A Quantitative Theory of Relationship Lending

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A Quantitative Theory of Relationship Lending*

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Abstract

Banks’ loan pricing decisions reflect the fact that borrowers tend to have long-lasting relationships with lenders. Therefore, pricing decisions have an inherently dynamic component: high interest rates may yield higher static profits per loan, but in the long run they erode a banks’ customer base and reduce future profitability. We study this tradeoff using a dynamic banking model which embeds lending relationships using deep habits (“customer capital”) and costs of adjusting loan portfolio composition. High customer capital raises the level and decreases the interest rate elasticity of loan demand. When faced with an adverse shock to net worth, banks with high customer capital recapitalize quickly by charging high interest rates and eroding customer capital in the short term, while banks with low customer capital face persistent financial distress. Using Call Report data to measure the franchise value of banks’ loan portfolios, we find that this effect has strong implications for how individual banks and the financial sector as a whole recover from shocks.

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1 Introduction

It is well known that banks operate in imperfectly competitive markets in their core business activities: deposit taking and loan making (Berger and Hannan, 1998). Banking industry concentration and market power is well documented both internationally (Fernández de Gayvarra et al., 2005) and in the U.S., which has experienced a stark secular decline in the number of banks (Prescott and Janicki, 2006) over the last several decades. Market power in the loan making and deposit taking industries allows banks to generate economic profits by lending at interest rates that exceed the fair (risk-adjusted) cost of capital and by borrowing at interest rates that are lower than prevailing risk-free rates. Such economic profits have long been considered a “feature, not a bug”, as they generate franchise value that curbs risk-taking by banks, thereby promoting financial stability (Demsetz et al., 1996).

In this paper, we study one potential source of market power in the lending market: the fact that banks tend to have long-lasting relationships with lenders. This introduces a strong dynamic component in banks’ loan pricing decisions, as it forces them to internalize the fact that higher interest rates may generate larger profits today at the expense of eroding customer capital and reducing the pool of borrowers tomorrow.

We develop a dynamic, general equilibrium model of heterogeneous banks with two key novel features. First, the loan demand a bank faces depends on its stock of customer capital, modeled as a deep habit on the part of borrowers. Second, the evolution of this stock of customer capital depends on the bank’s current pricing decision. We show that this simple model of customer capital interacts with other standard features of banking models to generate rich dynamics along multiple fronts. In particular, customer capital becomes an important factor affecting the speed at which banks can recapitalize themselves in the face of adverse shocks. When a bank receives a negative shock and has high customer capital, it can “expend” that capital by charging high interest rates in order to weather and adverse shock. Banks with more customer capital therefore recapitalize more quickly than other banks. Customer capital therefore functions as an extra buffer, on top of conventional equity. We show that this force has important implications for the speed of recovery of an economy from a financial crisis.

Empirically, we test these predictions in the cross-section of U.S. banks. Focusing on the financial crisis of 2007-08, we show that banks that faced larger negative shocks, measured in terms of the fall in their market value, experienced larger declines in measures of customer capital. This is consistent with the interpretation of customer capital as an intangible buffer, which banks tend to expend in face of adverse shocks. We then show that banks that lost the most customer capital during the crisis (i.e., banks that “spent” more of this buffer)
also tended to fare better in the post-crisis period, in terms of market value recovery. This result holds when controlling for the initial negative shock (the drop in market value) and other bank-level controls. We also show that this result is robust to the specific measure of customer capital.

**Related Literature**  This paper contributes to three distinct literatures in macroeconomics and finance: customer capital in macroeconomic models, structural models of banking, and empirical studies of the effects of bank market power.

While the dynamics of customer capital can be related to an older literature on consumption habits, a seminal formalization of customer capital in a macroeconomic model is the work of Gourio and Rudanko (2014). In the context of nonfinancial firms, Gilchrist et al. (2017) argue that the interaction between customer capital dynamics and financial constraints was key to explain the dynamics of inflation in the U.S. during the Great Recession. Our modeling of customer capital dynamics is reminiscent of theirs, also in the context of a model of heterogeneous firms. We argue that customer capital interacts with capital constraints that are specific to the banking industry. This is critical to understanding dynamics around recent recessions, since the aggregate capitalization of the banking sector has been argued to be a relevant state variable for macroeconomic performance (Adrian and Boyarchenko, 2012).

We study the effects of bank customer capital from a positive perspective in the context of a dynamic equilibrium model of heterogeneous banks that take deposits, make loans, and face constraints that depend on their net worth. We therefore contribute to an emerging literature that employs the tools of heterogeneous agent macroeconomic models to study questions that are related to the banking industry. Bigio and Bianchi (2014) use a quantitative model with heterogeneous banks and liquidity frictions in the interbank market to study monetary policy implementation. Corbae and D’Erasmo (2019) use a quantitative model of heterogeneous banks where size is correlated with market power to study the effects of capital requirements. We take these requirements as given, and study how their interaction with customer capital affects the overall stability of the banking system. (Neri et al., 2010) introduce a monopolistically competitive banking sector in an otherwise standard monetary DSGE model, and study how this affects the transmission of standard shocks.

Finally, our paper relates to a broader empirical literature that studies the efficiency and stability consequences of banking market power and concentration. Recent work on this topic has been focused on market power on the deposits market. Egan et al. (2017) use detailed branch-level data on deposit quantities and prices to estimate a demand system for secured and unsecured deposits at large U.S. banks. These estimates are then combined with a dynamic model of bank runs, which allows them to study the probabilities of counterfactual
runs on these large banks during the financial crisis. Drechsler et al. (2017) argue that bank market power in the deposit markets gives rise to a new channel of transmission for monetary policy in the U.S.

We focus instead on customer capital on the loan side of the balance sheet. Our interpretation of bank customer capital is closely linked to the notion of relationship lending, the fact that both banks and borrowers find it worthwhile to maintain long-standing relationships. A long-standing literature has found that banks smooth loan rates when faced with adverse cost of funding shocks (Berger and Udell, 1995; Berlin and Mester, 1998). There is also an extensive theoretical literature that derives conditions under which the optimal contract between a lender and a borrower shares some of those features under a variety of frictions, such as asymmetric information, search frictions, or switching costs. We take a different approach, by taking the lending contract and the process for customer capital dynamics as given, and instead study their macroeconomic implications.

2 A Model of Relationship Lending

We consider a model economy populated by a unit continuum of monopolistically competitive banks \( j \in [0, 1] \) and a representative borrower. Time is discrete and infinite, and there is a single good. The risk-free rate is \( r_t = \tau \) for all periods, which defines a risk-free discount price of \( q = (1 + \tau)^{-1} \), and the wage rate is \( w_t = w \) in all periods. Both these prices are specified exogenously. Proofs of all propositions are contained in Appendix (B).

2.1 Borrower

The representative borrower is a firm who operates a production technology which takes labor, \( n \), as its sole input and produces \( y = n^\alpha \) units of output, where \( \alpha \in (0, 1) \) is a parameter determining the firm’s decreasing returns to scale. Each period, this firm chooses how much labor at wage \( w \) to hire and how much to borrow both in aggregate and from each bank \( j \). The firm is subject to a working capital constraint: total lending must be at least a fraction \( \kappa \geq 0 \) of its total wage bill. We denote the firm’s total loan demand today by \( L' \), and the distribution of borrowing across banks by \( L' = \{\ell'_j\} \), where \( \ell'_j \) is the face value of this period’s loan from bank \( j \). The discount price of a loan from bank \( j \) is \( q_j \), and we denote the set of loan prices across banks by \( Q = \{q_j\} \).

We model lending relationships in the following way. For each bank \( j \), we summarize the borrower’s relationship or “habit” at that bank by \( s_j \), and denote the set of habits across all banks by \( S = \{s_j\} \). Habits determine adjustment costs: specifically, we assume that the
firm faces quadratic costs (with scale parameter $\phi \geq 0$) of adjusting its share of total lending at a given bank relative to its relative habit. We assume that habits are formed externally, in the sense that borrowers take the distribution of habits as given each period and do not internalize the effect of their choice of current loan demand on their evolution.

In what follows, it will be useful to define the relevant state variable for the borrower in this economy, which is the joint density of prices and habits across banks, $\mu(q, s) = \{q_j, s_j\}$. Combining these features, we can write the dynamic optimization problem of the firm as

$$V(\ell(q, s); \mu) = \max_{n, L', \{\ell'(q, s)\}} n^{\alpha} - wn + L' - \int \ell(q, s)d\mu(q, s)$$

subject to

$$\kappa wn \leq L'$$

$$L' \leq \int q\ell'(q, s)d\mu(q, s)$$

The firm’s objective function reflects the expected discounted net value of profits, which are net operating income $n^{\alpha} - wn$ plus net borrowing (new loans less repayments) and less adjustment costs, discounted at the risk free rate. The first constraint is the working capital constraint, and the second defines total loan demand as the total quantity of funds obtained from all banks. Note that the borrower does not take into account its choice of loan portfolio today on habits tomorrow: hence why we call our habits “external.”

We can establish the following results:

**Proposition 1. (Loan demand)** Bank-specific loan and aggregate demand satisfy

$$\frac{q\ell'(q, s; \mu)}{L'(\mu)} = \frac{s}{S(\mu)} - \frac{\bar{q}}{\phi} \left(\frac{1}{q} - R(\mu)\right)$$

$$L'(\mu) = \kappa w \left[\frac{\alpha/w}{1 + \kappa\bar{q}(R(\mu) + P(\mu)) - \kappa}\right]^{\frac{1}{1-\alpha}}$$

To justify this assumption, it is perhaps natural to appeal to a continuum of identical firms $i \in [0, 1]$, and define $s_j = \int s_{ij}d\mu$. It is in principle possible to accommodate “internal habits,” but this cannot be done in closed form and therefore becomes intractable when paired with the endogenous distribution of banks described below.
where $S(\mu)$, $R(\mu)$, and $P(\mu)$ are sufficient statistics which summarize $\mu(q,s)$:

$$
R(\mu) = \mathbb{E} \left[ \frac{1}{q_j} \right] = \int \frac{1}{q} d\mu(q,s) \quad \text{(6)}
$$

$$
S(\mu) = \mathbb{E} [s] = \int s d\mu(q,s) \quad \text{(7)}
$$

$$
P(\mu) = \text{Cov} \left( \frac{s}{S(\mu)}, \frac{1}{q} \right) - \frac{1}{2} \frac{\varphi}{\mu} \text{Var} \left( \frac{1}{q} \right) \quad \text{(8)}
$$

Equation (4) defines the demand curve faced by each bank with habit $s$ charging price $q$ as a function of three key aggregates $\{R, S, L'\}$ defined in (6), (7), and (8). Equation (5) determines the borrower’s aggregate loan demand taking as given the joint distribution of loan prices and habits across banks, summarized by $\mu$.

Proposition 1 has several properties worth noting. The loan demand at a given bank (4) is: (i) decreasing in the loan rate spread over the benchmark $\tau = q^{-1} - R$, with elasticity governed by the risk free rate and the adjustment cost; and (ii) increasing in the relative habit $s_j/S$. The first effect is a standard price effect: when a bank’s loans are cheaper, fixing total demand for funds and the prices at all other banks, that bank’s loan demand will increase. The second effect reflects the fact that existing lending relationships increase loan demand, all else equal. Finally, our specification also implies that habits affect the price elasticity of loan demand. Not only do banks with higher customer capital $s$ experience a higher level of loan demand, but also they face lower price elasticity of loan demand. This endows them with greater effective market power.

Moreover, Proposition 1 implies that aggregate loan demand depends not only prices – captured by the $R$ term –, but also on the joint distribution of prices and habits – captured by the $P$ term. This latter term has an intuitive interpretation: to the extent that there is dispersion in spreads the borrower can borrow more by going to cheaper lenders (variance term), but to the extent that spreads and customer capital are correlated, this has a dampening effect on aggregate loan demand. These results are summarized in the following proposition:

**Proposition 2. (Elasticities)** The elasticity of aggregate loan demand with respect to the covariance-adjusted average interest rate $R + P$ is

$$
\epsilon(L', R + P) = -\frac{\kappa \beta (R + P)}{1 - \kappa + \kappa \beta (R + P)} < 0. \quad \text{(9)}
$$
The elasticities of bank specific loan demand \( q' \) with respect to the interest rate spread \( \tau = 1/q - R \) and customer capital \( s \), respectively, are given by

\[
\epsilon(q', \tau) = -\frac{\beta \tau}{\phi_s^2 - \beta \tau} = -\frac{\beta \tau}{\phi q' L'} < 0 \quad (10)
\]

\[
\epsilon(q', s) = \frac{\phi_s}{\phi_s - \beta \tau} \quad (11)
\]

Further, we have \( \lim_{\phi \to 0} \epsilon(q', \tau) = +\infty \) and \( \lim_{\phi \to 0} \epsilon(q', s) = 0 \).

2.2 Banks

Balance sheet and lending Each bank \( j \) uses retained earnings, newly issued equity \( e_j < 0 \) (paid dividends if \( e_j > 0 \)), and deposits \( d_j \geq 0 \) to make risky loans \( \ell'_j \) at discount price \( q_j \) (interest rate \( r_j = 1/q_j - 1 \)) and invest in riskless securities \( a'_j \) at discount price \( \bar{q} \). Deposits are risk-free (insured) and issued at price \( q^d = \bar{q}(1 + \nu) \), where \( \nu > 0 \) is a small liquidity premium capturing a convenience yield on bank deposits that we take as exogenous. Each bank \( j \) has a maximal quantity of deposits \( d'_j \) that it can issue, and chooses actual deposits \( \delta'_j \leq d'_j \). The bound \( d'_j \) follows a Markov process, \( \pi_d(d'|d) \). In order to lend \( \ell'_j \), the bank must finance \( q_j \ell'_j \); then, tomorrow, the bank will receive payments of \( z'_j \ell'_j \), where \( z'_j \) is an idiosyncratic shock which shifts realized loan returns. Loan returns are persistent, drawn from a Markov process, \( \pi_z(z'|z) \). We denote the beginning of period net worth for bank \( j \) by

\[
n_j = z_j \ell_j + a_j - \delta_j. \quad (12)
\]

Preferences and regulation Banks value dividends according to the function \( \xi(e_j) \), which has the minimal property that \( \xi'(e_j) > 0 \) for all \( e \in \mathbb{R} \). The opportunity cost of funds to the bank is the risk-free rate \( \bar{q} \), and so the risk-neutral bank discounts the future at the rate \( \bar{q} \). In addition, banks face a regulatory capital constraint. This constraint specifies that the bank’s total lending, scaled by a factor \( \chi \), may not exceed its total net worth, reflecting the current period’s lending and financing decisions:

\[
\chi q_j \ell'_j \leq q_j \ell'_j + \bar{q} a'_j - q^d \delta'_j \quad (13)
\]

We assume that each bank is monopolistically competitive, setting its loan price taking the demand function of the borrower as given. Finally, we assume that banks exit with exogenous i.i.d. probability \( \pi \) each period and are replaced with identical “clones” at the start of the

\[^2\text{It may be further desirable to assume that } \xi(\cdot) \text{ is concave, at least over } e < 0; \text{ more on this below.}\]
next period.

**Bank problem** It will be useful to analyze banks’ decision problems recursively. Therefore, denote the state variable of bank \( j \) by

\[
x_j = (\ell_j, a_j, s_j, \delta_j, d_j', z_j).
\]

The first four state variables are endogenous; the last two are exogenous. Since banks are only differentiated by their state variables, we drop the superscript \( j \) in what follows. We define the mass of banks with state \( x \) by \( m(x) \).

Let \( X = \{L', S, R\} \) be a set of aggregates which specify the parameters of the bank-specific loan demand function (4). Taking this set and the function itself as given, the bank solves

\[
V_B(\ell, a, \delta, s; d', z) = \max_{q, e, \ell', a', \delta', s'} \xi(e) + \bar{q}(1 - \pi)\mathbb{E} [V_B(\ell', a', \delta', s'; d'', z')] 
\]

subject to

\[
q\ell' + \bar{q}a' + e \leq q^d\delta' + (1 + z)\ell + a - \delta \quad (15)
\]

\[
q^d\delta' \leq (1 - \chi)q\ell + \bar{q}a' \quad (16)
\]

\[
0 \leq \delta' \leq d', a' \geq 0, \ell' \geq 0 \quad (17)
\]

\[
s' = (1 - \rho)q\ell' + \rho s \quad (18)
\]

\[
\ell' = \ell(q, s; X) \quad (19)
\]

The optimal policies for the control variables associated with solving this problem are denoted \( g_y(x) \) for \( y \in \{q, e, \ell', \delta', a', s'\} \). Note that in certain instances (and in computation), it is desirable to reduce the state variable of the bank by replacing \( \ell, d, \) and \( a \) with \( n \) according to equation (12).

The bank’s objective function (14) reflects its valuation of dividends, \( \xi(\cdot) \), and discount rate \( \bar{q} \). Expectations are taken with respect to loan return and maximal deposit shocks. Constraint (15) is the bank’s balance sheet identity: total outflows (loan issuances, dividends, riskless savings, and repayments on deposits) cannot exceed total inflows (realized loan returns, savings returns, and new deposits. Constraint (16) is the capital requirement, which states that the value of the bank’s assets (its loans) cannot exceed a multiple \( \chi \) of its net worth, its assets minus liabilities. Constraint (17) enforces the upper bound on deposits, and lower bounds on deposits, loans, and securities. Constraint (18) critical: it captures the evolution of customer capital, which the banks – unlike the borrower – internalize. Finally constraint (19) imposes the relationship between loan demand, price, and customer capital.
We can establish the following result about the bank’s problem:

**Proposition 3. (Bank financing and lending)** Under the assumption that \( q^d > q \) (i.e. that \( \nu > 0 \)), banks always use all of their deposits, \( \delta' = d' \). Furthermore, in steady state banks’ loan pricing decisions are determined by the Euler equation

\[
\phi \frac{q_t \ell_{t+1}}{\beta L_{t+1}} \overline{q}(1 - \pi) \mathbb{E}_t \left[ (1 + z_{t+1}) \xi'(e_{t+1}) \right] = \rho \left( \lambda_t (1 - \chi) + \frac{\overline{q}}{q_t} (1 - \pi) \mathbb{E}_t \left[ (1 + z_{t+1}) \xi'(e_{t+1}) \right] - \xi'(e_t) \right) \\
+ (1 - \rho) \sum_{i=0}^{\infty} (\rho \overline{q} (1 - \pi))^i \mathbb{E}_t \left[ \lambda_{t+i} (1 - \chi) + \frac{\overline{q}}{q_{t+i}} (1 - \pi) (1 + z_{t+i+1}) \xi'(e_{t+i+1}) - \xi'(e_{t+i}) \right]
\]

where \( \lambda_t \) is the multiplier on the capital requirement in period \( t \) and \( \epsilon(q^\ell, q) \) is the elasticity of bank-specific loan demand with respect to the loan price.

While a long expression, equation (20) has an intuitive interpretation. Each term on the right hand side represents the net marginal benefit to the bank of lending more (i.e. raising its loan price) in a given period. This includes three terms: (i) the benefit of easing of the capital requirement; (ii) the benefit of receiving the per-unit return on loans on an additional unit of loans; and (iii) the cost of foregone dividends implied by the additional lending. Because of the persistence of customer capital and the link between its evolution and loan pricing decisions, banks weight current and future marginal benefits according to the persistence of customer capital: they weight the present at \( \rho \), and the discounted sum of all present and future periods at \( 1 - \rho \).

The left hand side reflects the fact that banks can extract markups due to the finite price elasticity of bank-specific loan demand to the extent that \( \phi > 0 \) (top line). In the perfectly competitive limit, when \( \phi = 0 \) so that there are no relationships, this elasticity is infinite, and the net benefits on the right hand side must be equal to zero, regardless of how persistent customer capital is.

**Evolution of bank distribution** We summarize banks’ state variables by \( x = (\ell, a, \delta, s, d', z) \). Let \( \mathcal{L}, \mathcal{D}, \mathcal{A}, \mathcal{S}, \mathcal{Z} \subseteq \mathbb{R} \) be subsets of the possible regions of loan supply, deposits, customer capital and demand / return shocks for the current period, and let these objects with primes denote the same for the next period. To ease notation, define \( \mathcal{X} = (\mathcal{L}, \mathcal{D}, \mathcal{D}, \mathcal{A}, \mathcal{S}, \mathcal{Z}) \) and
\( \mathcal{X}' = (\mathcal{L}', \mathcal{D}', \mathcal{A}', \mathcal{S}', \mathcal{Z}') \). The distribution of banks evolves according to the operator

\[
(T^* m)(\mathcal{X}'; X) = \int_{\mathcal{X}'} \left\{ \int_{\mathcal{X}} 1 \left[ \ell' = g_{\ell'}(x; X), \delta' = g_\delta(x; X), a' = g_{a'}(x; X) \right] \pi_d(d'|d) \pi_z(z'|z) \right\} \times \int_{\mathcal{X}'} \left[ s(x') = (1 - \rho) g_q(x; X) \ell(g_q(x; X), s(x); X) + \rho s(x) \right] dm(x; X) \right\} dx'.
\]

2.3 Equilibrium

Our equilibrium definition proceeds in two phases. First, we define an equilibrium within the banking industry, taking bank-specific loan demand as given. An industry equilibrium clears the loan market for the proposed aggregate demand and generates an endogenous distribution of banks, and therefore of prices and customer capital levels. Second, we define a general equilibrium, in which the borrower takes this distribution as given and chooses aggregate loan demand. The general equilibrium selects the banking industry equilibrium consistent with aggregate loan demand.

2.3.1 Banking industry equilibrium

Definition 1. A steady state banking industry equilibrium given a bank-specific loan demand function \( \ell(q, s; X) \) and demand parameters \( X = \{ L', S, R \} \) corresponding to aggregate loan demand, aggregate customer capital, and a benchmark interest rate is a list of:

1. bank optimization: taking \( X \) and \( \ell(q, s; X) \) as given, the value function \( V_B(x; X) \) solves the bank problem (14) through (19) with associated optimal policies \( g_y(x; X) \);
2. stationarity: the distribution \( m(x; X) \) is a fixed point of the \( T^* \) operator (21) given optimal policies \( g_y(x; X) \); and
3. consistency: the aggregates \( X \) are consistent with the distribution \( m(x; X) \) and the

\(^3\)Note we can also define, residually, policies for \( e, \ell', s', \delta' \), etc. I focus here on the ones we explicitly solve for (see note on solving the bank problem).
optimal policies $g_q(x;X)$:

\[
L'_S(X) = \int g_q(x;X) \ell (g_q(x;X), s(x); X) \, dm(x; X) \\
S(X) = \int s(x) \, dm(x; X) \\
R(X) = \int \frac{dm(x; X)}{g_q(x;X)}
\]

4. **market clearing**: aggregate loan supply equals aggregate loan demand:

\[L'_S(X) = L'\]  \hspace{1cm} (25)

We can establish two important results about banking industry equilibria. The first allows us to simplify the steady state analysis using consistency conditions.

**Proposition 4. (Stationarity conditions)** For any stationary distribution $m(x;X)$ implied by a set of demand parameters $X$, $S(X) = L'_S(X)$. Furthermore, in any steady state banking industry equilibrium, 

\[P(X) = -\frac{\rho}{2} \beta \varphi \text{Var}(\tau),\]

where $\text{Var}(\tau)$ is the cross-sectional dispersion of interest rate spreads implied by the equilibrium distribution and optimal pricing policies.

The second result establishes a fixed point procedure useful for computation.

**Proposition 5. (Fixed point for industry equilibrium)** Let the bank-specific loan demand function $\ell(q, s; X)$ be given by (4) and let $X = \{L'_S, S, R\}$ that satisfies $S = L'_S$ be given. Let the banking industry equilibrium aggregates implied by banks’ optimizing behavior under $X$ be denoted $X^*(X)$. Then a banking industry equilibrium is a fixed point of $X^*$.

### 2.3.2 General equilibrium

**Definition 2**. A **steady state general equilibrium** is a list of: (i) a joint distribution of prices and customer capital, $\mu(q, s)$; (ii) bank-specific loan demand $\ell(q, s; X)$; (iii) aggregate loan demand $L'_D(\mu)$; (iv) a distribution of banks $m(x;X)$; (v) a set of banking industry pricing policies $g_q(x;X)$; and (vi) aggregates $X = \{L'_S, S, R\}$ such that:

1. **banking industry equilibrium**: the distribution $m(x;X)$, policies $g_q(x;X)$, and aggregates $X$ comprise a banking industry equilibrium, taking the bank-specific loan demand function as given;
2. **consistency:** the joint distribution of prices and customer capital is consistent with the banking industry equilibrium:

\[
\mu(q, s) = \int \mathbf{1} \{q = g_q(x; X), s = s(x)\} \, dm(x; X);
\]  

(26)

3. **borrower optimization:** taking \(\mu(q, s)\) as given, bank-specific and aggregate loan demand \(\ell(q, s; \mu)\) and \(L'_D(\mu)\) satisfy (4) and (5), respectively; and

4. **market clearing:** aggregate demand for loans equals aggregate supply for loans:

\[L'_D(\mu) = L'_S(X).\]

### 3 Quantitative Properties of the Model

This section describes the key quantitative properties of the model. We first describe a simple benchmark calibration. Then, we describe bank decision rules to highlight the ways in which bank behavior varies by level of customer capital. Then, we consider simulations of shock realizations for specific banks to demonstrate how customer capital changes banks’ decisions through time. Finally, we use the equilibrium distribution of banks to compute key cross-sectional moments of the model.

In each subsection, we consider several model variants in addition to the baseline model presented in Section 2. First, we consider a model in which there is no customer capital (“No CC”). This version of the model can be thought of as setting \(\phi = 0\). In this version of the model, the state variable \(s\) is meaningless. Second, in order to highlight the role of critical customer capital parameter in the model, we consider an additional case with a higher customer capital elasticity of demand, \(\phi\).

### 3.1 Calibration

Table 1 details the parameters used for our quantitative analysis. The period of our model is quarterly.

**Bank preferences and regulation** The risk-free rate is set to be 3.41% in annualized terms, and the bank’s discount rate \(\bar{q}\) is set to be consistent with that rate at the quarterly frequency of the model.\(^4\) We assume that the liquidity premium \(\nu = 17\) bps to reflect banks’

\(^4\)Specifically, for a net interest rate in annualized terms \(r\), \(\bar{q} = (1 + r)^{-1/4}\).
Table 1: Parameters

Notes: The model period is quarterly. All functional forms for parameters are described in Section 3.1. Note that this parameterization, in the current draft, is intended as a numerical illustration. Subsequent work will discipline these parameters using empirical moments.

low funding costs. Banks’ dividend valuation / equity issuance cost function is given by

\[ \xi(e) = \bar{\xi} \left( 1 - \exp \left\{ -\frac{e}{\xi} \right\} \right) \]

where \( \bar{\xi} = \begin{cases} \xi^- & \text{if } e < 0 \\ \xi^+ & \text{if } e \geq 0 \end{cases} \) (27)

This flexible functional form allows for convex costs of equity issuance while still imposing smoothness at the (potential) kink at \( e = 0 \) (since \( \lim_{e \uparrow 0} \xi'(e) = 1 = \xi'(0) \) regardless of whether \( \xi^- = \xi^+ \)). Furthermore, it nests the case of pure linearity in preferences, since \( \lim_{\xi \to \infty} \xi'(e) = 1 \) for all \( e \).
Loan demand and customer capital evolution  Our adjustment cost parameter $\phi$ is consistent with an spread elasticity of loan demand of about $-4$. This reflects the fact that loans are highly substitutable, and therefore there is a substantial degree of price sensitivity. The elasticity of loan demand with respect to customer capital is sizable, but 5 times smaller than the interest rate elasticity. This implies that while customer capital or habits play a role, borrower behavior is more sensitive to prices than habits. Turning to the persistence of customer capital, we assume that $\rho = 0.81$ so that customer capital is highly persistent, reflecting the “stickiness” of lending relationships that is well documented in the empirical banking literature.

Shock processes  Loan returns and deposits follow a standard AR(1) process in logs:

$$\log x' = (1 - \rho_x) \log \mu_x + \rho_x \log x + \epsilon'_x, \text{ with } \epsilon_x \sim N(0, \sigma_x) \text{ for } x \in \{z, d\}$$  (28)

where $\epsilon$ is a normally distributed, i.i.d. shock with mean 0 and variance $\sigma^2_x$ and $\rho_x$ is a persistence parameter. These processes is discretized using the method of Adda and Cooper into $N_z = 5$ states.

3.2 Aggregate moments across model economies

In this section, we use the stationary distribution of the model to aggregate across banks and compare long run outcomes across model economies. Table 2 presents some key moments of the stationary equilibrium for different versions of the model. We consider three economies: our baseline, one in which there are lower costs to adjusting lending relationships, and one in which lending relationships are even more persistent. Note two key features of our baseline equilibrium: considerable interest rate dispersion and a positive but moderate correlation between customer capital and bank net worth.

When it is less costly for borrowers to adjust their portfolio of loans, banks have lower effective market power – recall that in the limit as $\phi \to 0$, our economy becomes perfectly competitive. Therefore, the average interest rate declines, as well as the amount of dispersion in each of interest rates, loans, and customer capital. Being less profitable, banks increase their capital buffers, and aggregate net worth increases.

Turning to the final column of the table, we see that raising the persistence of customer capital reduces the punishment from charging high spreads today because it limits the extent to which this pricing strategy erodes customer capital. As a result, the average interest rate increases sharply. These more profitable banks hold much less capital, with aggregate bank net worth dropping by half.
<table>
<thead>
<tr>
<th>moment</th>
<th>baseline</th>
<th>low $\phi$</th>
<th>high $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>loan market</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average interest rate</td>
<td>1.99</td>
<td>1.87</td>
<td>2.23</td>
</tr>
<tr>
<td>IR dispersion</td>
<td>0.72</td>
<td>0.32</td>
<td>0.68</td>
</tr>
<tr>
<td>total loan volume</td>
<td>0.40</td>
<td>0.40</td>
<td>0.39</td>
</tr>
<tr>
<td>loan volume dispersion</td>
<td>0.03</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>bank distribution</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>avg bank net worth</td>
<td>0.14</td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td>bank net worth dispersion</td>
<td>0.04</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>CC dispersion</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>corr(CC, net worth)</td>
<td>0.37</td>
<td>0.72</td>
<td>-0.11</td>
</tr>
<tr>
<td>loans / book equity</td>
<td>3.09</td>
<td>2.88</td>
<td>7.25</td>
</tr>
</tbody>
</table>

Table 2: **Aggregate moments**

**Note:** Moments are computed using the equilibrium distribution in each economy. The average interest rate is weighted only by the mass of banks charging each interest rate. The loan-weighted average interest rate weights instead by total loan volume. Interest rates are reported in annualize terms.

### 3.3 Response to an aggregate shock to bank net worth

In this section, we consider the effects of an unanticipated shock that wipes out 10% of banks’ net worth across two model variants; the baseline model from above, as well as a variant in which pricing is competitive and there are no relationships or customer capital. These results are presented in Figure 1.

In both cases, the shock induces an immediate rise in the average interest rate on loans. The magnitude of this surge, however, is largest in the competitive economy. This is because banks in the baseline model prefer, on the margin, to keep prices closer to their steady state levels to avoid any loss of customer base, which will have persistent costs. At the same time, though, prices stay further above their steady state levels for longer in the baseline model than in the competitive model. That is, banks choose to dampen the adjustment costs on impact, but spread them out more evenly over the recovery. These patterns are mirrored in total lending in the bottom left panel of the figure.

Next, consider banks’ recoveries. By construction, the top left panel shows that the initial drop in net worth is identical across the two economies. The baseline model, however, features a more sluggish recovery in net worth than the competitive model. This is because the relationship feature prevents banks from recapitalizing quickly by increasing loan rates sharply on impact. This is further reflected in total dividend payouts in the bottom right panel. In the competitive model, banks choose to incur equity issuance costs and cut dividends sharply on impact in order to finance the lending that will help them to recapitalize.
Figure 1: Aggregate shock to bank net worth
more quickly. Thereafter, banks cut their dividends less relative to steady state than in the relationship model, in which the cut in equity payouts is more persistent.

4 Empirical Analysis

We begin our analysis by documenting some facts about the joint behavior of bank market value and measures of bank customer capital in recent decades, with special emphasis on the recent financial crisis of 2007-08. We are particularly interested in studying how changes in bank market value and customer capital interact when banks are exposed to large shocks.

We hypothesize, based on the model presented in Section 2 below, that banks should expend customer capital (CC) to reduce the effects of shocks to more traditional measures of equity, such as book or market value. For example, a bank that has earned a large customer base by charging relatively low interest rates and operating at slimmer margins may respond to a capital crunch by raising rates and temporarily increasing their profit margins. This may cost the bank some customers on the margin, but may be worthwhile to weather an adverse shock. Banks that expend more CC should then be more resilient, i.e., their value should recover more quickly/by more following such shocks.

To test this claim, we need then to measure the following concepts:

1. **Shocks.** As a measure of the negative shocks experienced by banks, we use the change in market capitalization of a given bank between the end of the financial crisis (2009Q4) and its beginning (2006Q4).

2. **Resilience.** As a measure of resilience, we measure the change in market capitalization of a given bank between the end of the financial crisis (2010Q1) and a later period (2013Q1, three years later, to be consistent with the measure of shock).

3. **Customer Capital.** We employ two alternative measures of customer capital, (i) an estimate of the fair value of the bank’s loan portfolio, and (ii) the Lerner index. These two measures are described in more detail in the following subsections. We measure the change in customer capital during the crisis period, i.e. from 2006Q4 to 2009Q4.

4.1 Data sources

Our analysis is done at the bank holding company (BHC) - quarter level. Our primary source of data are the Consolidated Financial Statements for Holding Companies (FR Y-9C report), which include quarterly data on balance sheet and income items for BHC’s in the US. We complement this data with stock market prices and returns from CRSP. After applying some
standard sample selection criteria\(^5\) This leaves us with an unbalanced panel of 1,251 BHC’s, ranging from 1986Q3 to 2019Q4, for a total of 50,361 observations. Finally, we complement this data with time series aggregates taken from the Federal Reserve Bank of St. Louis FRED database.

### 4.2 Measuring customer capital

We construct two measures of CC: a measure of the fair value of a BHC’s loan portfolio, and the Lerner index. Both measures involve similar data inputs and yield similar results. The goal of the measurement exercise is to construct a proxy for the economic value in excess of the book value of a BHC’s loan portfolio.

**Fair value of loan portfolio (FVL)** This measure follows closely the methodology in Atkeson et al. (2019). In this paper, the authors develop a simple valuation model of banks’ core businesses – loans and deposits – that can be easily taken to the data. We extend the authors’ original work, which applies this model to the aggregate US banking system, and apply their methodology to the cross-section of US banks. The methodology is explained in more detail in A. The key output of this measurement exercise is a measure \(v_L\) of the fair value of a bank’s loan business per dollar of loan book value. For a bank \(i\) in quarter \(t\), this is computed as

\[
v_{L,\text{it}} = \frac{r_{L,\text{it}} + \mu_{L,\text{it}}}{\delta_{L,\text{it}}} = \frac{r_{L,\text{it}}}{i_t + \mu_{L,\text{it}} + \delta_{L,\text{it}}}
\]

\(^5\)We keep only BHC-quarter observations that are matched with CRSP, meaning that we only have publicly traded BHC’s in our sample. We drop BHC-quarter observations for which key income and expenditure items are missing, as well as observations for which book equity or market capitalization are either missing or non-positive.
where $r_{L,it}$ is interest income per dollar of loan, $\mu_{L,it}$ is the prepayment rate of the bank’s loan portfolio, $\delta_{L,it}$ is the default rate of the bank’s loan portfolio, and $i_t$ is the discount rate of the bank’s shareholders. $r_{L,it}$ and $\delta_{L,it}$ can be readily measured from the FR Y-9C. Unfortunately, there is no readily available data on duration, maturity, or prepayment rates of BHC assets (only for individual banks). For this reason, we set $\mu_{L,it}$ to a constant $\mu_L$ that reflects an average loan maturity of 3.5 years, an average value over the 1997-2014 period as per Drechsler et al. (2018). Finally, we set the bank’s shareholders discount rate to be equal to the 5-year Treasury rate as in Atkeson et al. (2019). The data inputs along with their sources are summarized in Table 3.

**Lerner index** The Lerner Index has been widely used in banking as a measure of market power, and employs similar inputs to those above. In this application, we closely follow Jiménez et al. (2013) and define the Lerner index for a BHC at a given point in time as

$$\ell_{it} = \frac{r_{L,it} - r_{C,it}}{1 + r_{C,it}}$$
where $r_{L,it}$ is a direct measure of loan revenue as above, and $r_{C,it}$ is a proxy for a bank’s cost of capital. The Lerner index can therefore be seen as a measure of the average markup charged by a bank on its loan portfolio. While $r_{L,it}$ is straightforward to measure as shown above, $r_{C,it}$ requires some additional assumptions: it can be inferred from a simple no arbitrage condition, as the interest rate that makes the bank indifferent between originating a dollar loan and investing a dollar at some prevailing interbank rate. Let $r_t$ denote such interbank rate, let $p_{it}$ denote the expected probability of default of a given loan, and let $LGD_{it}$ stand for the loss given default of the loan. Then, $r_{C,it}$ should obey the following indifference condition

$$(1 - p_{it}) (1 + r_{C,it}) + p_{it} \times LGD_{it} = 1 + r_t$$

which implies

$$r_{C,it} = \frac{1 + r_t - p_{it} \times LGD_{it}}{1 - p_{it}} - 1$$

We set the expected default rate to be equal to the average charge-off rate faced by the BHC at a given point in time, $p_{it} = \delta_{L,it}$. In the absence of good measures of $LGD_{it}$, we follow Jiménez et al. (2013) and set this number equal to 0.45. Finally, we use the 3-month LIBOR as a measure of $r_t$, as our banking data is quarterly.

**Summary** All data inputs along with their sources are summarized in table 3. Figure 2 plots the median paths for these two measures over our sample, showing that they are quite highly correlated. Table 4 presents some summary statistics for the sample. As is standard for the U.S. banking sector, the size distribution of banks in our sample is highly skewed. Banks tend to have relatively low values of both book and market equity to total assets. In terms of our main measures of customer capital, there is considerable variation over the distribution of banks. We utilize this variation in our regression analyses below.

### 4.3 Customer capital during the financial crisis

Armed with these measures of customer capital, we proceed to study the joint behavior of changes in bank market value and CC. First, we study how the change in CC varied in the cross-section with the size of the shock received by the bank, measured as the change in market value between the beginning and the end of the crisis. To that end, we first estimate the following specification

$$\Delta_{09Q4-06Q4} \log CC_i = \alpha + \beta \Delta_{09Q4-06Q4} \log MV_i + \gamma X_{06Q4,i} + \epsilon_i$$  \hspace{1cm} (29)
Table 4: **Summary statistics for selected variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>mean</th>
<th>5th percentile</th>
<th>50th percentile</th>
<th>95th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>assets, USD B</td>
<td>22.45</td>
<td>0.246</td>
<td>1.517</td>
<td>54.02</td>
</tr>
<tr>
<td>loans / assets</td>
<td>0.634</td>
<td>0.399</td>
<td>0.652</td>
<td>0.809</td>
</tr>
<tr>
<td>deposits / liabilities</td>
<td>0.854</td>
<td>0.644</td>
<td>0.884</td>
<td>0.984</td>
</tr>
<tr>
<td>book equity / assets</td>
<td>0.096</td>
<td>0.058</td>
<td>0.091</td>
<td>0.143</td>
</tr>
<tr>
<td>market value / assets</td>
<td>0.143</td>
<td>0.036</td>
<td>0.128</td>
<td>0.265</td>
</tr>
<tr>
<td>Lerner index</td>
<td>0.009</td>
<td>0.004</td>
<td>0.010</td>
<td>0.015</td>
</tr>
<tr>
<td>Fair Value of Loans</td>
<td>1.071</td>
<td>1.001</td>
<td>1.071</td>
<td>1.139</td>
</tr>
<tr>
<td>observations</td>
<td>50,361</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Statistics computed using Call Report data as described in the main text. The sample period is 1986Q3 through 2019Q4. All ratio values are computed at the individual bank level, and means and percentiles are computed by comparing across banks.

where \( \log CC_i \) is the log of a measure of customer capital, \( \log MV_i \) is the log of market capitalization, and \( X_i \) is a vector of bank-specific controls that includes bank characteristics in 2006Q4: the log of size as measured by total assets, loan-to-asset ratio, deposit-to-liability ratio, equity-to-assets ratio (book leverage), and the level of customer capital in 2006Q4.

The resulting estimates are shown in Table 5 for the FVL (columns (1) through (3)) and Lerner index (columns (4) through (6)) measures. For each independent variable, each column expands the set of controls included in \( X_i \); regardless of the set of included controls, there is a robust and statistically significant positive relationship between the change in market value and the change in customer capital in the cross-section of US BHCs. This means that BHCs that lost more market value during the 2006Q4-09Q4 period also tended to register larger losses in measures of customer capital. Consistent with the proposed mechanism, we take this as evidence that banks treat customer capital as an additional asset which can be expended in order to weather a financial shock.

We then proceed to study how changes in a BHC’s customer capital relate to subsequent growth in market value, in a period of equal length after the crisis (2010Q1-2013Q1). To that end, we estimate the following specification

\[
\Delta_{13Q1-10Q1} \log MV_i = \alpha + \delta \Delta_{09Q4-06Q4} \log CC_i + \delta X_{10Q1,i} + u_i
\]

Note that now the controls refer to variables measured in the beginning of the period in analysis, 2010Q1. Resulting estimates are presented in Tables 6 for the two measures of CC. The relationship between the change in market value in the post-crisis period and the change in customer capital in the crisis period is negative and statistically significant, regardless of
### Table 5: Estimation results for equation (29)

<table>
<thead>
<tr>
<th>Change in CC measure</th>
<th>Fair Value of Loans</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>change in MV, 06Q4-09Q4</td>
<td>0.066***</td>
<td>0.063***</td>
<td>0.060***</td>
<td>0.2854***</td>
<td>0.169***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.083)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>log size 06Q4</td>
<td>-0.015***</td>
<td>-0.016***</td>
<td>-0.057***</td>
<td>-0.064***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.008)</td>
<td>(0.009)</td>
<td></td>
</tr>
<tr>
<td>loans / assets 06Q4</td>
<td>-0.029</td>
<td>-0.213</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.131)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>deposits / liabilities 06Q4</td>
<td>-0.035</td>
<td>-0.167</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.142)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equity / assets 06Q4</td>
<td>-0.083</td>
<td>-0.908***</td>
<td>-0.904***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0058)</td>
<td>(0.030)</td>
<td>(0.031)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log CC 06Q4</td>
<td>-0.867***</td>
<td>-0.820***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.104)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>observations</td>
<td>237</td>
<td>237</td>
<td>237</td>
<td>233</td>
<td>233</td>
</tr>
<tr>
<td>adj $R^2$</td>
<td>0.175</td>
<td>0.462</td>
<td>0.474</td>
<td>0.047</td>
<td>0.803</td>
</tr>
</tbody>
</table>

**Note:** Standard errors in parentheses. * $p < 0.01$, ** $p < 0.05$, *** $p < 0.1$.

the set of controls. This suggests that banks that expended the most customer capital tended to recapitalize faster, and fare better in the post-crisis period. Importantly, this is also true when controlling for the size of the shock (the dependent variable in equation 29): in principle, we could expect that banks that suffered larger shocks would mechanically recapitalize faster. These results show that even when accounting for this effect, larger reductions in customer capital during the crisis led to faster recapitalizations.

### 4.4 Customer Capital and local markets

A third approach to the measurement of customer capital relies on the relationship between market shares, interest rate spreads, and customer capital that arises from model equation 4. This equation imposes that a bank’s market share should be (i) decreasing in the interest rate charged by the bank, relative to the average interest rate charged in the market, and (ii) increasing in that bank’s level of customer capital. We combine this relationship with data on local markets in which the BHC’s in our sample operate, in order to extract a measure of
<table>
<thead>
<tr>
<th>Change in CC measure</th>
<th>Fair Value of Loans</th>
<th>Lerner Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>change in CC, 06Q4-09Q4</td>
<td>-21.321***</td>
<td>-18.290***</td>
</tr>
<tr>
<td></td>
<td>(4.111)</td>
<td>(4.702)</td>
</tr>
<tr>
<td>change in MV, 06Q4-09Q4</td>
<td>-2.207***</td>
<td>-1.114</td>
</tr>
<tr>
<td></td>
<td>(0.709)</td>
<td>(0.792)</td>
</tr>
<tr>
<td>log size 10Q1</td>
<td>-0.421**</td>
<td>-0.279*</td>
</tr>
<tr>
<td></td>
<td>(0.172)</td>
<td>(0.168)</td>
</tr>
<tr>
<td>loans / assets 10Q1</td>
<td>-0.376</td>
<td>-1.6979</td>
</tr>
<tr>
<td></td>
<td>(2.147)</td>
<td>(2.3620)</td>
</tr>
<tr>
<td>deposits / liabilities 10Q1</td>
<td>-4.633*</td>
<td>-4.096</td>
</tr>
<tr>
<td></td>
<td>(2.429)</td>
<td>(2.525)</td>
</tr>
<tr>
<td>equity / assets 10Q1</td>
<td>-13.693***</td>
<td>-13.811***</td>
</tr>
<tr>
<td></td>
<td>(4.625)</td>
<td>(4.567)</td>
</tr>
</tbody>
</table>

| observations | 237 | 237 | 237 | 233 | 233 | 233 |
| adj $R^2$     | 0.102 | 0.148 | 0.205 | 0.020 | 0.124 | 0.170 |

Table 6: Estimation results for equation (30)

Note: Standard errors in parentheses. * $p < 0.01$, ** $p < 0.05$, *** $p < 0.1$.

BHC-specific customer capital.

Data To this end, we construct a BHC-year-MSA panel that contains data on BHC market shares in a given year in a specific MSA and on the interest rates charged by those BHCs. To measure market shares, we use data from the Home Disclosure Mortgage Act dataset (HMDA), a regulatory annual dataset that contains information on most mortgage applications in the US. Importantly, we observe the ZIP code associated with each mortgage application. This allows us to construct a measure of BHC mortgage market share in each MSA for a given year. We denote the market share for BHC $i$ in MSA $j$ in year $t$ as $\ell_{ijt}$, where $L_{jt} = \sum_i \ell_{ijt}$ are total mortgage loans originated on that MSA in a given year.

To measure interest rates, we use Ratewatch, which collects weekly data on interest rates offered by bank branches on different products on a weekly basis. Given that we measure market shares in the mortgage market, we focus on interest rates offered on 30-year conventional mortgages. Since Ratewatch includes branch coordinates, we compute
a measure of the interest rate offered by BHC $i$ on a given MSA $j$ in year $t$ as a simple average across weeks and branches, $r_{ijt}$. The average interest rate in a given MSA-year is the simple average of the interest rates offered by BHCs that are present in that market, $R_{jt} = \frac{1}{N_{jt}} \sum_i r_{ijt}$, where $N_{jt}$ is the number of BHCs in the market and that year.\(^6\)

**Measuring Customer Capital** To measure customer capital, we estimate the following specification:

$$\ell_{ijt} = \alpha_i + \gamma_{jt} + \beta (r_{ijt} - R_{jt}) + u_{ijt}$$

where $\alpha_i$ is a BHC fixed effect, and $\gamma_{jt}$ is a MSA-year fixed effect. We use the resulting estimates to compute a measure of BHC customer capital in a given MSA-year as follows:

$$cc_{ijt} = \hat{\alpha}_i + \hat{u}_{ijt}$$

that is, $cc_{ijt}$ is equal to the estimate fixed effect for that BHC plus the estimated residual. We then aggregate these measures across MSAs to compute a BHC-year measure, weighting the customer capital measure in each MSA by the BHC’s exposure to that particular market:

$$cc_{it} = \frac{\sum_j \ell_{ijt} cc_{ijt}}{\sum_j \ell_{ijt}}$$

**Results** We then use this resulting measure to perform the same type of analysis as in the previous subsection. The results for the shock regression are shown in Table (7), and the results for the resilience regression are in Table (8). Given that market share data is only available at the annual level, we adapt the regressions to compare changes between 2006 and 2009 (shock period) and 2010 and 2013 (resilience period). The fact that this measure of customer capital requires the matching of several different datasets also means that we lose a significant number of observations for this specification (our cross-sectional sample of BHCs around the crisis shrinks from 237 to 153 observations). Overall, however, the regressions show the same overall pattern: that drops in market value during the crisis period were positively associated with drops in customer capital during the crisis, and that BHCs with larger drops in customer capital during the crisis tended to recover faster in the post-crisis period, even when controlling for the initial drop in market value.

\(^6\)Unfortunately, Ratewatch does not include any data on the quantities of credit, which makes it difficult to compute weighted averages, for example.
### Shock Regression

<table>
<thead>
<tr>
<th>Dependent Variable: change in CC 06-09</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>change in MV, 06-09</td>
<td>0.060</td>
<td>0.060</td>
<td>0.105**</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>log size 06</td>
<td>-0.003</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
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Table 7: **Estimation results for equation (29)**

**Note:** Standard errors in parentheses. * $p < 0.01$, ** $p < 0.05$, *** $p < 0.1$.

## 5 Conclusion

We present a dynamic, general equilibrium theory of the role of customer capital in the banking industry. In the model, banks with high customer capital are those for whom lenders have a strong, endogenously built up lending relationship. These banks face a higher level of loan demand for a given loan price, and can therefore charge higher interest rates with a smaller cost to total loan demand than other banks. However, this pricing strategy is costly in the long run because charging high interest rates erodes customer capital as borrowers substitute away.

In the quantitative model, banks with high customer capital charge higher interest rates and lend more than banks with low customer capital. Motivated by the recent financial crisis, we simulate a shock to bank net worth, and show that banks with higher levels of customer capital experience a smaller decline in both net worth and market value, and also recover more quickly.

Motivated by the insights of the model, we use data on U.S. bank holding companies
### Resilience Regression

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Table 8: **Estimation results for equation (30)**

**Note:** Standard errors in parentheses. * $p < 0.01$, ** $p < 0.05$, *** $p < 0.1$.

from the Call Reports in the period surrounding the financial crisis to assess the relationship between customer capital, shocks to bank net worth, and resilience from those shocks. Across measures of customer capital, we find evidence that banks with high levels of customer capital expend this asset when a negative shock hits. This reduces the impact of the shock, and increases the speed at which the bank recovers from it.
References


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Appendix

A Data Appendix

A.1 Details on the Construction of Customer Capital Measures

Due to strong seasonal effects on BHC income measures, we use 4-quarter moving averages for all income statement variables: income from loans and charge-offs on loans.

A.1.1 Fair Value Derivations

We follow closely Atkeson et al. (2019), with some additional assumptions. In that paper, the authors are interested in computing a measure of fair value of bank equity. They argue that market valuations are contaminated by the implicit value of government guarantees, and cannot therefore be relied on as accurate measures of the economic value of banking operations. They compute the fair value of equity by using a simple valuation model to compute the fair value of the loan-making and deposit-taking businesses, and assuming that all other activities are fairly measured in bank’s balance sheets.

For a given bank, let $L$ denote the total book value of the loan portfolio. Every period, each dollar in $L$ yields a coupon equal to $r_L$. Each dollar in $L$ is prepaid with some probability $\mu_L$, and can default with probability $\delta_L$. Let $v_L \times L$ denote the fair value of the loan portfolio, where $v_L$ can be thought of as the “Q” for the current portfolio. Letting $i$ denote the discount rate of the bank’s shareholders, this fair value can then be computed recursively as

$$v_L = \frac{1}{1 + i} E[r_L + \mu'_L + (1 - \mu'_L - \delta'_L) v_L]$$

This equation can be solved for $v_L$ to yield

$$v_L = \frac{r_L + \bar{\mu}_L}{i + \bar{\mu}_L + \bar{\delta}_L}$$

where $\bar{\mu}_L, \bar{\delta}_L$ are the expected probabilities of repayment and default, respectively. When bringing the model to the data, we approximate these expected measures with average maturity of bank loans, and average charge-off rates, respectively.
B Model Appendix

B.1 Proofs

B.1.1 Proof of Proposition 1: Loan Demand

Taking first order conditions in the borrower’s problem (1) - (3), we get

\[ n^{\alpha n - 1} = w(1 + \lambda \kappa) \]

\[
[L'] 1 - \frac{\phi}{2} \int \left( \frac{q^\ell(q, s)}{L'} - \frac{s}{S} \right)^2 d\mu(q, s) - \phi L' \int \left( \frac{q^\ell(q, s)}{L'} \right) \left( \frac{q^\ell(q, s)}{L'} - \frac{s}{S} \right) d\mu(q, s) + \lambda - \zeta = 0
\]

\[
[L'] - \phi L' \frac{q}{L'} \left( \frac{q^\ell(q, s)}{L'} - \frac{s}{S} \right) d\mu(q, s) + \beta \mathbb{E} [V_\ell(\ell'; \mu)] + \zeta q d\mu(q, s) = 0
\]

With the envelope condition \( V_\ell(\ell'; \mu) = -d\mu(q, s) \), we obtain the optimality conditions:

- for labor demand:
  \[ n = \left[ \frac{w}{\alpha} (1 + \lambda \kappa) \right]^{\frac{1}{\alpha - 1}} \] (B.1)

- for total loan demand:
  \[ 1 + \lambda - \zeta = \frac{\phi}{2} \int \left( \frac{q^\ell(q, s)}{L'} - \frac{s}{S} \right)^2 d\mu(q, s) - \frac{\phi}{L'} \int q^\ell(q, s) \left( \frac{q^\ell(q, s)}{L'} - \frac{s}{S} \right) d\mu(q, s) \]

  \[ = \phi \int \left( \frac{q^\ell(q, s)}{L'} - \frac{s}{S} \right) \left( \frac{1}{2} \left( \frac{q^\ell(q, s)}{L'} - \frac{s}{S} \right) - \frac{q^\ell(q, s)}{L'} \right) d\mu(q, s) \]

  \[ = -\frac{\phi}{2} \left[ \int \left( \frac{q^\ell(q, s)}{L'} \right)^2 d\mu(q, s) - \int \left( \frac{s}{S} \right)^2 d\mu(q, s) \right] \]

  \[ = -\frac{\phi}{2} \left[ 1 + \text{Var} \left( \frac{q^\ell}{L'} \right) - 1 + \text{Var} \left( \frac{s}{S} \right) \right] \] (B.2)

- and for bank-specific loan demand:
  \[ \zeta = \frac{\beta}{q} + \phi \left( \frac{q^\ell(q, s)}{L'} - \frac{s}{S} \right) \text{ for all } (q, s) \] (B.3)

Integrate:
\[ \beta \int \frac{d\mu(q, s)}{q} + \frac{\phi}{L'} \int q^\ell(q, s) d\mu(q, s) - \frac{\phi}{S} \int s d\mu(q, s) = \beta R \] (B.4)

---

A useful thing here: if \( X = \mathbb{E}[x] \), then \( \mathbb{E}[x/X] = \mathbb{E}[x]/X = X/X = 1 \), and similarly

\[ \text{Var} \left( \frac{x}{X} \right) = \mathbb{E} \left[ \left( \frac{x}{X} - \mathbb{E} \left( \frac{x}{X} \right) \right)^2 \right] = \mathbb{E} \left[ \left( \frac{x}{X} - 1 \right)^2 \right] = \mathbb{E} \left[ \left( \frac{x}{X} \right)^2 \right] - 2 \mathbb{E} \left( \frac{x}{X} \right) + 1 = \mathbb{E} \left[ \left( \frac{x}{X} \right)^2 \right] + 1 \]
Plugging the binding working capital constraint (2) into (B.1) gives aggregate loan demand:

$$\kappa w n = L' = \kappa w \left[ \frac{\alpha}{w} \frac{1}{1 + \lambda \kappa} \right]^{1/\alpha}$$  \hspace{1cm} (B.5)

where all we need to do is solve for $\lambda$. Plugging (B.4) back into (B.3) gives us our bank-specific loan demand equation (4).

We can plug (4) and (B.4) back into (B.2) to obtain

$$\text{Var} \left( \frac{q^l L'}{L} \right) - \text{Var} \left( \frac{s}{S} \right) = \text{Var} \left( \frac{s}{S} - \frac{\beta}{\phi} \tau \right) - \text{Var} \left( \frac{s}{S} \right)$$

$$= \text{Var} \left( \frac{s}{S} \right) + \text{Var} \left( \frac{\beta}{\phi} \tau \right) + 2 \text{Cov} \left( \frac{s}{S}, \frac{\beta}{\phi} \tau \right) - \text{Var} \left( \frac{s}{S} \right)$$

$$= \left( \frac{\beta}{\phi} \right)^2 \text{Var}(\tau) - 2 \frac{\beta}{\phi} \text{Cov} \left( \frac{s}{S}, \tau \right)$$

where Var($\tau$) is the cross-sectional variance of interest rates. This delivers the aggregate demand equation (5) since we now have

$$1 + \lambda - \beta R = - \frac{\phi}{2} \left[ \left( \frac{\beta}{\phi} \right)^2 \text{Var}(\tau) - 2 \frac{\beta}{\phi} \text{Cov} \left( \frac{s}{S}, \tau \right) \right]$$

$$= \beta \left[ \text{Cov} \left( \frac{s}{S}, \tau \right) - \frac{1}{2} \frac{\beta}{\phi} \text{Var}(\tau) \right] \equiv \beta P$$

$$\implies \lambda = \beta (R + P) - 1$$

**B.1.2 Proof of Proposition 2: Elasticities**

*Proof.* Taking the derivative of (5) with respect to $R + P$, we obtain

$$\frac{\partial L'}{\partial (R + P)} = \kappa w \left[ \frac{\alpha/w}{1 + \kappa \beta (R + P) - \kappa} \right]^{1/\alpha - 1} \frac{-\alpha/w}{(1 + \kappa \beta (R + P) - \kappa)^2 \kappa \beta}$$

$$= \kappa w \left[ \frac{\alpha/w}{1 + \kappa \beta (R + P) - \kappa} \right]^{1/\alpha} \frac{-\kappa \beta}{1 + \kappa \beta (R + P) - \kappa}$$

$$= \frac{L'}{1 + \kappa \beta (R + P) - \kappa}$$

Then, defining $\epsilon(L', R + P) = \frac{\partial L'}{\partial (R + P)} / \frac{L'}{R + P}$ and again applying (5) yields the result.

Turning to the bank-specific demand elasticities, consider first the spread elasticity:

$$\frac{\partial q^l}{\partial \tau} = - \frac{\beta}{\phi} L'$$

$$\implies \epsilon(q^l, \tau) = \frac{\tau}{q^l} \frac{\partial q^l}{\partial \tau} = - \tau \frac{\beta}{\phi} \frac{s}{S - \frac{\beta}{\phi} \tau}$$
Applying the minus sign to the denominator and multiplying by \( \phi/\phi \) gives (10). Similarly for customer capital,

\[
\frac{\partial q\ell'}{\partial s} = \frac{L'}{S} \quad \Rightarrow \quad \epsilon(q\ell', s) = \frac{s}{q\ell'} \frac{\partial q\ell'}{\partial s} = s\frac{L'}{S} - \frac{1}{S} \frac{L'}{\phi}\tau
\]

Multiplying by \( \phi/\phi \) gives (11). Note that this multiplication step obscures some things for the limiting case when \( \phi \to 0 \). In this case, the spread elasticity becomes infinite, and the customer capital elasticity becomes 0.

\[\square\]

B.1.3 Proof of Proposition 3: Bank Financing and Lending

**Proof.** Since \( \xi'(e) > 0 \), the budget constraint (15) must bind, and so we can eliminate \( e \) from the set of control variables. Mechanically, conditions (18) and (19) must hold (with \( \ell(q, s; X) \) in (19) given by (4)), and so we may further eliminate \( s' \) and \( \ell' \). This leaves us with a problem in 3 variables (dropping \( X \) and \( X' \) to ease notation):

\[
V_B (\ell, a, \delta, s; d', z) = \max_{q, a', \delta'} q^d \delta' \leq (1 - \chi)q\ell(q, s) + \overline{q}a' \\
\text{subject to} \quad [\lambda] \quad q^d \delta' \leq (1 - \chi)q\ell(q, s) + \overline{q}a' \\
\quad \quad [q^d \eta] \quad \delta' \leq d'
\]

Taking first order conditions, we obtain:

\[
\frac{\partial q\ell(q, s)}{\partial q} \xi'(e) = \overline{q}(1 - \pi)\mathbb{E} \left[ \frac{\partial V_B (\ell', a', \delta', s'; d'', z')}{\partial \ell'} \frac{\partial \ell(q, s)}{\partial q} \right] \tag{B.7}
\]

\[
\overline{q} \xi'(e) = \overline{q}(1 - \pi)\mathbb{E} \left[ \frac{\partial V_B (\ell', a', \delta', s'; d'', z')}{\partial a'} \right] + \lambda \overline{q} \tag{B.8}
\]

\[
-q^d \xi'(e) = \overline{q}(1 - \pi)\mathbb{E} \left[ \frac{\partial V_B (\ell', a', \delta', s'; d'', z')}{\partial d'} \right] - \lambda q^d - \eta q^d \tag{B.9}
\]
The relevant envelope conditions are:

\[
\frac{\partial V_B}{\partial \ell} = (1 + z)\xi'(e) \quad \text{(B.10)}
\]

\[
\frac{\partial V_B}{\partial s} = -\frac{\partial q\ell(q, s)}{\partial s} \xi'(e) + \lambda (1 - \chi) \frac{\partial q\ell(q, s)}{\partial s} + \eta (1 - \pi) \frac{\partial V_B}{\partial \ell'} \frac{\partial \ell'(q, s)}{\partial s} \quad \text{(B.11)}
\]

\[
\frac{\partial V_B}{\partial a} = \xi'(e) \quad \text{(B.12)}
\]

\[
\frac{\partial V_B}{\partial \delta} = -\xi'(e) \quad \text{(B.13)}
\]

We turn first to the financing results. Plugging in \( q^d = (1 + \nu)q \) and applying the envelope conditions (B.12) and (B.13) to the first order conditions (B.8) and (B.9), we obtain

\[
\xi'(e) = (1 - \pi)\mathbb{E}[\xi'(e')] + \lambda \quad \text{(B.14)}
\]

\[
\xi'(e) = \frac{1 - \pi}{1 + \nu} \mathbb{E}[\xi'(e')] + \lambda + \eta \quad \text{(B.15)}
\]

Subtracting (B.14) from (B.15) yields

\[
\eta = (1 - \pi)\mathbb{E}[\xi'(e')] \left( 1 - \frac{1}{1 + \nu} \right),
\]

which is strictly positive as long as \( \xi'(e) > 0 \), \( \pi < 1 \), and \( \nu > 0 \), proving the first part of the result that constraint (17) binds and therefore \( \delta' = d' \).

Next, consider the pricing policy. It is useful to begin by simplifying the envelope condition for customer capital (B.11). First, under the law of motion for customer capital (18), we have \( \frac{\partial s'}{\partial s} = \rho \). Second, under the bank-specific demand equation (4), we have \( \frac{\partial q\ell'}{\partial s} = L'/S \) (and \( \frac{\partial q\ell'}{\partial \delta} = L'/(qS) \)). Note that this expression equals 1 in steady state under Proposition 4.
Combining these results and simplifying notation in (B.11), we obtain:

\[
\frac{\partial V_B}{\partial s} = \frac{L'}{S}(\lambda(1-\chi) - \xi'(e)) + \frac{\bar{q}}{q}(1-\pi)\mathbb{E}\left[\frac{\partial V'_B}{\partial \ell'} q S + \rho \frac{\partial V'_B}{\partial s'}\right]
\]

\[
= \frac{L'}{S}\left(\lambda(1-\chi) - \xi'(e) + \frac{\bar{q}}{q}(1-\pi)\mathbb{E}\left[(1+z')\xi'(e')\right]\right) + \rho\bar{q}(1-\pi)\mathbb{E}\left[\frac{\partial V'_B}{\partial s'}\right]
\]

\[
= \frac{L'}{S}\left(\lambda(1-\chi) - \xi'(e) + \frac{\bar{q}}{q}(1-\pi)\mathbb{E}\left[(1+z')\xi'(e')\right]\right)
\]

\[
+ \rho\bar{q}(1-\pi)\mathbb{E}\left[\frac{L''}{S'}\left(\lambda'(1-\chi) - \xi'(e') + \frac{\bar{q}}{q}(1-\pi)\mathbb{E}\left[(1+z'')\xi'(e'')\right]\right)\right]
\]

\[
+ \rho\bar{q}(1-\pi)\mathbb{E}\left[\frac{\partial V''_B}{\partial s''}\right]\right]
\]

\[
= \ldots[\text{iterate}]
\]

\[
\Rightarrow \frac{\partial V_{B,t}}{\partial s_t} = \sum_{i=0}^{\infty} (\rho\bar{q}(1-\pi))^i \mathbb{E}_t \left[\frac{L_{t+i+1}}{S_{t+i}} \left(\lambda_{t+i}(1-\chi) - \xi'(e_{t+i})\right)
\right.
\]

\[
+ \frac{\bar{q}}{q_{t+i}} (1-\pi)(1+z_{t+i+1})\xi'(e_{t+i+1})\right]\right]
\]

(B.16)

Note also that \(\frac{\partial e'}{\partial q} = (1-\rho)\frac{\partial e'(q,s)}{\partial q}\). Then, we can divide (B.7) through by \(q\ell'/q\) to express derivatives as elasticities and plug in the envelope conditions (B.10) and (B.16) to obtain

\[
\xi'(e)\epsilon(q\ell', q) = \bar{q}(1-\pi)\mathbb{E}\left[(1+z')\xi'(e')q\frac{\partial e}{\partial q}\right]
\]

\[
+(1-\rho)\epsilon(q\ell', q)\frac{\partial V_{B,t+1}}{\partial s_{t+1}} + \lambda(1-\chi)\epsilon(q\ell', q)
\]
Then we can rearrange the above expression and reintroduce time subscripts to get\(^8\)

\[
\xi'(e_t) = \lambda_t(1 - \chi) + \frac{\bar{q}}{q_t}(1 - \pi)E_t \left[ (1 + z_{t+1})\xi'(e_{t+1}) - \frac{\epsilon(q_{t+1}, e_t)}{\epsilon(q_{t+1}, q_t)} - \lambda_t(1 - \chi) - \xi'(e_{t+1}) + \frac{\xi'(e_{t+1})}{\epsilon(q_{t+1}, q_t)} \right]
\]

\[
+ (1 - \rho) \sum_{i=0}^{\infty} \left( \rho \bar{q}(1 - \pi) \right)^i E_{t+i} \left[ \frac{L_{t+i+2}}{S_{t+i+1}} \lambda_{t+i+1}(1 - \chi) - \xi'(e_{t+i+1}) + \frac{\xi'(e_{t+i+1})}{\epsilon(q_{t+i+1}, q_t)} \right]
\]

\[
\Rightarrow 0 = \lambda_t(1 - \chi) - \xi'(e_t) + \frac{\bar{q}}{q_t}(1 - \pi)E_t \left[ (1 + z_{t+1})\xi'(e_{t+1}) - \lambda_t(1 - \chi) - \xi'(e_{t+1}) + \frac{\xi'(e_{t+1})}{\epsilon(q_{t+1}, q_t)} \right]
\]

\[
- (1 + z_{t+1})\xi'(e_{t+1}) \frac{1}{\epsilon(q_{t+1}, q_t)}
\]

\[
+ (1 - \rho) \sum_{i=1}^{\infty} \left( \rho \bar{q}(1 - \pi) \right)^i \left[ \frac{L_{t+i+1}}{S_{t+i}} \lambda_{t+i}(1 - \chi) - \xi'(e_{t+i}) + \frac{\xi'(e_{t+i})}{\epsilon(q_{t+i}, q_t)} \right]
\]

Equation (B.17) is the most general form of the bank’s Euler equation.

We can simplify in two ways. First, recognizing that the spread implied the loan price \( q \) is \( \tau(q) = \frac{1}{q} - R \), the price implied by a given spread is \( q(\tau) = \frac{1}{R + \tau} \), and

\[
\epsilon(q(\tau))\ell(q(\tau), s, \tau) = \frac{\partial q(\tau)\ell(q(\tau), s)}{\partial \tau} \left/ \frac{q\ell'}{\tau} \right.
\]

\[
= \frac{q'(\tau)}{\tau/q} \left( \ell' + q \frac{\partial q(\tau)}{\partial q} \right)
\]

\[
= \epsilon(q, \tau)(1 + \epsilon(\ell', q))
\]

\[
= \epsilon(q, \tau)\epsilon(q\ell', q)
\]

We know that

\[
\epsilon(q, \tau) = \frac{q'(\tau)}{q/\tau} = -\frac{\tau(R + \tau)}{(R + \tau)^2} = -\frac{\tau}{R + \tau} = -\tau q
\]

and from equation (10), we know the left hand side of the expression above. Therefore,

\[
\epsilon(q\ell', q) = \frac{\epsilon(q\ell', \tau)}{\epsilon(q, \tau)} = -\frac{\beta q}{R + \tau} = -\frac{\beta(R + \tau)}{\beta\tau - \phi S} = -\frac{\beta}{\frac{q}{R} - \beta R - \phi S}
\]

\(^8\)Here we use the elasticity calculation that \( \epsilon(q\ell', q) = 1 + \epsilon(\ell', q) \).
which implies that
\[
\frac{1}{\epsilon(q^f, q)} = \frac{\beta R + \phi \frac{s}{\phi} - \frac{\beta}{q}}{\frac{s}{\phi}} = \frac{1}{\phi} \frac{\beta R + \phi \frac{s}{\phi} - \frac{\beta}{q}}{\frac{s}{\phi}} = \frac{\frac{s}{\phi} \left( \frac{1}{q} - R \right)}{\frac{s}{\phi} q} = \frac{1}{q} \frac{q^f L'}{L}
\]

Second, in steady state \( L_{t+1} = S_t \) for all \( t \), and so (B.17) may be simplified to

\[
\Rightarrow 0 = \lambda_t (1 - \chi) - \xi'(e_t) + \frac{\bar{q}}{q_t} (1 - \pi) \mathbb{E}(1 + z_{t+1}) \xi'(e_{t+1})
\]
\[
- \frac{\phi}{\beta} q_t L_{t+1} \frac{\bar{q}}{q_t} (1 - \pi) \mathbb{E}(1 + z_{t+1}) \xi'(e_{t+1})
\]
\[
+ (1 - \rho) \sum_{i=1}^{\infty} (\rho \bar{q} (1 - \pi))^i \left[ \lambda_{t+i} (1 - \chi) - \xi'(e_{t+i}) \right]
\]
\[
= \rho \left( \lambda_t (1 - \chi) - \xi'(e_t) + \frac{\bar{q}}{q_t} (1 - \pi) \mathbb{E}(1 + z_{t+1}) \xi'(e_{t+1}) \right)
\]
\[
- \frac{\phi}{\beta} q_t L_{t+1} \frac{\bar{q}}{q_t} (1 - \pi) \mathbb{E}(1 + z_{t+1}) \xi'(e_{t+1})
\]
\[
+ (1 - \rho) \sum_{i=1}^{\infty} (\rho \bar{q} (1 - \pi))^i \mathbb{E}(1 - \xi'(e_{t+i}))
\]
\[
= \rho \left( \lambda_t (1 - \chi) - \xi'(e_t) + \frac{\bar{q}}{q_t} (1 - \pi) \mathbb{E}(1 + z_{t+1}) \xi'(e_{t+1}) \right)
\]
\[
- \frac{\phi}{\beta} q_t L_{t+1} \frac{\bar{q}}{q_t} (1 - \pi) \mathbb{E}(1 + z_{t+1}) \xi'(e_{t+1})
\]
\[
+ (1 - \rho) \sum_{i=1}^{\infty} (\rho \bar{q} (1 - \pi))^i \mathbb{E}(1 - \xi'(e_{t+i}))
\]

which implies (20). This admits several special cases.

- \( \rho = 0 \). In this case, customer capital is dictated entirely by this period’s loans. Equation (20) simplifies to:

\[
\xi'(e_t) + \frac{\phi}{\beta} q_t L_{t+1} \frac{\bar{q}}{q_t} (1 - \pi) \mathbb{E}(1 + z_{t+1}) \xi'(e_{t+1})
\]
\[
= \lambda_t (1 - \chi) + \frac{\bar{q}}{q_t} (1 - \pi) \mathbb{E}(1 + z_{t+1}) \xi'(e_{t+1}) \tag{B.18}
\]

- \( \rho = 1 \). In this case, customer capital is completely sticky, i.e. a bank’s permanent type. The Euler equation is unchanged from (B.18)!

- \( \phi = 0 \). In this case, demand is infinitely elastic regardless of customer capital. We cannot directly take the limit here. But, we know the bank’s optimal pricing policy must be to set \( q = 1/R \) always.

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B.1.4 Proof of Proposition 4: Stationarity

Proof. Aggregating across banks and using the individual law of motion for customer capital (18), we obtain:

\[
S'(X) = \int s'(x)dm(x;X)
= \int [(1 - \rho)g_q(x;X)\ell (g_q(x;X), s(x); X) + \rho s(x)] dm(x; X)
= (1 - \rho) \int g_q(x)\ell (g_q(x;X), s(x); X) m(x; X) dx + \rho \int s(x)m(x; X)dx
= (1 - \rho)L_S(X) + \rho S(X)
\]

Applying stationarity, we know that \( S'(X) = S(X) \), yielding \( S(X) = L_S(X) \) directly.

The law of motion (18) and the properties of variance imply that

\[
\text{Var} \left( \frac{s'}{S'} \right) = \text{Var} \left( \frac{(1 - \rho)q\ell' + \rho s}{S'} \right)
= (1 - \rho)^2 \text{Var} \left( \frac{q\ell'}{S'} \right) + \rho^2 \text{Var} \left( \frac{s}{S} \right) + 2\rho(1 - \rho)\text{Cov} \left( \frac{q\ell'}{S'}, \frac{s}{S} \right)
\Rightarrow (1 - \rho^2) \text{Var} \left( \frac{s}{S} \right) = (1 - \rho)^2 \text{Var} \left( \frac{q\ell'}{L'} \right) + 2\rho(1 - \rho)\text{Cov} \left( \frac{q\ell'}{L'}, \frac{s}{S} \right)
\tag{B.19}
\]

where the last line applies stationarity arguments from Proposition 4: \( S' = S = L' \) and \( \text{Var}(s'/S') = \text{Var}(s/S) \). We can further analyze the variance term in (B.19) using the bank-specific loan demand (4):

\[
\text{Var} \left( \frac{q\ell'}{L'} \right) = \text{Var} \left( \frac{s}{S} - \frac{\beta}{\phi} \tau \right)
= \text{Var} \left( \frac{s}{S} \right) + \left( \frac{\beta}{\phi} \right)^2 \text{Var}(\tau) - 2\frac{\beta}{\phi} \text{Cov} \left( \frac{s}{S}, \tau \right)
\tag{B.20}
\]

and likewise the covariance in (B.19):

\[
\text{Cov} \left( \frac{q\ell'}{L'}, \frac{s}{S} \right) = \text{Cov} \left( \frac{s}{S} - \frac{\beta}{\phi} \tau, \frac{s}{S} \right)
= \text{Var} \left( \frac{s}{S} \right) - \frac{\beta}{\phi} \text{Cov} \left( \frac{s}{S}, \tau \right)
\tag{B.21}
\]
Then, we can plug (B.20) and (B.21) into (B.19) to obtain:

\[
(1 - \rho^2) \text{Var}\left(\frac{s}{S}\right) = (1 - \rho)^2 \left[ \text{Var}\left(\frac{s}{S}\right) + \left(\frac{\beta}{\phi}\right)^2 \text{Var}(\tau) - 2\frac{\beta}{\phi} \text{Cov}\left(\frac{s}{S}, \tau\right) \right] \\
+ 2\rho(1 - \rho) \left[ \text{Var}\left(\frac{s}{S}\right) - \frac{\beta}{\phi} \text{Cov}\left(\frac{s}{S}, \tau\right) \right] \\
= (1 - 2\rho + \rho^2 + 2\rho - 2\rho^2) \text{Var}\left(\frac{s}{S}\right) \\
+ (1 - \rho)^2 \left(\frac{\beta}{\phi}\right)^2 \text{Var}(\tau) - \left[ 2(1 - \rho)^2 \frac{\beta}{\phi} + 2\rho(1 - \rho) \frac{\beta}{\phi}\right] \text{Cov}\left(\frac{s}{S}, \tau\right)
\]

This condition implies that the \text{Var}(s/S) terms cancel and we are left with

\[
2(1 - \rho) \frac{\beta}{\phi} [(1 - \rho) + \rho] \text{Cov}\left(\frac{s}{S}, \tau\right) = (1 - \rho)^2 \left(\frac{\beta}{\phi}\right)^2 \text{Var}(\tau)
\]

\[
\implies \text{Cov}\left(\frac{s}{S}, \tau\right) = (1 - \rho)\frac{1}{2} \frac{\beta}{\phi} \text{Var}(\tau)
\]

(B.22)

Returning to the definition of \( P \) from (8), we obtain:

\[
P = \text{Cov}\left(\frac{s}{S}, \tau\right) - \frac{1}{2} \frac{\beta}{\phi} \text{Var}(\tau)
\]

\[
= (1 - \rho)\frac{1}{2} \frac{\beta}{\phi} \text{Var}(\tau) - \frac{1}{2} \frac{\beta}{\phi} \text{Var}(\tau)
\]

\[
= -\rho \frac{1}{2} \frac{\beta}{\phi} \text{Var}(\tau)
\]

which is the result from the main text. \( \square \)

**B.1.5 Proof of Proposition 5: Fixed Point**

*Proof*. Using the bank-specific loan demand function (4) we can plug in optimal policies to get:

\[
g_q(x; X) \ell (g_q(x; X), s(x); X) = L' \left[ \frac{s(x)}{S} - \frac{\beta}{\phi} \left( \frac{1}{g_q(x; X)} - R \right) \right]
\]

Then, integrating over the distribution implied by these policies, we obtain

\[
\int g_q(x; X) \ell (g_q(x; X), s(x); X) dm(x; X) = L' \int \left[ \frac{s(x)}{S} - \frac{\beta}{\phi} \left( \frac{1}{g_q(x; X)} - R \right) \right] dm(x; X)
\]

\[
L'_S(X) = L' \left[ \frac{S(X)}{S} - \frac{\beta}{\phi} (R(X) - R) \right]
\]

This last expression satisfies the market clearing condition \( L'_S(X) = L' \) if and only if: (i) \( S(X) = L \) and \( R(X) = R \) or (ii) \( \frac{L'_S(X)}{L} + \frac{2}{\phi} (R(X) - R) = \frac{S(X)}{S} \).

We can show that the former condition must hold and the latter condition cannot. That
is, assume $R - R(X) = \Delta R \neq 0$. Then, we must have $\frac{L_S'(X)}{L_S'} - \frac{S(X)}{S} = \frac{\beta}{\phi} \Delta R$. Under the assumption in the statement of the proposition, we know $S = L'$, so this can be written $L_S'(X) - S(X) = \frac{\beta}{\phi} L' \Delta R$. But, proposition (4) requires that $L_S'(X) = S(X)$ for all $X$, so we have reached a contradiction.

\[\Box\]

C Quantitative Appendix

C.1 Bank problem

1. Given a current guess of $V_B$, compute expected $V_B$ (over deposits) for all $(n, s, d', z)$:

$$V_B(n, s; d, z) = \overline{q}(1 - \pi) \sum_{d'} \pi_d(d'|d) V_B(n, s; d', z) \quad (C.1)$$

before entering in loop to compute policies. This will save on calculations later.

2. Fix $(n, s, d', z)$. Implement nested golden section at this point with the $a'$ choice as the outer loop.

(a) Set $q = 0$ and $\overline{a}$ to be a sufficiently high parameter.

(b) Within each outer GS step (i.e. for $a'$ fixed within the bounds above, which narrow step to step), loop over possible spreads $\tau \in [\overline{\tau}, \tau(s, d', a')]$ where $\tau$ is

$$\min \left\{ \frac{\phi s}{\beta S}, \frac{s}{\beta} \left( \frac{\overline{\tau} a' - q d'}{\overline{\tau} + (1 - \chi)L'} \right) \right\}$$

and $\overline{\tau} < 0$ is a parameter.

(c) Compute $n'(q(\tau), a', z', s, d')$ via

$$n'(q, a', z', s, d') = (1 + z') \ell(q, s) + a' - d' \quad (C.2)$$

for all $z'$, and $s'(q(\tau), s)$ via (18).

(d) Compute the value associated with the current $(\tau, a')$ according to

$$v(\tau, a'; x) = \xi \left( q d'(x) + n(x) - f(\tau, s(x)) - \overline{q} a' \right)$$

$$+ \overline{q} \sum_{z'} \pi(z'|z) V_B(n'(\tau, a', x, z'), s'(\tau, x), d', z')$$

Of course this step will require interpolation on $n'$ and $s'$.

(e) These values will be used to find $w(a'; x) = \max_\tau v(\tau, a'; x)$ in the inner GS loop and then $TV(x) = \max_{a'} w(a'; x)$ in the outer GS loop.

C.2 Steady state

1. Solve for the bank-specific loan demand function $\ell(q, s; X)$.
2. Make a guess of aggregate loan demand $L_D'$. 
3. Solve for a banking industry equilibrium \( \{g(\cdot), m(\cdot), X\} \) that results in aggregate loan supply $L_S'(X) = L_D'$. 
   \( \begin{align*} 
   & (a) \text{ Impose } S = L_D' \text{ and make a guess of } R. \text{ This fixes current } X = \{L_D', S, R\}. \\
   & (b) \text{ Solve for bank’s optimal policies } g_q(x; X) \text{ and the distribution of banks implied by those policies } m(x; X). \\
   & (c) \text{ Compute } L_S(X). \\
   & (d) \text{ If } L_S'(X) = L_D' \text{ (within tolerance) proceed to next step. Otherwise update } R \text{ in the direction to reduce excess demand (keeping } S = L_D' \text{ fixed) and return to 3b.} 
\end{align*} \)

4. Compute the distribution of prices and customer capital $\mu(q, s)$ associated with the banking industry equilibrium distribution $m(x; X)$ in step 3 according to (26). 
   \( \begin{align*} 
   & \text{• Note: in our quadratic-share case, this amounts to computing } P. 
\end{align*} \)

5. Compute aggregate loan demand $L_D'(\mu)$. 
6. If $L_D'(\mu) = L_D'$ (within tolerance), then we have an equilibrium. Otherwise, revise $L_D'$ and return to step 2.

### C.3 Transition

1. Solve for initial and terminal steady states.
2. Make a guess of the path of $\{L_t'\}_{t=1}^T$. 
3. Iterate to convergence on $\{R_t\}_{t=1}^T$ by solving for sequential banking industry equilibria. 
   \( \begin{align*} 
   & (a) \text{ Make a guess of the path } \{R_t\}_{t=1}^T. \text{ Impose } S_t = L_t' \text{ for all } t, \text{ completing } \{X_t\}_{t=1}^T. \\
   & (b) \text{ Solve backward for the sequence bank policy functions } g_{q,t}(x; X_t) \text{ date by date, imposing that } V_{T+1}(x) \text{ be the (terminal) steady state value function.} \\
   & (c) \text{ Solve forward for the sequence of equilibrium distributions } m_t(x; X_t) \text{ implied by the sequence of decision rules.} \\
   & (d) \text{ Compute implied loan supply } \{L_{S,t}(X_t)\}_{t=1}^T. \text{ To the extent that it deviates from the assumed path } \{L_t'\}_{t=1}^T \text{ from step 2, update } \{R_t\}_{t=1}^T \text{ in the direction that reduces excess demand.} 
\end{align*} \)

4. Compute $\{P_t\}_{t=1}^T$ using the sequence of banking industry equilibria.
5. Compute the aggregate loan demand $\{L_{D,t}'\}_{t=1}^T$ implied by the sequence $\{R_t, P_t\}_{t=1}^T$.\footnote{This is another place external habits matter: the distribution of banks and therefore prices / CC levels tomorrow doesn’t factor into the borrower’s decision today.} Compare this sequence to the guessed sequence from step 2. Update via relaxation.