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Lifetime Work Hours and the Evolution of the Gender Wage Gap*

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Abstract

The gender wage gap expanded between 1940 and 1975 but narrowed sharply between 1980 and 1995. We use a human capital accumulation model introduced in Ben-Porath (1967) to assess the role of gender differences in life-cycle profiles of market time and occupation sorting in explaining the gender wage gap dynamics over the long run. Men's aggregate hours profiles changed little across cohorts, but women's profiles converged to those of men, and especially so in higher-paying occupations. We calibrate the model to wage data by age, year, gender and occupation, and find that changing time allocation patterns induced human capital investments that account for nearly all of the gender wage gap dynamics. Occupation-specific human capital rental rates played a small role in helping close the gender gap since 1980. The roles of cohort-specific endowments, however, were less pronounced.

JEL codes: J16, J22, J24, J31
Keywords: gender wage gap, selection bias, female labor force participation, on-the-job investment, human capital, market hours

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1 Introduction

Figure 1 shows the evolution of hourly wages of men and women in Panel A and their percentage difference (i.e. the gender wage gap) in Panel B. While the wage gap widened between 1940 and 1975, it narrowed throughout the 1980s and the early 1990s. The focus of this paper is on the dynamics of the gender wage gap.

The literature on the gender wage gap often notes the dramatic change in the time allocation of women in the post-World War II era, characterized by women increasing their market hours, and especially so in higher-paying occupations (see Section 2). Mulligan and Rubinstein (2008), in particular, argue that changes in the composition of working women in terms of unobserved characteristics account for nearly all of the closing of the wage gap in the 1980s and the early 1990s.[1] Since human capital is complementary to labor market time, as they repeatedly emphasize, it likely represents one such unobserved characteristic that changed a great deal for working women.

Motivated by these observations, we examine the extent to which gender-specific human capital investments, induced by changing time allocation patterns, contributed to the observed dynamics of the gender wage gap. Our main hypothesis is that workers invest more in on-the-job learning if they expect to work longer hours in the future, and especially so in occupations that provide better training and career advancement opportunities. These investments translate into more human capital and higher wage growth.

In Section 2, we report the changing time allocation patterns and present an empirical decomposition of the aggregate wage gap dynamics. We find that changes in age- and occupation-specific wage gaps account for the bulk (more than 90%) of the 1940 to 1975 widening and most (60%) of the 1975 to 2010 closing of the aggregate gap. The age- and occupation-specific composition of workers accounts for most of the remaining dynamics. Goldin (2014) also emphasized the importance of understanding within-occupation gaps. In light of these facts, we chose to model the age- and occupation-specific wage gaps while taking as exogenous the evolution of the demographic and occupation composition

[1] While this particular result may be dependent on model specification (see Bar et al. (2015) for robustness), labor literature largely agrees that understanding unobserved characteristics is important for understanding the gender wage gap and its evolution. Among the observed characteristics, experience and occupation tend to have the most explanatory power. See Blau and Kahn (2017), Goldin (2014) for literature overview.

of the workforce. Specifically, we build on the model of human capital accumulation introduced in Ben-Porath (1967). Workers of each gender, cohort and occupation are endowed with initial human capital, learning productivity and a sequence of market hours over their working lives. They optimally allocate their market hours between work and human capital accumulation – which raises their future effective wages. Occupation and learning productivity matter for the human capital technology.[2]

To map our model variables to the data, we consider two groups of occupations (categorized according to pay) and measure market hours as cohort-, age-, gender- and occupation-specific hours per person. We organize market hours into life-cycle profiles for synthetic cohorts of men and women and use them as exogenous inputs in the model. Our exogenous treatment of market hours is by design – the goal is to understand their direct impact on human capital accumulation of different population groups.[3] The remaining exogenous model inputs are group-specific initial human capital endowments and learning productivity levels, as well as human capital rental rates which we assume to be occupation-specific but gender-neutral. We calibrate these alongside the human capital production parameters by fitting the model to hourly wage data by age, year, gender, and occupation from 1940 to 2010.

Our quantitative model successfully reproduces the aggregate wage dynamics as well as life-cycle wage profiles for different cohorts, gender and occupation groups. To understand the intuition behind it, consider the key determinant of human capital dynamics in our model – its marginal value (which we refer to as β). It is defined as the effect of one extra unit of human capital on the present value of labor income and comprises the combined effects of future human capital rental rates and market hours. The higher the β, the higher is the return to human capital accumulation and the steeper is the wage profile.

We find a particularly striking contrast between the dynamics of the marginal value of human capital of men in (lower paying) occupation 1 and that of women in (higher paying) occupation 2. These two β's played an important role for the aggregate dynamics of gender-specific human capital, and therefore, wages. In 1940, men allocated a lot of market time to occupation 1 and therefore faced high returns to human capital accumu-

[2]This assumption is motivated by the facts documented in Erosa et al. (2022b).

[3]There is a large body of literature aimed at understanding the rise of female market hours and/or their changing pattern over the life cycle. We defer to literature surveys for summary of this work, e.g. Doepke and Tertilt (2016), Greenwood et al. (2017).

lation. These returns declined over time because of both, falling market hours and falling rental rates of human capital. In contrast, women allocated low hours to occupation 2 in 1940 (which remained low throughout the baby boom period) and therefore faced low returns to human capital accumulation. These returns rose over time, driven predominantly by the dramatic increase in market hours allocated to occupation 2. Thus, in the early part of the sample, men accumulated human capital faster than women, which was conducive to the widening of the gender wage gap. By 1975, the rate at which women accumulated human capital surpassed that of men, and the wage gap began to shrink.

We perform several counterfactual experiments in the context of our calibrated model in order to examine separate contributions of market hours (our main hypothesis), human capital rental rates, initial human capital endowments and learning productivity levels.

Consistent with our hypothesis, we find that the largest impact was due to the change in gender-specific market hours. Indeed, with gender differences in hours eliminated, the model no longer generates the salient features of the gender wage gap dynamics. Precisely, we find that the gender hours difference accounted for about 104% to 123% of the post-1975 wage gap closing and predicted a substantially stronger widening of the gap than what was observed in the earlier period. It was within-occupation gender-specific hours dynamics that drove the dynamics of β's whose role was emphasized above. However, the timing of hours reallocation towards occupation 2 – which offered a more productive learning technology and higher rental rates – also mattered. Men leading the time reallocation process up until 1970 contributed to the wage gap widening, while women leading the process in the later period contributed to the wage gap closing.

Next in importance was the role of human capital rental rate dynamics, although it was relatively minor, most notably accounting for 3% to 34% of the post-1975 gender gap closing – the period when the occupation-2 rental rates exhibited relative growth. Women experienced relative wage growth in response to this change because their occupation-2 share of market hours was higher. But the main effect of rental rates was due to their differential levels and the fact that, since 1980, women continued to reallocate their hours towards higher paying occupations while men's reallocation had slowed down significantly.

The dynamics of initial human capital endowments, calibrated to grow for both men and women over time, accounted for a small part of the gender wage gap closing in the 1980s/90s, but worked largely against its widening in the earlier period. Finally, gender differences in learning productivity levels did not contribute to the observed gender gap dynamics, and in fact worked against it throughout the sample.

Our paper belongs to the strand of literature that studies the role of unobserved time investments in human capital – i.e. models that build on Ben-Porath (1967) – in explaining different aspects of the earnings distribution, or its evolution over time (e.g., Heckman et al., 1998, Guvenen and Kuruscu, 2010, Huggett et al., 2011, Hendricks, 2013, Guvenen et al., 2014, Kong et al., 2018, Fonseca et al., 2024). The key tradeoff in this class of models is between current and future earnings. These studies focus on the earnings distribution of men, while our focus is on women and across-gender inequality. To the best of our knowledge, ours is the first application of this type of model to the evolution of the gender gap phenomenon.

We enhance our model's ability to match the data by introducing two occupations that can potentially differ in human capital accumulation technology and rental rates of human capital. In addition to capturing occupation-specific factors related to career growth opportunities, human capital technology would reflect worker selection effects, such as those arising endogenously in gender-specific occupation choice models of Hsieh et al. (2019) and Erosa et al. (2022a). Importantly, the presence of two occupation categories allows our model to capture discrimination practices that took the form of barriers to entering certain occupations. Within occupations, however, rental rates of human capital are gender-neutral, and therefore any difference in earnings between two workers with the same experience would be attributed to human capital differences.[4]

Note that our model attributes a much larger role to market hours in explaining the gender wage gap dynamics compared to the role of labor market experience measured in regression-based analyses (e.g., O'Neill and Polachek, 1993, Blau and Kahn, 1997, Goldin, 2006). The key insight is that in models with unobserved investments in human capital like ours (as opposed to models of learning-by-doing), experience is an imperfect proxy for human capital. Two women with the same experience and initial human capital levels will have different levels of human capital (and wages) today if one of them plans to drop out of the labor force during her childrearing years and one plans to continue working. The woman expecting to drop out would have invested less time in on-the-job learning and, as a result, would embody less human capital today. As such, our work

[4]See Albanesi and Olivetti (2009) and Gayle and Golan (2012) for examples of additional market-based sources of the gender pay gap.

suggests that future expected market hours contain relevant information about present wages even after one controls for experience. A similar point is made in a related study by Erosa et al. (2016) as their framework also features unobserved investments in human capital, although it is modeled as a utility cost. That study focuses on explaining the life-cycle evolution of the gender wage gap for a single cohort, assuming exogenously given gender differences in childrearing time costs.

Two related quantitative studies emphasize the role of experience and gender-specific changes in returns to experience (i.e. learning technology) for the rise in female labor supply and closing of the gender wage gap. Olivetti (2006) employs a learning-by-doing model to estimate changes in gender-specific learning technology. Our model is consistent with her finding that women saw a greater increase in "returns to experience" during the 1980s – this is driven by hours' reallocation towards occupations with a more productive learning technology. Gayle and Golan (2012) estimate a general equilibrium structural model of labor supply and occupational sorting that features private information regarding workers' participation costs. Experience affects wage offers not only through its traditional learning-by-doing effect but also through its signaling value. Relative to these studies, our framework highlights the effect of future expected market hours on present wages even after one controls for experience, as explained above.

The rest of the paper is organized as follows. We describe our data work in Section 2 and our model in Section 3. The model calibration, identification, and quantitative analysis are presented in Section 4. We conclude in Section 5.

2 Data

2.1 Wages and hours

Beginning with 1940, we assemble data on population (P), employment (E), annual hours per worker (ℓ), and earnings per worker (y) at a five-year frequency. We employ the U.S. Population Census for years 1940, 1950 and 1960 and the Current Population Survey (CPS) for years 1965, 1970, 1975,...2010, using linear interpolation to construct observations for 1945 and 1955. All data are assembled for eleven age groups (21–25, 26–30,...71–75), two gender groups and two occupation groups among workers. Earnings data are adjusted for inflation and reported in terms of 2010 dollars.

To create the two occupation categories, we first categorized three-digit "OCC1990" codes into ten broader categories and ranked them according to the average wage of male workers (from highest to lowest paid): technicians, managers, professionals, sales, precision workers, administrative workers, mechanics and transportation workers, machine operators, service workers and farm workers. We then classified the top three categories as high paying occupations ($o = 2$) and the remaining seven categories as lower paying occupations ($o = 1$).[5]

We use $P_{s,t}$, $E_{s,t}$, $L_{s,t}$ and $Y_{s,t}$ to denote the population size, employment (i.e. number of workers),[6] annual work hours and annual earnings of gender s individuals in period t. Disaggregated into age groups, these quantities are denoted by $P^j_{s,t}$, $E^j_{s,t}$, $L^j_{s,t}$ and $Y^j_{s,t}$.[7] Work-related quantities can be further disaggregated by occupation: We use $E^{j,o}_{s,t}$, $L^{j,o}_{s,t}$ and $Y^{j,o}_{s,t}$ to denote employment, annual hours and annual earnings of age j, gender s, occupation o individuals in period t.[8]

We measure group-specific wages as annual earnings divided by annual hours, e.g. $W^{j,o}_{s,t} = \frac{Y^{j,o}_{s,t}}{L^{j,o}_{s,t}}$. We refer to $W^{j,o}_{s,t}$ as a "micro" wage to distinguish it from the aggregate wage, $W_{s,t} = \frac{Y_{s,t}}{L_{s,t}}$. It is straightforward to show that the aggregate wage is an hours-weighted aggregate of "micro" wages:

$$W_{s,t} = \frac{Y_{s,t}}{L_{s,t}} = \frac{\sum_{j,o} Y^{j,o}_{s,t}}{L_{s,t}} = \sum_{j,o} \frac{L^{j,o}_{s,t}}{L_{s,t}} W^{j,o}_{s,t}. \tag{1}$$

We define the gender wage gap as

$$\text{Gap}_t = 100 \times \frac{W_{f,t} - W_{m,t}}{W_{m,t}}. \tag{2}$$

Panel A of Figure 1 reports the time series of gender-specific aggregate wages, indicating a general upward trend except for the post-1975 stagnation of male wages. Panel B shows the gender wage gap – it expanded by 12.5pp between 1940 and 1975 and narrowed by 16.1pp between 1975 and 2010.

[5] In 2015, occupation 2 accounted for about 45% of female workers.

[6] We categorize individuals as workers if they report working at least 100 annual hours.

[7] These quantities aggregate by summing over age groups: $P_{s,t} = \sum_j P^j_{s,t}$, $E_{s,t} = \sum_j E^j_{s,t}$, ...

[8] These quantities aggregate to broader groups using the appropriate summation, e.g. $L^j_{s,t} = \sum_o L^{j,o}_{s,t}$, $L^o_{s,t} = \sum_j L^{j,o}_{s,t}$, and $L_{s,t} = \sum_{j,o} L^{j,o}_{s,t}$.

Figure 2 shows the time series of occupation-specific wages for men and women, i.e. $W^o_{s,t} = \frac{Y^o_{s,t}}{L^o_{s,t}}$.[9] While the 1980s' closing of the aggregate gender wage gap is equally pronounced in both occupations, its earlier expansion is much more pronounced in lower paying occupations ($o = 1$).

Our goal is to analyze the role of market hours, both at the intensive and extensive margin, in driving the dynamics of the gender wage gap. To this end, we focus on group-specific hours per person, referring to this measure as the "market time endowment:"

$$\tau^{j,o}_{s,t} = \frac{L^{j,o}_{s,t}}{P^j_{s,t}} = \frac{E^{j,o}_{s,t}}{P^j_{s,t}} \times \frac{L^{j,o}_{s,t}}{E^{j,o}_{s,t}}. \tag{3}$$

Note that $\tau^{j,o}_{s,t}$ combines the extensive margin of labor supply, i.e., the participation rate in occupation o, i.e. $E^{j,o}_{s,t}/P^j_{s,t}$, and the intensive margin, i.e., hours per worker $L^{j,o}_{s,t}/E^{j,o}_{s,t}$.

These "micro" time endowments aggregate to age-specific endowments by summing over occupations: $\tau^j_{s,t} \equiv \frac{L^j_{s,t}}{P^j_{s,t}} = \frac{\sum_o L^{j,o}_{s,t}}{P^j_{s,t}} = \sum_o \tau^{j,o}_{s,t}$. They aggregate to occupation-specific endowments by summing over age groups and applying population weights: $\tau^o_{s,t} \equiv \frac{L^o_{s,t}}{P_{s,t}} = \frac{\sum_j L^{j,o}_{s,t}}{P_{s,t}} = \sum_j \frac{P^{j,o}_{s,t}}{P_{s,t}} \tau^{j,o}_{s,t}$. Finally, total time endowments can be constructed by summing over occupation and age groups and applying population weights: $\tau_{s,t} \equiv \frac{L_{s,t}}{P_{s,t}} = \frac{\sum_{j,o} L^{j,o}_{s,t}}{P_{s,t}} = \sum_{j,o} \frac{P^{j,o}_{s,t}}{P_{s,t}} \tau^{j,o}_{s,t}$.

Panels A and B of Figure 3 display the age-specific endowments, $\{\tau^j_{s,t}\}$, organized into life-cycle profiles for synthetic cohorts of men and women, with labels (i.e. 1940, 1945,...) referring to the year the cohort is of 21 to 25 years of age. Note that, for each cohort, the hours of men are higher than those of women. The hours of women, however, grow considerably across cohorts, as is well known (c.f. McGrattan and Rogerson (2004, Charts 2 and 3)), partly converging to those of men. The convergence is slow though, especially for the earlier cohorts that represented mothers of the baby boom generation (babies born between 1946 and 1964). While the older mothers contributed to the initial surge in birth rates, it was the cohorts labeled here as 1955, 1960 and 1965 that experienced particularly high completed fertility and were slow to increase their market hours early in life.[10] While we do not model fertility, the baby boom phenomenon

[9]It is equivalent to an hours-weighted aggregate of "micro" wages: $W^o_{s,t} = \frac{Y^o_{s,t}}{L^o_{s,t}} = \sum_j \frac{L^j_{s,t}}{L_{s,t}} W^{j,o}_{s,t}$.

[10]Using a general equilibrium model of learning by doing, Doepke et al. (2015) argue that the high fertility (and low labor supply) of these cohorts resulted from increased competition with older women

certainly mattered for the market hours profiles. In turn, these time profiles, we argue, mattered for the gender wage gap dynamics.

Panel C shows the gender hours gap by age, $100 \times \frac{\tau^j_{f,t} - \tau^j_{m,t}}{\tau^j_{m,t}}$, for four groups of cohorts.[11] Note that, for a given cohort, the gender hours gap tends to widen at first, as women enter their childbearing years, and narrow later in life. Comparing the 1940-50 and the 1955-65 cohorts, it is clear that the early reduction in the hours gap was most pronounced at older ages. As discussed above, this is likely related to the high fertility rates of the 1955-1965 cohorts. Comparing the 1955-1965 and the 1970-1980 cohorts, it is clear that the more recent reduction in the hours gap was most pronounced at younger ages. Over the entire time period considered, the hours gap narrowed considerably at all stages of the life cycle.

Figure 4 displays the life-cycle profiles of hours per person by gender and occupation, $\{\tau^{j,o}_{s,t}\}$. The main observations are as follows: (i) for all cohorts of men and women, occupation-1 hours were higher than occupation-2 hours; (ii) female hours increased in both occupations, with the faster increase seen in occupation 2; (iii) occupation-2 hours also increased for men, while their hours in occupation 1 remained relatively stable across cohorts;

To see the timing of hours' reallocation across occupations more clearly, we examine the time series of occupation-specific hours by gender, $\{\tau^o_{s,t}\}$, in Figure 5. Panel A shows that, throughout the time period considered, most hours are worked by men in occupation 1. These hours increased slightly up until 1970, but have trended downward since then. Their occupation-2 hours increased rather sharply up until 1970, remaining stable since then. As a result, the occupation-2 share of hours for men increased throughout the period, but most of the increase happened early on, up until 1970 (see Panel C). Turning to females, we see that their hours in both occupations were substantially (65 to 70 percent) lower than men's but they increased throughout the period, especially in occupation 2. Panel B displays the dynamics of female hours relative to male hours in both occupations. In occupation 1, the hours gap narrowed gradually up until 1990, remaining stable at around 30 percent since then. In contrast, the occupation 2 hours gap remained stable around 65 percent until 1970, narrowing sharply since then and closing entirely by around 2005.

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who gained a lot of experience during World War II.

[11] Each cohort group in panel C comprises three smaller cohorts, e.g. the "1940-50" group comprises the 1940, 1945 and 1950 cohorts.

Panel C highlights gender differences in the timing of hours reallocation towards occupation 2. Men clearly led the process of hours reallocation towards occupation 2 up until 1970, but the occupation-2 share of their total hours stalled at around 30-35 percent since then. By contrast, the occupation-2 share of female hours grew substantially since 1970. By 1980, their share surpassed the male share. By 2010, women allocated over 40 percent of total market hours towards occupation 2.

2.2 Decomposition

It is convenient to rewrite Equation (1) as

$$W_{s,t} = \sum_{j,o} \frac{\tau_{s,t}^{j,o}}{\tau_{s,t}} \frac{P_{s,t}^{j}}{P_{s,t}} W_{s,t}^{j,o}, \tag{4}$$

which we obtained by substituting for $L_{s,t}^{j,o} = \tau_{s,t}^{j,o} P_{s,t}^{j}$ and for $L_{s,t} = \tau_{s,t} P_{s,t}$. We introduce the following shorthand for this expression:

$$W_{s,t} = \mathcal{W}(\bar{\tau}_{s,t}, \bar{P}_{s,t}, \bar{W}_{s,t}),$$

where $\bar{\tau}_{s,t}$ stands for the 11×2 matrix $\{\tau_{s,t}^{j,o}\}^{j,o}$, $\bar{P}_{s,t}$ stands for the 11×1 vector $\{P_{s,t}^{j}\}^{j}$, and $\bar{W}_{s,t}$ stands for the 11×2 matrix $\{W_{s,t}^{j,o}\}^{j,o}$.[12]

In Appendix C, we use Equation (1) to decompose the wage gap at time t into three components. The first component is due to gender differences in time endowments, the second is due to gender differences in the population distribution by age, and the third is due to gender differences in micro wages. For example, to evaluate the effect of the gender differences in $\bar{\tau}_{s,t}$'s, we need to provide values for $\bar{P}_{s,t}$ and $\bar{W}_{s,t}$. We evaluate the effect for each of the four possible value combinations, $(\bar{P}_{f,t}, \bar{W}_{f,t})$, $(\bar{P}_{m,t}, \bar{W}_{m,t})$, $(\bar{P}_{f,t}, \bar{W}_{m,t})$ and $(\bar{P}_{m,t}, \bar{W}_{f,t})$, and take the average. We apply this procedure to proxy each of the three components. We label our approximated components by Gap_t^{τ}, Gap_t^{P}, and Gap_t^{W}, respectively:

$$\frac{\text{Gap}_t}{100} \simeq \text{Gap}_t^{\tau} + \text{Gap}_t^{P} + \text{Gap}_t^{W}.$$

We then decompose the change in the wage gap between two arbitrary dates, say 0 and

[12]Note that $\mathcal{W}$ has only three arguments because $\tau_{s,t}$ and $P_{s,t}$ can be deduced from $\bar{\tau}_{s,t}$ and $\bar{P}_{s,t}$.

1, into changes of its three components:

$$\frac{\text{Gap}_1 - \text{Gap}_0}{100} \simeq \underbrace{\left(\text{Gap}_1^{\tau} - \text{Gap}_0^{\tau}\right)}_{\text{Endowment effect}} + \underbrace{\left(\text{Gap}_1^{P} - \text{Gap}_0^{P}\right)}_{\text{Population effect}} + \underbrace{\left(\text{Gap}_1^{W} - \text{Gap}_0^{W}\right)}_{\text{Micro wage effect}}.$$

We emphasize that our empirical decomposition does not speak to the causal effects of changes in $\bar{\tau}_{s,t}, \bar{P}_{s,t}$ and $\bar{W}_{s,t}$. We will argue that, in addition to the mechanical "endowment effect" seen in the above equation, the closing of the gender gap in time endowments will also play a causal role in driving the micro wage effect.

Table 1 reports the empirical decomposition of (1) the gender wage gap widening observed between 1940 and 1975 and (2) the gender wage gap closing observed between 1975 and 2010. The first lesson from the table is that most of the wage gap changes are accounted for by changes in micro wages – about 90% of the pre-1975 gap widening and 60% of the post-1975 gap closing. The endowment effect played a relatively minor role during the first sub-period, but accounted for close to 40% of the post-1975 gap closing.

We draw the following lessons from Table 1. First, a quantitative theory of the wage gap must include a theory of micro wages as a first order component. Second, a theory of the labor supply – at the intensive and extensive margins, over the life cycle and across occupations – *by itself* would account for only 10% to 40% of the wage gap dynamics. Third, the age distribution of the population played a minor role.

In what follows, we develop a theory in which we abstract from population differences between men and women and take the evolution of gender-specific time endowments as exogenously given. In our model, micro wages are endogenously determined through optimal human capital accumulation choices. Higher time endowment within occupations will encourage human capital accumulation. And so will the shift of market hours towards occupation 2 – which offers a more productive learning technology. Note this latter effect goes beyond the mechanical effect of reallocating hours towards higher paying occupations.

3 Model

We employ a model based on the one introduced in Ben-Porath (1967). Time is discrete and the economy is populated by overlapping generations of workers of age 1 to J. In this section, we describe the optimization problem of workers without reference to gender or occupation so as to simplify the notation. We introduce gender- and occupation-specific notation in the context of quantitative analysis, in Section 4.

Workers of cohort t (age 1 at date t) are endowed with a fixed ability to accumulate human capital, x_t, an initial stock of human capital, h_t^1, and an age-specific profile of market time, $\{\tau_{t+j-1}^j\}_{j=1}^J$. In our notation, superscripts indicate age and subscripts indicate time periods, except for the age-invariant ability endowment for which subscripts indicate cohorts.

Consider workers of age j at date t (cohort $t-j+1$) with human capital h_t^j. They allocate their market time between accumulating human capital and working. Their objective is to maximize the present value of their labor income, denoted by $V_t^j\left(h_t^j\right)$:

$$V_t^j\left(h_t^j\right) = \max_{0\leq n_t^j\leq \tau_t^j} \omega_t h_t^j\left(\tau_t^j - n_t^j\right) + \frac{1}{1+r}V_{t+1}^{j+1}\left(h_{t+1}^{j+1}\right) \tag{5}$$

$$\text{s.t.} \quad h_{t+1}^{j+1} = (1-\delta)\, h_t^j + x_{t-j+1}F\left(n_t^j, h_t^j\right), \tag{6}$$

$$h_{t-j+1}^1 \text{ given}, \tag{7}$$

$$V_{t-j+1+J}^{J+1} = 0. \tag{8}$$

In the formulation above, ω_t denotes the rental rate of human capital per unit of time worked. Equation (5) states that the present value of labor income comprises age j labor income, $\omega_t h_t^j\left(\tau_t^j - n_t^j\right)$, where n_t^j is the time allocated to human capital accumulation, and the present value of labor income at age $j+1$. We assume the interest rate r is constant. Equation (6) describes the law of motion for human capital, where δ denotes the depreciation rate. The product $x_{t-j+1}F\left(n_t^j, h_t^j\right)$ describes human capital production, mapping the ability endowment x_{t-j+1}, time spent learning n_t^j and the current stock of human capital h_t^j into additional units of human capital. We adopt the following form for F:

$$F(n,h) = (nh)^\gamma,$$

where $\gamma \in (0,1)$ is the elasticity parameter.[13] Equation (7) specifies age-1 (initial) human capital of workers. Finally, Equation (8) is the terminal condition stating that there are no earnings after the maximum age. (Cohort $t-j+1$ reaches age J in period

[13] Note that n and h share the same elasticity parameter. Heckman et al. (1998) estimate production functions for human capital, allowing for different elasticities with respect to time and human capital. They cannot reject the hypothesis that these elasticities are the same.

$t - j + J$).

In Appendix D.1, we formally analyze the interior solution to the maximization problem in (5)-(8) and derive closed-form expressions for the profiles of human capital h_t^j and present value of labor income $V_t^j\left(h_t^j\right)$.

To understand the shape of human capital profiles, consider the first-order condition:

$$n_t^j h_t^j = \left[\frac{\gamma}{1+r}\ \frac{\beta_{t+1}^{j+1}}{\omega_t}\ x_{t-j+1}\right]^{\frac{1}{1-\gamma}}, \tag{9}$$

where

$$\beta_t^j = \sum_{k=j}^{J}\left(\frac{1-\delta}{1+r}\right)^{k-j}\omega_{t+k-j}\tau_{t+k-j}^k \tag{10}$$

denotes the shorthand summarizing the marginal value of human capital for age-j workers at date t, i.e. the effect of one extra unit of human capital on the present value of labor income. It comprises the combined effects of future rental rates of human capital and future market time of workers. All else equal, the marginal value of human capital is higher for cohorts facing more market time and higher rental rates of human capital over their lifetime. It also declines with age as the remaining lifespan shortens. The marginal value of human capital plays a critical role in our analysis as it has a first-order effect on wage growth over the life cycle.

Equation (9) describes the optimal investment of resources (i.e. effective units of time $n_t^j h_t^j$) in learning, making it clear that it depends positively on the marginal value of human capital and learning ability but negatively on the current rental rate – which captures the opportunity cost of learning.

The growth rate of human capital between ages j and $j+1$ is given by

$$\frac{h_{t+1}^{j+1}}{h_t^j} = 1 - \delta + \frac{1}{h_t^j} x_{t-j+1}\left(n_t^j h_t^j\right)^{\gamma}. \tag{11}$$

Equations (9) and (11) reveal the roles of β_t^j and h_t^j in the growth rate of human capital over the life cycle. Consider workers A and B – two workers of the same cohort, age and ability. Suppose these two workers also have the same stock of human capital in the current period, but A faces a higher marginal value of human capital in the next period (β_{t+1}^{j+1}). Then A will allocate more resources to human capital accumulation, experiencing higher growth of human capital as a result. Alternatively, suppose these two workers face the same marginal value of human capital but A has a lower stock of human capital in the current period. In this case, both workers will allocate the same number of effective time units to learning. However, worker A will experience higher growth of human capital simply because of the lower starting value.

Note that without depreciation, human capital would grow throughout the life cycle due to the positive investment at all but the last age. Its growth rate would decline with age for the two reasons highlighted above – the declining β_{t+1}^{j+1} and increasing h_t^j. Depreciation introduces a constant negative growth component which is more likely to dominate later in life, thereby implying a hump-shaped profile of human capital.[14] The shape of the human capital profile will allow the model to match the empirical wage profiles.

With τ_t^j representing market time and $w_t h_t^j(\tau_t^j - n_t^j)$ representing earnings, their ratio represents our empirical measure of wages:[15]

$$\hat{W}_t^j \equiv \omega_t h_t^j \underbrace{\frac{\tau_t^j - n_t^j}{\tau_t^j}}_{m_t^j}. \tag{12}$$

We refer to the term m_t^j in the above expression as the "effective labor supply" because it measures the fraction of market time allocated to active production rather than human capital accumulation. Our model will produce lower wages early in life because of both, lower starting values of human capital and lower levels of effective labor supply.

Even though, for any given cohort, we work with a representative agent economy, its optimal outcomes can be thought of as aggregated outcomes that emerge in a heterogeneous agent economy. In a model with heterogeneous agents, each cohort consists of many agent types of known measures, each described by a tuple of initial human capital,

[14] If depreciation is sufficiently high, human capital will decrease throughout the life cycle.

[15] Our mapping between model and data earnings and model and data wages implicitly assumes that the job training costs are borne by workers, and wage contracts in the data take that into account. This mapping allows for a straightforward interpretation of the optimization problem - as workers' earnings optimization. An alternative choice would be to map wh and $wh\tau$ into data wages and earnings, respectively. In this case, workers get paid for each unit of market time even while learning, so the training costs are borne by the firm. This choice would then necessitate a reinterpretation of the optimization problem as arising from a broader (unmodeled) firms' problem. We have repeated our analysis under this alternative mapping and, broadly speaking, arrived at a similar main message.

ability, and a profile of market time. In Appendix D.2, we show that these individual endowments aggregate into the corresponding endowments for the representative agent in a way that leads to the aggregation result for optimal outcomes: (i) the age j human capital of the representative agent equals the average human capital of age-j individual types; (ii) the age-j wage of the representative agent equals the average wage of age-j individual types. This result holds under fairly general conditions.

4 Quantitative analysis

In this section, we describe our calibration methodology, calibrated parameters and model fit. We then describe the insights obtained from the calibrated model.

4.1 Calibration

We assume that model periods correspond to five-year periods in the data and focus on the time period between 1940 and 2010. Thus, model periods $t = 1, 2, \dots 15$ correspond to years $1940, 1945, \dots 2010$.

We use our model of human capital accumulation to represent population groups defined by occupation (o) and gender (s). Period t market times in the model, $\{\tau_{s,t}^{j,o}\}$, are taken exogenously from the data and computed for the corresponding years as average annual market hours per person within population groups defined by gender, occupation and five-year age groups.

The real rate of interest is set to 4% per year: $1 + r = 1.04^5$.

With regard to human capital technology, we assume the depreciation rate is common across occupations and set it to 1.5% per year ($1 - \delta = 0.985^5$) following estimates in the literature (e.g. Huggett et al., 2006). We allow for $\{\gamma^o\}$, the curvature of F, to be occupation-specific. We choose to be more general and allow for the occupation-specific productivity parameters $\{x_{s,t}^o\}$ to also depend on gender and cohort. This is because $x_{s,t}^o$ captures a combination of both, job-level characteristics (e.g. quality of training and career advancement opportunities) and worker characteristics (e.g. learning ability, work ethic, ambition), the latter of which has surely changed over the years, especially for women. To minimize the number of parameters, we impose a smooth shape on the productivity parameters of cohorts born in $t = 1, 2, ...15$,

$$x^o_{s,t} = x^o_{s,1} \exp\left[g^{o,X}_{s,1}(t-1) + g^{o,X}_{s,2}(t-1)^2\right].$$

We also set productivity parameters of age $j \geq 2$ cohorts in period 1 to that of the first cohort, $x^o_{s,1-j+1} = x^o_{s,1}$, as we do not have pre-1940 data needed to identify those parameters. To sum this up, we are to calibrate the 2×2 matrix of period 1 productivity, $x_1 \equiv \{x^o_{s,1}\}$, and the $2 \times 2 \times 2$ array of growth parameters, $g^X \equiv \{g^{o,X}_{s,1}, g^{o,X}_{s,2}\}$.

Further, we postulate that the exogenously given human capital stocks – i.e. the initial human capital stocks of cohorts born in $t = 1, 2, ...15$, $\{h^1_{s,t}\}$, and period-1 human capital stocks of age $j = 2, 3, ...11$ cohorts, $\{h^j_{s,1}\}$ – are gender-specific but follow Erosa et al. (2022b) in assuming occupation-neutrality. Table 2 helps visualize the panel of human capital stocks in the model. The exogenous stocks are highlighted in gray. We explain below that these are all separately identified by our calibration approach. Nonetheless, we impose a smooth shape on their values:

$$h^1_{s,t} = h^1_{s,1} \exp\left[g^H_{s,1}(t-1) + g^H_{s,2}(t-1)^2 + g^H_{s,3}(t-1)^3\right]$$

for $t = 1, 2, ...15$ and

$$h^j_{s,1} = h^1_{s,1} \exp\left[g^H_{s,4}(j-1) + g^H_{s,5}(j-1)^2\right]$$

for $j = 1, 2, ...11$. Thus, we are to calibrate the 2×1 vector of initial human capital stocks, $h^1 \equiv \{h^1_{s,1}\}$, and the 2×5 matrix of growth parameters, $g^H \equiv \{g^H_{s,i}\}_{i=1,2...5}$.

We assume that the rental rates of effective labor units are occupation-specific but gender-neutral, $\{\omega^o_t\}$. Thus, there is no gender pay bias when comparing two equally qualified (same h) workers of different gender within a given occupation. The gender wage gap will arise from women accumulating less human capital and spending relatively more time in lower paying occupations.

We normalize the rental rate in occupation 1 in period 1 to 1: $\omega^1_1 = 1$. Our model necessitates this normalization, as wage data cannot identify rental rates from human capital levels. Importantly, this normalization is innocuous and makes no difference for our results. To be precise, we formally show in Appendix D.3 that the level to which we set ω_1 is irrelevant for the following outcomes in the calibrated model: wages, optimal levels of human capital investment, and the response of optimal human capital investment levels and wages to changes in market time endowments. To be calibrated are the remaining occupation-specific rental rates, $\{\omega_t^1\}$ for $t = 2, 3, ...15$, and $\{\omega_t^2\}$ for $t = 1, 2, ...15$. We denote this 29×1 vector by ω^o.

Finally, even though period 15 is the last period in our model, we need to make a projection for the evolution of time endowments and rental rates in order to solve the model for cohorts that would live beyond period 15. We set their time endowments $\{\tau_{s,t}^{j,o}\}_{t\geq 15}$ to the most recent available data. We assume that the rental rates grow at occupation-specific constant rates: $g^\omega = \{g^{o,\omega}\}$. Thus, there is a total of $29 + 2 = 31$ rental rate-related parameters to calibrate.

In sum, there are 57 parameters to calibrate:

$$\Theta = \left\{ \underbrace{\gamma^o}_{2}, \underbrace{h^1}_{2}, \underbrace{g^H}_{10}, \underbrace{x_1}_{4}, \underbrace{g^X}_{8}, \underbrace{\omega^o}_{29}, \underbrace{g^\omega}_{2}, \right\}.$$

We calibrate these parameters by targeting wage data, $\{W_{s,t}^{j,o}\}$, computed as average annual earnings divided by average annual market hours within the population groups defined by gender, occupation, age and year – that is a total of $2 \times 2 \times 11 \times 15 = 660$ wage observations. Formally, we find Θ by solving

$$\min_{\Theta} \frac{1}{\sqrt{660}} \left[\sum_{s\in\{m,f\}} \sum_{o=1}^{2} \sum_{j=1}^{11} \sum_{t=1}^{15} \left(W_{s,t}^{j,o} - \hat{W}_{s,t}^{j,o}(\Theta) \right)^2 \right]^{1/2}, \tag{13}$$

where $\hat{W}_{s,t}^{j,o}(\Theta)$ are model-implied micro wages.

4.2 Identification

Given the normalization we made, wage data successfully identify rental prices, human capital and other model parameters. While we calibrate all of the parameters simultaneously, it is helpful to outline the identification as if it were a step-by-step procedure.

Step 1 With δ calibrated outside of the model, the observed growth of wage profiles later in life identifies the growth rates of rental rates, for all t and both occupations, $\{w_t^o\}$. Indeed, when investment $n^{j,o}$ is close to zero (for high enough j), the observed wage growth late in life is given by the product of rental rate growth and human capital growth while the growth rate of human capital is proxied by $1-\delta$: $W^{j+1,o}_{s,t+1}/W^{j,o}_{t} \approx \frac{\omega^{o}_{t+1} h^{j+1,o}_{s,t+1}}{\omega^{o}_{t} h^{j,o}_{s,t}} = \frac{(1-\delta)\omega^{o}_{t+1}}{\omega^{o}_{t}}$. Since our algorithm targets all of the "micro-wages," wage growth of older population groups identify $\omega^{o}_{t+1}/\omega^{o}_{t}$ for all t and both occupations. If anything, there is over-identification due to the presence of several older cohorts and two genders in each period.

Step 2 With occupation-1 rental rate growth rates, $\omega^{1}_{t+1}/\omega^{1}_{t}$, identified and w^{1}_{1} normalized to 1, we obtain the entire path of rental rates in occupation 1, $\{w^{1}_{t}\}$.

Step 3 Now suppose we are given the curvature parameter γ^{1} and productivity parameters in occupation 1, $\{x^{1}_{s,t}\}_{t=1\ldots15}$. Then the observed age-1 period t wages in occupation 1, $W^{1,1}_{s,t} = w^{1}_{t} h^{1}_{s,t}\left(\tau^{1,1}_{s,t} - n^{1,1}_{s,t}\right)/\tau^{1,1}_{s,t}$, pinpoint the initial human capital for each cohort t, i.e. $h^{1}_{s,t}$. This is because $n^{1,1}_{s,t}$ is just a function of $h^{1}_{s,t}$, future time endowments and rental rates (identified in Step 2), so that $h^{1}_{s,t}$ is the only unknown in the above equation.

Step 4 Applying the same argument as in Step 3, the cross-section of age-j human capital levels in period 1, $\{h^{j}_{s,1}\}$, is identified by age-j occupation-1 wages in period 1, $W^{j,1}_{s,1} = w^{1}_{s,1} h^{j}_{s,1}\left(\tau^{j,1}_{s,1} - n^{j,1}_{s,1}\right)/\tau^{j,1}_{s,1}$. Again, this is because $n^{j,1}_{s,1}$ is a function of $h^{j}_{s,1}$ and future time endowments and rental rates. This argument goes through for given values of γ^{1} and occupation-1 productivity parameters of cohorts born prior to period 1, $\{x^{1}_{s,1-j+1}\}_{j=1,2\ldots11}$ (which we have set to $x^{1}_{s,1}$).

Step 5 With period-1 age-1 human capital ($h^{1}_{s,1}$) now in hand, the observed period-1 occupation-2 wage, $W^{1,2}_{s,1} = w^{2}_{1} h^{1}_{s,t}\left(\tau^{1,2}_{s,1} - n^{1,2}_{s,1}\right)/\tau^{1,2}_{s,1}$, identifies w^{2}_{1}. This is true for given γ^{2} and $\{x^{2}_{s,t}\}_{t=1\ldots15}$. Again, we have an over-identification here because of the two genders. With w^{2}_{1} in hand, the remaining $\{w^{2}_{t}\}$ are implied by their growth rates obtained in Step 1.

Step 6 Finally, with $\{w^{o}_{t}\}$ and $\{h^{1}_{s,t}\}$ in hand, $\{\gamma^{o}\}$ and $\{x^{o}_{s,t}\}$ are identified by the shape of wage profiles earlier in life of cohorts $t \geq 1$.

Notice that Steps 3-6 are interdependent. In other words, $\{w^{2}_{t}\}$, $\{h^{1}_{s,t}\}$, $\{h^{j}_{s,1}\}$, $\{\gamma^{o}\}$ and $\{x^{o}_{s,t}\}$ are jointly determined.

4.3 Model Fit and Calibrated Parameters

Table 3 reports the calibrated parameters. Panels A and B of Figure 6 show the calibrated levels of exogenous human capital stocks for men and women – age-1 human capital levels by cohort (Panel A) and age-j human capital levels in period 1 (Panel B). In both cases, the human capital of men is calibrated to be higher than that of women. Age-1 human capital levels grow over time which helps account for real wage growth.

Panel C shows the calibrated gender- and occupation-specific learning productivity parameters by cohort, whereas Panel D shows these parameters by age in 1940. Learning productivity is higher in occupation 2 for both genders. This is likely a reflection of two patterns – that occupation 2 offers better learning opportunities and that workers selecting into occupation 2 tend to be more productive learners. Note that, over successive cohorts, learning productivity of women in occupation 1 trends slightly upwards while the opposite is true for occupation 2. Panel E shows the implied hours-weighted average of learning productivity levels by gender. The dynamics is closely related to the dynamics of occupation-2 share of hours (Panel C, Figure 5). As men led the process of hours reallocation towards occupation 2, their aggregate productivity grew substantially up until 1970. By contrast, female average productivity stagnated during this period as they expanded their labor supply in both occupations, but it grew considerably since 1970 as their market time shifted towards occupation 2.

These gender-specific learning productivity trends are qualitatively consistent with an improvement of female selection on unobserved productivity (relative to men's) estimated by Mulligan and Rubinstein (2008) for the post-1975 time period. We will also show this to be true for human capital – which better corresponds to their measure of unobserved productivity.[16] There is scarce evidence on changing female selection prior to 1975. Smith and Ward (1989) estimate that, between 1940 and 1970, female selection into the labor force worsened relative to men's in terms of schooling. Our results are qualitatively consistent with this finding. We will also show this to be true for human capital.

[16]One possible narrative that helps interpret these patterns is that, in the 1970s, compared to men who had already transitioned to occupation-2 jobs, women still faced barriers to those jobs and so productive learners chose to forego market work except the few that gained access to professional occupations. Over time, returns to human capital accumulation increased and a wide adoption of anti-discrimination practices made high growth careers more accessible to women. Most women joined the labor force, especially the highly productive learners. This narrative helps interpret our finding that female productivity declined in occupation 2 and their average productivity caught up to men's.

Panel F of Figure 6 displays the time series of calibrated rental rates of human capital by occupation. The rental rate in occupation 2 is consistently higher than that in occupation 1. The rental rates are relatively flat up until 1970 and trend down since then. Our finding of decreasing rental rates is similar to that of Bowlus and Robinson (2012, Figure 3) and Hendricks (2013, Figure 4) for the rental rates of education-specific human capital.

Panel A of Figure 7 shows that the model fits the micro wage data well. Each point in the plot represents the gender-, occupation-, age- and year-specific average wage in the data and in the model. This plot is a condensed alternative to demonstrating the model fit to wage profile data, cohort by cohort.[17] Panels B and C show the evolution of male and female average wages as well as the gender wage gap, showing that the model closely tracks the observed wage dynamics. To compute these, the model- and data-generated micro wages are aggregated using hours and population data according to Equation (4). The wage gap declines by 8 pp between 1940 and 1975 in the model, accounting for 60% of the observed 13 pp decline (see Table 4). From 1975 to 2010, the model-generated increase in the wage gap is 14 pp, accounting for 84% of the 16 pp observed increase.

Average wage dynamics in the model is aggregated from cohort-specific wage profiles. In turn, changes in cohort-specific wage profiles are driven by the exogenous variables, i.e. the initial human capital, occupation-specific hours profiles, rental rates and learning productivities. All of these components affect wages through human capital accumulation (although hours and rental rates also have a direct effect). Thus, it is important to understand the dynamics of human capital. Figure 9 reports the evolution of average human capital for men and women (Panel A) and the corresponding gender gap in human capital (Panel B). The dynamics of the human capital gap is clearly an important driver of the gender wage gap dynamics in the baseline model (Panel C of Figure 7). Human capital of men increases faster than that of women in the earlier part of the sample and, thus, the human capital gap widens. The opposite is true in the latter part of the sample. In what follows, we explain this pattern.

Figure 10 shows the (normalized) marginal value of human capital (β in Equation 10). Recall that β captures the additional future income one could obtain from gaining an extra unit of human capital today, and it is a function of future hours and rental rates.

[17]Nonetheless, we include the profile plots for three arbitrarily selected male cohorts (1940, 1960 and 1980), so as to demonstrate the model produces an equally good fit over the lifecycle and across occupations. See Figure 8.

Panel A shows this value at age 1 across cohorts and Panel B shows its hours-weighted average by year. A few observations are in order. First, the two panels display similar patterns, which means it suffices to focus on easier-to-understand age-1 values. Second, the most noticeable difference is between the patterns of human capital values for men in occupation 1 and women in occupation 2. Men in occupation 1 start off with the highest value among all groups but see it decline substantially over time. The opposite is true for women in occupation 2. Their β grows slow initially, but accelerates later in the period. We focus our discussion on these two groups as they are the main drivers of changes in aggregate human capital. The other two groups (men in occupation 2 and women in occupation 1) face more similar and relatively stable marginal values of human capital.

Several factors must be considered to understand differences in levels and rates of change of human capital values between the two groups. First, the rental rate of human capital is higher in occupation 2 (Panel F of Figure 6) which is conducive to a higher β. Why, then, is the β for women in occupation 2 lower than the β for men in occupation 1? Recall our data discussion in Section 2. The reason is that male hours in occupation 1 were initially sufficiently high relative to female hours in occupation 2 to offset the effect of the rental rate difference.

Second, the hours of women in occupation 2 caught up to those of men in occupation 1, which was conducive to their β catching up. We discussed that male hours in occupation 1 declined over time while female hours in occupation 2 increased, albeit the increase started off slowly and was initially driven by hours later in the life cycle (which have a smaller impact on β). Third, the rental rate of human capital in occupation 2 increased relative to that in occupation 1 after 1980 (Panel F of Figure 6). Our final observation is that the marginal value of human capital for women increased despite the overall decline in the rental rate. This implies that the dramatic increase in female hours allocated to occupation 2 played a dominant role in its dynamics. For men, however, the decline in the rental rate was only exacerbated by their hours' dynamics, and therefore their β decreased.

Consistent with β dynamics, our baseline model implies that the older cohorts of men had a strong incentive to accumulate human capital in occupation 1 and spent a lot of time doing so, but that incentive got weaker for the younger cohorts. The opposite is true for women in occupation 2. Figure 11 makes this point clear by plotting their time allocation to on-the-job learning, cohort by cohort.

Panel C of Figure 10 displays the evolution of the marginal value of human capital computed for the entire population. Panel D shows the corresponding gender gap. Driven by the two groups discussed above, our model implies a strong convergence of marginal values of human capital across genders, which partly helps us understand the dynamics of human capital accumulation.

Importantly, the incentive to accumulate human capital is determined not only by its marginal value but also by the productivity of learning technology (recall Equation 9). Therefore, the timing of hours reallocation towards occupation 2 – which offered a more productive learning technology and higher rental rates – also mattered. Men leading the hours reallocation process up until 1970 (see Section 2) disproportionately increased their incentive to accumulate human capital early on, while women leading the process since then disproportionately increased their incentive to accumulate human capital in the 1980s and 1990s.

Thus, the evolution of marginal values of human capital and the timing of time reallocation towards occupation 2 both help us understand the human capital accumulation of men and women represented in Figure 9. Initially, men had a higher level of human capital than women, which works to discourage human capital accumulation in our model (see Equation 11). This effect, however, was dominated by the fact that men faced a stronger incentive to accumulate human capital. As a result, the human capital of men increased faster than that of women in the early part of the sample, implying a widening of the gender human capital gap (Panel B of Figure 9). Over time, the incentive to accumulate human capital weakened for men but strengthened for women (Panel C of Figure 10). Eventually, the rate at which women accumulated human capital surpassed that of men, and the human capital gap began to shrink (Panel B of Figure 9). It is this dynamics of human capital accumulation by gender that explains the dynamics of the gender wage gap in the model in Panel C of Figure 7.

4.4 Experiments

We have discussed that the evolution of gender-specific marginal values of human capital was an important driver of the wage gap dynamics. In this section, we conduct several counterfactual experiments in order to examine separate contributions of occupation-specific rental rates, gender-specific market hours, and gender- and occupation-specific productivity terms.

4.4.1 *The role of occupation-specific rental rates*

It may seem plausible that the ∪-shaped dynamics of the relative rental rates, ω_2/ω_1 (Panel F of Figure 6), played an important role in generating the ∪-shaped pattern of the gender wage gap. This would be the case if men benefited more from the relative rise of occupation-1 rental rates seen up until 1980 but women benefited more from the post-1980 relative rise of occupation-2 rental rates.

To assess this channel quantitatively, we conduct two counterfactual experiments within the context of the baseline model. In the first experiment, which we refer to as "ω A," we solve the model assuming all exogenous variables evolve as in the baseline model except the rental rates are fixed at their initial values:

$$\tilde{\omega}_t^o = \omega_1^o \text{ for } \; o = 1, 2 \text{ and } t \geq 1.$$

Note we use "~" to denote counterfactual parameters throughout this section. Thus, we eliminate the rental rate dynamics from the baseline model and examine how much of the gender gap dynamics is lost. The resulting gender gap dynamics is reported in Panel A of Figure 12 alongside the baseline case – it is hardly distinguishable from the baseline dynamics, implying that the rental rate dynamics did not matter for the gender wage gap dynamics.

The results are presented separately for the 1940-1975 and 1975-2010 periods in columns entitled "Baseline-Experiment" in Table 4. We find that the occupation-specific rental rate dynamics accounted for about 3% of the GG closing during the 1975-2010 period. Compared to men, women's relative preference for occupation-2 jobs was high and growing during this period (Panel A of Figure 5), and therefore they experienced a slightly higher wage growth in response to its relative rental rate growth.

The opposite was true during the 1940-1975 period. Women were more likely to work in occupation 1, which was also the occupation that saw the relative gain in rental rates. Thus, the rental rate dynamics worked to shrink rather than widen the wage gap in the earlier period, accounting for -6% of its empirical dynamics.

In the second experiment, labeled "ω B," we solve the model assuming all exogenous variables evolve as in the baseline model except we lower occupation-2 rental rates to match occupation-1 values:

$$\tilde{\omega}_t^2 = \omega_t^1 \text{ for } t \geq 1.$$

Thus, we maintain the general downward trend in rental rates but eliminate any difference in their relative values. This experiment essentially eliminates the mechanical effect of time reallocation across occupations (similar to the endowment effect from Table 1 that we measured directly in the data). As a result, compared to Experiment ω A, slightly more of the benchmark model success is lost in this counterfactual experiment (compare Panels A and B of Figure 12). Note this experiment does not eliminate the incentive effect of time reallocation – with more hours employed in occupation 2, which features a more productive learning technology, the incentive to accumulate human capital increases.

We conclude that the rental rate dynamics played a relatively minor role in driving the aggregate gender gap dynamics, most notably accounting for 3% to 34% of its post-1975 closing.

4.4.2 *The role of gender-specific time endowments*

Our main hypothesis is that female wages increased primarily because, generation after generation, women gradually began to expect working longer hours over their lifecycle and allocating a greater fraction of those hours to higher paying occupations. As a result, they invested more in on-the-job learning early in their careers and accumulated more human capital.

To test this hypothesis, we conduct two experiments, which we label "τWM" and "τMW." In Experiment τWM, all exogenous variables evolve as in the benchmark model except that time endowment of women (by age, occupation and year) is assigned to men in place of their baseline values, i.e. $\{\tilde{\tau}_{m,t}^{j,o}\} = \{\tau_{f,t}^{j,o}\}$. Conversely, in Experiment τMW, time endowment of men is assigned to women, $\{\tilde{\tau}_{f,t}^{j,o}\} = \{\tau_{m,t}^{j,o}\}$. In both experiments, the only gender difference eliminated from the baseline model is in terms of hours. If it were important for the gender wage gap dynamics, we should see a significant loss in the model's explanatory power. This is exactly what we find.

Figure 13 shows the results for Experiment τWM. Female wages are identical to their baseline values by design (Panel A). Similarly, human capital (Panel C) and effective hours (Panel E) of women are identical to their baseline values. Male wages, however, are noticeably different from their baseline levels: They start off at a higher level and grow at a slower rate (Panel A). In fact, the shape of male wage dynamics now resembles the shape of female wage dynamics, making it immediately clear that it is the gender difference in hours dynamics that is responsible for the gender difference in wage dynamics.

Panel B confirms that the baseline wage gap dynamics is lost in this experiment: the initial gender wage gap is higher, it closes during the 1940-1975 period (in contrast to its widening in the baseline model) and remains relatively flat during the 1975-2010 period (in contrast to closing in the baseline). Table 4 summarizes the results: the gender hours difference accounted for 200% of the wage gap widening in the earlier period and 104% of the wage gap closing in the later period.

To understand this dynamics, note that the marginal value of human capital (β) faced by men is substantially reduced compared to the baseline model. It is, in fact, identical to women's β in the baseline model (shown in Panel C of Figure 10). This is because β depends only on future hours and rental rates, and in this experiment, hours are the same for men and women by design while the rental rates are gender-neutral by assumption. Since the 1940 human capital is unchanged, lower β implies that, compared to the baseline model, men invest less in human capital accumulation (Panel C) and work longer effective hours (Panel E). This explains higher male wages in 1940 (Panel A), and slower subsequent human capital accumulation and wage growth.

In addition, the timing of hours reallocation towards occupation 2 – which features a more productive human capital accumulation technology – is delayed compared to the baseline model. This effects explains why, relative to the baseline, the incentives to accumulate human capital are weakened early in the period but strengthened later (Panel C).

Compared to women, men have the same β but higher initial human capital. As a result, they invest less in human capital early on and work longer effective hours (Panels C and E). This is conducive to a large initial wage gap which also begins closing right away. Panels D and F make it clear that both the human capital gap and the effective hours gap contributed to the early closing of the wage gap. Near the end of the sample period, however, male wages (and therefore the gender wage gap) do not differ remarkably from their baseline levels. At this point, men have accumulated less human capital (Panel C) but they allocate a greater fraction of their hours to occupation 2 and work slightly longer effective hours (Panel E).

Figure 14 displays the results of Experiment τMW, in which the importance of gender differences in hours is evaluated by assuming both genders face male hours, $\{\tau_{m,t}^{j,o}\}$. Male wages are identical to their baseline values by experimental design (Panel A). Female wages, however, start off at a lower level and grow faster, resembling the shape of male wage dynamics. Again, this is because women are now endowed with male hours, and therefore face the same marginal value of human capital and the same timing of hours reallocation as men. As a result, women spend less time in effective work (Panel E) and accumulate human capital faster (Panel C). This is conducive to an earlier closing of a larger initial wage gap.

The takeaway point is the same: With gender differences in time endowments eliminated, the model no longer generates the salient features of the gender wage gap dynamics (Panel B). In fact, the role of gender differences in hours appears stronger in this experiment (see Table 4). These findings provide strong support for our main hypothesis.

4.4.3 *The role of age-1 human capital levels*

As new cohorts enter the economy, their age-1 human capital levels impact both their current wages and choices that affect future wages. It is therefore possible that the gender difference in human capital endowments at age 1 (reported in Panel A of Figure 6) has an effect on the aggregate gender wage gap. We conduct two experiments to assess the magnitude and direction of this effect. In the first one, labeled "h1WM," we assign age-1 human capital of women to men, cohort by cohort, so that $\{\tilde{h}_{m,t}^1\} = \{h_{f,t}^1\}$. In the second, labeled "h1MW," the assignment is reversed, so that $\{\tilde{h}_{f,t}^1\} = \{h_{m,t}^1\}$. All other exogenous variables are set to their baseline values.

Panels A and B of Figure 15 show the results for Experiment h1WM. Women have the same wages as in the baseline model by design, while men's aggregate wage series is shifted down in a parallel-like fashion. This immediately implies that the aggregate gender wage gap is reduced compared to the baseline model but its behavior over time retains a similar shape, leading us to conclude that gender differences in age-1 human capital were not important in shaping the gender wage gap dynamics. Table 4 shows they actually worked against the gap opening in the early period and helped explain only 10% of the gap closing in the later period.

To understand the parallel-like downward shift in the aggregate male wage series, note that both men and women face the same marginal values of human capital as in the baseline model. This is because time endowments and rental rates remain at their baseline values. Thus, men in each cohort start off with a lower endowment of human capital but face the same incentives for accumulating it. Our previous discussion of Equations (9) and (11) explains why this leads to a lower human capital in 1940 with a slightly higher growth, resulting in a parallel-like downward shift of the human capital and wage series.[18]

Experiment h1MW yields similar results (see Table 4), although via a converse mechanism (Panels C and D of Figure 15). Cohort after cohort, women face higher initial levels but the same marginal value of human capital. This leads to a higher aggregate human capital level in 1940 that grows somewhat slower, resulting in a parallel-like upward shift of the wage series.

In sum, we find that the initial human capital has little bearing on the dynamics of the wage gap.

4.4.4 *The role of gender-specific learning productivity levels*

It remains to explore the effect of gender differences in learning productivity levels, which were reported in Figure 6 and discussed in detail in Section 4.3. We explained that the dynamics of average learning productivity (Panel E of Figure 6) was closely related to the dynamics of occupation-2 share of total market hours, with relative female productivity worsening up until 1970 or so and improving since then.

In terms of levels, female learning productivity was slightly higher than men's up until 1965. Our interpretation is that most women opted out of market work during this period, with an exception of the highly productive learners who also favored occupation-2 jobs. Male workers, however, were not positively selected into the labor force and most of them worked in occupation 1. By the 1970s, gender-specific productivity levels

[18] The shape of the wage gap in the experiment is driven almost entirely by the shape of the human capital gap, with the effective hours gap playing a very minor role, and only early in the time period considered.

became reversed as men had transitioned to occupation-2 jobs (which offered a more productive learning technology) while women increased their market hours more evenly across occupations. It is not until the end of our sample that learning productivity equalized across genders as women gained more access to occupation-2 jobs.

It is this trend in relative learning productivity levels that worked to close the gap early in the sample and increase it later on, working against the actual gender gap pattern.

Figure 16 shows the results of two counterfactual experiments. In Experiment xWM, we endow men with female productivity levels, $\{\tilde{x}^o_{m,t}\} = \{x^o_{f,t}\}$. In Experiment xMW, we endow women with male productivity levels, $\{\tilde{x}^o_{f,t}\} = \{x^o_{m,t}\}$. In both experiments, the early opening and later closing of the gender wage gap are exacerbated relative to the baseline model (Panels B and D). This leads us to conclude that gender differences in learning productivity levels did not contribute to the observed gender gap dynamics, and in fact worked against it (see Table 4).

To understand these findings, consider Experiment xWM. Compared to the baseline model, men face a greater learning productivity in the initial period. As a result, they spend more time accumulating human capital early on. Wages fall initially because of lower effective hours but rise as the human capital stock builds up. This process is reversed in the later period as men face a lower learning productivity compared to their baseline level.

The same relative wage dynamics emerges in the context of Experiment xMW, except it is driven by women accumulating less (more) human capital early on (later on) as they face lower (higher) learning productivity relative to its baseline level.

5 Conclusion

Labor economists have argued that understanding workers' unobserved characteristics is important for understanding the evolution of the gender wage gap (see Section 1). Motivated by these arguments, we have employed a Ben-Porath (1967)-based model of unobserved time investments in human capital to study the gender wage gap dynamics. As in the original model, life-cycle profiles of market hours are exogenously given, but workers optimally allocate them between effective work and human capital accumulation – which raises their future effective wages. Higher hours profiles induce greater investments in human capital.

Starting with the cohort that entered the labor market in 1940, we documented that men's aggregate hours profiles changed little across cohorts, although the share of male hours allocated to higher paying occupations increased sharply between 1940 and 1970. Women's aggregate hours profiles increased dramatically over time, although the increase was slow to start and mainly affected middle-aged women – likely because of high fertility rates of the younger women in the 1950s and 1960s. As the baby boom phenomenon concluded, younger women drove the rise in market hours and the share of female hours allocated to higher paying occupations accelerated.

We employed our model to examine the extent to which gender-specific human capital investments, induced by these changes in the time allocation patterns across cohorts and occupations, contributed to the observed dynamics of the gender wage gap. In addition to the exogenously given market hours profiles, male and female cohorts within each occupation are endowed with initial human capital and learning productivity, and face occupation-specific human capital rental rates. We calibrated these time series alongside the human capital production parameters by fitting the model to hourly wage data by age, gender, occupation and year, for the time period between 1940 and 2010.

We performed several counterfactual experiments in the context of our calibrated model in order to examine separate contributions of market hours (our main hypothesis), rental rates, initial human capital endowments and learning productivity levels.

We found that changing time allocation patterns accounted for about 104% to 123% of the post-1975 wage gap closing and predicted a substantially stronger widening of the gap than what was observed in the early part of the sample.

Human capital rental rate dynamics also played a role, most notably accounting for up to 34% of the post-1975 gender wage gap closing – the period when the occupation-2 rental rates exhibited relative growth. Women experienced relative wage growth in response to this change because their occupation-2 share of market hours was higher. But the main effect of rental rates was due to their differential levels and the fact that, since 1980, women continued to reallocate their hours towards higher paying occupations while men did not.

The roles of cohort-specific endowments, however, were less pronounced or even worked against the observed wage dynamics. We have also provided model-based insights for these findings.

Our work complements the strand of literature that has emphasized the role of unobserved time investments in human capital in explaining earnings inequality for men (see discussion in Section 1), as we apply the same approach to study earnings inequality between genders.

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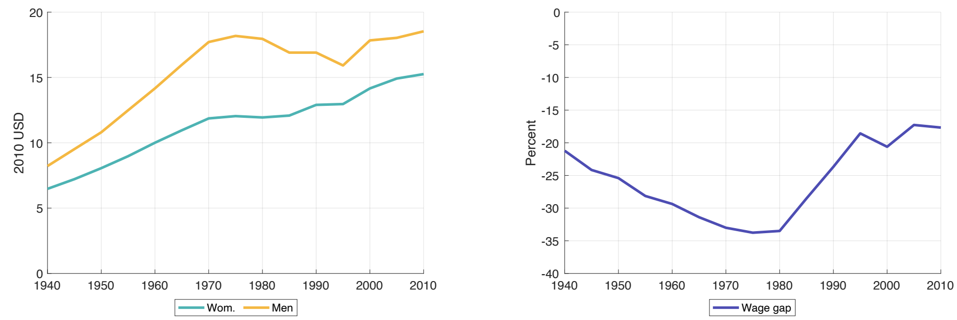
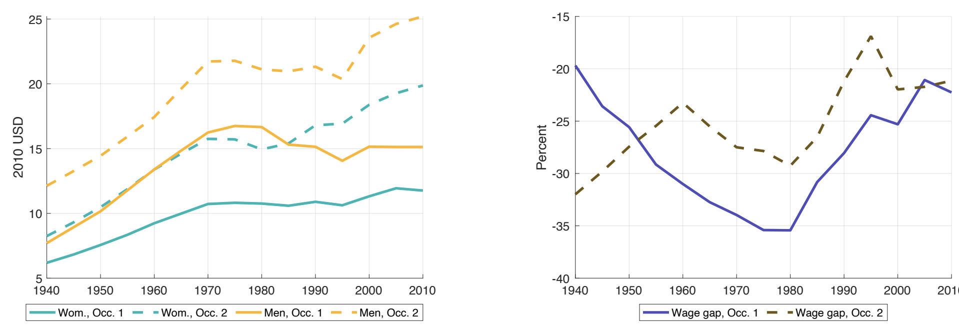
A Figures



A – Wage by gender B – Gender wage gap

Figure 1: Wages and the gender wage gap

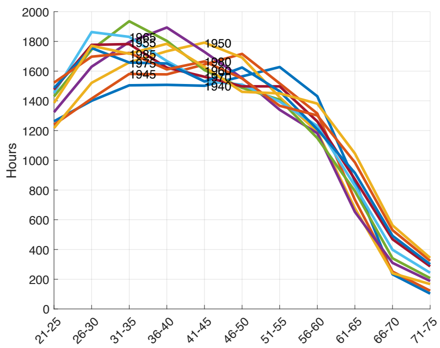
Note: See Section 2 for details on wage rate and gender wage gap calculations.
Source: Census, CPS, and authors' calculations.



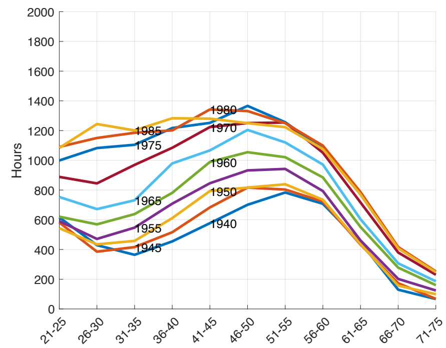
A – Wage by gender and occupation B – Gender wage gap by occupation

Figure 2: Wages and the gender wage gap by occupation

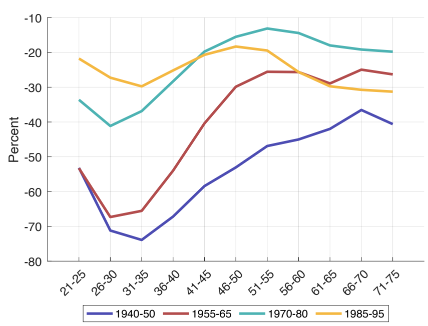
Note: See Section 2 for details on wage rate and gender wage gap calculations.
Source: Census, CPS, and authors' calculations.



A – Hours, Men



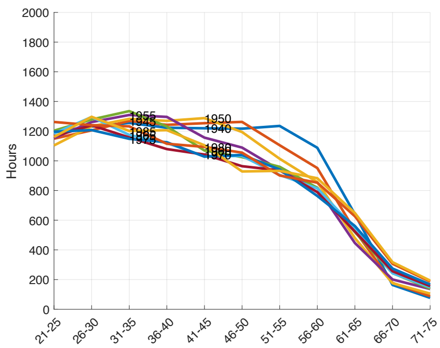
B – Hours, Women



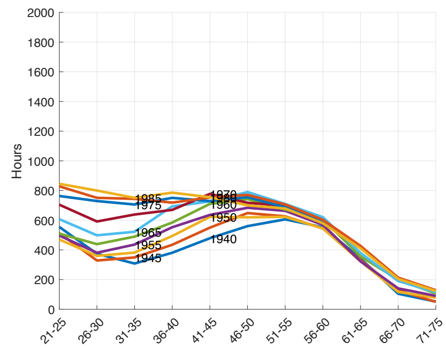
C – Gender hours gap

Figure 3: Age profiles of time endowments and gender endowment gaps

Note: Panels A and B display age profiles of average time endowments, $\{\tau^j_{m,t}\}$ and $\{\tau^j_{f,t}\}$, for cohorts of men and women that are between 21 and 25 years of age in 1940,1945,...1985. Panel C displays age profiles of gender endowment gap, for four large cohorts: the "1940-50" cohort comprises 21- to 25-year old individuals in 1940, 1945 and 1950, and so on.
Source: Census, CPS, and authors' calculations.



A – Occupation 1 hours, Men



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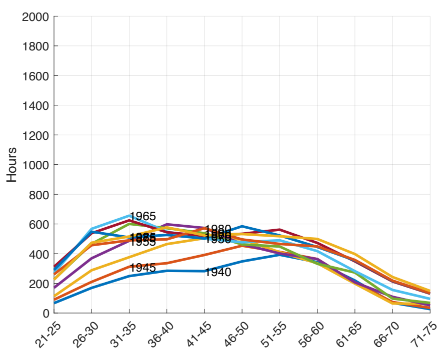
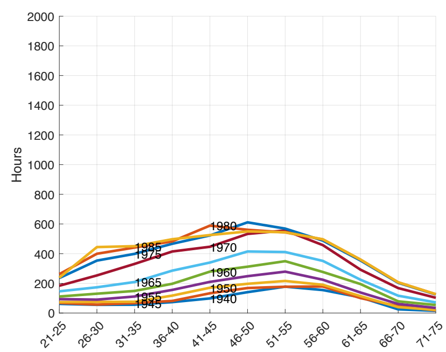


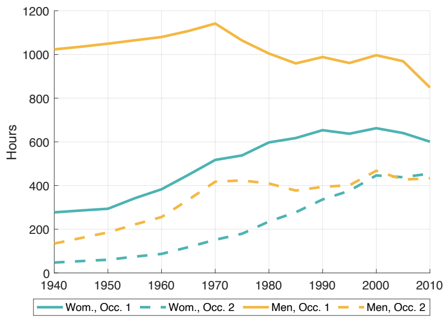
C – Occupation 2 hours, Men



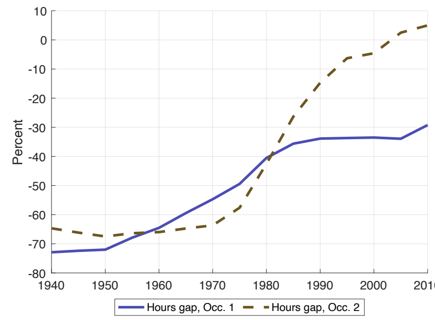
D – Occupation 2 hours, Women

Figure 4: Age profiles of time endowments by occupation

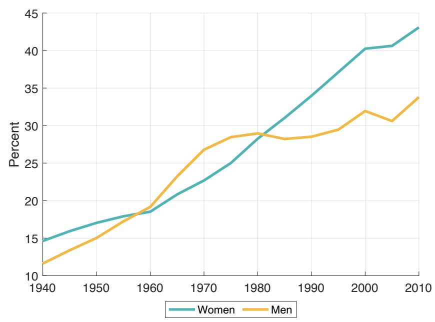
Note: Panels A and B display age profiles of average time endowments in occupation 1, $\{\tau^{j,1}_{m,t}\}$ and $\{\tau^{j,1}_{f,t}\}$, for cohorts of men and women that are between 21 and 25 years of age in 1940,1945,...1985. Panels C and D display age profiles of average time endowments in occupation 2, $\{\tau^{j,2}_{m,t}\}$ and $\{\tau^{j,2}_{f,t}\}$, for the same cohorts of men and women.
Source: Census, CPS, and authors' calculations.



A – Hours by gender and occupation



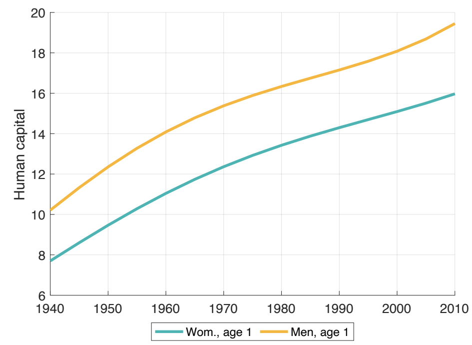
B – Gender hours gap by occupation



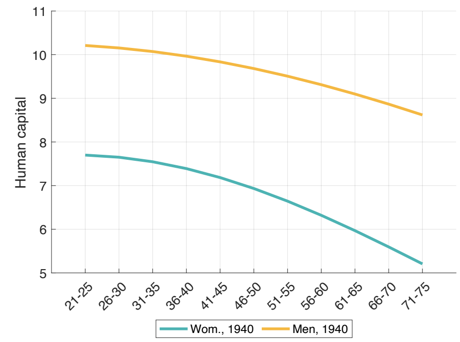
C – Occupation-2 share of hours

Figure 5: Time allocation patterns over time

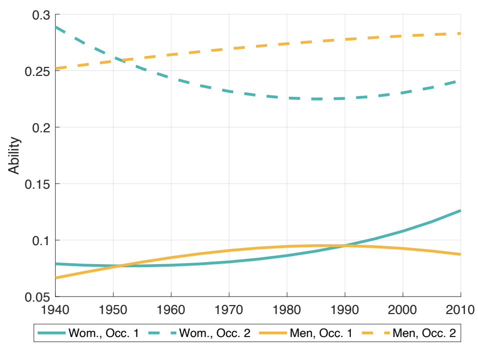
Note: Panel A shows $\tau^{o}_{s,t}$ over time, for each occupation and gender. Panel B shows the gender gap in hours, within each occupation. Panel C displays share of hours allocated to occupation 2, by gender.
Source: Census, CPS, and authors' calculations.



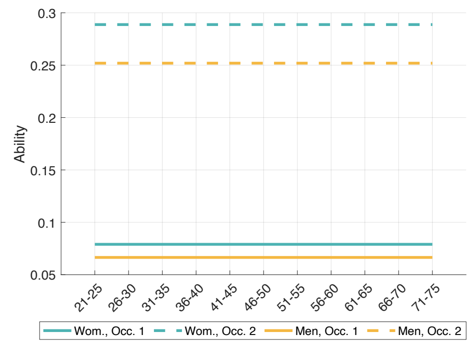
A – Age-1 human capital by cohort



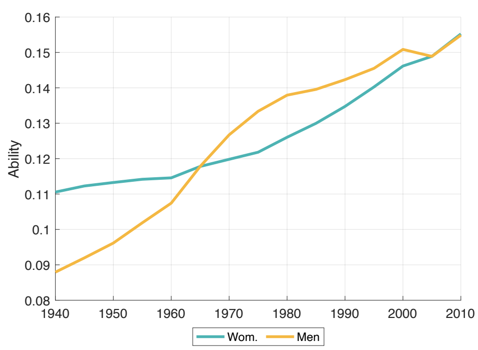
B – Human capital in 1940 by age



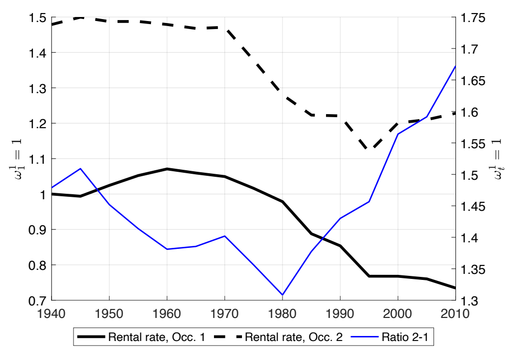
C – Learning productivity by cohort



D – Learning productivity in 1940 by age



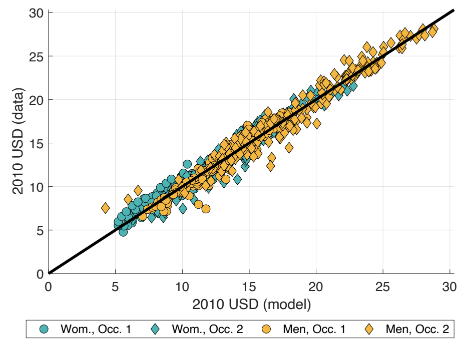
E – Average Learning Productivity



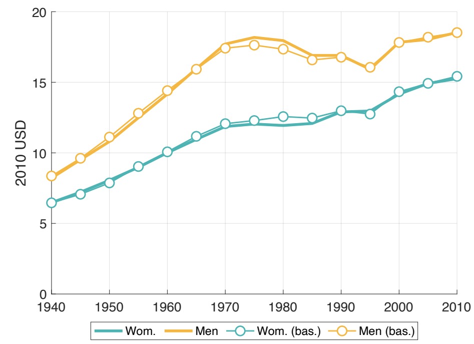
F – Rental rate by occupation

Figure 6: Calibrated exogenous variables

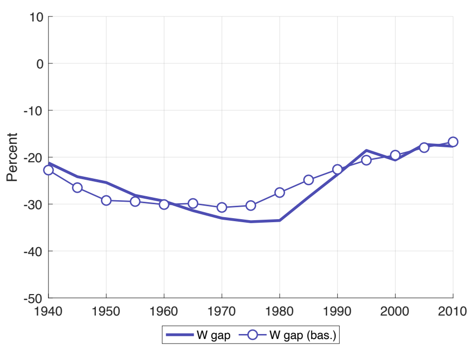
Note: The figure shows the following calibrated parameters: age-1 human capital stocks $\{h^1_{m,t}, h^1_{f,t}\}$ for cohorts born in $t = 1, 2, ...15$ (Panel A), period-1 human capital stocks of age $j \geq 2$ cohorts $\{h^j_{m,1}, h^j_{f,1}\}$ (Panel B), learning productivity levels $\{x^o_{m,t}, x^o_{f,t}\}$ for cohorts born in $t = 1, 2, ...15$ (Panel C), learning productivity levels of age $j \geq 2$ cohorts $\{x^o_{m,1-j+1}, x^o_{f,1-j+1}\}$ (Panel D), hours-weighted aggregate learning productivity levels (Panel E), rental rates $\{\omega^1_t, \omega^2_t\}$ for $t = 1, 2, ...15$ (Panel F).
Source: Authors' calculations.



A – Wages by gender, age, year, occupation



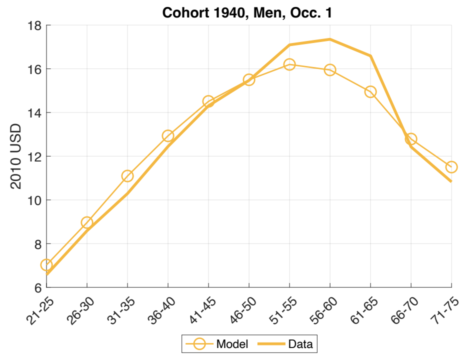
B – Wages by gender and year



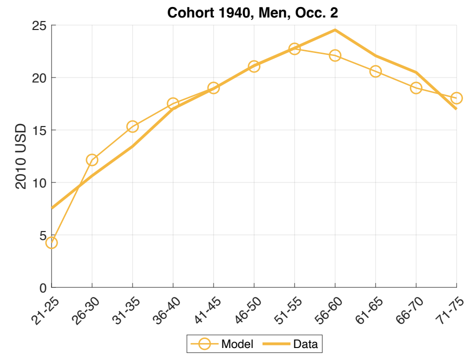
C – Wage gap

Figure 7: Model fit

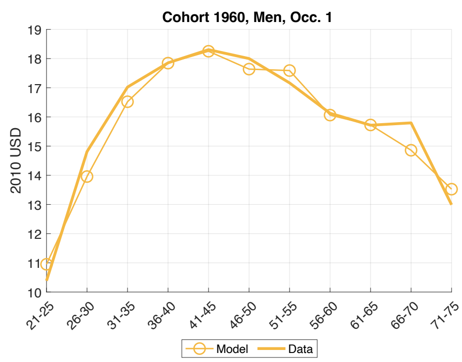
Note: Each point in Panel A represents a pair of model and data wages, for a given gender, occupation, year and age. Panels B and C report the aggregate (hours-weighted) wage series by gender and the gender wage gap, in the model and in the data.
Source: Census, CPS, and authors' calculations.



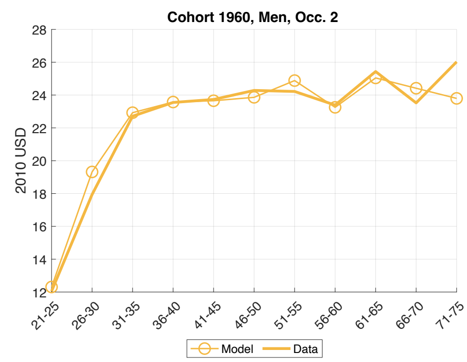
A – Model Fit, Occ 1, 1940



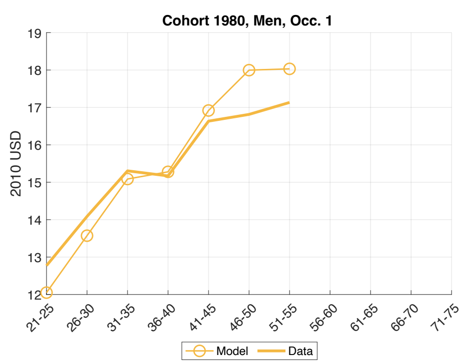
B –Model Fit, Occ 2, 1940



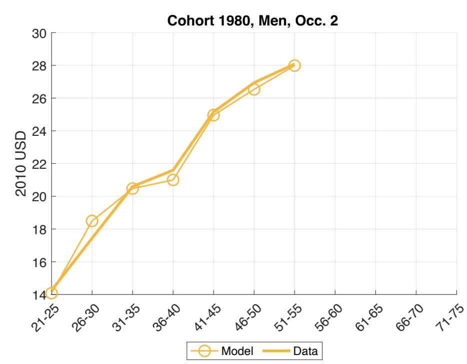
C – Model Fit, Occ 1, 1960



D – Model Fit, Occ 2, 1960



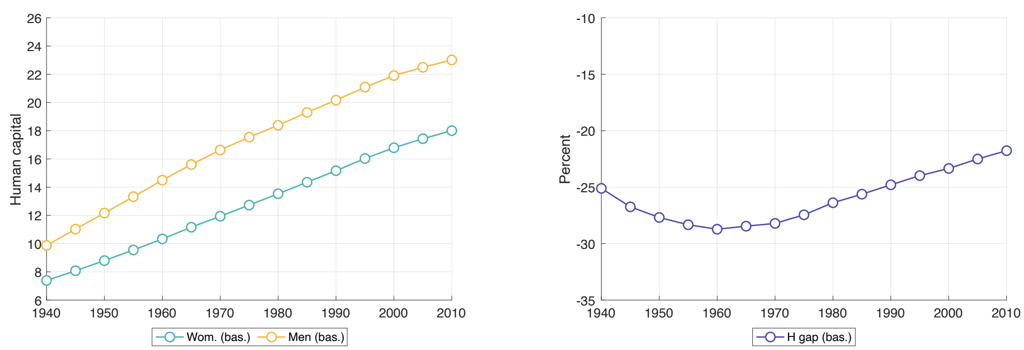
E – Model Fit, Occ 1, 1980



F – Model Fit, Occ 2, 1980

Figure 8: Model fit: Selected cohorts

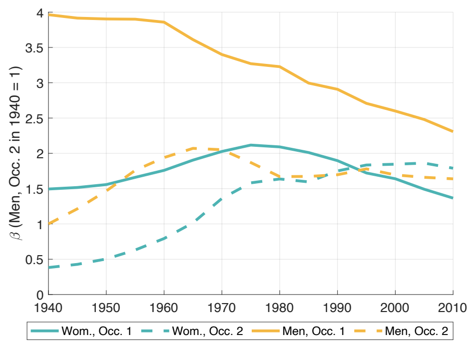
Note: The figure reports the model fit to occupation-specific wage profiles for three arbitrarily selected male cohorts (1940, 1960 and 1980).
Source: Census, CPS, and authors' calculations.



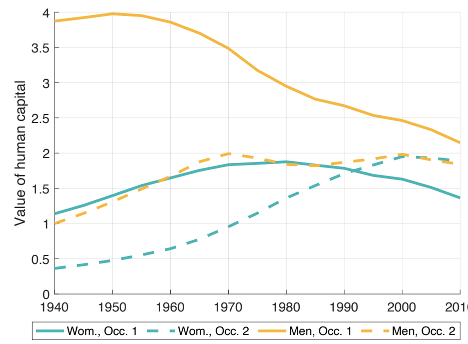
A – Human capital by gender B – Gender gap in human capital

Figure 9: Baseline model: Average human capital levels by gender and their ratio

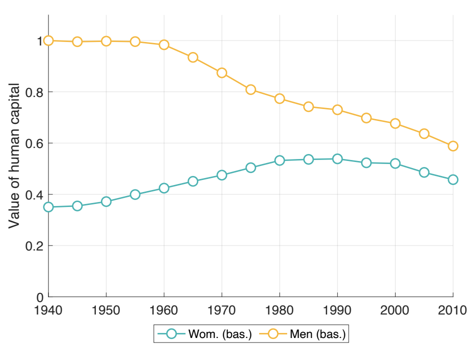
Note: Human capital is hours-weighted.
Source: Authors' calculations.



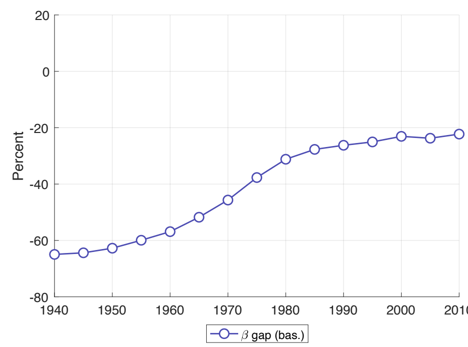
A – Age-1 β



B – Average β



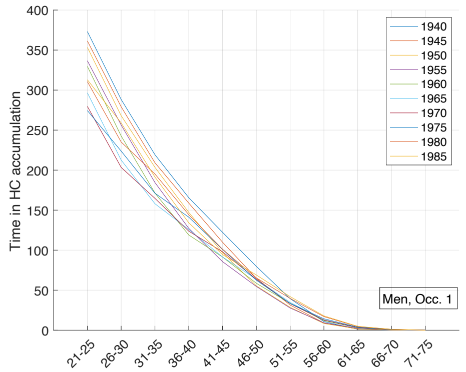
C – Average β by gender



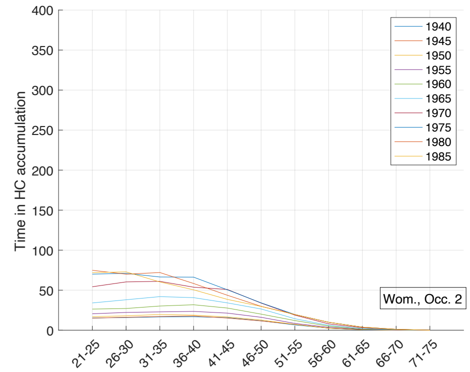
D – Gender gap in average β

Figure 10: Baseline model: Marginal value of human capital, β

Note: The marginal value of human capital (Equation 10) is normalized to 1 for men in occupation 2 in 1940. Panel A shows the marginal value of human capital at age 1 by gender and occupation, for each cohort. Panel B shows the hours-weighted marginal value of human capital by gender and occupation, over time. Panel C shows the hours-weighted marginal value of human capital by gender, over time. Panel D shows the gender gap in the hours-weighted marginal value of human capital over time.
Source: Authors' calculations.



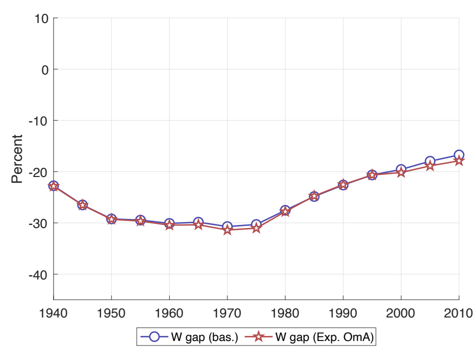
A – Men in occupation 1, $n_m^{j,1}$



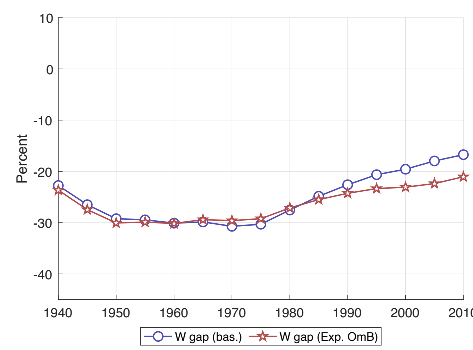
B – Women in occupation 2, $n_f^{j,2}$

Figure 11: Baseline model: Age profiles of time spent learning

Source: Authors' calculations.



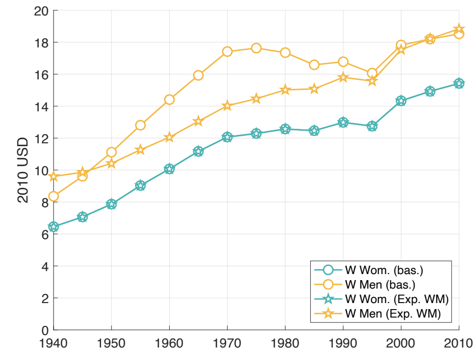
A – Wage gap, Experiment "ω A"



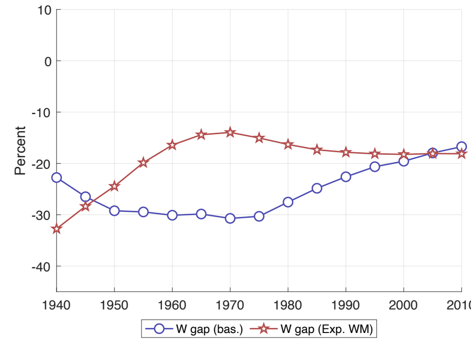
B – Wage gap, Experiment"ω B"

Figure 12: The role of occupation-specific rental rates

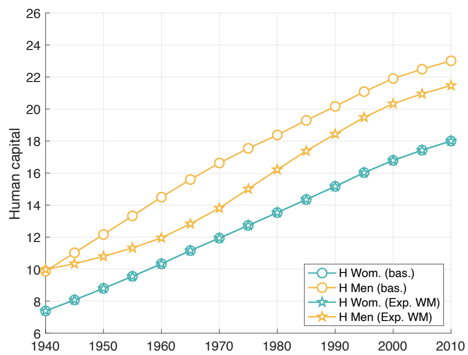
Note: Panel A compares the baseline model and Experiment "ω A" in which ω_t^1, ω_t^2 are set to their 1940 values. Panel B compares the baseline model and Experiment "ω B" in which ω_t^2 is set to ω_t^1 for all t.
Source: Authors' calculations.



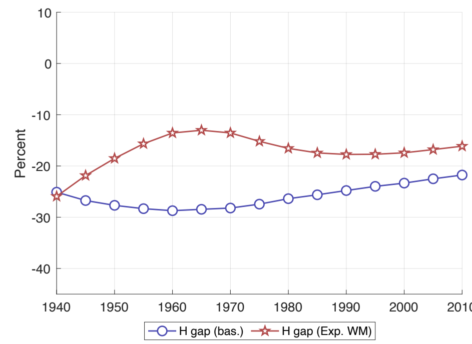
A – Wages



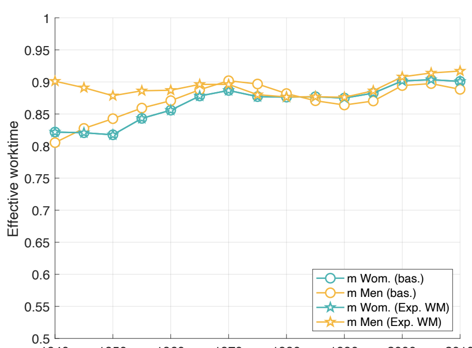
B – Wage gap



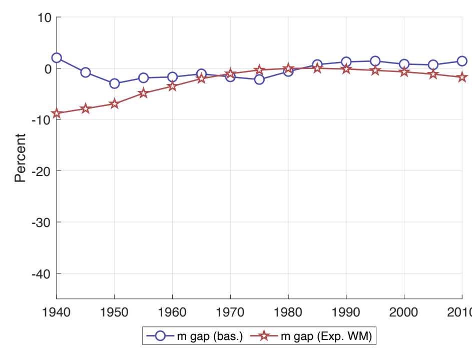
C – Human capital



D – Human capital gap



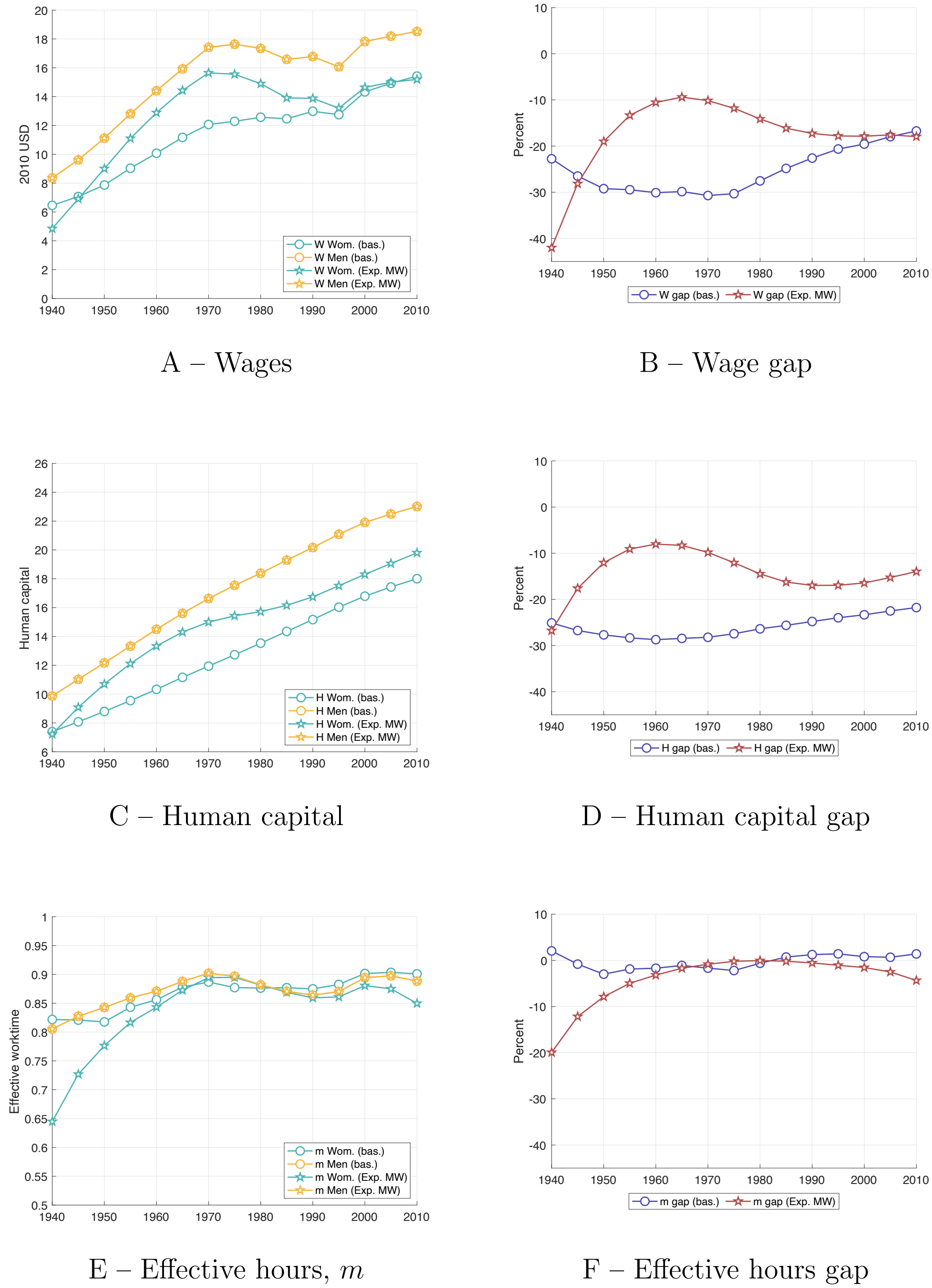
E – Effective hours, m



F – Effective hours gap

Figure 13: The role of gender-specific time endowments (Experiment "τWM")

Note: The figure compares the results of the baseline model and Experiment "τWM" in which men are assigned female time endowments.
Source: Authors' calculations.



A – Wages

B – Wage gap

C – Human capital

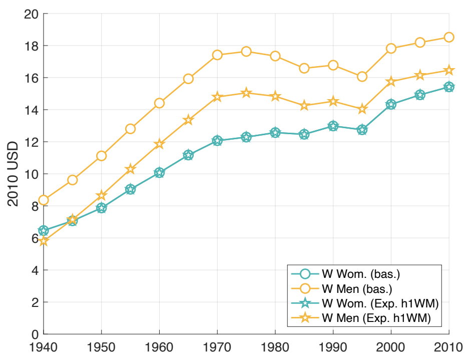
D – Human capital gap

E – Effective hours, m

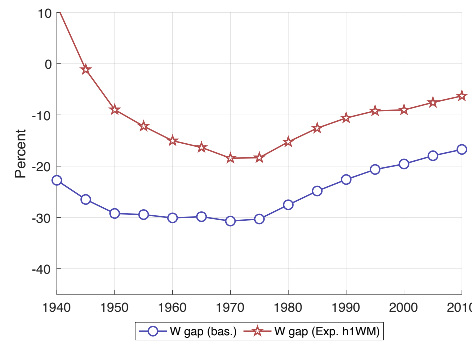
F – Effective hours gap

Figure 14: The role of gender-specific time endowments (Experiment “τMW”)

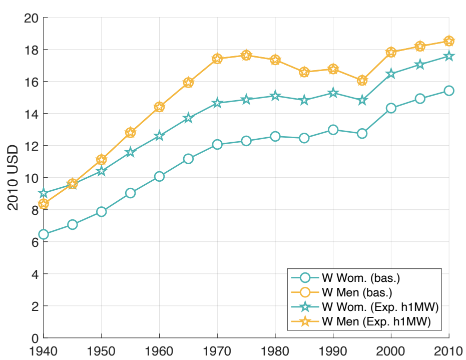
Note: The figure compares the results of the baseline model and Experiment “τMW” in which women are assigned male time endowments.
Source: Authors’ calculations.



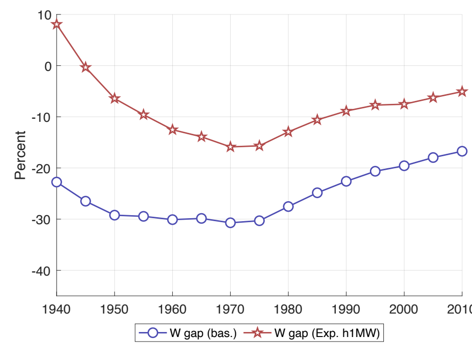
A – Wages (Exp. h1WM)



B – Wage gap (Exp. h1WM)



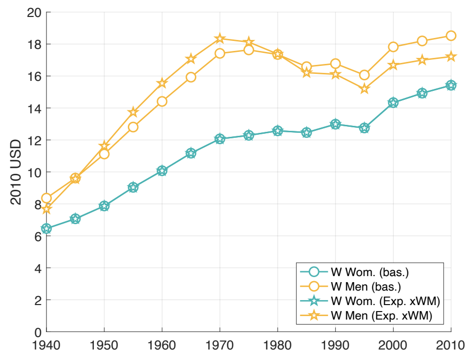
C – Wages (Exp. h1MW)



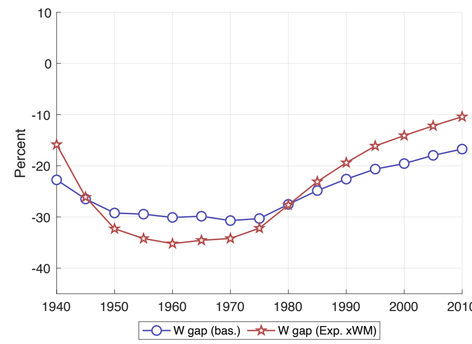
D – Wage gap (Exp. h1MW)

Figure 15: The role of age-1 human capital levels

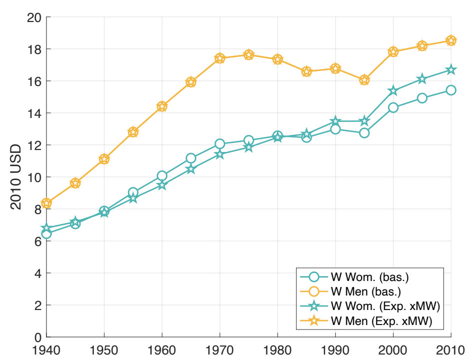
Note: The figure compares the baseline model to Experiment "$h1WM$," in which men are assigned age-1 human capital of women, and Experiment "$h1MW$," in which women are assigned age-1 human capital of men.
Source: Authors' calculations.



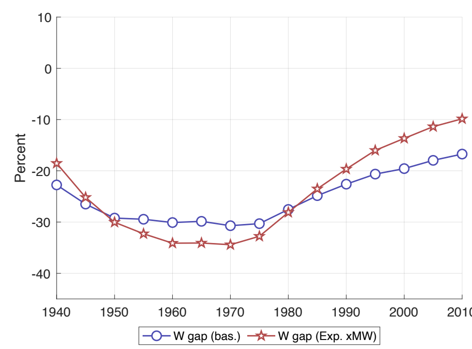
A – Wages (Exp. xWM)



B – Wage gap (Exp. xWM)



C – Wages (Exp. xMW)



D – Wage gap (Exp. xMW)

Figure 16: The role of gender-specific learning productivity levels

Note: The figure compares the baseline model to Experiment "xWM," in which men are assigned female productivity parameters and Experiment "xMW," in which women are assigned male productivity parameters.
Source: Authors' calculations.

B Tables

Table 1: Changes in the wage gap and its decomposition

	Change in wage gap	Endowment effect	Population effect	Micro wage effect	Interaction effects
1940 → 1975	−12.5	−1.0	+0.2	−11.7	
	+100.0	+7.9	−1.3	+93.4	0.050
1975 → 2010	+16.1	+6.2	+0.3	+9.6	
	+100.0	+38.5	+1.8	+59.6	0.059

Note: The "1940 → 1975" panel reports the decomposition results for the time period between 1940 and 1975. The first row reports the change in the wage gap (in percentage point) in column 2, and the contribution of each effect in columns 3 to 5. The second row reports the normalized contribution of each effect. Positive signs in columns 3 to 5 of the second row indicate that the effects operate in the same direction as the change in the gender gap. Negative signs indicate that the effects operate in the opposite direction. The "1975 → 2010" panel reports these results for the time period between 1975 and 2010.
Source: Census, CPS, and authors' calculations.

Table 2: The panel of exogenous and endogenous human capital stocks in the model

	$j=1$	$j=2$	$\dots$	$j=J$
$t=1$	$h^1_{s,1}$	$h^2_{s,1}$	$\dots$	$h^J_{s,1}$
$t=2$	$h^1_{s,2}$	$h^2_{s,2}$		$h^J_{s,2}$
$\vdots$	$\vdots$			
$t=15$	$h^1_{s,15}$	$h^2_{s,15}$	$\dots$	$h^J_{s,15}$

Note: The highlighted human capital stocks are exogenous parameters determined by the calibration of the initial stocks, h^1, and the growth parameters, g^H—see text for details. The non-highlighted stocks are endogenous.

Table 3: Calibrated parameters

δ, r	$+0.073, +0.217$	γ^1, γ^2	$+0.430, +0.327$
$h^1_{f,1}$	$+7.699$	g^H_f	$+0.118, -0.008, +0.000, -0.003, -0.004$
$h^1_{m,1}$	$+10.211$	g^H_m	$+0.113, -0.009, +0.000, -0.004, -0.001$
$x^1_{f,1}$	$+0.079$	$g^{1,X}_f$	$-0.019, +0.004,$
$x^2_{f,1}$	$+0.289$	$g^{2,X}_f$	$-0.055, +0.003,$
$x^1_{m,1}$	$+0.066$	$g^{1,X}_m$	$+0.076, -0.004,$
$x^2_{m,1}$	$+0.252$	$g^{2,X}_m$	$+0.013, -0.000,$
$g^{1,\omega}, g^{2,\omega}$	$+0.960, +0.906$		

Source: Authors' calculations.

Table 4: Main results

	Change in wage gap 1940 - 1975	% Data	Baseline - Exp	Change in wage gap 1975 - 2010	% Data	Baseline - Exp
Data	**-0.13**			**0.16**		
Baseline	**-0.08**	**60**		**0.14**	**84**	
Exp.						
ω A	-0.08	65	**-6**	0.13	81	**3**
ω B	-0.06	44	**16**	0.08	51	**34**
τ WM	0.18	-140	**200**	-0.03	-19	**104**
τ MW	0.30	-240	**299**	-0.06	-39	**123**
h^1 WM	-0.30	236	**-176**	0.12	75	**10**
h^1 MW	-0.24	189	**-129**	0.11	66	**19**
x WM	-0.16	130	**-71**	0.22	135	**-51**
x MW	-0.14	113	**-54**	0.23	142	**-58**

Note: The table reports results from the baseline model and eight counterfactual experiments, each one introducing a single change in the context of the baseline model: (Exp "ω A") ω^1_t, ω^2_t are set to their 1940 values; (Exp "ω B") ω^2_t is set to ω^1_t for all t; (Exp "τ WM") men's $\tau^{j,o}_t$ are set to women's $\tau^{j,o}_t$; (Exp "τ MW") women's $\tau^{j,o}_t$ are set to men's $\tau^{j,o}_t$; (Exp "h^1 WM") men's h^1_t are set to women's h^1_t; (Exp "h^1 WM") women's h^1_t are set to men's h^1_t; (Exp "x WM") men's x^o_t are set to women's x^o_t; (Exp "x MW") women's x^o_t are set to men's x^o_t

Source: Authors' model-based calculations.

C Decomposition

To simplify notation in this appendix, it is convenient to abandon time subscripts. The gender gap is

$$\text{Gap} = 100 \times \frac{W_f - W_m}{W_m},$$

that is

$$\begin{aligned}
\frac{\text{Gap}}{100} W_m &= W_f - W_m, \\
&= \mathcal{W}(\bar{\tau}_f, \bar{P}_f, \bar{W}_f) - \mathcal{W}(\bar{\tau}_m, \bar{P}_m, \bar{W}_m), \\
&\equiv \mathcal{W}_{fff} - \mathcal{W}_{mmm},
\end{aligned}$$

where the last row simplifies the notation further. Then

$$\begin{aligned}
\frac{\text{Gap}}{100} W_m \simeq\ & \frac{1}{4}\left[(\mathcal{W}_{fff} - \mathcal{W}_{mff}) + (\mathcal{W}_{fmm} - \mathcal{W}_{mmm}) + (\mathcal{W}_{ffm} - \mathcal{W}_{mfm}) + (\mathcal{W}_{fmf} - \mathcal{W}_{mmf})\right] \\
+\ & \frac{1}{4}\left[(\mathcal{W}_{fff} - \mathcal{W}_{fmf}) + (\mathcal{W}_{mfm} - \mathcal{W}_{mmm}) + (\mathcal{W}_{ffm} - \mathcal{W}_{fmm}) + (\mathcal{W}_{mff} - \mathcal{W}_{mmf})\right] \\
+\ & \frac{1}{4}\left[(\mathcal{W}_{fff} - \mathcal{W}_{ffm}) + (\mathcal{W}_{mmf} - \mathcal{W}_{mmm}) + (\mathcal{W}_{fmf} - \mathcal{W}_{fmm}) + (\mathcal{W}_{mff} - \mathcal{W}_{mfm})\right].
\end{aligned}$$

The first row measures the role of gender differences in $\bar{\tau}$. To see this, note that each of the four differences in the first row reflect the effect of gender differences in $\bar{\tau}$, holding $\bar{P}$ and $\bar{W}$ constant, at one of the four possible combinations: The first difference, $(\mathcal{W}_{fff} - \mathcal{W}_{mff})$, holds $(\bar{P}_f, \bar{W}_f)$ constant, the second difference holds $(\bar{P}_m, \bar{W}_m)$ constant, etc. Similarly, the second row measures the role of gender differences in $\bar{P}$, and the third row measures the role of gender differences in $\bar{W}$. The formula is an approximation because the effects of arguments of $\mathcal{W}$ changing simultaneously are not accounted for.

We denote the effect of $\bar{\tau}$, given by the first row in the above decomposition, by Gap^τ:

$$\text{Gap}^\tau \equiv \frac{1}{4W_m}\left[(\mathcal{W}_{fff} - \mathcal{W}_{mff}) + (\mathcal{W}_{fmm} - \mathcal{W}_{mmm}) + (\mathcal{W}_{ffm} - \mathcal{W}_{mfm}) + (\mathcal{W}_{fmf} - \mathcal{W}_{mmf})\right].$$

Similarly, we denote by Gap^P and Gap^W the effects given in the second and third rows, respectively. Then,

$$\frac{\text{Gap}}{100} \simeq \text{Gap}^\tau + \text{Gap}^P + \text{Gap}^W.$$

D Model Solution

D.1 Interior Solution

Consider the optimization problem (5)-(8) for generation $t-j+1$ at age j (period t). The terminal condition in (8) implies no learning at age J (period $t-j+J$). The value function in the last period of life can then be stated as follows:

$$V_{t-j+J}^{J}\left(h_{t-j+J}^{J}\right) = \beta_{t-j+J}^{J} h_{t-j+J}^{J} + \alpha_{t-j+J}^{J},$$

where $\beta_{t-j+J}^{J} = \omega_{t-j+J}\tau_{t-j+J}^{J}$ and $\alpha_{t-j+J}^{J} = 0$.

We conjecture that, at age $j+1$ (period $t+1$), the value function is also of linear form:

$$V_{t+1}^{j+1}\left(h_{t+1}^{j+1}\right) = \beta_{t+1}^{j+1} h_{t+1}^{j+1} + \alpha_{t+1}^{j+1},$$

and proceed to derive the slope and intercept.

Substituting the above expression along with the law of motion for human capital (6) into the problem of age j workers, we obtain

$$V_t^j\left(h_t^j\right) = \max_{0 \le n_t^j \le \tau_t^j} \omega_t h_t^j\left(\tau_t^j - n_t^j\right) + \frac{1}{1+r}\beta_{t+1}^{j+1}\left((1-\delta)\, h_t^j + x_{t-j+1} F\left(n_t^j h_t^j\right)\right) + \frac{1}{1+r}\alpha_{t+1}^{j+1}.$$

The first-order condition is given by:

$$n_t^j h_t^j = \left[\frac{\gamma}{1+r}\,\frac{\beta_{t+1}^{j+1}}{\omega_t}\, x_{t-j+1}\right]^{\frac{1}{1-\gamma}}. \tag{14}$$

Substituting it into $V_t^j\left(h_t^j\right)$ yields

$$\begin{aligned} V_t^j\left(h_t^j\right) &= \omega_t h_t^j \tau_t^j + \omega_t\left(\frac{1}{\gamma} - 1\right)\left[\frac{\gamma}{1+r}\,\frac{\beta_{t+1}^{j+1}}{\omega_t}\, x_{t-j+1}\right]^{\frac{1}{1-\gamma}} + \frac{1}{1+r}\beta_{t+1}^{j+1}(1-\delta)\, h_t^j + \frac{1}{1+r}\alpha_{t+1}^{j+1}, \\ &= \beta_t^j h_t^j + \alpha_t^j, \end{aligned}$$

where

$$\begin{aligned} \beta_t^j &= \omega_t \tau_t^j + \frac{1-\delta}{1+r}\beta_{t+1}^{j+1}, \\ \alpha_t^j &= \omega_t\left(\frac{1}{\gamma} - 1\right)\left[\frac{\gamma}{1+r}\,\frac{\beta_{t+1}^{j+1}}{\omega_t}\, x_{t-j+1}\right]^{\frac{1}{1-\gamma}} + \frac{1}{1+r}\alpha_{t+1}^{j+1}. \end{aligned}$$

Thus, the slope and intercept can be obtained recursively, given their age J values found above. This yields the following expression for β_t^j, referenced in the main text:

$$\beta_t^j = \sum_{k=j}^{J}\left(\frac{1-\delta}{1+r}\right)^{k-j} \omega_{t+k-j}\tau_{t+k-j}^{k}. \tag{15}$$

D.2 Aggregation of a Heterogeneous Agent Economy

We derive the conditions under which aggregate outcomes in a heterogeneous agent model (for a given cohort) can be characterized by solving the representative agent model considered in this paper. We show this aggregation result holds under fairly general conditions. There are no restrictions on heterogeneity with respect to initial capital stocks, learning abilities, or market time levels. However, the shape of time endowment profiles must be the same for all agents.

To simplify the notation, we consider cohort 1 so that $t = j$ throughout the life cycle.

Consider the following heterogeneous agent economy. Suppose that each cohort is populated by N types of individuals indexed by i and living from age 1 to J. There is a proportion α_i of each type such that $\sum_{i=1}^{N} \alpha_i = 1$. Type i agents are characterized by a tuple $\left(h_i^1, x_i, \{\tau_i^j\}_{j=1}^J\right)$, where h_i^1 is the initial human capital, x_i is the ability, and the sequence $\{\tau_i^j\}_{j=1}^J$ gives the age profile of market time.

Result Consider the heterogeneous agent model presented above. Suppose the ratio of market time across any two agents stays constant over the life cycle.[19] Using type 1 agents' profile as the numeraire profile, this assumption can be formalized as

$$\tau_i^j / \tau_1^j = z_i \text{ for all } j. \tag{16}$$

Consider the optimal solution for each agent type and the implied average profiles for human capital and wages, $\bar{h}^j = \sum_i \alpha_i h_i^j$ and $\bar{W}^j = \sum_i \alpha_i W_i^j$, for all j.

Our representative agent model yields identical optimal profiles, $h^j = \bar{h}^j$ and $W^j = \bar{W}^j$ for all j, under the following assumptions on endowments:

$$h^1 = \bar{h}^1, \;\; x = \left[\sum_{i=1}^{N} \alpha_i x_i^{\frac{1}{1-\gamma}} (z_i/\bar{z})^{\frac{\gamma}{1-\gamma}} \right]^{1-\gamma}, \;\; \tau^j = \bar{\tau}^j \text{ for all } j, \tag{17}$$

where $\bar{h}^1 = \sum_i \alpha_i h_i^1$, $\bar{\tau}^j = \sum_i \alpha_i \tau_i^j$ and $\bar{z} = \sum_{i=1}^{N} \alpha_i z_i$.

Proof Consider the heterogeneous agent economy. We proceed to derive the average profiles for human capital and wages.

Following the results in Section D.1, the interior solution of the income maximization problem of type i individuals at age j yields

$$\beta_i^j = \sum_{k=j}^{J} \left(\frac{1-\delta}{1+r} \right)^{k-j} \omega_k \tau_i^k \;\; \text{and} \;\; n_i^j h_i^j = \left[\frac{\gamma}{1+r} \frac{\beta_i^{j+1}}{\omega_j} x_i \right]^{\frac{1}{1-\gamma}}.$$

Accordingly, human capital for these individuals evolves in line with

$$h_i^{j+1} = (1-\delta)\, h_i^j + x_i^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{1+r} \frac{\beta_i^{j+1}}{\omega_j} \right]^{\frac{\gamma}{1-\gamma}}. \tag{18}$$

[19] Note there are no restrictions on the shape of time profiles $\{\tau_1^j\}_{j=1}^J$.

Employing the assumption made in (16), the average market time at age j can be written as

$$\bar{\tau}^j = \sum_{i=1}^{N} \alpha_i \tau_i^j = \sum_{i=1}^{N} \alpha_i z_i \tau_1^j = \bar{z}\tau_1^j,$$

where $\bar{z} = \sum_{i=1}^{N} \alpha_i z_i$. It follows that the type i agents' market time at age j is a constant fraction of the average market time,

$$\tau_i^j = \frac{z_i}{\bar{z}} \bar{\tau}^j.$$

Using this result, we show that the average value of human capital can be expressed in terms of the average market time profile:

$$\bar{\beta}^j = \sum_{i=1}^{N} \alpha_i \beta_i^j = \sum_{i=1}^{N} \alpha_i \sum_{k=j}^{J} \left(\frac{1-\delta}{1+r}\right)^{k-j} \omega_k z_i \frac{\bar{\tau}^k}{\bar{z}} = \sum_{k=j}^{J} \left(\frac{1-\delta}{1+r}\right)^{k-j} \omega_k \bar{\tau}^k. \tag{19}$$

It follows that, just like in the case of market time, the value of human capital for a type i agent at age j is a constant fraction of its average value:

$$\beta_i^j = \sum_{k=j}^{J} \left(\frac{1-\delta}{1+r}\right)^{k-j} \omega_k z_i \frac{\bar{\tau}^k}{\bar{z}} = \frac{z_i}{\bar{z}} \bar{\beta}^j. \tag{20}$$

Substituting the above expession into the law of motion (18) yields

$$h_i^{j+1} = (1-\delta)\, h_i^j + x_i^{\frac{1}{1-\gamma}} \left(\frac{z_i}{\bar{z}}\right)^{\frac{\gamma}{1-\gamma}} \left[\frac{\gamma}{1+r}\ \frac{\bar{\beta}^{j+1}}{\omega_j}\right]^{\frac{\gamma}{1-\gamma}}.$$

Averaging across types yields

$$\sum_{i=1}^{N} \alpha_i h_i^{j+1} = (1-\delta) \sum_{i=1}^{N} \alpha_i h_i^j + \sum_{i=1}^{N} \alpha_i x_i^{\frac{1}{1-\gamma}} \left(\frac{z_i}{\bar{z}}\right)^{\frac{\gamma}{1-\gamma}} \left[\frac{\gamma}{1+r}\ \frac{\bar{\beta}^{j+1}}{\omega_j}\right]^{\frac{\gamma}{1-\gamma}},$$

or

$$\bar{h}^{j+1} = (1-\delta)\, \bar{h}^j + \tilde{x}^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{1+r}\ \frac{\bar{\beta}^{j+1}}{\omega_j}\right]^{\frac{\gamma}{1-\gamma}}, \tag{21}$$

where $\bar{h}^j = \sum_i \alpha_i h_i^j$ denotes average hours at age j and $\tilde{x} \equiv \left[\sum_{i=1}^{N} \alpha_i x_i^{\frac{1}{1-\gamma}} (z_i/\bar{z})^{\frac{\gamma}{1-\gamma}}\right]^{1-\gamma}$ is a shorthand. Thus, Equation (21) characterizes the evolution of average human capital in the heterogeneous agent economy.

Now consider the representative agent model, whose optimal human capital stock evolves according to

$$h^{j+1} = (1-\delta)\, h^j + x^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{1+r}\ \frac{\beta^{j+1}}{\omega_j}\right]^{\frac{\gamma}{1-\gamma}}. \tag{22}$$

By Assumption (17), the representative agent faces the market time profile equal to the average profile in the heterogeneous agent economy, and therefore the same must hold for the value of human capital: $\beta^j = \sum_{k=j}^{J} \left(\frac{1-\delta}{1+r}\right)^{k-j} \omega_k \bar{\tau}^k = \bar{\beta}^j$, where the last equality is due to Equation (19). This result, along with the remaining assumptions on endowments, $h^1 = \bar{h}^1,\ x = \tilde{x}$, make it clear that, for $j = 1$, the right hand sides of Equations (21) and (22) are the same, and therefore $h^2 = \bar{h}^2$, and so on. By induction, we obtain that our representative agent model yields the same profile, $h^j = \bar{h}^j$ for all j.

It remains to show that the wage profiles also aggregate, i.e. $W^j = \bar{W}^j$.

In the heterogeneous agent economy, type i wages are given by

$$W_i^j = \omega_j h_i^j \frac{\tau_i^j - n_i^j}{\tau_i^j} = \omega_j h_i^j - \frac{\omega_j h_i^j n_i^j}{\tau_i^j}.$$

Using the optimality condition and expression in (20), the term $h_i^j n_i^j / \tau_i^j$ can be rewritten as follows:

$$\begin{aligned} \frac{h_i^j n_i^j}{\tau_i^j} &= \frac{1}{\tau_i^j} \left(\frac{\gamma}{1+r} \frac{\beta_i^{j+1}}{\omega_j} x_i \right)^{\frac{1}{1-\gamma}}, \\ &= \frac{x_i^{\frac{1}{1-\gamma}} \left(\frac{z_i}{\bar{z}}\right)^{\frac{1}{1-\gamma}}}{(z_i/\bar{z})\, \bar{\tau}^j} \left(\frac{\gamma}{1+r} \frac{\bar{\beta}^{j+1}}{\omega_j} \right)^{\frac{1}{1-\gamma}}, \\ &= \frac{x_i^{\frac{1}{1-\gamma}} \left(\frac{z_i}{\bar{z}}\right)^{\frac{\gamma}{1-\gamma}}}{\bar{\tau}^j} \left(\frac{\gamma}{1+r} \frac{\bar{\beta}^{j+1}}{\omega_j} \right)^{\frac{1}{1-\gamma}}. \end{aligned}$$

Averaging across individuals and using the above result, we obtain the average wage profile:

$$\begin{aligned} \bar{W}^j = \sum_{i=1}^{N} \alpha_i W_i^j &= \omega_j \sum_{i=1}^{N} \alpha_i h_i^j - \omega_j \sum_{i=1}^{N} \alpha_i \frac{h_i^j n_i^j}{\tau_i^j}, \\ &= \omega_j \bar{h}^j - \omega_j \frac{\tilde{x}^{\frac{\gamma}{1-\gamma}}}{\bar{\tau}^j} \left(\frac{\gamma}{1+r} \frac{\bar{\beta}^{j+1}}{\omega_j} \right)^{\frac{1}{1-\gamma}}. \end{aligned}$$

Using the optimality condition in the representative agent's wage profile, we obtain

$$\begin{aligned} W^j &= \omega_j h^j - \omega_j \frac{h^j n^j}{\tau^j}, \\ &= \omega_j h^j - \omega_j \frac{x^{\frac{\gamma}{1-\gamma}}}{\tau^j} \left(\frac{\gamma}{1+r} \frac{\beta^{j+1}}{\omega_j} \right)^{\frac{1}{1-\gamma}}. \end{aligned}$$

The two profiles are identical because of the assumptions on the representative agent's endowments ($x = \tilde{x}$ and $\tau^j = \bar{\tau}^j$) and the already established results that $\beta^j = \bar{\beta}^j$ and $h^j = \bar{h}^j$.

D.3 On Implications of Normalizing ω_1

Result The level to which we normalize ω_1^1 is irrelevant for the following optimal outcomes in the calibrated model: (1) wages $\{W_{s,t}^{j,o}\}$, (2) time investments in human capital $\{n_{s,t}^{j,o}\}$ and (3) the response of time investments (and wages) to changes in time endowment.

Proof Let us compare two model economies, both calibrated to wage data. The first is the benchmark economy defined by the normalized ω_1^1 and corresponding calibrated parameters. The second is the economy defined by an alternative normalization, say $\tilde{\omega}_1^1 = \kappa\omega_1^1$, and calibrated parameters denoted by "˜". Let us also use "˜" to denote optimal outcomes in the second model economy. We want to show that optimal wages and human capital investments are the same in the two economies. To simplify the notation, we drop the gender indices.

Given the benchmark calibration, we will show how to select parameters for the second economy that would yield the same wages, i.e. $\{\tilde{W}_t^{j,o}\} = \{W_t^{j,o}\}$. The reverse is also true. Since both model economies are calibrated by targeting wage data, it follows that the same fit to wage data will be attained and wage outcomes will be the same in the two model economies.

We select parameters for the second economy as follows: set the remaining rental rates to the same multiple of their benchmark values, so that we have $\tilde{\omega}_t^o = \kappa\omega_t^o$ for all t and both occupations, set the initial human capital levels to $\tilde{h}_t^1 = h_t^1/\kappa$ and learning ability levels to $\tilde{x}_t{}^o = x_t^o/\kappa^{1-\gamma}$ while keeping the rest of the parameters as in the benchmark model.

To see that this calibration yields the desired result, recall the expression for the optimal wages,

$$W_t^{j,o} = \omega_t^o h_t^{j,o} \frac{\tau_t^{j,o} - n_t^{j,o}}{\tau_t^{j,o}},$$

and the key equations that matter for it. The first equation gives the value of human capital:

$$\beta_t^{j,o} = \sum_{k=j}^{J} \left(\frac{1-\delta}{1+r}\right)^{k-j} \omega_{t+k-j}^o \tau_{t+k-j}^{k,o}.$$

The second is the first-order condition for human capital investments, Equation (9):

$$n_t^{j,o} h_t^{j,o} = \left[\frac{\gamma}{1+r} \frac{\beta_{t+1}^{j+1,o}}{\omega_t^o} x_{t-j+1}^o\right]^{\frac{1}{1-\gamma}}.$$

The third is the law of motion for human capital, Equation (6),

$$h_{t+1}^{j+1,o} = (1-\delta)\, h_t^{j,o} + x_{t-j+1}^o \left(n_t^{j,o} h_t^{j,o}\right)^{\gamma} = (1-\delta)\, h_t^{j,o} + \left(x_{t-j+1}^o\right)^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{1+r} \frac{\beta_{t+1}^{j+1,o}}{\omega_t^o}\right]^{\frac{\gamma}{1-\gamma}},$$

where we substituted for nh from the above first order condition.

Given our assumption on the calibrated rental prices in the second economy, it is clear that $\tilde{\beta}_t{}^{j,o} = \kappa\beta_t^{j,o}$. Using this expression along with our assumption regarding the calibrated values for $\tilde{h}_t^1$, $\tilde{x}_t^1$ and $\tilde{\omega}_t^o$ in the above equation, we can derive the expression for age 2 human capital of cohort t in the second economy:

$$\begin{aligned}
\tilde{h}_{t+1}^{2,o} &= (1-\delta)\, \tilde{h}_t^{1,o} + \left(\tilde{x}_t^o\right)^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{1+r} \frac{\tilde{\beta}_{t+1}^{2,o}}{\tilde{\omega}_t^o}\right]^{\frac{\gamma}{1-\gamma}} \\
&= (1-\delta)\, h_t^{1,o}/\kappa + \left(x_t^o/\kappa^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{1+r} \frac{\kappa\beta_{t+1}^{2,o}}{\kappa\omega_t^o}\right]^{\frac{\gamma}{1-\gamma}} \\
&= h_{t+1}^{2,o}/\kappa.
\end{aligned}$$

Iterating forward implies that $\tilde{h}_{t+1}^{j+1,o} = h_{t+1}^{j+1,o}/\kappa$ for all j.

Using this result along with our assumptions on the calibrated values for $\tilde{x}_t^1$ and $\tilde{\omega}_t^o$ in the first-order condition for n, we rewrite it as:

$$\begin{aligned}
\tilde{n}_t^{j,o}\tilde{h}_t^{j,o} &= \left[\frac{\gamma}{1+r}\,\frac{\tilde{\beta}_{t+1}^{j+1,o}}{\tilde{\omega}_t^o}\,\tilde{x}_{t-j+1}^o\right]^{\frac{1}{1-\gamma}}, \\
\tilde{n}_t^{j,o}h_{t+1}^{j+1,o}/\kappa &= \left[\frac{\gamma}{1+r}\,\frac{\kappa\beta_{t+1}^{j+1,o}}{\kappa\omega_t^o}\,x_{t-j+1}^o/\kappa^{1-\gamma}\right]^{\frac{1}{1-\gamma}} = \left[\frac{\gamma}{1+r}\,\frac{\beta_{t+1}^{j+1,o}}{\omega_t^o}\,x_{t-j+1}^o\right]^{\frac{1}{1-\gamma}}/\kappa,
\end{aligned}$$

which implies that $\tilde{n}_t^{j,o} = n_t^{j,o}$.

It then follows that

$$\tilde{W}_t^{j,o} = \tilde{\omega}_t^o\tilde{h}_t^{j,o}\frac{\tau_t^{j,o} - \tilde{n}_t^{j,o}}{\tau_t^{j,o}} = \kappa\omega_t^o(h_t^{j,o}/\kappa)\frac{\tau_t^{j,o} - n_t^{j,o}}{\tau_t^{j,o}} = W_t^{j,o}.$$

Thus, the second economy yields the same time investments in human capital and wages as in the benchmark. One lesson from this discussion is that we cannot use wage data to distinguish between the two model economies, i.e. to uniquely identify human capital and rental prices. Since neither of these quantities is observable, no other data can help either. The upside is that it does not matter for our results. Both economies yield the same prediction for how time investment in learning and wages respond to changes in time endowment. This is what remains to show.

Differentiating time investment with respect to the time endowment yields

$$\frac{dn}{d\tau} = \left[\frac{\gamma}{1+r}\,\frac{1}{\omega_j}\,x\right]^{\frac{1}{1-\gamma}}\frac{1}{h}\frac{1}{1-\gamma}\beta^{\frac{\gamma}{1-\gamma}}\frac{d\beta}{d\tau}.$$

Rewriting this expression for the second economy in terms of the benchmark variables yields the same expression:

$$\begin{aligned}
\frac{d\tilde{n}}{d\tau} &= \left[\frac{\gamma}{1+r}\,\frac{1}{\tilde{\omega}_j}\,\tilde{x}\right]^{\frac{1}{1-\gamma}}\frac{1}{\tilde{h}}\frac{1}{1-\gamma}\tilde{\beta}^{\frac{\gamma}{1-\gamma}}\frac{d\tilde{\beta}}{d\tau} \\
&= \left[\frac{\gamma}{1+r}\,\frac{1}{\kappa\omega_j}\,\frac{x}{\kappa^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}\frac{\kappa}{h}\frac{1}{1-\gamma}\beta^{\frac{\gamma}{1-\gamma}}\kappa^{\frac{\gamma}{1-\gamma}}\frac{d\beta}{d\tau}\kappa \\
&= \left[\frac{\gamma}{1+r}\,\frac{1}{\omega_j}\,x\right]^{\frac{1}{1-\gamma}}\frac{1}{h}\frac{1}{1-\gamma}\beta^{\frac{\gamma}{1-\gamma}}\frac{d\beta}{d\tau} = \frac{dn}{d\tau}.
\end{aligned}$$

Differentiating wages with respect to the time endowment yields

$$\frac{dW^j}{d\tau} = -wh(\frac{dn}{d\tau}\tau^{-1} - n\tau^{-2}).$$

Rewriting this expression for the second economy in terms of the benchmark variables and using the results that $\frac{d\tilde{n}}{d\tau} = \frac{dn}{d\tau}$ and $\tilde{n} = n$ allows us to show that the wage response is also identical across the two economies:

$$\frac{d\tilde{W}^j}{d\tau} = -\tilde{w}\tilde{h}(\frac{d\tilde{n}}{d\tau}\tau^{-1} - \tilde{n}\tau^{-2}) = -\omega w\frac{h}{\omega}(\frac{dn}{d\tau}\tau^{-1} - n\tau^{-2}) = \frac{dW^j}{d\tau}.$$