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Welfare-enhancing inflation and liquidity premia

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Abstract

We investigate what principles should govern the evolution and maturity structure of the national debt when nominal government securities constitute an important form of exchange media. Even in the absence of government funding risk, we find a rationale for issuing nominal debt in different maturities, purposely mispricing long-term debt, and growing the nominal debt to support a strictly positive inflation target. The policy of discounting long-term debt and supporting a strictly positive inflation target provides superior risk-sharing arrangements for clienteles characterized by different degrees of patience. Pareto improvements are possible only if these policies are offered jointly.

Keywords: Term premium, liquidity, inflation target.

JEL Codes: E4, E5

1 Introduction

What principles should govern the size, evolution, and maturity composition of the national debt? Virtually all theoretical investigations of this question approach it from the perspective of optimal public finance. Barro (1979), for example, explains how debt should be used to smooth distortionary taxes as government outlays vary over time. Numerous papers explain how debt issued at different maturities can substitute for the lack of contingent debt; see, for example, Angeletos (2002), Barro (2003), Buera and Nicolini (2004), and Nosbusch (2008), among others. In the same vein, Siu (2004) explains how the use of nominal debt and state-contingent inflation can be used to hedge fiscal risk. Lustig, Sleet, and Yeltekin (2008) extend that analysis to permit a maturity structure and find that long-term nominal debt is a preferred vehicle for hedging fiscal risk. To the extent that expected inflation is discussed, it is either taken as exogenous to fiscal decisions or strongly discouraged as a revenue device.[1]

While the problem of debt management as it relates to public finance is clearly important, it is not the only factor that policymakers may want to consider in determining best practices. The securities that national governments issue often constitute *exchange media*—i.e., objects that facilitate intertemporal trading arrangements (Woodford, 1990). The liquidity benefits of government debt are incorporated into a model of optimal maturity structure in a recent paper by Greenwood, Hanson, and Stein (2015).[2] They demonstrate that when short-term debt provides liquidity services, then it is optimal to shorten average duration relative to a standard benchmark that emphasizes rollover risk. Absent rollover risk, it is optimal to finance the entire debt with short-term securities.[3]

In this paper, we investigate the problem of debt management purely from the perspective of the role government securities play as exchange media. We abstract from the public-finance considerations only to highlight what we think are heretofore neglected benefits related to a growing national debt and a policy-induced term premium on long-term debt. These benefits are available in any economy that satisfies the following conditions. First, the timing of at least some spending opportunities for at least some agents is random (there is *ex post* variation in the demand for present versus future consumption). Second, agents vary in the way they value present *vis-à-vis* future consumption (there is *ex ante* variation in the demand for present versus future consumption). Third, agents are not able to fully-insure themselves against idiosyncratic expenditure-timing risks (incomplete financial markets). Fourth, the government lacks a lump-sum

[1] Barro (2003) argues than the nominal debt should grow at the rate of expected inflation. Chari, Christiano, and Kehoe (1996) argue that the Friedman rule remains optimal even in the presence of distortionary taxes.

[2] Angeletos, Collard, and Dellas (2023) consider the liquidity value of debt from the perspective of a Ramsey planner, although they abstract from maturity structure.

[3] This result is reminiscent of the recommendation once made by Friedman (1948); namely, that deficits should be financed entirely with money.

tax instrument. These conditions, which we believe are empirically plausible, imply that a strictly positive inflation target—supported by a growing supply of nominal debt—combined with a strict discount policy on long-maturity bonds are *jointly* welfare-improving. That is, neither a positive term premium nor a positive inflation target implemented independently generate a Pareto improvement.

Our framework of analysis consists of an overlapping generations model populated by *ex ante* heterogeneous groups of individuals subject to idiosyncratic "liquidity shocks" in the spirit of Diamond and Dybvig (1983). As in Samuelson (1958), a supply of zero-interest nominal debt rolled over indefinitely can be used as an exchange medium to overcome a dynamic inefficiency. At the same time, such a policy allows agents to self-insure when liquidity insurance is unavailable. Self-insurance, however, is only partial. In the Samuelson (1958) equilibrium, impatient agents allocate too much purchasing power to distant events that may never arise, while patient agents allocate too much purchasing power to near events that may never arise.

An improved risk-sharing arrangement requires equilibrium purchasing power to be reallocated to states in which consumption is highly-valued by agents. For impatient agents, this requires tilting purchasing power from distant to nearer events. For patient agents, the opposite is needed. The former effect can be induced through a strictly positive inflation target implemented through a cash-transfer program financed with an ever-expanding supply of nominal debt. That is, an expected inflation reduces the return on money held to meet distant contingencies, which encourages the desired substitution of purchasing power to nearer contingencies. The latter effect can be induced by issuing debt in two different maturities and *preventing* any arbitrage that would eliminate yield differentials between short- and long-duration securities. One way to inhibit arbitrage is by ensuring that the secondary market for off-the-run bonds remains subject to trading frictions (Lagos and Rocheteau, 2007, and Bigio, Nuno, and Passadore, 2017). Another way is to render long-bonds non-marketable, but with an option for early redemption at a properly-calibrated discount. Because an illiquid bond sells at a discount on the primary market, patient agents are able to weigh their portfolios more heavily with high-yielding long-bonds. The implied higher yield more than compensates for inflation and thus induces an increase in their desire to save for longer-term contingencies.

It is of some interest to compare our paper to Guibaud, Nosbusch, and Vayanos (2013) who, like us, motivate the use of debt as an exchange medium in an overlapping generations model. Maturity structure in their setting facilitates *intergenerational* risk-sharing against aggregate risk. In our model, maturity structure and inflation *jointly* facilitate *intragenerational* risk-sharing against idiosyncratic risk. As in their setup, we are able to identify "clienteles" that produce a preferred-habitat structure over the demand for securities with different maturities. For example, our impatient agents can be thought of money market funds with asset portfolios more heavily weighted toward bills.

Our patient agents can be thought of as pension funds with asset portfolios more heavily weighted toward bonds.

Our results rest on important idea from Kocherlakota (2003), a paper that in a sense endogenizes a cash-in-advance constraint. That is, bonds in his model can only enhance risk-sharing if they are illiquid in the sense they cannot be used (either by accident or by design) to make purchases. In our application of this idea, long-term bonds can only enhance risk-sharing if their off-the-run counterparts trade in a "frictional" secondary market. There is an important implication that follows from this insight for policies designed to enhance bond market liquidity, but we reserve this discussion for later.

2 The environment

Time is denoted by $t = 1, 2, ..., \infty$. At each date $t \geq 1$, the economy is populated by individuals that live for three periods. At any given point in time, the population consists of an equal number of *young*, *middle-aged* and *old* individuals. There is zero population growth, so the total population is fixed with population size equal to 3 (a unit measure of each cohort). At $t = 1$, there is an initial middle-aged population with a two-period time horizon and an initial old population with a one-period time horizon. The young have a nonstorable endowment, $\omega > 0$. The middle-aged and old have no endowments.

The young do not value consumption in their youth, but anticipate wanting consumption in either middle or old age. Specifically, young individuals anticipate wanting to consume "early" (in middle age) or "late" (in old age) with probability β and $1 - \beta$, respectively. In aggregate, a fraction β of the population wants to consume early, and the remaining fraction $1 - \beta$, late. The young will want to insure against this idiosyncratic preference risk, if possible.

The population is also characterized by *ex ante* preference heterogeneity. In particular, the population consists of types $i = 1, \ldots, N$, with $N \geq 2$. Let $0 \leq \pi(i) \leq 1$ denote the fraction of type i individuals, where $\sum_{i=1}^{N} \pi(i) = 1$. A type i individual has preference parameter $0 < \alpha(i) < 1$, where $\alpha(i)$ indexes the degree to which a person values early consumption relative to late consumption on an *ex ante* basis. For convenience and without loss of generality, we assume $\alpha(1) > \ldots > \alpha(N)$. That is, lower-numbered types are *ex ante* more impatient than higher-numbered types.

The preference ordering over consumption allocations of a type i individual born in period t is given by $U_t(i) = [\alpha(i)\beta u(c^m_{t+1}(i)) + (1-\alpha(i))(1-\beta)u(c^o_{t+2}(i))]$, where $c^m_t(i), c^o_t(i)$ denote the consumption of middle- and old-aged individuals at date t, respectively. The flow utility u is strictly increasing and strictly concave. We assume that U is homothetic.

2.1 Golden rule allocation

The Golden rule is an allocation $\{c^m(i), c^o(i)\}_{i=1}^N$ that maximizes the welfare of a representative generation as defined by

$$\sum_i \pi(i) \left[\alpha(i)\beta u(c^m(i)) + (1-\alpha(i))(1-\beta)u(c^o(i))\right] \tag{1}$$

subject to the resource constraint,

$$\sum_i \pi(i) \left[\beta c^m(i) + (1-\beta)c^o(i)\right] = \omega \tag{2}$$

The Golden rule is characterized by (2) and

$$\begin{aligned} \alpha(i)u'(c^m(i)) &= (1-\alpha(i))u'(c^o(i)) & (3)\\ \alpha(i)u'(c^m(i)) &= \alpha(1)u'(c^m(1)) & (4) \end{aligned}$$

for all i. Condition (3) equates the marginal utility of consumption across time, while condition (4) equates the marginal utility across *ex ante* types. Let $\{c_*^m(i), c_*^o(i)\}_{i=1}^N$ denote the Golden rule allocation, which solves (2)–(4) for all $i = 1, \ldots, N$. Since $\alpha(1) > \ldots > \alpha(N)$, (3)–(4) imply $c_*^m(1) > \ldots > c_*^m(N)$ and $c_*^o(1) < \ldots < c_*^o(N)$.

Rearranging (3) we obtain

$$\frac{u'(c_*^o(i))}{u'(c_*^m(i))} = \frac{\alpha(i)}{1-\alpha(i)} \tag{5}$$

Hence, the desired type-i consumption profile depends on $\alpha(i)$. If $\alpha(i) = 1/2$ then $c_*^m(i) = c_*^o(i)$, that is, from an *ex ante* perspective, type-i individuals desire a flat consumption profile. If $\alpha(i) > 1/2$ then $c_*^m(i) > c_*^o(i)$, that is, type-i individuals want to frontload consumption (they are *ex ante* impatient). Finally, if $\alpha(i) < 1/2$ then $c_*^m(i) < c_*^o(i)$, that is, type-i individuals want to backload consumption (they are *ex ante* patient). Of course, from an *ex post* perspective, individuals will strictly prefer to consume *either* early *or* late.

For $u'(c) = c^{-\sigma}$, $\sigma > 0$, the Golden rule allocation can be solved in closed-form,

$$c_*^m(i) = \alpha(i)^{\frac{1}{\sigma}}(\omega/\Gamma) \qquad c_*^o(i) = (1-\alpha(i))^{\frac{1}{\sigma}}(\omega/\Gamma) \tag{6}$$

where $\Gamma \equiv \sum_i \pi(i)[\beta\alpha(i)^{\frac{1}{\sigma}} + (1-\beta)(1-\alpha(i))^{\frac{1}{\sigma}}]$. For the special case $\sigma = 1$ and $\beta = 1/2$, the expressions in (6) further simplify to $c_*^m(i) = \alpha(i)2\omega$ and $c_*^o(i) = (1-\alpha(i))2\omega$.

3 A monetary economy

3.1 Government policy

The government issues three types of securities: money, bills, and bonds. All securities exist as digital entries in a central ledger of money/security accounts managed costlessly by a trusted agency. Money is a zero-interest security used to make payments. A bill issued at date t turns into money at date $t+1$. A bond issued at date t turns into money at date $t+2$, unless it is presented for early redemption at date $t+1$. Both bills and bonds are discount securities—they pay no coupon. Both securities are risk-free. Because money is (weakly) dominated in rate of return by bills, it is not held as a store of value—it is only used to make intra-period payments. Individuals only carry bills and bonds over time.

Let M_t, B_t denote the nominal value of bills and bonds issued at date t, respectively. Without loss of generality, we set the discount on bills to zero.[4] Let q_t denote the price of newly-issued bonds. We assume that off-the-run bonds (bonds with one period left to maturity) are non-marketable.[5] However, off-the-run bonds can be liquidated at a standing facility operated by the government. Let δ_t denote the discount applied to off-the-run bonds. Let $0 \leq X_t \leq B_{t-1}$ denote the quantity of off-the-run bonds presented for early redemption at date t. Finally, let Z_t denote government money transfers in periot t. Unless otherwise specified, there are no taxes and there are no other forms of government spending.

The money injected into the economy at any given point in time may be spent within the period but is not carried over time. Instead, end-of-period money holdings are converted into bills and/or bonds.[6] With this in mind, the government's flow budget constraint is given by,

$$M_t + q_t B_t = M_{t-1} + [B_{t-2} - X_{t-1}] + \delta_t X_t + Z_t \tag{7}$$

for $t \geq 1$. The right-hand-side of (7) records the government's nominal obligations at date t. M_{t-1} represents maturing bills, $[B_{t-2} - X_{t-1}]$ represents maturing bonds, $\delta_t X_t$ represents off-the-run bond purchases, and Z_t represents the government's promised transfer to the private sector. The left-hand-side of (7) represents the nominal debt that must be issued at date t to cover the government's nominal obligations. As of period $t = 1$, there is an outstanding supply

[4] Normalizing the discount on bills to zero helps to economize on notation without affecting our main results; see Andolfatto (2020). Our focus is on the discount applied to bonds of longer maturities.

[5] We assume the extreme case where a secondary market for bonds is missing. Our results would continue to hold in the more realistic case where bonds traded in a decentralized search-for-counterparty market. Note that while the market for off-the-run U.S. Treasury securities is considered to be deep, it is not frictionless—as the events of March 2020 clearly demonstrated.

[6] Since the discount on bills is set to zero, bills only weakly dominate money in rate of return. If the discount on bills is positive (as in Andolfatto, 2020), then bills would strictly dominate money in rate of return. As mentioned earlier, the discount on bills plays no role in our analysis and so it is normalized to zero for the sake of notational simplicity.

of bills and bonds given by history, M_0 and $B_{-1} - X_0$, respectively. Since these initial conditions do not matter for the steady-stae analysis below, we can assume without loss that these securities are distributed in some arbitrary manner across the initial population.

The total stock of government securities at date t is given by $D_t \equiv M_t + B_t$. Define $\mu_t \equiv D_t/D_{t-1}$ and $\theta_t \equiv B_t/M_t$. In what follows, we assume that the central bank targets nominal interest rates and that the treasury lets the composition of its liabilities to adjust passively to market demand. A government policy then is characterized by parameters $\{\mu_t, q_t, \delta_t : t \geq 1\}$ and nominal quantities $\{D_t, X_t, Z_t : t \geq 1\}$ consistent with (7). We assume below that the money transfer Z_t is distributed to the middle-aged. Since there is a unit measure of each age cohort, each middle-aged individual receives Z_t dollars.

3.2 Individual decision-making

Insurance markets are unavailable and there is no secondary market for off-the-run bonds.[7] Individuals can trade money for goods at price p_t on a sequence of competitive spot markets. The pattern of trades here will entail young individuals selling their endowment to impatient middle-aged individuals and old individuals (who were patient in their middle age) in exchange for money that is then invested in bills and bonds.

Let $m_t(i)$ and $b_t(i)$ denote the bill and bond purchases made by a young type i individual. These purchases are financed out of the income generated from the sale of their endowment, $p_t\omega$. Hence, a type i young person's budget at date t is given by

$$m_t(i) + q_t b_t(i) = p_t \omega \tag{8}$$

With probability β a young person will want to consume in middle age. For the people that do want to consume, their middle age budget constraint will be given by,

$$p_{t+1} c^m_{t+1}(i) = m_t(i) + \delta_{t+1} b_t(i) + Z_{t+1} \tag{9}$$

That is, a middle-aged wanting to consume at $t+1$ will spend all accumulated wealth plus their transfer income. Accumulated wealth at date $t+1$ consists of maturing bills $m_t(i)$, plus bonds presented for early redemption $b_t(i)$ at the standing facility with discount rate δ_{t+1}. Transfer income is equal to Z_{t+1}.

For the middle-aged people wanting to postpone consumption to old age, their budget constraint in period $t+2$ will be given by,

$$p_{t+2} c^o_{t+2}(i) = m_t(i) + b_t(i) + Z_{t+1} \tag{10}$$

The only difference in (10) relative to (9) is that the long-bond matures at face value and the money transfer received in middle age is carried into old age at

[7] Again, our results would continue to hold if there was a secondary market subject to trading frictions as in Lagos and Rocheteau (2007).

zero interest (in the form of bills).[8]

It will prove convenient to recast the constraints (8)–(10) in real terms. To this end, define $\hat{m}_t(i) \equiv m_t(i)/p_t$, $\hat{b}_t(i) \equiv b_t(i)/p_t$, $z_t \equiv Z_t/p_t$ and $\Pi_t \equiv p_t/p_{t-1}$. A type i young person at date $t \geq 1$ faces the following sequence of budget constraints

$$\omega = \hat{m}_t(i) + q_t\hat{b}_t(i) \tag{11}$$

$$c^m_{t+1}(i) = \Pi^{-1}_{t+1}\left[\hat{m}_t(i) + \delta_{t+1}\hat{b}_t(i)\right] + z_{t+1} \tag{12}$$

$$c^o_{t+2}(i) = \Pi^{-1}_{t+1}\Pi^{-1}_{t+2}\left[\hat{m}_t(i) + \hat{b}_t(i)\right] + \Pi^{-1}_{t+2}z_{t+1} \tag{13}$$

From (11) we can write $\hat{b}_t(i) = q_t^{-1}(\omega - \hat{m}_t(i))$ so (12) and (13) may alternatively be expressed as,

$$c^m_{t+1}(i) = \Pi^{-1}_{t+1}\left[(\delta_{t+1}/q_t)\omega + (1 - \delta_{t+1}/q_t)\hat{m}_t(i))\right] + z_{t+1} \tag{14}$$

$$c^o_{t+2}(i) = \Pi^{-1}_{t+1}\Pi^{-1}_{t+2}\left[(1/q_t)\omega + (1 - 1/q_t)\hat{m}_t(i)\right] + \Pi^{-1}_{t+2}z_{t+1} \tag{15}$$

A young type i individual at date t has the following decision-problem,

$$\max_{\hat{m}_t(i)} \; \alpha(i)\beta u(c^m_{t+1}(i)) + (1 - \alpha(i))(1 - \beta)u(c^o_{t+2}(i)) \tag{16}$$

where $c^m_{t+1}(i)$ and $c^o_{t+2}(i)$ are given by (14) and (15), respectively. We also impose non-negativity constraints on bill and bond holdings, $0 \leq \hat{m}_t(i) \leq \omega$. These latter constraints are assumed to be slack for now, but we will check for possible corner solutions in our calculations below.

Note that if $\delta_{t+1} = q_t = 1$, then bills and bonds are perfect substitutes. In this case, there would be no point in issuing different maturities, as bills can be costlessly rolled over and bonds can be costlessly liquidated. In what follows, we assume that bonds are illiquid in the sense they can be redeemed early, but only at a penalty rate $\delta_{t+1} < 1$. Since investors anticipate the penalty rate for early redemption, bonds are purchased in the primary market only at an appropriate discount $\delta_{t+1} < q_t < 1$. At an interior solution for $\hat{m}_t(i)$, the first-order condition implies

$$\alpha(i)\beta u'(c^m_{t+1}(i))(1 - \delta_{t+1}/q_t) = (1 - \alpha(i))(1 - \beta)u'(c^o_{t+2}(i))\Pi^{-1}_{t+2}(1/q_t - 1) \tag{17}$$

for all $t \geq 1$ and all i, where $c^m_{t+1}(i)$ and $c^o_{t+2}(i)$ are given by (14) and (15), respectively.

[8] Patient middle-aged individuals will prefer to rollover their maturing bills and store their money transfer as bills, relative to storing wealth as newly-issued bonds that will have to be liquiditated at a penalty rate.

3.3 Market-clearing

Use (14) and (15) to write,

$$\begin{aligned} c_t^m(i) &= \Pi_t^{-1}\left[(\delta_t/q_{t-1})\omega + (1-\delta_t/q_{t-1})\hat{m}_{t-1}(i))\right] + z_t && (18) \\ c_t^o(i) &= \Pi_{t-1}^{-1}\Pi_t^{-1}\left[(1/q_{t-2})\omega + (1-1/q_{t-2})\hat{m}_{t-2}(i)\right] + \Pi_t^{-1}z_{t-1} && (19) \end{aligned}$$

where $\{\hat{m}_t(i) : i = 1, 2, ..., N\}$ are the bill-demand functions defined implicitly by conditions (17) for $t \geq 1$, and $\hat{m}_0$ and $\hat{m}_{-1}$ are the real bill-holdings at $t = 1$ of the initial middle-aged and old, respectively.

The aggregate demand for output must, in equilibrium, be consistent with the aggregate supply of output:

$$\omega = \sum_i \pi(i)[\beta c_t^m(i) + (1-\beta)c_t^o(i)] \tag{20}$$

for all $t \geq 1$.

At the end of each period, bills are held by the young (via their sales of output for money, which is then stored as bills) and are also held by the patient middle-aged (who receive a money transfer and do not spend any of their wealth). As a result, we have $\sum_i \pi(i)\left[m_t(i) + (1-\beta)m_{t-1}(i)\right] + (1-\beta)Z_t = M_t$, or

$$\sum_i \pi(i)\left[\hat{m}_t(i) + (1-\beta)\Pi_t^{-1}\hat{m}_{t-1}(i)\right] + (1-\beta)z_t = \hat{M}_t \tag{21}$$

where $\hat{M}_t \equiv M_t/p_t$.

There is also a market-clearing condition for bonds. In equilibrium, the young hold all newly-issued bonds,

$$\sum_i \pi(i)\hat{b}_t(i) = \theta_t\hat{M}_t \tag{22}$$

By Walras' Law, one of (20), (21) and (22) is redundant.

Let $\hat{D}_t \equiv D_t/p_t$. The equilibrium inflation rate must therefore satisfy,

$$\Pi_t = \mu_t(\hat{D}_{t-1}/\hat{D}_t) \tag{23}$$

where $\hat{D}_t = [1+\theta_t]\hat{M}_t$. The equilibrium price-level is given by $p_t = D_t/\hat{D}_t$.

Finally, in equilibrium, impatient middle-aged individuals present their bonds for redemption. Because a measure β of the population turns out to be impatient, $X_t = \beta B_{t-1}$. This permits us to rewrite the government budget constraint(7) as $M_t - M_{t-1} + q_tB_t = \beta\delta_tB_{t-1} + (1-\beta)B_{t-2} + Z_t$ or, stated in real terms,

$$\hat{M}_t[1+q_t\theta_t] = \hat{M}_{t-1}\Pi_t^{-1}[1+\beta\delta_t\theta_{t-1}] + \hat{M}_{t-2}\Pi_{t-1}^{-1}\Pi_t^{-1}(1-\beta)\theta_{t-2} + z_t \tag{24}$$

3.4 Stationary equilibrium

We restrict attention to stationary equilibria. In a stationary equilibrium, $c_{t+1}^m(i) = c^m(i)$, $c_{t+2}^o(i) = c^o(i)$, $\hat{m}_t(i) = \hat{m}(i)$, $\hat{b}_t(i) = \hat{b}(i)$ for all i and t. In addition, we have $z_t = z$, $q_t = q$, $\delta_t = \delta, \theta_t = \theta$ and $\mu_t = \mu$ for all t.

From (22), stationarity implies $\hat{M}_t = \hat{M}$ and so, $\hat{D}_t = \hat{D}$. Then, by condition (23), the equilibrium inflation rate is given by $\Pi = \mu$.

Imposing stationarity on (17)–(19), we have

$$\begin{aligned}
\alpha(i)\beta u'(c^m(i))(1-\delta/q) &= (1-\alpha(i))(1-\beta)u'(c^o(i))\mu^{-1}(1/q-1) && (25)\\
c^m(i) &= \mu^{-1}\left[(\delta/q)\omega + (1-\delta/q)\hat{m}(i))\right] + z && (26)\\
c^o(i) &= \mu^{-2}\left[(1/q)\omega + (1-1/q)\hat{m}(i)\right] + \mu^{-1}z && (27)
\end{aligned}$$

Conditions (25)–(27) determine the demand for real bill holdings $\hat{m}(i)$ for all i in a steady state, conditional on (q, δ, μ, z). Using (11), we can derive the equilibrium value of real bond holdings, $\hat{b}(i) = (1/q)\left[\omega - \hat{m}(i)\right]$. Combine this latter expression with the stationary version of (22) to derive

$$\sum_i \pi(i)(1/q)\left[\omega - \hat{m}(i)\right] = \theta\hat{M} \tag{28}$$

Imposing stationarity on (21), we have

$$\left[1 + (1-\beta)\mu^{-1}\right]\sum_i \pi(i)\hat{m}(i) + (1-\beta)z = \hat{M} \tag{29}$$

Finally, from the government budget constraint (24) we obtain

$$z = (1-1/\mu)\hat{M} + [q - \beta(\delta/\mu) - (1-\beta)(1/\mu^2)]\theta\hat{M} \tag{30}$$

Note that for $\theta = 0$ (a bills-only economy), condition (29) reduces to $z = \left[1-\mu^{-1}\right]\hat{M}$, which is the standard seigniorage formula for zero-interest money.

Conditions (25)–(30) characterize the steady-state values

$$\{c^m(i), c^o(i), \hat{m}(i)\}_{i=1}^N, \hat{M}, z, \theta$$

for a given set of policy parameters (q, δ, μ). We are interested in whether there exists a policy (q, δ, μ) that implements the Golden rule allocation as an equilibrium.

4 Golden rule implementation

Recall that the Golden rule allocation, denoted by $\{c_*^m(i), c_*^o(i)\}_{i=1}^N$, solves (2)–(4) for all $i = 1, \ldots, N$. Comparing Golden rule condition (3) to the equilibrium condition (25), we see the following condition is necessary for implementation,

$$q = \frac{\delta\mu\beta + 1 - \beta}{\mu\beta + 1 - \beta} \tag{31}$$

Condition (31) ensures that $c^o(i)/c^m(i) = c^o_*(i)/c^m_*(i)$ for all i. Next, combine the Golden rule allocation with the individual budget constraints (26) and (27) to form,

$$c^m_*(i) = \mu^{-1}\left[(\delta/q)\omega + (1-\delta/q)\hat{m}(i))\right] + z \tag{32}$$
$$c^o_*(i) = \mu^{-2}\left[(1/q)\omega + (1-1/q)\hat{m}(i)\right] + \mu^{-1} z \tag{33}$$

Conditions (31)–(33) require that an implementation policy (q, δ, μ) generates conditions which ensure that individuals can afford the Golden rule allocation in equilibrium.

Next, we need to ensure that individual choices are consistent with market-clearing and the government budget constraint. The Golden rule allocation satisfies market-clearing condition (20) by construction, since (20) also corresponds to the resource constraint (2). In addition, the allocation will have to satisfy the market-clearing conditions (28) and (29), and the government budget constraint (30). When government policies satisfy (31), we can use these conditions to solve explicitly for the aggregate equilibrium variables.

Lemma 1 *For policies (q, δ, μ) that satisfy* (31)*, the market clearing conditions* (28) *and* (29)*, and the government budget constraint* (30) *imply:*

$$z = \left[\frac{1-\beta+\mu}{1-\beta+\beta\mu}\right]\left[1-\frac{1}{\mu}\right]\omega \tag{34}$$
$$\hat{M} = \left[\frac{1-\beta+\mu}{1-\beta+\beta\mu}\right]\left(\frac{\mu}{\mu+\Omega}\right)\omega \tag{35}$$
$$\theta = \left[\frac{1-\beta+\beta\mu}{1-\beta+\mu}\right]\left(\frac{\Omega}{1-\beta+\delta\beta\mu}\right) \tag{36}$$

where $\Omega \equiv 0$ *if* $\sum_i \pi(i)\hat{m}(i) = \omega$ *and* $\Omega \equiv \left\{\frac{\omega}{(1-\beta+\beta\mu)[\omega-\sum_i \pi(i)\hat{m}(i)]} - \frac{1}{\mu}\right\}^{-1}$ *otherwise.*

The proof of Lemma 1 is relegated to the appendix. Lemma 1 derives the equilibrium values of (z, M, θ) when policies satisfy (31), which is necessary for implementation of the Golden rule. Notably, real transfers, z, only depend on inflation, μ, and are independent of the discount rate, δ. The real value of the aggregate stock of bills, $\hat{M}$, depend on both inflation and the discount rate—the latter indirectly through its effect on individuals' portfolio choice, $\hat{m}(i)$. Finally, both μ and δ impact the bonds-to-bills ratio, θ. We now derive some properties of these variables.

Corollary 1 *For policies (q, δ, μ) that satisfy* (31)*: (i) $z \gtreqless 0$ when $\mu \gtreqless 1$; (ii) $\hat{M} > 0$ and $\theta = 0$ if $\sum_i \pi(i)\hat{m}(i) = \omega$; and (iii) $\hat{M} > 0$, $\theta > 0$ if $\mu \geq 1$ and $0 < \sum_i \pi(i)\hat{m}(i) < \omega$.*

See proof in appendix. The first result in Corollary 1 states that inflation (deflation) needs to be supported with lump-sum transfers (taxes). Although intuitive, this result does not hold for arbitrary policies. The first term in the government budget constraint (30) is the seigniorage from bills, while the second term is the net revenue from issuing new bonds and buying off-the-run bonds. The sign of both terms depends on inflation, but the latter can be further manipulated by setting appropriate discount rates. Corollary 1 assures us that when policies satisfy (31), both terms have the same sign as (net) inflation.

The second result in Corollary 1 guarantees $\hat{M} > 0$ when individuals do not hold any bonds and so $\theta = 0$. The third result states that $\mu \geq 1$ is a sufficient condition to guarantee $\hat{M} > 0$ and $\theta > 0$ when individuals hold both bills and bonds, a relevant case in the analysis below. There is also a necessary condition, effectively a lower bound on μ, that more generally ensures $\hat{M} > 0$ and $\theta > 0$.[9]

4.1 Homogeneous discount factors

We begin by demonstrating implementation for the special case $\alpha(i) = \alpha$ for all i. This implies $(c_*^m(i), c_*^o(i)) = (c_*^m, c_*^o)$ for all i. We conjecture that the Golden rule can be implemented via an appropriate choice of μ with $\delta = q = 1$. These latter parameter settings, which satisfy (31), imply that bills and bonds are perfect substitutes, so that one of the securities redundant. Without loss of generality, we focus on a bills-only economy (so that $\theta = 0$) which, by Corollary 1 implies

$$\hat{m}(i) = \hat{m} = \omega \tag{37}$$

for all i. Using (37), conditions (32) and (33) reduce to $c_*^m = \mu^{-1}\omega + z$ and $c_*^o = \mu^{-1} c_*^m$, respectively. These conditions imply the following policy settings:

$$\mu = c_*^m / c_*^o \tag{38}$$

$$z = c_*^m - (c_*^o / c_*^m)\omega \tag{39}$$

Condition (38) implies that if $\alpha > 1/2$, then $\mu > 1$ (inflation is necessary for implementation). If $\alpha < 1/2$, then $\mu < 1$ (deflation is necessary for implementation). Condition (39) delivers the associated tax/transfer necessary to support the desired inflation policy. Note that (39) implies the same transfer as that derived in Lemma 1—combining (2), (38) and (39) yields (34). By Corollary 1, inflation is associated with positive transfers, while deflation is financed with lump-sum taxes, i.e., $z > 0$ if $\mu > 1$ and $z < 0$ if $\mu < 1$.

Thus, implementation for any range of α is straightforward if a lump-sum tax instrument is available. We turn next to case in which lump-sum taxes are infeasible.

[9] Specifically, when $0 < \sum_i \pi(i)\hat{m}(i) < \omega$, the restriction $\mu > \frac{(1-\beta)(1-\sum_i \pi(i)\hat{m}(i)/\omega)}{1-\beta(1-\sum_i \pi(i)\hat{m}(i)/\omega)}$ guarantees $\hat{M} > 0$ and $\theta > 0$.

4.1.1 Implementation without a lump-sum tax

If $\alpha < 1/2$ and lump-sum taxes are not available ($z \geq 0$), some other instrument is needed to restore efficiency. Following Andolfatto (2020), set $\mu = 1$ and introduce a long-maturity bond with off-the-run redemptions discounted at some $\delta < 1$.

When $\mu = 1$, condition (31) reduces to,

$$q = \delta\beta + 1 - \beta \tag{40}$$

so that $\delta < q < 1$. This manner of discounting on- and off-the-run bonds ensures that individuals choose $c^m/c^o = c_*^m/c_*^o$. By Corollary 1, setting $\mu = 1$ implies $z = 0$ so that the non-negativity constraint on z is satisfied. The policy of using sales of long-maturity bonds here generates revenue that is just sufficient to finance early and late redemptions of long-maturity bonds (bills are simply rolled over at zero interest expense).

Next, use either (32) or (33) to solve for $\hat{m}$ using the resource constraint (2) to eliminate ω, imposing $\mu = 1$, $z = 0$ and (40) to derive,

$$\hat{m} = (c_*^m - \delta c_*^o)/(1 - \delta) \tag{41}$$

If we restrict bill-holdings to be non-negative, then (41) implies the following restriction on the discount rate,

$$\delta \leq c_*^m/c_*^o < 1 \tag{42}$$

Combining (41) with (2), one can verify that $\hat{b} \geq 0$. Next, combine (41) with (35) and (36) to obtain $\hat{M}$ and θ.

If we insert (41) back into either (32) or (33) we obtain the resource constraint (2) and so, there are no further restrictions on δ; i.e., any $0 < \delta < c_*^m/c_*^o$ is consistent with implementation of the Golden rule. That is, there is no unique optimal discount policy—the optimal allocation can be implemented by discounting long-maturity bonds more or less severely. Discounting off-the-run bonds severely means bond-holders do not receive much money when they turn their bonds in for early redemptions. On the other hand, they will not have paid very much for these bonds on the primary market. In terms of supporting an efficient allocation, these two effects are a wash.

The overall lesson here is that inflation is optimal when consumption profiles are frontloaded while discounting bonds is necessary when consumption profiles are backloaded. The lack of fiscal support (no lump-sum taxes) is critical for the latter result, but not the former. If individuals were *ex ante* heterogenous, with different desired consumption profiles, we expect that inflation should help those relatively impatient, while illiquid (discounted) bonds should help those relatively patient. We now turn our analysis to this more general case.

4.2 Heterogeneous discount factors

Assume now that there are $N > 1$ types of individuals with $\alpha(1) > \ldots > \alpha(N)$, which implies $c_*^m(1) > \ldots > c_*^m(N)$ and $c_*^o(1) < \ldots < c_*^o(N)$. Furthermore, assume that $\alpha(N) < 1/2 < \alpha(1)$ so that the population has some individuals wanting to frontload consumption and others wanting to backload consumption (on an *ex ante* basis). Because we rule out lump-sum taxation, the existence of *ex ante* patient types ($\alpha(i) < 1/2$) implies that both bills and bonds will be necessary for implementation. Since borrowing is not permitted, we restrict $\hat{m}(i), \hat{b}(i) \geq 0$ for all i. Drawing on our analysis from above, we know that the existence of impatient types requires $\mu > 1$ ($z > 0$) and that the existence of patient types requires $\delta < 1$.

Let us now establish the restrictions on policy necessary to ensure implementation of the Golden rule. To begin, and as before, condition (31) ensures that $c^o(i)/c^m(i) = c_*^o(i)/c_*^m(i)$ for all i. Combine (31) with the individual budget constraints (32) and (33) to solve for $\hat{m}(i)$ and z:

$$\hat{m}(i) = \frac{\mu[1+\beta(\delta\mu-1)][c_*^m(i)-\mu c_*^o(i)]}{(1-\delta)[1+\beta(\mu-1)]} + \omega \tag{43}$$

$$z = \frac{\mu[\beta c_*^m(i)+(1-\beta)c_*^o(i)]}{1+\beta(\mu-1)} - \frac{\omega}{\mu} \tag{44}$$

In addition, use the budget constraint of the young, $\hat{m}(i) + q\hat{b}(i) = \omega$, to obtain

$$\hat{b}(i) = \frac{\mu[\mu c_*^o(i) - c_*^m(i)]}{1-\delta} \tag{45}$$

Recall that $c_*^m(i)$ is decreasing in i and that $c_*^o(i)$ is increasing in i. It follows that $\mu c_*^o(i) - c_*^m(i)$ is increasing in i. Using (45), the restriction $\hat{b}(i) \geq 0$ for all i implies $\mu c_*^o(i) - c_*^m(i) \geq 0$ for all i, which implies $\min_i \mu c_*^o(i) - c_*^m(i) \geq 0$. Since $\mu c_*^o(i) - c_*^m(i) = \mu c_*^o(1) - c_*^m(1)$, a necessary condition for implementation is,

$$\mu \geq c_*^m(1)/c_*^o(1) > 1 \tag{46}$$

That is, since $\alpha(1) > 1/2$, it follows that a strictly positive inflation is necessary for first-best implementation.

Let us next turn to the restriction $\hat{m}(i) \geq 0$ for all i. Imposing this latter restriction on condition (43) implies

$$\frac{(1-\delta)[1+\beta(\mu-1)]\omega}{\mu[1+\beta(\delta\mu-1)]} \geq \mu c_*^o(N) - c_*^m(N) > 0 \tag{47}$$

where $\mu c_*^o(N) - c_*^m(N) = \max_i \ \mu c_*^o(i) - c_*^m(i) > 0$. Note that, for a given μ, condition (47) implies an upper bound on the discount rate, δ.

Conditions (35) and (36) in Lemma 1 provide the expressions for $\hat{M}$ and θ, respectively. The restrictions (46) and (47) imply $\mu > 1$ and $0 < \sum_i \pi(i)\hat{m}(i) < \omega$. Hence, by Corollary 1 we obtain $\hat{M} > 0$ and $\theta > 0$.

Let us finally consider condition (44). Because transfers are not contingent on type, condition (44) implies that implementation (for this class of policies) is only possible for Golden rule allocations that satisfy the property that $\beta c_*^m(i) + (1-\beta)c_*^o(i)$ is constant across all types. Combined with the resource constraint (2), this property must satisfy

$$\beta c_*^m(i) + (1-\beta)c_*^o(i) = \omega \tag{48}$$

for all i. Alas, condition (48) holds only in a set of special circumstances, which we now describe.

4.2.1 Restrictions on the Golden rule allocation

Suppose $u'(c) = c^{-\sigma}$. We can then apply the analytical solutions to $\{c_*^m(i), c_*^o(i)\}_{i=1}^N$, as given by the expressions in (6), to further characterize the restrictions (46)–(48).

The first restriction, condition (46), implies

$$\mu \geq \left(\frac{\alpha(1)}{1-\alpha(1)}\right)^{\frac{1}{\sigma}} > 1$$

The second restriction, condition (47), implies

$$\frac{(1-\delta)[1+\beta(\mu-1)]}{\mu[1+\beta(\delta\mu-1)]} \geq \frac{\mu(1-\alpha(N))^{\frac{1}{\sigma}} - \alpha(N)^{\frac{1}{\sigma}}}{\sum_i \pi(i)\left[\beta\alpha(i)^{\frac{1}{\sigma}} + (1-\beta)(1-\alpha(i))^{\frac{1}{\sigma}}\right]}$$

Note that the condition is now independent of ω.

The third restriction, condition (48), can be expressed as

$$\beta\alpha(i)^{\frac{1}{\sigma}} + (1-\beta)(1-\alpha(i))^{\frac{1}{\sigma}} = \sum_i \pi(i)\left[\beta\alpha(i)^{\frac{1}{\sigma}} + (1-\beta)(1-\alpha(i))^{\frac{1}{\sigma}}\right]$$

for all i. This condition is satisfied for the case $\sigma = 1$ and $\beta = 1/2$. For general values of σ, we need $\beta = 1/2$ and $\alpha(i)^{\frac{1}{\sigma}} + (1-\alpha(i))^{\frac{1}{\sigma}}$ constant across types. We now summarize these results in the following proposition.

Proposition 1 *Assume $u'(c) = c^{-\sigma}$, for $\sigma > 0$. The Golden rule allocation $\{c_*^m(i), c_*^o(i)\}_{i=1}^N$ is implementable for a policy $q < 1$, $\delta < 1$, and $\mu > 1$ satisfying* (31), (46) *and* (47)*, if there are equal measures of ex post impatient and patient types ($\beta = 1/2$) for: (i) log preferences ($\sigma = 1$); and (ii) general preferences ($\sigma \neq 1$) if $\alpha(i)^{\frac{1}{\sigma}} + (1-\alpha(i))^{\frac{1}{\sigma}}$ is constant for all i.*

Note that the condition $\alpha(i)^{\frac{1}{\sigma}} + (1-\alpha(i))^{\frac{1}{\sigma}}$ constant across all types holds when there are two symmetric types; i.e., $\alpha(1) = \alpha$ and $\alpha(2) = 1-\alpha$.

There is no need to re-hash the intuition here. While it is true that exact implementation is possible only under very strict conditions, there is every reason to believe that even if the policies we restrict ourselves to here cannot implement the Golden rule, they can nevertheless improve welfare.[10] We now turn to some numerical calculations to verify this conjecture.

4.2.2 Welfare-improving policies under more general conditions

The competitive equilibrium (*laissez faire*) allocation under a constant money supply ($\delta = \mu = 1$) delivers $c^m(i) = c^o(i) = \omega$ for all i. Hence, the expected utility payoff is

$$Q(\beta) = u(\omega) \sum_i \pi(i) \left[\alpha(i)\beta + (1-\alpha(i))(1-\beta)\right]$$

Let $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$, with $\sigma > 0$, and normalize $\omega = 1$, so that $Q(\beta) = 0$. Next, compute the expected utility payoff associated with the Golden rule allocation for different values of β,

$$W(\beta) = \sum_i \pi(i) \left[\alpha(i)\beta u(c_*^m(i)) + (1-\alpha(i))(1-\beta)u(c_*^o(i))\right]$$

where, recall, the allocations $\{c_*^m(i), c_*^o(i)\}$ also depend on β.

Proposition 1 asserts that individuals achieve $W(1/2)$ in equilibrium in a bills and bonds economy for some policy (q, δ, μ) satisfying $\delta^* < 1$ and $\mu^* > 1$. Let $q^*(\beta)$ satisfy (31) for any given β under policy (δ^*, μ^*), i.e.,

$$q^*(\beta) = \frac{\delta^* \mu^* \beta + 1 - \beta}{\mu^* \beta + 1 - \beta}$$

Define the expected utility payoff individuals attain under the policy $(q^*(\beta), \delta^*, \mu^*)$ as

$$E(\beta) = \sum_i \pi(i) \left[\alpha(i)\beta u(c^m(i)) + (1-\alpha(i))(1-\beta)u(c^o(i))\right]$$

Again, Proposition 1 asserts $E(1/2) = W(1/2)$. But how is welfare affected under policy $q^*(\beta)$ for economies where $\beta \neq 1/2$? Our numerical results show that $E(\beta) > Q(\beta) = 0$ for all β.

Suppose there are two *ex ante* symmetric types, $\alpha(1) = \alpha$ and $\alpha(2) = 1 - \alpha$, with $\alpha = 0.55$, and an equal measure of *ex post* types, i.e., $\beta = 1/2$. Then, by Proposition 1 the Golden rule is implementable under an appropriate policy. Assume there is an equal measure of each type so, $\pi(1) = \pi(2) = 1/2$ and let $\omega = 1$ and $\sigma = 2$.

[10] One might consider additional instruments, like type-dependent transfers. If types are private information (which seems likely in the present context), one would have to ensure that the optimum (or constrained-efficient allocation) is incentive-compatible.

For this parametrization, the lower bound on μ given by (46) is 1.106. Hence, we calibrate a period to 10 years and set $\mu^* = 1.02^{10} \approx 1.219$ to target 2% annual inflation. The upper bound on δ implied by (47) is 0.691, but this discount would imply a very low nominal interest rate on bonds.[11] Thus, we set $\delta^* = 0.100$ which implies $q^*(1/2) = 0.506$, which translates to a nominal interest rate of 3.5% annual. Note that picking other values of δ^* does not significantly alter our results.

Figure 1: Ex ante welfare under policy $(q^*(\beta), \delta^*, \mu^*)$

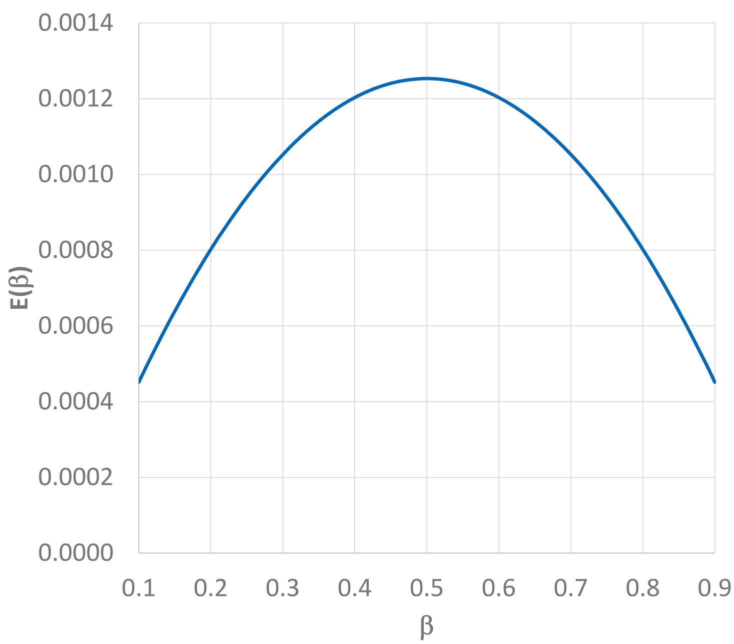

Figure 1 shows the expected utility $E(\beta)$, for the parameterization described above. Expected utility peaks at $\beta = 1/2$, the Golden rule allocation, and remains positive for all β. Recall that, given our assumptions, the competitive equilibrium ($\delta = \mu = 1$) delivers an expected utility payoff of zero for all β, i.e., $Q(\beta) = 0$. Thus, the policy $q^*(\beta)$ dominates the competitive equilibrium for all degrees of *ex post* heterogeneity.

[11] Recall that $1/q - 1$ is the interest rate on a bond over two periods; hence the annual rate of interest on a bond is $q^{-20} - 1$. Note that, given our parameterization, the highest nominal interest rate, given by $\delta^* = 0$, is 4.1% annual.

4.2.3 Robust policy when β is unknown

Imagine the policymaker discounting on-the-run bonds at price $q^*(1/2)$ when the actual value of β is, in fact, unknown. Then we can show that the "optimal" policy is welfare-improving over a range of β in the neighborhood of $\beta = 1/2$, but that welfare is lower than $Q(\beta) = 0$ when β falls outside this range.

In this (more realistic) case, it may make sense for the policymaker to choose a "suboptimal" policy (q, δ, μ) in the interest of robustness. That is, while the welfare gains will be smaller for β in the neighborhood of $1/2$, the welfare losses will be smaller if the actual β falls outside this range.

Figure 2: Ex ante welfare under policy $((1-\varepsilon) \times q^*(1/2), \delta^*, \mu^*)$

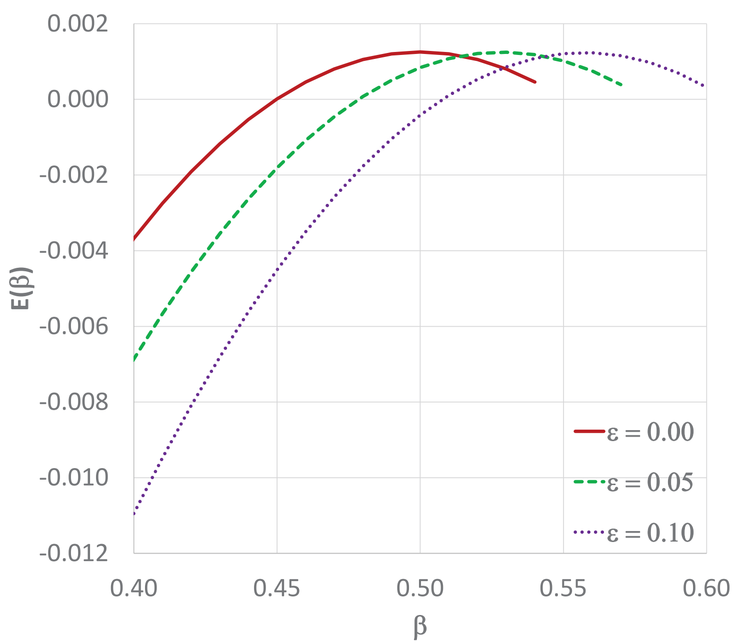

Figure 2 shows expected utility under policy (δ^*, μ^*), when combined with a suboptimal interest rate policy, q. We focus on an interest rate policy that does not vary with β, $q = (1-\varepsilon) \times q^*(1/2)$, and consider the effects of changing ε. As β rises above $1/2$ these fixed interest rate policies hit the bound on non-negative bond holdings; hence, we stop drawing welfare when the lower bound is hit—i.e., we only consider policies with interior solutions to individuals' portfolio decisions. When β falls below $1/2$ welfare drops significantly, so we stop drawing welfare at $\beta = 0.4$ to keep the chart legible.

When $\varepsilon = 0$, so that $q = q^*(1/2)$ (the solid line), *ex ante* welfare is higher than in the competitive equilibrium, $E(\beta) > 0$, for $\beta \in (0.45, 0.54)$. Next, we consider policies with higher ε, i.e., lower q (the dashed and dotted lines). These higher nominal interest rate policies dominate the competitive equilibrium for ranges of β that are shifted to the right relative to the case $\varepsilon = 0$. Furthermore, higher ε policies are preferred to lower ε policies as β increases. In contrast, when β is low, welfare losses increase with ε. Hence, though the policymaker may be uncertain about the exact value of β it is still useful to know whether β is low or high.

5 Discussion and conclusions

Our analysis departs in some significant and potentially important ways from the literature. First, there is no exogenous government spending program to finance. Second, there is no aggregate uncertainty. Third, we do not *assume* that some government securities are intrinsically disadvantaged relative to each other in terms of their liquidity properties. Our framework does, however, share with other analyses the assumption of incomplete financial markets.

We think our departure in terms of what we assume about the liquidity properties of government securities is important. In the Greenwood, Hanson, and Stein (2015) framework, the absence of aggregate risk and symmetric liquidity properties of short- and long-term debt would render the optimal maturity structure indeterminate. This is not the case in our model. If short-term debt is valued for its liquidity properties in their framework, then it is optimal to finance the entire debt with short-term securities. Again, this is not the case in our model. Moreover, their model has nothing to say about how the nominal debt should grow over time. By way of contrast, in our model, maturity structure and the expected evolution of nominal government debt matters. Let us now explain how all these things can be true.

We begin by noting that government securities issued by monetary sovereigns in advanced economies represent nominally risk-free claims to government money.[12] Securities issued at different maturities promise to return the principal at different future dates. In modern day economies, nominal values of outstanding debt are recorded in an electronic ledger accessible by the public. In the United States, all U.S. persons have access to online U.S. Treasury accounts at *Treasury Direct*.[13] There is literally nothing—in principle, at least—from connecting these online accounts to a payment rail permitting Treasury Direct "depositors" to use U.S. Treasury securities as payment instruments.[14] There is, of course, a relatively deep market for U.S. Treasuries at the wholesale level. But this over-the-counter market does not operate seamlessly and, indeed, is occasionally subject to market freezes. While liquidity in this market is enhanced by

[12] At the very least, there is no *economic* reason for a monetary sovereign to default on nominal claims against domestic currency.

[13] The website is located at www.treasurydirect.gov

[14] This would be akin to the Federal Reserve permitting banks to make interbank payments on their Reserve accounts via the payment service *Fedwire*.

the Federal Reserve through its standing repo facilities, its existence does not eliminate all frictions in over-the-counter repo markets.

In light of the discussion above, it is tempting to conclude that government debt is rendered illiquid by design. Aiyagari, Wallace, and Wright (1996) go so far as to assert that any rate of return differential observed between two nominally risk-free government securities must be policy-induced. But whether bond illiquidity exists by accident or by design, it remains a potentially welfare-enhancing property. This is an important observation because the natural inclination (among economists, at least) is to view financial market liquidity as an unambiguously good thing. Bohn (1999) implicitly expresses this view in suggesting that policy should aim to eliminate expected return differentials between bonds only to the extent these differentials are driven by liquidity or market imperfections. This compelling logic is overturned in our environment.[15] An implication of our analysis is that policies designed to enhance financial market liquidity—like the Federal Reserve's standing repo facility—may actually inhibit the ability of intermediaries to manufacture attractive financial products. Pension funds, for example, value the term premium presumably because it lets them structure attractive defined-benefit plans for their clients. Monetary policies that eliminate that premium, say, through yield-curve control, inhibit the ability of pension funds to deliver that service for their clients. In an open economy, domestic pension funds may be compelled to purchase cheaper foreign bonds, boosting expected returns, but at the same time exposing domestic policy holders to foreign exchange risk.[16]

Finally, our paper is also related to a large literature that asks what principles should be employed to determine an optimal inflation target. The answers to this question typically range anywhere in between a moderate deflation and a low, but positive, rate of inflation. Our analysis provides another rationale for favoring a low, but positive, rate of inflation. We speculate that by adding idiosyncratic endowment risk, our rationale for a strictly positive inflation rate would be strengthened for the reason highlighted in Levine (1991) and Molico (2006).

Let us conclude with a few thoughts on future work. We think our framework is sufficiently simple that it may be easily extended to interpret important historical episodes in monetary history. In particular, over the period 1942-51, Federal Reserve monetary policy resembled a form of yield curve control; see Eichengreen and Garber (1991) and Garbade (2020). The Bank of Japan has effectively been engaged in yield curve control for a number of years now. The idea of yield curve control periodically appears as a serious policy proposal, as it did recently in the United States prior to the COVID-19 pandemic. It would be of some practical use, we think, to examine the policy in the context of a

[15]This argument is reminiscent of the one in Jacklin (1987) where the introduction of a competitive secondary market in claims to firm equity can potentially destroy *ex ante* efficient risk-sharing arrangements.

[16]This has been the case, for example, with Japanese pension funds purchasing U.S. Treasury securities.

monetary model with a non-trivial role for government securities of different maturities.

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A Proofs

Proofs of Lemma 1 and Corollary 1. From (28) we can write $\sum_i \pi(i)\hat{m}(i) = \omega - q\theta\hat{M}$. Combining this latter expression with (29) and rearranging, we obtain

$$[1 + (1-\beta)\mu^{-1}]\omega + (1-\beta)z = [1 + q\theta(1 + (1-\beta)\mu^{-1})]\hat{M} \tag{49}$$

Use (30) to substitute out z and we get

$$[1 + (1-\beta)\mu^{-1}]\omega = \left\{\beta(1+q\theta) + \frac{(1-\beta)[1+\theta(q+\beta\delta)]}{\mu} + \frac{\theta(\beta-1)^2}{\mu^2}\right\}\hat{M}$$

Now use (31) to substitute out q and solve for $\hat{M}$:

$$\hat{M} = \frac{\mu\omega(1-\beta+\mu)}{\mu(1-\beta+\beta\mu) + \theta(1-\beta+\mu)[1+\beta[\delta\mu-1)]} \tag{50}$$

Conditions (49) and (50) imply

$$z = \frac{(\mu-1)(1-\beta+\mu)\omega}{(1-\beta+\beta\mu)\mu}$$

It follows that $z \gtrless 0$ when $\mu \gtrless 1$.

Now, combine (28) and (50) to obtain:

$$\begin{aligned} \hat{M} &= \frac{(1-\beta+\mu)\mu\omega}{(1-\beta+\beta\mu)(\mu+\Omega)} \\ \theta &= \frac{(1-\beta+\beta\mu)\Omega}{(1-\beta+\delta\beta\mu)(1-\beta+\mu)} \end{aligned}$$

where $\Omega \equiv 0$ if $\sum_i \pi(i)\hat{m}(i) = \omega$ and

$$\Omega \equiv \left\{\frac{\omega}{(1-\beta+\beta\mu)[\omega - \sum_i \pi(i)\hat{m}(i)]} - \frac{1}{\mu}\right\}^{-1}$$

otherwise.

When $\sum_i \pi(i)\hat{m}(i) = \omega$, $\Omega = 0$ and so $\hat{M} = \frac{(1-\beta+\mu)\omega}{1-\beta+\beta\mu} > 0$ and $\theta = 0$.

When $0 < \sum_i \pi(i)\hat{m}(i) < \omega$, $\Omega > 0$ implies $\mu > (1-\beta+\beta\mu)[1-(1/\omega)\sum_i \pi(i)\hat{m}(i)]$. Given $0 < (1/\omega)\sum_i \pi(i)\hat{m}(i) < 1$, a sufficient condition for $\Omega > 0$ is $\mu \geq 1-\beta+\beta\mu$, which simplifies to $\mu \geq 1$. ■