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Authors	B. Ravikumar, Raymond Riezman, and Yuzhe Zhang
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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Private Information and Optimal Infant Industry Protection *

B. Ravikumar†, Raymond Riezman‡, and Yuzhe Zhang§

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Abstract

We study infant industry protection using a dynamic model in which the industry's cost is initially higher than that of foreign competitors. The industry can stochastically lower its cost via learning by doing. Whether the industry has transitioned to low cost is private information. Using a mechanism-design approach, we solve for the optimal protection policy that induces the industry to reveal its true cost. We show that (i) optimal protection, measured by infant industry output, declines over time and is less than that under public information and (ii) eventual viability of the infant industry is not necessary for optimal protection under private information.

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†Research Division, Federal Reserve Bank of St. Louis. Email: b.ravikumar@wustl.edu

‡Aarhus University, University of California-Santa Barbara, and University of Iowa, raymond-riezman@uiowa.edu

§Department of Economics, Texas A&M University. Email: zhangeager@tamu.edu

1 Introduction

Protection of infant industries is perhaps the longest-lived exception to free trade. The rationale for infant industry protection is that a newer, smaller domestic industry cannot survive against mature foreign competitors who have a superior technology. Protection provides the industry the time to develop so it can compete in the world market. Examples of such protection are ubiquitous, ranging from 19th-century cases in the U.S. for steel rails (Head, 1994) and tinplate (Irwin, 2000) to more recent instances, such as support for the chemical industry in South Korea (Choi and Levchenko, 2023) and the U.S. and EU tariffs on electric vehicles (EV).

The Mill and Bastable tests provide some criteria for protecting an infant industry: "The Mill test requires that the protected sector can eventually survive international competition without protection, whereas the Bastable test requires that the discounted future benefits compensate the present costs of protection" (Harrison and Rodriguez-Clare, 2010). Assessing the costs and benefits is fraught with private information problems, especially in developing economies where the rationale for infant industry protection is most applicable.

Our paper is the first attempt to study the dynamic incentive problems in infant industry protection under private information: The industry knows its cost of production, but the government does not.[1] We use a mechanism-design approach to examine how private information affects protection policies.

We develop a dynamic model where foreign firms have zero cost of production and where a domestic infant industry initially has a positive marginal cost c for producing the same good. Our model features external benefits and learning, two crucial elements in the infant-industry argument (Succar, 1987). First, the benefit is external to the infant industry. Second, the industry's cost may stochastically transition from c to zero at a Poisson arrival rate that is increasing in its output. This process embodies stochastic learning by doing: The expected marginal cost of the industry declines over time as long as it continues to produce. After the

[1] Private information plays an important role in many areas of economics: Lemons (Akerlof, 1970), public goods (Groves and Ledyard, 1977), auctions (Myerson, 1981), regulation (Laffont and Tirole, 1993), and trade (Grossman, 2004).

transition, the industry's cost is zero forever. The government knows the initial cost c, but the time at which the industry transitions from c to zero is private information and random.

We restrict the demand, learning by doing, and external benefit such that whether to protect the infant industry is *not* the question, i.e., the Bastable test is satisfied. The questions in our paper are *how* to protect the industry optimally when it has private information and *whether* the optimal policy satisfies the Mill test. Our mechanism has access to all instruments—tariffs, domestic production/import quotas, rewards, taxes, and subsidies—in order to maximize social welfare. In our dynamic setting, the optimal mechanism achieves truth telling via a higher reward for reporting an early transition to zero cost than for reporting a late transition.

Our first result is that, under private information, before the transition to zero cost the optimal domestic output declines (or the optimal import quota increases) over time and the domestic industry receives a per-unit subsidy. That is, the protection is front loaded: The subsidies are the highest initially and decline subsequently. To incentivize truth telling, the government pays a reward when the industry reports a transition to zero cost. The reward is what would have been the total cost of domestic output if the industry had not transitioned. After the transition, the industry receives no subsidies. In contrast, under public information, the optimal policy before the transition is a constant level of domestic output. Furthermore, the protection is more generous: The import quota is lower than that under private information at every point in time. This is because protection is more costly under private information since the government has to reward truth telling. There is no such reward under public information.

Second, the optimal policy under private information does not guarantee that the domestic industry can compete internationally. In other words, the policy cannot guarantee that the Mill test would be satisfied. Our model is stochastic, so passing the Mill test is a probabilistic event. With a declining path of domestic output, the probability of eventual transition to zero cost, although positive, is below 1. Under public information, with constant output over time, the infant industry will eventually transition to zero cost and be able to compete internationally with probability 1.

Three remarks are in order here. First, our model is about infant-industry protection in developing economies, but it is also applicable to recent industrial policies in developed economies. For instance, the U.S. and EU offer protection to the domestic EV industry, while cheaper imports from China are available. The protection is often justified on the grounds of national security or spillovers to related sectors such as battery and semiconductors.

Second, our model has two features that are uncommon in the dynamic mechanism design literature on *persistent* private information: Externalities and strategic use of outside options. Our optimal policy uses imports to incentivize the domestic industry, balancing less expensive imports against more expensive domestic output that confers external benefits.

Third, we model private information in the infant-industry problem. Other papers either model only public information (e.g., Bardhan, 1971; Melitz, 2005) or make additional assumptions that render private information inconsequential (e.g., Dinopoulos, Lewis, and Sappington, 1995).

2 Model

As a benchmark, we first study the case in which the government observes the infant industry's cost at all times. In this full-information setting, the optimal policy prescribes a constant level of domestic output until the industry becomes competitive. This policy guarantees that the Mill test is satisfied.

There is a unit measure of buyers with the inverse demand function $p(Q)$, where Q is the total quantity of the good. This could be a derived demand for an intermediate good or final demand by consumers. A domestic infant industry can produce the good at cost cq, where q is the quantity produced by the industry and $c > 0$ is the cost per unit. Foreign firms can produce the same good at zero cost.

Learning by doing The infant industry's cost may stochastically transition from c to zero (absorbing state) at arrival rate $\pi(q)$ that satisfies:

Assumption 1 *$\pi(\cdot)$ is increasing, strictly concave, $\pi(0) = 0$, and $\pi'(0) < \infty$.*

Let Ω_t denote the event that the infant industry's cost remains high at t; i.e., no transition has occurred until t. *Conditional* on Ω_t, the probability of a transition during time interval $(t, t + dt]$ is $\pi(q_t)dt$, where $dt > 0$ is small. Denote the *unconditional* (i.e., as of time 0) probability of Ω_t as $\Pr(\Omega_t)$; $\Pr(\Omega_0) = 1$ and $\Pr(\Omega_t) = e^{-\Pi_t}$, where $\Pi_t \equiv \int_0^t \pi(q_s)ds$ is the cumulative transition up to t. The industry's marginal cost at time t, c_t, equals 0 with unconditional probability $1 - e^{-\Pi_t}$. Its mean $E[c_t] = ce^{-\Pi_t}$ is monotonically decreasing over time. Thus, our specification is one of stochastic learning by doing. The growth rate of $E[c_t]$ is $-\pi(q_t)$, so the cost cannot be reduced further if the industry stops production.

Externality Output q implies an external benefit $\Gamma(q)$ that satisfies:

Assumption 2 *$\Gamma(q)$ is increasing and concave in q, $\Gamma(0) = 0$, $\Gamma'(0) > 0$, and $\Gamma'(\bar{q}) = 0$ for some $\bar{q} \in (0, p^{-1}(0))$.*

Assumption 2 implies that $\Gamma(q)$ is strictly increasing for $q < \bar{q}$ and remains constant for $q \geq \bar{q}$. External benefits being bounded above is in the spirit of Mill (1848): "*...it is essential that the protection should be confined to cases in which there is ground of assurance that the industry which it fosters will after a time be able to dispense with it...*"[2] Had we modeled the external benefits as unbounded, the subsidies would continue even after the infant industry is ready to compete internationally and there would be no such thing as "dispense with" protection in Mill's sense, by construction.

Three remarks about our model are in order here. First, our environment before the transition is time invariant. That is, conditional on event Ω_t, the environment at t is identical to that at time 0. In particular, the transition rate at t, $\pi(q_t)$, depends only on q_t; the cumulative production before t does not affect the *conditional* probability of transition in the interval $(t, t + dt]$, which resembles the memoryless property of stationary Markov processes.

[2] The statement in its entirety is reprinted in Irwin (1996). Juhasz (2018) examines a case of "natural" protection for the cotton industry in France during the Napoleonic Wars and documents a reduction in French firms' marginal cost.

The time-invariant nature of the physical environment, however, does not necessarily imply that the optimal policy is time invariant.

Second, there is no contradiction between the memoryless property and learning by doing. The independence between the past and future, *conditional* on the current state, is a property of models with a Markovian structure. For example, consider a stochastic version of the learning-by-doing model in Arrow (1962): A firm's productivity depends on the capital stock, which is the sum of past investments and random shocks. The capital stock is a state variable in this model because future dynamics depend on the history through the current capital stock. Since a history with large investments but bad shocks can result in the same capital stock as another history with small investments but good shocks, the two histories imply the same productivity in the future despite different cumulative investments. Similarly, in our model, the state at time t is the infant industry's cost. Conditional on cost c at t, the probability of transition to zero cost after t is independent of cumulative output until t. However, the *unconditional* probability of the transition, $1 - e^{-\Pi_t}$, depends on the history and increases over time due to learning by doing and, hence, the expected marginal cost decreases over time.

Third, if there are no benefits external to the industry, i.e., $\Gamma \equiv 0$, then the laissez-faire allocation is efficient and there is no need for government intervention. When $\Gamma(q) > 0$ for $q > 0$ then the laissez-faire allocation is inefficient and government intervention achieves the efficient allocation. Bounding Γ above with the assumption that $\bar{q}$ is below $p^{-1}(0)$ allows for the possibility that both domestic production and imports can coexist even after the infant industry transitions to zero cost. All of our results go through even if $\bar{q} = p^{-1}(0)$.

2.1 Protection under public information

The government knows when the domestic industry transitions to zero cost and precommits to a protection policy at time 0 through a direct mechanism by choosing paths of several variables. The government

1. provides a subsidy τ_t to the industry and asks it to produce q_t;

2. provides a subsidy τ_t^f to the foreign firm and asks it to produce q_t^f; and
3. charges p_t per unit to each consumer and collects a tax $\tau_t + \tau_t^f$.[3]

While the subsidy τ_t in each period helps the domestic industry to compete with foreign firms, τ_t^f can be negative, in which case a natural interpretation of τ_t^f is tariff. Note that the mechanism is not restricted to choosing one instrument at a time. It can simultaneously use tariffs, import quotas, taxes, and production subsidies. The constraints on the direct mechanism are limited liability of the domestic industry and foreign firms. Formally, for all t, (i) $\tau_t + p_t q_t \geq c_t q_t$ and (ii) $\tau_t^f + p_t q_t^f \geq 0$.

The government's utility flow is the consumer surplus plus external benefit:

$$\int_0^{q_t + q_t^f} p(Q)dQ - p_t(q_t + q_t^f) - (\tau_t + \tau_t^f) + \Gamma(q_t).$$

Given that the government has access to taxes and subsidies in our mechanism, the price p_t is redundant. We can define an alternative tax/subsidy $\tilde{\tau}_t = \tau_t + p_t q_t$ and $\tilde{\tau}_t^f = \tau_t^f + p_t q_t^f$ and set the price to zero; the alternative would yield the same sequence of import quota and domestic output.

Remark 1 *Without loss of generality we can set $p_t = 0$ with τ_t and τ_t^f adjusted accordingly for all $t \geq 0$.*

Remark 2 *With $p_t = 0$, limited liability of foreign firms implies $\tau_t^f \geq 0$ $\forall t$. It is suboptimal to subsidize the foreign firms with a tax on domestic consumers; hence, $\tau_t^f = 0$. In general, $\tau_t^f = -p_t q_t^f < 0$ whenever $p_t > 0$ so that the foreign firms' rents are completely extracted.*

Remark 3 *With $p_t = 0$, limited liability of the domestic industry implies $\tau_t \geq c_t q_t$. It is*

[3] In our model, an economy-wide representative consumer pays the tax, purchases the good produced by the infant industry, and enjoys the externality generated by it. Alternatively, one could imagine industry-specific consumers purchasing the good and paying the tax and consumers outside the industry benefiting from the externality. As long as the government maximizes the total surplus of all consumers, our analysis remains the same.

suboptimal to provide rents to the domestic industry—$\tau_t - c_t q_t > 0$—financed by a tax τ_t on consumers. Thus, $\tau_t = cq_t$ before the transition and $\tau_t = 0$ after the transition.

REMARK 4 *Our externality Γ is a reduced form for productivity spillovers to other industries that ultimately generate benefits to consumers. It can be micro founded using inter-industry externalities as in Succar (1987) and Harrison and Rodriguez-Clare (2010).*

The first three remarks imply that even though the mechanism has access to all instruments, a subset of instruments—domestic production/import quota and subsidy to the infant industry financed by a tax on consumers—is sufficient to describe the optimal protection policy. For the rest of this section, we set $p_t = 0$, $\tau_t^f = 0$, and $\tau_t = c_t q_t \ \forall t$.

It is useful to divide the government's problem into two parts: (i) after the domestic industry has transitioned to zero cost and (ii) before the domestic industry has transitioned.

After the transition, the domestic industry and the foreign firms are equally efficient: The marginal cost is 0 for both. The government solves a sequence of static problems, each of which is

$$\max_{q,q^f} \int_0^{q+q^f} p(Q)dQ + \Gamma(q).$$

Two results emerge immediately. (i) The import quota satisfies $q^f = p^{-1}(0) - q$. (ii) Since q confers an external benefit, it is optimal to set q to be any value weakly above $\bar{q}$: The domestic output is such that the marginal social benefit equals 0. Denote the optimal value in the above problem as

$$S \equiv \int_0^{p^{-1}(0)} p(Q)dQ + \Gamma(\bar{q}).$$

Before the transition to zero cost, the government's problem is

$$\max_{q_t,q_t^f} \quad E\left[\int_0^T e^{-rt}\left(\int_0^{q_t+q_t^f} p(Q)dQ - cq_t + \Gamma(q_t)\right)dt + \int_T^\infty e^{-rt}Sdt\right], \tag{1}$$

where T is the random transition time and $\int_0^{q_t+q_t^f} p(Q)dQ - cq_t + \Gamma(q_t)$ and S are the government's payoff flows at $t < T$ and $t \geq T$, respectively. Note that q_t^f affects only the payoff flow at t, while q_t affects not only the payoff flow but also the random variable T. Consequently, the optimal q_t^f before the transition is a solution to a static optimization problem, while the optimal q_t requires a dynamic analysis and is the focus of this paper.

Import quota For any given q_t, the optimal import quota solves

$$\max_{q_t^f} \quad \int_0^{q_t+q_t^f} p(Q)dQ - cq_t + \Gamma(q_t).$$

The first-order condition for q_t^f is $0 = p(q_t + q_t^f)$, which implicitly defines $q^f = p^{-1}(0) - q$, a *stationary* function of q. The optimal import quota offsets domestic output one-for-one both before and after the transition. That is, the optimal protection policy implies there is *no distortion of consumer surplus.*

Dynamic protection After changing the order of integration, we can rewrite the objective in (1) as

$$\begin{aligned}
&\int_0^\infty e^{-rt}\left[\Pr(T\geq t)\left(\int_0^{p^{-1}(0)} p(Q)\,dQ + \Gamma(q_t) - cq_t\right) + \big(1-\Pr(T\geq t)\big)S\right]dt \\
&\quad = \frac{S}{r} + \int_0^\infty e^{-rt}\Pr(T\geq t)\left(\int_0^{p^{-1}(0)} p(Q)\,dQ + \Gamma(q_t) - cq_t - S\right)dt \\
&\quad = \frac{S}{r} - \int_0^\infty e^{-rt-\Pi_t}\big(\Gamma(\bar{q}) - \Gamma(q_t)\big)dt - \int_0^\infty e^{-rt-\Pi_t} cq_t\,dt. \qquad (2)
\end{aligned}$$

where $\Pr(T \geq t) \equiv e^{-\Pi_t}$ is the probability that the transition has *not* arrived until t. We note two features of (2). First, the optimal q_t involves a dynamic tradeoff due to learning by doing: q_t affects not only the current payoff, but also future payoffs through probabilities $e^{-\Pi_s}$ $\forall s > t$. Second, the objective before transition is written as the post-transition value, $\frac{S}{r}$, minus two losses: deviation of the external benefit of q_t from its post-transition counterpart

$\Gamma(\bar{q})$ and the production subsidy from 0. To simplify the notation, let $L(q) \equiv \Gamma(\bar{q}) - \Gamma(q)$.

Before characterizing the dynamic protection policy, it is useful to know whether the infant industry is worth protecting. This amounts to checking whether the industry satisfies the

$$\text{Bastable condition: } c < \bar{c} \equiv -L'(0) + \pi'(0)\frac{L(0)}{r}. \tag{3}$$

To understand the condition, consider a variation around the scenario where the domestic industry produces zero output forever: Increase q_0 from 0 to $\epsilon > 0$ but fix $q_t = 0$ for all $t > 0$. This increase costs the government c, on the margin, to protect. The benefits are: (i) an immediate gain of $\Gamma'(0) = -L'(0)$ and (ii) an increase in the probability of transition resulting in a gain *forever* yielding the second term $\pi'(0)\frac{L(0)}{r}$.

Recall from Assumption 1 that $\pi'(0) < \infty$. If $\pi'(0) = \infty$, then $\bar{c} = \infty$ and the Bastable condition will always be satisfied by the industry. For the rest of this section, we will assume that (3) is satisfied: $c < \bar{c} < \infty$.

Theorem 1 below characterizes the optimal policy under public information. Since $\frac{S}{r}$ is a constant, we minimize the sum of the two losses in (2).

THEOREM 1 (Permanent protection under public information) *Before the transition to zero cost, the optimal domestic output is time invariant: $q_t = q^{pub} > 0$, where q^{pub} is the unique minimizer of*

$$\frac{L(q) + cq}{r + \pi(q)} = \frac{\Gamma(\bar{q}) - \Gamma(q) + cq}{r + \pi(q)}. \tag{4}$$

Theorem 1 has three implications. First, the optimal protection policy is time invariant because, if the infant industry has not transitioned to zero cost until t, the continuation problem faced by the government at t is identical to that at time 0. Hence, whatever is optimal at time 0 will continue to be optimal at time t, conditional on high cost at t.

Second, since the infant industry's cost is public information, optimality combined with the stationary nature of the environment requires that the protection continues as long as the industry has not transitioned to zero cost. (This is what we mean by "permanent protection.") The permanent protection implies that the transition will occur eventually with probability 1. In other words, the protection policy would pass the Mill test and the infant industry would be able to compete internationally.

Third, the government embarks on a protection policy (i.e., $q^{pub} > 0$) if and only if $c < \bar{c}$. That is, if the infant industry is "too inefficient" relative to the foreign firms, then it is optimal to let the imports serve the entire domestic demand forever. In the minimization problem (4), higher c raises the cost of protection and reduces q^{pub}. If $c \geq \bar{c}$, then the loss in (4) is weakly increasing at $q = 0$, so $0 \in \arg\min \frac{L(q)+cq}{r+\pi(q)}$.

3 Protection under private information

When the transition to zero cost is private information, protection is strictly lower than that in the public-information case. The optimal policy does not guarantee transition to zero cost, so the Mill test fails with positive probability.

Our model in this section is identical to that in Section 2 except for the presence of private information: The government knows the infant industry's initial cost c but does not observe when the transition to zero cost occurs. Since zero cost is an absorbing state, the private information is persistent. The government observes the domestic output and imports.

The government precommits to a protection policy at time 0 through a direct mechanism by choosing paths of several variables. The government

1. provides a subsidy τ_t to the infant industry and asks it to produce q_t;
2. provides a subsidy τ_t^f to the foreign firm and asks it to produce q_t^f;
3. sets a price p_t for consumers and collects a tax $\tau_t + \tau_t^f$; and
4. provides a one-time reward M_t to the industry, financed by a lump-sum tax, *if* the industry reports a transition to zero cost at t.

Under the revelation principle, we can focus on direct mechanisms that are *incentive compatible*, i.e., protection polices that induce the infant industry to truthfully report its transition. The newly introduced instrument M_t is important for ensuring incentive compatibility, but it is not needed in Section 2.1, where there are no incentive constraints. Thus, the absence of M_t in Section 2.1 is without loss of generality. The constraints on the mechanism are (i) $\tau_t + p_t q_t \geq c_t q_t$, (ii) $\tau_t^f + p_t q_t^f \geq 0$, and (iii) incentive compatibility.

Several observations from Section 2 carry over to the private-information setup. As in Remarks 1-3, we can set $p_t = 0$, $\tau_t^f = 0$, and $\tau_t = c_t q_t$ for all t.[4] The optimal import quota is still $q^f = p^{-1}(0) - q$ both before and after the transition because, conditional on q, there are no dynamic or incentive problems associated with choosing q^f. The optimal q still equals $\bar{q}$ after the transition because there are no incentive problems. If the transition occurs at a random time T, the government's flow payoff is still S after T, leading to a continuation value of $\frac{S}{r} - M_T$ at T.

Thus, the government's objective at time 0 is

$$E\left[\int_0^T e^{-rt}\left(\int_0^{p^{-1}(0)} p(Q)dQ + \Gamma(q_t) - cq_t\right)dt + \int_T^\infty e^{-rt}Sdt - e^{-rT}M_T\right].$$

Using $E\left[e^{-rT}M_T\right] = \int_0^\infty e^{-rt-\Pi_t}\pi(q_t)M_t dt$ and (2), the objective becomes

$$\frac{S}{r} - \int_0^\infty e^{-rt-\Pi_t}\left(L(q_t) + cq_t + \pi(q_t)M_t\right)dt. \tag{5}$$

So the government's problem is to maximize (5) subject to incentive compatibility. As in Section 2, we exclude $\frac{S}{r}$ and minimize the losses in (5).

[4]Technically, we have to make sure these simplifications of policy instruments do not violate any incentive constraints. The formal analysis requires messy notation, but it is easy to see (i) setting $p_t = 0$ and $\tau_t^f = 0$ for all t does not affect the domestic industry's payoff whether it cheats or not, (ii) setting $\tau_t = cq_t$ before the transition reduces the industry's payoff from cheating, thus relaxing the incentive constraint, and (iii) setting post-transition subsidy to 0 is innocuous since the subsidies, if any, can be subsumed into the one-time reward M, so the industry's continuation payoff upon transition does not change.

Incentive compatibility If the industry transitions at time t but reports the transition at $\tilde{t} > t$, then it continues to receive subsidies from t to $\tilde{t}$. The payoff from cheating is $\int_t^{\tilde{t}} e^{-r(s-t)} cq_s ds + e^{-r(\tilde{t}-t)} M_{\tilde{t}}$. Incentive compatibility requires that

$$M_t \geq \int_t^{\tilde{t}} e^{-r(s-t)} cq_s ds + e^{-r(\tilde{t}-t)} M_{\tilde{t}}, \quad \forall t \geq 0, \forall \tilde{t} > t. \tag{6}$$

In particular, when $\tilde{t} = \infty$ the above constraint becomes

$$M_t \geq \int_t^{\infty} e^{-r(s-t)} cq_s ds, \quad \forall t \geq 0. \tag{7}$$

There is no incentive to cheat in the other direction, i.e, report a transition to zero cost when the cost is c. We prove this in Lemma 5 in Appendix A.

In the following analysis, we consider a "relaxed" problem, in which we minimize the losses in (5) subject to (7). That is, we replace incentive constraints (6) with (7), a strict subset of those in (6). The following lemma shows that (7) binds for all $t \geq 0$ in the relaxed problem, which implies (6) holds for all $t < \tilde{t}$ in the original problem.

Lemma 1 (Relaxed problem) *Minimizing the losses in* (5) *subject to* (7), *the optimal solution satisfies*

$$M_t = \int_t^{\infty} e^{-r(s-t)} cq_s \, ds, \quad \forall t \geq 0. \tag{8}$$

Thus, (6) *holds with equality, and the solution to the relaxed problem is incentive compatible and also solves the original problem. Moreover,* (8) *allows us to simplify the losses in* (5) *as*

$$\int_0^{\infty} e^{-rt-\Pi_t} L(q_t) \, dt + \int_0^{\infty} e^{-rt} cq_t \, dt. \tag{9}$$

Unlike equation (2), where the probability $e^{-\Pi_t}$ appears in front of both the social loss $L(q_t)$ and the subsidy cq_t, here the probability $e^{-\Pi_t}$ is only in front of $L(q_t)$. This is because the government does not observe the transition and must offer a reward to induce truth telling. The reward M_t equals the present value of the subsidies that the domestic industry would have received if it had cheated after the transition. So, it is as if the industry receives the subsidy both before and after transition; i.e., the subsidy is *unconditional.* Thus, private information makes protection more costly.

Differentiating (9) with respect to $q_t \geq 0$, the first-order condition is

$$\begin{aligned} e^{-rt-\Pi_t} L'(q_t) - \int_t^\infty e^{-rs-\Pi_s} \pi'(q_t) L(q_s) ds + e^{-rt} c &\geq 0, \text{ or} \\ -e^{-\Pi_t} L'(q_t) + \left[\int_t^\infty e^{-r(s-t)-\Pi_s} L(q_s) ds \right] \pi'(q_t) &\leq c, \end{aligned} \tag{10}$$

which holds as an equality if $q_t > 0$. The right-hand side of (10) is the cost of protection: For each unit of output produced by the domestic industry, the government provides a subsidy c. On the left-hand side of (10) are the two benefits of protection: The first term is the increased external benefit of higher q_t $(-L'(q_t) = \Gamma'(q_t))$, while the second term is the benefit of learning by doing. Specifically, higher q_t increases the probability of transition and $\int_t^\infty e^{-r(s-t)-\Pi_s} L(q_s) ds$ is the social gain due to the transition.

To determine whether the industry is worth protecting at all under private information (i.e., the Bastable condition), consider the benefit from protection in equation (10) at $t = 0$, i.e., a variation $q_0 = \epsilon > 0$ but $q_t = 0 \ \forall t > 0$. The benefit as of time 0 would be the sum of the marginal social gain, $-e^0 L'(0)$, and the permanent gain due to the increase in probability of a transition to zero cost $\left[\int_0^\infty e^{-rs} L(0) ds\right] \pi'(0)$. The sum is exactly the same as in the public-information model; see the description below equation (3). For the benefit to exceed the cost of protection, c must be less than $\bar{c} \equiv -L'(0) + \pi'(0) \frac{L(0)}{r}$. This is the same as the Bastable condition (3) under public information.[5]

[5] One might wonder why M_0, the reward for reporting a transition at time 0 under private information, does not appear in the Bastable condition. For the variation ϵ at time 0, the difference between the rewards under private and public information is M_0. Since $M_0 = c\epsilon$ and the probability of transition is $\pi(\epsilon)$, the expected value of reward for truthful reporting $M_0 \pi(\epsilon)$ is a higher-order infinitesimal than ϵ and does not have a first-order effect. Thus, the Bastable condition is the same under private- and public-information scenarios.

Lemma 2 (Bastable condition) *The optimal protection satisfies* $q_t = 0\ \forall t \geq 0$ *if and only if* $c \geq \bar{c}$.

Hereafter, we shall impose the Bastable condition, $c < \bar{c} < \infty$, so that the optimal-protection problem is nontrivial.

Our first private-information result is that optimal protection is always less than that under public information, monotonically decreasing over time, and disappears in the long run.

Theorem 2 (Decreasing protection under private information) *The optimal* $\{q_t\}_{t \geq 0}$ *is decreasing over time with* $q_0 < q^{pub}$. *Furthermore,* $\lim_{t\to\infty} q_t = 0$ *and* $\lim_{t\to\infty} \pi(q_t) = 0$.

Declining q_t helps ensure that the industry does not postpone the report of transition. Declining q_t and $q_0 < q^{pub}$ together imply $q_t < q^{pub}\ \forall t$. That is, the infant industry gets less protection under private information *at all times*. This is not surprising since the cost of protection under private information is more than that under public information; see (9) and (2).[6]

Our next result shows that an eventual transition to zero cost is not guaranteed under the optimal protection policy.

Theorem 3 (Mill test) $\lim_{t\to\infty}(1 - e^{-\Pi_t}) = (\bar{c} - c)/\bar{c} < 1$.

Under private information, the probability that the infant industry would eventually transition to zero cost is positive, but it is less than 1. That is, the protection policy does not pass the Mill test with certainty. Recall that under public information, the optimal domestic output is stationary, so the arrival rate of transition is constant $\pi(q^{pub}) > 0$ and the industry will eventually pass the Mill test.

[6] In our model, the incentive constraints bind and the optimal allocation under private information is different from that under public information. In contrast, in the two-period model of Dinopoulos et al. (1995), the incentive constraints do not bind, so the optimal allocations under private and public information are the same.

We demonstrate the robustness of Theorems 1–3 through two extensions. Appendix B extends the government's objective function to also include the infant industry's payoff. Appendix C extends the infant industry's cost from two to three values.

Implementation The optimal allocation in the previous section can be implemented through the establishment of a fund for the infant industry at $t = 0$. The implementation scheme is as follows:

1. The government sets up a fund $M_0 \equiv \int_0^\infty e^{-rt} c q_t dt$ for the infant industry at time 0, which is a pool of money from which the industry is paid c per unit of output until the fund runs out. The government also mandates that the industry chooses output from interval $[q_t, \bar{q}]$ as long as the fund has a positive balance.
2. The industry produces $\tilde{q}_t \in [q_t, \bar{q}]$ until the fund runs out. Afterwards the industry has no obligation to produce.

The implementation is simple: The government sets up a fund M_0 initially after which it neither collects the report of privately observed transition nor offers subsidies or rewards to the industry. We show in Appendix D that given the scheme above the infant industry would replicate the optimal allocation.

4 Conclusion

We study infant industry protection in a dynamic model where initially the industry cannot compete with foreign firms that possess a superior technology. The industry can stochastically reduce its cost through learning by doing, but the transition to low cost is private information. We use a mechanism-design approach to establish that the optimal protection declines over time and can be implemented with minimal information requirements.

Passing the Mill test has been a central criterion to justify infant industry protection. The Mill test requires that the industry must eventually be able to compete on the world market without protection. In a private-information setting this criterion is not necessary for optimality.

The optimal protection policy under private information is not time consistent. To see this, suppose a new government arrives at date t. Promised subsidies after t affect the incentives before t, which have to be taken into account by the time-0 government, but not by the time-t government. Consequently, the time-t government would offer more protection and not continue the path of the time-0 government. In contrast, the optimal policy under public information is time consistent since the transition is observable and incentive constraints are not an issue. One can find a time-consistent protection policy under private information but the policy would be suboptimal.

We have focused on the incentive problem when the infant industry's transition is not observable, but other incentive problems might also be plausible. For example, the government may provide funds for the industry's R&D activities but cannot monitor how the industry uses these funds.

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Appendix A: Proofs

PROOF OF THEOREM 1: We show that $\frac{L(q)+cq}{r+\pi(q)}$ has a unique minimum. Its derivative with respect to q is

$$\frac{(L'(q)+c)(r+\pi(q))-(L(q)+cq)\pi'(q)}{(r+\pi(q))^2}.$$

The derivative of the numerator is

$$L''(q)(r+\pi(q))-(L(q)+cq)\pi''(q)>0,$$

which means the numerator $(L'(q)+c)(r+\pi(q))-(L(q)+cq)\pi'(q)$ is strictly increasing in q. Moreover, it is negative at $q=0$:

$$\begin{aligned} & (L'(q)+c)(r+\pi(q))-(L(q)+cq)\pi'(q)|_{q=0} \\ = \; & (L'(0)+c)r-L(0)\pi'(0)<(L'(0)+\bar{c})r-L(0)\pi'(0)=0, \end{aligned}$$

but positive at $q=\bar{q}$:

$$\begin{aligned} & (L'(q)+c)(r+\pi(q))-(L(q)+cq)\pi'(q)|_{q=\bar{q}} \\ = \; & c(r+\pi(\bar{q})-\pi'(\bar{q})\bar{q})>0. \end{aligned}$$

Therefore, $(L'(q)+c)(r+\pi(q))-(L(q)+cq)\pi'(q)$ is first negative and then positive. The optimal $q^{pub}\in(0,\bar{q})$ is the solution to $(L'(q)+c)(r+\pi(q))-(L(q)+cq)\pi'(q)=0$ and is unique. ■

PROOF OF LEMMA 1: First, consider a relaxed problem where the government minimizes the losses in (5) subject to (7).

$$\min \quad \int_0^\infty e^{-rt-\Pi_t}\left(L(q_t)+cq_t+\pi(q_t)M_t\right)dt,$$

$$s.t. \qquad M_t \geq \int_t^\infty e^{-r(s-t)} c q_s ds.$$

Since the above objective function is increasing in M_t, it is obvious that (7) binds in the optimal solution.

Second, if $M_t = \int_t^\infty e^{-r(s-t)} c q_s ds$ for all t, then the incentive constraint (6) holds as equality for all t and $\tilde{t}$. Therefore, the solution to the relaxed problem is indeed incentive compatible.

Finally, equation (8) allows us to simplify the losses in (5) as

$$\begin{aligned}
& \int_0^\infty e^{-rt-\Pi_t} \left(L(q_t) + c q_t + \pi(q_t) M_t \right) dt \\
= & \int_0^\infty e^{-rt-\Pi_t} (L(q_t) + c q_t) dt + \int_0^\infty \pi(q_t) e^{-rt-\Pi_t} \left(\int_t^\infty e^{-r(s-t)} c q_s ds \right) dt \\
= & \int_0^\infty e^{-rt-\Pi_t} (L(q_t) + c q_t) dt + \int_0^\infty \left(e^{-rs} \int_0^s \pi(q_t) e^{-\Pi_t} dt \right) c q_s ds \\
= & \int_0^\infty e^{-rt-\Pi_t} (L(q_t) + c q_t) dt + \int_0^\infty (1 - e^{-\Pi_t}) e^{-rt} c q_t dt \\
= & \int_0^\infty e^{-rt-\Pi_t} L(q_t) dt + \int_0^\infty e^{-rt} c q_t dt,
\end{aligned}$$

where the change of the order of integration in the third line follows from Fubini's theorem. ■

PROOF OF LEMMA 2: If $c \geq \bar{c}$, then $q_t = 0 \ \forall t \geq 0$ is optimal because it satisfies the first-order condition in (10). To verify (10), note

$$\begin{aligned}
& -e^{-\Pi_t} L'(q_t) + \left[\int_t^\infty e^{-r(s-t)-\Pi_s} L(q_s) ds \right] \pi'(q_t) \\
= & -L'(0) + \left[\int_t^\infty e^{-r(s-t)} L(0) ds \right] \pi'(0) = \bar{c} \leq c.
\end{aligned}$$

If $c < \bar{c}$, then consider a stationary plan where $q_t = q, \forall t \geq 0$ for some $q \geq 0$. Then the government's cost function is

$$\mathcal{L}_q = \int_0^\infty e^{-rt-\Pi_t} L(q_t)dt + c\int_0^\infty e^{-rt} q_t dt = \frac{L(q)}{r+\pi(q)} + \frac{c}{r}q.$$

We have

$$\begin{aligned}
\frac{d\mathcal{L}_q}{dq}|_{q=0} &= \frac{c}{r} + \frac{L'(q)(r+\pi(q)) - L(q)\pi'(q)}{(r+\pi(q))^2}|_{q=0} \\
&= \frac{c}{r} - \left(\frac{\pi'(0)}{r^2}L(0) - \frac{L'(0)}{r}\right) = \frac{c-\bar{c}}{r} < 0.
\end{aligned}$$

Hence, $\mathcal{L}_q < \mathcal{L}_0$ for small $q > 0$, implying that $q_t = 0\ \forall t \geq 0$ is suboptimal. ■

A recursive formulation To prove Theorem 2, we formulate the government's problem recursively. The standard method for solving models with persistent private information (e.g., Fernandes and Phelan, 2000) is to formulate the principal's problem recursively and use a vector of agent's continuation utilities as the state variable. The first-order approach (e.g., Williams, 2011; Farhi and Werning, 2013) reduces the state vector to a pair—continuation utilities of only the truth teller and his nearest neighbor in the type space—which leads to nonlinear partial differential equations in two variables. In contrast, our state variable is *not* continuation utilities, but rather the industry's cumulative transition rate up to the current period. Since the latter is one-dimensional, our optimal policy is characterized by an ordinary differential equation, rather than a partial differential equation. This methodological simplification allows us to derive analytical results–such as the monotonicity of protection–that are unavailable under conventional methods.

The government faces an optimal-control problem with state variable Π_t and control variable q_t:

$$\mathcal{L}(\Pi) \equiv \min_{\{q_t\}_{t\geq 0}} \quad \int_0^\infty e^{-rt-\Pi_t} L(q_t)dt + \int_0^\infty e^{-rt} c q_t dt \tag{11}$$

$$\text{s.t.} \qquad \frac{d\Pi_t}{dt} = \pi(q_t), \quad \Pi_0 = \Pi.$$

The state variable Π is the cumulative learning until the current period. Although $\Pi_0 = 0$, we treat $\Pi_0 = \Pi$ as any nonnegative number in the following analysis so that $[0, \infty)$ is the state space in a recursive formulation. Lemma 3 below will be used in other proofs in this appendix.

LEMMA 3 $-\mathcal{L}(\Pi) \leq \mathcal{L}'(\Pi) < 0$, *where the equality holds iff* $q_t = 0\ \forall t \geq 0$.

PROOF: Differentiating the objective function (11) with respect to Π yields

$$\mathcal{L}'(\Pi) = -\int_0^\infty e^{-rt-\Pi_t} L(q_t)dt, \tag{12}$$

where $\{q_t\}_{t\geq 0}$ is the path of optimal controls in problem (11). We show $L(q_t) > 0$ for some t. Suppose to the contrary that $L(q_t) = 0\ \forall t$. Then $q_t \geq \bar{q}\ \forall t$, which violates the first-order condition (10). Since $L'(q_t) = L(q_s) = 0\ \forall s \geq t$, the left side of (10) is 0 and less than the right side, but (10) needs to hold as an equality since $q_t \geq \bar{q} > 0$. Then, $L(q_t) > 0$ for some t and (12) imply $\mathcal{L}'(\Pi) < 0$.

That $-\mathcal{L}(\Pi) \leq \mathcal{L}'(\Pi)$ holds because $\mathcal{L}'(\Pi) = -\int_0^\infty e^{-rt} e^{-\Pi_t} L(q_t)dt$ and

$$-\mathcal{L}(\Pi) = -\int_0^\infty e^{-rt}(e^{-\Pi_t} L(q_t) + cq_t)dt \leq -\int_0^\infty e^{-rt} e^{-\Pi_t} L(q_t)dt.$$

Clearly, the equality holds if and only if $\int_0^\infty e^{-rt} q_t dt = 0$ or $q_t = 0\ \forall t \geq 0$. ■

The Hamilton-Jacobi-Bellman equation is

$$r\mathcal{L}(\Pi) = \min_{q\geq 0} e^{-\Pi} L(q) + cq + \mathcal{L}'(\Pi)\pi(q). \tag{13}$$

The right-hand side of (13) is convex in q because L is convex, $\mathcal{L}' < 0$, and π is concave. Therefore, the following first-order condition for $q \geq 0$ is both necessary and sufficient for optimality:

$$-e^{-\Pi}L'(q) - \mathcal{L}'(\Pi)\pi'(q) \le c, \tag{14}$$

with an equality if $q > 0$. Lemma 4 shows that the policy function $q(\Pi)$ defined by (14) is monotonic.

LEMMA 4 (Monotonicity of domestic output) $q'(\Pi) < 0$ *as long as* $q(\Pi) > 0$.

PROOF: First, the HJB equation (13) is

$$r\mathcal{L}(\Pi) = e^{-\Pi}L(q) + cq + \mathcal{L}'(\Pi)\pi(q), \tag{15}$$

where $q \equiv q(\Pi)$ is the optimal policy (the dependence of q on Π is suppressed to simplify notation). Applying the Envelope theorem to (15), we have

$$r\mathcal{L}'(\Pi) = -e^{-\Pi}L(q) + \mathcal{L}''(\Pi)\pi(q). \tag{16}$$

Summing up (15) and (16), we have, for $q > 0$,

$$\mathcal{L}'(\Pi) + \mathcal{L}''(\Pi) = \frac{r(\mathcal{L}(\Pi) + \mathcal{L}'(\Pi)) - cq}{\pi(q)}. \tag{17}$$

Second, totally differentiating (14) with respect to Π and $q > 0$ yields

$$\begin{aligned}
\frac{dq}{d\Pi} &= -\frac{-e^{-\Pi}L'(q) + \mathcal{L}''(\Pi)\pi'(q)}{e^{-\Pi}L''(q) + \mathcal{L}'(\Pi)\pi''(q)} \\
&= -\frac{\mathcal{L}'(\Pi)\pi'(q) + c + \mathcal{L}''(\Pi)\pi'(q)}{e^{-\Pi}L''(q) + \mathcal{L}'(\Pi)\pi''(q)} \\
&= -\frac{\frac{(\mathcal{L}(\Pi)+\mathcal{L}'(\Pi))r\pi'(q)+(\pi(q)-\pi'(q)q)c}{\pi(q)}}{e^{-\Pi}L''(q) + \mathcal{L}'(\Pi)\pi''(q)} < 0,
\end{aligned}$$

where the second equality follows from (14), the third equality from (17), and the last inequality follows from $\mathcal{L}(\Pi) + \mathcal{L}'(\Pi) \geq 0$ (Lemma 3) and the assumption that $q > 0$ and $\pi(q) > \pi'(q)q$. ■

PROOF OF THEOREM 2: First, because Π_t is increasing over time and $q'(\Pi) < 0$ (Lemma 4), q_t is decreasing over time. By contradiction, suppose $\underline{q} \equiv \lim_{t\to\infty} q_t > 0$ and $\underline{\pi} \equiv \lim_{t\to\infty} \pi(q_t) > 0$. Then $\frac{d\Pi_t}{dt} = \pi(q_t) > \underline{\pi} > 0$ and $\lim_{t\to\infty} \Pi_t = \infty$. Taking limit $\Pi \to \infty$ in (14) yields

$$0L'(\underline{q}) - 0\pi'(\underline{q}) = c,$$

which cannot hold as long as $\underline{q} > 0$.

Second, we show $q_0 < q^{pub}$. We first prove

$$\mathcal{L}'(0) > -\frac{cq^{pub} + L(q^{pub})}{r + \pi(q^{pub})}. \tag{18}$$

By contradiction, suppose $\mathcal{L}'(0) \leq -\frac{cq^{pub}+L(q^{pub})}{r+\pi(q^{pub})}$. Then

$$\begin{aligned} r\mathcal{L}(0) &= \min_q L(q) + cq + \mathcal{L}'(0)\pi(q) \\ &\leq \min_q L(q) + cq - \frac{cq^{pub} + L(q^{pub})}{r + \pi(q^{pub})}\pi(q) \\ &= L(q^{pub}) + cq^{pub} - \frac{cq^{pub} + L(q^{pub})}{r + \pi(q^{pub})}\pi(q^{pub}) = r\frac{cq^{pub} + L(q^{pub})}{r + \pi(q^{pub})}. \end{aligned}$$

This contradicts the fact that private-information cost $\mathcal{L}(0)$ must exceed the public-information cost. That $q_0 < q^{pub}$ follows from (18), $q_0 \in \arg\max_q L(q) + cq + \mathcal{L}'(0)\pi(q)$, and $q^{pub} \in \arg\max_q L(q) + cq - \frac{cq^{pub}+L(q^{pub})}{r+\pi(q^{pub})}\pi(q)$. ■

PROOF OF THEOREM 3: It is sufficient to show $\lim_{t\to\infty} \Pi_t = \log(\frac{\bar{c}}{c})$. Define $\bar{\Pi} \equiv \log(\frac{\bar{c}}{c})$. On $[\bar{\Pi}, \infty)$, we can verify that $\mathcal{L}(\Pi) = \frac{e^{-\Pi}L(0)}{r}$ and $q(\Pi) = 0 = \pi(q(\Pi))$ solve the HJB equation. So Π_t cannot exceed $\bar{\Pi}$, and therefore $\Pi_\infty \equiv \lim_{t\to\infty} \Pi_t \leq \bar{\Pi}$. Since Theorem 2 shows $\lim_{t\to\infty} q_t = \lim_{t\to\infty} q(\Pi_t) = 0$, the monotonicity of $q(\Pi_t)$ implies $q(\Pi) = 0$ for all $\Pi \geq \Pi_\infty$, therefore, $\mathcal{L}(\Pi) = \frac{e^{-\Pi}L(0)}{r}$ on $[\Pi_\infty, \infty)$. Therefore, condition (14) at Π_∞ implies

$$\begin{aligned} c \geq -e^{-\Pi_\infty}L'(0) - \mathcal{L}'(\Pi_\infty)\pi'(0) &= -e^{-\Pi_\infty}L'(0) + \frac{e^{-\Pi_\infty}L(0)\pi'(0)}{r} \\ &= e^{-\Pi_\infty}\bar{c} \geq e^{-\bar{\Pi}}\bar{c} = c, \end{aligned}$$

which implies $\Pi_\infty = \bar{\Pi}$. ■

Lemma 5 *In the direct mechanism, a domestic industry without a transition does not have the incentive to report one.*

PROOF: First, we prove

$$\int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds \leq \frac{\bar{q}}{r+\pi(\bar{q})}, \tag{19}$$

where $\Pi_t \equiv \int_0^t \pi(q_s)ds$. We have

$$\begin{aligned} \int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds &\leq \int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} \frac{\bar{q}}{r+\pi(\bar{q})}(r+\pi(q_s))dt \\ &= \frac{\bar{q}}{r+\pi(\bar{q})} \int_t^\infty (-1)d\left(e^{-r(s-t)-(\Pi_s-\Pi_t)}\right) = \frac{\bar{q}}{r+\pi(\bar{q})}, \end{aligned}$$

where the inequality follows from $\frac{q_t}{r+\pi(q_t)} \leq \frac{\bar{q}}{r+\pi(\bar{q})}$, which is implied by the concavity of $\pi(q)$.

Second, suppose the domestic industry has no transition up to time t in the direct mechanism. The continuation values for a truth teller and a liar (who reports a transition at t) are, respectively,

$$\int_t^\infty e^{-r(s-t)}(1-e^{-(\Pi_s-\Pi_t)})cq_s ds = \int_t^\infty e^{-r(s-t)}cq_s ds -$$

$$c \int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds,$$

$$M_t - \frac{c\bar{q}}{r+\pi(\bar{q})} = \int_t^\infty e^{-r(s-t)} c q_s ds - c\frac{\bar{q}}{r+\pi(\bar{q})}.$$

The first value is higher than the second because of (19). ■

Appendix B: Social surplus

Suppose the government cares about both the consumers and the domestic industry, and puts a weight $\delta \leq 1$ on the latter. If $\delta < 1$, the optimal policy under public information is the same as that in Theorem 1: The optimal payoff of the domestic industry is still zero because the government puts a higher weight on the consumers. Under private information, the government's objective function changes to

$$\begin{aligned} & -\int_0^\infty e^{-rt-\Pi_t} \left(L(q_t) + cq_t + \pi(q_t)(1-\delta)M_t\right) dt \\ = & -\int_0^\infty e^{-rt-\Pi_t}(L(q_t) + cq_t)dt - \int_0^\infty (1-e^{-\Pi_t})(1-\delta)e^{-rt} cq_t dt \\ = & -\int_0^\infty e^{-rt}(e^{-\Pi_t}(L(q_t) + \delta cq_t) + (1-\delta)cq_t)dt \\ = & -(1-\delta)\int_0^\infty e^{-rt}(e^{-\Pi_t}\tilde{L}(q_t) + cq_t), \end{aligned}$$

where $\tilde{L}(q) \equiv \frac{L(q)+\delta cq}{1-\delta}$. In other words, the government's problem with a positive weight on the domestic industry is equivalent to another problem in which the weight is zero but the social loss function increases from $L(q)$ to $\tilde{L}(q)$. Intuitively, the government provides more protection with either a positive weight on the domestic industry or a higher social loss function. Our Theorems 2 and 3 continue to hold, but the parameter value of $\bar{c}$ in Theorem 3 has to change with the new social loss function $\tilde{L}$.

If $\delta = 1$, the optimal policies under both public and private information are the same as in Theorem 1: $q_t = q^{pub}$ before the transition and $q_t = \bar{q}$ after the transition. Under private information, the government can choose $M_t = cq^{pub}/r$ for all $t \geq 0$ to satisfy the incentive constraint (8). Providing this reward to the domestic industry does not affect the government's payoff.

Appendix C: Three-cost model

Suppose the domestic industry's cost can take one of three values, i.e., the time-t cost is given by $c_t \in \{c_l, c_m, c_h\}$, where $0 = c_l < c_m < c_h$. The industry's initial cost c_0 is equal to c_h and is observable to the government, but future costs c_t for $t > 0$ are not observable. Let $\pi(q_t)$ denote the Poisson transition rate from state c_i to a lower-cost state $c_j < c_i$; that is, $\Pr(c_{t+dt} = c_j | c_t = c_i) = \pi(q_t)dt$. Clearly, the transition rate from c_i to c_j is zero if $c_j > c_i$.

The time-t history of the domestic industry, c^t, is the realizations of its cost up to t; that is, $c^t \equiv \{c_s : 0 \leq s \leq t\}$. A time-$t$ reporting strategy of the industry, σ_t, is a mapping from c^t to $\{c_l, c_m, c_h\}$, i.e., $\sigma_t(c^t) \in \{c_l, c_m, c_h\}$. A reporting strategy $\sigma^* \equiv \{\sigma_t^* : 0 \leq t < \infty\}$ is said to be *truth telling* if $\sigma_t^*(c^t) = c_t$ for all c^t.

The set of all possible histories reported before the realization of c_t is

$$\Big\{\sigma^{t-} : [0,t) \to \{c_l, c_m, c_h\} \quad \Big| \quad \sigma^{t-} \text{ is a right continuous and monotonically decreasing function on } [0,t)\Big\}.$$

Denote a reported history up to t as $\sigma^t \equiv (\sigma^{t-}, \sigma_t)$, i.e., σ^{t-} is followed by a report σ_t at t. A mechanism is a history-contingent plan $\{(q_t, \mu_t) : 0 \leq t < \infty\}$, where $q_t(\sigma^t)$ denotes domestic output and $\mu_t(\sigma^t)$ denotes the flow monetary reward to the industry, both contingent on the reported history σ^t at time t. A mechanism is *incentive compatible* if the industry's payoff under truth telling, σ^*, is weakly greater than under any alternative reporting strategy σ. To ensure incentive compatibility, the standard approach is to formulate the mechanism design problem as a dynamic programming problem with the industry's continuation utility as the state variable. In this recursive formulation, incentive compatibility corresponds to a set of inequality constraints on the law of motion of continuation utilities, which we discuss next.

The continuation utility of the industry, $w_i(\sigma^{t-})$, is defined as the expected payoff conditional on reported history σ^{t-}, current cost $c_t = c_i$, and truth telling from time t onward:

$$w_i(\sigma^{t-}) \equiv \mathbb{E}\left[\int_t^{\infty} e^{-r(s-t)}\mu_s\left(\sigma^{t-}, c_{[t,s]}\right)\, ds \,\middle|\, c_t = c_i\right],$$

where $\left(\sigma^{t-}, c_{[t,s]}\right)$ denotes the time-s history formed by appending the industry's true cost path $c_{[t,s]}$ to the reported history σ^{t-}. The expectation depends on c_i, since the distribution of future costs $c_{[t,s]}$ is conditional on the realization $c_t = c_i$, and it depends on σ^{t-}, since the reward μ_s may depend on the entire reported history up to time s, including reports made before t. The continuation utility of the government, $V_i(\sigma^{t-})$, is defined analogously:

$$V_i(\sigma^{t-}) \equiv \mathbb{E}\left[\int_t^{\infty} e^{-r(s-t)}\left(\Gamma(q_s) - c_s q_s - \mu_s\right)\, ds \,\middle|\, c_t = c_i\right],$$

where the dependence of (q_s, μ_s) on history $\left(\sigma^{t-}, c_{[t,s]}\right)$ has been suppressed to simplify notation.

We next consider incentive compatibility under three reports of σ_t.

1. $\sigma_t = c_l$. Since c_l is an absorbing state, no further transitions occur after time t. There are no incentive constraints. The low-cost industry receives continuation utility $w_l(\sigma^{t-})$.

2. $\sigma_t = c_m$. Suppose a low-cost industry misreports and reports the middle cost at time t. To compute the utility from such misreporting, assume it continues to report c_m during the interval $[t, t+dt)$, and then truthfully reveals c_l at time $t+dt$. In that case, its payoff from cheating is:

$$((c_m - c_l)q_t + \mu_t)\, dt + e^{-rdt} w_l(\sigma^{t-}, c_m^{[t,t+dt)}),$$

where $(\sigma^{t-}, c_m^{[t,t+dt)})$ denotes the history where σ^{t-} is followed by reports of c_m during $[t, t+dt)$. The corresponding incentive constraint is:

$$w_l(\sigma^{t-}) \geq ((c_m - c_l)q_t + \mu_t)\, dt + e^{-rdt} w_l(\sigma^{t-}, c_m^{[t,t+dt)}). \tag{20}$$

If the industry instead reports truthfully, the promise-keeping constraint for the middle-cost industry is:

$$\begin{aligned} w_m(\sigma^{t-}) = \mu_t\, dt + e^{-rdt}\Big[&\pi(q_t)dt \cdot w_l(\sigma^{t-}, c_m^{[t,t+dt)}) \\ &+ (1 - \pi(q_t)dt) \cdot w_m(\sigma^{t-}, c_m^{[t,t+dt)})\Big], \end{aligned} \tag{21}$$

where $\pi(q_t)dt$ is the probability of transitioning to the low-cost state at $t + dt$, given a current cost of c_m.

As shown in Theorem 1 of Zhang (2009), conditions (20) and (21) are equivalent to the following system:

$$\frac{dw_l(\sigma^{s-})}{ds} \leq rw_l(\sigma^{s-}) - (c_m - c_l)q_s - \mu_s, \tag{22}$$

$$\frac{dw_m(\sigma^{s-})}{ds} = rw_m(\sigma^{s-}) + \pi(q_s)\left(w_m(\sigma^{s-}) - w_l(\sigma^{s-})\right) - \mu_s, \tag{23}$$

where $\sigma^{s-} \equiv (\sigma^{t-}, c_m^{[t,s)})$. In other words, as long as the industry reports c_m, the state vector (w_l, w_m) must satisfy (22)-(23) to ensure both incentive compatibility and promise keeping.

3. $\sigma_t = c_h$. As long as the industry reports c_h, the state vector (w_l, w_m, w_h) must satisfy a system analogous to (22)-(23):

$$\frac{dw_l}{ds} \leq rw_l - (c_h - c_l)q_s - \mu_s, \tag{24}$$

$$\frac{dw_m}{ds} \leq rw_m + \pi(q_s)(w_m - w_l) - (c_h - c_m)q_s - \mu_s, \tag{25}$$

$$\frac{dw_h}{ds} = rw_h + \pi(q_s)(w_h - w_l) + \pi(q_s)(w_h - w_m) - \mu_s. \tag{26}$$

For simplicity, we have suppressed the dependence of (w_l, w_m, w_h) on the history $(\sigma^{t-}, c_h^{[t,s)})$.

Next, we present the HJB equations for the government's value function. As before, we consider three cases of σ_t:

1. $\sigma_t = c_l$. The state variable is the continuation utility w_l for a low-cost industry. The government sets $q_t = \bar{q}$ to maximize externality Γ, provides no production subsidy since $c_l = 0$, and delivers w_l via monetary transfer: $\int_t^\infty e^{-r(s-t)}\mu_s\, ds = w_l$. Thus, the government's payoff is:
$$V_l(w_l) \equiv \frac{\Gamma(\bar{q})}{r} - w_l.$$

2. $\sigma_t = c_m$. The state variables are the continuation utilities (w_l, w_m) for the low-cost and middle-cost industries. Given the law of motion in (22)–(23), the HJB equation for the value function $V_m(w_l, w_m)$ is:
$$\begin{aligned} &rV_m(w_l, w_m) \\ = \max_{\substack{q\geq 0,\mu\geq 0\\ \lambda_l\geq 0}} \Bigg\{ &\Gamma(q) - c_m q - \mu + \frac{\partial V_m}{\partial w_l}\left(rw_l - (c_m - c_l)q - \mu - \lambda_l\right) \\ &+ \frac{\partial V_m}{\partial w_m}\left(rw_m + \pi(q)(w_m - w_l) - \mu\right) \\ &+ \pi(q)\left[V_l(w_l) - V_m(w_l, w_m)\right] \Bigg\}, \end{aligned}$$
where the control variable $\lambda_l \geq 0$ represents the slackness in the incentive constraint (22).

3. $\sigma_t = c_h$. The state variables are the continuation utilities (w_l, w_m, w_h) for all three types. Based on the dynamics in (24)–(26), the HJB equation for the value function

$V_h(w_l, w_m, w_h)$ is:

$$\begin{aligned} & rV_h(w_l, w_m, w_h) \\ = \max_{\substack{q \geq 0,\, \mu \geq 0 \\ \lambda_l \geq 0, \lambda_m \geq 0}} & \Bigg\{ \Gamma(q) - c_h q - \mu + \frac{\partial V_h}{\partial w_l} (r w_l - (c_h - c_l) q - \mu - \lambda_l) \\ & + \frac{\partial V_h}{\partial w_m} (r w_m + \pi(q)(w_m - w_l) - (c_h - c_m) q - \mu - \lambda_m) \\ & + \frac{\partial V_h}{\partial w_h} (r w_h + \pi(q)(w_h - w_l) + \pi(q)(w_h - w_m) - \mu) \\ & + \pi(q) \left[V_l(w_l) - V_h(w_l, w_m, w_h) \right] \\ & + \pi(q) \left[V_m(w_l, w_m) - V_h(w_l, w_m, w_h) \right] \Bigg\}, \end{aligned} \tag{27}$$

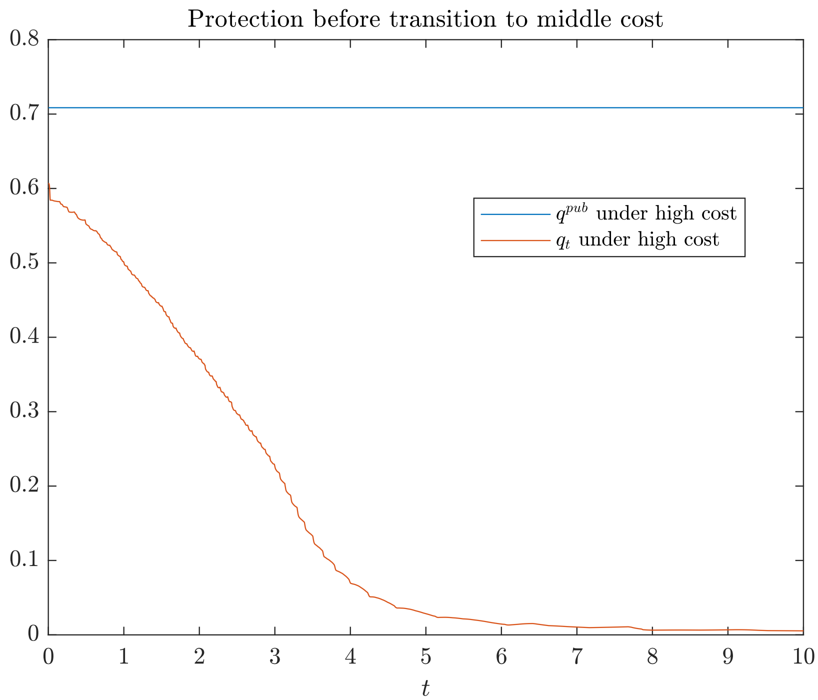
where the control variables λ_l and λ_m capture the slackness in constraints (24) and (25), respectively.

Because we cannot solve equation (27) analytically, we turn to numerical methods and solve it using value function iteration. We find that the main results of our two-cost model–Theorems 1, 2, and 3–continue to hold in the three-cost setting, broadly speaking. In particular, domestic output q_t declines over time until the zero-cost state is reported, and remains strictly below q^{pub} throughout; see Figure 1.

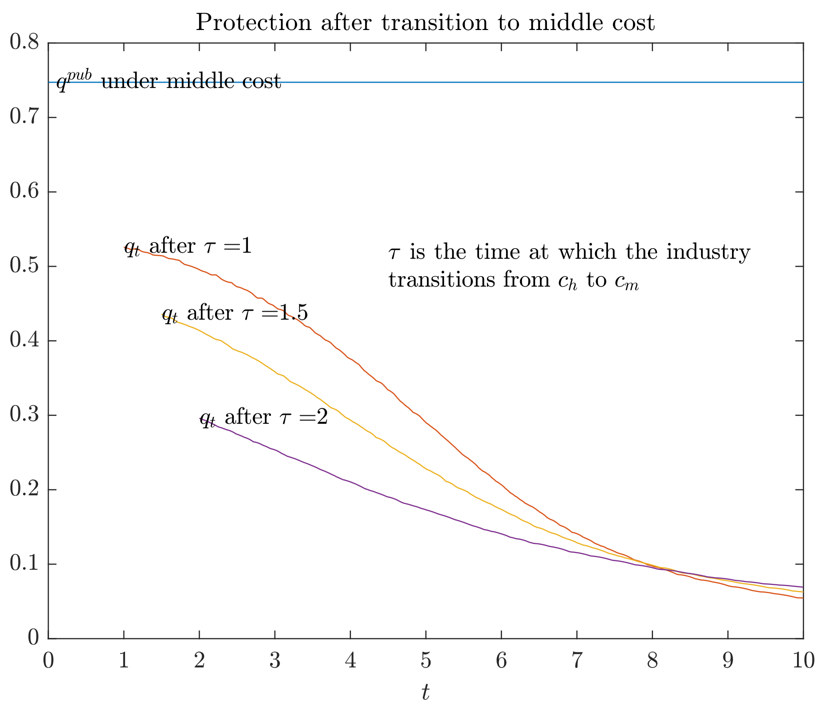
Appendix D: Implementation

THEOREM 4 (Implementation) *Facing the constraint $\tilde{q}_t \in [q_t, \bar{q}]$, it is optimal for the infant industry to replicate the allocation in the direct mechanism, i.e., produce q_t before the transition and $\bar{q}$ after the transition.*

PROOF: We do not need to study the domestic industry's strategy after the transition, because any feasible strategy is optimal. In our implementation, the domestic industry



(a)



(b)

Figure 1: The time path of q_t before and after transition to the middle cost. Parameters: $c_l = 0$, $c_m = 1$, $c_h = 2$, $r = 0.05$, and $\Gamma(q) = 1.6q - q^2$. The transition rate equals 0 if $c_j > c_i$ and $q^{0.95}$ if $c_j < c_i$, where $i \in \{l, m, h\}$ and $j \in \{l, m, h\}$.

does not receive any subsidy after time 0 and is fully responsible for its production cost. Therefore the industry's objective is to minimize the present value of its production cost before transition.

Consider any feasible production plan $\{\tilde{q}_t\}_{t\geq 0}$ and let $\tilde{T}$ denote the time when the fund runs out, that is $\int_0^{\tilde{T}} e^{-rt}\tilde{q}_t dt = M_0 = \int_0^{\infty} e^{-rt} q_t dt$. Feasibility here means

$$\tilde{q}_t \begin{cases} \geq q_t, & \text{if } t \in [0, \tilde{T}]; \\ = 0, & \text{if } t > \tilde{T}. \end{cases}$$

In the rest of this proof, we will show $\int_0^{\tilde{T}} e^{-rt-\tilde{\Pi}_t}\tilde{q}_t dt \geq \int_0^{\infty} e^{-rt-\Pi_t} q_t dt$, which means choosing $\{q_t\}_{t\geq 0}$ is optimal.

First, construct a production plan that equals $\tilde{q}_s$ for all $s \in [0, t]$ and equals q_s for $s \geq t$, where $t \in [0, \tilde{T}]$ is a parameter. Under this plan, the time when the fund runs out, T_t, satisfies $\int_t^{T_t} e^{-rs} q_s ds = \int_t^{\tilde{T}} e^{-rs}\tilde{q}_s ds$. The total production cost under this plan is $V_t \equiv \int_0^t e^{-rs-\tilde{\Pi}_s}\tilde{q}_s ds + \int_t^{T_t} e^{-rs-\tilde{\Pi}_t-(\Pi_s-\Pi_t)} q_s ds$.

Second, we show that V_t is increasing in t, or equivalently

$$\begin{aligned} \frac{dV_t}{dt} &= e^{-rt-\tilde{\Pi}_t}\tilde{q}_t - e^{-rt-\tilde{\Pi}_t} q_t + e^{-rT_t-\tilde{\Pi}_t-(\Pi_{T_t}-\Pi_t)} q_{T_t}\frac{dT_t}{dt} \\ &\quad + \left(\int_t^{T_t} e^{-rs-\tilde{\Pi}_t-(\Pi_s-\Pi_t)} q_s ds\right)\frac{d(\Pi_t-\tilde{\Pi}_t)}{dt} \geq 0. \end{aligned}$$

Substituting $e^{-rT_t} q_{T_t}\frac{dT_t}{dt} = e^{-rt}(q_t - \tilde{q}_t)$ and $\frac{d(\Pi_t-\tilde{\Pi}_t)}{dt} = \pi(q_t) - \pi(\tilde{q}_t)$ yields

$$\begin{aligned} \frac{dV_t}{dt} &= e^{-rt-\tilde{\Pi}_t}\tilde{q}_t - e^{-rt-\tilde{\Pi}_t} q_t + e^{-rt-\tilde{\Pi}_t-(\Pi_{T_t}-\Pi_t)}(q_t - \tilde{q}_t) \\ &\quad + \left(\int_t^{T_t} e^{-rs-\tilde{\Pi}_t-(\Pi_s-\Pi_t)} q_s ds\right)(\pi(q_t) - \pi(\tilde{q}_t)) \\ &= e^{-rt-\tilde{\Pi}_t}\Bigg(\tilde{q}_t - q_t + e^{-(\Pi_{T_t}-\Pi_t)}(q_t - \tilde{q}_t) \end{aligned}$$

$$+\left(\int_t^{T_t} e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds\right)\left(\pi(q_t)-\pi(\tilde{q}_t)\right)\Bigg).$$

To show the inequality

$$\begin{aligned} f(\tilde{q}_t) \equiv \tilde{q}_t - q_t &+ e^{-(\Pi_{T_t}-\Pi_t)}(q_t-\tilde{q}_t) \\ &+ \left(\int_t^{T_t} e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds\right)\left(\pi(q_t)-\pi(\tilde{q}_t)\right) \geq 0, \end{aligned}$$

it is sufficient to show $f'(q_t) \geq 0$ since $f(q_t) = 0$ and function $f(\tilde{q}_t)$ in convex in $\tilde{q}_t$. We have

$$\begin{aligned} f'(q_t) &= 1 - e^{-(\Pi_{T_t}-\Pi_t)} - \left(\int_t^{T_t} e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds\right)\pi'(q_t) \\ &= \int_t^{T_t} e^{-(\Pi_s-\Pi_t)}\pi(q_s)ds - \left(\int_t^{T_t} e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds\right)\pi'(q_t) \\ &\geq \int_t^{T_t} e^{-(\Pi_s-\Pi_t)}\pi'(q_t)q_s ds - \left(\int_t^{T_t} e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds\right)\pi'(q_t) \\ &= \pi'(q_t)\int_t^{T_t} e^{-(\Pi_s-\Pi_t)} q_s(1-e^{-r(s-t)})ds \geq 0, \end{aligned}$$

where the first inequality follows from $\pi(q_s) \geq \pi'(q_s)q_s \geq \pi'(q_t)q_s$, which is implied by the concavity of $\pi(q)$.

Third, the fact that V_t is increasing in t implies $V_0 \leq V_{\tilde{T}}$, or

$$\int_0^{\infty} e^{-rs-\Pi_s} q_s ds = V_0 \leq V_{\tilde{T}} = \int_0^{\tilde{T}} e^{-rs-\tilde{\Pi}_s}\tilde{q}_s ds.$$

This concludes the proof of Theorem 4. ∎

To understand our implementation, four features are worth noting. First, in the presence of positive imports the competitive market price is 0. The total quantity demanded is $Q_t = p^{-1}(0)$. The infant industry supplies $\tilde{q}_t$ and the foreign firms supply $p^{-1}(0) - \tilde{q}_t$.

Second, before the transition time T, the fund balance, $B_t \geq 0$, evolves as a function of domestic output $\tilde{q}_t$:

$$\frac{dB_t}{dt} = rB_t - c\tilde{q}_t, \quad B_0 = M_0. \tag{28}$$

For the infant industry, increasing $\tilde{q}_t$ depletes the fund at t but increases the odds of transition due to learning by doing. The government has already internalized the benefit of learning when it chose q_t, so the industry cannot gain by producing more than q_t. However, the industry does not take into account the higher external benefits, which is internalized only by the government. Therefore, the industry prefers an output below the socially optimal q_t. The constraint $\tilde{q}_t \in [q_t, \bar{q}]$ ensures that the industry's choice is q_t.

Third, one can easily verify that M_t defined in (8) satisfies the constraint (28) when $\tilde{q}_t = q_t$, meaning that the fund balance B_t equals M_t. If the transition arrives at T, then the balance B_T can be interpreted as a reward. Note that the industry is not allowed to withdraw B_T as a lump sum at T. Instead, the payments to the industry follow the sequence $\{c\tilde{q}_t\}_{t \geq T}$ whose present value is B_T. This is because the transition is not observable and our implementation does not rely on the report of transition. In contrast, the lump-sum reward and sequence of payments are equivalent in the direct mechanism.

Fourth, after the transition at T, the industry is indifferent to any output in $[q_t, \bar{q}]$ and does not gain by deviating from $\bar{q}$. If the industry produces $\bar{q}$, then imports are $p^{-1}(0) - \bar{q}$, and the social welfare is maximized.