \times 100, \end{aligned}$$

for $1992 \leq \tau \leq 2015$, where E_{ijt} and W_{ijt} are real earnings and wealth, respectively, in period t for individual i that was employed in occupation j in 1991, and $\bar{W}_{ij0}$ is the average wealth of

[2]Information on wealth after the year 2000 is every four years and there is no information on wealth in 1991.

[3]Disaggregated measures of U.S. imports from China by industry or goods are not available prior to 1991, which conditions the choice of the starting period.

that individual between 1988 and 1990, and $\bar{E}_{ij0}$ is average yearly earnings of that individual between 1988 and 1991. Since information on earnings is every two years after 1994, I multiply by two this variable for $t \geq 1996$. In this way, $\tilde{E}_{ij\tau}$ is the cumulative earnings since 1992 relative to earnings before the China shock and $\tilde{W}_{ij\tau}$ is the change in wealth relative to wealth before the shock. Since initial wealth is negative for an important fraction of individuals, around 15% of the sample, I use the absolute value in the denominator.

I run the following regression for different periods,

$$Y_{ij\tau} = \beta_0 + \beta_1\, \Delta IP_{ij} + \beta_2\, IP_{ij} + Z'_{ij}\, \beta_4 + e_{ij\tau}, \tag{2}$$

where $Y_{ij\tau}$ is either E_{ijt} or W_{ijt}, and Z'_{ij} is a set of controls that include dummies for workers' gender, race, education, and foreign born status, tenure at the firm in 1991, dummies for firm size in 1991, and average earnings between 1988 and 1991. As before, subindex j denotes worker i industry of employment in 1991. Variable IP_{ij} denotes the level of import penetration of industry j in 1991, which is computed similar to equation (1) but using the level of imports in 1991 in the numerator rather than the changes. The sample consists of all individuals with average earnings between 1988 and 1991 larger than $\$1,000$ constant dollars of the year 2007. Keep in mind that the NLSY79 follows a cohort of workers and this cohort was between 26 and 34 years old in 1991, that is, individuals in the early part of their prime work years.

Similar to Autor, Dorn, Hanson, and Song (2014), I instrument variable ΔIP_{ij} in the regressions with ΔIPO_{ij}, which is constructed in a similar way but replacing ΔM_j^{U-C} in equation (1) with the change in imports from China between 1991 to 2007 of non-U.S. high-income countries.

Figure 1 shows the estimated values of coefficient β_1 in the regression for different periods of time, together with 90% confidence intervals. Panel (a) in the Figure shows the effect of the "China shock" as measured by import penetration on workers' cumulative earnings as a percent of average earnings of the worker before the shock, while panel (b) shows the effect on the change in wealth relative to average wealth before the shock. Panel (a) is similar to the left panel of Figure III in Autor, Dorn, Hanson, and Song (2014) and the point estimates on the coefficient for cumulative earnings are similar to those estimated in that work up to 2007, which is the last year they study. Point estimates for the effects on wealth are negative and large, but estimates have wide confidence bands and are statistically significative only in 1996 and 2004.

The economic impact can be quantified in the following way. The median exposure to Chinese imports of a manufacturing worker in 1991 (median value of ΔIP_j) is 3.8. By defini-

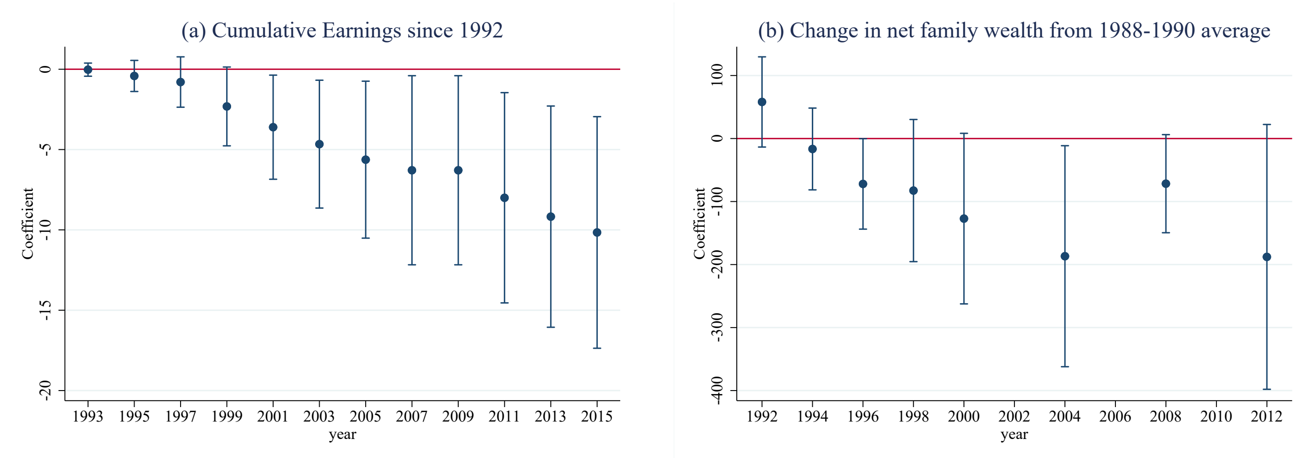

Figure 1: Estimated coefficients on Import Penetration from China

tion, a non-manufacturing worker in 1991 has zero exposure to trade with China. Since the estimated coefficient in 2007 is -6.3, the median exposure translates to a decrease in cumulative earnings up to 2007 of -24% of initial average earnings. In the sample, initial average earnings between 1988 and 1991 are around 34 thousand dollars of the year 2007, thus the decrease in earnings due to trade exposure is around 8 thousand dollars. Over time the effect is larger, with a point estimate of -10 in 2015, which translates into a drop of 38% of cumulative earnings for the median exposure, or a drop in 13 thousand dollars for the median exposed worker in manufacturing.

The decrease in wealth is larger, with a point estimate in 2004 of -186. Given the median exposure of 3.8, this translates into a decrease of 700% in wealth relative to the average level of wealth in 1988-1990. Median average wealth in the sample is 19 thousand dollars of 2007, which translates into a drop in wealth for a manufacturing worker with the median exposure of 133 thousand dollars of 2007.

The empirical evidence in this section shows that workers more exposed to trade endure long-lasting earnings and wealth loses and that workers. A few economic mechanisms may explain these facts. On the one hand, trade exposure may lead to job displacement, which destroys industry and/or occupation specific human capital. Thus, compared to non-displaced workers, those affected by trade will find reemployment in positions for which they are poorly matched and with a low labor productivity. On the other hand, if workers seek reemployment in industries and occupations similar to those they had in the past in order to avoid human capital loses, labor demand and wages in exposed industries and occupations will remain depressed, affecting earnings. In terms of wealth, if the periods of job loss are perceived as transitory, workers will use their savings as a way to smooth consumption over time. Moreover, an important part of worker's wealth is in the form of real estate. Regions with more exposed industries will see a relative decline in the value of housing as local economic conditions remain weak, affecting workers' wealth.

3 Labor market choice, human capital accumulation and wealth

To understand the empirical evidence of the previous section, I propose a model of labor reallocation and regional migration where workers invest in their human capital and save in financial assets. The model extends the setup developed in Caliendo, Dvorkin, and Parro (2019) and Dvorkin and Monge-Naranjo (2019) to an economy with heterogeneous agents facing idiosyncratic shocks affecting their human capital or efficient units of labor and the returns of their assets.

I consider an overlapping generations model in which workers with characteristics e enter the economy at an initial age, $a = 0$, and work for a fixed number of periods before retiring at age $a = A$. Workers have standard log-preferences.[4] At time t, the utility of a worker of type e and age a is given by,

$$U_t^{a,e} = \log\left(c_t^{a,e}\right) + E\left[\sum_{s=1}^{A-a-1} \beta^s \log\left(c_{t+s}^{a+s,e}\right)\right] + \beta^{A-a} U_{R,t}^{A,e},$$

where $0 < \beta < 1$ is the worker's discount factor and $U_{R,t}^{A,e}$ is a level of utility in retirement.[5]

The economy has N geographic regions and I industries or sectors. For age $1 \leq a \leq A-1$, the worker starts each period (year of life) attached to one of $j = 1, ..., J$ labor markets, where a labor market is an industry-region pair such that $J = N \times I$. The worker carries over from the previous period a level of human capital and assets which describe, respectively, the total efficiency units of labor of the worker in her labor market (industry/region) and her stock of savings in units of physical capital up to that date. At the beginning of each period, the worker may switch to a different labor market, ℓ. As in Dvorkin and Monge-Naranjo (2019), the decision to switch to a different labor market is shaped by costs affecting workers' human capital and assets. In particular, I assume there is a matrix $\tau^{a,e}$ of size $J \times J$ that captures

[4]The analysis extends to CRRA preferences, but some expressions are more involved.

[5]The discount factor may incorporate a constant death probability or labor market exit probability. For simplicity I abstract from heterogeneity in the duration of workers lifetimes, but this can be easily extended with a discount factor that varies with age and workers characteristics.

the average transferability of human capital across labor markets. That is, each element of this matrix, $\tau_{j\ell}^{a,e} > 0$, determines the fraction of human capital h that can be transferred from the current labor market j to a new one ℓ. Some labor markets may use a similar set of skills, and the loss of human capital for those transitions is low, which implies a value of $\tau_{j,\ell}^{a,e}$ close to one. For other transitions, the skills or experience in one labor market may not be useful in other activities, and $\tau_{j\ell}^{a,e}$ will be low.[6]

In addition, there is a vector of labor market opportunities, or idiosyncratic shocks, $\check{\epsilon}^{a,e} \in \mathbb{R}_+^J$, which affect workers human capital in the chosen labor market and, thus, reflect the effective unit of human capital they can transfer if they switch to occupation ℓ. Thus, given a level of human capital at the beginning of the period, h, last period's labor market j, and a vector $\check{\epsilon}_t^{a,e} \in \mathbb{R}_+^J$ of labor market opportunities, the level of human capital, or efficient units of labor, a worker that switches to labor market ℓ has available is, $h\,\tau_{j\ell}^{a,e}\,\check{\epsilon}_\ell^{a,e}$. In each labor market ℓ there is a real *gross* rate of return of human capital, $w_{\ell,t}$.[7] Thus, the worker's gross real earnings for the period, after selling all their human capital in her chosen labor market ℓ, are $w_{\ell,t} h\,\tau_{j\ell}^{a,e}\,\check{\epsilon}_\ell^{a,e}$.

Migrating across regions or switching to a different labor market affects the return on workers' assets. There are several possible reasons for this. On the one hand, workers may incur in important monetary costs when moving across regions. In other cases, a labor market switch may reflect a change in employer, enduring a short spell with no earnings or involving a signing bonus that increases disposable income. Alternatively, the worker may spend resources in conducting interviews or obtaining certifications required for a new job. Finally, workers can spend resources in the sale and purchase of real estate or stakes in privately owned businesses with an important geographic or sectoral component. As in the case of human capital, I also assume there is an idiosyncratic shock, reflected in the random vector $\epsilon^{a,e} \in \mathbb{R}_+^J$, that affects how costly or rewarding the reallocation or migration across labor markets are for the worker in terms of assets. The matrix $\psi^{a,e}$, of size $J \times J$ and with strictly positive elements, reflects a common component to asset changes for switchers. In each market there is a *gross* real rental rate per unit of the asset, $r_{\ell,t}$, that represents a common component of the returns. Then, for the individual worker, the gross real return generated by the her assets in market ℓ is $r_{\ell,t}\,k\,\psi_{j\ell}^{a,e}\,\epsilon_\ell^{a,e}$.

It is important to highlight that individuals in the model face both idiosyncratic earnings risk and capital-income risk. In this way my model connects with an important number of

[6] Note that $\tau_{j\ell}^{a,e}$ may be larger than one in the model.

[7] In each region, nominal variables are deflated by the cost of the final consumption basket to express them in real terms.

works that develop incomplete markets models with idiosyncratic capital-income risk.[8] Empirically, fluctuations in capital-income over time for the same individual and large differences in the level of capital income across individuals are a prevalent feature of the data. Moreover, while some of the risks related to the return on assets in the real world can be diversified or hedged, individuals tend to have a poorly diversified portfolio, and a very large share of the capital stock in the United States is in the form of private businesses and private equity, and housing. In this way, my assumptions on idiosyncratic stochastic earnings and capital-income connect to a recent literature and find support in the data. As I discuss later, it is possible to extend the model to include different types of assets, with different levels of risk, volatility and frictions.

For tractability, I now make the following assumption on the distribution of random variables $\epsilon_\ell^{a,e}$ and $\tilde{\epsilon}_\ell^{a,e}$.

Assumption 1: Random variables $\epsilon_\ell^{a,e}$ are distributed i.i.d. Frechet with scale parameter $\lambda_\ell^{a,e}$ and shape parameter α. Moreover, I assume that $\tilde{\epsilon}_\ell^{a,e} = \epsilon_\ell^{a,e}\,\nu$, where ν is a *scalar* random variable independently distributed log-normal across time and across individuals.

In this way, shocks affecting the evolution of human capital and assets are correlated, but not perfectly due to the effects of variable ν.[9]

After making their labor market choice and obtaining their earnings and returns to assets, workers use available gross income to consume and invest in human capital and assets for the next period. Thus, the budget constraint, in real terms, for a worker that chose labor market ℓ is,

$$c_{j\ell,t}^{a,e} + h_{j\ell,t+1}^{a+1,e} + k_{j\ell,t+1}^{a+1,e} = w_{\ell,t}\,\nu\,\tau_{j\ell}^{a,e}\,\epsilon_\ell^{a,e}\,h_t + r_{\ell,t}\,\psi_{j\ell}^{a,e}\,\epsilon_\ell^{a,e}\,k_t. \tag{3}$$

where $h_{j\ell,t+1}^{a+1,e}$ and $k_{j\ell,t}^{a+1,e}$ are the levels of human capital and assets of the worker at the beginning of next period, but optimally chosen in the current period, and $c_{j\ell,t}^{a,e}$ is the consumption of a basket of final goods from all industries in that region.[10] Note that I am treating the

[8]See, for example, Angeletos (2007), Moll (2014) and Guvenen et al. (2022). In addition, Benhabib et al. (2011) develops a tractable incomplete markets model with both stochastic labor and capital income processes.

[9]Since shocks $\epsilon_\ell^{a,e}\,\nu$ are i.i.d., I omit the time subscript.

[10]Note that I write the budget constraint using gross expenditures in real terms, where the transactions in market ℓ are in units of the final consumption aggregate of the region market ℓ is part of, and investment in human capital or assets are in units of that final good. Alternatively, we can write it using net income and outlays it in the following way,

$$\begin{aligned}
c_{j\ell,t}^{a,e} + i_{j\ell,t}^{h,a+1,e} + i_{j\ell,t}^{k,a+1,e} &= (\tilde{w}_{\ell,t} + \tilde{\nu} + \tilde{\epsilon}_\ell^{a,e})\,h_t + (\tilde{r}_{\ell,t} + \tilde{\epsilon}_\ell^{a,e})\;k_t; \\
h_{j\ell,t+1}^{a+1,e} &= \left(1 + \tilde{\tau}_{j\ell}^{a,e}\right)\,h_t + i_{j\ell,t}^{h,a+1,e}, \\
k_{j\ell,t+1}^{a+1,e} &= \left(1 + \tilde{\psi}_{j\ell}^{a,e}\right)\,k_t + i_{j\ell,t}^{k,a+1,e},
\end{aligned}$$

investment in human capital in a similar fashion as the investment in assets. In this way I am following a large literature on human capital models in the tradition of Ben-Porath (1967), in which human capital is an asset that workers invest in, contributing to an expanding stock of knowledge and skills useful in the labor market. However, the human capital investment function I assume here is a restricted version of that in Ben-Porath (1967), as I am assuming that workers invest only resources to accumulate human capital, while in the seminal paper workers also invest time to increase human capital.

Entering cohort. At the beginning of each period, a new generation of individuals enter the economy with $a = 0$. They arrive with some level of human capital and assets, and face a labor market choice similar to that of other workers in the economy, except that new individuals are not attached to any past labor market. In particular, new individuals observe a vector of labor market opportunities $\epsilon^{0,e}$ and ν^0 and decide in which labor market to participate, and their consumption/savings decision. I assume that $\epsilon^{0,e}$ and ν^0 are distributed i.i.d. Frechet and log-normal, respectively, but the parameters of the distribution for the entering cohort may differ from older workers.

For the entering cohort the budget constraint is,

$$c_{\ell,t}^{0,e} + h_{\ell,t+1}^{1,e} + k_{\ell,t+1}^{1,e} = w_{\ell,t}\, \nu^0\, \epsilon_{\ell}^{0,e}\, h + r_{\ell,t}\, \epsilon_{\ell}^{0,e}\, k_t. \tag{4}$$

Finally, I assume that individuals aggregate final consumption goods from all industries in their region using a Cobb-Douglas aggregator with parameters ζ_i.

3.1 Recursive problem of the worker

I set up the problem of the worker recursively in individual state variables and characterize the optimal choices under some additional assumptions.[11]

Denote by $V_{j,t}^{a,e}(h_t, k_t, \epsilon, \nu)$ the expected life-time discounted utility of the worker of type e with age a current labor market j, a level of human capital h_t, savings k_t and with a vector of labor market opportunities ϵ. The Bellman Equation (BE) that defines this value function

where variables with a tilde are the log of the respective variable with no tilde and $i_{j\ell,t}^{h,a+1,e}$ and $i_{j\ell,t}^{k,a+1,e}$ are investments (in terms of the final consumption aggregate of a region) in human capital and assets, respectively. The first term in parenthesis on the right is the rental rate of human capital and the second parenthesis is the rental rate of assets, both inclusive of the idiosyncratic shocks. After proper substitution and rearrangement we have,

$$c_{j\ell,t}^{a,e} + h_{j\ell,t+1}^{a+1,e} + k_{j\ell,t+1}^{a+1,e} = \exp\left[\log\left(1 + \tilde{w}_{\ell,t} + \tilde{\tau}_{j\ell}^{a,e} + \tilde{\nu} + \tilde{\epsilon}_{\ell}^{a,e}\right)\right] h_t + \exp\left[\log\left(1 + \tilde{r}_{\ell,t} + \tilde{\psi}_{j\ell}^{a,e} + \tilde{\epsilon}_{\ell}^{a,e}\right)\right] k_t.$$

[11] Thus, I keep using the subindex t as this captures changes in aggregate economic conditions.

can be written as,

$$V_t^{a,e}\left(h_t,k_t,\epsilon^{a,e},\nu^a\right)=\begin{cases}\max\limits_{\ell,c_{\ell,t}^{0,e},h_{\ell,t+1}^{1,e},k_{\ell,t+1}^{1,e}}\left\{\log(\chi_{\ell,t}^{0,e})+\log\left(c_{\ell,t}^{0,e}\right)+\beta\,E\left[V_{\ell,t+1}^{1,e}\left(h_{\ell,t}^{1,e},k_{\ell,t}^{1,e},\epsilon^{1,e},\nu'\right)\right]\right\}, & \text{if } a=0;\\ \max\limits_{\ell,c_{j\ell,t}^{a,e},h_{j\ell,t+1}^{a+1,e},k_{j\ell,t+1}^{a+1,e}}\left\{\log(\chi_{j\ell}^{a,e})+\log\left(c_{j\ell,t}^{a,e}\right)+\beta\,E\left[V_{\ell,t+1}^{a+1,e}\left(h_{j\ell,t+1}^{a+1,e},k_{j\ell,t+1}^{a+1,e},\epsilon^{a+1,e},\nu'\right)\right]\right\}, & \text{if } 1\le a<A;\\ \frac{1}{1-\beta}\log\left(\varsigma_R\left(w_{j,t}\,\nu\,h_t+r_{j,t}\,k_t\right)\right) & \text{if } a=A.\end{cases} \tag{5}$$

Subject to,

$$c_{j\ell,t}^{a,e}+h_{j\ell,t+1}^{a+1,e}+k_{j\ell,t+1}^{a+1,e} = w_{\ell,t}\,\tau_{j\ell}^{a,e}\,\nu\,\epsilon_\ell^{a,e}\,h_t+r_{\ell,t}\,\psi_{j\ell}^{a,e}\,\epsilon_\ell^{a,e}\,k_t, \qquad \text{for } 1\le a<A, \tag{6}$$

$$c_{\ell,t}^{0,e}+h_{\ell,t+1}^{1,e}+k_{\ell,t+1}^{1,e} = w_{\ell,t}\,\nu^0\,\epsilon_\ell^{0,e}\,h_t+r_{\ell,t}\,\epsilon_\ell^{0,e}\,k_t, \qquad \text{for } a=0, \tag{7}$$

where $E[\cdot]$ is the expectation over the next period's idiosyncratic shocks, ϵ^{a+1}, ν'. $\chi_{j\ell}^{a,e}$ are non-pecuniary costs of switching labor markets in terms of utility, as usually assumed in the literature. In retirement, workers make no choices and simply receive a final utility that depends on their level of human capital, assets and the shock ν at the time of retirement. This particular functional form can be derived from a model where workers optimally consume a constant fraction of their wealth every period under log-preferences. For tractability, I am assuming that utility in retirement is multiplied by the inverse of $(1-\beta)$, as this simplifies the expressions.[12]

Equation (6) is the budget constraint of the worker, with consumption and gross investment on the left hand side and sources of income on the right. As discussed before, human capital accumulation is affected by labor market reallocation decisions and the labor market opportunity shocks through $\tau_{j\ell}^{a,e}\epsilon_\ell$. Similarly for assets.

Note the timing of the model. The worker starts the period and observes the realization of idiosyncratic shocks and the aggregate state and decides where to relocate. The human capital and assets the worker is able to rent in markets are net of any reallocation costs or depreciation. At the end of the period, the worker decides the amount of consumption and the investment in human capital and assets.

To characterize this Bellman Equation and find a tractable solution to the problem I take a few steps. I follow Krebs (2003, 2006) and write the problem in terms of effective wealth, that is, the sum of human and financial, as in Angeletos (2007). This way of setting up the problem relates closely to the optimal portfolio choice problem in Merton (1969); Samuelson (1969).[13] Define effective wealth as $W_t = h_t + k_t$, and let $\theta_t = h_t/W_t$ be the share of effective

[12] This can be interpreted not only as consumption in retirement but also as some utility from bequest. Parameter ς_R can be used to adjust the importance of this assumption.

[13] See also Toda (2014) for a recent generalization of the results.

wealth that is human capital, then we can write the optimization problem as,

$$V_{j,t}^{a,e}\left(W,\theta,\epsilon^{a,e},\nu\right)=\begin{cases}\max_{\ell,c_{\ell,t}^{0,e},W_{\ell,t+1}^{1,e},\theta_{\ell,t+1}^{1,e}}\left\{\log(\chi_{\ell}^{0,e})+\log(c_{\ell,t}^{0,e})+\beta\,E\left[V_{\ell,t+1}^{1,e}\left(W_{\ell,t+1}^{1,e},\theta_{\ell,t+1}^{1,e},\epsilon^{1,e},\nu'\right)\right]\right\}, & \text{if } a=0;\\ \max_{\ell,c_{j\ell,t}^{a,e},W_{j\ell,t+1}^{a+1,e},\theta_{j\ell,t+1}^{a+1,e}}\left\{\log(\chi_{j\ell}^{a,e})+\log(c_{j\ell,t}^{a,e})+\beta\,E\left[V_{\ell,t+1}^{a+1,e}\left(W_{j\ell,t+1}^{a+1,e},\theta_{j\ell,t+1}^{a+1,e},\epsilon^{a+1,e},\nu'\right)\right]\right\}, & \text{if } 1\leq a<A;\\ \frac{1}{1-\beta}\log\left(\varsigma_R\left(\omega_j^{A,e}(\theta,\nu)\right)W\right) & \text{if } a=A.\end{cases}\tag{8}$$

Subject to,

$$\begin{aligned}c_{j\ell,t}^{a,e}+W_{j\ell,t+1}^{a+1,e} &= \omega_{j\ell,t}^{a,e}(\theta,\nu)\,\epsilon_{\ell}^{a,e}\,W, && \text{for } 1\leq a<A,\\ c_{\ell,t}^{0,e}+W_{\ell,t+1}^{1,e} &= \omega_{\ell,t}^{0,e}(\theta,\nu^0)\,\epsilon_{\ell}^{0,e}\,W, && \text{for } a=0,\end{aligned}$$

where $\omega_{j\ell,t}^{a,e}(\theta,\nu)=\left[w_{\ell,t}\,\tau_{j\ell}^{a,e}\,\nu\,\theta+r_{\ell,t}\,\psi_{j\ell}^{a,e}\,(1-\theta)\right]$, for $1\leq a<A$ and a similar expression for retirement and entering cohorts, with the exception of the frictions, $\omega_{j,t}^{A,e}(\theta,\nu)=w_{j,t}\,\nu\,\theta+r_{j,t}\,(1-\theta)$, and $\omega_{\ell,t}^{0,e}(\theta,\nu)=w_{\ell,t}\,\nu^0\,\theta+r_{\ell,t}\,(1-\theta)$.

In this way, the problem boils-down to an optimal portfolio decision problem with CRRA utility, where the portfolio decision is how much to invest in human capital and how much in assets. Different from other settings in the spirit of Merton (1969) and Krebs (2003), there is a discrete choice decision each period and the return on human capital and assets and the optimal portfolio allocation will be affected by this.

The necessary conditions that characterize an optimal recursive decision on future effective wealth and portfolio shares, conditional on the choice of labor market ℓ, are,

$$c_{j\ell,t}^{a,e}(W,\theta,\epsilon,\nu)^{-1}=\beta\frac{\partial E[V_{\ell,t+1}^{a+1,e}\left(W_{j\ell,t+1}^{a+1,e},\theta_{j\ell,t+1}^{a+1,e},\epsilon',\nu'\right)]}{\partial W_{j\ell,t+1}^{a+1,e}}\quad\text{if } 1\leq a<A,\forall\,j\ell;\tag{9}$$

$$\frac{\partial E[V_{\ell,t+1}^{a+1,e}\left(W_{j\ell,t+1}^{a+1,e},\theta_{j\ell,t+1}^{a+1,e},\epsilon',\nu'\right)]}{\partial\theta_{j\ell,t+1}^{a+1,e}}=0\quad\text{if } 1\leq a<A,\forall\,j\ell;\tag{10}$$

and similar conditions for the entering cohort. Condition (9) is the derivative of problem (8) with respect to end of the period effective wealth $W_{j\ell,t+1}^{a+1,e}$, equated to zero, and (10) is the derivative with respect to next period human capital share $\theta_{j\ell,t+1}^{a+1,e}$ equated to zero, both chosen in period t and conditional on the selected labor market ℓ.

3.2 Optimal decisions

The following proposition characterizes the optimal policies of the individual problem.

Proposition 1 ***-Optimal consumption, portfolio and reallocation choices:*** *The following policies on consumption, total next period effective wealth and human capital share, satisfy the necessary conditions for the problem of the worker for $a < A$,*

$$
\begin{aligned}
c^{a,e}_{j\ell,t}\left(W,\theta,\epsilon,\nu\right) &= (1-\beta)\,\omega^{a,e}_{j\ell,t}(\theta,\nu)\,\epsilon^{a,e}_{\ell}W, \\
W^{a+1,e}_{j\ell,t+1}\left(W,\theta,\epsilon,\nu\right) &= \beta\,\omega^{a,e}_{j\ell,t}(\theta,\nu)\,\epsilon^{a,e}_{\ell}W, \\
\theta^{a+1,e}_{j\ell,t+1}\left(W,\theta,\epsilon,\nu\right) &= \theta^{a+1,e}_{\ell,t+1},
\end{aligned}
$$

where $\theta^{a+1,e}_{\ell,t+1}$ depends only on worker's characteristics (age, type), the chosen labor market ℓ, and aggregate economic conditions.

Moreover, the value function is log-additive in effective wealth, W, and the ex-ante value function, $E_\epsilon\left[V^{a,e}_{j,t}\left(\theta,\nu,\epsilon,W\right)\right] = v^{a,e}_{j,t}\left(\theta,\nu\right)+\frac{1}{1-\beta}\log(W)$, for $1 \le a \le A-1$ can be characterized recursively as,

$$
v^{a,e}_{j,t}\left(\theta,\nu\right) = \mathbb{C} + \frac{1}{\alpha(1-\beta)}\log\left[\sum_{\ell=1}^{J}\exp\left(\alpha(1-\beta)\left[\log(\chi^{a,e}_{j\ell}) + \frac{1}{1-\beta}\log\left(\omega^{a,e}_{j\ell,t}(\theta,\nu)\right) + \beta\,E\left[v^{a+1,e}_{\ell,t+1}(\theta^{a+1,e}_{\ell,t+1},\nu')\right]\right] + \alpha\log(\lambda^{a,e}_{\ell})\right)\right],
$$

and for $a = 0$,

$$
v^{0,e}_{t}\left(\theta_t,\nu,\right) = \mathbb{C} + \frac{1}{\alpha^0(1-\beta)}\log\left[\sum_{\ell=1}^{J}\exp\left(\alpha^0(1-\beta)\left[\log(\chi^{0,e}_{\ell}) + \frac{1}{1-\beta}\log\left(\omega^{0,e}_{\ell,t}(\theta,\nu)\right) + \beta\,E\left[v^{1,e}_{\ell,t+1}(\theta^{1,e}_{\ell,t+1},\nu')\right]\right] + \alpha^0\log(\lambda^{0,e}_{\ell})\right)\right],
$$

where $\mathbb{C} = \frac{\bar{\gamma}}{\alpha(1-\beta)}+\log\left(1-\beta\right)+\frac{\beta}{1-\beta}\log(\beta)$, $\bar{\gamma}$ is Euler's constant, and $v^{A,e}_{j,t}\left(\theta,\nu\right) = \frac{1}{1-\beta}\log\left(\varsigma_R\,\omega^{A,e}_{j\ell,t}(\theta,\nu)\right)$.

In addition, the share of individuals that choose labor market ℓ for age $a = 0$,

$$
\mu^{0,e}_{\ell,t}\left(\theta,\nu\right) = \frac{\exp\left(\alpha^0(1-\beta)\left[\log(\chi^{0,e}_{\ell}) + \frac{1}{1-\beta}\log\left(\omega^{0,e}_{\ell}(\theta,\nu)\right) + \beta\,E\left[v^{1,e}_{\ell,t}(\theta^{1,e}_{\ell}(\Upsilon),\nu')\right]\right] + \alpha^0\log(\lambda^{0,e}_{\ell})\right)}{\sum_{m=1}^{J}\exp\left(\alpha^0(1-\beta)\left[\log(\chi^{0,e}_{m}) + \frac{1}{1-\beta}\log\left(\omega^{0,e}_{m}(\theta,\nu)\right) + \beta\,E\left[v^{1,e}_{m,t}(\theta^{1,e}_{\ell}(\Upsilon),\nu')\right]\right] + \alpha^0\log(\lambda^{0,e}_{m})\right)}
$$

and for age $1 \le a \le A-1$ are,

$$
\mu^{a,e}_{j\ell,t}\left(\theta,\nu\right) = \frac{\exp\left(\alpha(1-\beta)\left[\log(\chi^{a,e}_{j\ell}) + \frac{1}{1-\beta}\log\left(\omega^{a,e}_{j\ell}(\theta,\nu)\right) + \beta\,E\left[v^{a+1,e}_{\ell,t}(\theta^{a+1,e}_{\ell}(\Upsilon),\nu')\right]\right] + \alpha\log(\lambda^{a,e}_{\ell})\right)}{\sum_{m=1}^{J}\exp\left(\alpha(1-\beta)\left[\log(\chi^{a,e}_{jm}) + \frac{1}{1-\beta}\log\left(\omega^{a,e}_{jm}(\theta,\nu)\right) + \beta\,E\left[v^{a+1,e}_{m,t}(\theta^{a+1,e}_{\ell}(\Upsilon),\nu')\right]\right] + \alpha\log(\lambda^{a,e}_{m})\right)}.
$$

And the optimal human capital share satisfies the following conditions,

$$E_{\nu'}\left[\frac{w_{\ell,t+1}\,\nu - r_{\ell,t+1}}{w_{\ell,t+1}\,\nu'\,\theta^{A,e}_{\ell,t+1} + r_{\ell,t+1}\,(1-\theta^{A,e}_{\ell,t+1})}\right] = 0, \qquad \textit{for } a = A, \tag{11}$$

$$E_{\nu'}\left[\sum_{m=1}^{J}\mu^{a,e}_{\ell m,t+1}\left(\theta^{a,e}_{\ell,t+1},\nu'\right)\frac{w_{m,t+1}\,\tau^{a,e}_{\ell m}\,\nu' - r_{m,t+1}\,\psi^{a,e}_{\ell m}}{w_{m,t+1}\,\tau^{a,e}_{\ell m}\,\nu'\,\theta^{a,e}_{\ell,t+1} + r_{m,t+1}\,\psi^{a,e}_{\ell m}\,(1-\theta^{a,e}_{\ell,t+1})}\right] = 0, \qquad \textit{for } 1 \le a \le A-1 \tag{12}$$

The first step in the proof, fully derived in the Appendix, is to compute the expression for the ex-ante value function and its derivatives. I show that policy functions for consumption and effective wealth accumulation are homogeneous of degree one in the initial wealth of the period and on shocks ϵ, and that the Bellman equation is log-additive in effective wealth. Intuitively, this says that the lifetime utility and consumption/investment decision of individuals with the same initial state variables, except effective wealth W, are proportional to each other. In addition, since this is a finite horizon dynamic programing problem, the ex-ante value function (expectation with respect to the ϵ shcoks) always exists, is finite and unique, and is differentiable with respect to W and θ.

Moreover, equation (12) shows that the policy function for the share of human capital does not depend on last period labor market j, the effective wealth W at the beginning of the period (due to the homogeneity of the problem in effective wealth) or the labor market opportunity shocks ϵ. Moreover, since the shocks ν' are independently distributed, we have that the optimal decision on the share of human capital depends only on the aggregate state and the chosen labor market ℓ, i.e. $\theta^{a+1,e}_{j\ell,t+1}(W,\theta,\epsilon,\nu) = \theta^{a+1,e}_{\ell,t+1}$.

Note that the optimality condition that characterize the optimal share of human capital for workers in this economy is different from that in Krebs (2003). While equation (11) is similar to that in Krebs (2003), here there are many potential portfolio choices, one for each labor market the worker chooses at age $A-1$. The main reason for the similarity is that I assume that the worker cannot select a different labor market at age A. On the other hand, equation (12) differs from Krebs (2003) since the optimal choice for the share of human capital for period one takes into account that the worker may subsequently move to a different labor market and future reallocation affects the returns to human capital and assets. Clearly, if $J=1$, $\mu=1$ and the expressions are similar to Krebs (2003).

Simplification of equilibrium conditions.- I now obtain simpler expressions for equilibrium conditions using $\log(1+x)\approx x$, or $1+x\approx e^{x}$ for x small. Let a variable with a tilde be related to the original variable in the following way, $x=e^{\tilde{x}}$, or $\tilde{x}=\log(x)$, then we can write,

$$\omega^{a,e}_{j\ell,t}(\theta,\nu) \;\approx\; (1+\tilde{w}_{\ell,t}+\tilde{\nu}+\tilde{\tau}^{a,e}_{j\ell})\theta+(1+\tilde{r}_{\ell,t}+\tilde{\psi}^{a,e}_{j\ell})(1-\theta),$$

and

$$w_{m,t+1}\,\tau^{a,e}_{\ell m}\,\nu'-r_{m,t+1}\,\psi^{a,e}_{\ell m} \;\approx\; \tilde{w}_{m,t+1}+\tilde{\tau}^{a,e}_{\ell m}+\tilde{\nu}'-(\tilde{r}_{m,t+1}+\tilde{\psi}^{a,e}_{\ell m}).$$

It is important to highlight that these approximations do not imply a linearization or a Taylor expansion of equilibrium conditions around some particular point. In fact, if the model was written in continuous time, the previous expressions would be exact. I prefer to use discrete time as most works in the literature proceed in this way.

Then, the following Lemma characterizes simpler equilibrium conditions,

Lemma 1 *Using* $\log(1+x)\approx x$*, optimal conditions for the individual problem can be expressed as,*

$$v^{a,e}_{j,t}(\theta,\tilde{\nu}) \;\approx\; \mathbb{C}+\log\left(\sum_{\ell=1}^{J}\exp\left(\alpha\left[(1-\beta)\tilde{\chi}^{a,e}_{j\ell}+(\tilde{w}_{\ell,t}+\tilde{\tau}^{a,e}_{j\ell})\theta+(\tilde{r}_{\ell,t}+\tilde{\psi}^{a,e}_{j\ell})(1-\theta)\right]+\right.\right.$$
$$\left.\left.+\alpha\tilde{\lambda}^{a,e}_{\ell}+\beta E_{\nu'}\left[v^{a+1,e}_{\ell,t+1}(\theta^{a+1,e}_{\ell,t+1},\tilde{\nu}')\right]\right)\right)+\tilde{\nu}\,\theta.$$

$$\mu^{a,e}_{j\ell,t}(\theta)\approx\frac{\exp\left(\alpha\left[(1-\beta)\tilde{\chi}^{a,e}_{j\ell}+(\tilde{w}_{\ell,t}+\tilde{\tau}^{a,e}_{j\ell})\theta+(\tilde{r}_{\ell,t}+\tilde{\psi}^{a,e}_{j\ell})(1-\theta)\right]+\alpha\tilde{\lambda}^{a,e}_{\ell}+\beta E_{\tilde{\nu}'}\left[v^{a+1,e}_{\ell,t+1}(\theta^{a+1,e}_{\ell,t+1},\nu')\right]\right)}{\sum_{n=1}^{J}\exp\left(\alpha\left[(1-\beta)\tilde{\chi}^{a,e}_{jn}+(\tilde{w}_{n,t}+\tilde{\tau}^{a,e}_{jn})\theta+(\tilde{r}_{n,t}+\tilde{\psi}^{a,e}_{jn})(1-\theta)\right]+\alpha\tilde{\lambda}^{a,e}_{n}+\beta E_{\nu'}\left[v^{a+1,e}_{n,+1}(\theta^{a+1,e}_{n},\nu')\right]\right)}.$$

$$\theta^{A,e}_{m,t+1}\approx\frac{\tilde{w}_{m,t+1}-\tilde{r}_{m,t+1}}{\sigma^2_{\nu}},$$

$$\sum_{m=1}^{J}\mu^{a+1,e}_{\ell m,t+1}(\theta^{a+1,e}_{\ell,t+1})\frac{\left[\tilde{w}_{m,t+1}+\tilde{\tau}^{a+1,e}_{\ell m}-\sigma^2_{\nu}\,\theta^{a+1,e}_{\ell,t+1}-\tilde{r}_{m,t+1}-\tilde{\psi}^{a+1,e}_{\ell m}\right]}{1+\tilde{r}_{m,t+1}+\tilde{\psi}^{a+1,e}_{\ell m}+(\tilde{w}_{m,t+1}+\tilde{\tau}^{a+1,e}_{\ell m}-\tilde{r}_{m,t+1}-\tilde{\psi}^{a+1,e}_{\ell m})\theta^{a+1,e}_{\ell,t+1}-\frac{\sigma^2_{\nu}}{2}(\theta^{a+1,e}_{\ell,t+1})^2}=0$$

where $\tilde{\nu}=\log(\nu)$ *is i.i.d normal with zero mean and variance* σ^2_{ν}*.*

Note that under perfect foresight of wages and rental rates, we obtain a closed-form solution for the optimal human capital share, which is the result by Merton (1969) for the case of log-utility.

Note also that the mobility matrix μ does not depend on the shock ν. The intuition is that the earnings shock ν is a common component of returns across all choices, and thus does not influence labor market choices. In addition, the state variable θ will be identical across all individuals in j with past labor market j and with the same age and type. This implies that the probability of making a transition from j to ℓ is the same for all individuals in j with the same age and type. This property is useful to connect the model with the data, as the model-implied mobility rates do not vary with the realization of (potentially unobserved) idiosyncratic shocks, only by origin-destination and demographic characteristics.

Moreover, we can further characterize the optimal portfolio choices by computing the expectation over future values of ν. In this way, finding the optimal human capital share, for each labor market, age and type, boils down to finding the zero of a (well-behaved) non-linear function of a single variable.

The results in Proposition 1 and 1 are important not only to characterize the solution to the individual problem, but also for aggregation across heterogeneous individuals. As mentioned before, conditional on a labor market choice, consumption and savings decision (effective wealth) are proportional to the level of wealth of individuals at the start of the period and their ϵ shock. In addition, conditional choosing labor market ℓ, all workers (of the same age and type) will invest the same fraction of their effective wealth in human capital and assets, as $\theta^{a+1,e}_{\ell,t+1}$ only depends on the chosen labor market. The following proposition shows how to characterize aggregate supply of labor and assets and the dynamics of effective wealth.

Dynamics of aggregate labor and assets supply. Let $\Lambda^{a,e}_j$ be the share of workers with characteristics a, e and past labor market j at the beginning a period, with $\sum_{j=1}^{J} \Lambda^{a,e}_j = 1$. At each period of time, all workers that chose labor market ℓ have the same share of effective wealth in the form of human capital, θ_j. Since $\mu_{j\ell}(\theta_j)$ denotes the fraction of workers with past labor market j that chose market ℓ in period t, we have that $\Lambda^{a+1,e}_\ell(\Upsilon) = \sum_{j=1}^{J} \mu_{j\ell}(\theta_j)\, \Lambda^{a,e}_j = 1$.

Proposition 2 -*Aggregate capital, labor supply and effective wealth dynamics:* *The law of motion of effective wealth across labor markets can be characterized recursively as,*

$$E[W^{a+1,e}_{\ell,t+1}] = \sum_{j=1}^{J} \frac{\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\, \Lambda^{a,e}_{j,t}}{\Lambda^{a+1,e}_{\ell,t+1}} \Bigg(\beta \left[\left(w_{\ell,t}\, e^{\sigma^2_\nu/2}\, \tau^{a,e}_{j\ell} \right) \theta^{a,e}_{j,t} + \left(r_{\ell,t}\, \psi^{a,e}_{j\ell} \right) (1-\theta^{a,e}_{j,t}) \right] \times \Gamma(1-1/\alpha)\, \lambda^{a,e}_\ell\, (\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha} E[W^{a,e}_{j,t}] \Bigg).$$

The total supply of labor (human capital) and assets by workers of characteristics a, e takes *the following expression,*

$$
\begin{aligned}
h^{a,e}_{\ell,t} &= \sum_{j=1}^{J} \mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}, \Upsilon)\, \Lambda^{a,e}_{j,t} \left(e^{\sigma^2_\nu/2}\, \tau^{a,e}_{j\ell}\, \Gamma(1-1/\alpha)\, \lambda^{a,e}_{\ell}\, (\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha}\, \theta^{a,e}_{j,t}\, E[W^{a,e}_{j,t}] \right), \\
k^{a,e}_{\ell,t} &= \sum_{j=1}^{J} \mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}, \Upsilon)\, \Lambda^{a,e}_{j,t} \left(\psi^{a,e}_{j\ell}\, \Gamma(1-1/\alpha)\, \lambda^{a,e}_{\ell}\, (\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha}\, (1-\theta^{a,e}_{j,t})\, E[W^{a,e}_{j,t}] \right).
\end{aligned}
$$

Finally, the total consumption of final goods by workers og age a and type e is,

$$
\begin{aligned}
c^{a,e}_{\ell,t} &= \sum_{j=1}^{J} \mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\, \Lambda^{a,e}_{j,t}\, (1-\beta) \Bigg(\left[\left(w_{\ell,t}\, e^{\sigma^2_\nu/2}\, \tau^{a,e}_{j\ell} \right) \theta^{a,e}_{j,t} + \left(r_{\ell,t}\, \psi^{a,e}_{j\ell} \right) (1-\theta^{a,e}_{j,t}) \right] \times \\
&\qquad \Gamma(1-1/\alpha)\, \lambda^{a,e}_{\ell}\, (\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha} E[W^{a,e}_{j,t}] \Bigg).
\end{aligned}
$$

The importance of Proposition 2 can be grasped by highlighting the heterogeneity in the problem. In this economy, agents are heterogeneous in the level of human capital and assets they bring from the previous period. In addition, are subject to idiosyncratic shocks affecting their labor supply and assets (or their returns). The impact of these shocks, depend on reallocation and migration decision of workers and their pattern of selection, and the total supply of human capital and assets in labor market ℓ depends on the aggregation across workers arriving from all previous labor markets j, across all these sources of heterogeneity and selection patterns. Proposition 2 tells us that in this model, it is possible to characterize the aggregate supply of labor and assets and their dynamics over time.

Total factor supply and consumption for the whole market ℓ can easily be obtained by aggregating age and type level expressions. For instance, let $\Psi^{a,e}$ be the mass of individuals with age a and type e in the population, such that $\sum_{a,e} \Psi^{a,e} = 1$, then total labor and capital supply in market ℓ are, $L_{\ell,t} = \sum_{a,e} \Psi^{a,e} h^{a,e}_{\ell,t}$ and $K_{\ell,t} = \sum_{a,e} \Psi^{a,e} k^{a,e}_{\ell,t}$, respectively, and total consumption of final goods by individuals is $C_{\ell,t} = \sum_{a,e} \Psi^{a,e} c^{a,e}_{\ell}$

Proposition 3 ***-Dynamic exact hat algebra:*** *If $\tau^{ae}_{j\ell} = \psi^{ae}_{j\ell}$ and initial mobility rates $\mu^{ae}_{j\ell,0}$, initial human capital transferability matrix $\mathcal{M}^{ae}$, initial stocks of aggregate human capital and assets, and initial wages and rental rates are observed, then we can express the equilibrium conditions that characterize the solution to this economy over time, in changes. Moreover, equilibrium conditions in changes do not require information on the level of parameters, or* *fundamentals,* $\tau_{j\ell}^{ae}$*,* $\psi_{j\ell}^{ae}$*, and* $\chi_{j\ell}^{ae}$*.*

3.3 Production and the demand for labor and capital

Production follows closely the open economy multisector model of Caliendo, Parro, Rossi-Hansberg, and Sarte (2018) and Caliendo, Dvorkin, and Parro (2019), but with a demand for capital as in Kleinman, Liu, and Redding (2023). Firms in industry i and region n are able to produce many varieties of intermediate goods. The technology to produce these intermediate goods requires efficient units of labor and capital, which are the primary factors of production, and materials, which consist of goods from all sectors. Total factor productivity (TFP) of an intermediate good is composed of two terms, a time-varying sectoral-regional component, A_t^{ni}, which is common to all varieties in ni, and a variety-specific component, z^{ni}.

Intermediate Goods Producers: The output for a producer of an intermediate variety with efficiency z^{ni} is given by,

$$q_t^{ni} = z^{ni}\left(A_t^{ni}\,(k_t^{ni})^{\xi^n}(l_t^{ni})^{1-\xi^n}\right)^{\gamma^{ni}}\prod_{k=1}^{J}(M_t^{ni,nk})^{\gamma^{ni,nk}},$$

where l_t^{ni}, k_t^{ni} are labor and capital inputs, respectively, and $M_t^{ni,nk}$are material inputs from industry k and region n demanded by a firm in industry i to produce q units of an intermediate variety with efficiency z^{ni}. Material inputs are final goods from region n and industry k. Parameter $\gamma^{ni} \geq 0$ is the share of value added in the production of sector i and region n, and $\gamma^{ni,nk} \geq 0$ is the share of materials from sector k in the production of sector i and region n. The production function exhibits constant returns to scale such that $\sum_{k=1}^{J}\gamma^{ni,nk} = 1-\gamma^{ni}$. The parameter ξ^n is the share of capital in value added. The nominal unit price of an input bundle is

$$x_t^{ni} = B^{ni}\,\left(P_t^n(\tilde{r}_{ni,t})^{\xi^n}\,(\tilde{w}_{ni,t})^{1-\xi^n}\right)^{\gamma^{ni}}\prod_{k=1}^{J}(P_t^{nk})^{\gamma^{ni,nk}}, \tag{13}$$

where B^{ni} is a constant and P_t^{ni} also applies to goods used as materials in production, as described below.[14] Then, the unit cost of an intermediate good z^{ni} at time t is $\frac{x_t^{ni}}{z^{ni}\,(A_t^{ni})^{\gamma^{nj}}}$.

Trade costs are represented by $\kappa_t^{ni,mi}$ and are of the iceberg type. One unit of any variety of intermediate good i shipped from region m to n requires producing $\kappa_t^{ni,mi} \geq 1$ units in region m. If a good is nontradable, then $\kappa = \infty$. Competition implies that the price paid for a particular variety of good i in region n is given by the minimum unit cost across regions, taking into account trade costs, and where the vector of productivity draws received by the

[14]Since I defined factor prices, $\tilde{r}_{ni,t}$ and $\tilde{w}_{ni,t}$, in real terms, I multiply by the cost of the final good basket, P_t^n, to express them in nominal terms.

different regions is $z^i = (z^{1i}, z^{2i}, \dots, z^{Ni})$. That is, using z^i to index varieties,

$$p_t^{ni}(z^i) = \min_m \left\{ \frac{\kappa_t^{ni,mi} \; x_t^{mi}}{z^{mi}(A_t^{mi})^{\gamma^{mi}}} \right\}.$$

Local Sectoral Aggregate Goods. Intermediate goods demanded from sector i and from all regions are aggregated into a local sectoral good denoted by Q and that can be thought as a bundle of goods purchased from different regions. In particular, let Q_t^{ni} be the quantity produced of aggregate sectoral goods i in region n and $\tilde{q}_t^{ni}(z^i)$ be the quantity demanded of an intermediate good of a given variety from the lowest-cost supplier. The production of local sectoral goods is given by

$$Q_t^{ni} = \left(\int (\tilde{q}_t^{ni}(z^i))^{1-1/\eta^{ni}} \; d\phi^i(z^i) \right)^{\eta^{ni}/(\eta^{ni}-1)},$$

where $\phi^j(z^i) = \exp\left\{ -\sum_{n=1}^N (z^{ni})^{-\vartheta^i} \right\}$ is the joint distribution over the vector z^i, with marginal distribution given by $\phi^{ni}(z^{ni}) = \exp\left\{ -(z^{ni})^{-\vartheta^i} \right\}$, and the integral is over R_+^N. For nontradable sectors, the only relevant distribution is $\phi^{ni}(z^{ni})$ since sectoral good producers use only local intermediate goods. There are no fixed costs or barriers to entry and exit in the production of intermediate and sectoral goods. Competitive behavior implies zero profits at all times.

Local sectoral aggregate goods are used as materials for the production of intermediate varieties as well as for final consumption. Note that the fact that local sectoral aggregate goods are not traded does not imply that consumers are not purchasing traded goods. On the contrary, both intermediate goods producers and households, via the direct purchase of the local sectoral aggregate good, purchase tradable varieties.

Given the properties of the Fréchet distribution, the price of the sectoral aggregate good i in region n at time t is

$$P_t^{ni} = \Gamma^{ni} \left(\sum_{m=1}^N (x_t^{mi} \kappa_t^{ni,mi})^{-\vartheta^i} (A_t^{mi})^{\vartheta^i \gamma^{mi}} \right)^{-1/\vartheta^j}, \quad (14)$$

where Γ^{ni} is a constant.[15] To obtain (14), we assume that $1 + \vartheta^i > \eta^{ni}$. Following similar steps as earlier, we can solve for the share of total expenditure in market (n, i) on goods i from

[15] In particular, the constant Γ^{ni} is the Gamma function evaluated at $1 + \left(1 - \eta^{ni}/\vartheta^i\right)$.

market m. In particular,

$$\pi_t^{ni,mi} = \frac{(x_t^{mi}\kappa_t^{ni,mi})^{-\vartheta^i}(A_t^{mi})^{\vartheta^i\gamma^{mi}}}{\sum_{m=1}^N (x_t^{mj}\kappa_t^{ni,mj})^{-\vartheta^i}(A_t^{mj})^{\vartheta^i\gamma^{mj}}}. \tag{15}$$

This equilibrium condition reflects that the more productive market mi is, given factor costs, the cheaper the cost of production is in market mi and, therefore, the more region n purchases sector j goods from region i. In addition, the easier it is to ship sector j goods from region i to n (lower $\kappa^{ni,mi}$), the more region n purchases sector j goods from region i. This equilibrium condition resembles a gravity equation.

The price of the final good basket in region n is, $P_t^n = \prod_{i=1}^I \left(\frac{P_t^{ni}}{\zeta^i}\right)^{\zeta^i}$.

3.4 General equilibrium

Let X_t^{ni} be the total expenditure on sector i good in region n. Then, goods market clearing implies

$$X_t^{ni} = \sum_{k=1}^J \gamma^{nk,ni} \sum_{m=1}^N \pi_t^{mk,nk} X_t^{mk} + \zeta^i C_{n,t}, \tag{16}$$

where the first term on the right-hand side is the value of the total demand for sector i goods produced in n used as materials in all sectors and regions in the economy, and $\zeta^i C_{n,t}$ is the value of the final demand of good i in region n, with $C_{n,t} = \sum_{i=1}^I C_{ni,t}$.

Labor market clearing in region n and sector i is,

$$L_t^{ni} = \frac{\gamma^{ni}(1-\xi^n)}{P_t^n \tilde{w}_t^{ni}} \sum_{m=1}^N \pi_t^{mi,ni} X_t^{mi}, \tag{17}$$

while the market clearing for capital in region n and sector i must satisfy,

$$K_t^{ni} = \frac{\gamma^{ni}\xi^n}{P_t^n \tilde{r}_t^{ni}} \sum_{m=1}^N \pi_t^{mi,ni} X_t^{mi}. \tag{18}$$

Definition 1 *General equilibrium.- Equilibrium in this economy is defines as a sequence of prices for goods, P_t^n, P_t^{ni}, and $p_t^{ni}(z^i)$, and rental rates of factors of production, $\tilde{r}_{ni,t}$ and $\tilde{w}_{ni,t}$, sequences of quantities of goods produced, $q_t^{ni}(z^{ni})$ and Q_t^{ni}, sequences of aggregate consumption, labor, and assets, $C_{\ell,t}$, $L_{\ell,t}$, and $K_{\ell,t}$, sequences of optimal individual decision rules and value functions that depend on time and individual state variables, $c_{\ell,t}^{a,e}$, $h_{\ell,t}^{a,e}$, $k_{\ell,t}^{a,e}$, $W_{j\ell,t}^{a,e}$, $\mu_{j\ell,t}^{a,e}$, $\theta_{\ell,t}^{a,e}$, $a, e_{j,t}$, for all ages a and types e, and a sequence of distributions of effective wealth across labor markets ℓ, $F_{W,t}^{a,e}$, such that,*

- *Given prices, individuals optimally chose consumption, investment in human capital and assets, and migration/reallocation and satisfy the conditions in Lemma 1.*
- *Aggregate consumption, supply of human capital and assets is the result of aggregating individual supply according to $F^{a,e}_{W,t}$ together with the measure of individuals by age and type.*
- *Given prices, firms optimally chose labor, capital and materials demand to produce individual goods varieties and aggregate varieties by industry and region.*
- *Goods and factor markets clear.*
- *The sequence of distributions of effective wealth over time and across industries and regions is consistent with individual decisions.*

4 Quantitative Analysis

In this section I conduct a simple exercises to understand the mechanism driving mobility, earnings and wealth changes in the model after a shock. The next subsection abstracts from general equilibrium and studies the effects of an exogenous change. A full quantitative exercise is currently underway and will be updated shortly.

4.1 Simple example

Take an economy with three industries such that $J = 3$. These industries are fully symmetric before the shock hits. I calibrate a period to be three years. Individuals are identical in all respects except their age. A=10 such that individuals have a worklife of 30 years. I make $\lambda = 0.95$ and $\alpha = 20$ such that earnings and assets have an upward trajectory over the life of individuals and the variance of (log) earnings and assets resembles that of the U.S. economy. Frictions are such that individuals that the elements outside of the diagonal of matrices τ and ψ are 0.86 and 0.87 respectively. In this way industry switchers see their stock of human capital and assets fall by approximately 15%, all else equal (slightly higher for assets). This implies that industry mobility is around 15% over three years. The earnings shock nu has a variance of 0.1. Finaly, the initial returns on human capital and assets, $\tilde{w}$ and $\tilde{r}$ are 10% and 7%, respectively, in all industries.[16] In this way, initial human capital share of effective wealth θ is 0.3. Due to symmetry, employment is evenly distributed across all industries.

[16]Thus, on average the wage is 1.1.

In period 1 the economy is hit by a shock that lowers the return on human capital and assets in industry one. In particular, the shock is a decrease of 100 basis points in the return of human capital and assets in that industry. I solve the worker's optimization problem given these fixed set of returns and simulate employment histories for a large number of individuals. For each of these simulated agents, I compute accumulated earnings since the shock (returns to human capital) and percent change in assets since initial period to connect with the analysis of Section 2. Then, I run the following regression for different periods since the shock,

$$Y_{ij} = \beta_0 + \beta_1 \, dummy_{i1} + e_{ij},$$

where $Y_{ij\tau}$ is either the accumulated earnings of individual i since period zero and relative to that initial period, or the percent change in the stock of assets since period zero. The variable dummy is equal to one if the individual was employed in industry one at the time of the shock.

Figure 2 shows the estimates of coefficient β_1 for different periods since the shock. The estimates show the differential effects on earnings and assets of workers exposed to the shock relative to those that were not exposed. As the figure shows, the exposure to the shock has a (relative) negative effect on the evolution of earnings and assets.

Figure 2: Estimated coefficients on cumulative earnings and assets regressions

The effects on earnings and assets on exposed workers are the result of different forces. On the one hand, for workers that stay in industry one, the effects of the shock are small initially since returns on human capital and assets change modestly. However, over time, the differentially lower rates of return in industry one imply a lower growth rate of effective wealth for workers that stay in that industry, and the effects on the stocks increases over time, leading to lower levels of human capital and assets. On the other hand, workers that switch to a different industry, can benefit from the higher returns, but they must endure a drop in their human capital and assets of a considerable magnitude, in this example of around to 15%. Total employment in industry 1 gradually contracts, partly due to an increased outflow and partly due to a decrease in inflows. The higher outflow implies a higher decline in effective wealth across workers exposed and a lower-than-normal decline in effective wealth among non-exposed workers as some of them do not switch.

Towards the end of their work life workers load more of their effective wealth into human capital due to the higher frictions in this example, and this is the reason why the effect on earnings moderates in period 9, but increases for assets.

5 Conclusion

I develop an heterogeneous agent dynamic spatial general equilibrium model of labor market choice (industry/occupations) and migration (regions) with human capital and assets accumulation and use it to quantify the effects of increased import competition on earnings and wealth. In my setting, the evolution of a worker's human capital and assets is driven by his labor market choices, idiosyncratic labor-market-specific shocks and the costs of switching/migrating. I characterize the worker's human capital and asset investment choices and equilibrium assignment of workers to labor markets, which involves a set of discrete-continuous decisions over time. I show how individual consumption/investment decisions can be written as a optimal portfolio problem given labor market choices, leading to decision rules that are homogeneous in wealth and portfolio choices that are similar across individuals. Exploiting properties of extreme value distributions I show how the ensuing discrete problem leads to a tractable characterization. A key result is that individual decisions can be aggregated and the resulting evolution of the aggregate supply of human capital and assets across labor markets can be represented in closed-form.

I then use the model to quantitatively study how worker's individual choices change with the exposure to import competition from China, and in turn how this choices shape the equilibrium allocation of workers to different labor markets, the dynamics of aggregate human capital and assets, the behavior of earnings inequality, and the welfare of the different workers in the economy. A full quantitative exercise is currently underway.

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