# Gender Gap

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Gender Gap*

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WORK IN PROGRESS – COMMENTS WELCOME

Abstract

We employ the Ben-Porath (1967) human capital model to study the evolution of the gender wage gap over the long run. We consider the effect of changing lifecycle profiles of female market hours. We find that the implied response in unobserved investment in human capital accumulation accounts for most of the long run gender wage gap dynamics. This finding is consistent with the labor economists’ view that changing selection on unobservables played a critical role in the gender wage gap dynamics. Our contribution is to make explicit and quantify the link between market hours and (unobserved) investment in human capital.

JEL: J16; J24; J31

Key words: Gender wage gap; Selection bias; Female labor force participation; On-the-job investment; Human capital

*The views expressed in this article are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.
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1 Introduction

Figure 1 illustrates the dynamics of the gender wage gap – defined as the ratio of female to male hourly earnings – for working-age Americans in the labor force. While the observed gender wage gap in the U.S. declined steadily since the beginning of the century, it experienced a setback during the 1950s and 1960s before closing rapidly throughout the 1980s and the early 1990s. The dramatic closing of the gender wage gap that took place during the 1980s and early 1990s has been the main focus in related literature, and as seen from Figure 1, it is concurrent with a significant slowdown of growth in male wages.

Figure 1: Wage Dynamics for Individuals in the Labor Force

Note: The figure represents average income per hour for working men and women (and its ratio), calculated as population-weighted averages of age-specific wages.
Source: U.S. Census, CPS and authors’ calculations

Understanding the forces behind the dynamics of the observed GG requires measurement of the joint evolution of prices of the observed and unobserved characteristics of men and women and the composition of the labor force in terms of those characteristics. Mulligan and Rubinstein (2008) argued that the change in the composition of working women in terms of their unobservables – all characteristics that are not
easily captured by observable factors such as education or years of experience – accounted for nearly all of the closing of the observed gender wage gap. Their main hypothesis is that an exogenous rise in the market value of unobservable characteristics was the driving force, which is appealing because the same force was also found to underlie the concurrent rise in inequality within gender (e.g. Juhn et al. (1993), Katz and Autor (1999)). Of course, other factors – for example, changes in discrimination (Jones et al. (2015)), composition of women on observable characteristics (e.g. O’Neill and Polachek (1993), Goldin (2006)), attitudes towards women (Fernandez and Fogli (2009), Fortin (2015)), and availability of birth control (Bailey et al. (2012)) – must have mattered too. In fact, Bar et al. (2015) reconciled the Mulligan and Rubinstein (2008)’s drastic findings with much of related literature while showing further support for their main hypothesis, namely, that an exogenous rise in the market value of unobservable characteristics contributed to the closing of the gender gap.

In this paper, we take a step further and inquire into the nature of changing unobservable characteristics of women in the labor force. Specifically, we employ the Ben-Porath (1967) model to quantify the role of unobservable on-the-job investment in human capital in driving the dynamics of the gender wage gap. Thus, our paper belongs to the strand of literature that studies the role of unobservable time investments in human capital in helping us understand different aspects of the earnings distribution (e.g. Guvenen and Kuruscu (2010), Huggett et al. (2011), Hendricks (2013), Guvenen et al. (2014), Kong et al. (2018)).

In our model, agents face a tradeoff between current and future earnings. Agents that expect to work more in the future spend a greater fraction of their work time on human capital accumulation. In practice this may mean working lower salary jobs (such as a medical residency or a low-paid internship) in exchange for more on-the-job learning and greater career growth opportunities. Note that experience – an observable characteristic widely recognized for its contribution to wage growth\(^1\) – is a poor proxy of such unobserved investment as two agents with the same number of

\(^1\text{e.g. Gayle and Golan (2008)}\)
years may undertake different investments if they plan to work different hours in the future. Erosa et al. (2016) also recognize the importance of unobserved investment (effort in their model) in explaining wage growth. They focus on lifecycle evolution of the gender wage gap for a single cohort.

Our model economy is comprised of representative overlapping cohorts of men and women, of measures taken from the data. Our approach is to assume each cohort of men and women face the same human capital accumulation process, the same skill prices and the same initial human capital. The only difference between men and women, for a given birth cohort, is in the lifecycle profiles of time endowments – the total time allocated to working and human capital investment while at work. For each cohort and gender, these profiles are measured as age-specific average hours worked. While these profiles are relatively stable across cohorts for men, they differ substantially for women with older cohorts exhibiting very low market hours during child bearing years. Figure 2 shows that the oldest four cohorts we consider (21-25 year old women in 1940, 1945, .. 1955) represent mothers of the baby boomers – they significantly reduced work hours during childbearing years and their labor supply remained low even later in life. The 5th cohort represents the cohort for whom competed fertility returned to the level prior to the baby boom.

We discipline changes in initial human capital endowments and growth in skill prices by using the observed cohort-specific wage profiles for men. We then assess the contribution of gender-specific time endowment profiles to the evolution of the gender wage gap by modeling women as men with female time endowment profiles. In other words, we simply ask what the evolution of male wages would look like if they were to face the same time profiles as their female counterparts. This is the cleanest way to assess the importance of changing time endowment profiles on unobservable investment in human capital and its contribution to gender wage gap dynamics. The advantage of our approach is that we take the labor supply profiles for women as given, without attempting to explain their evolution.\(^2\) This allows us to focus on the

\(^2\)There is a large body of literature concerned with explaining the rise in female labor supply and/or its changing shape over the lifecycle. Eckstein and Wolpin (1989), Caucutt et al. (2002),
time allocation at work and more precisely measure the importance of unobservable investment in human capital.

Figure 2: Labor Supply Profiles

Note: The figure represents lifecycle labor supply profiles, computed as the average annual hours per person divided by 5600, for cohorts that are 21-35 in 1940, 1945, ...1985.
Source: U.S. Census, CPS and authors’ calculations

We find that the trend in unobserved investment in human capital accumulation, which results from steepening of the time endowment profiles across cohorts over time, accounts for most of the long run gender wage gap dynamics. This finding is consistent with the labor economists’ view that changing selection on unobservables played a critical role in the gender wage gap dynamics.

We have also examined the effects of different “learning abilities” as measured by high school test scores of girls and boys and found little effect on gender gap dynamics.

Greenwood et al. (2005), Olivetti (2006), Attanasio et al. (2008), Doepke et al. (2015) comprise few examples of many important papers.
2 Data

We consider 5-year time periods and the following 11 age groups: 21–25, 26–30, …, 71–75. We employ data on labor income, hours worked and education attainment from the U.S. Population Census for years 1940, 1950 and 1960 and from CPS data for years 1965, 1970, 1975,…, 2015. For the two missing years (1945 and 1955), we impute variable values based on a linear interpolation between adjacent years. All nominal variables are converted into 2000 dollars using the consumer price index. All flow variables are reported in annual terms.

We compute the following quantities for each (year,age,sex) cell, denoted by \((y,a,s)\):

- \( pop (y,a,s) \) – total population,
- \( hou (y,a,s) \) – average annual hours of workers,
- \( lfp (y,a,s) \) – fraction working, defined as fraction of population working at least 200 annual hours,
- \( inc (y,a,s) \) – average annual labor income among workers,
- \( col (y,a,s) \) – fraction of workers with a college degree,
- \( hcg (y,a,s) \) – average annual hours of workers with a college degree,
- \( hnc (y,a,s) \) – average annual hours of workers without a college degree.

Each variable is a \((16,11,2)\) table as we construct 16 years worth of data (1940, 1945,…, 2015) for 11 age groups and 2 genders.

We define the time endowment as average annual hours per person divided by annual productive hours:

\[
ee (y,a,s) = \frac{hou (y,a,s) \times lfp (y,a,s)}{(24 - 8) \times 7 \times 50}.
\]
These are the data underlying cohort-specific labor supply profiles depicted in Figure 2.

Wages by year, depicted in Figure 1, are defined by aggregating across age-specific average wages for workers.

\[ w(y, s) = \sum_a \left( \sum_k \text{pop}(y, a, s) \right) \times \frac{\text{inc}(y, a, s)}{\text{hous}(y, a, s)}. \]

3 Model

Our model is a version of Ben-Porath (1967). Individuals work from age 1 to \( J \) and seek to maximize the present value of their consumption. Let \( V_{t,j}(h, x) \) denote the present value of labor income for a worker of cohort \( t \) at age \( j \)—that is in period \( t + j - 1 \) (cohort \( t \) refers to people who are age 1 in period \( t \)). The variable \( x \) represents an “ability” to accumulate human capital. Let \( w_t \) denote the wage rate per unit of human capital-hour in period \( t \). Finally, let \( e_{t,j} \) denote the time endowment of a worker of cohort \( t \) at age \( j \). Assume the interest rate, \( r \), is constant.

\[
V_{t,j}(h, x) = \max_{n \leq e_{t,j}} w_{t+j-1}h\left(e_{t,j} - n\right) + \frac{1}{1 + r}V_{t,j+1}(h', x)
\]

subject to

\[
\begin{align*}
h' &= (1 - \delta) h + x(nh)^\gamma \\
V_{t,J+1} &= 0
\end{align*}
\]

where \( J \) is the last period of work and \( n \) is the time allocated to human capital accumulation. The variable \( \delta \) is the depreciation rate of human capital. The function \( x(nh)^\gamma \) is a production function combining learning ability, \( x \), time, \( n \), and human capital, \( h \), to produce more human capital for the next period. The condition \( V_{t,J+1} = 0 \) stipulates a terminal condition.
In appendix A, we show that the interior solution to optimal investment in human capital is given by

\[ n = \left( \frac{x^{\gamma} \beta_{t,j+1}}{1 + r w_{t+j-1}} \right)^{\frac{1}{1-\gamma}} / h. \]  

(2)

where

\[ \beta_{t,j} = \sum_{k=j}^{J} \left( \frac{1 - \delta}{1 + r} \right)^{k-j} w_{t+k-1} e_{t,k} \]  

(3)

is the shorthand that represents the marginal contribution of human capital to lifetime income.

What is immediately clear from the solution is that \( n \) is inversely proportional to \( h \) and depends positively on learning ability as well as the labor supply and wage growth over one’s lifecycle.

It is also clear that multiplying \( h_1 \) by a constant \( \kappa \) and learning ability \( x \) by \( \kappa^{1-\gamma} \) does not alter the optimal investment. It follows from 1 that the entire human capital profile is multiplied by \( \kappa \).

Substituting for \( n \) from 2 into the law of motion 1 gives

\[ h_{t,j+1} = (1 - \delta) h_{t,j} + x \left( \frac{1}{1 + r w_{t+j-1}} \right)^{\frac{1}{1-\gamma}} \]  

(4)

where the second term represents the human capital increment due to learning at age \( j \) which we denote by \( \Omega_{t,j+1} \).

Iterating forward on the law of motion of human capital gives the closed-form solution for human capital of cohort \( t \) and age \( j \):
\[ h_{t,j} = (1 - \delta)^{j-1} h_{t,1} + \sum_{\tau=2}^{j} (1 - \delta)^{j-\tau} \Omega_{t,\tau}, \]  

(5)

where the human capital increment due to learning at age \( \tau - 1 \),

\[
\Omega_{t,\tau} = \frac{x_{t+j+1}}{w_{t+j-1}} \left( \frac{\gamma}{1 + r} \right)^{\tau-j+1} \times
\]
\[
\times \left[ w_{t+j-1} \epsilon_{t,\tau} + \left( \frac{1 - \delta}{1 + r} \right) w_{t+j-1} \epsilon_{t,\tau+1} + \ldots + \left( \frac{1 - \delta}{1 + r} \right)^{J-j} w_{t+J-1} \epsilon_{t,J} \right]^{\frac{\gamma}{1 - \delta}} \right],
\]  

(6)

was obtained by substituting for \( \beta_{t,j+1} \) into the second term of 4.

Thus, human capital at age \( j \) is just the sum of properly depreciated initial stock and properly depreciated age-specific increments resulting from past learning. To understand the shape of the human capital profile by age, consider the case of constant skill prices. Without depreciation of human capital, \( h \) would increase throughout one’s lifecycle, as at every age a new (positive) increment \( \Omega_{t,j} \) is added on. Of course, human capital increments \( \Omega_{t,j} \) also decrease with age as future opportunities to employ human capital decline – agents face fewer periods of life left and endowment profiles typically decline with age. This means that \( h \) increases at a decreasing rate. Now, with depreciation added on, \( h \) increases from one age to the next only if the added increment \( \Omega_{t,j} \) offsets the depreciated stock. This is less likely to happen later in life because the increments get smaller. Therefore, \( h \) exhibits a hump-shaped profile.³

The hourly earnings are obtained by multiplying \( h \) by the appropriate skill price. So the hourly wage of cohort \( t \) at age \( j \) is given by

\[ y_{t,j} = w_{t+j-1} h_{t,j}. \]  

(7)

³If depreciation is sufficiently high, human capital profile will decrease throughout the lifecycle.
4 Quantitative analysis

4.1 Calibration

A model period is 5 years. We consider 11 age groups: 21–25, 26–30, \ldots, 71–75.

The first period for which we have a complete cross section of age-specific hourly earnings data is 1940, which makes the cohort of (model) age 11 in 1940 the oldest cohort for which we have any earnings data. We begin our simulations in year 1890 so as to capture the full lifecycle of this cohort. We refer to 1940 as model period 1, implying that 1890 corresponds to model period −9. The youngest cohort that we simulate is model age 1 in 2010 so that the model can generate a complete cross section of age-specific hourly earnings in 2010. The youngest cohort is, therefore, model cohort 15. In what follows, we indicate time and cohort in model period (instead of years), and we refer to a cohort by the period in which it is age 1 (that is age 21–25 in the data).

We assume that \( w_t \) evolves exogenously according to

\[
 w_t = w_{-9} \exp \left( g_1 (t - 1) + g_2 (t - 1)^2 \right).
\]

We set the real rate of interest to 4 percent per year: \( 1 + r = 1.04^5 \). Following the estimates in related literature (for example, Huggett et al. (2011) and Huggett et al. JME use 1% or 2% depreciation), we set the rate of human capital depreciation to 1.5 percent per year: \( 1 - \delta = 0.985^5 \). We normalize the initial human capital stock of cohort 1, \( h_{1,1} = 1 \). We calibrate the remaining parameters by targeting all available observations of hourly earnings data for men in the labor force, i.e. average hourly earnings by age in periods 1, 2, \ldots, 15.

First, we explain why the normalization of initial human capital (of any cohort) is warranted. This can be seen from model wage definition 7 and human capital solution 5. For any change in \( h_{1,1} \), say an increase by a factor of \( \kappa \), the same exact
wage profiles are attained by raising all other $h_{1,t}$ by the same factor, lowering $w_{-9}$ by the same factor and raising learning ability by $\kappa^{1-\gamma}$ while keeping the rest of the parameters unchanged. As a result, the $h$ profiles shift up by $\kappa$ for all cohorts, while the $n$ profiles remain unchanged. Because the skill price path is shifted down by $\kappa$, wage profiles indeed remain unchanged.

It is helpful to discuss how the hourly earnings identify model parameters. In order to clarify the model identification, we break our minimization procedure into two components.

Let $y_{t,j}^M$ denote the hourly earnings of an age $j$ man of cohort $t$,

$$y_{t,j}^M = w_{t+j-1} h_{t,j}^M,$$

and $y_{t,j}$ denote its empirical counterpart.

First, for a given path of $w_t$ (to be calibrated below) and normalized value for $h_{t,1}^M$, we can determine the initial human capital of all younger cohorts, $h_{t,1}, t = 2, \ldots, 15$, by exactly reproducing age-1 earnings growth across these cohorts. Precisely, to ensure that $y_{t-1,1}^M = y_{t-1,1}^M w_{t-1} / w_t$, we set

$$h_{t,1} = h_{t-1,1} \frac{y_{t,1}^M}{y_{t-1,1}^M} \frac{w_{t-1}}{w_t} \quad \text{for } t = 2, \ldots, 15. \tag{8}$$

Second, we calibrate the remaining parameters, including the two parameters that determine the path of $w_t$, to match the rest of the observed age-year earnings. Denote by $\omega$ the list of parameters, $\omega := (w_{-9}, g_1, g_2, \{h_{t,1}^M\}_{t=-9}^{15}, \gamma, x)$, that is, the initial level of efficiency wage ($w_{-9}$) and the two constants governing its growth rate ($g_1, g_2$), the levels of initial human capital stock for cohorts born in $t < 1$, $\{h_{t,1}^M\}_{t=-9}^0$, the human capital technology parameter $\gamma$ and learning ability $x$, which we keep fixed across cohorts and genders in the benchmark model. We choose $\omega$ that minimizes the Euclidean norm of the matrix with elements representing the time $t$ age $j$ difference.
between model and data earnings:

$$\min_\omega \sum_{t=1}^{15} \sum_{j=1}^{11} (y_{t,(j-1),j}^M(\omega) - y_{t,(j-1),j}^M)^2$$

subject to equations (8).\(^4\)

The dynamics of skill prices is governed by two parameters. With depreciation calibrated outside of the model, we think of growth of wage profiles later in life as identifying skill price growth. Indeed, with investments close to zero, human capital later in life declines at the preset depreciation rate and therefore the observed wage growth would have to be matched via the growth rates of skill prices.

With growth rates of \(w_t\) identified and \(h_{t,1}\) normalized to 1, it becomes clear that \(w_{-9}\) (which controls the level of the path of \(w_t\)) is identified from \(y_{t,1}^M\). With \(w_t\) path in hand, equations (8) identify \(h_{t,1}\) for cohorts \(t > 1\).

With skill price dynamics and initial human capital stocks identified, we argue that \(\gamma\) and \(x\) are identified by the shape of wage profiles earlier in life of cohorts \(t \geq 1\). As seen from equation 6 describing \(\Omega_{t,\tau}\), ability scale \(x\) increases human capital growth over the lifecycle by shifting all human capital increments \(\Omega_{t,\tau}\) up by the same factor. This is not the case for \(\gamma\). If \(\gamma > 0.5\), which introduces convexity to the bracketed term in equation 6, \(\gamma\) raises larger human capital increments (those made earlier in life) by more. This is because it raises the exponent on the bracketed term. It may even lower \(\Omega_{t,\tau}\) if the bracketed term falls below 1. So it changes the shape of human capital profiles by making it steeper earlier in life and flatter later on.\(^5\) Thus, both \(x\) and \(\gamma\) have independent effects on the shape of wage profiles provided learning is still sufficiently important.

Finally, parameters \(\{h_{t,1}\}_{t=-9}^0\) represent initial human capital values of cohorts for which wages in the beginning of life are not observed. They are identified by end of

\(^4\)Note that equations (8) ensure that \(y_{t,j}^M(\omega) - y_{t,j}^M = 0\) for \(j = 1,2,\ldots,15\).

\(^5\)The opposite is true for \(\gamma < 0.5\) although this is not an empirically relevant case.
life wages.

Our calibration results are reported in Table 1 and Figure 3 which depicts \( \{h_{t,1}^M\}_{t=-9}^{15} \) and dynamics of \( w_t \) since 1940. The calibration procedure yields a very small change in skill prices, they barely move. To be specific, the skill prices fall by less than 5% since 1940.

As explained above, the dynamics of \( w_t \) is determined by data earnings profiles late in life and the set value of \( \delta \). The larger the \( \delta \) we set, the higher is the growth rate of \( w_t \). We picked \( \delta \) from the literature because it allowed us to explain the identification clearly. However, if we throw \( \delta \) into the minimization routine, it increases only very slightly.

<table>
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<th>Life cycle</th>
<th>( J = 11 )</th>
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<tr>
<td>Human capital</td>
<td>( \gamma = 0.59, \delta = 0.073 ) (0.015 annual)</td>
</tr>
<tr>
<td></td>
<td>( x = 1.085 )</td>
</tr>
<tr>
<td>Wage process</td>
<td>( w_{-9} = 6.455, g_1 = -0.00566, g_2 = 0.000047 )</td>
</tr>
<tr>
<td>Interest rate</td>
<td>( r = 0.217 )</td>
</tr>
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</table>
Figure 3: Calibration Results

Note: The figure shows the calibrated values of $\{h_{i,1}^M\}_{i=9}^{15}$ and calibrated dynamics of $w$.

4.2 Model Fit

Figure 4 shows the model’s fit of selected profiles, omitting every other cohort and a few of the younger ones for which we observe earnings only early in life. We also omit the negative cohorts for which we observe earnings only later in life. But the fit is similar across all cohorts.
Figure 4: Model Fit: Selected Wage Profiles for Men in the Labor Force

Note: The figure shows the model fit of average income per hour for men working over 200 hours per year.

Source: U.S. Census, CPS and authors’ calculations
Figure 5 summarizes the model fit focusing on cohorts born in 1940 and later and plotting all age-cohort-specific earnings in the model and in the data. The fit is very good.

![Figure 5: Model Fit: All age and time specific wages](image)

*Note:* The figure shows the model fit of average income per hour for men working over 200 hours per year.

### 4.3 Experiments

#### 4.3.1 Main

We assess the contribution of gender-specific time endowment profiles to the evolution of the gender wage gap by modeling women as men with female time endowment profiles. In other words, we simply ask what the evolution of male wages would look like if they were to face the same market time profiles as their female counterparts. This is the cleanest way to assess the importance of changing time endowment profiles on unobservable investment in human capital and its contribution to gender wage gap dynamics.

Thus, for each cohort, we introduce the appropriate market time endowment from Figure 2.
Figure 6 shows the results of our main experiment. We plot the time series of average hourly earnings, and the gender gap in hourly earnings. Specifically, the first panel shows the population weighted average earnings of men (and women) at date $t$ defined as

$$y_t^M = \frac{\sum_{j=1}^{11} p_{t-j+1,j}^M y_{t-j+1,j}}{\sum_{k=1}^{11} p_{t-k+1,k}^M},$$

(9)

where $p_{t-j+1,j}^M$ is the population of male workers of age $j$ from generation $t - j + 1$. This is consistent with how we defined average wages by year in the data. We apply the same population weights to U.S. data and model-generated data. The second panel shows the resulting wage gap defined as

$$\frac{y_t^W}{y_t^M}.$$

Although we used all of the available wage profiles, including the end-of-life profiles for the negative cohorts, our main focus is on cohorts born in 1940 and those that are younger. We observe more data for these cohorts and therefore feel more confident about the identification of age 1 human capital stocks – which should be evident from our discussion above. Therefore, when reporting the average wage by year implied by the model in 9, we use data wages for negative cohorts. The numbers in the plot indicate the fraction of population for which model wages are used, which shows that, for the period of the gender wage gap closing (after 1980), over 90% of population is accounted for by the positive cohorts.
We find that the trend in unobserved investment in human capital accumulation accounts for most of the long run gender wage gap dynamics, accounting for 88% of the closing of the gender wage gap that took place since 1975. Younger cohorts of women differ from older cohorts along three dimensions: labor supply profiles depicted in Figure 2, initial human capital endowments depicted in panel a of Figure 3 and skill price dynamics depicted in panel b of the same figure. Figure 7 reveals that the younger cohort responded overwhelmingly by increasing the amount of time they devote to human capital investments early in life. The 21-25 year olds even spent a higher fraction of their time at work on learning.
Figure 7: Benchmark Model: Human Capital Investment Over Life Cycle

Note: The figure presents human capital investment over the life cycle in the main experiment, where women differ from men only in their endowment.

Figure 8 illustrates the resulting wage profiles of an older (1960 cohort), which worked primarily during the period before closing of the gender gap, and a younger (1985) cohort, which worked after the gap declined. We also plotted the empirical wage profile for females as a point of reference.

It is important to emphasize that we did not intend to reproduce female wage profiles. To do so, we would need to take a stance on how the initial human capital endowment changed across cohorts for women and how their skill prices evolved. Women surely face different skill prices whether it is due to differences in occupations and job types or statistical discrimination. Quantifying that differential is difficult. Instead, we merely ask how much men would earn over their lifecycle if they were to face female time endowment profiles.

The results are astounding. The differential time endowment seems to get us close to the actual female wage profiles. The model profiles for females are somewhat flatter compared to their empirical counterparts, again because the starting human capital stock is that of men.
4.3.2 The importance of changing time endowment profiles

Several things vary in the benchmark model, including time endowment, skill prices and initial human capital stocks. In this counterfactual, we seek to clarify the role of changing time endowment profiles of women. To do so, we assume that women of all cohorts face the same life cycle labor supply as one faced by women born in 1940 – the first cohort for which we observe the complete time endowment profile.

The main results are presented in Figure 9. We assess the contribution of changing time endowment profiles by comparing the results to the benchmark model. Clearly, the closing of the gender wage gap, seen in the benchmark model, is no longer present. If anything, the gender wage gap widens as women invest less and less in human capital due to slightly declining skill prices.
(a) Main Experiment: Wages by Gender, Model v. Data

(b) Main Experiment: Gender Wage Gap, Model v. Data

Figure 9: Counterfactual Results: Edowment profiles of the 1940 cohort

Note: The figure presents the results of the following counterfactual experiment. In the benchmark model, we assume that women of all cohorts face the endowment profile of the 1940 cohort women.

5 Conclusion

We find that the trend in unobserved investment in human capital accumulation, which results from changing time endowment profiles of women, accounts for most of the long run gender wage gap dynamics. The 1940-1960 cohorts of women gave birth to the baby boom generation. This is reflected in the shape of their labor supply profiles, as they would reduce their work during their child rearing years. Younger women faced steeper profiles and warranted more aggressive investment in human capital. Our version of the Ben Porath model allowed us to quantify the consequences of changing labor supply profiles for the gender wage gap.

This finding is consistent with the labor economists’ view that changing selection on unobservables played a critical role in the gender wage gap dynamics.

We have also examined the effects of different “learning abilities” as measured by high school test scores of girls and boys and found little effect on gender gap dynamics.
REFERENCES


A Optimization

Solution at age \(J\)

There is no human capital accumulation, so \(n = 0\) and

\[
V_{t,J} (h, x) = w_{t+J-1} he_{t,J}.
\]

Write this as \(V_{t,J} (h, x) = \beta_{t,J} h + \alpha_{t,J} (x)\) where

\[
\beta_{t,J} = w_{t+J-1} e_{t,J}, \quad \alpha_{t,J} (x) = 0.
\]

Solution at age \(j < J\)

Guess that the value function is of linear form for all ages, as was the case for age \(J\). Thus, we write \(V_{t,j+1} (h, x) = \beta_{t,j+1} h + \alpha_{t,j+1} (x)\). Then

\[
V_{t,j} (h, x) = \max_n w_{t+j-1} h (e_{t,j} - n) + \frac{1}{1 + r} [\beta_{t,j+1} ((1 - \delta) h + x (nh)^{\gamma}) + \alpha_{t,j+1} (x)].
\]

The first-order condition for an interior solution for \(n\) is

\[
w_{t+j-1} = \frac{1}{1 + r} \beta_{t,j+1} x^{\gamma} (nh)^{\gamma-1}.
\]

This implies

\[
w_{t+j-1} nh = \frac{1}{1 + r} \beta_{t,j+1} x^{\gamma} (nh)^{\gamma}
\]

and

\[
nh = \left( \frac{1}{1 + r w_{t+j-1}} \right)^{\frac{1}{1-\gamma}}.
\]
The value function writes

\[ V_{t,j}(h, x) = w_{t+j-1}e_{t,j} - w_{t+j-1}hn + \frac{1}{1+r} \beta_{t,j+1} (1 - \delta) h + \frac{1}{1+r} \beta_{t,j+1} x (nh)^\gamma \]

\[ + \frac{1}{1+r} \alpha_{t,j+1} (x), \]

\[ V_{t,j}(h, x) = w_{t+j-1}e_{t,j} - \frac{1}{1+r} \beta_{t,j+1} x (nh)^\gamma + \frac{1}{1+r} \beta_{t,j+1} (1 - \delta) h \]

\[ + \frac{1}{1+r} \beta_{t,j+1} x (nh)^\gamma + \frac{1}{1+r} \alpha_{t,j+1} (x), \]

\[ V_{t,j}(h, x) = w_{t+j-1}e_{t,j} + \frac{1}{1+r} \beta_{t,j+1} (1 - \delta) h + \frac{1}{1+r} \beta_{t,j+1} x (nh)^\gamma + \frac{1}{1+r} \alpha_{t,j+1} (x), \]

\[ V_{t,j}(h, x) = w_{t+j-1}e_{t,j} + \frac{1}{1+r} \beta_{t,j+1} (1 - \delta) h + \frac{1}{1+r} \beta_{t,j+1} x (1 - \delta) h + \frac{1}{1+r} \beta_{t,j+1} x (nh)^\gamma + \frac{1}{1+r} \alpha_{t,j+1} (x), \]

It follows the value function can be written in the form of

\[ V_{t,j}(h, x) = \beta_{t,j} h + \alpha_{t,j} (x) \quad (13) \]

with \( \beta_{t,j} \) and \( \alpha_{t,j} (x) \) recursively defined, starting with their terminal values, given by equations 10 and 11, and according to

\[ \beta_{t,j} = w_{t+j-1}e_{t,j} + \frac{1 - \delta}{1 + r} \beta_{t,j+1}, \quad (14) \]

\[ \alpha_{t,j} (x) = \frac{1 - \gamma}{\gamma} w_{t+j-1} \left( \frac{1}{1 + r} \frac{\beta_{t,j+1}}{w_{t+j-1}} x \gamma \right) + \frac{1}{1+r} \alpha_{t,j+1} (x). \]

At a corner solution, that is when \( n = e_{t,j} \), the value function is

\[ V_{t,j}(h, x) \bigg|_{n=e_{t,j}} = \frac{1}{1+r} V_{t,j+1} ((1 - \delta) h + x (e_{t,j} h)^\gamma, x). \]
The term $\beta_{t,j}$ is the marginal value of human capital, i.e., the effect of a unit of human capital on lifetime income at age $j$.

**Lifecycle profiles of human capital**

It is convenient to solve $\beta_{t,j}$ forward, which yields a closed form solution:

$$
\beta_{t,j} = w_{t+j-1} e_{t,j} + \frac{1 - \delta}{1 + r} \beta_{t,j+1}
$$

$$
= w_{t+j-1} e_{t,j} + \frac{1 - \delta}{1 + r} \left[ w_{t+j} e_{t,j+1} + \frac{1 - \delta}{1 + r} \beta_{t,j+2} \right]
$$

$$
= \ldots
$$

$$
= \sum_{\tau=j}^{J} \left( \frac{1 - \delta}{1 + r} \right)^{\tau-j} w_{t+\tau-1} e_{t,\tau}.
$$

Substituting from equation (12) into the law of motion for human capital obtains

$$
h_{t,j+1} = (1 - \delta) h_{t,j} + x (n_{t,j} h_{t,j})^\gamma 
$$

$$
= (1 - \delta) h_{t,j} + x \left( \frac{1}{1 + r} \frac{\beta_{t,j+1}}{w_{t+j-1}} x^\gamma \right)^\frac{1}{1-\gamma}.
$$

Introducing another shorthand to denote the second term above,

$$
\Omega_{t,j+1} = x \left( \frac{1}{1 + r} \frac{\beta_{t,j+1}}{w_{t+j-1}} x^\gamma \right)^\frac{1}{1-\gamma} = x^{\frac{1}{1-\gamma}} \left( \frac{\gamma}{1 + r} \sum_{\tau=j+1}^{J} \left( \frac{1 - \delta}{1 + r} \right)^{\tau-j-1} \frac{w_{t+\tau-1} e_{t,\tau}}{w_{t+j-1}} \right)^\frac{1}{1-\gamma}
$$

we rewrite the human capital of a worker of cohort $t$ and age $j$ as

$$
h_{t,j+1} = (1 - \delta) h_{t,j} + \Omega_{t,j+1}(x). \quad (17)
$$

Iterating on (17), we can derive the closed form solution for the human capital of
cohort $t$ and age $j$

\begin{align*}
h_{t,2} &= (1 - \delta) h_{t,1} + \Omega_{t,2} \\
h_{t,3} &= (1 - \delta) ((1 - \delta) h_{t,1} + \Omega_{t,2}) + \Omega_{t,3} \\
&= (1 - \delta)^2 h_{t,1} + (1 - \delta) \Omega_{t,2} + \Omega_{t,3} \\
\vdots \\
h_{t,j} &= (1 - \delta)^{j-1} h_{t,1} + \sum_{\tau=2}^{j} (1 - \delta)^{j-\tau} \Omega_{t,\tau}. \quad (18)
\end{align*}