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Liquidity and Investment in General Equilibrium*

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Abstract

This paper studies how trading frictions in financial markets impact firms' investment and dividend policies, and explores its aggregate consequences. When equity shares trade in frictional asset markets, the firm's problem is time-inconsistent, and it is as if it faces quasi-hyperbolic discounting. The transmission of trading frictions to the real economy crucially depends on the firms' ability to commit. In a calibrated economy without commitment, larger trading frictions imply lower capital. In contrast, if firms can commit, trading frictions affect asset prices but have little effect on aggregate capital. Our findings rationalize several empirical regularities on liquidity and investment.

JEL Classifications: E44, G32, G12.

Keywords: Liquidity, Investment, Present bias.

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1 Introduction

Asset liquidity matters for a wide range of macroeconomic outcomes, such as asset prices (e.g., Aiyagari and Gertler, 1991; Amihud et al., 2005), consumption risk-sharing (e.g., Kaplan et al., 2014, 2018), and firm investment, the capital structure, and the transmission of monetary policy (e.g., Amihud and Levi, 2022; Cui and Radde, 2020; Jeenas and Lagos, 2024). In this paper, we show that liquidity frictions also generate heterogeneous valuations: agents disagree on how to discount future cash flows. This disagreement provides a novel channel through which liquidity frictions impact investment decisions. In particular, we find that the aggregation of owners' preferences leads to a discount factor that is *time-inconsistent*. Our findings help rationalize several empirical regularities, including the corporate discount rate wedge (Gormsen and Huber, 2023), the relationship between stock market liquidity and investment in the cross-section and in the aggregate (Amihud and Levi, 2022; Naes et al., 2011), and the short-termism observed among stock market-listed firms (Asker et al., 2015; Graham et al., 2005; Terry, 2023).

We present a theory of investment in an incomplete markets production economy in which households trade firms' shares subject to transaction costs. Because of financial frictions, agents disagree on the valuation of the firm and, crucially, on the discount factor used to value cash flows. Our main result is that trading frictions generate a disagreement between owners' discount factors that leads to a problem for the firms that is time-inconsistent: the optimal investment plan at any point in time is suboptimal at future dates. Moreover, we show that, under plausible assumptions, the firms' problem is equivalent to one in which there are no financial frictions but the firm uses a stochastic discount factor with *quasi-hyperbolic discounting*. Notably, we obtain this result from the frictions in financial markets rather than behavioral assumptions. In a calibrated economy, we find that the transmission of trading frictions to the real economy is highly sensitive to the firms' ability to commit to an investment plan. If firms cannot commit, trading frictions can have significant adverse effects on investment and output due to present bias. In contrast, if firms can commit, trading frictions affect asset prices but have little effect on the aggregate level of capital.

We build a model with two standard blocks: incomplete markets as in Aiyagari (1994), and neoclassical firms. Households face income risk and have access to two financial markets: a market for the stock of the representative firm and a market for risk-free one-period bonds. In the spirit of Aiyagari and Gertler (1991) and Kaplan et al. (2014), we assume that stock trading is subject to transaction costs while the bond market is frictionless but agents can borrow only up to a limit. Unlike Aiyagari and Gertler (1991) and Kaplan et al. (2014), we consider a production economy, which allows us to study the implications of financial market frictions on aggregate investment. Crucially, we assume that firms make the investment decisions and, therefore, they face a dynamic problem. This requires us to find an appropriate discount factor for the firms. However, the presence of transaction costs poses a challenge, as households' expected rates of return for holding the firm's stock do not equalize. As a consequence, the firm's owners disagree on the appropriate discount factor the firm should use and the implied investment plan. There is a long tradition in microeconomics that studies ways to specify the firm's objective in this scenario (see DeMarzo, 1993; Dreze, 1974; Grossman and Hart, 1979; Makowski, 1983). In this paper, we assume that the firm maximizes the weighted valuation of owners, where the weights are given by the initial stock holdings.[1]

Disagreement about the discount factor leads to the paper's main result: the firm's problem is *time-inconsistent*. In particular, we show that, under plausible assumptions, the firm's problem is equivalent to one in which the firm exhibits quasi-hyperbolic discounting. To understand why, consider the tradeoffs involved in the design of the firm's investment plan. Investment is an intertemporal decision: agents give up consumption in period t (through the distribution of dividends) for a higher level of production in $t+1$. However, if the discount factor of owners and buyers are not equal, they will disagree on the optimal investment plan for the firm. Sellers prefer to be paid out through frictionless dividends as opposed to asset sales, for which they incur a transaction cost. This implies that sellers are more impatient than buyers. Hence, owners (including sellers) will favor a higher level of dividends and a lower level of investment than buyers. In contrast, when planning investment multiple periods into the future, the firm uses the discounting implied by asset prices: the risk-free rate plus a liquidity premium. Since asset prices reflect buyers' preferences, if buyers expect to be more patient than current owners (which they will typically be in our economy), the firm would like to commit to a higher level of investment multiple periods into the future. However, if the decisions are revised in $t+1$ by the new set of owners, they will be tempted to again lower the investment level in $t+1$ and promise a higher level of investment starting in $t+2$. Thus, the problem of the firm is time-inconsistent. Notably, transaction costs are crucial for these results. In the absence of transaction costs (e.g., as in Aiyagari, 1994), owners' and buyers' expected returns from holding the stock are equalized, so the problem of the firm is time-consistent.

We then calibrate the model and study its quantitative properties in general equilibrium. Most of the parameters are standard and we take them from existing literature. The main parameter is the transaction cost, which we discipline to match a liquidity premium of about 40 basis points following recent estimates by van Binsbergen et al. (2022). As non-targeted moments, the model is consistent with data on the corporate discount rate (Gormsen and Huber, 2023), the composition of liquid and illiquid assets and the fraction of borrowing constrained households (Kaplan et al., 2014).

We find that present bias is the empirically relevant case. When there are no trading frictions, the model is isomorphic to the one-asset economy in Aiyagari (1994). In this case, the steady-state level of capital is higher than in a complete markets economy due to precautionary savings. With trading frictions and commitment, we find that trading frictions affect asset prices but have minor consequences for aggregate capital. On the one hand, trading frictions depress

[1] We show that our results are similar under alternative weights such as Dreze (1974), or majority voting (DeMarzo, 1993).

asset prices, implying a lower steady-state level of capital. On the other hand, there is a higher precautionary motive for saving, implying a higher steady-state level of capital. Quantitatively, these two forces are of similar magnitude, and, as a result, capital does not change significantly. However, without commitment, there is a third force at play: present bias. Quantitatively, this force strongly favors more discounting and, as a result, we obtain a lower level of capital than in the complete markets economy, contrary to the overaccumulation result in Aiyagari (1994).

Our results are consistent with recent empirical evidence from Gormsen and Huber (2023) on the firm's discount rate *wedge*, that is, the difference between the discount rate firms' use to evaluate investment projects and the financial cost of capital. Notably, we find that only the model without commitment generates a corporate discount wedge, providing empirical evidence in favor of lack of commitment. As a sanity check, we first verify that less liquid firms have higher discount rates in the data. This is to be expected, because these firms have a higher cost of capital (which shows up as a *liquidity premium* in our setting). We then show that less liquid firms also have a higher discount rate wedge. Our model suggests that present bias is a factor behind this empirical finding.

Furthermore, we extend the main framework in several dimensions. First, we allow the firm to borrow in illiquid corporate bonds markets. We find that the problem of the firm is still time-inconsistent. Corporate bonds can affect the firm's financial structure but have no impact on investment decisions, which are made by the stockholders. Even if corporate bonds are more illiquid than stocks, a firm without commitment may issue bonds due to present bias. This result provides a reason for corporate borrowing that does not rely on the tax advantage of debt.

In a second extension, we study how the demand and supply of liquid assets affect aggregate capital. On the demand side, we consider an increase in idiosyncratic uncertainty. More uncertainty implies that the illiquid asset is traded more, which increases the severity of the time-inconsistency problem and reduces aggregate capital. On the supply side, we incorporate liquid government bonds into the model. A higher supply of government bonds leads to the exact opposite—a higher level of capital—because the illiquid asset is traded less. Thus, under no commitment, a higher supply of government debt *crowds-in* capital investment.

Related Literature. The paper is related to several strands of the literature.

First, there is an ample empirical, theoretical, and quantitative literature arguing that liquidity matters for asset pricing (e.g., Amihud et al., 2005; Duffie et al., 2005; Lagos and Rocheteau, 2009; Lagos, 2010). More specifically, our results relate to the literature that studies the effects of transaction costs on financial markets (e.g., Constantinides, 1986; Aiyagari and Gertler, 1991; Heaton and Lucas, 1996; Vayanos, 1998; Gârleanu and Pedersen, 2013; Abel et al., 2013).[2] This literature considers either exogenous dividend streams or endowment economies and focuses on the asset-pricing implications, abstracting from issues such as how

[2] See Vayanos et al. (2012) for a survey of this literature.

trading frictions affect the firm's problem and the implications for investment.

Second, recent papers in macroeconomics incorporate asset liquidity frictions. For example, Kaplan and Violante (2014) and Kaplan et al. (2018) study the importance of asset liquidity for consumption risk-sharing. Cui and Radde (2020) and Jeenas and Lagos (2024) study the effects of liquidity on firms' investment decisions.[3] However, this literature does not study the implications of owner disagreement. Cui and Radde (2020) assumes that households directly own the capital of the economy, so that the problem of the firm is static. Jeenas and Lagos (2024) assumes that investment decisions are made by a single entrepreneur. As a consequence, liquidity frictions do not directly affect the firms' problem but only through their impact on asset prices. In our paper, liquidity frictions affect asset prices but also generate disagreement among owners, which leads to a problem for the firms that is time-inconsistent. Closest to our paper is Carceles-Poveda and Coen-Pirani (2010), who study the implications of household and firm ownership of capital in heterogeneous agents models. Their focus is on cases that lead to an equivalence result: whether the households or the firms own the capital is irrelevant for equilibrium under the appropriate choice of the firms' discount factor. We focus instead on cases where the distinction matters. Transaction costs are crucial for our analysis; absent transaction costs, we recover the equivalence result.[4]

Third, the paper is also related to the literature that studies the problem of the firm in economies with incomplete markets (see Diamond, 1967; Dreze, 1974; Grossman and Hart, 1979; DeMarzo, 1993; Makowski, 1983).[5] These papers focus on aggregate risk and study models with a finite number of time periods. Our paper considers an infinitely lived economy with idiosyncratic risk and trading frictions but no aggregate risk. In a steady-state equilibrium, we show that the firms' problem can be expressed as featuring beta-delta discounting. Notably, we do not assume beta-delta discounting as a behavioral phenomenon (as in Laibson et al., forthcoming; Grenadier and Wang, 2007; Kang and Ye, 2019), but it arises endogenously from trading frictions in asset markets (see also Amador, 2012; Azzimonti, 2011). Recent papers have also encountered the problem of the appropriate discount factor of the firm in incomplete market economies. For example, Favilukis (2013) and Favilukis et al. (2017) similarly use a portfolio-weighted average of the agents' intertemporal marginal rates of substitution (IMRS) as the firms' discount factor. However, they do not consider the potential time-inconsistency problem. Jackson and Yariv (2015) show, in a very different setting than ours, that preference aggregation naturally leads to time inconsistency. Present bias obtains because, in the distant future, weighted discount rates are determined by the most patient individual. Espino et al. (2018) and Bisin et al. (2022) study the insurance properties of firm ownership. Espino et al. (2018) consider privately owned firms and focus on the solution of the centralized allocation with a finite number of agents and private information. Bisin et al. (2022) study how investors'

[3] Relatedly, see Jermann (1998) and Cochrane (2008) for production-based asset pricing models.

[4] See also Carceles-Poveda and Coen-Pirani (2009).

[5] See Magill and Quinzii (2002) for a review of the literature.

hedging demand shapes the firms' capital structure. Instead, our paper focuses on how trading frictions affect firms' investment decisions. Moreover, as we have already emphasized, our model provides a rationalization for the corporate discount rate wedge documented by Gormsen and Huber (2023).

Finally, there is an extensive literature on short-termism showing that managers use increased hurdle rates to evaluate investment projects, resulting in lower investment levels (see Asker et al. (2015)). Several explanations have been offered for this behavior: reputation concerns as in Narayanan (1985), external analyst benchmarks as in Graham et al. (2005) and Terry (2023), and other agency problems (see Stein (2003) for a review). We find that disagreement amongst shareholders alone, even absent these additional mechanisms, leads to an increased hurdle rate and lowered investment.

The paper is organized as follows. Section 2 presents the household and firm problems. Section 3 considers a simple three-period example to understand the sources of time inconsistency. Section 4 defines the equilibrium in the infinite horizon economy, and Section 5 describes the solution to the firm problem with beta-delta discounting. Section 6 presents the quantitative evaluation. Finally, Section 7 concludes.

2 The Model

We study an Aiyagari economy augmented to incorporate transaction costs on financial assets. Time is discrete and denoted by $t = 0, 1, 2, \ldots$. Households face labor income risk, and markets are incomplete. In particular, households have access to only two financial markets: a market for the stock of the representative firm and a market for a risk-free bond. There is no aggregate risk.

Households. The economy is populated by a measure one of households indexed by $j \in [0, 1]$. Households are subject to idiosyncratic labor shocks that determine the number of hours they can sell in the labor market, which we label as their *employment status* and denote by h. We assume that h is drawn from a finite set $\mathbb{H} = \{h_1, h_2, \ldots, h_S\}$, where $h_1 < h_2 < \cdots < h_S$, with associated transition density $dF(h_{s'}|h_s)$. Households' preferences can be represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where $\mathbb{E}_t$ represents the expectation operator conditional on the information set at period t, β is the households' subjective discount factor, c_t denotes consumption in period t, and $u(\cdot)$ is a continuously differentiable and strictly concave function that satisfies Inada conditions.

Markets in this economy are incomplete. Households can trade two classes of financial assets: the stock of the representative firm and a one-period risk-free bond. Trade in the stock market entails a transaction cost, which we model as a wasteful use of resources.[6] Let q_t denote the price of a share, and Δ_t^- and Δ_t^+ denote the shares sold and bought by a household. We assume that the transaction cost incurred in a sale is $\frac{\phi}{2}(\Delta_t^-)^2 q_t$. Quadratic transaction costs simplify the solution of the agents problem, as it avoids the existence of inaction regions.[7] Short-selling is not allowed in this market. In contrast, trading bonds is frictionless but households face a borrowing limit $\overline{b}$.[8] Thus, the constraints faced by a household in period t are

$$c_t + q_t\Delta_t^+ + \frac{b_{t+1}}{1+r_t} \leq w_t h_t + d_t\theta_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t + b_t, \tag{2}$$

$$\theta_{t+1} = \theta_t + \Delta_t^+ - \Delta_t^-, \tag{3}$$

$$\Delta_t^+ \geq 0, \quad \theta_t \geq \Delta_t^- \geq 0, \tag{4}$$

$$b_{t+1} \geq \overline{b}, \tag{5}$$

where b_t denotes the holdings of one-period risk-free bonds, r_t is the real interest rate, w_t is the real wage, θ_t denotes the household's holdings of stock at the beginning of period t, and d_t denotes the dividends distributed by the representative firm. Equation (2) is a standard budget constraint. The household receives labor income, $w_t h_t$, dividend income, $d_t\theta_t$, the proceeds from the sale of stock net of transaction costs, $\left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t$, and the maturing bonds, b_t. They use their income to consume, c_t, buy stock, $q_t\Delta_t^+$, and buy bonds, $\frac{b_{t+1}}{1+r_t}$. Equation (3) represents the law of motion of stock holdings. Condition (4) imposes natural constraints on trades, ensuring that purchases and sales are non-negative and that the household does not sell more shares than it owns. Finally, equation (5) represents the borrowing constraint. We assume that $\overline{b} > -\sum_{t=1}^{\infty}\prod_{s=1}^{t-1}\frac{w_t h_1}{1+r_s}$.

Thus, the problem of a household is to choose processes $\{c_t, b_{t+1}, \theta_{t+1}, \Delta_t^+, \Delta_t^-\}_{t=0}^{\infty}$ in order to maximize (1) subject to (2), (3), (4), and (5) for every $t \geq 0$, and given an initial portfolio (θ_0, b_0) and prices and dividends $\{w_t, q_t, r_t, d_t\}_{t=0}^{\infty}$.

Firms. There is a representative firm that operates a non-increasing returns to scale technology that combines labor and capital to produce the final consumption good, given by $y_t = \left(l_t^{\gamma} k_t^{1-\gamma}\right)^{\psi}$, where l_t denotes the amount of labor hired, k_t denotes the amount of capital operated, $\gamma \in (0,1)$ and $\psi \leq 1$.[9] Moreover, the firm operates the economy's investment technology, $k_{t+1} = (1-\delta)k_t + i_t$, where i_t denotes the level of investment in period t and $\delta \in (0,1)$ is the

[6] Our results would not change if we assumed that transaction costs are paid to a financial intermediary.

[7] See Heaton and Lucas (1996). Our theoretical results go through with any continuously differentiable, strictly convex transaction cost function $\phi\mathscr{T}(\Delta_t^-)q_t$ where $\mathscr{T}(0) = \mathscr{T}'(0) = 0$.

[8] We think of bonds as bank deposits and loans rather than assets that trade in frictional markets like corporate bonds. See Aiyagari and Gertler (1991).

[9] In the background, we are assuming that there is a measure one of identical firms with non-increasing returns to scale. Since all firms make the same choices in equilibrium, we have $k_i = k$ and $l_i = l$ for all i. Then, there exists a representative firm with the same technology that operates all the capital and labor, i.e., $\int_0^1 k_i di = k$ and $\int_0^1 l_i di = l$. Finally, we assume that the entry costs are such that there is no firm entry in equilibrium.

depreciation rate.

As is standard, we assume that the firm acts in the best interest of its shareholders. Because the choice of labor is an intratemporal decision, optimality implies that $l_t = \gamma\psi \frac{y_t}{w_t}$. Let $\pi_t = y_t - w_t l_t$ denote the firm's per-period profits. Then, $\pi_t = (1-\gamma\psi)y_t$, or $\pi_t = z_t k_t^{\alpha}$, where $z_t = (1-\gamma\psi)\left(\frac{\gamma\psi}{w_t}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$ and $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$. Dividends are then given by $d_t = \pi_t - i_t$, or

$$d_t = F_t(k_t, k_{t+1}) \equiv z_t k_t^{\alpha} + (1-\delta)k_t - k_{t+1}.$$

Investment is an intertemporal decision that requires knowledge of shareholders' intertemporal preferences. Because of the assumed financial frictions, shareholders may have conflicting preferences about the firm's investment plans. We proceed in two steps. First, we compute the shareholder's valuations. Then, we specify the firm's objective.

Shareholders' valuations. Let $\tilde{q}_t(\theta,b,h)$ denote the shareholder's valuation of the firm in units of the consumption good, which is given by

$$\tilde{q}_t(\theta,b,h) \equiv \frac{V_{\theta t}(\theta,b,h)}{\lambda_t(\theta,b,h)} = d_t + \frac{\mu_t(\theta,b,h) + \overline{\eta}_t^-(\theta,b,h)}{\lambda_t(\theta,b,h)},$$

where $V_t(\theta,b,h)$ denotes the value function in period t of a household with stock holdings θ, bond holdings b, and employment status h, $V_{\theta t}(\theta,b,h)$ denotes the envelope condition of a shareholder (i.e., a household with $\theta > 0$) with respect to θ, $\lambda_t(\theta,b,h)$ denotes the marginal utility of wealth (measured as the Lagrange multiplier associated to the budget constraint (2)), $\mu_t(\theta,b,h)$ denotes the marginal utility of an extra unit of the stock (measured by the Lagrange multiplier of constraint (3)), and $\overline{\eta}_t^-(\theta,b,h)$ denotes the Lagrange multiplier associated to the short-selling constraint in (4).

The shareholder's valuation has two components. First, shareholders value the dividend they receive, d_t. Since the dividend is in units of the consumption good, all shareholders agree on its valuation. The second term denotes the *ex-dividend* value of the stock. Since the stock is a long-lived asset, agents value holding shares of the firm above and beyond the dividend they receive in the current period. This term may differ across agents.

There are two types of shareholders: those who keep all their holdings (and potentially buy more) and those who sell at least part of their portfolio. We call them buyers and sellers, respectively. Absent transaction costs, this distinction would be inconsequential: at the margin, the valuation of buyers and sellers would always coincide. However, transaction costs introduce a wedge in the agents' valuation, generating disagreement. The following lemma characterizes the shareholders' valuation. All proofs are in Appendix A.

Lemma 1. *The shareholders' valuation of the firm is given by*

$$\tilde{q}_t(\theta,b,h) = d_t + (1-\phi\Delta_t^-(\theta,b,h))q_t. \tag{6}$$

Lemma 1 characterizes the valuation of all agents that start the period with $\theta > 0$. Note that since buyers choose $\Delta_t^- = 0$, their valuation simplifies to $\tilde{q}_t(\theta, b, h) = d_t + q_t$, which is independent of agent-specific variables. In contrast, the seller's valuation is decreasing in the amount of stock they sell. The more the agent sells, the higher the transaction cost and the lower the benefits of holding the stock. Next, we use Lemma 1 to specify the firm's objective.

The firm's problem. There is a vast literature studying the problem of the firm when shareholders disagree.[10] In this paper, we assume that the firm maximizes an *ownership-weighted valuation*, which can be thought of as giving shareholders votes in proportion to their holdings, in the spirit of Grossman and Hart (1979) (see also Favilukis, 2013).[11]

Assumption 1. *The firm maximizes an ownership-weighted valuation given by:*

$$\int_{\theta,b,h} \theta \left[d_t + (1-\phi\Delta_t^-(\theta,b,h))q_t\right] d\Gamma_t(\theta,b,h) = d_t + (1-\overline{\Phi}_t)q_t, \tag{7}$$

where $\Gamma_t(\theta,b,h)$ denotes the cross-section distribution over the portfolio holdings and employment status, $d_t = F_t(k_t, k_{t+1})$, and

$$\overline{\Phi}_t \equiv \phi \int_{\theta,b,h} \theta\Delta_t^-(\theta,b,h)d\Gamma_t(\theta,b,h). \tag{8}$$

Maximizing the firms value involves choosing the path of capital that trades off the effects on the dividend distributed in period t, d_t, and the continuation market value, q_t. For example, higher investment today implies a lower current dividend but potentially higher dividends in the future. Crucially, the continuation value q_t is discounted by $\overline{\Phi}_t$, which represents the weighted-average marginal cost faced by *current owners*. Consider the valuations of owners who sell some of their holdings relative to owners who keep everything. Since the firm can distribute dividends costlessly, those who sell tend to favor the distribution of dividends at the expense of a higher continuation value. The firm aggregates these differences by internalizing a weighted-average transaction cost in its problem.

Even with Assumption 1, the firm still needs to understand the sensitivity of the stock price, q_t, to its investment plan. Naturally, the firm does not directly control q_t, which is determined in equilibrium. However, the firm understands that the equilibrium value of q_t depends on its investment decisions through their consequences on the dividend payout plan. To tackle this problem, we first solve a simplified 3-period version of the model in Section 3. We turn back to the infinite-horizon setting in Section 4.

[10] See, e.g, Diamond (1967), Dreze (1974), Grossman and Hart (1979).

[11] In Appendix B, we generalize the weights used in the firm's problem to encompass weighting by future ownership as in Dreze (1974) and majority voting as in DeMarzo (1993). We show that our results are consistent with these alternative specifications.

3 Motivating Example: A 3-Period Model

With only three time periods, $t = 0,1,2$, we have that $c_t = 0$ for $t \geq 3$ and equations (2) to (5) only hold for $t = 0,1,2$. Note that $q_2 = 0$ because there is no need to hold assets beyond $t = 2$.[12]

Besides assuming a shorter time horizon, we make two additional simplifying assumptions relative to the model described in Section 2. First, we assume that households face no income risk. Instead, we follow Woodford (1990) and assume that households' employment status oscillates in a deterministic fashion between a low value h_{low} and a high value h_{high}. We assume that half of the households receive the low labor endowment in period $t = 0$, and the other half receive the high labor endowment. We refer to these two groups as the low and high groups $j \in \{l,h\}$, respectively. Second, we assume that households cannot borrow, that is, the borrowing limit is $\overline{b} = 0$. Thus, there effectively is a single financial asset in the economy, the illiquid stock. All other aspects of the example are as described in the previous section.[13]

Our main object of interest is the firm's intertemporal problem. We first characterize the solution for a firm that commits in period 0 to an investment path for capital in periods 1 and 2. Then, we show that if the firm is allowed to reoptimize in period 1, it chooses a different allocation for capital in period 2. Hence, the problem is not time-consistent.

The firm maximizes shareholder value, which, by Assumption 1, is given by:

$$\sum_{j\in\{l,h\}} \frac{\theta_t^j}{2} \left[d_t + (1-\phi\Delta_t^{j-})q_t\right], \quad t \in \{0,1\}. \tag{9}$$

As discussed in Section 2, while q_t is determined in equilibrium, the firm understands that its decisions (investment and dividend payout policy) will impact its value. Here, we assume that the firm knows that the stock price satisfies the following households' Euler equations

$$\left(1-\phi\Delta_t^{j-}\right) q_t = \beta \frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)} d_{t+1} + \beta \frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)} \left(1-\phi\Delta_{t+1}^{j-}\right) q_{t+1}, \quad t \in \{0,1\}. \tag{10}$$

Note that we focus on equilibria where the short-selling constraint does not bind. In the numerical illustration below, we choose parameter values that satisfy this requirement. Then, introducing (10) into (9), we get that the firm's value in period 0 is given by

$$V_0^F(k_0) = \max_{k_1,k_2\geq 0} \sum_{j\in\{l,h\}} \frac{\theta_0^j}{2} \left[d_0 + \beta \frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)} d_1 + \beta^2 \frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)} d_2\right], \tag{11}$$

[12] When we solve the infinite-horizon in Section 5, we will focus on a stationary equilibrium where aggregate variables are constant over time. Since the 3-period model has a final period where the stock price is zero, a stationary equilibrium does not exist. In Appendix C.1, we present a natural extension of this model that generates a stationary equilibrium. All of our results go through in that setting.

[13] In Appendix C.2, we also consider a version of this 3-period model with symmetric transaction costs for buyers and sellers. All of our results go through in that setting.

where $d_t = F_t(k_t, k_{t+1})$ and the firm takes the households' consumption plans as given.[14] The optimal investment plan from period 0's perspective is the solution to problem (11).

The stochastic discount factor of the firm in problem (11) corresponds to a weighted average of the shareholders' IMRS implied by the household's Euler equations (10), where the weights correspond to the shareholder's proportional ownership in the firm. Note that this discount factor corresponds to the assumption in other papers such as Favilukis (2013) and Favilukis et al. (2017).

Time Inconsistency. Consider now the problem of the firm in period 1. The firm starts the period with capital stock k_1, which was chosen in period 0. Suppose we give the firm the option to reoptimize its investment plan, i.e., the choice of k_2. If the plan in period 0 is time-consistent, the firm would not have an incentive to change its choice. However, we show next that the firm may choose to change its plan; that is, its problem may be time-inconsistent. To see this, note that the firm's problem in period 1 along the equilibrium path is

$$\begin{aligned} V_1^F(k_1) &= \max_{k_2 \geq 0} \sum_{j \in \{l,h\}} \frac{\theta_1^j}{2} \left[d_1 + (1 - \phi \Delta_1^{j-}) q_1 \right] \\ &= \max_{k_2 \geq 0} \sum_{j \in \{l,h\}} \frac{\theta_1^j}{2} \left[d_1 + \beta \frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)} d_2 \right] \end{aligned} \tag{12}$$

where $\theta_1^j = \theta_0^j + \Delta_1^{j+} - \Delta_1^{j-}$. Then, the firm's problem is time-consistent if and only if the discounting between $t = 1$ and $t = 2$ in problem (11) and problem (12) coincide, or, equivalently, if

$$\frac{\sum_{j \in \{l,h\}} \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_{j \in \{l,h\}} \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} = \sum_{j \in \{l,h\}} \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}. \tag{13}$$

First consider the case with $\phi = 0$. The Euler equation (10) becomes

$$\frac{q_t}{d_{t+1} + q_{t+1}} = \beta \frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)}.$$

Crucially, in this case, the IMRS are equalized across agents. In this case, the firm's problem is time-consistent:

$$\frac{\sum_{j \in \{l,h\}} \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_{j \in \{l,h\}} \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} = \frac{\frac{q_0}{d_1+q_1} \frac{q_1}{d_2+q_2}}{\frac{q_0}{d_1+q_1}} = \frac{q_1}{d_2 + q_2} = \sum_{j \in \{l,h\}} \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}.$$

[14] This is an application of the standard *"big D little d"* argument. The firm takes aggregate consumption—which depends on dividends—as given, but optimizes on its own dividend payments.

Table 1: Equilibrium in the 3-Period Model

Household type	c_0	c_1	c_2	Δ_0^-	Δ_1^-	θ_1	θ_2
low	2.6226	2.7290	2.6030	0.3358	0.0000	0.6642	1.6031
high	3.3667	3.2271	3.3970	0.0000	0.9389	1.3358	0.3969

When financial markets are frictionless, agents' valuations of consumption across periods coincide. As a consequence, they will agree on the optimal investment plan for the firm across periods, and the firm's problem is time-consistent.

In contrast, when $\phi > 0$, the agents' IMRS might not equalize if there is positive trade (i.e., if $\Delta_t^{j-} > 0$ for some households). In that case, there is no guarantee that condition (13) will hold. In particular, as agents' valuation and ownership change, the optimal plan for the firm may also change. We illustrate this with a numerical example.

Suppose $w_t = 1$ and $d_t = 1$ for all t, $u = ln(c)$, $\phi = 0.1$, $\beta = 0.95$, $h_{low} = 1$, and $h_{high} = 3$.[15] Initial asset holdings are $\theta_0^j = 1$ for $j \in \{l, h\}$. The household equilibrium is summarized in Table 1, and asset prices are given by $q_0 = 1.8856$ and $q_1 = 0.9960$.

The problem of the firm in period 0 becomes

$$V_0^F(k_0) = \max_{k_1, k_2 \geq 0} d_0 + 0.9520 d_1 + 0.9019 d_2,$$

and the problem of the firm in period 1 becomes

$$V_1^F(k_1) = \max_{k_2 \geq 0} d_1 + 0.9335 d_2.$$

Let us compare the firm's problem in $t = 0$ and $t = 1$ along the equilibrium path. From its period 0 perspective, the firm's discount factor between $t = 1$ and $t = 2$ is 0.9474, which is the ratio between 0.9019 and 0.9520. When period 1 arrives, the firm's discount factor between $t = 1$ and $t = 2$ is 0.9335. Hence, the problem of the firm is time-inconsistent: the time preference between $t = 1$ and $t = 2$ is different in $t = 0$ than in $t = 1$. Furthermore, in this calibration, the direction of inconsistency is towards *present bias*. That is, when period 1 arrives, the firm discounts period 2 more than it did in period 0.

The purpose of this example was to show, in a simple and transparent model, that time-inconsistency is a natural outcome in the absence of IMRS equalization across agents. Intratemporal disagreement among current shareholders, as well as intertemporal disagreement among current and future shareholders, can make the optimal plan—from period 0's perspective—suboptimal in period 1.

The key step that allowed us to characterize the solution was to replace the price of the stock, q_t, from the firm's objective by substituting the households' Euler equation in (12),

[15]We can obtain $w_t = 1$ and $d_t = 1$ by choosing the parameters of the production function and the capital accumulation technology appropriately.

which holds true when $q_2 = 0$ and short-selling constraints are not binding. Note, however, that this substitution requires a great deal of information on the part of the firm. The firm needs to know not only the current preferences of its owners but also their *future* preferences. Crucially, the firm cannot recover these valuations from asset prices. While the firm's stock market price is observable, the *sensitivity* of the price to changes in the investment plan is unobservable.

Unfortunately, these problems get exacerbated in an infinite-horizon setting. The firm would need information on its shareholders' valuation from the present to the infinite future. Moreover, realistic settings will allow for the set of shareholders to change over time (rather than the same shareholders changing the amount they own), and some shareholders might be constrained by the short-selling constraint. These issues substantially complicate the firm's problem and the computation of equilibrium.[16] To make progress, Section 4 discusses an additional assumption that renders the infinite-horizon problem tractable.

4 The Firm in Infinite-Horizon

In this section, we set up the problem of the representative firm in the infinite horizon setting. A simplifying assumption will allow us to sidestep the aforementioned complications; however, two outcomes from the previous section remain: time-inconsistency and present bias. Moreover, under this additional assumption, we show that the firm behaves as if it faced beta-delta discounting.

4.1 The Stock Price Elasticity

A crucial element of the firm's problem is its understanding of how its stock price, q_t, changes in response to its future dividend choices, $\{d_{t+s}\}_{s\geq 1}$. Like in the 3-period model of Section 3, the firm understands that the answer to this question lies in the households' Euler equation. Let $\mathscr{C}_t(\theta,b,h)$, $\Theta_{t+1}(\theta,b,h)$, and $\mathscr{B}_{t+1}(\theta,b,h)$ denote the policy functions for consumption, stock holdings, and bonds, given (θ,b,h). To avoid excessive clutter, we simplify the notation by removing the dependence of $\mathscr{C}_t$, Θ_{t+1}, and $\mathscr{B}_{t+1}$ on (θ,b,h) and, instead, including the household's index j as a superscript.

The Euler equation of household j is given by

$$\left(1-\phi\Delta_t^{j-}\right)q_t = \mathbb{E}_t\left[\beta\frac{u'\left(\mathscr{C}_{t+1}^j\right)}{u'\left(\mathscr{C}_t^j\right)}\left(d_{t+1}+\left(1-\phi\Delta_{t+1}^{j-}q_{t+1}\right)\right)\right]+\eta_t^j,$$

where η_t^j is the Lagrange multiplier on the households' short-selling constraint for stocks in

[16] See Moll (2023) on why this (i) makes it very hard/imposible to solve, and (ii) it is an unrealisitic assumption.

(4), and the expectation is taken with respect to h_{t+1}. Let

$$\Phi_t^j \equiv \mathbb{E}_t\left[\phi\Delta_{t+1}^{j-}\right] + \phi\frac{\mathrm{cov}_t\left(u'(\mathscr{C}_{t+1}^j), \Delta_{t+1}^{j-}\right)}{\mathbb{E}_t\left[u'(\mathscr{C}_{t+1}^j)\right]}, \tag{14}$$

Then, we can rewrite the households' Euler equation as

$$\left(1-\phi\Delta_t^{j-}\right)q_t = \mathbb{E}_t\left[\beta\frac{u'\left(\mathscr{C}_{t+1}^j\right)}{u'\left(\mathscr{C}_t^j\right)}\right]\left[d_{t+1} + \left(1-\Phi_t^j\right)q_{t+1}\right] + \eta_t^j. \tag{15}$$

The household uses its expected IMRS between t and $t+1$ to discount future cash flows. But the stock valuation has an extra component, Φ_t^j, which reflects the household's expected marginal transaction cost. A higher transaction cost (indexed by ϕ) reduces the household's valuation, as it lowers the stock return in a sale. This effect is amplified by the fact that households will sell (i.e. $\Delta_{t+1}^{j-} > 0$) when their marginal utility of consumption, $u'(\mathscr{C}_{t+1}^j)$, is relatively high. Hence, a positive covariance between marginal utility and quantity sold further depresses asset prices.

Equation (15) is at the core of the firm's investment decisions. In the 3-period model of Section 3, we used (15) to iteratively replace $(1-\phi\Delta_t^{j-})q_t$ from the firm's problem (9). This strategy is not helpful in the infinite-horizon version of the model for two reasons. First, recall that in the simple model, it was important that there were no agents against their short-selling constraints, so that $\eta_t^j = 0$ for all agents and periods. However, this will not be true in the infinite-horizon model with income risk. Households that suffer long spells of low employment will eventually hit the constraint. Thus, the Lagrange multiplier associated with the short-selling constraint will be positive for some agents. Second, after introducing the Euler equations into the firm's problem, we obtained a characterization of the firm's objective that depended on the *whole distribution of marginal utilities at all horizons*. This would require the firm to obtain information on agents' *individual* marginal utilities over time, information that is not readily available from aggregate variables or the price system.

Our strategy will use equation (15), but it will leverage the fact that if buyers are not borrowing constrained, it is possible to recover the sensitivity of the stock price to the firm's investment plan from two financial prices: the risk-free interest rate and a *liquidity premium*.

4.2 The Liquidity Premium

For buyers, condition (15) reduces to

$$q_t = \mathbb{E}_t\left[\beta\frac{u'(\mathscr{C}_{t+1}^j)}{u'(\mathscr{C}_t^j)}\right]\left[d_{t+1} + (1-\Phi_t^j)q_{t+1}\right],$$

because $\Delta_t^{j-} = \eta_t^j = 0$ for them. In principle, buyers may disagree on how much they value the dividend d_{t+1} relative to the *ex-dividend* market price q_{t+1}.[17] That is, $\mathbb{E}_t\left[\beta \frac{u'(\mathscr{C}_{t+1}^j)}{u'(\mathscr{C}_t^j)}\right]$ and Φ_t^j could vary depending on the buyer's initial portfolio and realization of the employment shock. However, their valuations coincide if their borrowing constraint does not bind. To see this, note that the agents' optimality condition for bonds is given by

$$\frac{1}{1+r_t} = \mathbb{E}_t\left[\beta \frac{u'(\mathscr{C}_{t+1}^j)}{u'(\mathscr{C}_t^j)}\right] + \gamma_t^j,$$

where γ_t^j is the Lagrange multiplier associated with the borrowing constraint (5). For unconstrained agents $\gamma_t^j = 0$, and hence

$$\mathbb{E}_t\left[\beta \frac{u'(\mathscr{C}_{t+1}^j)}{u'(\mathscr{C}_t^j)}\right] = \frac{1}{1+r_t} \implies q_t = \frac{d_{t+1}}{1+r_t} + \frac{(1-\Phi_t^b)q_{t+1}}{1+r_t},$$

so Φ_t^b must be the same for all unconstrained buyers, who we denote with the superscript b. In what follows, we focus on economies where the borrowing constraint does not bind for buyers. We verify their existence in the numerical exercise.

Let the yield of the stock be $r_t^\theta \equiv \frac{d_{t+1}+q_{t+1}}{q_t} - 1$. Define the liquidity premium as the excess return of the stock over the return of the bond, i.e. $r_t^\theta - r_t = \Phi_t^b \frac{q_{t+1}}{q_t}$, which reduces to Φ^b in steady-state. Then, the pricing equation of the firm's stock depends on two widely used financial prices: the risk-free rate, r_t, and the liquidity premium, Φ_t^b. With this, we are ready to set up the firm's intertemporal problem.

4.3 The Firm's Problem

Recall that Lemma 1 shows that the objective of the firm is to maximize a weighted valuation given by

$$d_t + (1-\overline{\Phi}_t)q_t. \tag{16}$$

The discussion in Section 4.2 implies that the stock price can be expressed as

$$q_t = \frac{d_{t+1}}{1+r_t} + \frac{(1-\Phi_t^b)q_{t+1}}{1+r_t}. \tag{17}$$

The following assumption provides the final piece necessary to solve the firm's problem.

Assumption 2. *The firm takes* $\{\overline{\Phi}_t, \Phi_t^b\}_{t\geq 0}$ *as given.*

Assumption 2 has two parts. First, it states that the firm takes the owner's average transaction cost, $\overline{\Phi}_t$, as given. This variable can be computed by a firm with access to data on

[17] All buyers agree on the stock's valuation; if they did not, they would not be willing to buy at the common price q_t.

how many stocks were bought or sold. Second, it states that the firm also takes the liquidity premium, Φ_t^b, as given. This implies that the firm does not anticipate how its decisions impact households' trading patterns, summarized by the cross-sectional distribution of Δ_t^- (see the definitions of $\overline{\Phi}_t$ and Φ_t^b in equations (8) and (14), respectively). The firm always has the correct expectation with respect to the *levels* of these variables but ignores the potential *changes* that arise from the effect of its investment decisions on the households' trading strategies.[18]

The resulting pricing equation (17) is intuitive. Since dividends are distributed without frictions and the economy features no aggregate risk, the firm's dividend is discounted by the same discount factor as the bonds, i.e., $\frac{1}{1+r_t}$. Note that if $\phi = 0$, the pricing equation simplifies to $q_t = \frac{d_{t+1}+q_{t+1}}{1+r_t}$ and the problem of the firm is to maximize $d_t + q_t$. In contrast, when $\phi > 0$, the resale price q_{t+1} is further discounted by the liquidity premium Φ_t^b. Thus, the firm's problem will be to maximize (16) subject to (17).

4.4 Equilibrium

An equilibrium for this economy consists of household allocations $\left\{\left(c_{j,t}, b_{j,t+1}, \theta_{j,t+1}, \Delta_{j,t}^+, \Delta_{j,t}^-\right)_{\forall j\in[0,1]}\right\}_{t=0}^{\infty}$, firm allocations $\{l_t, k_{t+1}, d_t\}_{t=0}^{\infty}$ and aggregates $\{w_t, r_t, q_t, \overline{\Phi}_t, \Phi_t^b, \Gamma_t\}_{t=0}^{\infty}$, such that, given $(\theta_{j,0}, b_{j,0})_{\forall j\in[0,1]}$ and k_0,

1. Given $\{d_t, w_t, r_t, q_t\}_{t=0}^{\infty}$, households optimize;
2. Given $\{w_t, r_t, \overline{\Phi}_t, \Phi_t^b\}_{t=0}^{\infty}$, firms optimize [19];
3. $\{\overline{\Phi}_t, \Phi_t^b\}_{t=0}^{\infty}$ are consistent with the cross-sectional distribution $\{\Gamma_t\}_{t=0}^{\infty}$ according to (8) and (14);
4. Markets clear: $\int_{j\in[0,1]} h_{j,t} dj = l_t$, $\int_{j\in[0,1]} \theta_{j,t} dj = 1$, $\int_{j\in[0,1]} b_{j,t} dj = 0$, for all $t \geq 0$. By Walras Law, the goods market also clears.

A steady-state equilibrium is an equilibrium in which firm allocations and aggregates are constant over time. Our analysis will focus on steady-state equilibria.

5 Equilibrium Characterization

In a steady-state equilibrium, $\overline{\Phi}_t = \overline{\Phi}$ and $\Phi_t^b = \Phi^b$ for all t. Iterating the price (17) forward and replacing it in the objective function, the problem of the firm simplifies to

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \frac{1-\overline{\Phi}}{1-\Phi^b} \sum_{s=1}^{\infty} \left(\frac{1-\Phi^b}{1+r}\right)^s F(k_{t+s}, k_{t+s+1}). \tag{FP}$$

[18] Grossman and Hart (1979) argue that since such an assumption involves an elasticity, the firm's beliefs are neither verified nor falsified in equilibrium.

[19] The optimization of the firm will depend on whether we consider the solution with or without commitment. This will be made explicit in the next section.

5.1 Beta-Delta discounting

The program (FP) features beta-delta discounting. To see this, let $\tilde{\delta} = \frac{1-\Phi^b}{1+r}$ and $\tilde{\beta} = \frac{1-\bar{\Phi}}{1-\Phi^b}$. Then, the value of the firm can be written as

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1}).$$

On the one hand, $\tilde{\delta}$ represents the "natural" discount factor obtained from the pricing equation (17): it combines the risk-free rate, r, and the liquidity premium, Φ^b. That is, $\tilde{\delta}$ reflects the discounting implied in asset prices. On the other hand, $\tilde{\beta}$ summarizes the disagreement between current and future owners. If $\bar{\Phi} = \Phi^b$, then current and future owners agree on how to value the firm, so $\tilde{\beta} = 1$ and the problem simplifies to a standard one with exponential discounting. However, if $\bar{\Phi} > \Phi^b$ ($\bar{\Phi} < \Phi^b$), then current owners are more impatient (patient) than the market, which implies that $\tilde{\beta} < 1$ ($\tilde{\beta} > 1$) and the firm exhibits present (future) bias. In the next proposition, we decompose the difference $\Phi^b - \overline{\Phi}$ to identify the factors contributing to time-inconsistency.

Proposition 1. *The difference $\Phi^b - \overline{\Phi}$ is equal to the sum of a* persistence effect *and a* risk premium*:*

$$\Phi^b - \bar{\Phi} = \underbrace{\phi\left(\widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\Delta^{j-}_{t+1}\right]\middle| buyer\right] - \widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\Delta^{j-}_{t+1}\right]\right]\right)}_{persistence\ effect} + \underbrace{\phi\,\widetilde{\mathbb{E}}_t\left[\frac{\mathrm{cov}_t\left(u'\left(\mathscr{C}^j_{t+1}\right), \Delta^{j-}_{t+1}\right)}{\mathbb{E}_t\left[u'\left(\mathscr{C}^j_{t+1}\right)\right]}\middle| buyer\right]}_{risk\ premium}$$

where tilde moments are taken with respect to the cross-sectional weighted density $\Theta_{t+1}(\theta, b, h)$ $d\Gamma(\theta, b, h)$, and the non-tilde moments are taken with respect to the density $dF(h'|h)$.

Proposition 1 presents several results. First, it shows that time-inconsistency can only emerge in an economy with transaction costs, i.e., $\phi > 0$. When $\phi = 0, \Phi^b = \bar{\Phi}$ and $\tilde{\beta} = 1$, recovering the standard exponential discounting problem. Second, it shows that the degree and direction of time-inconsistency (i.e., present or future bias) depends on the interaction of two economic forces.

The persistence effect captures the difference in transaction costs for different agents. On the one hand, we have the expected transaction costs next period for those that are buyers today. On the other hand, we have the expected transaction cost for owners—which includes both buyers and sellers. If there is *persistence* in trades, so that buyers in t expect to also be buyers in $t+1$, then the expected transaction cost of buyers is smaller than the average transaction cost in the economy, i.e. $\widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\Delta_{t+1}^{j-}\right]\middle|\text{buyer}\right] < \widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\Delta_{t+1}^{j-}\right]\right]$, which represents a force towards present bias. This is the case we expect to obtain in our model, as purchasing the illiquid asset is profitable only if there is a relatively high chance that the agent will *not* sell the asset immediately (the bond is a better asset for that case). We confirm that this is true in our quantitative exercise.

The second term in the expression is the risk premium effect, which captures the fact that it is precisely when the agents need the resources the most, i.e., high marginal utility states, that households sell the most. This positive covariance makes the equity a risky investment, which is priced in by the buyers and represents a force toward future bias. The net effect of these two competing forces (persistence versus risk) depends on parameter values. In our calibrated economy of Section 6, we find that the persistence effect significantly outweighs the risk premium, leading to present bias.

The main force in Proposition 1, the persistence effect, can be understood as comparing the rate of impatience of buyers and owners. While agents' IMRS does not appear explicitly in Proposition 1, their impatience is reflected in the quantity sold, Δ^{j-}. Since selling stock is costly while selling bonds (i.e., borrowing) is not, sellers of the stocks are likely borrowing constrained. Due to the presence of constrained sellers, we have that $\widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\frac{\beta u'(\mathscr{C}_{t+1}^j)}{u'(\mathscr{C}_t^j)}\right]\middle| seller\right] < \frac{1}{1+r_t}$. In contrast, buyers are not borrowing constrained today, which implies that their IMRS is $\frac{1}{1+r_t}$. This implies that owners (i.e., both buyers and sellers) are more impatient than buyers. Moreover, as we argued above, buyers today are likely to be buyers tomorrow (the persistence effect), so their expected IMRS is close to $\frac{1}{1+r_t}$. Thus, in equilibrium, we expect to have $\widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\frac{\beta u'(\mathscr{C}_{t+1}^j)}{u'(\mathscr{C}_t^j)}\right]\right] < \widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\frac{\beta u'(\mathscr{C}_{t+2}^j)}{u'(\mathscr{C}_{t+1}^j)}\right]\middle| buyer\right]$, where the LHS is the (weighted) average IMRS across owners (i.e., both buyers and sellers) in period t, while the RHS is the (weighted) average IMRS across buyers in $t+1$. Note that the firm's discount factor between $t+1$ and $t+2$ in period t is determined by the stock price q_t, and so is the expected IMRS of the buyers. However, the discount between $t+1$ and $t+2$ *in period* $t+1$ is determined by the owners. Using that owners are likely to be more impatient than buyers, we conclude that the firm's problem is time-inconsistent and, in particular, it suffers from present bias.

Next, we solve the problem of the firm analytically. First, we consider the problem of a firm that can commit to future policies. This firm chooses an investment plan in the initial period and never reoptimizes. Then, we study the problem of a firm that cannot commit to an investment plan but fully anticipates its future incentives and understands how present decisions can affect its future self.

5.2 Firm with commitment

A firm that can commit to a future investment policy chooses a sequence of capital $\{k_{t+s}\}_{s=1}^{\infty}$ in period t to maximize its value:

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1}).$$

The first-order condition with respect to k_{t+1} is

$$F_2(k_t, k_{t+1}) + \tilde{\beta}\tilde{\delta} F_1(k_{t+1}, k_{t+2}) = 0$$

and the first-order condition with respect to k_{t+s+1} for $s \geq 1$ is

$$F_2(k_{t+s}, k_{t+s+1}) + \tilde{\delta} F_1(k_{t+s+1}, k_{t+s+2}) = 0$$

where $F_1(k_t, k_{t+1}) \equiv z\alpha k_t^{\alpha-1} + (1-\delta)$ and $F_2(k_t, k_{t+1}) \equiv -1$. Note that when $\tilde{\beta} \neq 1$, the choice of initial investment involves a different trade-off than future investment.

Focusing on steady-state equilibria and using that $z = (1-\gamma\psi)\left(\frac{\gamma\psi}{w}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$, $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$, and $w = \gamma\psi k^{(1-\gamma)\psi} H^{\gamma\psi-1}$ (where H is the unconditional expectation of the employment process), we get that the level of capital when the firm has commitment is

$$k^C = \left(\frac{(1-\gamma)\psi\tilde{\delta}}{1-\tilde{\delta}(1-\delta)} H^{\gamma\psi}\right)^{\frac{1}{1-(1-\gamma)\psi}}. \tag{18}$$

5.3 No commitment

We now turn to the problem of a firm that cannot commit to an entire sequence of investments but optimizes period-by-period. We assume that the firm fully understands its future-self incentives and takes them into account when making its choice. This is known as a *sophisticated solution* in the literature on time-inconsistent preferences.

We solve for a Markov perfect equilibrium (MPE). Given an initial level of capital k, the firm takes the future policy function, $k' = g(k)$, as given. Thus, we can write the firm's problem as

$$V^F(k) = \max_{k'} F(k, k') + \tilde{\beta}\tilde{\delta} W(k')$$

subject to

$$W(k') = F(k', g(k')) + \tilde{\delta} W(g(k')).$$

Let $k' = \zeta(k)$ denote the solution of this equation for a given level of initial capital k. A MPE requires $\zeta(k) = g(k)$. The function $W(\cdot)$ takes into account the preference disagreement between the current and the future firm. If $\tilde{\beta} = 1$, the problem simplifies to a standard Bellman equation, with $V^F(k) = W(k)$. However, when $\tilde{\beta} \neq 1$, there is disagreement between the current and future firm. In particular, the current firm uses the rate $\tilde{\delta}$ to discount the future two periods ahead yet, the future firm intends to discount the immediate future at the rate $\tilde{\beta}\tilde{\delta}$ (because $\zeta(k) = g(k)$).

The following proposition shows that there is a unique differentiable solution that is the limit of the finite-horizon version of this problem. Moreover, it shows that this solution features a lower level of capital than with commitment if and only if the firm suffers from present bias.

Proposition 2. *The firm's problem with no commitment can have multiple, differentiable, Markovian solutions, but only one is the limit of the unique finite-horizon equilibrium. This solution is given by*

$$k^N = \left(\frac{(1-\gamma)\,\psi\tilde{\beta}\tilde{\delta}}{1-\tilde{\beta}\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}. \tag{19}$$

Finally, $k^N < k^C$ if and only if $\tilde{\beta} < 1$.

Notice that (19) is directly comparable to (18), where $\tilde{\beta}\tilde{\delta}$ replaces $\tilde{\delta}$. When $\tilde{\beta} < 1$, Proposition 2 shows that firms with no commitment under-invest compared to those with commitment. It is as if the firm without commitment is more impatient, which then delivers a lower level of capital. Intuitively, firms have an incentive to increase dividend payouts today and leave little capital for future selves. This is a best response, considering that future selves have the same incentive to over-distribute dividends.

5.4 Commitment and the Corporate Discount Rate

These results provide a rationale to the findings in Gormsen and Huber (2023), which document the existence of a wedge between the discount rate used by firms in investment decisions and their cost of capital. To see this in our model, let Λ denote the (log of the) firm's discount factor. When the firm cannot commit to an investment plan, Λ is given by

$$\Lambda \equiv -\log\tilde{\beta}\tilde{\delta} \approx r + \overline{\Phi}.$$

Gormsen and Huber (2023) decompose the firm's discount factor into a *financial cost of capital*, r^{fin}, and a *discount rate wedge*, κ, such that

$$\Lambda = r^{fin} + \kappa.$$

In the model without commitment, we have

$$r^{fin} \equiv \log\left(\frac{1}{\tilde{\delta}}\right) \approx r + \Phi^b, \quad \text{and} \quad \kappa \equiv \log\left(\frac{1}{\tilde{\beta}}\right) \approx \overline{\Phi} - \Phi^b.$$

Thus, present bias generates a positive discount rate wedge in the case of no commitment. In contrast, a firm that can commit to an investment plan discounts cashflows at $\tilde{\delta}$, so the wedge is zero. Therefore, only the model without commitment is consistent with a positive wedge. In our quantitative evaluation below, we quantify the relative importance of the wedge in the firm's discount rate.

5.5 Corporate Bonds

Up to now, we have assumed that the firm funds itself from retained earnings. We now explore the consequences of allowing the firm to also issue illiquid corporate bonds. Suppose the firm can borrow up to an exogenous limit at the gross interest rate $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$. The parameter $\tilde{\phi}$ captures a premium that corporate bonds pay relative to the risk-free rate in reduced form. This is motivated by the fact that corporate bond markets tend to be more illiquid than stock markets. In contrast, we assume that the rate of return from savings is equal to r. Because $\frac{1}{1+r} > \tilde{\beta}\tilde{\delta}$ and $\frac{1}{1+r} > \tilde{\delta}$, the firm never finds it optimal to save.

The firm's borrowing decision depends on how $\frac{1}{1+r^{cb}}$ compares to $\tilde{\beta}\tilde{\delta}$ and $\tilde{\delta}$. Note that the discount rate the firm uses for this comparison depends on its ability to commit to an investment plan. It might seem natural to conclude that, if the premium on corporate bonds is higher than the premium on stocks ($\tilde{\phi} > \Phi^b$), the firm does not issue corporate debt. However, this logic is correct only if the firm has commitment. If it does not, the firm uses $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r}$ to discount future cash flows and, hence, the firm may borrow even if the corporate rate is high relative to the stock market discount. This result provides a rationalization for corporate bond issuance that does not rely on the canonical tax advantage of debt.

Important for our results is that incorporating corporate bonds does not affect our conclusions: the firm still discounts futures cash flows using $\tilde{\beta}\tilde{\delta}^s$.[20] The following proposition formalizes this discussion.[21]

Proposition 3. *Suppose the firm suffers from present bias and it has access to the bond market with an interest rate of $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$. If $\tilde{\phi} < \Phi^b$ the firm always borrows to the limit independently of its degree of commitment. If $\Phi^b < \tilde{\phi} < \overline{\Phi}$ the firm without commitment borrows up to the limit, while the firm with commitment does not borrow. Furthermore, optimal levels of capital are determined according to* (18) *and* (19) *with and without commitment, respectively.*

6 Quantitative Evaluation

We now calibrate and solve the model numerically to study how liquidity affects aggregate capital in general equilibrium. The analysis in Section 5.4 showed that only the model without

[20]Naturally, whether the firm can borrow or not can affect the general equilibrium determination of $\tilde{\beta}$ and $\tilde{\delta}$. However, its qualitative properties do not change.

[21]Note that the firm's borrowing problem also suffers from time inconsistency. We focus on the same refinement of MPE as in Proposition 2.

Table 2: Parameter Values

Parameter	Value
Discount factor β	0.95
Risk aversion σ	2.00
Production weight on labor γ	0.80
Returns to scale ψ	0.95
Depreciation δ	0.05
Borrowing limit $\overline{b}$	-1.00
Labor persistence ρ_h	0.50
Labor st dev σ_h	0.30
Transaction cost ϕ	4.00

commitment is consistent with a positive wedge between the corporate discount rate and the cost of capital. Hence, we consider the model without commitment as the benchmark case, and analyze the commitment economy as a counterfactual.

6.1 Calibration

We have three sets of parameters: (i) standard parameters that we take from the literature, (ii) parameters governing the income process and borrowing constraint, and (iii) transaction cost, which we discipline with estimates of the liquidity premium. Table 2 summarizes the calibration.

Standard parameters. We borrow standard parameter values from the literature. We set the discount factor equal to 0.95, which is standard for annual calibrations, and the coefficient of relative risk aversion equal to 2. For the production function, we assume $\gamma = 0.80$ and $\psi = 0.95$ (e.g., Gavazza et al., 2018). Finally, we set the depreciation rate to 5%.

Income process. For the income process, we approximate a stationary AR(1) process with persistence ρ_h and a standard deviation of innovations σ_h using the Rouwenhorst method and assuming that labor h can take two values, interpreted as high and low income states.[22] We select conservative values for the income process and borrowing constraint. Based on the persistence effect decomposition from Proposition 1, that means selecting a relatively low value of ρ_h. Moreover, a small value of σ_h and large absolute value of $\overline{b}$ result in less trade of the illiquid stock, which tends to dampen our results. In particular, we set $\rho_h = 0.5$, $\sigma_h = 0.3$, and a borrowing limit equal to negative one.[23] The resulting standard deviation of log income in

[22]For the transition matrix, we define p as the probability of staying in the same income state. The Rouwenhorst method implies that $p = (1+\rho_h)/2$. For the values of $log(h)$ define $\Sigma_h = \sigma_h/\sqrt{1-\rho_h^2}$. Then $log(h)$ can take on values Σ_h or $-\Sigma_h$. We then normalize h so it has mean one.

[23] The average income is w which is equal to 0.86 in the benchmark calibration. Hence, the household borrowing capacity equals to 116% of the average annual income. As a reference, Kaplan et al. (2018) considers a borrowing constraint of 1 period of labor income but their model is quarterly, so effectively we are allowing for

the model is 0.3. The comparison with the data is not straightforward because we should only consider the fraction of households participating in the stock market, which is less than 50% (Morelli, 2021). To get a rough comparison, the standard deviation and lag-one autocorrelation of residual log income for the U.S. population are 0.9 and 0.8, respectively (Guvenen et al., 2022).

Transaction costs. To discipline the range for the transaction cost parameter ϕ, we rely on estimates of the liquidity premium by van Binsbergen et al. (2022). They estimate a convenience yield of 40 basis points using the put-call parity relationship for European-style options in the S&P500 index. To generate a liquidity premium consistent with this evidence, we consider values of ϕ in $[0, 10]$ in our numerical simulations, which generates a liquidity premium below 50 basis points, consistent with the empirical evidence.[24] Our calibration of the transaction cost is also consistent with high-frequency estimates in Goyenko et al. (2009) and Abdi and Ranaldo (2017).[25]

Non-targeted moments. Although the model is stylized, several non-targeted moments, reported in Table 3, are consistent with the data. First, using data from Gormsen and Huber (2023) we estimate that the corporate discount rate wedge (i.e., the wedge between the discount rate and the cost of capital) is 2.1 percent. Following the decomposition in Section 5.3, the corporate discount rate wedge in the model, κ, is equal to 1.5 percent. Hence, the corporate discount rate wedge is broadly consistent with data.

Second, the variance of log consumption relative to log income, a measure of market incompleteness, is about 0.2 in the model while it is 0.3 in the data (Krueger and Perri, 2006). Hence, the degree of market incompleteness is consistent with the data.

Third, the composition of liquid and illiquid assets is also consistent with the data. The third and fourth rows of Table 3 show that the average ratio of illiquid assets to GDP is 3.5 in the model and 2.9 in the data, while the average ratio of liquid assets is 0.5 in the model and 0.3 in the data. Moreover, both in the model and data, the fraction of savers (i.e., $b_t > 0$) is equal to 50%.

Finally, the share of stock owners that are at the borrowing constraint equals 5.4 percent in the model. We follow Kaplan et al. (2014) and, using the 2019 SCF, we estimate that the corresponding figure is 5.7 percent in the data. Therefore, the model is consistent along several

more borrowing.

[24]Most of the exercises below do not rely on a specific value of ϕ but rather on a range of values as we study the effects of the variation in transaction costs. Nevertheless, we set $\phi = 4$ as a benchmark when needed, as it equals the average cost considered in Heaton and Lucas (1996), and it generates a liquidity premium of 30 basis points, lying on the conservative side of the estimates by van Binsbergen et al. (2022).

[25]Goyenko et al. (2009) measure transaction costs at the trade level using BBO quotes. This represents the spread between the highest bid and lowest ask for a particular stock at a point in time. They use TAQ data which corresponds to the New York Stock Exchange between 1993 and 2005. The estimator proposed by Abdi and Ranaldo (2017) is based on daily CRSP data.

Table 3: Non-targeted Moments

	Model	Data
Corporate discount rate wedge, percent	1.5	2.1
Variance log consumption / variance log income	0.2	0.3
Mean illiquid assets	3.5	2.9
Mean liquid assets	0.5	0.3
Frac. with $b > 0$	0.5	0.5
Stock owners at the borrowing constraint, percent	5.4	5.7

Note: The benchmark model has $\phi = 4$. Means are expressed as ratios to annual output. Corporate discount rates from Gormsen and Huber (2023), consumption and income data from Krueger and Perri (2006), asset data from Kaplan et al. (2018), and stock owners moment from SCF 2019.

dimensions not targeted in the calibration.

6.2 Liquidity and Investment in General Equilibrium

Relative to a complete markets economy, our model features three forces impacting the firm's discount factor: (i) precautionary savings, (ii) transaction costs, and (iii) lack of commitment. We now analyze the importance of each channel numerically.

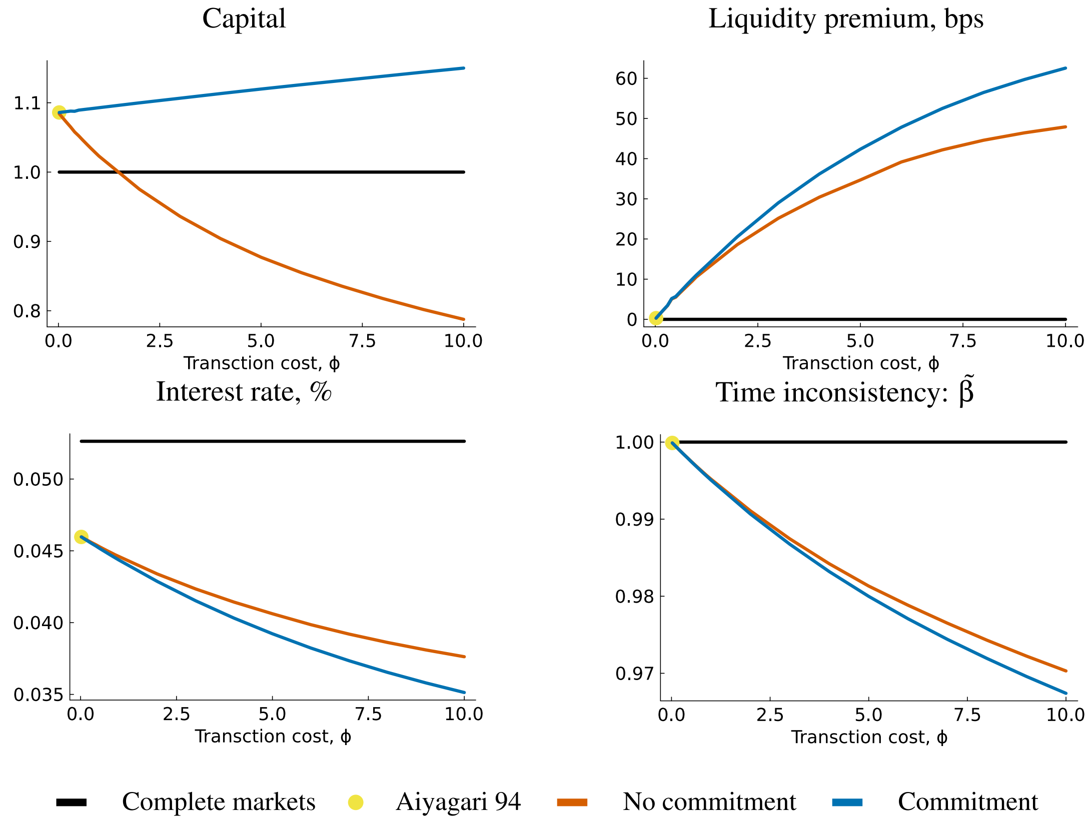
Figure 1 shows the equilibrium allocations for economies with different values of the transaction cost, ϕ. First, the black line shows the solution under complete markets, when the firm discounts at rate $r = 1/\beta - 1$ (independent of ϕ). Second, the yellow point highlights the case with no transaction costs, i.e., $\phi = 0$, but with incomplete markets, so the model is analogous to the economy studied in Aiyagari (1994). The absence of trading frictions implies that the firm's problem is time-consistent. In this economy, the interest rate is lower and capital is higher than with complete markets due to precautionary savings. In our parametrization, capital is 9% higher than under complete markets.

Third, the red line shows the solution when markets are incomplete, $\phi > 0$, and the firm cannot commit to future policies. In this case, capital is decreasing in ϕ. For example, when $\phi = 4$, the liquidity premium is about 30 basis points, and capital is 10% lower than with complete markets. Hence, the combination of transaction costs and lack of commitment generates significant changes in aggregate capital relative to the standard Aiyagari (1994) economy. Lack of commitment is critical for this result. To see this, the blue line shows the solution when the firm can commit. In this case, capital increases moderately in ϕ.

To quantify the effect of the liquidity premium we estimate the elasticity of capital to the liquidity premium. We find that an increase of 10 basis points in the liquidity premium reduces capital by about 7% if firms cannot commit, while it increases by less than 1% if firms can commit.

To understand the discrepancy between firms with and without commitment, consider again Proposition 1. When the firm has commitment, the discount factor is $\tilde{\delta} = (1 - \Phi^b)/(1 + r)$,

Figure 1: Liquidity and Investment in General Equilibrium

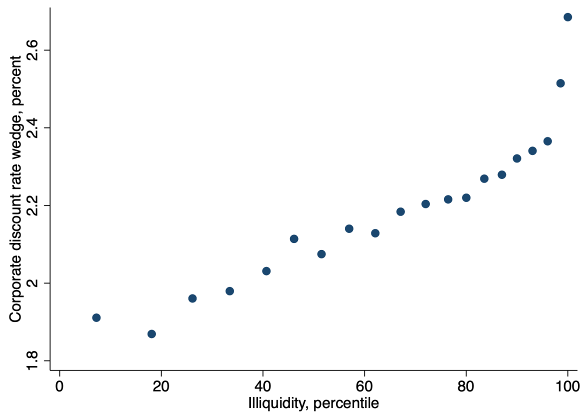


Note: The figures show the allocations under different transaction costs, ϕ.

which, in our numerical exercise, increases modestly in ϕ. The reason is that, as ϕ increases, bonds become more desirable assets than stocks so (i) the liquidity premium increases, but (ii) the the interest rate decreases. Both forces approximately offset each other, with a small net effect on the steady-state level of capital. When the firm lacks commitment, the disagreement among owners is captured by $\tilde{\beta} = \frac{1-\bar{\Phi}}{1-\Phi^b}$, which is decreasing in the trading friction ϕ (see last panel of Figure 1). Because Proposition 2 showed that capital is an increasing function of $\tilde{\beta}$, without commitment, an increase in ϕ decreases capital. Thus, the model predicts that when firms cannot commit, trading frictions can have large and negative effects on the real economy because of present bias. Interestingly, the economy can end up with less capital than with complete markets. That is, the over-accumulation of capital present in Aiyagari (1994) is overturned once we introduce trading frictions and lack of commitment. In contrast, if firms can commit, trading frictions affect asset pricing with almost no consequences for aggregate capital.

Moreover, our model can be used to rationalize the findings in Amihud and Levi (2022), who use cross-sectional data for U.S. public firms to show that there is a negative relation between stock illiquidity and firm investment. Appendix D extends the model to include firms with liquid and illiquid stock. We show that while both the commitment and no-commitment cases can qualitatively replicate the cross-sectional findings in Amihud and Levi (2022), only

Figure 2: Illiquidity and Corporate Discount Rate Wedge



Note: This figure is a binscatter. Each dot is the mean for a bin of each percentile of the relative spread data.

the case without commitment can generate a negative relation between aggregate stock liquidity and aggregate capital in our calibration.

Finally, our results are consistent with the literature on short-termism, which shows that public firms are concerned about meeting short-term targets. Laurence Fink, the CEO of BlackRock, one of the largest money managers, wrote that "the effects of the short-termist phenomenon are troubling ... more and more corporate leaders have responded with actions that can deliver immediate returns to shareholders, such as buybacks or dividend increases, while underinvesting in innovation, killed workforces or essential capital expenditures necessary to sustain long-term growth." Our theory shows that this short-termism can be attributed to trading frictions in financial markets and asset liquidity.

6.3 Liquidity and the Corporate Discount Rate

In this subsection, we show that there is a strong empirical relationship between firms' illiquidity and their discount rate wedge. For the firm's discount rate wedge, we use the recent empirical evidence by Gormsen and Huber (2023). They use corporate conference calls to provide a firm-quarter panel of corporate discount rates and the cost of capital. We need a cross-sectional measure of the liquidity premium, so we rely on data on relative spreads. We follow Naes et al. (2011) and consider daily stock data from the Center for Research in Security Prices (CRSP). For each day, we compute the relative spread as the quoted spread (the difference between the best ask and bid quotes) as a fraction of the midpoint price (the average of the best ask and bid quotes). We then define the quarterly data as the average within the quarter.

We start by showing that the corporate discount rate wedge (discount rate net of cost of capital) and firm's illiquidity are positively correlated. Figure 2 shows a binscatter of relative spreads and corporate discount rate wedges: more illiquid firms are associated with higher discount rate wedges, consistent with the lack of commitment theory presented in this paper.

Next, Table 4 exploits within- and across-firm variation to show the strong relationship between illiquidity and firms' discount rate. The first column of Panel A regresses the corporate discount rate on relative spreads and includes time fixed effects. Hence, this specification focuses on cross-sectional variation across firms, controlling by aggregate variation across quarters. The second column controls for both firm and time fixed effects, which isolates variation within firms over time (controlling for aggregate changes over time). The regressors are standardized, so the coefficients estimate the impact of a 1 standard deviation increase in relative spreads. We find that firms with higher relative spreads have higher discount rates, with coefficients that are statistically significant and economically large. When we look at variation across firms, we find that a 1 standard deviation increase in relative spreads implies about a 50 basis point increase in the corporate discount rate, while within-firm variation implies about a 30 basis point increase.

One potential concern is that the results might be driven by variation in the cost of capital. Liquidity affects the cost of capital, which, in turn, could be passed through to the discount rate. To address this, panel B of Table 4 regresses the discount rate *wedge* on relative spreads. Focusing first on variation across firms (first column), we find that a 1 standard deviation increase in relative spreads implies about a 23 basis point increase in the corporate discount rate wedge. Within-firm variation (second column) implies about an 18 basis point increase. Finally, the third and fourth columns of both panels show that the results are robust to including the market capitalization. These findings alleviate potential concerns that our results are driven by variation in the market capitalization and/or the cost of capital.

To conclude, the empirical evidence is in line with the main prediction of the theory of this paper. There is a strong empirical relationship between firm's illiquidity and the discount rate: more illiquid firms tend to have higher discount rates, consistent with the theory.

6.4 Demand and Supply of Liquidity

We conclude this section with comparative statics with respect to changes in the demand and supply of assets. In particular, we consider the effects of changes in the level of idiosyncratic risk and the supply of government bonds.

Demand for Liquidity. First, we evaluate the role of increasing uncertainty, which is akin to increasing the demand for liquidity. In particular, we consider increases in σ_h, which increases the uncertainty of the income process while keeping the average income and the persistence of the process constant (i.e., a mean preserving spread).

A higher level of uncertainty increases the agents' precautionary savings motive and—as a result—represents a force toward a higher level of capital, as in Aiyagari (1994). This effect dominates when firms can commit to an investment plan. However, when firms cannot commit, more uncertainty implies more trade of the illiquid asset and greater time-inconsistency

Table 4: Relative Spreads and Corporate Discount Rates

Panel A: Corporate Discount Rate

Relative spread	0.509***	0.281***	0.497***	0.278***
	(0.026)	(0.016)	(0.027)	(0.016)
Observations	27163	27158	27163	27158
R-squared	0.236	0.805	0.238	0.805
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

Panel B: Corporate Discount Rate Wedge

Relative spread	0.228***	0.184***	0.230***	0.181***
	(0.016)	(0.012)	(0.016)	(0.012)
Observations	27163	27158	27163	27158
R-squared	0.266	0.668	0.266	0.669
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

*Notes: The dataset is at the firm-quarter level and runs from 2002 to 2021. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase. The specification includes fixed effects for time, or time and firm. Statistical significance is denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.*

problems, which is a force towards a lower level of capital.

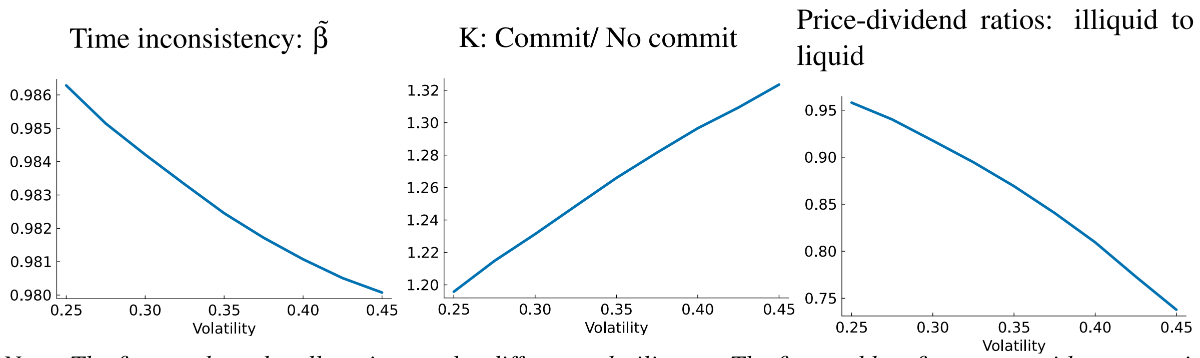
Figure 3 shows how the economy responds to changes in σ_h. The first panel shows that more uncertainty implies more time-inconsistency problems: $\tilde{\beta}$ decreases with volatility, σ_h. The second panel shows the ratio of capital with commitment relative to no commitment. An increase in uncertainty implies more capital with commitment and less without commitment. As a consequence, the capital ratio increases with volatility.

Finally, a useful way to evaluate the return of illiquid and liquid assets is to compare their price-dividend ratios. The price-dividend ratio is equal to $(r+\Phi^b)^{-1}$ for the stock and $(r)^{-1}$ for the bond. Hence, the illiquid-to-liquid ratio of the price-dividend ratios is $\left(1+\frac{\Phi^b}{r}\right)^{-1}$. The third panel of Figure 3 shows that the illiquid-to-liquid ratio of the price-dividend ratios decreases with volatility. These results reflect a "*flight to liquidity*": as uncertainty increases, investors shift their portfolios into liquid assets, consistent with the empirical evidence in Naes et al. (2011).

Supply of Liquidity. Now suppose that the government can issue one-period bonds, denoted by B^G, and it uses lump-sum taxes to pay for the debt services. As B^G increases, the economy has a larger supply of liquid assets.[26] As the supply of bonds increases, bond prices decrease

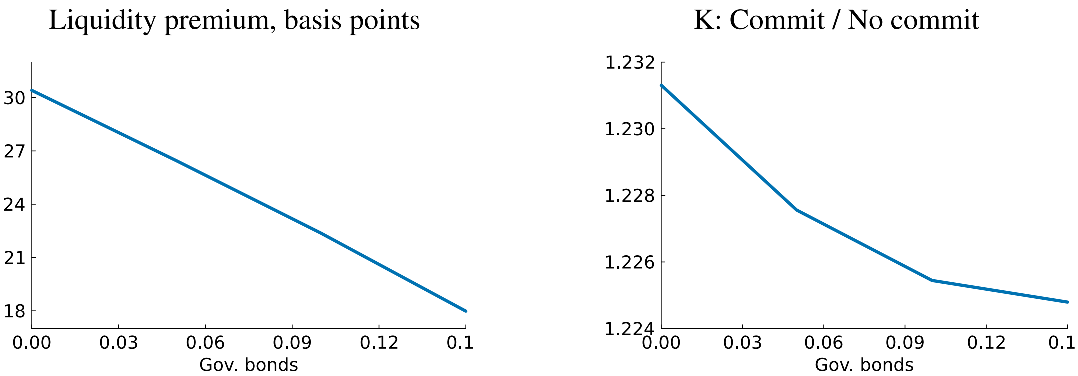
[26] We assume that the return on government bonds is the risk-free rate r. While government bonds trade in over-the-counter markets, short-term U.S. bonds are among the most liquid securities.

Figure 3: Uncertainty



Note: The figures show the allocations under different volatility, σ_h. The first and last figures are without commitment. The middle figure is the ratio of commitment to no commitment.

Figure 4: Shortage of Liquid Assets



Note: The figures show the allocations under different government bonds, B^G.

and the interest rate increases. Not surprisingly, then, the illiquid stock is traded less and the liquidity premium decreases. The first panel of Figure 4 shows this decrease in liquidity premium as the supply of liquid assets increases.

Once again, the effect on aggregate capital depends on the firm's ability to commit. On the one hand, when firms can commit, a higher supply of liquid assets implies a lower residual demand for illiquid assets and—as a result—less capital. On the other hand, when firms cannot commit, a higher supply of liquid assets implies a less severe time-inconsistency problem and—as a result—a higher level of capital. The second panel of Figure 4 shows that the ratio of capital with—relative to without—commitment decreases as the supply of liquid assets increases.

7 Conclusion

This paper studies the implications of trading frictions in financial markets for firm investment and dividend choices. The main result is that when equity shares trade in frictional asset markets, the firm's problem is time-inconsistent, and the empirically relevant direction of inconsistency is present bias. Financial transaction costs cause buyers to value future firm investment more than owners (as if they were more patient) and sellers to prefer immediate dividend distribution (as if they were more impatient). While future investment plans are made based on buyers preferences that are reflected in asset prices, owners (including sellers) each period renege on such plans in favor of high contemporaneous dividend payouts.

When firms are able to commit to future investments, present bias has large effects on asset prices but small consequences on real variables. However, without commitment, firms over-issue dividends and under-invest in capital. The no-commitment case is consistent with recent empirical findings pertaining to the wedge between firms' discount rate and their cost of capital.

While market incompleteness is a natural way to generate investor disagreement, it is not the only one. Any disagreement amongst traders leading to trade drives a transaction cost wedge between buyers and sellers. Transaction costs, on the other hand, are necessary for our results. This is clear from Section 5, where the firm's problem is shown to be time consistent with $\phi = 0$, despite the presence of borrowing constraints and incomplete markets.

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A Proofs

Proof of Lemma 1. In what follows, the dependence of each variable on (θ, b, h) is suppressed for brevity. The first-order condition for Δ_t^+ when household are buyers ($\overline{\eta}_t^- = 0$) is given by $\mu_t = \lambda_t q_t$. The first-order condition for Δ_t^- when households are sellers is given by $\mu_t = \lambda_t q_t(1 - \phi\Delta_t^-) - \overline{\eta}_t^-$. Expression (6) follows from plugging these conditions into the envelope condition $\tilde{q}_t = d_t + \frac{\mu_t + \overline{\eta}_t^-}{\lambda_t}$. $\square$

Proof of Propositions 1 and 4. We prove Proposition 4, which collapses to Proposition 1 when $w(\theta, b, h) = \theta$. Begin with the definitions of Φ^b and $\overline{\Phi}$, suppressing dependence on (θ, b, h) whenever it does not lead to confusion.

$$
\begin{aligned}
\Phi^b =& \phi \left[\mathbb{E}_t[\Delta_{t+1}^-] + \frac{\mathrm{cov}_t(u'(\mathscr{C}_{t+1}), \Delta_{t+1}^-)}{\mathbb{E}_t[u'(\mathscr{C}_{t+1})]} \right] \\
\overline{\Phi} =& \phi \sum_h \int_\theta \int_b w(\theta, b, h) \Delta_t^-(\theta, b, h) d\Gamma(\theta, b, h)
\end{aligned}
$$

By stationarity, $\overline{\Phi}$ also equals:

$$
\begin{aligned}
\overline{\Phi} =& \phi \sum_{h'} \int_{\theta_{t+1}} \int_{b_{t+1}} w(\theta_{t+1}, b_{t+1}, h') \Delta_{t+1}^-(\theta_{t+1}, b_{t+1}, h') d\Gamma(\theta_{t+1}, b_{t+1}, h') \\
=& \phi \sum_h \sum_{h'} \int_\theta \int_b w(\Theta_{t+1}, \mathscr{B}_{t+1}, h') \Delta_{t+1}^-(\Theta_{t+1}, \mathscr{B}_{t+1}, h') dF(h'|h) d\Gamma(\theta, b, h)
\end{aligned}
$$

where the second equality follows from plugging in policy functions Θ_{t+1} and $\mathscr{B}_{t+1}$. Because we focus on equilibria where buyers are unconstrained, Φ^b does not depend on (θ, b, h) for buyers. Hence we make use of the following tautology:

$$
\Phi^b = \frac{\sum_h \sum_{h'} \int_\theta \int_b w(\Theta_{t+1}, \mathscr{B}_{t+1}, h') \Phi^b 1_{\{\Delta_t^+ > 0\}} dF(h'|h) d\Gamma(\theta, b, h)}{\sum_h \sum_{h'} \int_\theta \int_b w(\Theta_{t+1}, \mathscr{B}_{t+1}, h') 1_{\{\Delta_t^+ > 0\}} dF(h'|h) d\Gamma(\theta, b, h)}
$$

We call the denominator B. Now take the difference between Φ^b and $\overline{\Phi}$:

$$
\begin{aligned}
&\Phi^b - \overline{\Phi} \\
=& \frac{1}{B} \sum_h \sum_{h'} \int_\theta \int_b w(\Theta_{t+1}, \mathscr{B}_{t+1}, h') \Phi^b 1_{\{\Delta_t^+ > 0\}} dF(h'|h) d\Gamma(\theta, b, h) - \overline{\Phi} \\
=& \frac{\phi}{B} \sum_h \sum_{h'} \int_\theta \int_b w(\Theta_{t+1}, \mathscr{B}_{t+1}, h') \mathbb{E}_t[\Delta_{t+1}^-] 1_{\{\Delta_t^+ > 0\}} dF(h'|h) d\Gamma(\theta, b, h) \\
&+ \frac{\phi}{B} \sum_h \sum_{h'} \int_\theta \int_b w(\Theta_{t+1}, \mathscr{B}_{t+1}, h') \frac{cov_t(u'(\mathscr{C}_{t+1}), \Delta_{t+1}^-)}{\mathbb{E}_t[u'(\mathscr{C}_{t+1})]} 1_{\{\Delta_t^+ > 0\}} dF(h'|h) d\Gamma(\theta, b, h) \\
&- \phi \sum_h \sum_{h'} \int_\theta \int_b w(\Theta_{t+1}, \mathscr{B}_{t+1}, h') \Delta_{t+1}^- dF(h'|h) d\Gamma(\theta, b, h)
\end{aligned}
\tag{20}
$$

where we have plugged in the definitions of Φ^b and $\overline{\Phi}$. Notice that the second term in condition

(20) is the risk premium. Focusing on the first and third term in (20):

$$
\begin{aligned}
&\text{Persistence effect} + \text{Weight covariance} \\
=&\phi \sum_h \sum_{h'} \int_\theta \int_b w_{t+1} \mathbb{E}_t[\Delta^-_{t+1}] \frac{1_{\{\Delta^+_t > 0\}}}{B} dF(h'|h) d\Gamma(\theta, b, h) \\
-&\phi \sum_h \int_\theta \int_b (\mathbb{E}_t[w_{t+1}]\mathbb{E}_t[\Delta^-_{t+1}] + cov_t(w_{t+1}, \Delta^-_{t+1})) d\Gamma(\theta, b, h) \\
=&\phi \left(\widetilde{\mathbb{E}}_t[\mathbb{E}_t[\Delta^-_{t+1}] \mid \text{buyer}] - \widetilde{\mathbb{E}}_t[\mathbb{E}_t[\Delta^-_{t+1}]] \right) - \phi \sum_h \int_\theta \int_b cov_t(w_{t+1}, \Delta^-_{t+1}) d\Gamma(\theta, b, h)
\end{aligned}
$$

This completes the proof of Proposition 4. To specialize this result to Proposition 1, let $w(\Theta_{t+1}, \mathscr{B}_{t+1}, h') = \Theta_{t+1}$, which is not measurable with respect to h'. This makes the weight covariance equal to zero. □

Proof of Proposition 2. We prove the proposition for a more general firm problem with any twice-differentiable production function $y_t = f(l_t, k_t)$ that is weakly concave in both variables (jointly), and strictly concave in k_t. Profits are given by $\pi_t = y_t - w_t l_t$ so the intratemporal decision satisfies

$$f_1(l_t, k_t) = w_t \tag{21}$$

and the subscript denotes the partial derivative. Condition (21) implicitly defines the labor function, which we denote $l(k_t)$. Next we solve the firm's intertemporal problem with no commitment. The first-order condition of the firm's problem is given by

$$F_2(k, k') + \tilde{\beta}\tilde{\delta} W'(k') = 0 \tag{22}$$

where

$$W'(k') \equiv F_1(k', g(k')) + g'(k') \left(F_2(k', g(k')) + \tilde{\delta} W'(g(k')) \right) \tag{23}$$

First we guess that $g'(k') = 0$. Plugging in the function $F(k, k') = f(l(k), k) - wl(k) + (1 - \delta)k - k'$:

$$\frac{1}{\tilde{\beta}\tilde{\delta}} = \pi'(k') + 1 - \delta$$

where $\pi'(k) = f_1(H, k) l_1(k) + f_2(H, k) - w l_1(k) = f_2(H, k)$ and the second equality follows from the intratemporal condition (21), and H is the unconditional expectation of the labor process. Because f is strictly concave in k, $\pi'(k)$ is a strictly decreasing function. This yields the steady-state level of capital $k^N = (\pi')^{-1} \left(\frac{1}{\tilde{\beta}\tilde{\delta}} - 1 + \delta \right)$ where π' has an inverse because π' was shown to be strictly decreasing. By the same reasoning, $(\pi')^{-1}$ must be strictly decreasing. Similar logic for the commitment case yields $k^C = (\pi')^{-1} \left(\frac{1}{\tilde{\delta}} - 1 + \delta \right)$ and so we have shown that $k^N < k^C$ if and only if the firm has present bias, $\tilde{\beta} < 1$. Now say $g'(k') \neq 0$, and one of two scenarios can occur. First, (22) and (23) could pin down an optimal value of k' (notice that this value does not depend on the value of k). But since $g'(k') \neq 0$, $g(k')$ is not a constant function. By definition of MPE, $\zeta(k)$ is also not a constant function, which contradicts the optimal choice k' (which was irrespective of the value of k). Second, we cannot rule out a policy $g(k')$ where (22) and (23) are satisfied for any value of k'. Such a $g(k')$ would imply that any $\zeta(k)$ is optimal, and in particular one could select $\zeta(k) = g(k)$. Now consider a resource-exhausting finite-horizon version of the same problem.[27] Because there is zero investment in the final period, the firm objective in the preceding period would be

$$F(k,k') + \tilde{\beta}\tilde{\delta}F(k',0)$$

and first-order conditions yield k^N. Using backward induction, the firm in each preceding period would also choose k^N and hence our claim is proved. $\square$

Proof of Proposition 3. The problem of the firm that can borrow up to a limit is given by

$$V^F(k_t, b_t^-, b_t^+) = \max_{\{k_{t+s}, b_{t+s}^-, b_{t+s}^+\}_{s\geq 1}} d_t + \tilde{\beta}\sum_{s=1}^{\infty}\tilde{\delta}^s d_{t+s}$$

subject to

$$d_t = F(k_t, k_{t+1}) + \frac{b_{t+1}^-(1-\tilde{\phi})}{1+r} - b_t^- - \frac{b_{t+1}^+}{1+r} + b_t^+, \quad 0 \leq b_{t+1}^- \leq \overline{b}, \quad 0 \leq b_{t+1}^+$$

The firm can borrow the amount b_{t+1}^-, up to a limit $\overline{b}$. The parameter $\tilde{\phi} \geq 0$ captures the illiquidity of corporate bonds. Firms can save the amount b_{t+1}^+ at the interest rate r. First-order conditions for capital do not change from Proposition 2. First-order conditions for bonds are

$$\begin{aligned}
(b_{t+1}^-) &: \frac{1-\tilde{\phi}}{1+r} - \tilde{\beta}\tilde{\delta} + \mu_t^1 - \mu_t^2 = 0 \\
(b_{t+s}^-) &: \frac{1-\tilde{\phi}}{1+r} - \tilde{\delta} + \mu_{t+s-1}^1 - \mu_{t+s-1}^2 = 0 \\
(b_{t+1}^+) &: -\frac{1}{1+r} + \tilde{\beta}\tilde{\delta} + \mu_t^3 = 0 \\
(b_{t+s}^+) &: -\frac{1}{1+r} + \tilde{\delta} + \mu_{t+s-1}^3 = 0
\end{aligned}$$

where μ_t^1, μ_t^2, and μ_t^3 are the Lagrange multipliers on the three inequality constraints, respectively. Because $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} < \frac{1}{1+r}$ and $\tilde{\delta} = \frac{1-\Phi^b}{1+r} < \frac{1}{1+r}$, we have that $b_{t+s}^+ = 0$ for all $s \geq 1$. In other words, no savings occurs. The firm's borrowing decision depends on the illiquidity parameter $\tilde{\phi}$. We consider the empirically relevant case of present bias, $\overline{\Phi} > \Phi^b$.

<u>Case 1</u>: $\tilde{\phi} > \overline{\Phi}$ and, hence, corporate bonds are extremely illiquid. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} > \frac{1-\tilde{\phi}}{1+r}$ and $\tilde{\delta} = \frac{1-\Phi^b}{1+r} > \frac{1-\tilde{\phi}}{1+r}$ so $b_{t+s}^- = 0$ for all $s \geq 1$.

[27] Laibson (1997) uses the same equilibrium refinement.

Case 2: $\tilde{\phi} < \Phi^b$ and, hence, corporate bonds are extremely liquid. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} < \frac{1-\tilde{\phi}}{1+r}$ and $\tilde{\delta} = \frac{1-\Phi^b}{1+r} < \frac{1-\tilde{\phi}}{1+r}$ so $b^-_{t+s} = \overline{b}$ for all $s \geq 1$.

Case 3a: $\Phi^b = \tilde{\phi} < \overline{\Phi}$ and, hence, corporate bonds are characterized by intermediate liquidity. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} < \frac{1-\tilde{\phi}}{1+r}$ yet $\tilde{\delta} = \frac{1-\Phi^b}{1+r} = \frac{1-\tilde{\phi}}{1+r}$, so $b^-_{t+1} = \overline{b}$ yet $0 \leq b^-_{t+s} \leq \overline{b}$ for $s > 1$. This problem is time-inconsistent.

Case 3b: $\Phi^b < \tilde{\phi} < \overline{\Phi}$ and, again, corporate bonds are characterized by intermediate liquidity. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} < \frac{1-\tilde{\phi}}{1+r}$ yet $\tilde{\delta} = \frac{1-\Phi^b}{1+r} > \frac{1-\tilde{\phi}}{1+r}$, so $b^-_{t+1} = \overline{b}$ yet $b^-_{t+s} = 0$ for $s > 1$. This problem is time-inconsistent.

Case 3c: $\Phi^b < \tilde{\phi} = \overline{\Phi}$ and, again, corporate bonds are characterized by intermediate liquidity. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} = \frac{1-\tilde{\phi}}{1+r}$ yet $\tilde{\delta} = \frac{1-\Phi^b}{1+r} > \frac{1-\tilde{\phi}}{1+r}$, so $0 \leq b^-_{t+1} \leq \overline{b}$ yet $b^-_{t+s} = 0$ for $s > 1$. This problem is time-inconsistent.

In the time-inconsistent parameter range, we must consider a MPE when the firm cannot commit. Letting $g(b)$ denote the policy function, the first-order condition with respect to b^-_{t+1} is

$$\frac{1-\tilde{\phi}}{1+r} + \mu^1 - \mu^2 + \tilde{\beta}\tilde{\delta}W'(b') = 0$$

where $W'(b') \equiv -1 + g'(b')\left[\frac{1-\tilde{\phi}}{1+r} + \mu^{1'} - \mu^{2'} + \tilde{\delta}W'(g(b'))\right]$ and one of the solutions is found by setting $g'(b') = 0$. This collapses the first-order condition into those of Case 3 above. One can apply similar arguments to those in Proposition 2 to show that this is, in fact, the only equilibrium that is the limit of a finite-horizon version of the same problem. □

B Generalized Weights

B.1 Theoretical Analysis

In this appendix, we generalize Assumption 1 to encompass three important voting rules: that of Grossman and Hart (1979), Dreze (1974), and DeMarzo (1993). Grossman and Hart (1979) is the familiar case, discussed in the main text, of the firm maximizing an ownership-weighted valuation. Dreze (1974) makes a similar assumption, except that the weighting is done by *the next period's* (that is, future) ownership. Finally, DeMarzo (1993) suggests a majority voting rule that works in the following way. A majority stable allocation is defined by the absence of an alternative allocation preferred by at least half of the shares. Due to the continuum of traders in our setting, for tractability, we strengthen this definition to the absence of any alternative allocation preferred by *more than* half of the shares.

Assumption 3. *The firm maximizes a weighted valuation given by:*

$$\int_{\theta,b,h} w(\theta,b,h)\left[d_t+(1-\phi\Delta_t^-(\theta,b,h))q_t\right]d\Gamma_t(\theta,b,h)=d_t+(1-\overline{\Phi}_t)q_t, \tag{24}$$

where $\Gamma_t(\theta,b,h)$ denotes the cross-section distribution over the portfolio holdings and employment status, $d_t=F_t(k_t,k_{t+1})$, and

$$\overline{\Phi}_t\equiv\phi\int_{\theta,b,h} w(\theta,b,h)\Delta_t^-(\theta,b,h)d\Gamma_t(\theta,b,h). \tag{25}$$

Weights $w(\theta,b,h)$ are assumed to be non-negative and integrate to one. Current shareholder weighting from Grossman and Hart (1979) is given by $w(\theta,b,h)=\theta$, and future shareholder weighting from Dreze (1974) is given by $w(\theta,b,h)=\Theta_{t+1}(\theta,b,h)$. For the case of DeMarzo (1993), consider the median m satisfying

$$\sum_h\int_\theta\int_b 1_{\{\Delta_t^-(\theta,b,h)\le m\}}\theta\, d\Gamma_t(\theta,b,h)=\frac{1}{2}$$

and let

$$w(\theta,b,h)=\begin{cases}k \text{ if } \Delta_t^-(\theta,b,h)=m\\ 0 \text{ otherwise}\end{cases}$$

where $k>0$ is chosen so that weights integrate to one. Then the optimum of (24) satisfies majority stability. To see why, fix two households such that $\Delta_t^-(\overline{\theta},\overline{b},\overline{h})>m$ and $\Delta_t^-(\underline{\theta},\underline{b},\underline{h})<m$. Then if the above-median household prefers a particular deviation $(\tilde{d}_t,\tilde{q}_t)$,

$$\tilde{d}_t+(1-\phi\Delta_t^-(\overline{\theta},\overline{b},\overline{h}))\tilde{q}_t>d_t+(1-\phi\Delta_t^-(\overline{\theta},\overline{b},\overline{h}))q_t$$

where $(\tilde{d}_t,\tilde{q}_t)$ satisfy constraints of the firm problem and $(d_t,q_t)\in\text{argmax } d_t+(1-\phi m)q_t$ subject to constraints of the firm problem, then the below-median household must not,

$$\tilde{d}_t+(1-\phi\Delta_t^-(\underline{\theta},\underline{b},\underline{h}))\tilde{q}_t<d_t+(1-\phi\Delta_t^-(\underline{\theta},\underline{b},\underline{h}))q_t$$

For if this were not the case, we could take an appropriate convex combination of the two inequalities above to contradict the initial optimality of (d_t,q_t). Similar arguments apply to deviations by the below-median household.

Next we show that our theoretical results from the main text hold under this generalized setup. This next proposition generalizes Proposition 1, which decomposed the sources of time inconsistency.

Proposition 4. *The difference $\Phi^b-\overline{\Phi}$ is equal to the sum of a* persistence effect, *a* risk premium, *and a* weight covariance*:*

$$\Phi^b - \bar{\Phi} = \underbrace{\phi\left(\widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\middle|\, buyer\right] - \widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\right]\right)}_{persistence\ effect}$$
$$+\underbrace{\phi\widetilde{\mathbb{E}}_t\left[\frac{\mathrm{cov}_t\left(u'\left(\mathscr{C}_{t+1}\right),\Delta^-_{t+1}\right)}{\mathbb{E}_t\left[u'\left(\mathscr{C}_{t+1}\right)\right]}\middle|\, buyer\right]}_{risk\ premium} \underbrace{-\phi\sum_h\int_\theta\int_b \mathrm{cov}_t(w_{t+1},\Delta^-_{t+1})d\Gamma(\theta,b,h)}_{weight\ covariance}$$

where tilde moments are taken with respect to the cross-sectional weighted density $w(\Theta_{t+1},\mathscr{B}_{t+1},h')$ $dF(h'|h)d\Gamma(\theta,b,h)$*, and the non-tilde moments are taken with respect to the density* $dF(h'|h)$*.*

Just like in the special case of Grossman and Hart (1979) from the main text, Proposition 4 shows that time-inconsistency can only emerge in an economy with transaction costs, i.e., $\phi > 0$. The new term in the decomposition above is the weight covariance. Recall that in the Grossman and Hart (1979) setup, $w(\theta,b,h) = \theta$. In this case, $w_{t+1}(\Theta_{t+1},\mathscr{B}_{t+1},h') = \Theta_{t+1}(\theta,b,h)$, which is not measurable with respect to h'. This makes the weight covariance equal to zero. The weight covariance only appears under more complex weighting schemes such as that of Dreze (1974). In the Dreze (1974) case, the weight covariance is negative because those who sell (Δ^-_{t+1} large) have smaller portfolios (Θ_{t+2} small) by construction. This represents a force towards future bias.

In the case of majority voting from DeMarzo (1993), instead of studying the decomposition in Proposition 4, we directly consider whether the median shareholder is a seller. In numerical simulations, we find that sixty to seventy percent of shares belong to sellers. That is, a large number of households sell small amounts, while a small number of households buy large amounts; this is partly due to the quadratic nature of transaction costs. The result is a large value of $\bar{\Phi}$. When buyers tend to stay buyers, Φ^b is small; altogether we get a force towards present bias.

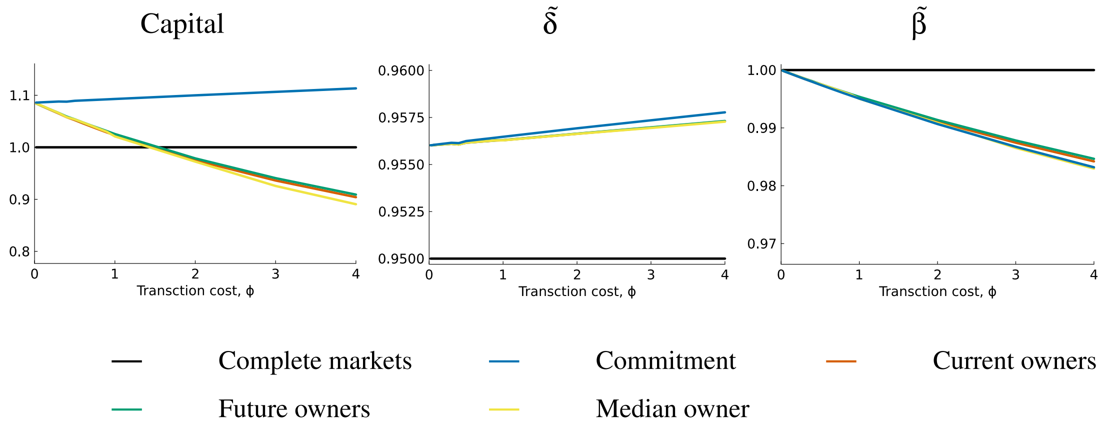
The net effect of all three competing forces depends on parameter values. However, in our numerical simulations, we find that the risk premium and weight covariance are negligible in comparison to the persistence effect, leading to present bias.

B.2 Quantitative Analysis

In this section, we quantitatively solve the general equilibrium model under alternative assumptions for the weights in the firms' problem. First, we consider the case with commitment. Note that the solution with commitment is independent of the specific weights assumed in the problem of the firm. The reason is that the specific weights matter for $\tilde{\beta}$ but not for $\tilde{\delta}$, and with commitment the firm does not use $\tilde{\beta}$. Second, we solve the model without commitment for three alternative weights: (i) current owners (the benchmark model, Grossman and Hart, 1979), (ii) future owners (Dreze, 1974), and (iii) median owner (DeMarzo, 1993).

Figure 5 shows the equilibrium allocations for capital, and the discount factors $\tilde{\delta}$ and $\tilde{\beta}$ for economies with different values of the transaction cost, ϕ. The solution with commitment as well as the solution without commitment and current owners' weight correspond to the benchmark case considered in the main text. The main result of this exercise is that both the case of future owners as well as the median owner generates qualitatively the same results of present-bias and under-accumulation of capital due to trading frictions and lack of commitment.

Figure 5: Generalized Weights



Note: The figures show the allocations under different transaction costs, ϕ.

C 3-Period Model

C.1 Stationary Model

Consider a stationary version of the 3-period model laid out in Section 3. Like before, there are only three time periods $t = 0,1,2$ and $c_t = 0$ for $t \geq 3$; however $q_2 \neq 0$. The reason households hold stock beyond period 2 is due to an exogenous "continuation value" we append to the household's objective. This trick allows us to induce a stationary 3-period equilibrium with constant asset prices. The household objective becomes

$$\sum_{t=0}^{2} \beta^t u(c_t^j) + \beta^2 \mu^j \theta_3^j$$

The Euler equation in $t = 2$ becomes

$$q_2 u'(c_2^j)(1 - \phi \Delta_2^{j,-}) = \mu^j$$

If μ^j and initial assets θ_0^j are selected appropriately, then the resulting household equilibrium will be stationary. In this stationary equilibrium, consumption will simply fluctuate (i.e. c^l, c^h, c^l), asset sales will also fluctuate (i.e. $\Delta^-, 0, \Delta^-$), and asset prices will be a constant q.

Using this notation, the required μ^j are given by

$$\mu^l = \beta u'(c^h)(d+q), \text{ and } \mu^h = \beta u'(c^l)(d+(1-\phi\Delta^-)q)$$

We now re-solve the numerical example in Section 3 with all the same parameter values but one. Moving to this stationary setup, asset prices are larger in magnitude (because we mimic the first-order conditions of an infinite-horizon problem). With larger asset prices, asset trades are smaller and time-inconsistency issues become more difficult to discern. For no other reason than to reduce asset prices, we let $d_t = 0.1$ for all t. The household equilibrium is summarized in Table 5, and the stationary asset price is given by $q = 1.9480$.

Table 5: Stationary Equilibrium

Household type	c_0	c_1	c_2	Δ_0^-	Δ_1^-	Δ_2^-	θ_0
low	2.0385	2.1389	2.0385	0.4810	0.0000	0.4810	1.2405
high	2.1389	2.0385	2.1389	0.0000	0.4810	0.0000	0.7595

The problem of the firm is defined as before, which, in period 0, becomes

$$V_0^F(k_0) = \max_{k_1,k_2\geq 0} d_0 + 0.9401 d_1 + 0.9025 d_2$$

and, in period 1, becomes

$$V_1^F(k_1) = \max_{k_2\geq 0} d_1 + 0.9401 d_2$$

Due to the stationary nature of our new equilibrium, the one-period-ahead discount rate is always the same, 9.401, and the two-period-ahead discount rate is simply β^2 (that is, consumption at time t and time $t+2$ agree). From its period 0 perspective, the firm discounts between time $t=1$ and $t=2$ at the rate 0.9600, which is the ratio between 0.9025 and 0.9401. When period 1 arrives, the firm discounts between time $t=1$ and $t=2$ at the rate 0.9401. Hence the problem of the firm is time-inconsistent and, like in Section 3, the direction of inconsistency is towards present bias.

C.2 Symmetric Transaction Costs

Consider a 3-period economy, as in Section 3, but with symmetric transaction costs. That is, the budget constraint (2) now has one additional term $\frac{\phi}{2}(\Delta_t^+)^2 q_t$, which denotes the transaction cost incurred for buying the stock. The new budget constraint is

$$c_t + q_t\Delta_t^+ + \frac{b_{t+1}}{1+r_t} \leq w_t h_t + d_t\theta_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2 - \frac{\phi}{2}(\Delta_t^+)^2\right) q_t + b_t$$

Table 6: Equilibrium with Symmetric Transaction Costs

Household type	c_0	c_1	c_2	Δ_0^-	Δ_1^-	θ_1	θ_2
low	2.5939	2.7925	2.5635	0.3253	0.0000	0.6747	1.5635
high	3.3865	3.1324	3.4365	0.0000	0.8888	1.3253	0.4365

and the Euler equation of the household becomes

$$(1+\phi\Delta_t^j)q_t = \beta\frac{u'(c_{t+1}^j)}{u'(c_t^j)}d_{t+1} + \beta\frac{u'(c_{t+1}^j)}{u'(c_t^j)}(1+\phi\Delta_{t+1}^j)q_{t+1}, \quad t \in \{0,1\}$$

where $\Delta_t^j \equiv (\Delta_t^{j+} - \Delta_t^{j-})$. Intuitively, buyers now have an even higher valuation of the stock than in the asymmetric setting because they must incur a transaction cost to buy. Calculations similar to those of Lemma 1 yield a firm objective equal to

$$\sum_{j\in\{l,h\}}\frac{\theta_t^j}{2}[d_t + (1+\phi\Delta_t^j)q_t]$$

With this new setup, we solve a numerical example using the same parameter values as in Section 3. The household equilibrium is summarized in Table 6, and asset prices are given by $q_0 = 1.8560$ and $q_1 = 0.9504$.

The problem of the firm in period 0 becomes

$$V_0^F(k_0) = \max_{k_1,k_2\geq 0} d_0 + 0.9547d_1 + 0.9013d_2$$

and the problem of the firm in period 1 becomes

$$V_1^F(k_1) = \max_{k_2\geq 0} d_1 + 0.9229d_2$$

so that, from its period 0 perspective, the firm discounts between time $t=1$ and $t=2$ at the rate 0.9441, which is the ratio between 0.9013 and 0.9547. When period 1 arrives, the firm discounts between time $t=1$ and $t=2$ at the rate 0.9229. Hence the problem of the firm remains time-inconsistent, with a direction of inconsistency towards present bias. Transaction costs on buyers further restrict asset trading (beyond the case of asymmetric transaction costs), which creates larger differences in IMRS across agents; this exacerbates problems of time-inconsistency.

D Heterogeneous Firms: Liquid and Illiquid

In this appendix we extend the model to have heterogeneous firms. We quantitatively analyze the agregate implications and show that the model is consistent with the cross-sectional evidence.

D.1 Model

Consider an economy with both liquid and illiquid stocks. To make the comparison stark, we assume that liquid stocks trade in markets with zero transaction costs but are still subject to the same short-selling constraint. The proportion of liquid stocks is $(1-\omega)$, and the proportion of illiquid stocks is ω, so that the total mass of stocks is still normalized to one. The special case of $\omega = 1$ corresponds to our main model.

With two types of firms in the economy (with potentially differing discount rates), we must restrict our analysis to decreasing returns technologies to retain an interior solution to both firms' problems. The household budget constraint becomes

$$c_t + q_t \Delta_t^+ + \frac{b_{t+1}}{1+r_t} + \hat{q}_t \hat{\theta}_{t+1} \leq w_t h_t + d_t \theta_t + (\hat{d}_t + \hat{q}_t)\hat{\theta}_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t + b_t$$

where $\hat{q}_t$ denotes the price of the liquid stock, $\hat{\theta}_{t+1}$ denotes the amount of liquid stock purchased, and $\hat{d}_t$ denotes dividends paid by the liquid firm. Beyond the standard constraints on illiquid stocks, we have the short-selling constraint on the liquid stock and the borrowing constraint

$$\hat{\theta}_{t+1} \geq 0, \quad b_{t+1} \geq \overline{b} \tag{26}$$

To simplify this problem, we note that liquid stocks and bonds are equivalent, and hence we can combine them into one state variable $W_{t+1} \equiv (\hat{d}_{t+1} + \hat{q}_{t+1})\hat{\theta}_{t+1} + b_{t+1}$. so that the budget constraint can be rewritten

$$c_t + q_t \Delta_t^+ + \frac{W_{t+1}}{1+r_t} \leq w_t h_t + d_t \theta_t + W_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t.$$

The constraints (26) can be combined

$$W_{t+1} \geq \overline{b}$$

Market clearing conditions for bonds, illiquid stocks, and labor become

$$\int_{j\in[0,1]} W_{j,t} dj = (1-\omega)(\hat{d}_t + \hat{q}_t), \quad \int_{j\in[0,1]} \theta_{j,t} dj = \omega, \quad \int_{j\in[0,1]} h_{j,t} dj = \omega l_t + (1-\omega)\hat{l}_t.$$

On the first line above, the equality follows from the fact that the proportion of liquid stocks is $(1-\omega)$. On the third line above, $\hat{l}_t$ denotes labor demanded by the liquid firm, which is defined by the intratemporal first-order condition, $\hat{l}_t = \psi\gamma\frac{\hat{y}_t}{w_t}$, where $\hat{y}_t = [(\hat{l}_t)^{\gamma}(\hat{k}_t)^{1-\gamma}]^{\psi}$. Labor demanded by the illiquid firm, l_t, is defined by similar conditions, $l_t = \psi\gamma\frac{y_t}{w_t}$, where $y_t = [(l_t)^{\gamma}(k_t)^{1-\gamma}]^{\psi}$. Combining the expression for output with the optimality condition for labor, we get $l_t = (\gamma\psi)^{\frac{1}{1-\gamma\psi}} \frac{k_t^{\frac{(1-\gamma)\psi}{1-\gamma\psi}}}{w_t^{\frac{1}{1-\gamma\psi}}}$, and $\hat{l}_t = (\gamma\psi)^{\frac{1}{1-\gamma\psi}} \frac{\hat{k}_t^{\frac{(1-\gamma)\psi}{1-\gamma\psi}}}{w_t^{\frac{1}{1-\gamma\psi}}}$. Thus, market clearing in the labor market implies

$$\omega l_t + (1-\omega)\hat{l}_t = H = \omega(\gamma\psi)^{\frac{1}{1-\gamma\psi}} \frac{k_t^{\frac{(1-\gamma)\psi}{1-\gamma\psi}}}{w_t^{\frac{1}{1-\gamma\psi}}} + (1-\omega)(\gamma\psi)^{\frac{1}{1-\gamma\psi}} \frac{\hat{k}_t^{\frac{(1-\gamma)\psi}{1-\gamma\psi}}}{w_t^{\frac{1}{1-\gamma\psi}}}$$

or

$$w_t = \frac{\gamma\psi}{H^{1-\gamma\psi}} \left(\omega k_t^{\frac{(1-\gamma)\psi}{1-\gamma\psi}} + (1-\omega)\hat{k}_t^{\frac{(1-\gamma)\psi}{1-\gamma\psi}}\right)^{1-\gamma\psi}.$$

Now, let's use the optimality conditions for capital. Focusing on a steady-state equilibrium, the analysis in Section 5, implies

$$M\left(z\alpha k^{\alpha-1} + (1-\delta)\right) = 1$$

for the illiquid firms, and

$$\hat{M}\left(z\alpha \hat{k}^{\alpha-1} + (1-\delta)\right) = 1$$

for the liquid firms, where $M = \tilde{\delta}$ if the firms can commit to an investment plan and $M = \tilde{\beta}\tilde{\delta}$ if they cannot, $\hat{M} = \frac{1}{1+r}$, and $z = (1-\gamma\psi)\left(\frac{\gamma\psi}{w}\right)^{\frac{\gamma\psi}{1-\gamma\psi}} = \frac{(1-\gamma\psi)H^{\gamma\psi}}{\left(\omega k^{\frac{(1-\gamma)\psi}{1-\gamma\psi}} + (1-\omega)\hat{k}^{\frac{(1-\gamma)\psi}{1-\gamma\psi}}\right)^{\gamma\psi}}$. That is, by no-arbitrage, the liquid stock has the same return as the bond, so it discounts the dividends it generates at the risk-free rate $\frac{1}{1+r}$, with standard exponential discounting. In contrast, the discount factor of illiquid firms is $\frac{1-\Phi^b}{1+r} < \frac{1}{1+r}$ with commitment, and $\frac{1-\overline{\Phi}}{1+r} < \frac{1}{1+r}$ without commitment.

Combining the two optimality conditions, we get $\hat{k} = \left(\frac{\frac{1}{M}-(1-\delta)}{\frac{1}{\hat{M}}-(1-\delta)}\right)^{\frac{1}{1-\alpha}} k$. Since, $M < \hat{M}$, then $\hat{k} > k$. To solve for k, we plug the expressions for z and $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$ into the optimality condition for illiquid firms,

$$k = \left[\frac{1}{\left(\omega + (1-\omega)\left(\frac{\frac{1}{M}-(1-\delta)}{\frac{1}{\hat{M}}-(1-\delta)}\right)^{\frac{(1-\gamma)\psi}{1-\psi}}\right)^{\gamma\psi}} \frac{(1-\gamma)\psi M}{1-M(1-\delta)} H^{\gamma\psi}\right]^{\frac{1}{1-(1-\gamma)\psi}}.$$

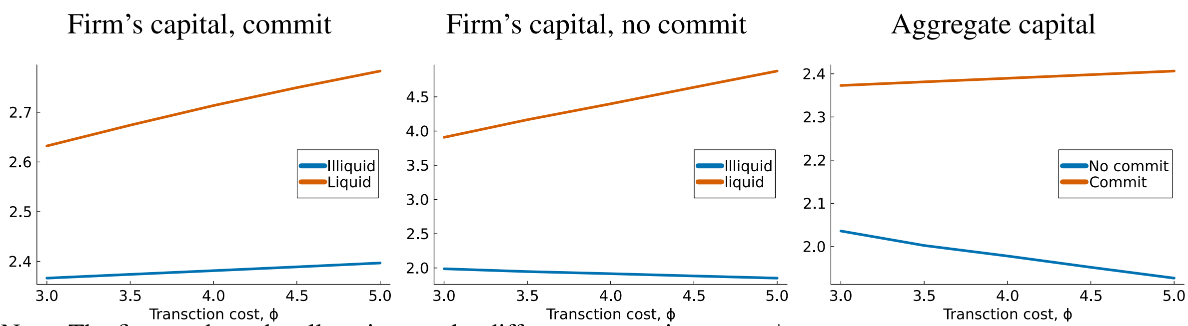
Thus, firms with illiquid stock always have less capital than firms with liquid stock. This result is consistent with Amihud and Levi (2022), who use cross-sectional data for US public firms to compare liquid and illiquid firms and conclude that illiquid firms invest less than liquid ones. Next, we quantitatively analyze this extension.

D.2 Quantitative Results

The first two panels of Figure 6 confirm our theoretical results about the cross-section.[28] The first panel shows that, in the economy with commitment, firms with illiquid stock have less capital than firms with liquid stock. Moreover, the second panel shows that the difference in capital is more pronounced if the illiquid firms cannot commit. These results are consistent with Amihud and Levi (2022), who use cross-sectional data for US public firms to compare liquid and illiquid firms and conclude that illiquid firms invest less than liquid ones.

It is important to note that the cross-sectional evidence is not enough to understand the *aggregate* effects of liquidity on investment. When firms can commit to an investment plan, an aggregate increase in transaction costs generates an increase in capital for both liquid and illiquid firms, which is driven by the decline in the risk-free rate. Hence, aggregate capital increases with the aggregate transaction cost when firms can commit. In contrast, when firms cannot commit, an increase in the aggregate transaction cost generates an increase in capital for liquid firms but a reduction in capital for illiquid firms, as the increase in the present bias dominates the interest rate effect. Thus, the aggregate effect on capital depends on the composition of liquid and illiquid firms in the economy. This observation is important to understand, for example, the consequences of an aggregate liquidity dry-up. The response of the economy could vary substantially depending on the firms' ability to commit to an investment plan, even though they would all be qualitatively consistent with the cross-sectional evidence in Amihud and Levi (2022).

Figure 6: Liquidity and Investment in the Cross-Section and the Aggregate



Note: The figures show the allocations under different transaction costs, ϕ.

[28]We set the fraction of illiquid firms, ω equal to 0.975 so the economy is close to our benchmark calibration of $\omega = 1.0$. Results are robust to alternative values of ω as long as the amount of liquid assets in the economy is not too large.