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An Elementary Model of VC Financing and Growth

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Abstract

This article uses an endogenous growth model to study how the improvements in financing for innovative start-ups brought by venture capital (VC) affect firm innovation and growth. Partial equilibrium results show how lending contracts change as financing efficiency improves, while general equilibrium results demonstrate that better screening and development of projects by VC investors leads to higher aggregate productivity growth.

Keywords: endogenous growth, financial development, innovation, IPO, screening, research and development, startups, venture capital.

JEL Codes: E13, E22, G24, L26, O16, O31, O40.

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1 Introduction

There is a strong link between venture capital (VC) financing and firm growth. Akcigit, Dinlersoz, Greenwood, and Penciakova (2019) shows that companies with VC financing grow faster over time than companies using other sources of financing. Greenwood, Han, and Sánchez (2022b) show that in the first years after going public, VC-backed firms have employment and sales growth higher (5 and 7 percent, respectively) than publicly traded firms that did not receive VC financing. These authors also show that there exists a connection between VC financing and innovation. They show that VC investment increases the number of patents that a firm will file in the following years: A 10 percent increase in VC funding will induce a 7.9 percent increase in the numbers of patents expected by the firm in the three years after funding.

There is also evidence suggesting that VC financing is important for aggregate economic growth. Greenwood et al. (2022a) build a model in which financing occurs though an optimal contract similar to Bergemann and Hege (1998), Clementi and Hopenhayn (2006), and Cole et al. (2016). Then, the VC contract is embedded in a Romer (1986)-style model of endogenous growth to analyze the connection between VC financing and economic growth. The complexity of the dynamic contracting problem forces the analysis of the link between financing efficiency and productivity that is performed mainly with the numerical solution of the model. Quantitative results performed with a model calibrated to the US economy suggest that debasing VC financing would reduce the economy's growth rate by almost 20 percent (from 1.8 percent to 1.5 percent).

This article aims to expand the analysis of the relationship between VC financing and growth by both presenting a simpler model of VC financing and investigating how VC activity affects innovation and economic growth. The results demonstrate that more-efficient VC financing–due to improvements in either project screening or development–increases both the probability that new projects are successfully developed and the share of the value of the project that the entrepreneur keeps. This same improvement in VC financing encourages entrepreneurs to choose a more-innovative project. In turn, these results imply that on a balanced growth path, more efficient VC financing leads to a higher productivity growth rate.

2 Setup

There are good and bad projects in this economy. Only good projects can become successful. The share of good projects is ρ. An entrepreneur starts a project (or an idea) of productivity x in an economy with average productivity X that is growing at the rate g_x. The research cost for an idea of productivity x is $R(x, X)$, which is an increasing and convex function of how far the new idea x is from average productivity X (which will be defined later). To obtain a closed-form solution, the R&D cost function R is assumed to follow the functional form of

$$R(x, X) = \frac{1}{z_R}\left(\frac{x}{X}\right)^{\iota}, \ \iota > 2,$$

where x/X is the step size of innovation or g.

Successful firms with a project of productivity x use labor l, capital k, and the following technology to produce and maximize profits:

$$S(x, X) = \max_{k,l}\{x^{\zeta}k^{\alpha}l^{1-\alpha-\zeta} - lw(X) - rk\},$$

taking as given wages w and the interest rate r. While the interest rate, r, is endogenously determined in Greenwood et al. (2022a) by the growth rate of the economy, here we make the simplifying assumption that they are constant.

Note that S depends on aggregate productivity X because wages are a function of aggregate productivity. The surplus they obtain in a given period is given by

$$S(x, X) = x\zeta\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha-\zeta}{w(X)}\right)^{1-\alpha-\zeta}\right]^{1/\zeta}.$$

Using the expression for profits, the value of a successful project is given by the discounted sum of future profits,

$$\begin{aligned} I(x; X, g_w) =& \sum_{t=1}^{\infty}(\boldsymbol{s}\delta)^{t-1}S(x, wg_w^{t-1}), \\ =& \frac{x}{w(X)^{(1-\alpha-\zeta)/\zeta}}\sum_{t=1}^{\infty}(\boldsymbol{s}\delta)^{t-1}\zeta\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha-\zeta}{g_w^{t-1}}\right)^{1-\alpha-\zeta}\right]^{1/\zeta}, \end{aligned}$$

where $\boldsymbol{s}$ is the probability that the project survives to the next period; δ is the rate at which the entrepreneurs discount the future; and g_w is the growth rate of wages. Dividing and multiplying by X, and referring to a large term that depends on several parameters and the endogenous growth rate of wages as $\Gamma(g_w)$, the value of an IPO can be rewritten as

$$\begin{aligned} I(x; X, g_w) =& \frac{x}{X} \frac{X}{w(X)^{(1-\alpha-\zeta)/\zeta}} \sum_{t=1}^{\infty} (\boldsymbol{s}\delta)^{t-1} \zeta \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{1-\alpha-\zeta}{g_w^{t-1}}\right)^{1-\alpha-\zeta} \right]^{1/\zeta}, \\ =& g \frac{X}{w(X)^{(1-\alpha-\zeta)/\zeta}} \Gamma(g_w), \end{aligned}$$

where the firm will take the term $\Gamma(g_w)$ as given at the time of selecting the size of innovation, g, which is equal to x/X.

3 Productivity and labor market equilibrium

To obtain average productivity (i.e., X), sum across the productivities of all firms and divide it by the number of firms. The number of entrants is constant and equal to $\boldsymbol{n}$.[1] Then, the number of firms of age 1 with productivity x_{-1} is then $\boldsymbol{n}$.[2] Similarly, the number of firms of age 2 with productivity x_{-1}/g_x is equal to $\boldsymbol{n} \times s$. Continuing with this logic, we can add up the productivity across all firms today by

$$x_{-1}\boldsymbol{n}(1 + \boldsymbol{s}/g_x + (\boldsymbol{s}/g_x)^2 + ...) = x_{-1} \frac{\boldsymbol{n}}{1 - \boldsymbol{s}/g_x}.$$

Another way of obtaining the sum of productivity across all firms is by multiplying the number of firms $N = \frac{\boldsymbol{n}}{1-\boldsymbol{s}}$ by average productivity X; i.e., $\frac{\boldsymbol{n}}{1-\boldsymbol{s}}X$. Combining these two expressions we obtain

$$X = x_{-1} \frac{1 - \boldsymbol{s}}{1 - \boldsymbol{s}/g_x},$$

where we recall that x_{-1} is given but g_x is the key variable that will be determined in the equilibrium of the model.

[1] In Greenwood et al. (2022a), the number of entrants is endogenously determined as a function of the expected profits of becoming an entrepreneur.

[2] x_{-1} is the productivity where the idea was generated 1 period ago. We assume that it takes one period to materialize the idea (if it is good).

Labor demand for a project with productivity x is

$$l(x,w) = x\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha}\right]^{1/\zeta}.$$

Hence, we can write the labor market-clearing condition by adding the labor demand of all firms and setting it equal to the aggregate labor supply, which is assumed to be 1:

$$\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha}\right]^{1/\zeta} x_{-1}\boldsymbol{n}(1+\boldsymbol{s}/g_x+(\boldsymbol{s}/g_x)^2+...)=1$$

or

$$\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha-\zeta}{w}\right)^{1-\alpha}\right]^{1/\zeta} x_{-1}\boldsymbol{n}\frac{1}{1-\boldsymbol{s}/g_x}=1.$$

Since the last term is very close to average productivity, we can use this equation to solve for w as a function of X and parameters,

$$\begin{aligned} w(X) =& \left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)}(1-\alpha-\zeta)\left(\boldsymbol{n}\frac{x_{-1}}{1-\boldsymbol{s}/g_x}\right)^{\zeta/(1-\alpha)}, \\ =& \left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)}(1-\alpha-\zeta)\left(\frac{\boldsymbol{n}}{1-\boldsymbol{s}}X\right)^{\zeta/(1-\alpha)}, \\ =& B\times\left(\frac{\boldsymbol{n}}{1-\boldsymbol{s}}X\right)^{\zeta/(1-\alpha)} \end{aligned}$$

where $B=\left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)}(1-\alpha-\zeta)$.

4 VC contract

Though equipped with ideas, the entrepreneurs have no money, so they must seek financing from VC investors. Both the entrepreneur and the VC investors discount future payoffs at the rate δ. If the project becomes successful and can start producing in the next period, the value of the project is $I(x;X,g_x)$, which is the discounted sum of future profits. The probability for a project to succeed (σ) hinges on the amount of money spent on developing the project, $D(\sigma)$. Since a higher success probability implies a higher cost, the development cost function $D(\sigma)$ is assumed to be increasing and convex in σ. VC investors will choose σ when deciding how much to spend on developing the project. In return for financing the project, VC investors ask for a share e of the projects value when the project succeeds. VC investors are able to detect a share β of the bad projects, which they shut down. Thus, β is a measure of the evaluation efficiency of the VC industry.

The VC industry is modeled as a competitive industry, so the contract is written to maximize the payoff of the entrepreneurs. From the perspective of the entrepreneurs, the expected value of starting a project with VC financing is

$$C(x; X, g_w) = \max_{(\sigma,e)\in[0,1]^2} \delta\rho\sigma \left[I(x; g_x X, g_w)\,(1-e)\right]$$

subject to

$$0 = \underbrace{e\rho\sigma\delta I(x; g_x X, g_w)}_{\text{Revenue from successful projects}} - \underbrace{w(X)R(x,X) - [\rho + (1-\beta)(1-\rho)]w(X)D(\sigma)}_{\text{Financing cost}}.$$

The function $C(x; X, g_w)$ describes the value the entrepreneur will receive if the project is good and is is successful, which occurs with probability $\rho\sigma$. Note that what the entrepreneur receives is a share $(1-e)$ of the value of the IPO, $I(x; g_x X, g_w)$, which was described above. Note that the other part of the value of the IPO is received by the VC investor and is therefore part of the zero-profit constraint (i.e., $\rho\sigma I(x; g_x X, g_w)$). But, VC investors are facing two types of expenses. To be specific, they will cover the research expenses for all projects, as well as the development cost for projects remaining in the funding pool (i.e., all good projects and the bad ones that are not shut down).[3] Recall that the evaluation efficiency of VC is not perfect, so they detect only a share β of the bad projects.[4]

Note also that the maximization problem is precisely the same as a problem where entrepreneurs maximize their profits by spending the research and development costs by themselves (if entrepreneurs and banks have the same ability to select and develop

[3]Note that both the R&D cost (R) and development cost (D) are expressed in terms of labor. Hence, both terms are multiplied by the wage rate (w) in the zero profit constraint.

[4]While the contract here is a one-period interaction between the VC investor and the entrepreneur, Greenwood et al. (2022a) analyze a multi-period version of this problem that also incorporates moral hazard.

projects). This fact means that the result will not change if only one decision maker decides on the success probability and firms' innovation.

To obtain the optimal solution for the characteristics of the contract, (σ^*, e^*), we derive their first-order conditions (FOCs),

$$\rho\delta I(x; g_x X, g_w)(1-e) + \lambda\rho\delta I(x; g_x X, g_w)e - \lambda[\rho + (1-\beta)(1-\rho)]\,w(X)D'(\sigma) = 0, \quad (\sigma)$$

and

$$-\rho\delta\sigma I(x; g_x X, g_w) + \lambda\rho\delta\sigma I(x; g_x X, g_w) = 0, \quad (e)$$

where λ is the multiplier associated with the above problem. The optimal solution for e comes from the zero-profit constraint. An intuitive expression emerges for the equity share of the VC investors, as follows:

$$e = \frac{[\rho + (1-\beta)(1-\rho)]\,w(X)D(\sigma) + wR(x, X)}{\rho\delta\sigma I(x; g_x X, g_w)} = \frac{\text{expenses}}{\text{expected discounted profits}}.$$

The FOC for e gives an expression for the Lagrange multiplier, $\lambda = 1$. In addition, combining the FOC for σ with the expression for the Lagrange multiplier, we can obtain the following intuitive expression for the optimal choice of σ:

$$\underbrace{[\rho + (1-\beta)(1-\rho)]\,w(X)D'(\sigma)}_{\sigma,\text{ marginal cost}} = \underbrace{\rho\delta I(x; g_x X, g_w)}_{\sigma,\text{ marginal benefit}}.$$

In order to derive an analytical solution, the development cost function D is assumed to follow the functional form of

$$D(\sigma) = \frac{\sigma^2}{2z_D},$$

where z_D is the efficiency of VC in developing projects.[5] This implies that $D'(\sigma) = \frac{\sigma}{z_D}$.

[5] We can also derive an analytical solution with a more general form of cost function $D(\sigma) = \sigma^n/nz_D$ $(n > 1)$. In this case, the necessary condition for ι is $\iota > n/(n-1)$, instead of $\iota > 2$.

Using this in the efficiency for σ yields

$$\sigma^* = \frac{z_D \rho \delta I(x; g_x X, g_w)}{w(X)\,[1-\beta(1-\rho)]}.$$

This expression for σ^* delivers the first set of results taking as given aggregate variables X and g_w (i.e., in partial equilibrium).

Lemma 1 *The probability for a successful project to succeed (σ^*) is increasing in the following:*

- *The efficiency of VC to detect bad projects, β*
- *The efficiency of VC to lend expertise to entrepreneurs, z_D*
- *The share of good projects, ρ*
- *The value of the project, I.*

The results of Lemma 1 are very intuitive. As either the cost of financing declines (higher values of β or z_D) or the project's quality improves (higher ρ or I), it is more likely that a project financed by a VC investor succeeds.

Substituting the solution for σ^* into the expression for e, we obtain

$$e^* = \frac{1}{2} + \frac{w(X)^2\,[1-\beta(1-\rho)]\,R(x,X)}{z_D\,(\rho\delta I(x; g_x X, g_w))^2}.$$

Interestingly, note that there is an upper bound value of x that can be financed because $e^* \leq 1$. With this expression at hand, we can derive the second set of partial equilibrium results as follows.

Lemma 2 *The venture capitalists' share of equity, e^*, is decreasing in the following:*

- *The evaluation efficiency of VCs to detect bad projects, β*
- *The development efficiency of VCs to lend expertise to entrepreneurs, z_D*
- *The share of good projects, ρ*

- *The value of the project, I.*

To understand the results in Lemma 2, recall that VC is modeled as a competitive industry. Hence, a higher level of VC efficiency (higher β or z_D) contributes to a lower level of VC expenses, and, thus, the VCs can break-even at a lower level of VC's share of equity. Analogously, since VCs are chasing the same pool of projects, competition among VCs will transfer the surplus associated with better quality of projects (higher ρ or I) to the entrepreneurs, whereas the VCs end up with a lower share of equity in equilibrium.

5 Innovation choice

The choice of the step size for innovation is made by an entrepreneur choosing x to maximize its payoff; i.e,

$$V(X, g_x) = \max_x C(x; X, g_w).$$

With the solutions for (σ^*, e^*) and the expression for the value of a successful project $I(x; g_x X, g_w)$, we can rewrite $C(x, X, g_w)$ in terms of the step size for innovation, $g = x/X$:

$$\begin{aligned} C(x, X, g_w) &= \delta\rho\sigma^* I(x; g_x X, g_w)\,(1 - e^*)\,, \\ &= g^2 \left(\frac{X}{w(g_x X)^{\frac{1-\alpha-\zeta}{\zeta}}} \right)^2 \frac{z_D(\rho\delta)^2 \Gamma(g_w)^2}{2w(X)\,[1 - \beta(1-\rho)]} - \frac{w(X)}{z_R} g^\iota . \end{aligned}$$

To find the optimal step size of an innovation compute the FOC with respect to g:

$$g \left(\frac{X}{w(g_x X)^{\frac{1-\alpha-\zeta}{\zeta}}} \right)^2 \frac{z_D(\delta\rho\Gamma(g_w))^2}{w(X)[1 - \beta(1-\rho)]} - \iota \frac{w(X)}{z_R} g^{\iota-1} = 0.$$

Solving gives the step size of innovation chosen by the entrepreneur:

$$g^* = \left(\frac{z_R z_D(\delta\rho\Gamma(g_w))^2}{\iota[1 - \beta(1-\rho)]w(X)^2} \left(\frac{X}{w(g_x X)^{\frac{1-\alpha-\zeta}{\zeta}}} \right)^2 \right)^{1/(\iota-2)} .$$

This solution is now used to derive the following partial equilibrium results about the optimal step size of innovation.

Lemma 3 *The step size of innovation, g^*, is increasing in the following:*

- *The evaluation efficiency of VCs to detect bad projects, β*
- *The development efficiency of VCs to lend expertise to entrepreneurs, z_D*
- *The share of good projects, ρ*

and it is decreasing in the growth rate of wages, g_w, which the entrepreneur takes as given.

Recall that entrepreneurs choose how innovative (how far ahead of the pack) their project is by considering alternative values of g given the aggregate conditions of the economy. Intuitively, Lemma 3 suggests that more-efficient VC financing (higher β and z_D) and a better pool of projects (higher ρ) encourage an entrepreneur to choose a more-innovative project. Finally, the last result relates the choice of innovation to the aggregate conditions of the economy. In particular, the last result indicates that the entrepreneurs will choose less-innovative projects when the wage rate grows faster because growing wages erodes the expected profits from the project.

6 Economic Growth

We can embed this VC contract into a growth model as long as the economy is growing at a constant rate. In particular, the next lemma demonstrates that along a balanced growth path the contract's characteristics and the innovation choice will be constant over time.

Lemma 4 *There is a balanced growth path where the growth rate of wages, $g_w = g_x^{\frac{\zeta}{1-\alpha}}$, the equity share of entrepreneurs, e^*, the probability for a good project to succeed, σ^*, and the step size of innovation, g^*, are constant.*

So far we have taken the choice of innovation by entrepreneurs, g, and the growth of average productivity in the economy, g_x, as two different things. To ensure we have an equilibrium, we find a growth rate of average productivity g_x consistent with the choice of innovation, g. This consistency can be obtained by pinpointing the relationship between g_x and g. In particular, note that the growth rate of productivity is

$$g_x = X'/X = x'/x,$$

and that $g = x/X$. Therefore, a link between aggregate productivity growth, g_x, and the step size of innovation, g, can be established as follows:

$$g_x = \frac{x}{x_{-1}} = \frac{gX}{x_{-1}} = g\frac{1-\boldsymbol{s}}{1-\boldsymbol{s}/g_x}.$$

This provides an expression for g as a function of g_x,

$$g = \frac{g_x - \boldsymbol{s}}{1-\boldsymbol{s}}.$$

In addition, the consistency between the growth rate of wages, g_w, and aggregate productivity growth, g_x, is also key to ensuring the existence of an equilibrium. A link between g_w and g_x is written as

$$g_w = \frac{w'}{w} = \left(\frac{X'}{X}\right)^{\frac{\zeta}{1-\alpha}} = g_x^{\frac{\zeta}{1-\alpha}}.$$

We will use this relationship between g, g_w, and g_x in the solution for g^* to find the equilibrium growth rate of the economy. But first we need to rewrite g^* as a function of g_x. We start by substituting $\Gamma(w)$ in the expression for g^*, which is referred to as g for simplicity,

$$g = \left(\frac{\delta\rho}{w(X)}\left(\frac{z_R z_D}{\iota[1-\beta(1-\rho)]}\right)^{1/2}\frac{X}{w(g_x X)^{(1-\alpha-\zeta)/\zeta}}\sum_{t=1}^{\infty}\left(\frac{\boldsymbol{s}\delta}{g_w^{(1-\alpha-\zeta)/\zeta}}\right)^{t-1}\right)^{2/(\iota-2)} \times A,$$

where

$$A = \left(\zeta\left(\frac{\alpha}{r}\right)^{\alpha/\zeta}(1-\alpha-\zeta)^{(1-\alpha-\zeta)/\zeta}\right)^{2/(\iota-2)}.$$

Replacing $w(X)$ and solving the series for g_w,

$$g = \left(\delta\rho\left(\frac{z_R z_D}{\iota[1-\beta(1-\rho)]}\right)^{1/2}\left(\frac{1-\boldsymbol{s}}{\boldsymbol{n}}\right)\left(\frac{1}{g_x^{\frac{1-\alpha-\zeta}{1-\alpha}}}\right)\left(\frac{g_w^{(1-\alpha-\zeta)/\zeta}}{g_w^{(1-\alpha-\zeta)/\zeta}-\boldsymbol{s}\delta}\right)\right)^{2/(\iota-2)} \times \hat{A},$$

where $\hat{A} = A/B^{2(1-\alpha)/(\iota-2)\zeta}$ for the value of B defined above.

Now we need to use the expressions for g, X, g_w to find an equation for g_x. We obtain

$$\frac{g_x - \boldsymbol{s}}{1-\boldsymbol{s}} = \hat{A}\times$$

$$\left(\delta\rho\left(\frac{z_R z_D}{\iota[1-\beta(1-\rho)]}\right)^{1/2}\left(\frac{1-\boldsymbol{s}}{\boldsymbol{n}}\right)\left(\frac{1}{g_x^{(1-\alpha-\zeta)/(1-\alpha)}-\boldsymbol{s}\delta}\right)\right)^{2/(\iota-2)}$$

To gain more intuition from this equation, let

$$\Gamma = \left(\delta\rho\left(\frac{z_R z_D}{\iota[1-\beta(1-\rho)]}\right)^{1/2}\left(\frac{1}{\boldsymbol{n}}\right)\right)^{2/(\iota-2)} \times \hat{A}.$$

It transpires that Γ is increasing in research efficiency (z_R), VC's development efficiency (z_D), VC's evaluation efficiency (β), and the share of good ideas (ρ).

Γ is useful to rewrite the equation for g_x as

$$\frac{g_x - \boldsymbol{s}}{1-\boldsymbol{s}} = \Gamma \times \left(\frac{1-\boldsymbol{s}}{g_x^{(1-\alpha-\zeta)/(1-\alpha)}-\boldsymbol{s}\delta}\right)^{2/(\iota-2)}.$$

In light of this, to understand the impact of the efficiency of VC financing, the quality of the projects, and the R&D efficiency on the growth of the general model, we just need to figure out the impact of Γ on g_x.[6] The results are summarized in the next lemma.

Lemma 5 *The equilibrium growth rate of productivity in the economy, g_x, is increasing in the following:*

[6]It is simple to see that if the Γ increases the RHS increases, and what would lead to an increase in g_x to increase the LHS and reduce the RHS.

- *The evaluation efficiency of VCs to detect bad projects, β*
- *The development efficiency of VCs to lend expertise to entrepreneurs, z_D*
- *The share of good projects, ρ*
- *The efficiency of R&D, z_R.*

Finally, check that there indeed exists a balanced growth path. It will be shown that $\{\sigma^*, e^*\}$, which is the solution for the old problem, solves the new one for $x' = g_x x$ and $X' = g_x X$. First, observe that if $x' = g_x x$ and $X' = g_x X$, then $I(x'; X', g_w) = g_w I(x'; X', g_w)$. This can be seen because x will rise by g_x and wages by g_w. Then, it is immediate that $C(x'; X', g_w) = g_w C(x'; X', g_w)$. Now, consider the zero-profit constraint. At the conjectured solution, the right-hand side will inflate by the factor g_w since $I(x'; X', g_w) = g_w I(x'; X', g_w)$, $w' = g_w w$, and $R(x', X') = R(x, X)$. This is trivially true for the left-hand side. Hence, the zero-profit constraint holds at the new allocations. To sum up, at the conjectured new solution, the objective function and the constraint scale up by the same factor of proportionality g_w. By canceling out this factor of proportionality, the new problem reverts back to the old one. Likewise, it is easy to deduce that if x solves the choice problem of the step size for X, then $x' = g_x x$ solves it when $X' = g_x X$. This occurs because this problem also scales up by the factor of proportionality g_w, and so $V(X, g_x)$ will grow at the same rate as wages, w. Additionally, g* is constant since it doesn't depend on either x or X. Therefore, there exists a balanced growth path where the growth rate of wages, $g_w = g_x^{\frac{\zeta}{1-\alpha}}$, the equity share of entrepreneurs, e^*, the probability that a good project succeeds, σ^*, and the step size of innovation, g^*, are constant, as in Lemma 4.[7]

Taking stock of all these lemmas, our findings have unveiled exactly how VC contributes to innovation and economic growth. To be specific, enhanced VC efficiency encourages VCs to invest more in financing ventures (Lemma 1), and the entrepreneurs are rewarded with a larger share of the pie because of competition among VCs (Lemma 2). Consequently, the entrepreneurs are incentivized to pursue

[7]For the optimally chosen g_x, the venture capitalists' share of equity, e^*, is written as $1/2 + 1/\iota$, which is only dependent on the convexity of the R&D cost function, ι. Since ι is larger than 2 to have a general equilibrium, e^* is less than one. As such, the upper bound of e ($e \leq 1$) doesn't matter if g_x is optimally chosen.

more radical innovations (Lemma 3), which contributes to a higher level of long-run productivity growth (Lemma 5).

7 Conclusion

This article explores the connection between VC financing and economic growth. It introduces a simple endogenous growth model with a VC financing contract along the lines of Greenwood et al. (2022a). The emphasis is on the model's simplicity and its analytical solutions that allow for a characterization of the model. The results describe how the VC activity changes as project screening and development improve and how this better financing affects aggregate productivity growth.

One of the crucial questions left unanswered is the role of VC financing to account for differences in development across countries. Although more research is needed in this area, the findings in Cole, Greenwood, and Sánchez (2016) and Greenwood, Han, and Sánchez (2022a) suggest that differences in the cost of enforcing contracts, as well as the taxation of successful entrepreneurs, may be behind such cross-country differences.

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