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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Voluntary participation in a terror group and counterterrorism policy

Subhayu Bandyopadhyay[a]
[a] Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63102 USA
Todd Sandler[b]
[b]School of Economic, Political & Policy Sciences, University of Texas at Dallas, Richardson, TX, 75080 USA

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Abstract

A three-stage game investigates how counterterrorism measures are affected by volunteers' choice in joining a terrorist group. In stage 1, the government chooses both proactive and defensive countermeasures, while looking ahead to the anticipated size and actions of terrorist groups. After radicalized individuals choose whether to join a terrorist group in stage 2, group members then allocate their time between work and terrorist operations. Based on wages and government counterterrorism, the game characterizes the extensive margin determining group size and the intensive margin indicating the group's level of attacks. Comparative statics show how changes in wages or radicalization impact the optimal mix between defensive and proactive countermeasures. Higher (lower) wages favor a larger (smaller) mix of proactive measures over defensive actions. In the absence of backlash, enhanced radicalization of terrorist members calls for a greater reliance on defensive actions. The influence of backlash on counterterrorism is also examined.

JEL Codes: C72, D71, H56

Keywords: terrorist supporters' occupational choice, rational terrorist supply, radicalization and wage rate, optimal mix between proactive and defensive countermeasures, backlash

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The views expressed are those of the authors and do not necessarily represent the official positions of the Federal Reserve Bank of St. Louis or the Federal Reserve System

Corresponding Author: Subhayu Bandyopadhyay
Email: subhayu.bandyopadhyay@stls.frb.org

Voluntary participation in a terror group and counterterrorism policy

1. Introduction

In the last two decades, the hotspots of terrorism have shifted from Latin America and the Caribbean (LAC) and Europe and Central Asia (ECA) to South Asia (SAS), Middle East and North Africa (MENA), and sub-Saharan Africa (SSA), with the latter three regions accounting for 35.2%, 33.8%, and 13.7% of worldwide attacks during 2002–2020, respectively (Gaibulloev and Sandler 2022).[1] The current geographical concentrations of terrorists are primarily driven by religious fundamentalist and nationalist/separatist terrorist groups. Many of the former are jihadist terrorist organizations located in Afghanistan, Indonesia, Iraq, Kenya, Mali, Nigeria, Pakistan, the Philippines, Somalia, Yemen, and elsewhere (Hou, Gaibulloev, and Sandler 2020).

In their ongoing war on terror, targeted countries' authorities must understand the drivers of terrorist groups' recruitment and the motivating role played by economic conditions and counterterrorism campaigns. Generally, counterterrorism falls into two categories: proactive and defensive measures. Proactive operations confront the terrorists and their supporters directly by eliminating terrorist resources (weapons and operatives), finances, safe havens, infrastructure (e.g., training camps), or sponsors (Enders and Sandler 2012, 105). By contrast, defensive counterterrorism actions harden or protect potential targets, thereby making an attack more costly and less likely to succeed. For instance, the installation of metal detectors at US airports on January 5, 1973, made skyjackings less apt to succeed, thereby resulting in a huge immediate fall in attempted US hijackings (Enders and Sandler 1993; Landes 1978). By differentially affecting the fate of the terrorists or the success likelihood of their operations, the proactive-defensive mix of measures can influence the extensive and intensive margins for volunteers to

[1]Those percentages account for the sum of domestic, transnational, and uncertain terrorist attacks recorded in the Global Terrorism Database (GTD) (National Consortium for the Study of Terrorism and Responses to Terrorism 2020) and classified by Enders, Sandler, and Gaibulloev (2011) and updates. Uncertain incidents cannot be characterized as domestic or transnational based on the information in GTD.

the terrorist groups. The joining decision is tied to the extensive margin and the resulting group size, while volunteers' effort allocation to attacks is associated with the intensive margin.

Our main purpose is to present a game-theoretic model of counterterrorism policy where some radicalized individuals in a nation volunteer to participate in a terror group after weighing associated costs and benefits. In so doing, a targeted government chooses its mix of counterterrorism policies in stage 1, followed by radicalized individuals deciding whether to join the terrorist group in stage 2. In stage 3, the group members choose their terrorism and work efforts, given radicalization and the mix of counterterrorism measures. There is some similarity in our approach to the influential club model of terror proposed by Berman (2009), Berman and Laitin (2008), Iannaccone (1992), and Iannaccone and Berman (2006) in which a religious terrorist group extracts loyalty and effort from members by offering them excludable public goods. In the case of Hamas in Gaza or Hezbollah in Lebanon, the terrorist group provides public services such as schools, health care, disaster relief, and social services to members and their dependents.

Although our approach shares some features of the club approach as later shown, the model here is fundamentally different. First, if (radicalized) individuals are sympathetic to a terror group, then they derive utility from aggregate terror created, even when they do not volunteer to serve in the group. In that context, the terrorist group creates a public "good" in terms of attacks that is *nonexcludable* to sympathizers, unlike in a club setting where nonmembers cannot consume the group-provided public good(s). In the club model, this excludability provides leverage over the members that is absent from the model here. Second, our model allows radicalized individuals to choose whether or not to volunteer their time to the group based on an occupational-choice type model where this decision is affected by proactive counterterror policy and *labor market opportunity cost*. In the model here, proactive measures affect the number of volunteers of the terror group (the *extensive margin*). Additionally, our model allows for terror reduction achieved through defensive policy, which complements proactive measures by hardening potential targets without radicalizing the population through collateral damage and the resulting backlash.[2] As shown by Berman and Laitin (2008) and Iannaccone (1992), their terror club ideally eliminates the outside occupational options of their members to foster dependence and loyalty. Our model of rational terrorism supply, based on individual level decision making and occupational choice, is novel to the literature and applies to terrorist organizations that do not supply excludable public goods.[3]

The analysis delivers insights on the optimal defensive-proactive policy combination that is also new to the rapidly evolving game-theoretic literature on counterterrorism policy (see recent surveys by Gaibulloev and Sandler 2019; Schneider, Brück, and Meierrieks 2015). Specifically, we show that wage increases favor the use of proactive measures over defensive actions, while enhanced radicalization in the absence of backlash, instead, favors the application of defensive actions over proactive measures. In the former case, the mix of proactive-defensive policies to be deployed against resident terrorist groups will differ between regions displaying different wage levels. By contrast, policy mixes must be tailored to the degrees of radicalization in different regions or countries..

The remainder of the paper contains five main sections. Section 2 offers a brief review of the game-theoretic literature, while Section 3 presents the basic three-stage game. Comparative statics involving the wage rate and radicalization are indicated in Section 4. Model extensions

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[2] On backlash models, see Arce and Sandler (2010), Bloom (2005), Rosendorff and Sandler (2004), and Siqueira and Sandler 2007).

[3] Bueno de Mesquita (2005) allows individuals to choose between becoming a terrorist or not but does not permit their time allocation between producing a private good and terrorism. Other differences between his model and the current one exists – e.g., our distinguishing between defensive and proactive countermeasures. Moreover, our model allows for individual heterogeneity in terms of radicalization, while his model allows for heterogeneity in terms of individual abilities. Other differences abound between the two approaches that seek to answer different questions.

consider backlash impact of radicalization and counterterrorism actions in Section 5, followed by concluding remarks in Section 6.

2. A brief literature review

Game theory models have been applied to a variety of situations. For instance, Hausken (2012), Hausken and Bier (2011), and Bandyopadhyay and Sandler (2022) examine government counterterrorism measures when there are two or more terrorism groups that vary in size and the risks that they pose. Another line of game-theoretic modeling investigates how at-risk governments choose their proactive and defensive mix when confronted with one or more terrorist groups (see, e.g., Bandyopadhyay and Sandler 2011; 2022; Cárceles-Poveda and Tauman 2011; Das and Chowdhury 2014). Other interfaces between targeted countries and terrorists allow for the independent choice of proactive or defensive actions (e.g., Bier, Oliveros, and Samuelson 2007; Hausken 2008; Sandler and Siqueira 2006). Unlike the representation here, many extant game models consider the government and a terrorist group as the adversaries or players without considering the decision calculus of the terrorist operatives. Another set of game models examine how targeted governments choose their defensive allocations among alternative targets when confronting terrorist groups (e.g., Heal and Kunreuther 2007; Hausken and Bier 2011; Hausken, Bier, and Zhuang 2009; Kunreuther and Heal 2003; Sandler and Lapan 1988) . In some models, the terrorist group resides in one of the two targeted countries, thereby opening the door to taxing policies or foreign aid by a targeted country to induce the country hosting the terrorist group to do more to counter the threat (Bandyopadhyay and Sandler 2021; Bandyopadhyay, Sandler, and Younas 2011). In most of the earlier game models, the terrorist group is a passive entity, whose inner drivers and workings do not figure into the analysis. As a consequence, the occupational choice of the terrorist recruits and their aggregate level of terrorism are not considered in contrast to the approach here.

Yet another game-theoretic approach to terrorism examines the formation of coalitions among targeted governments when deciding their counterterrorism policy against a terrorist adversary (e.g., Cárceles-Poveda and Tauman 2011; Das and Chowdhury 2014; Rossi de Oliveira, Faria, and Silva 2018). When determining their counterterrorism actions independently, targeted governments tend to underprovide proactive measures and overprovide defensive ones (Arce and Sandler 2005; Sandler and Lapan 1988), leaving the possibility for cooperative gains through coalition formation. In such theoretical frameworks, a terrorist group or groups confront a government, but again there is no micro foundations laid for how the individual volunteers determine whether to join the group or their allocation of effort once a member. Those individuals' time allocation decisions are what, in part, distinguishes the current exercise from past models in the game-theory literature.

3. The model

Suppose that individuals of mass *M* are sympathetic to a resident terrorist group in a nation. A subset of these individuals $i \in [0, n \leq M]$ volunteers to join a terrorist group for which volunteer *i* provides effort level $r(i)$ to the group's operations. We denote terrorism corresponding to zero defensive actions by the government as $\tilde{T}$, produced through the collective effort of the volunteers. The terror production function features diminishing returns, so that

$$\tilde{T} = \tilde{T}(R),\ \tilde{T}' > 0,\ \tilde{T}'' < 0, \tag{1}$$

where $R = \int_{i=0}^{n} r(i)di$ is the aggregate terror effort. Suppose that counterterrorism defensive policy a reduces the terrorist group's success rate θ $(0 < \theta \leq 1)$ at a diminishing rate, such that the effective terrorism level is:

$$T = \theta(a)\tilde{T}(R), \tag{2}$$

where $\theta = \theta(a)$, $\theta' < 0$, and $\theta''(a) > 0$.

Terrorist group volunteers derive utility from their private consumption, group participation, and group-generated terrorism output *T*.[4] Private consumption corresponds to a numéraire good *x*. A radicalized individual *i*'s utility function is:

$$U(i) = x(i) + \beta(i)\{u[r(i)] + B(T)\},\ u' > 0, u'' < 0; B > 0, B' > 0 \text{ ; and } \beta(i) > 0, \tag{3}$$

where $x(i)$ is the consumption of good *x*, $u[r(i)]$ is *i*'s utility from volunteering terrorism effort, $B(T)$ is utility derived from successful terror attacks, and $\beta(i)$ is a measure of radicalization of individual *i*. Individuals with higher β derive greater utility from their provision of terror effort and from the incidence of aggregate terror.

We propose a three-stage game. In stage 1, the targeted government chooses proactive policy E and defensive action a for which the former attacks the terrorist group and its assets, while the latter hardens targets (Gaibulloev and Sandler 2019). A radicalized individual chooses whether to join the terror group in stage 2. Terrorist group members then choose how to allocate their time between production of good *x* and volunteering for the terrorist operations in stage 3. As is standard, the game is solved by backward induction.

3.1. Stage 3

All individuals possess a unit labor endowment. Terror group members allocate their endowment between terror operations $r(i)$ and work effort $h(i)$, such that $h(i) = 1 - r(i)$ goes for producing good x. For an assumed linear production function, $x = wh$ for good *x* in which

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[4] This is similar to the utility functions used in Iannaccone (1992) and Berman and Laitin (2008), except that terrorist attacks replace the excludable local public good..

the wage rate is *w*. Thus, individual *i*'s consumption of good *x* equals:

$$x(i) = wh(i) = w[1 - r(i)]. \tag{4}$$

Substituting Eq. (4) in Eq. (3), we have:

$$U(i) = w[1 - r(i)] + \beta(i)\{u[r(i)] + B(T)\}. \tag{5}$$

Individual *i*'s utility-maximizing choice of volunteering terror effort $r(i)$ is defined implicitly by the following first-order condition (FOC):[5]

$$-w + \beta(i)u'[r(i)] = 0 \Rightarrow u'[r(i)] = \frac{w}{\beta(i)}. \tag{6}$$

Based on the implicit function rule, Eq. (6) yields:

$$r(i) = \rho\left[\frac{w}{\beta(i)}\right],$$

where $\rho'\left[\frac{w}{\beta(i)}\right] = \frac{1}{u''[r(i)]} < 0$, $\frac{\partial r(i)}{\partial w} = \frac{\rho'}{\beta(i)} < 0$, and $\frac{\partial r(i)}{\partial \beta(i)} = -\frac{w\rho'}{[\beta(i)]^2} > 0$. (7)

From Eq. (7), we find that all individuals will reduce their terror effort when they get a higher market wage. Moreover, individual-supplied terror effort increases with individual's radicalization β. Using Eq. (7) in Eq. (2) and suppressing group size *n* from the functional form, we get:

$$T = \theta(a)\tilde{T}[R(w)] \equiv T(a,w) \text{ with } R(w) = \int_{i=0}^{n} r(i)di = \int_{i=0}^{n} \rho\left[\frac{w}{\beta(i)}\right]di. \tag{8}$$

Given Eq. (7), we know that $\rho'[w/\beta(i)] < 0$, so that differentiating Eq. (8) yields:

$$T_a = \theta'(a)\tilde{T}[R(w)] < 0 \text{ and} \tag{9a}$$

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[5] Since individual *i* is of measure zero, we make the atomistic assumption that individual *i* takes aggregate effort R as given when choosing $r(i)$. Also, notice that the second-order condition (SOC) is satisfied because $u'' < 0$. Throughout the paper, we check to ensure that all SOCs are satisfied.

$$T_w = \theta(a)\tilde{T}'\left[R(w)\right]R'(w) < 0\,. \tag{9b}$$

Eq. (9a) indicates that defensive effort reduces terrorism, and thus its damage by decreasing the terrorists' success rate. Additionally, Eq. (9b) shows that aggregate terror effort must fall when each volunteer faces a higher opportunity cost (*w*) of providing terror effort. At a given defense level *a* and at a group size *n*, aggregate terror must fall when the wage rate rises.

The volunteer pool size *n* is endogenous to the model, and we turn next to describe how this pool size is determined.

3.2. Stage 2

A radicalized individual chooses whether to join the terrorist group as an active participant by weighing the net benefit of joining over working in the productive sector. For non-participating individuals working full time in the productive sector, the associated wage income is *w*, which covers consumption of good *x*, such that $x(i) = w$. Even when individuals do not join the terror group, they derive nonexcludable terror-related utility, $\beta(i)B(T)$. Given Eq. (5), the utility of such non-participating individuals is $U^{NP} = w + \beta(i)B(T)$. Now consider the utility of a person who joins the terror group. The joiner has to contend with the government's proactive effort E, where the individual is detected through proactive intelligence or surveillance with probability $p(E)$, where $p'(E) > 0$ and $p''(E) < 0$. If detected, the individual is incarcerated and cannot provide either terrorist effort or labor to produce good *x*, such that Eq. (5) yields utility $\beta(i)B(T)$. If, on the contrary, the individual is not caught, the person's utility is: $w\left[1 - r(i)\right] + \beta(i)\left\{u\left[r(i)\right] + B(T)\right\}$ from working and terrorism. Thus, accounting for the possibility of getting caught, the expected utility of an individual contemplating membership in the terrorist group is:

$$\tilde{U}(i) = p(E)\beta(i)B(T) + \left[1 - p(E)\right]\left\{w\left[1 - r(i)\right] + \beta(i)\left\{u\left[r(i)\right] + B(T)\right\}\right\},$$

$$= \beta(i)B(T) + \left[1 - p(E)\right]\left\{w\left[1 - r(i)\right] + \beta(i)u\left[r(i)\right]\right\}. \tag{10}$$

An individual joins the terror group if and only if $\tilde{U}(i)$ exceeds the certain utility, $U^{NP} = w + \beta(i)B(T)$, of not joining. Using Eq. (10), the condition $\tilde{U} > U^{NP}$ reduces to:

$$\left[1 - p(E)\right]\left\{w\left[1 - r(i)\right] + \beta(i)u\left[r(i)\right]\right\} > w. \tag{11a}$$

Dividing Eq. (11a) through by $(1 - p)\beta(i)$ and using Eq. (6) to substitute $u'\left[r(i)\right]$ for $\left[w / \beta(i)\right]$, we derive (after some rearrangement of terms):

$$\tilde{U} > U^{NP} \Leftrightarrow \psi\left[r(i)\right] > \frac{p(E)}{1 - p(E)}, \text{ where } \psi(r) = \frac{u(r)}{u'(r)} - r, \ \psi'(r) = -\frac{uu''}{(u')^2} > 0. \tag{11b}$$

Consider individuals for whom $\beta \to 0^+$ such that Eq. (6) suggests that $r(i) \to 0$. For these individuals, the left-hand side of Eq. (11a) tends to $(1 - p)w$, where the inequality *cannot* be satisfied for any $p > 0$. For these individuals, $\psi\left[r(i)\right] < p(E) / \left[1 - p(E)\right]$. Similarly, when $\beta(i) \to \beta^S$, where β^S is the upper limit of radicalization, group participation must dominate with $\tilde{U} > U^{NP}$ in which case $\psi\left[r(i)\right] > p(E) / \left[1 - p(E)\right]$. From Eq. (11b), we know that ψ is monotonically increasing in *r*, such that, assuming continuity, a critical level $r = r^c$ (see Figure 1) exists where

$$\psi\left(r^c\right) = \frac{p(E)}{1 - p(E)}. \tag{12a}$$

[Figure 1 near here]

In Figure 1, $\psi(r)$ is measured on the vertical axis and the individual level of contributed terror effort *r* is measured on the horizontal axis. The intersection between the increasing net surplus value, $\psi(r)$, from participating in the terrorist group[6] and the relative odds of being incarcerated indicates the critical terror effort for the extensive margin for joining.

Eq. (12a) and $\psi'(r) > 0$ imply that for all $r > r^c$, $\psi[r(i)] > p(E)/[1 - p(E)]$, where Eq. (11b) is satisfied and an individual joins the terrorist group. Now, given Eq. (6), $\beta(i)$ is an increasing function of $r(i)$. Therefore, $r(i) > r^c \Rightarrow \beta(i) > \beta^c$, where β^c is the radicalization parameter corresponding to the critical terror effort level r^c. Using Eqs, (6) and (7), we have $r^c = \rho(w/\beta^c)$, such that from Eq. (12a) we get:

$$\psi\left[\rho\left(\frac{w}{\beta^c}\right)\right] - \frac{p(E)}{1 - p(E)} = 0 . \qquad (12b)$$

Eq. (12b) implicitly defines:

$$\beta^c = \beta^c(E, w),\ \beta^c_E > 0, \text{ and } \beta^c_w > 0 . \qquad (12c)$$

The critical radicalization parameter, dependent on proactive measures and the wage rate, determines the extensive margin of the terror group separating joiners from non-joiners. Assume that β is distributed in the radicalized population (*M*) with the probability density function $f(\beta)$, distribution function $F(\beta)$, and support $(0, \beta^S]$. Hence, the proportion of *M* joining the terror group is given by $1 - F(\beta^c)$, such that the number of volunteers joining the group is $N = [1 - F(\beta^c)]M$. However, a fraction $p(E)$ of these volunteers is caught and incarcerated, so that based on Eq. (12c) the active number of volunteers in stage 3 is:

$$n = [1 - p(E)]\{1 - F[\beta^c(E, w)]\}M . \qquad (13)$$

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[6] $u/u' = u \times \partial r/\partial u$ denotes the value to volunteering in effort units *r*. When effort is subtracted from this expression, we get the net surplus value from participating in the terror group.

Next, we turn to the supply of *aggregate* terror effort. Because supplying terror effort is conditional on $\beta \geq \beta^c$, the expected volunteer effort (conditional on joining) is:

$$\bar{r} = \int_{\beta^c}^{\beta^S} \frac{\rho\left(\frac{w}{\beta}\right) f(\beta)}{1 - F(\beta^c)} d\beta . \tag{14a}$$

Using Eqs. (13) and (14a), suppressing *M* from the functional form, and noting that $\beta^c = \beta^c(E, w)$, we have that the aggregate terror effort level as:

$$R(E, w) \equiv [1 - p(E)] M \int_{\beta^c}^{\beta^S} \rho\left(\frac{w}{\beta}\right) f(\beta) d\beta . \tag{14b}$$

Because β_E^c and β_w^c are positive while $\rho'(w/\beta)$ is negative, differentiating Eq. (14b) yields:

$$R_E = -\left[\frac{p'R}{(1-p)} + M(1-p) r^c f(\beta^c) \beta_E^c\right] < 0 \text{ and} \tag{15a}$$

$$R_w = M(1-p)\left[-r^c f(\beta^c)\beta_w^c + \int_{\beta^c}^{\beta^S} \frac{\rho'(w/\beta) f(\beta)}{\beta} d\beta\right] < 0 , \tag{15b}$$

indicating essential results on how aggregate terrorism levels fall with enhanced proactive measures and higher wages.

Eq. (15a) indicates that greater proactive enforcement reduces aggregate terror effort for two reasons. First, enhanced proactive measures increase the fraction of terrorists incarcerated [i.e., reduces $(1-p)$ or those not caught]. Second, as *E* raises β^c, the range of β over which radicalized individuals join the terror group shrinks, thereby limiting the volunteer pool. Similarly, Eq. (15b) shows that higher wages reduce aggregate terror effort on two grounds. First, as shown earlier in Eq. (7), each volunteer's effort falls when faced with a higher wage rate, thus capturing the traditional *intensive margin* aspect of greater work effort. Second, through the *extensive margin*, a larger wage rate increases the attractiveness of the productive sector by raising β^c, thereby driving the marginal individuals fully to the productive sector.

Using Eq. (14b) in Eq. (2), we have that the incidence of terrorism is:

$$T = \theta(a)\tilde{T}\left[R(E;w)\right], \tag{16}$$

where defensive effort *a* reduces aggregate terrorism as in Eq. (9a) and proactive effort reduces terrorism through the two effects discussed in the preceding paragraph. Given stages 2 and 3 micro foundations for voluntary participation and the determination of aggregate terrorism, we now turn to how participation in terrorist organizations impact optimal counterterrorism policy in stage 1.

3.3. Stage 1

In stage 1, the government chooses its defensive and proactive measures *a* and *E*, respectively, with constant marginal unit cost for each.[7] The government seeks to minimize the terrorism-related loss function inclusive of counterterrorism expenses. Based on Eq. (16), the loss function equals

$$V(a,E;w) = \theta(a)\tilde{T}\left[R(E;w)\right] + a + E\ . \tag{17}$$

Assuming sufficient diminishing returns in both types of counterterrorism policies to ensure interior solutions, and noting that $\theta'(a)$ and R_E are negative, we have that Eq. (17) yields the FOCs of the loss-minimization problem as:

$$V_a(a,E;w) = 0 \Rightarrow \left|\theta'(a)\right|\tilde{T}\left[R(E;w)\right] = 1 \text{ and} \tag{18a}$$

$$V_E(a,E;w) = 0 \Rightarrow \theta(a)\tilde{T}'\left[R(E;w)\right]\left|R_E(E;w)\right| = 1. \tag{18b}$$

Eqs. (18a) and (18b) determine optimal defensive and proactive levels $a^*(w)$ and

[7] This assumption can be easily relaxed by assuming different but constant marginal costs for the two types of counterterrorism. However, equalizing the marginal costs of these two types of countermeasures allows for a more transparent comparison of their relative efficiency in controlling terror.

$E^*(w)$, respectively, thereby completing the description of the equilibrium of the 3-stage policy game. The right-hand sides of both FOCs reflect marginal enforcement costs of unity. In Eq. (18a), the left-hand side captures that the marginal benefit of defense depends on two considerations. First, at any level of terrorist effort *R*, target protection confers a marginal benefit of $|\theta'(a)|$, where the latter is the technological efficiency with which defense reduces the terrorist success rate. Similarly, for a given level of defensive actions, a higher scale of terrorism implicit in a larger aggregate terror effort also increases the marginal benefit from target hardening. Turning to proactive policy [in Eq. (18b)], we note that its marginal benefit depends critically on $|R_E(E;w)|$, which captures proactive measures working through reducing the *extensive margin* of supply of aggregate terror effort. As later shown, another crucial factor in the choice of optimal proactive policy is the scaling effect of $\tilde{T}'(R)$, reflecting the marginal productivity of terrorist effort. Proactive policy is also affected directly by the level of defense, where greater target protection dampens the need for a proactive response by curbing terrorist success $\theta(a)$.

4. Comparative statics: altering wage or radicalization

4.1. Change in the wage rate w

In the terror supply context, an interesting question concerns how labor market conditions in the productive sector may influence terrorism. As some literature note (Berman and Laitin, 2008, Blomberg, Hess, and Weerapana 2004; Landes 1978; Iannaccone 1992), a higher wage rate raises the opportunity cost of terrorists and is likely to curtail terrorism by raising the opportunity cost of non-terrorist activities. A practical implication of this is that, *ceteris paribus*, countries or subnational regions with good employment opportunities should expect to see less terrorism. However, from a policy perspective, the critical question involves how the wage rate may affect the optimal defensive-proactive counterterrorism mix, while keeping the terrorists' occupational choice decision in mind. That issue has not yet been addressed in the counterterrorism literature. Proposition 1 fills this gap.

Proposition 1: A sufficient, but not necessary, pair of conditions for a wage increase to reduce aggregate terrorism, lower optimal defensive action, and raise optimal proactive measures require that (i) the probability density function of β is uniform and (ii) there are sufficient diminishing returns in the terror production function, such that $\varepsilon(R) = -\frac{R\tilde{T}''(R)}{\tilde{T}'(R)} \geq 1$.

Proof:

See appendix at the end.

Comment:

This is a novel result showing that the optimal mix of proactive and defensive policy depends on the countries' wage rate, which alters the favored policy mix as follows: Recall from the discussion following Eq. (15b) that higher wages reduce aggregate terrorism, *R*, as terrorist operatives' efforts fall both on the extensive and the intensive margins. At a lower *R*, the government's incentive to defend is smaller [see Eq. (18a)], tending to reduce optimal defensive actions. For proactive measures, three terms in Eq. (18b) determine the overall marginal benefit from such actions. First, a lower defense level tends to raise the marginal benefit of proactive measures through a larger terrorist success likelihood, θ. Second, as *R* is reduced, diminishing returns in terror production raises the productivity of aggregate terror effort, $\tilde{T}'$, such that there are greater proactive gains from limiting terrorist efforts. Third, there is an effect that can go either way because among other factors affecting $|R_E|$, there is substitutability between w and E stemming from the fact that both reduce R.[8] The appendix shows that when the density function of β is uniform and when there are sufficiently strong diminishing returns in terror production, such that $\varepsilon(R) = -\frac{R\tilde{T}''(R)}{\tilde{T}'(R)} \geq 1$, the aggregate of all the different wage-induced effects is an increased marginal benefit of proactive measures. Hence, the optimal proactive level rises relative to defensive action with the wage rate w. The opposite is the case for the optimal policy mix as wages fall.

The heightened desirability of proactive to defensive countermeasures in light of wage levels has policy implications on the war on terror both at home and abroad. In the Philippines, Abu Sayyaf, whose base is in Mindanao, has attacked mainly in the south on Jolo and Brasilan islands.[9] At times, attacks have been directed to more prosperous Manila in the north. Given Proposition 1, counterterrorism measures against Abu Sayyaf should rely more heavily on proactive measures in the north relative to favoring defensive measures in the poorer south. In India, terrorist groups are based and attack alternative regions that have different income and wealth levels.[10] Jammu and Kashmir Liberation Front and Tehrik-ul-Mujahedeen stage attacks in Jammu and Kashmir, while Harkat-ul-Mujahideen and Muslim United Liberation Tigers of Assam conduct attacks in Assam. Jammu and Kashmir is relatively richer than Assam so that Proposition 1 suggests that a greater reliance on proactive measures is appropriate for Jammu and Kashmir compared to Assam.

In Nigeria, Boko Haram and Islamic State in Iraq and Syria (ISIS)-West Africa primarily

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[8] There are other effects of w on $|R_E|$ that are complementary, such as the fact that, as higher w pushes up β^c, the marginal volunteer provides more effort, which increases the incentive to raise E to take the marginal volunteer out.
[9] Information on terrorist groups is from Hou, Gaibulloev, and Sandler (2020).
[10] Information here is derived from South Asia Terrorism Portal (2022).

attack the northern region of Nigerian, which is mostly rural and poor so that Proposition 1 favors the relative reliance on defensive countermeasures rather than proactive measures to protect schools and other prized targets of Boko Haram. Pakistan is home to a wide range of terrorist groups (e.g., Lashkar-e-Islam and Jaish-e-Mohammad) so that the mix of counterterrorism policies must be geared to relative wage and prosperity differences among Pakistani regions sustaining terrorism attacks. A counterterrorism mix suited to an affluent city or province may be unwise for a less affluent province. Since both India and Pakistan are home to many different terror groups operating throughout the country and even into neighboring states, a diverse mix of counterterrorism policies are needed in both countries.

4.2. Effect of increased radicalization, not due to backlash

The extent of radicalization of the population in different parts of a nation or the world may influence terrorism and counterterrorism policy, much like wage considerations. As such, changes of radicalization have implications for the general population and policy makers. In our model, increased radicalization may be associated with some variable *k*, whose higher values shift the probability mass in the distribution of β toward larger values of β. That idea can be formalized by assuming that the distribution function is $F(\beta,k)$, where $F_k(\beta,k)<0$, such that if $k^{**}>k^{*}$, $F(\beta,k^{**})<F(\beta,k^{*})$.[11] The latter inequality implies that for the distribution parameterized by k^{**}, the probability mass for higher values of β given by $1-F(\beta,k^{**})$ exceeds the corresponding probability mass $1-F(\beta,k^{*})$ for the distribution parameterized by k^{*}. We note that increased radicalization has no direct effect on Eq. (12b), which determines the critical level β^{c}. Using this fact, suppressing *w* from the functional form, and noting the effect

[11] This suggests that increased radicalization makes the resulting radicalization distribution stochastically dominate the initial distribution to the first order.

of the parameter *k*, we reduce Eq. (14b) to:

$$R(E,k) \equiv \left[1 - p(E)\right] M \int_{\beta^c}^{\beta^S(k)} \rho\left(\frac{w}{\beta}\right) f(\beta,k) d\beta . \tag{19}$$

With Eq. (19), we derive the counterparts of Eqs. (17), (18a), and (18b) where the focus is now on *k* rather than *w*. Proposition 2 summarizes the effects of increased radicalization on the terror level and the optimal policy mix.

Proposition 2: A sufficient pair of conditions for enhanced radicalization $\left[i.e., F_k(\beta,k) < 0\right]$ to augment aggregate terrorism, raise optimal defensive action, and lower optimal proactive measures are that (i) the probability density function is nonincreasing in *k* at the critical level $\beta = \beta^c$, i.e., $f_k(\beta^c,k) \le 0$, and (ii) $\varepsilon(R) = -\dfrac{R\tilde{T}''(R)}{\tilde{T}'(R)} \ge 1$.

Proof:

See appendix at the end.

Comment:

With increased radicalization, one would expect more terrorism and, thus, enhanced levels of counterterrorism effort. Proposition 1 can be used to suggest that the latter conclusion is not necessarily valid because at a *lower* wage there is more terrorism but less proactive measures if there are strong diminishing returns to those actions. Although the details are different, Proposition 2 shows that radicalization may similarly reduce optimal proactive policy in the face of strong diminishing returns.

At a given proactive enforcement level, increased radicalization has three effects. First, as $F(\beta,k)$ falls, the proportion of the radicalized population, $1 - F(\beta^c,k)$, joining the terrorist group rises. Second, the larger number of terrorists tends to pull down the average effort in Eq. (14a) by raising the denominator. Finally, more probability mass at higher levels of β pulls aggregate effort up because more radicalized volunteers provide more effort compared to less radicalized volunteers. We can show that, when $F_k(\beta,k)<0$, the sum of the aforementioned effects is positive, so that radicalization raises aggregate terror effort along with terrorism at given policy levels.

Turning to the effect of radicalization on optimal proactive measures, we focus on the product $\tilde{T}'\left[R(E;k)\right]\left|R_E(E;k)\right|$, which reflects the marginal benefit from proactive responses at zero defense level. As radicalization pushes up *R*, diminishing returns in terror production pulls $\tilde{T}'\left[R(E;k)\right]$ down. In Eq. (15a), $\left|R_E\right|$ is affected by two terms. The first term inside the bracket of Eq. (15a) rises because greater radicalization augments *R* and thereby increases the yield from an enhanced probability of detection. The second term works through how proactive E influences the critical β^c. If an increase in *k* reduces $f(\beta,k)$, then the probability mass around the marginal β^c is smaller, meaning there is less proactive gains from shrinking the volunteer range $\left[\beta^c,\beta^S\right]$. Even if the first term dominates such that $\left|R_E\right|$ rises, we can show that the overall effect, $\tilde{T}'\left[R(E;k)\right]\left|R_E(E;k)\right|$, must decline if $\varepsilon(R)\geq 1$ and $f_k(\beta^c,k)\leq 0$.[12] With lower proactive measures and higher radicalization, aggregate terror effort $R(E,k)$ must increase, which by Eq. (18a) implies that optimal defensive countermeasures must increase. Thus, our representation of enhanced radicalization leads to defensive measures being favored

[12] Consider a uniform distribution with support $\left[\beta^0>0,\beta^S(k)\right]$, where $\beta^{S\prime}(k)>0$ implies that greater radicalization shifts the probability mass to the right. The corresponding probability density function is $f(\beta,k)=1/\left[\beta^S(k)-\beta^0\right]$, so that $f_k(\beta^c,k)=-\beta^{S\prime}(k)/\left[\beta^S(k)-\beta^0\right]^2<0$.

over proactive ones, which is a novel result.

Proposition 2 implies that the optimal mix of counterterrorism policy needs to be tailored to how radicalization differs at alternative locations in a country or among countries hosting terrorist groups that pose a threat. Countries like India and Pakistan contain multiple terror groups located in different provinces that reflect varying degrees of radicalization among terrorist adherents so that counterterrorism policy mixes must account for such radicalization differences. This is also true when a terrorist group attacks at home and in a foreign country using domestic and foreign volunteer pools, respectively, which differ in their levels of radicalization.

5. Greater radicalization stemming from proaction-induced backlash

Early studies on retaliatory raids, a form of proactive countermeasure against terrorists, found that such actions may anger terrorists, thereby resulting in more terrorist attacks. For instance, Enders and Sandler (1993) display how the US retaliatory bombing of Libya on the morning of April 15, 1986, for Libya's alleged involvement in the terrorist bombing of the La Belle discotheque in West Berlin resulted in a wave of terrorist attacks against US and UK interests. The UK attacks were motivated by its logistical support for the US bombing raid on Libyan targets. In a similar study on Israeli retaliatory attacks, Brophy-Baermann and Conybeare (1994) also find that Israeli retaliatory responses resulted in enhanced terrorist attacks as a protest. In general, proactive measures can energize and radicalize a population leading to a "backlash" campaign of attacks (Bloom 2005). Often, this backlash leads to terrorist groups recruiting new members (Rosendorff and Sandler 2004). A wide range of proactive measures – e.g., killing the group's leader, drone attacks on key terrorist operatives (e.g., a master bomb maker), or destroying terrorist training camps – can unleash backlash attacks as group members become more committed or radicalized. For instance, in August 2022, Israel launched attacks in Gaza against the Palestinian Islamic Jihad (PIJ), killing and arresting senior PIJ officials and leading to three days of violence (BBC News 2022). Given collateral deaths among some non-terrorist Palestinians, such proactive operations often radicalize and entice young Palestinians to join the terrorist group.

In this section, we reconsider the government's stage 1 counterterrorism decisions when proaction is the motivator of radicalization. Earlier, Proposition 2 considered radicalization of a population in terms of the shift in the distribution of β without tying distributional changes to counterterrorism actions. A simple and easily interpretable way to model backlash is to assume that the radicalization parameter k equals:

$$k = bE + \bar{k}\ , \tag{20}$$

where b and $\bar{k}$ are positive constants, such that an increase in proaction induces backlash through increased radicalization at a constant rate of b. When b is zero, the model reverts to the case considered earlier where there is no proaction-induced backlash. Substituting Eq. (20) in Eq. (19), we get:

$$R \equiv R\left(E, k = bE + \bar{k}\right). \tag{21}$$

Given Eq. (21), the government's loss function in Eq. (17) becomes:

$$V = \theta(a)\tilde{T}\left[R\left(E, bE + \bar{k}\right)\right] + a + E\ , \tag{22}$$

where in stage 1 the government anticipates that proactive measures affect radicalization and accordingly adjusts its choice of optimal counterterrorism policy as indicated below.

The optimal defense rule remains qualitatively similar to Eq. (18a), but the optimal proactive response changes to:[13]

$$\theta(a)\tilde{T}'(R)\left|R_E\left(E,bE+\bar{k}\right)\right|=1+b\theta(a)\tilde{T}'(R)R_k\left(E,bE+\bar{k}\right). \tag{23}$$

The second term on the right-hand side of Eq. (23) is the marginal backlash cost of proaction. An increase in E enhances k to the scale of b, where greater k raises aggregate terror effort R and hence terrorism $\tilde{T}(R)$. The government anticipates this backlash cost and in equilibrium holds back its optimal proactive response compared to the no-backlash $(b=0)$ case.

Alternatively, consider an increase in $\bar{k}$, which allows for a novel comparison of optimal proactive policy in the presence of backlash in two regions (say A and B) with inherently different levels of radicalization. If region A is more radicalized than B, then we have $\bar{k}^A > \bar{k}^B$, implying that $k^A > k^B$ via Eq. (20) for the same proactive response and common backlash parameter b. Greater backlash parameterized by an increase in $\bar{k}$ affects both the marginal benefit and the marginal cost of proactive measures through several channels. Consider the marginal cost on the right-hand side of Eq. (23). As $\bar{k}$ augments R, it tends to limit $\tilde{T}'(R)$ because of diminishing returns in the terror production function. In addition, if there is diminishing returns to radicalization (i.e., $R_{kk} < 0$), then R_k also falls. Thus, *ceteris paribus*, an increase in $\bar{k}$ raises optimal proactive measures by reducing *backlash marginal cost*. However, the marginal benefit on the left-hand side of Eq. (23) changes too, so we must dig deeper to explore how the optimal proactive response is affected by an increase in $\bar{k}$. Proposition 3 summarizes the findings.

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[13] We note that $R_E(E,k) \equiv \left[\frac{\partial R(E,k)}{\partial E}\right]_{|k}$ is the marginal effect of E on R, holding the second argument of $R(E,k)$ constant. Similarly, $R_k(E,k) \equiv \left[\frac{\partial R(E,k)}{\partial k}\right]_{|E}$ is the marginal effect of k on R, keeping the first argument of $R(E,k)$ constant.

Proposition 3**:** Diminishing returns to radicalization ensure that an exogenous rise in radicalization is more likely to augment the implementation of proactive measures in the presence of backlash terrorism compared to a no-backlash scenario. Optimal defense increases with greater radicalization if aggregate terror effort rises.

Proof: In the appendix.

Comment:

The direct effect of an increase in $\bar{k}$ is to lift aggregate terror effort. However, there is an indirect effect operating through the $\bar{k}$ -induced change in the optimal proactive response, which could be negative or positive. If proaction falls, then the indirect effect complements the direct effect and expands terror effort further. If, however, optimal proaction rises, then the direct effect must dominate the indirect effect for aggregate terror effort to increase. When aggregate terror effort rises, the marginal benefit of defensive actions rises, calling for greater hardening of targets. We can show that despite a greater optimal defensive level, enhanced radicalization will increase the incidence of terrorism.

Turning to the proactive response, we consider the effect of $\bar{k}$ on the marginal effect of proaction. We can show that:

$$\frac{V_{E\bar{k}}}{\theta(a)} = \left(\tilde{T}''R_E - \frac{\tilde{T}'p'}{1-p}\right)R_k - \tilde{T}'M\left(1-p\right)r^c\beta_E^c f_k\left(\beta^c, k\right) + b\left[\left(R_k\right)^2\tilde{T}'' + \tilde{T}'R_{kk}\right]. \qquad (24)$$

The first two terms on the right-hand side of Eq. (24) can be shown to be positive if $\varepsilon \geq 1$ and $f_k\left(\beta^c, k\right) \leq 0$, which drive Proposition 2. However, because $b > 0$, the third term pulls in an opposing direction, depending on two types of diminishing returns alluded to in the paragraph following Eq. (23). While $\tilde{T}'' < 0$ represents diminishing returns in terror production, diminishing returns to radicalization (i.e., $R_{kk} < 0$) occurs when $F_{kk}(\beta, k) > 0$.[14] The latter condition implies that, even though increased radicalization shifts cumulative density *F* downward, it does so at a diminishing rate. If the two types of diminishing returns are sufficiently strong, the backlash marginal costs may fall sufficiently such that optimal proactive policy may rise when $\bar{k}$ increases. Thus, despite the sufficiency conditions of Proposition 2 being satisfied, we may see an increase in optimal proactive effort (in the presence of backlash) when there is an increase in the radicalization of the population. The possible rise in proactive measures in the presence of backlash is surprising and stems from diminishing returns of terror production and radicalization. When comparing two regions or countries with different $\bar{k}$, the more radicalized region requires a larger optimal proactive response if the last term of Eq. (24) is dominant and also sufficiently large to overcome the cross-substitutability influence of a higher optimal defense provision. In practice, this says that, when a targeted country is determining its proactive and defensive measures, the recruitment possibilities for the indigenous terrorist groups comes into play. What is the best mix of policies for an al-Qaida affiliate like al-Qaida in the Islamic Maghreb in Mali may be quite different than the ideal mix of policies for an al-Qaida affiliate like al-Shabaab in Somalia owing to differences in radicalization among its potential recruits. In the post-9/11 era, terror networks like al-Qaida or Islamic State draw their operatives from geographically diverse pools of terrorists that possess differing extent of radicalization, thereby influencing the optimizing mix of counterterrorism actions.

6. Concluding remarks

The paper examines the optimal mix between proactive and defensive counterterrorism actions

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[14] It can be checked that for the uniform distribution described in footnote 12, if $\beta^S(k)$ rises with *k* but at a non-increasing rate, i.e., $\beta^{S\prime}(k) > 0$ and $\beta^{S\prime\prime}(k) \leq 0$, then $F_{kk} > 0$.

from a novel vantage by accounting for policy-induced reactions of potential terrorist recruits. Unlike the extant game-theoretic literature, the model here permits a targeted government's antiterrorism campaign mix to influence the terrorist group's size by changing the extensive margin at which the marginal individual decides to join the group. Moreover, the government countermeasures affect the time allocation of joiners between working in the private sector and engaging in terrorist attacks (i.e., the intensive margin). In the first stage of the game, the government looks ahead and adjusts its mix of proactive and defensive actions to account for the terrorist group's extensive and intensive margins. Under various scenarios, we show how increased wages or enhanced radicalization of the population impacts the optimal counterterrorism policy mix between hardening targets and launching an offensive against the terrorists. We identify sufficient conditions whereby wage increases not only reduce aggregate terrorism but also promote proactive responses over defensive ones. Also, in the absence of backlash, we find that radicalization favors a greater reliance on defensive measures. In addition, we investigate how backlash-induced radicalization can somewhat tip the balance of countermeasures in the direction of greater proactive responses. Because wages and radicalization differ among subregions in a country, our analysis indicates that counterterrorism mixes must be tailored to each subregion. A similar conclusion would apply to how a targeted country determines its counterterrorism mix in foreign countries hosting terrorist groups posing threats to the country's interests at home or abroad.

By tying the potential terrorist decision to the government's counterterrorism mix, we endogenize the potential terrorist actions beyond what is found in the literature. In so doing, our model and analysis offers a firmer micro-foundation to the actions of the population in joining and then supplying their labor to the terrorist group.

Appendix[15]

(A) *Proof of Proposition 1:*

Differentiating Eqs. (18a) and (18b), solving via Cramer's rule, and using the signs of the second-order partials of the loss function, we can show that the sufficient condition for $da^*/dw < 0$ and $dE^*/dw > 0$ is that:

$$V_{Ew}(a,E,w) = \theta(a)\left\{\tilde{T}'\left[R(E,w)\right]R_{Ew}(E,w) + R_E(E,w)\tilde{T}''(\cdot)R_w(E,w)\right\} < 0. \tag{A1}$$

For a uniform distribution for $f(\beta)$, Eq. (A1) reduces to:

$$V_{Ew}(a,E,w) = \theta(a)\left[R_w(\cdot)\left(\tilde{T}''R_E - \frac{\tilde{T}'p'}{1-p}\right) - \tilde{T}'M(1-p)r^c f\left(\beta^c\right)\beta^c_{Ew}\right], \tag{A2}$$

where Eqs. (12b) and (12c) implies that $\beta^c_{Ew} > 0$. Because $R_w < 0$, we conclude from Eq. (A2) that if $\tilde{T}''R_E - \frac{\tilde{T}'p'}{1-p} > 0$, then $V_{Ew}(a,E,w) < 0$. Using the expression for R_E from Eq. (15a), we can show that:

$$\tilde{T}''R_E - \frac{\tilde{T}'p'}{1-p} > 0 \quad \text{if} \quad \varepsilon(R) \geq 1, \tag{A3}$$

where $\varepsilon(R) = -R\tilde{T}''(R)/\tilde{T}'(R) > 0$ is a measure of the degree of concavity of the terror production function. Eqs. (A1) through (A3) establish that $\frac{da^*}{dw} < 0$ and $\frac{dE^*}{dw} > 0$ when $\varepsilon(R) \geq 1$. Finally, recall from Eq. (14b) that $R = R(E,w)$, such that

$$\frac{dR}{dw} = R_E\frac{dE^*}{dw} + R_w < 0, \tag{A4}$$

because R_E and R_w are negative and $dE^*/dw > 0$. Thus, aggregate terror effort must fall when

[15] This appendix provides the outlines of the proofs of the three propositions, with further details in the accompanying online appendix.

the wage rate w rises.

Because optimal counterterror defense falls, it is not clear that the equilibrium level of terror falls without more analysis, which follows. From Eq. (18a), we note that, at an interior policy optimum, there is a one-to-one relationship between optimal defensive action a^* and the equilibrium aggregate terror effort R^*:

$$\theta'\left(a^*\right)\tilde{T}\left(R^*\right)+1=0 \Rightarrow \frac{dR^*}{da^*}=-\frac{\theta''\left(a^*\right)\tilde{T}\left(R^*\right)}{\theta'\left(a^*\right)\tilde{T}'\left(R^*\right)}>0\,, \tag{A5}$$

such that R^* can be expressed as $R^*\left(a^*\right)$. Given Eq. (2), we can write aggregate terror as $T^*\left(a^*\right)\equiv\theta\left(a^*\right)\tilde{T}\left[R^*\left(a^*\right)\right]$. Differentiating $T^*\left(a^*\right)$ and using Eq. (A5), we get:

$$\frac{dT^*}{da^*}>0 \Leftrightarrow \theta\left(a^*\right)\theta''\left(a^*\right)-\left[\theta'\left(a^*\right)\right]^2>0\,, \tag{A6}$$

where the inequality on the right-hand side of Eq. (A6) is satisfied when $\theta''\left(a\right)$ is sufficiently large to ensure that the own effect V_{aa} exceeds the cross effect V_{aE}. Thus, $dT^*/dw=\left(dT^*/da^*\right)\left(da^*/dw\right)<0$, when $da^*/dw<0$ so that aggregate terror must fall in response to higher w when the sufficient conditions in Proposition 1 hold.

(B) *Proof of Proposition 2:*

Given Eq. (19), Eq. (17) can be written as:

$$V\left(a,E;k\right)=\theta\left(a\right)\tilde{T}\left[R\left(E;k\right)\right]+a+E\,. \tag{A7}$$

Since the density $f\left(\beta,k\right)$ is the derivative of $F\left(\beta,k\right)$, Eq. (19) can be expressed as:

$$R\left(E,k\right)\equiv\left[1-p\left(E\right)\right]M\int_{\beta^c}^{\beta^S(k)}\rho\left(\frac{w}{\beta}\right)dF\left(\beta,k\right). \tag{A8}$$

It can be shown that

$$R_k\left(E,k\right)=\left[1-p\left(E\right)\right]M\left[-\rho\left(\frac{w}{\beta^c}\right)F_k\left(\beta^c,k\right)+\int_{\beta^c}^{\beta^S(k)}\left(\frac{w\rho'}{\beta^2}\right)F_k\left(\beta,k\right)d\beta\right]>0, \qquad \text{(A9)}$$

because $\rho'<0$ and $F_k\left(\beta,k\right)<0$ for all β, where $\beta<\beta^S\left(k\right)$. Given Eqs. (A7) and (A9), we have that $V_{ak}=\theta'\left(a\right)\tilde{T}'\left[R\left(E;k\right)\right]R_k<0$. Differentiating the FOCs and solving after using the known signs of the second-order partials of the loss function, we have that $\frac{da^*}{dk}>0$ and $\frac{dE^*}{dk}<0$ if $V_{Ek}>0$. Using Eqs. (A7) and (15a), and some routine algebra, we get:

$$V_{Ek}\left(a,E,k\right)=\theta\left(a\right)\left[R_k\left(E,k\right)\left(\tilde{T}''R_E-\frac{\tilde{T}'p'}{1-p}\right)-\tilde{T}'M\left(1-p\right)r^c\beta_E^c f_k\left(\beta^c,k\right)\right]. \qquad \text{(A10)}$$

Recall from Eq. (A3) that $\tilde{T}''R_E-\frac{\tilde{T}'p'}{1-p}>0$ if $\varepsilon\left(R\right)\geq 1$. Thus, when $\varepsilon\left(R\right)\geq 1$ and $f_k\left(\beta^c,k\right)\leq 0$, Eqs. (A9) and (A10) imply that $V_{Ek}>0$, such that $\frac{da^*}{dk}>0$ and $\frac{dE^*}{dk}<0$.

(C) *Proof of Proposition 3:*

Consider Eq. (A7) when $k=bE+\bar{k}$, so that

$$V\left(a,E;\bar{k}\right)\equiv\theta\left(a\right)\tilde{T}\left[R\left(E,bE+\bar{k}\right)\right]+a+E. \qquad \text{(A11)}$$

Differentiating Eq. (A11) with respect to *E* yields Eq. (23) of the paper. Furthermore, we have:

$$V_{E\bar{k}}\left(a,E;\bar{k}\right)=\theta\left(a\right)\left\{\tilde{T}'\left(R\right)R_{Ek}+\tilde{T}''R_E R_k+b\left[\tilde{T}'\left(R\right)R_{kk}+\tilde{T}''\left(R\right)\left(R_k\right)^2\right]\right\}. \qquad \text{(A12)}$$

Based on Eqs. (A7) and Eq. (15a), we get an expression for R_{Ek}, which when substituted in Eq. (A12) gives Eq. (24) of the paper.

Differentiating Eq. (A9) and noting that $\rho'<0$, we get:

$$R_{kk}\left(E,k\right)=\left(1-p\right)M\left[-\rho\left(\frac{w}{\beta^{c}}\right)F_{kk}\left(\beta^{c},k\right)+\int_{\beta^{c}}^{\beta^{S}(k)}\left(\frac{w\rho'}{\beta^{2}}\right)F_{kk}\left(\beta,k\right)d\beta\right]<0\,, \tag{A13}$$

if $F_{kk}\left(\beta,k\right)>0$. Thus, the third term on the right-hand side of Eq. (A12) is necessarily negative when $b>0$, making it possible for proaction to rise with enhanced radicalization even if the sufficiency conditions of Proposition 2 are satisfied.

Finally, notice that $R=R\left(E,bE+\bar{k}\right)$, such that

$$\frac{dR}{d\bar{k}}=\left(R_{E}+bR_{k}\right)\frac{dE}{d\bar{k}}+R_{k}\,. \tag{A14}$$

The proaction FOC yields $R_{E}+bR_{k}=-1/\left[\theta\tilde{T}'\left(R\right)\right]$, which when used in Eq. (A14) gives:

$$\frac{dR}{d\bar{k}}>0 \text{ if } R_{k}>\left[\frac{1}{\theta\tilde{T}'\left(R\right)}\right]\left(\frac{dE}{d\bar{k}}\right). \tag{A15}$$

Since $R_{k}>0$, Eq. (A15) is always satisfied for $\frac{dE}{d\bar{k}}\leq 0$. However, when $\frac{dE}{d\bar{k}}>0$, we need the direct effect R_{k} to exceed the indirect effect, given on the right-hand side of Eq. (A15) for R, to rise with an increase in $\bar{k}$. If R rises, then Eqs. (18a) and (A5) imply that optimal defensive action must rise. In turn, using Eq. (A6), we know that aggregate terrorism must rise.

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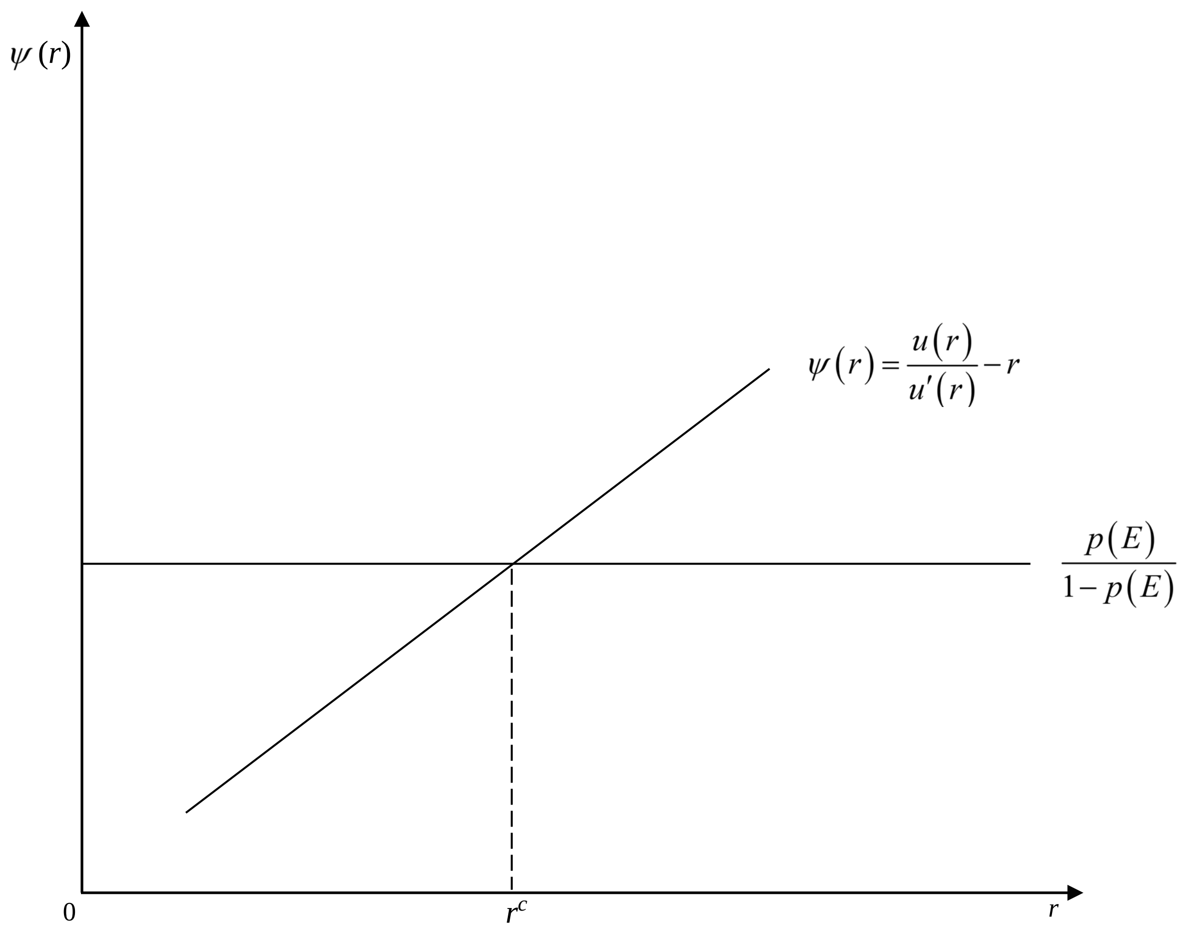

Figure 1. Critical terror effort corresponding to the extensive terror margin

Online Appendix for: Voluntary participation in a terror group and counterterrorism policy

This online appendix contains details of the derivations of Eqs. (15a) and (15b), and the second-order conditions (SOCs) for optimal policy. Finally, the appendix provides supporting details for the proofs of Proposition 1-3.

1. Equations (15a) and (15b) of the paper:

Differentiating Eq. (14b) with respect to *E*, recalling from Eq. (12c) that $\beta^c = \beta^c\left(E, w\right)$, and applying the Leibniz integral rule, we get:

$$\frac{R_E}{M} = -(1-p)\rho\left(\frac{w}{\beta^c}\right) f\left(\beta^c\right)\beta_E^c - p'\left(E\right)\int_{\beta^c}^{\beta^S}\rho\left(\frac{w}{\beta}\right) f\left(\beta\right) d\beta \,. \tag{B1}$$

From Eq. (14b), we note that $\int_{\beta^c}^{\beta^S}\rho\left(\frac{w}{\beta}\right) f\left(\beta\right) d\beta$ equals $R/\left[(1-p)M\right]$. Substituting the latter expression for $\int_{\beta^c}^{\beta^S}\rho\left(\frac{w}{\beta}\right) f\left(\beta\right) d\beta$ in Eq. (B1) and using $r^c = \rho\left(w/\beta^c\right)$, we reorganize terms to get Eq. (15a). Similarly, differentiating Eq. (14b) with respect to w yields:

$$\frac{R_w}{(1-p)M} = -\rho\left(w/\beta^c\right) f\left(\beta^c\right)\beta_w^c + \int_{\beta^c}^{\beta^S}\frac{\rho'\left(w/\beta\right) f\left(\beta\right)}{\beta} d\beta \,. \tag{B2}$$

Reorganizing terms and using $r^c = \rho\left(w/\beta^c\right)$ in Eq. (B2), we obtain Eq. (15b).

2. *SOCs for Optimal Policy:*

Given $\theta' < 0$, $\theta'' > 0$, $R_E < 0$, and $\tilde{T}' > 0$, second-order differentiation of Eq. (17) yields:

$$V_{aa} = \tilde{T}\theta''\left(a\right) > 0,\ \ V_{aE} = \tilde{T}'\theta' R_E > 0, \text{ and } V_{EE} = \theta\left(a\right)\left[\tilde{T}' R_{EE} + \left(R_E\right)^2 \tilde{T}''\right]. \tag{B3}$$

Differentiating Eq. (15a) with respect to *E* gives:

$$R_{EE} = -\frac{Rp''}{1-p} - \left[\frac{p'R_E}{1-p} + \frac{(p')^2 R}{(1-p)^2}\right] - M\frac{\partial Z}{\partial E}, \text{ where } Z = (1-p)r^c f\left(\beta^c\right)\beta_E^c. \quad \text{(B4)}$$

Multiplying Eq. (15a) through by $p'/(1-p)$ and reorganizing terms, Eq. (B4) reduces to:

$$R_{EE} = -\frac{Rp''}{1-p} + Mp'r^c f\left(\beta^c\right)\beta_E^c - M\frac{\partial Z}{\partial E}. \quad \text{(B5)}$$

Using Eqs. (6), (7), (11b), (12b), and (12c), we can show that

$$\beta_E^c = Yp'(E) > 0, \text{ where } Y = \frac{u'\left[r^c(E)\right]\beta^c(E,w)}{(1-p)^2 u\left[r^c(E)\right]} > 0. \quad \text{(B6)}$$

Eqs. (B4) and (B6) imply that $\partial Z/\partial E$ includes a derivative of β_E^c, which, in turn, involves $Yp''(E)$ among other terms. Given that negative signs precede p'' in both the first and the last terms on the right-hand side of Eq. (B5) where $p'' < 0$, sufficient diminishing returns in proaction ensure that $R_{EE} > 0$. Additionally, sufficient diminishing returns in proaction imply that $\tilde{T}'R_{EE} + (R_E)^2\tilde{T}'' > 0$, such that $V_{EE} > 0$ in Eq. (B3). Finally, we assume that diminishing returns to defense represented by $\theta'' > 0$ and in proaction by $p'' < 0$ are sufficiently enough to ensure that $V_{aa}V_{EE} - (V_{aE})^2 > 0$.

3. *Supporting Details of Proof of Proposition 1*

Differentiating Eqs. (18a) and (18b) and solving via Cramer's rule, we have:

$$\frac{da^*}{dw} = \frac{V_{Ew}V_{aE} - V_{aw}V_{EE}}{D}, \quad \text{(B7)}$$

$$\frac{dE^*}{dw} = \frac{V_{aw}V_{aE} - V_{Ew}V_{aa}}{D}, \quad \text{(B8)}$$

where $V_{aa} > 0$, $V_{EE} > 0$, and $D = V_{aa}V_{EE} - (V_{aE})^2 > 0$ from the SOC of the loss minimization problem, and V_{aE} is also positive [see Eq. (B3)]. Using Eq. (17), we obtain:

$V_{aw} = \theta'(a)\tilde{T}'(R)R_w > 0$, because $\theta' < 0$, $R_w < 0$, and $\tilde{T}' > 0$. Therefore, Eqs. (B7) and (B8) show that a sufficient condition for $\frac{da^*}{dw} < 0$ and $\frac{dE^*}{dw} > 0$ is that $V_{Ew} < 0$. Given Eq. (17), we get:

$$V_E(a,E,w) = \theta(a)\tilde{T}'\left[R(E,w)\right]R_E(E,w) + 1. \tag{B9}$$

Differentiating Eq. (B9), we get Eq. (A1) of the in-text appendix. Noting that Eq. (12a) defines r^c as a function of E only and assuming a uniform distribution for β, we can partially differentiate Eq. (15a) with respect to the wage rate to obtain:

$$R_{Ew} = -\frac{p'}{1-p}R_w - M(1-p)r^c f\left(\beta^c\right)\beta^c_{Ew}. \tag{B10}$$

Substituting the right-hand side expression in Eq. (B10) for R_{Ew} in Eq. (A1) and reorganizing terms, we have Eq. (A2). As claimed following in-text Eq. (A2), β^c_{Ew} is positive because using Eq. (B6) yields $\beta^c_{Ew} = \frac{p'(E)u'\left[r^c(E)\right]\beta^c_w(E,w)}{(1-p)^2 u\left[r^c(E)\right]}$. This last expression is strictly positive since $\beta^c_w(E,w) = \beta^c / w > 0$ from Eqs. (12b) and (12c). Using the expression for R_E from Eq. (15a), and noting that $\tilde{T}'' < 0$, we have:

$$\tilde{T}''R_E - \frac{\tilde{T}'p'}{1-p} = -\frac{p'}{(1-p)}\left(\tilde{T}' + R\tilde{T}''\right) - \tilde{T}''M(1-p)r^c f\left(\beta^c\right)\beta^c_E > 0$$

$$\text{if } \tilde{T}' + R\tilde{T}'' \leq 0 \Leftrightarrow \tilde{T}'(.)\left[1-\varepsilon(R)\right] \leq 0 \Leftrightarrow \varepsilon(R) \geq 1, \tag{B11}$$

as claimed in Eq. (A3).

Differentiating $T^* = \theta\left(a^*\right)\tilde{T}\left[R^*\left(a^*\right)\right]$, we have:

$$\frac{dT^*}{da^*} = \tilde{T}\left(R^*\right)\theta'\left(a^*\right) + \theta\left(a^*\right)\tilde{T}'\left(R^*\right)\frac{dR^*}{da^*}$$

$$= \tilde{T}\left(R^*\right)\left[\theta'\left(a^*\right) - \frac{\theta\left(a^*\right)\theta''\left(a^*\right)}{\theta'\left(a^*\right)}\right] > 0 \Leftrightarrow \theta\left(a^*\right)\theta''\left(a^*\right) - \left[\theta'\left(a^*\right)\right]^2 > 0, \tag{B12}$$

as claimed in in-text appendix Eq. (A6).

Finally, recall from Eq. (B3) that $V_{aa} = \tilde{T}\left(R\right)\theta''\left(a\right)$ and $V_{aE} = \theta'\left(a\right)\tilde{T}'\left(R\right)R_E$. We assume that diminishing returns in defense are sufficiently strong to ensure that $V_{aa} > V_{aE}$. Evaluating these second-order partials at the policy optimum defined by Eqs. (18a) and (18b), we apply $\tilde{T}\left(R^*\right) = -1/\theta'\left(a^*\right)$ and $\tilde{T}'\left(R^*\right)R_E = -1/\theta\left(a^*\right)$, so that

$$V_{aa} - V_{aE} = \tilde{T}\left(R^*\right)\theta''\left(a^*\right) - \theta'\left(a^*\right)\tilde{T}'\left(R^*\right)R_E = -\frac{\theta''\left(a^*\right)}{\theta'\left(a^*\right)} + \frac{\theta'\left(a^*\right)}{\theta\left(a^*\right)} > 0$$

$$\Leftrightarrow \theta\left(a^*\right)\theta''\left(a^*\right) - \left[\theta'\left(a^*\right)\right]^2 > 0, \text{ because } \theta'\left(a^*\right) < 0. \tag{B13}$$

Eq. (B13) supports the claim made in the two lines following Eq. (A6) in paper's appendix.

4. *Supporting Details of Proof of Proposition 2:*

Differentiating Eqs. (18a) and (18b) after replacing *w* by *k*, and solving, we have:

$$\frac{da^*}{dk} = \frac{V_{Ek}V_{aE} - V_{ak}V_{EE}}{D} \text{ and } \frac{dE^*}{dk} = \frac{V_{ak}V_{aE} - V_{Ek}V_{aa}}{D}. \tag{B14}$$

Second-order differentiation of Eq. (A7) yields:

$$V_{ak} = \theta'\left(a\right)\tilde{T}'\left[R\left(E;k\right)\right]R_k \text{ and} \tag{B15}$$

$$\frac{V_{Ek}\left(a,E,k\right)}{\theta\left(a\right)} = \tilde{T}'\left[R\left(E,k\right)\right]R_{Ek}\left(E,k\right) + R_E\left(E,k\right)\tilde{T}''\left(.\right)R_k\left(E,k\right). \tag{B16}$$

The integral in Eq. (A8) can be integrated by parts to obtain:

$$\int_{\beta^c}^{\beta^S(k)} \rho\left(\frac{w}{\beta}\right) dF(\beta,k) = \left[\rho\left(\frac{w}{\beta}\right)F(\beta,k)\right]_{\beta^c}^{\beta^S(k)} + \int_{\beta^c}^{\beta^S(k)} \left(\frac{w\rho'}{\beta^2}\right)F(\beta,k)\,d\beta \,. \tag{B17}$$

Using Eq. (B17) in (A8) and noting that $F\left[\beta^S(k),k\right] \equiv 1$, we get:

$$\frac{R(E,k)}{\left[1-p(E)\right]M} = \rho\left(\frac{w}{\beta^S}\right) - \rho\left(\frac{w}{\beta^c}\right)F\left(\beta^c,k\right) + \int_{\beta^c}^{\beta^S(k)} \left(\frac{w\rho'}{\beta^2}\right)F(\beta,k)\,d\beta \,. \tag{B18}$$

Partially differentiating Eq. (B18) with respect to *k* and reorganizing terms, we have $R_k > 0$ in Eq. (A9). Based on $R_k > 0$, Eq. (B15) establishes that $V_{ak} < 0$. Furthermore, Eq. (B3) and the SOCs imply that V_{EE}, V_{aa}, and V_{aE} are all positive. Therefore, as claimed in the text following Eq. (A9), Eq. (B14) establishes that if $V_{Ek} > 0$, so that $\frac{da^*}{dk} > 0$ and $\frac{dE^*}{dk} < 0$.

Turning to the expression for V_{Ek} given in Eq. (B16), we differentiate Eq. (15a) with respect to *k* to derive:

$$R_{Ek} = -\left[\frac{p'R_k}{1-p} + M\left(1-p\right)r^c\beta_E^c f_k\left(\beta^c,k\right)\right]. \tag{B19}$$

Substituting the right-hand side expression in Eq. (B19) for R_{Ek} in Eq. (B16) and reorganizing terms, we have Eq. (A10).

5. *Supporting Details of Proof of Proposition 3:*

Partially differentiating Eq. (A11) with respect to *E* yields:

$$V_E\left(a,E;\bar{k}\right) = \theta(a)\tilde{T}'(R)\left[R_E\left(E,bE+\bar{k}\right) + bR_k\left(E,bE+\bar{k}\right)\right] + 1 \,. \tag{B20}$$

Using Eq. (B20), we get the FOC for optimal proactive measures as Eq. (23) of the paper. The defense FOC is similar to Eq. (18b).

Differentiating the FOCs and solving via Cramer's rule, we have:

$$\frac{da^*}{d\bar{k}} = \frac{V_{E\bar{k}}V_{aE} - V_{a\bar{k}}V_{EE}}{D} \text{ and } \frac{dE^*}{d\bar{k}} = \frac{V_{a\bar{k}}V_{aE} - V_{E\bar{k}}V_{aa}}{D}. \tag{B21}$$

Second-order differentiation of Eq. (A11) yields $V_{a\bar{k}} = \theta'(a)\tilde{T}'(R)R_k < 0$ since $R_k > 0$.

Similarly, we have $V_{aE} = \theta'(a)\tilde{T}'(R)(R_E + bR_k) > 0$ because $\theta'(a) < 0$ and Eq. (B20) ensures that an interior solution for proactive measures requires that $R_E + bR_k < 0$. Differentiating Eq. (B20) yields:

$$\frac{V_{E\bar{k}}(a, E; \bar{k})}{\theta(a)} = \tilde{T}'(R)\left[\frac{\partial R_E}{\partial \bar{k}} + b\frac{\partial R_k}{\partial \bar{k}}\right] + (R_E + bR_k)\tilde{T}''R_k. \tag{B22}$$

Notice that $\frac{\partial R_E(E, bE + \bar{k})}{\partial \bar{k}}$ is the same as R_{Ek} [in Eq. (B19)] because $\partial k / \partial \bar{k} = 1$ and $R_E(E,k)$ is the same as defined in Eq. (15a). Similarly, $\frac{\partial R_k(E, bE + \bar{k})}{\partial \bar{k}} = R_{kk}(E,k)$. Based on these facts and some reorganization, Eq. (B22) reduces to Eq. (A12). Using Eq. (B19) to substitute the expression for R_{Ek} in Eq. (A12) and reorganizing terms, we obtain Eq. (24) of the paper.