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Politically influenced counterterrorism policy and welfare efficiency

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Abstract

The paper examines how two targeted countries strategically deploy their counterterror forces when lobbying defense firms influence counterterror provision. For proactive measures, lobbying activities in a single targeted country lessen underprovision, raise overall counterterrorism, and reduce terrorism. Welfare decreases in the politically influenced country but increases in the other targeted country owing to enhanced free riding. Lobbying influence on the targeted countries’ welfare is tied to terrorists’ targeting preferences and how the lobbied government weighs citizens’ welfare. For key parametric values, lobbying in both targeted countries may result in the first-best equilibrium. With two-country lobbying, international policy coordination by at-risk governments may lead, surprisingly, to less efficient outcomes than the noncooperative equilibrium. Additionally, lobby-influenced defensive countermeasures generally affect efficiency adversely.

\textit{JEL Codes}: D74, H23, H41
\textit{Keywords}: proactive counterterror and lobbying, drones, unilateral Nash equilibrium, politically influenced Nash equilibrium, welfare efficiency

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1. Introduction

Despite the passage of over two decades since the four skyjackings on September 11, 2001, (henceforth, 9/11) with the loss of nearly 3,000 people, the wounding of over 6,000 people, and tens of billions of US dollars in damages, the threat of transnational terrorism looms large and results in the spending of tens of billions of dollars annually (Sandler et al. 2009). Terrorism is the premeditated use or threat to use violence against noncombatants by individuals or subnational groups to obtain a political objective through the intimidation of a large audience beyond that of the immediate victims (Enders and Sandler 2012, p. 4; Hoffman 2012). Terrorists aim to generate sufficient audience costs through ghastly attacks that may put virtually anyone at risk, so that some governments feel constituent pressure to concede to terrorists’ political demands in order to bring tranquility. Because most governments are loath to grant terrorist demands, intransigent governments must resort to enhanced counterterrorism spending to protect their civilians’ lives and property.

A rich and varied literature developed during the last couple of decades, spurred in part by 9/11 (see, e.g., Arce and Sandler 2005; Bandyopadhyay et al. 2011; Bandyopadhyay and Sandler 2021; Carter 2016; Garcia-Alonso et al. 2016; Heal and Kunreuther 2007; Jindapon and Neilson 2009; Kunreuther and Heal 2003; Rosendorff and Sandler 2004; Schneider at al. 2015). The literature distinguishes between two kinds of counterterrorism actions. Proactive measures are offensive responses against terrorist organizations’ assets (i.e., training camps, sponsors, safe havens, resources, and personnel) with the goal to weaken the groups’ capabilities so that they pose a much-diminished threat, thereby resulting in fewer operations. Moreover, launched terrorist operations are less formidable if proactive means are effective. By contrast, defensive actions are protective, intended to limit the success or consequences of terrorist incidents by
fortifying potential targets (Bier et al. 2007; Bandyopadhyay and Sandler 2011; Hausken et al. 2009; Landes 1978). Such defensive measures can also allow targeted entities to recover faster in the aftermath of terrorist attacks. The fortification of potential targets (e.g., metal detectors in airports) is a prime defensive example. Other instances include air marshals on flights, cement barriers around buildings, installation of surveillance systems, and intelligence gathering on potential terrorist targets.

For multiple at-risk countries, theoretical studies of proactive and defensive counterterrorism identify two important policy tendencies: (1) proactive measures are generally underprovided by commonly targeted countries, and (2) defensive responses are oversupplied by commonly targeted countries (e.g., Arce and Sandler 2005; Bandyopadhyay and Sandler 2011; Cárceles-Poveda and Tauman 2011; Gaibulloev and Sandler 2019; Hausken et al. 2009; Rossi de Oliveira et al. 2018; Sandler and Lapan 1988). The undersupply of proactive operations stems from their purely public good properties (i.e., nonrival and nonexcludable benefits) that motivate targeted countries to free ride, when possible, on the proactive response of other countries threatened by a common terrorist adversary. By sitting back and waiting for other countries to weaken the terrorist group, a country can save its scarce proactive resources for other expenditures. When a nation takes proactive measures, it accounts for its own derived marginal benefits but not for those that its actions confer on other targeted countries, so that derived marginal benefits are not equated with marginal costs (Cornes and Sandler 1996; Enders and Sandler 2012).

Alternatively, the overprovision of defensive measures comes from an attempt by targeted countries to raise the price to terrorists from attacking them (Bier et al. 2007; Enders and Sandler 2004), which leads to terror transference to other countries. As countries engage in similar counterterrorism efforts, a deterrence race ensues whereby defensive actions increase
well beyond the point at which marginal benefits equal marginal costs, thereby resulting in overprovision. Two targeted countries’ proactive measures are strategic substitutes that vary inversely, while two targeted countries’ defensive actions are strategic complements that vary directly (Sandler and Siqueira 2006).

The purpose of the current study is to show theoretically that lobbying activities by defense contractors, say drone manufacturers, can affect the conventional wisdom with respect to the under- and overprovision of proactive and defensive counterterror measures, respectively. For proactive measures, lobbying by the defense firm reduces underprovision and, in some cases, may even result in overprovision when one of two targeted country is lobbied. The single-country lobbying scenario may actually lift the targeted countries’ combined welfare even as the politically influenced country experiences a welfare decline, but a first-best optimum cannot occur. The effects on the two countries’ welfare are dependent on the weight that the lobbied country assigns to its citizens’ welfare and on the targeting preferences of the terrorist group, who operates from some third country (e.g., al-Qaida in Afghanistan or Islamic State in Iraq). Those terrorists may favor attacks against one country over the other. In the two-country noncooperative lobbying scenario, surprising results arise where a first-best optimum may follow and outdo a two-country cooperative lobbying situation.

Since our main story hinges on the lobbying by defense firms that produce counterterror weapons, we must provide some background. Most of our analysis involves lobbying for proactive measures. An apt instance comes from producers of unmanned aerial vehicles (UAVs) or drones, which can target terrorists’ assets in the field. An increased demand for drones grew during the post-9/11 conflicts in Afghanistan and Iraq as part of the War on Terror (Hall 2015).  

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1 Hall and Coyne (2014) provide a fascinating history of UAVs that date back to 1913 with the development of radio-controlled aircraft. Drone development picked up after the first Gulf War in 1991.
During 2001–2013, US spending on drones increased from $36.3 million to $2.9 billion. By 2012, UAVs accounted for one-third of all US military aircrafts (Hall and Coyne 2014). Drones played a vital role in US proactive counterterror attacks with hundreds of missions authorized by the Obama and Trump administrations in Afghanistan, Pakistan, Somalia, and Yemen during 2015–2018 (see the list of operations by location in The Bureau of Investigative Journalism 2022). In part, drone attacks are favored because they pose much smaller risks to US military personnel. Ideally, but not always, such attacks can avoid collateral civilian casualties, which could result in backlash. Currently, combat drones are possessed by an ever-growing number of countries including the United States, the UK, Israel, and the United Arab Emirates (UAE) (New America 2022; Weiss 2017). Those and other countries have used drones against terrorists—e.g., US drone strikes involved al-Qaida, Haqqani Network, and Tehrik-i-Taliban in Pakistan (TTP) (Jaeger and Siddique 2011; Johnston and Sarbahi 2016).

In the United States, the four major producers of drones—Boeing, General Atomics, Lockheed Martin, and Northrop Grumman—spent millions in lobbying the Congress to promote UAVs (Conradis 2013a, 2013b; Karbal 2020). Hartley (2017) recognizes that the War on Terror offered profitable opportunities for arms producers to promote their specialized weapons systems. Drones became one of the favored weapons in proactive responses to terrorists (The Bureau of Investigative Journalism 2020). Johnston and Sarbahi (2016) find that drone strikes in Pakistan generally reduced the rate and lethality of terrorist attacks. Moreover, those authors present evidence that drone attacks limited the Taliban’s targeting of tribal elders, which was viewed favorably by the local population.

The remainder of the paper has five sections. In Section 2, we formulate a baseline

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2 However, Smith and Walsh (2013) show that drone strikes against al-Qaida in Pakistan has not reduced the group’s propaganda output.
model containing two targeted countries – $A$ and $B$ – and a terrorist group, based in country $C$. This model involves the targeted countries strategically deciding their levels of proactive measures, given their counterpart’s proactive choice and the targeting bias of the common terrorist group. Each targeted country produces a general consumption good and counterterror proactive good. A welfare-optimizing choice of the two goods is determined when the targeted countries act independently and there is no lobbying. Section 3 allows country $A$, but not $B$, to have its counterterror proactive production influenced by lobbying. With political lobbying, total proactive measures increase despite some free riding in $B$, leading to a decrease in overall terrorism in the two targeted-country scenario. Not only does lobbying limit proactive underprovision but it may, under identified circumstances, bolster the two countries’ welfare, which, however, remains suboptimal. We compare the targeted countries’ unilateral Nash equilibrium with their politically influenced equilibrium. In Section 4, the investigation permits lobbying in both countries to unleash a complex set of externalities that hinges on how targeted countries weight their citizens’ welfare, the countries’ capital shares, and the terrorists’ targeting decision. Under specified parameter values, the two-country lobbying scenario may achieve a first best and outdo a cooperative two-country lobbying equilibrium. Section 5 considers lobbying when at-risk countries are providing defensive counterterror actions. Now, lobbying exacerbates the routine tendency of overprovision, leading to further welfare losses. Concluding remarks comprise Section 6.

2. The Model: Proactive Counterterrorism

We commence with the baseline model that captures our three-country, terrorism-plagued environment. Nations $A$ and $B$ are potential targets of terrorists originating in nation $C$. Terrorism suffered by nation $j$ ($j = A, B$) is represented by $T^j$, with total terrorism then
equivalent to \( T = T^A + T^B \). The terrorist group in nation \( C \) has a resources pool, \( E \), which can be depleted by targeted governments’ aggregate proactive measures, \( G = g^A + g^B \), for which \( g^j (j = A, B) \) corresponds to those actions by nation \( j \). The terrorist group’s payoff function \( V \left( T^A, T^B \right) \) increases in its terrorist attacks on either targeted country. In the appendix, we use the terrorist group’s resource constrained optimization problem to provide the micro-foundation for its terrorism function,

\[
T = T \left( g^A + g^B \right), \ T' < 0, \text{ and } T'' > 0, \tag{1}
\]

which is reduced by total counterterror proactive measures at a diminishing rate. Further, the micro-foundations indicate that the national terrorism incidence functions are \( T^A = \theta T \) and \( T^B = (1-\theta)T \), where \( 0 < \theta < 1 \) is the terrorist group’s target parameter independent of counterterror provision levels.

Let nation \( j \) be populated by \( N^j \) individuals, who consume a good \( x^j \), and let person \( i \) in \( j \) be endowed with a unit of labor. A subset of individuals \( K^j < N^j \) also owns a unit each of specific capital, used in producing counterterror effort, such that the aggregate capital endowment is \( K^j \). We use the term counterterror effort broadly to include use of counterterrorism equipment like drones used to take out terrorist leaders (e.g., US drone attack against Abdul Hamid al-Matar, a senior al-Qaida leader, on October 23, 2021, in Syria) and operatives in the field (US drone attacks in Afghanistan and Iraq during post-9/11). The production of good \( x^j \) is assumed to follow a linear technology,

\[
x^j = l^j_x, \tag{2}
\]

where \( l^j_x \) is labor used to produce the good. Eq. (2) fixes the wage rate in both nations at unity. The utility function of individual \( i \) in nation \( j \) equals
\[ u^{ij} = x^{ij} - \frac{T^j}{N^j}, \]  
where \( x^{ij} \) is the consumption of good \( x \) in nation \( j \) by person \( i \), and \(-T^j/N^j\) is person \( i \)'s disutility from home terrorism. Utilitarian representation of nation \( j \) welfare is denoted by

\[ W^j = X^j - T^j, \]  
where \( \sum_{i=1}^{N^j} u^{ij} = W^j \) and \( \sum_{i=1}^{N^j} x^{ij} = X^j \), so that national welfare consists of aggregate consumption of good \( x \), net of the aggregate indigenous terrorist incidents.

A constant returns to scale production function, \( g^j = g^j (l^j, K^j) \), is assumed for which \( l^j \) is labor applied in the counterterrorism sector \( g \); hence, a given sector-specific capital \( K^j \) implies that the production function of good \( j \) can be written inversely as:

\[ l^j = \phi (g^j), \quad \phi' > 0, \quad \phi'' > 0. \]  
For suitably chosen units, the unit price of counterterror effort is unity.\(^3\) Denoting the excess of revenue of sector \( g \) over labor costs in \( j \) as \( \pi^j_g \) and recalling the wage rate is unity, we have

\[ \pi^j_g = g^j - l^j = g^j - \phi (g^j). \]  
Owners of specific capital earn \( \pi^j_g / K^j \) as their capital returns along with wage income of unity.

For a subsequent meaningful discussion of lobbying pressures on the government by the counterterror producers, we assume that, at the policy-constrained level of \( g \), say \( g^{j_0} \), \( \pi^j_g \) is increasing in \( g \); i.e.,

\[ \left( \frac{\partial \pi^j_g}{\partial g^j} \right)_{g^j = g^{j_0}} > 0 \iff \left[ \phi' (g^j) \right]_{g^j = g^{j_0}} < 1. \]  
\(^3\) This assumption requires a fixed price of the counterterror good, paid by the government to the producers of \( g \). As long as aggregate government expenses translate to greater real levels of counterterror effort, this assumption is for expository convenience.
In targeted country \( j \), the per-capita taxes, \( t^j \), finance the country’s counterterror efforts. The net income of individuals without any capital is \( 1-t^j \), while the net income of those with capital is \( 1+\frac{\pi_g^j}{K^j}-t^j \), which they spend on good \( x \). Based on Eq. (3), the utility of non-capital-endowed and capital-endowed individuals are, respectively,

\[
\begin{align*}
  u^{ij} = 1-t^j - \frac{T^j}{N^j} \\
  u^{ij} = 1+\frac{\pi_g^j}{K^j}-t^j - \frac{T^j}{N^j}.
\end{align*}
\]

(7)

Summing over all individuals in the two groups and using \( t^j = g^j / N^j \) and Eq. (6), we derive

\[
W^j = N^j - \phi(g^j) - T^j,
\]

(8)

in which \( N^j - \phi(g^j) = N^j - l^j_g \) is the labor available to produce good \( x \), allowing for production of \( g \). Given Eq. (2), the aggregate level of the consumption good is \( X^j = N^j - l^j_g \). Thus, Eq. (8) captures the resource tradeoff for the nation in which a larger amount of \( g \) reduces terror but at the expense of labor to make consumption good \( x \). That tradeoff guides the unilateral welfare-maximizing choice of each country’s counterterror \( g^j \) to which we now turn.

2.1. Welfare-maximizing counterterror effort

Given the public good nature of proactive counterterrorism, each targeted country must account for their counterpart’s choice of such actions (Bandyopadhyay and Sandler 2011; Rossi de Oliveira et al. 2018; Sandler and Lapan 1988; Sandler and Siqueira 2006; Schneider et al. 2015). That is, a targeted country, which pursues offensive actions to weaken a common foreign terrorist threat, makes the other targeted country safer, thus limiting its needed offensive effort.

Let at-risk nations \( A \) and \( B \) choose their respective unilateral counterterror effort levels simultaneously. Substituting \( T^A = \theta T \), \( T^B = (1-\theta) T \), and \( T = T \left( g^A + g^B \right) \) into Eq. (8) gives
\[ W^A = N^A - \phi(g^A) - \theta T(g^A + g^B), \]  

or \( A \)'s welfare level, net of counterterrorism expense and terrorism. We first consider nation \( A \)'s marginal welfare effect of an increase in its counterterror effort, \( g^A \):

\[ W^A_{g^A} = -\phi'(g^A) - \theta T'(g^A + g^B). \]

As \( g^A \to 0 \), without any fixed labor costs in sector \( g \), \( l^A_g \to 0 \). Given sector-specific capital \( K^j \), the sector’s capital to labor ratio rises sharply such that \( \phi'(g^A \to 0) \to 0 \). This implies that

\[ \left( W^A_{g^A} \right)_{g^A \to 0} = -\theta T'(g^A + g^B) > 0, \]

so that we can rule out a corner solution for \( A \)'s counterterror decision. Similar reasoning also rules out a corner solution for nation \( B \).

Consequently, the first-order conditions (FOCs) of unilateral welfare maximization for the two nations are, respectively,

\[ W^A_{g^A} = -\phi'(g^A) - \theta T'(g^A + g^B) = 0, \]

which implicitly define the Nash reaction functions of nations \( A \) and \( B \), respectively, in \((g^A, g^B)\) space. If \( \theta = 1/2 \) and both targeted countries possess identical \( \phi \) functions, then Eqs. (11a) and (11b) jointly determine a symmetric Nash equilibrium \((g^A = g^N, g^B = g^N)\). If, however, \( A \) is the

---

4 Consider as an example the Cobb-Douglas production function \( g^A = (l^A_g)^\mu (K^A)^{1-\mu} \), \( 0 < \mu < 1 \), such that

\[ l^A_g = \phi(g^A) = (K^A)^{\mu \gamma} (g^A)^{\frac{\mu}{\gamma}}. \]

One can readily checked that \( \phi'(g^A) = \frac{1}{\mu} \left( \frac{l^A_g}{K^A} \right)^{\frac{1-\mu}{\gamma}} \to 0 \) when \( l^A_g \to 0 \).

5 Note that second-order conditions (SOCs) \( W^A_{g^A} = -\phi'(g^A) - \theta T^* < 0 \) and \( W^B_{g^B} = -\phi'(g^B) - (1-\theta)T^* < 0 \) are satisfied. Throughout the paper, our modeling assumptions ensure that SOCs are satisfied – see online appendix.
preferred target of the terrorists, i.e., \( \theta > 1/2 \), then \(-\theta T'(g^A + g^B) > -(1-\theta)T'(g^A + g^B)\). As a consequence, Eqs. (11a) and (11b) imply that \( \phi'(g^A) > \phi'(g^B) \), which given \( \phi'' > 0 \), yields \( g^A > g^B \). Thus, although both countries provide counterterror effort in an asymmetric Nash equilibrium, the preferred targeted country provides a larger level of counterterror effort.

To foster intuition, we characterize the Nash reaction functions and their equilibrium. First, we note that the equilibrium is both interior and unique, the latter because the stability condition \( W_A^A g^A g^A - W_B^A g^A g^A > 0 \) can be shown to be globally satisfied (see Bergstrom et al. 1986; Cornes and Sandler 1996; Cornes et al. 1999). Second, the reaction functions are negatively sloped, which follows immediately from Eqs. (11a) and (11b), where, given that the SOC's are satisfied, \( A \)'s and \( B \)'s reaction function’s respective slopes are determined by the cross partials \( W_A^A g^A g^A < 0 \) and \( W_B^A g^A g^A < 0 \). Thus, counterterror provisions of the two countries are strategic substitutes, where a greater provision by either incentivizes the other to reduce its provision (i.e., free ride).

### 2.2. The Targeted Countries’ First-Best Outcome

At the “first-best” outcome, national proactive levels maximize the targeted countries’ welfare. In a symmetric case, maximizing the two countries’ welfare is the same as maximizing national welfare levels. Under asymmetry, joint welfare maximization remains relevant because international transfers can potentially be made to yield Pareto improvement compared to a two-country inefficient outcome. Given Eq. (8), joint welfare \( W(g^A, g^B) = W_A^A + W_B^B \) is maximized when:

\[
W_A^A = -\phi'(g^A) - T' = 0 \quad \text{and} \quad W_B^B = -\phi'(g^B) - T' = 0.
\]  
(12)
Eq. (12) implies that:
\[ \phi'(g^A) = \phi'(g^B) = \left| T'(g^A + g^B) \right|, \tag{13} \]
where (a) marginal provision costs are equal across targeted nations, and (b) the marginal provision costs equal the marginal two-countries’ benefit from terrorism reduction. Consequently, first-best provision levels are equalized across nations \( g^A = g^B = g^f \), independent of the target parameter \( \theta \). Eq. (13) implies that \( \phi'(g) + T'(2g) = 0 \), where the left-hand side is monotonically increasing in \( g \), because \( \phi'' > 0 \) and \( T'' > 0 \) so that the first best \( g^f \) is unique.

The Nash equilibrium of Section 2.1 is inefficient because Eqs. (11a) and (11b) show that \( \phi'(g^j) < |T'|, \quad j = A, B \). In addition, consider a symmetric Nash equilibrium, where \( g^A = g^B = g^N \). From Eqs. (11a) or (11b), we have \( -\phi'(g^j) = T'/2, \quad j = A, B \). Given Eq. (12), we get \( W_{g^j} = -T'/2 > 0, \quad j = A, B \) upon substitution so that there are gains from raising \( g^j \), \( j = A, B \), above the underprovided Nash level, \( g^N \).

3. Politically Influenced Counterterror Proactive Effort in Country A

We now explore how counterterror lobbying can have unanticipated consequences on total welfare and the level of overall counterterrorism in our two-targeted country scenario. The counterterror proactive industry in either targeted country can lobby the government – e.g., Boeing in the United States lobbies for battlefield use of its drones in counterterror missions. Our first task is to characterize the Nash equilibrium for countries \( A \) and \( B \) with lobby-influenced counterterrorism involving just \( A \). At this juncture, we assume that counterterrorism takes the form of proactive measure intended to deplete the assets of the terrorist group, based in nation \( C \).
In a subsequent section, we consider lobbying and defensive counterterrorism measures. We follow the Grossman and Helpman (1994) political contributions approach. For now, we assume that country $B$ is free of special-interest political considerations and hence pursues its true national welfare maximization. In the following section, we allow for lobbying in both nations. As in Grossman and Helpman (1994), the lobby represents the collective interests of specific capital and provides a contribution schedule to the government to elicit its action, desired by the lobby. The lobby’s contribution schedule is $H$, and the government’s payoff function is $\Omega^A$:

$$\Omega^A = H + \alpha W^A, \quad \alpha > 0,$$

(14)

where $\alpha$ is a measure of the relative weight that the government puts on welfare (gross of contribution) compared to the contribution $H$.

Formally, there are two stages in this game. In stage 1, the lobby announces the contribution schedule $H$, and in stage 2, the two governments choose their respective counterterror effort levels.

The contribution schedule $H$ can be derived by invoking Grossman and Helpman’s (1994, pg. 840, Eq. 8) truthful contribution schedule. This consists of the excess (if any) of the lobbying group’s gross welfare over some base welfare level $Z$. Using Eq. (7) and $i^j = g^i / N^j$, we obtain the gross welfare level of $A$’s capitalists, who supply counterterror goods, as:

$$\left(\sum_{i=1}^{K^A} w^i\right)_{Capital-Owners} = \sum_{i=1}^{K^A} \left(1 + \frac{\pi^A}{K^A} - i^{i^A} - \frac{T^A}{N^A}\right) = K^A + \pi^A - s^A \times \left(g^A + T^A\right),$$

(15)

where $s^A = \left(K^A / N^A\right) < 1$ denotes the share of capital owners in economy $A$. Given Eq. (6), we have $\pi^A = g^A - \phi\left(g^A\right)$. Substituting the latter expression in Eq. (15), we get:

---

As noted in Grossman and Helpman (1994), optimizing the objective function in Eq. (14) is equivalent to optimizing a function $F^A = a_1 H + a_2 (W^A - H)$, where $a_1$ and $a_2$ are weights applied to contributions and net of contributions welfare, and where $a_1 > a_2$, reflecting the fact that the government cares more about contributions than net welfare.
Thus, the truthful contribution schedule is:

\[
H = \left( \sum_{i=1}^{n} u_{ij} \right) - Z = K^A + g^A - \phi(g^A) - s^A \times \left[ g^A + \theta T \left( g^A + g^B \right) \right] - Z \equiv H \left( g^A, g^B \right).
\]  

(17)

Substituting Eqs. (9) and (17) in Eq. (14) yields:

\[
\Omega^A \left( g^A, g^B \right) = H \left( g^A, g^B \right) + \alpha W^A \left( g^A, g^B \right).
\]  

(18)

Using these functional forms of \( H \) and \( W^A \), we get government \( A \)'s Nash FOC for \( g^A \) as:

\[
\Omega_{g^A}^A \left( g^A, g^B \right) = \left( 1 - s^A \right) \left( 1 + \theta T' \right) - \alpha \phi' \left( g^A \right) + \theta T' \left( g^A + g^B \right) = 0.
\]  

(19)

Eq. (19) implicitly defines country \( A \)'s politically influenced reaction function for \( g^A \), which is negatively sloped because \( \Omega_{g^A}^A \left( g^A, g^B \right) = - \left( \alpha + s^A \right) \theta T'' < 0 \). Country \( B \) pursues welfare maximization; hence, \( B \)'s Nash reaction function is the same as Eq. (11b). Thus, Eqs. (19) and (11b) jointly determine a political Nash equilibrium \( \left( g^{AP}, g^{BP} \right) \).

The equilibrium contribution made by the lobby to government \( A \) is such that the lobby pays as little as possible to entice the government to participate in the contribution game. For the government, the payoff in the absence of any contribution \( \left( i.e., H \equiv 0 \right) \) is \( \alpha \tilde{W}^A \), where \( \tilde{W}^A \) is equilibrium welfare with zero contributions. In the absence of lobbying, we can find \( \tilde{W}^A \) by first obtaining \( \left( \tilde{g}^A, \tilde{g}^B \right) \) as the Nash equilibrium from Eqs. (11a) and (11b). For the government to participate at the politically influenced counterterror vector \( \left( g^A = g^{AP}, g^B = g^{BP} \right) \), the following

---

7 Analogous to Eq. (11a), a corner solution can be ruled out.
must hold:

\[ \Omega^A (g^{AP}, g^{BP}) \geq \Omega^A (\tilde{g}^A, \tilde{g}^B) \Rightarrow H(g^{AP}, g^{BP}) + \alpha W^A (g^{AP}, g^{BP}) \geq \alpha \tilde{W}^A. \] (20)

The lobby pushes the government to the participation constraint, such that:

\[ H = \alpha \left[ \tilde{W}^A - W^A (g^{AP}, g^{BP}) \right]. \] (21)

Now, we can determine the equilibrium level of \( Z \), which is the welfare of the lobby group net of political contributions from Eqs. (17) and (21).

3.1. Comparing unilateral welfare-maximizing equilibrium with the politically influenced equilibrium

In this subsection, we compare the Nash equilibrium from Section 2.1 with the Nash equilibrium where \( A \)'s government is politically influenced. Given Eqs. (11a) and (19), we evaluate \( \Omega^A \), at the unilateral welfare-maximizing level \( \left[ \phi'(g^A) + \theta T' = 0 \right] \) to obtain:

\[ \left( \Omega^A \right)_{g^A > 0} = (1 - s^A)(1 + \theta T') > 0, \] (22)

because \( \phi'(g^A) + \theta T' = 0 \Rightarrow 1 + \theta T' = 1 - \phi'(g^A) > 0 \). Given that \( \Omega^A_{g^A > 0} \) is globally negative, it must be that the FOC outlined in Eq. (19) can only be satisfied for \( g^A \) strictly larger than that satisfying the unilateral welfare-maximizing counterterror effort of nation \( A \). In other words, in \( (g^A, g^B) \) space, the Nash politically influenced reaction path of \( A \) must lie to the right of its Nash welfare-maximizing reaction path. We use this path displacement to provide a graphical analysis of the comparison of Nash welfare-maximizing and Nash politically influenced equilibriums.

[Figure 1 near here]

In Figure 1, for expositional simplicity we assume that \( \theta = 1/2 \), such that \( R^A \) and \( R^B \) are
the two nations’ respective Nash welfare-maximizing reaction paths, drawn linear for convenience, whose intersection determines the symmetric Nash equilibrium

$$N \left( g^A = g^B = g^N \right).$$

To consider the political equilibrium, we label $A$’s reaction path derived from Eq. (19) as $R^{ap}$, which lies to the right of $R^A$. The politically influenced Nash equilibrium is at $N^{p} \left( g^{ap}, g^{bp} \right)$, which is to the southeast of $N$ along $R^B$.

**Proposition 1:** Political influence in targeted nation $A$ raises its counterterror provision, reduces the other targeted nation $B$’s counterterror provision, and limits terrorist attacks in both nations. Nation $A$’s welfare must fall, nation $B$’s welfare must rise, while the effect on the targeted countries’ welfare is ambiguous.

**Proof:** All proofs are gathered in the appendix at the end of the paper.

The weight attached to political contributions incentivizes $A$’s government to trade off some national welfare for higher counterterror proactive provision. With greater effort from $A$, nation $B$’s marginal benefit from terror containment falls, so it reduces its counterterror effort. Aggregate terrorism falls because the absolute value of the slope of $B$’s reaction path is less than unity, so that a unit increase in $g^A$ results in a less than a unit decline in $g^B$, leading overall counterterror effort to rise. Since $\theta$ is given, terror in each nation must also fall. The rise in $A$’s counterterror provision provides a positive welfare-augmenting externality to $B$. By contrast, $A$’s welfare loss due to political influence is compounded by a negative externality of $B$’s reduced counterterror provision. Aggregate welfare effect depends on the relative strengths of these conflicting effects, an issue that we analyze in detail in the ensuing subsection. However, since Eq. (11b) still holds, Section 2.2 reminds us that even an overall welfare improvement is
inefficient for the case of unilateral political influence on the choice of proactive measures.

Finally, recall that in Section 2.1 we showed that the preferred targeted nation provides larger counterterror effort. Proposition 1 indicates that political influence tends to lift counterterror effort beyond the Nash welfare-maximizing level. This creates the possibility that even if \( B \) is the preferred target (i.e., \( \theta < 1/2 \)), it may provide less than the politically influenced nation \( A \). In this event, the nation experiencing more terror provides relatively less counterterror effort.

### 3.2. Comparative statics and aggregate efficiency considerations

Section 3.1 involved comparison of two discretely different equilibriums – one without any political influence in any nation and another with political influence in one of the two nations. Now, we look at a political Nash equilibrium and explore how a small increase in \( \alpha \) (\( A \)'s relative evaluation of \( W_A^A \)) affects that equilibrium. Since Proposition 1 provides a roadmap of what to expect in terms of the effects of political influence on counterterror provision, terrorism, and national welfare, we focus on the effect on the targeted countries’ combined welfare about which Proposition 1 is not informative. However, to complete the welfare analysis, we must first obtain the comparative static effects of a change in \( \alpha \) on \( g_A^A \) and \( g_B^B \).

Recall that Eqs. (11b) and (19) jointly define the political Nash equilibrium levels

\[ g_A^A = g_A^A(\alpha) \quad \text{and} \quad g_B^B = g_B^B(\alpha). \]

Differentiating Eqs. (11b) and (19) and solving via Cramer’s rule, we get:

\[
\frac{dg_A^A}{d\alpha} = -\frac{\Omega_A^{g_A} W_B^B g_A^B g_A^B}{D} < 0 \quad \text{and} \quad \frac{dg_B^B}{d\alpha} = \frac{\Omega_A^{g_A} W_B^B g_A^B g_A^B}{D} > 0.
\]

Those inequalities follow because \( W_B^B g_A^B \) and \( W_B^B g_A^B \) are both negative, Nash stability requires
that \(D = \Omega_{g^A}^A W_{g^B}^B - \Omega_{g^A}^B W_{g^B}^A > 0\), and Eq. (19) implies that
\[
\Omega_{g^A}^A = -\left[\phi'(g^A) + \theta T'(g^A + g^B)\right] = -\frac{(1 - s^A)(1 + \theta T')}{1 + \alpha} < 0.
\]
Based on Eq. (23), it follows that
\[
G^A = \frac{dG}{d\alpha} = \frac{dg^A}{d\alpha} + \frac{dg^B}{d\alpha} < 0.
\]
Thus, when \(\alpha\) increases, overall counterterror effort falls, terror in each nation rises, A’s welfare increases, and B’s welfare falls.

We now turn to aggregate welfare in A and B:
\[
W(\alpha) = W^A[g^A(\alpha), g^B(\alpha)] + W^B[g^A(\alpha), g^B(\alpha)].
\]
(24)

Differentiating Eq. (24) and noting that \(W^B_{g^A} = 0\) on B’s reaction path, we get:
\[
\frac{dW}{d\alpha} = W^A_{g^A} \frac{dg^A}{d\alpha} + W^B_{g^A} \frac{dg^A}{d\alpha} + W^A_{g^B} \frac{dg^B}{d\alpha}.
\]
(25)

Based on Eqs. (10), (19), (23), and (25), the first term on the right-hand side of Eq. (25) is necessarily positive because \(W^A_{g^A} = -\left[\phi'(g^A) + \theta T'(g^A + g^B)\right] < 0\) at the political equilibrium.

That means that a rise in \(\alpha\), which reduces \(g^A\), will tend to increase the targeted countries’ welfare by limiting A’s excessive counterterror provision on its own welfare. The second term in Eq. (25) is negative because \(W^B_{g^A} = -(1 - \theta)T' > 0\), which implies that a rise in \(\alpha\) reduces total welfare through a reduction in B’s welfare arising from a reduction in \(g^A\). The final term in Eq. (25) is positive because \(W^A_{g^B} = -\theta T' > 0\), which bolsters overall welfare stemming from A benefiting from an increase in B’s counterterror provision. Proposition 2 identifies sufficient conditions for a fall in aggregate welfare due to an increase in \(\alpha\).

**Proposition 2:** If the politically influenced nation A is an equal or lesser target \((\theta \leq 1/2)\), then
a greater weight on A’s welfare (a rise in \( \alpha \)) is necessarily total welfare reducing when \( \alpha \) exceeds a critical threshold \( \alpha^0 \). For a preferred targeted nation \((\theta > 1/2)\), if \( \theta \) is at least as large as a critical level \( \theta^c (>1/2) \), total welfare always rises with \( \alpha \).

When nation A is a strongly preferred target of the terrorists (i.e., \( \theta \) is large), the positive external effect on nation B of a rise in \( g^A \), \( W_{g^A}^B = -(1-\theta)T' \), is dampened by a lower target parameter \((1-\theta)\). Furthermore, the rise in \( g^A \) elicits a reduction in \( g^B \), with a negative external effect on A’s welfare, \( W_{g^A}^A \frac{dg^B}{dg^A} = -\theta T' \rho^B \), which is amplified by a higher \( \theta \). Thus, for larger values of \( \theta \), the sum of the external welfare effects of a politically induced increase in \( g^A \) is more apt to be negative. The sum of the externalities coupled with the excess provision by A on its own welfare (i.e., \( W_{g^A}^A < 0 \)) assures that any rise in \( g^A \) for a sufficiently large \( \theta \) must reduce aggregate welfare. Therefore, a rise in \( \alpha \), which reduces \( g^A \), is welfare enhancing at sufficiently high levels of \( \theta > \theta^c \). Along the same lines, we can see that at lower levels of \( \theta \), the net external effects are more likely to be positive, and these effects dominate A’s own welfare effect \( W_{g^A}^A \), where the latter is approximately zero for very low levels of political influence (i.e., when \( \alpha \to \infty \)).

The above considerations indicate that starting from relatively low political influence \((\alpha > \alpha^0)\), where country A’s own unilateral welfare distortion is small, an increase in political influence (i.e., fall in \( \alpha \)) that raises A’s counterterror provision must lift aggregate welfare by mitigating underprovision (arising out of net positive externalities). Since A’s welfare must fall with greater political influence, B’s welfare must rise to more than offset the fall in A’s welfare.
The ultimate impact on the targeted countries’ welfare depends on the terrorists’ targeting preferences and how the lobby-affected country weighs its citizens’ welfare.

A finding that is true for any level of the target parameter $\theta$ is that greater political influence necessarily hurts the politically influenced country $A$ while benefitting the other targeted country $B$, if $B$ is lobby-free. We next consider a perfectly symmetric case where both nations are influenced by lobbying and explore how a symmetry-preserving simultaneous increase in political influence in the two nations affect provisions and welfare levels.

4. Politically Influenced Proactive Counterterrorism in Symmetric Targeted Nations

When both targeted nations are influenced by lobbying, Eqs. (14)-(21) can describe each nation’s proactive decision, with minor notation changes. Defining $\alpha^j$ as the political-influence parameter for nation $j$, we get the two nations’ proactive choice as satisfying the following FOCs:

$$
\Omega_{g^j}^j \left( g^A, g^B, \alpha^j \right) = \left(1-s^j\right) \left[1+\theta^j T'(G)\right] - \left(1+\alpha^j\right) \left[ \phi'(g^j) + \theta^j T'(G) \right] = 0, \quad (26)
$$

where $j = A, B$, $\theta^A = \theta$, and $\theta^B = 1 - \theta$. Eq. (26) determines the political Nash equilibrium $\left( g^{Ap}, g^{Bp} \right)$, where suppressing the target and share parameters, we can express nation $j$’s equilibrium proactive provision as $g^{ip} = g^j \left( \alpha^A, \alpha^B \right)$, $j = A, B$. Consequently, aggregate proactive provision is $G = G^P \left( \alpha^A, \alpha^B \right)$. Differentiating Eq. (26) for $j = A, B$ and solving via Cramer’s rule, we get:

$$
\frac{\Omega^B_{g, g^A, g^B}}{D^A} < 0, \quad g^{Bp}_{\alpha^i} = \frac{\Omega^B_{g^A, g^B, g^A^i, g_{\alpha^i}}}{D^A} > 0, \quad \text{and} \quad (27)
$$

$$
G^{Pp}_{\alpha^A} = g^{Ap}_{\alpha^A} + g^{Bp}_{\alpha^A} = \frac{\Omega^B_{g^A, g^B, \Omega^A_{g^A, g^B, g^A^i, g_{\alpha^i}}} - \Omega^B_{g^A, g^B, g^A^i, g_{\alpha^i}}}{D^A} = \frac{-\Omega^B_{g^A, g^B, \Omega^A_{g^A, g^B, g^A^i, g_{\alpha^i}}}}{D^A} \left(1+\rho^B\right) < 0, \quad (28)
$$
where $D^g = \Omega^A_{g^g} - \Omega^B_{g^g} > 0$, $\Omega^A_{g^g} > 0$, $\Omega^B_{g^g} < 0$, $\Omega^B_{g^g} < 0$, $\Omega^B_{g^g} < 0$, $\Omega^B_{g^g} < 0$, and $\rho_{BP} > 0$. Similarly, we have $g^{AP}_{\alpha^A} > 0$, $g^{BP}_{\alpha^B} < 0$, and $G^{BP}_{\alpha^B} < 0$. Eqs. (27) and (28) imply that greater political influence in a country (i.e., $\text{fall}$ in $\alpha^j$) augments that country’s counterterror proactive provision, reduces the other country’s proactive provision, and raises aggregate proactive provision. This scenario would apply to the use of drones by the United States and the United Kingdom to attack Islamic State assets in the field. Drone manufacturers in both targeted countries have lobbied their respective governments to deploy their weapon platforms (Conradis 2013a, 2013b; Hall 2015; New America 2022) against foreign-based terrorist groups that target US and UK people and property.

We now consider simultaneous and symmetry-preserving increases in $\alpha^A$ and $\alpha^B$, where $\overline{\alpha} = \alpha^A = \alpha^B$ and $d\alpha = d\alpha^A = d\alpha^B$. Under symmetry, a simultaneous rise in $\alpha^j$, which reduces aggregate proactive provision, must also reduce national proactive provision levels. Based on the share of capitalists and the weight assigned to welfare, we have:

**Proposition 3:** For sufficiently low shares of capitalists in two symmetric countries so that $s^A = s^B = s < s^c$, there exists a critical welfare weight $\overline{\alpha} = \overline{\alpha}^c$, at which the political Nash equilibrium is a first-best outcome. For $s < s^c$ and $\overline{\alpha} > \overline{\alpha}^c \left( \overline{\alpha} < \overline{\alpha}^c \right)$, the political Nash equilibrium is associated with underprovision (overprovision) relative to the first-best outcome. If $s \geq s^c$, the political Nash equilibrium necessarily features underprovision.

When producer lobbies result in sufficiently high weights on national welfare $\left( \overline{\alpha} > \overline{\alpha}^c \right)$,
the ensuing equilibrium converges to the unilateral welfare-maximizing equilibrium, which, see Section 2.2, is characterized by underprovision of counterterror proactive measures. If the share of capitalists is small, their lobbying gains are concentrated while the tax burdens are dispersed across the population. As in Olson’s (1965) *The Logic of Collective Action*, such asymmetry of benefit and cost recipients from collective action creates an effective incentive to lobby the government for deploying the proactive-equipment firms’ hardware. That is, collective action for promoting this hardware thrives when interests are concentrated to overcome organizing costs on the lobbying side, but not on the tax-payers side. In turn, at a critical influence level of $\alpha^c$, two-country lobbying can push up counterterror proactive provision to correspond to the first-best level. By contrast, if the share of the capitalist owners of the counterterror good production is large, more of the tax burden is internalized by the lobby, and lobbying is insufficiently strong to yield a political equilibrium that is first-best. We have identified when the share of capitalists and the weight on national welfare can in a two-targeted-country scenario correct the tendency to undersupply proactive measures with their public benefits on mutually targeted countries.

**Corollary to Proposition 3:** The noncooperative politically influenced equilibrium can generate greater welfare than an equilibrium where the two governments cooperate in the presence of lobbying.

Under certain parameter configurations, the politically influenced cooperative outcome results in lower welfare than the politically influenced noncooperative outcome. Representatives of governments coordinate in international forums like the North Atlantic Treaty Organization (NATO) or the European Union (EU) to cooperate on policy that involves international
externalities and transnational public goods (e.g., counterterror proactive measures). To the extent that the policies involve politically influenced governments’ interests, the forums seek to maximize the joint governments’ payoffs. Because of the weight on political contributions, the proactive provision levels will be pushed beyond the joint welfare-maximizing provision level, overshooting the targeted-countries’ first-best level. In contrast, as we saw in Proposition 3, the noncooperative politically influenced Nash equilibrium may or may not be associated with overprovision. If \( s < s^c \), then the two-country noncooperative equilibrium yields a first-best outcome at \( \bar{\alpha}^c \). This implies that at values of \( \alpha \) in the neighborhood of \( \bar{\alpha}^c \), noncooperative behavior may help the countries reduce the overprovision associated with cooperative government policies.

We usually think of policy coordination through international organizations as participants’ efficient because the cooperation internalizes international externalities. However, our results show that when we recognize that countries are represented in these forums by representatives of politically influenced national governments, international organizations need not necessarily augment the members’ welfare levels given distorting lobbying interests.

5. Lobbying for Defensive Counterterror Measures

Defensive counterterror hardware may include metal detectors, bomb-sniffing equipment, or biometric servers at airports and embassies. The manufacturers of such equipment lobby US Department of Homeland Security (DHS) or similar authorities abroad for installation and upgrades. For example, the SureScan Corporation is a registered lobbyist that promotes its luggage-screening devices with the Transportation Security Administration of the US DHS (Propublica 2022). For ease of exposition, we focus solely here on defensive counterterror measures and abstract from jointly supplied proactive ones. Accordingly, we relabel
counterterror provision of country \( j \) (\( j = A, B \)) as defense provision \( g^j \). The appendix derives the following terrorism functions of the two targeted nations:

\[
T^j = T^j\left(g^A, g^B\right), \quad T^i_{g^j} < 0, \quad T^j_{g^j} > 0, \quad i, j = A, B, \quad i \neq j,
\]

where a nation’s defensive actions reduce its own terrorism incidence while raising that of the other targeted nation. In the literature, defense-induced terrorism transference across countries creates a negative transference international externality (e.g., Hausken et al. 2009; Kunreuther and Heal 2003).

Once the \( T^j \) functions are represented by Eq. (29), all equations of Section 2 then apply for counterterror defense provision. Based on Eq. (8), country \( j \)’s unilateral welfare-maximizing defense choice satisfies:

\[
W^j_{g^j}\left(g^A, g^B\right) = -\phi^j\left(g^j\right) - T^j_{g^j}\left(g^A, g^B\right) = 0, \quad j = A, B,
\]

which implicitly defines \( j \)’s Nash defense reaction function, \( g^j = g^j\left(g^i\right), \quad i, j = A, B, \quad i \neq j \). The simultaneous satisfaction of the two countries’ reaction functions gives the Nash unilateral welfare-maximizing defense equilibrium. In the appendix, we establish that \( W^j_{g^j} > 0 \), such that the slope \( \rho^j \) of country \( j \)’s reaction function at the symmetric Nash equilibrium is positive. Such strategic complementarity in defensive choices of targeted nations characterizes the literature (e.g., Eaton 2004; Gaibulloev and Sandler 2019, Sandler and Siqueira 2006).

Recalling that the joint welfare of the targeted countries is \( W \) and using Eqs. (8) and (29), we have that the first-best counterterror defense levels of the two countries consists of

\[
W^j_{g^j} = -\phi^j\left(g^j\right) - T^A_{g^j}\left(g^A, g^B\right) - T^B_{g^j}\left(g^A, g^B\right) = 0, \quad j = A, B.
\]

Finally, we consider a political equilibrium where both countries are influenced by defense firms’ lobbies. The FOCs corresponding to Eq. (26) of Section 4 are:
\[ \Omega_{g'}^{j}(g^A, g^B, \alpha^A) = \left(1 - s^j\right) \left[1 + T_{g'}^j(\cdot)\right] - (1 + \alpha^j) \left[H(g^j) + T_{g'}^j(\cdot)\right] = 0, \; j = A, B, \] (32)

which implicitly define the respective governments’ defense reaction functions that jointly determine the Nash political equilibrium. At a symmetric Nash political equilibrium, the associated reaction functions are upward sloping as shown in the appendix.

**Proposition 4:** For symmetric targeted countries, counterterror defensive levels at the Nash political equilibrium exceed those at the Nash unilateral welfare-maximizing equilibrium, and both equilibriums overprovide defensive measures compared to the targeted countries’ first-best outcome. Welfare at the Nash political equilibrium is lower than at the Nash unilateral welfare-maximizing equilibrium.

When a nation raises its counterterror defense, terrorists find the other nation to be the relatively softer target and allocates more of their attacks there, creating a negative international externality from further defense provision. Within-country lobbies push provision of their defensive hardware even greater than the inefficient unilateral welfare-maximizing levels, thus amplifying defense overprovision. As a result, the Nash political equilibrium is characterized by larger provision levels and lower welfare levels for the two nations compared to the Nash unilateral welfare-maximizing equilibrium. Defense Proposition 4 contrasts greatly with Proposition 3 for which lobbying could potentially raise a nation’s welfare above its unilateral welfare-maximizing level and improve welfare.

6. **Concluding Remarks**

To date, there is little work on examining how terrorism affects a targeted country’s defense
firms and vice versa. A notable exception is the innovative article by Berrebi and Klor (2010) who show that, in a heavy period of terrorism in Israel during 2000–2001, defense firms’ stock values rose while nondefense firms’ stock values fell. The authors’ identification strategy matched Israeli firms to their US counterparts to isolate the influence of terrorism from other common industrial shocks. Their results indicate that Israeli defense firms stood to profit and grow when the threat of terrorism is high. Although Berrebi and Klor (2010) do not consider lobbying activities by Israeli defense firms and their consequences on sales, their findings imply that such firms have a clear rationale to lobby the Israeli government to use their products.

From a theoretical perspective, our paper investigates the effects on counterterror provision from alternative viewpoints when two countries are targeted by a terrorist group residing in a third country. We are particularly interested in how lobbying defense firms affect counterterror proactive provision and the resulting impact on national and joint welfare of targeted countries. When such firms lobby one of the two targeted countries, proactive counterterror supply increases in the lobbied country whose welfare decreases. The opposite applies for the other targeted country as its proactive supply decreases, but its welfare increases from enhanced overall counterterrorism and its own free riding. However, the two countries’ total proactive response increases, leading to an overall decrease in terrorism but an ambiguous influence on their combined welfare. This welfare ambiguity hinges on the targeting preferences of the terrorists and the lobbied governments’ weights on their citizens’ welfare. There is a clear tendency for lobbying to reduce the underprovision generally associated with the independent choice of proactive measures by targeted governments.

If both targeted countries’ governments are influenced by lobbying firms producing proactive weapons, there exists a critical welfare weight in the symmetric-country scenario that raises undersupplied proactive measures sufficiently to obtain a first-best outcome. This novel
result hinges on there being low shares of capitalists owning the counterterror-equipment firms so that associated collective action problems of lobbying can be overcome. In fact, the noncooperative politically influenced equilibrium provides a better welfare outcome than a cooperative equilibrium owing to the proactive provision being pushed too far in the latter case. Thus, lobbying can make intergovernmental cooperation undesirable in some instances, which is unexpected.

There is ample reason to focus on proactive over defensive counterterror provision and lobbying. As shown in Section 5, defense suppliers lobbying tends to worsen the general tendency for independent actions by the targeted countries to overprovide protective counterterrorism and to lower welfare. The standard transference race among commonly targeted countries is worsened with lobbying in either or both of the targeted countries.

The case of both proactive and defensive measures with lobbying results in a complex set of opposing externalities. If the military hardware is primarily used for proactive supply as in the case of drones, then the general tendencies identified in Sections 3 and 4 will hold with some softening of the results. Other extensions could relax the Nash assumption and, say, allow for leadership by the lobbied country. Drawing from Sandler and Siqueira (2006), we have that proactive leadership worsens underprovision of the Nash equilibrium, making the lobbying-induced increased provision even more welcome from a welfare standpoint. Another extension can allow for more targeted countries with a subset being lobbied.
Appendix

1. Deriving Eq. (1) and the Countries’ Terrorism Functions

The terrorist group in \( C \) experiences positive but diminishing marginal utility from its terror attacks in a targeted country, such that:

\[
V(T^A, T^B) = \psi(T^A)\mu + (1-\psi)(T^B)\mu, \quad 0 < \psi < 1, \quad 0 < \mu < 1,
\]

where \( \psi \) and \( \mu \) are parameters reflecting the group’s targeting preferences and diminishing marginal utility, respectively. Let the respective terrorist efforts directed at the two countries be \( E^A \) and \( E^B \) with the following terrorism production functions:

\[
T^A = E^A \quad \text{and} \quad T^B = E^B.
\]

(E2)

The total resources of the terrorist group are \( \bar{E} \), which is negatively impacted by the governments’ aggregate proactive measures, \( G = g^A + g^B \), at a diminishing rate; namely,

\[
\bar{E} = \bar{E}(G), \quad \bar{E}'(G) < 0, \quad \text{and} \quad \bar{E}''(G) > 0.
\]

(E3)

The terrorist group chooses \( E^A \) and \( E^B \) to maximize \( V \) subject to the resource constraint, \( E^A + E^B = \bar{E}(G) \). Given Eqs. (A1) and (A2), the FOCs of this constrained optimization problem imply:

\[
\psi(E^A)^{\mu-1} = (1-\psi)(E^B)^{\mu-1} \quad \text{and} \quad E^A + E^B = \bar{E}(G).
\]

(E4)

Eqs. (A2) through (A4) yield terrorism levels in the two nations and aggregate terrorism as:

\[
T^A = E^A = \frac{\beta \bar{E}(G)}{1+\beta}, \quad T^B = E^B = \frac{\bar{E}(G)}{1+\beta}, \quad \text{and} \quad T = \bar{E}(G),
\]

(E5)

where \( \beta = \left( \frac{\psi}{1-\psi} \right)^{1-\mu} \) is independent of the proactive effort levels, \( g^A \) and \( g^B \). Thus, using Eqs. (A3) and (A5), we get Eq. (1) of the paper. In turn, those equations also yield \( T^A = \theta T(G) \) and
\[ T^b = (1 - \theta) T(G), \] where \( \theta = \frac{\beta}{1 + \beta} \) is fixed by the parameters \( \psi \) and \( \mu \).

2. Proof of Proposition 1

In Figure 1, a larger politically induced \( g^A \) elicits a reduction in \( g^B \) as a strategic substitution response. (Although we use Figure 1 for the proof, none of the proof’s arguments relies on symmetry and applies to any value of the target parameter \( \theta \).) Given that both \( N \) and \( N^p \) lie on \( B \)'s unilateral welfare-maximizing reaction path, we can use Eq. (11b), \( g^B = g^B(g^A) \).

Hence, we have:
\[
\frac{dG}{dg^A} = (1 + \rho^B) d\rho^A, 
\]
(A6)

where \( \rho^B = g^B(g^A) = -\frac{(1-\theta)T^*}{\phi^*(g^B) + (1-\theta)T^*} \) is the slope of \( B \)'s Nash reaction path. Given that \( 1 + \rho^B > 0 \), Eq. (A6) establishes that a rise in \( g^A \) along \( B \)'s reaction path must increase aggregate provision \( G \), thus reducing terrorism overall and in each country. (The increase in \( G \) is also seen by dropping lines with slope \(-1\) from the equilibriums to the horizontal axis on Figure 1, since the lines’ intercepts correspond to the equilibriums’ values of \( G \).) Along nation \( j \)'s Nash reaction path, we have \( \left( \frac{dW^A}{dg^A} \right)_{R^i} = -\theta T' > 0 \) and \( \left( \frac{dW^B}{dg^A} \right)_{R^B} = -(1-\theta)T' > 0 \) since \( W^j_{R^j} = 0 \). In moving from \( N \) to \( N^p \) along \( B \)'s reaction path, the augmented \( g^A \) leads to an increase in \( B \)'s welfare. For targeted country \( A \), a movement from \( N \) to \( M \) along \( A \)'s welfare-maximizing reaction path, \( W^A \) must fall as \( g^B \) falls. Furthermore, \( A \)'s welfare must be lower at \( N^p \) than at \( M \) because \( M \) represents \( A \)'s welfare-maximizing choice at \( g^B = g^B_{M} \). Therefore, \( A \)'s welfare at the political equilibrium \( N^p \) must be lower than at the unilateral welfare-maximizing Nash
equilibrium at \( N \). Because the welfare of the two countries moves in opposing directions when we compare the two Nash equilibriums, aggregate welfare \( W = W^A + W^B \) may rise or fall.

Q.E.D.

3. Proof of Proposition 2

Using Eqs. (23) and (25), we have:

\[
\frac{dW}{d\alpha} = \left( W_{g^A}^A + W_{g^B}^B + \rho^B W_{g^A}^A \right) \frac{dg^A}{d\alpha}.
\]  

(A7)

Substituting the expressions for \( W_{g^A}^A \), \( W_{g^B}^B \), and \( W_{g^A}^A \) in Eq. (A7) and using Eqs. (10) and (19), we transform Eq. (A7) to:

\[
\frac{dW}{d\alpha} = - \left[ \frac{(1-s^A)(1+\theta T')}{1+\alpha} + (1-\theta + \theta \rho^B) T' \right] \frac{dg^A}{d\alpha}.
\]  

(A8)

Case 1: \( \theta \geq \frac{1}{1+|\rho^B|} = \theta^c > \frac{1}{2} \).

When \( \theta \geq \frac{1}{1+|\rho^B|} \), we have \( 1-\theta + \theta \rho^B \leq 0 \). The term inside the bracket on the right-hand side of Eq. (A8) must then be positive because \( T' < 0 \). Given \( \frac{dg^A}{d\alpha} < 0 \), we get \( \frac{dW}{d\alpha} > 0 \), which establishes the last part of Proposition 2.

Case 2: \( \theta < \theta^c \).

In this case, \( 1-\theta + \theta \rho^B > 0 \), such that the bracketed term in Eq. (A8) has, \textit{ex ante}, an ambiguous sign. Define the bracketed term as:

\[
\chi(\alpha) \equiv \frac{(1-s^A)(1+\theta T'[G(\alpha)])}{1+\alpha} + (1-\theta + \theta \rho^B) T'[G(\alpha)],
\]  

(A9)
where we ignore third-order derivatives such that $\rho^B$ defined after Eq. (A6) is independent of $\alpha$.

We show below that $\chi'(\alpha) < 0$, so that there are two possibilities to consider: (i) $\chi(\alpha) < 0$ for all $\alpha$, and (ii) $\chi(\alpha) > 0$ for some $\alpha$. For the first possibility, Eqs. (A8) and (A9) imply that $\frac{dW}{d\alpha} < 0$ for all $\alpha$ for a given $\theta$. In this event, overall welfare is always declining in $\alpha$. The second possibility requires more careful analysis, because if $\chi(\alpha) > 0$ for some $\alpha$, then at that $\alpha$ total welfare is increasing in $\alpha$. Differentiating Eq. (A9) gives:

$$\chi'(\alpha) = \left(1-s^4\right)\frac{1+\theta T'G^*_\alpha - \frac{1+\theta T''}{1+\alpha}}{1+\alpha + \left(1-\theta + \theta \rho^B\right)T^*G^*_\alpha} < 0.$$  \hfill (A10)

We note that as $\alpha \to \infty$, we approach the case where nation $A$ has a welfare-maximizing government (as in Section 2.1), such that $G \to \tilde{G} = \tilde{g}^A + \tilde{g}^B$, where $(\tilde{g}^A, \tilde{g}^B)$ is the unilateral welfare-maximizing Nash equilibrium. For Eq. (A9), we see that as $\alpha \to \infty$,

$$\chi(\alpha) \to \left(1-\theta + \theta \rho^B\right)T'(\tilde{G}) < 0.$$  Therefore, for some $\alpha$, $\chi(\alpha)$ is strictly positive, and for $\alpha \to \infty$, $\chi(\alpha)$ is strictly negative. Moreover, given that $\chi$ is monotonically declining in $\alpha$, continuity ensures that there is a finite $\alpha = \alpha^0$, where $\chi(\alpha^0) = 0$. Using Eq. (A9), we find that $\alpha^0$ is implicitly defined by,

$$\left(1-s^4\right)\frac{1+\theta T'\left[G(\alpha)\right]}{1+\alpha} + \left(1-\theta + \theta \rho^B\right)T'\left[G(\alpha)\right] = 0.$$  \hfill (A11)

Because $\chi'(\alpha) < 0$, Eq. (A11) implies that $\chi(\alpha) < 0$ for all $\alpha > \alpha^0$. Given Eqs. (A8) and (A9), we have $\frac{dW}{d\alpha} < 0$ for all $\alpha > \alpha^0$. If $\theta \leq \frac{1}{2}$, then $\theta < \theta^*$ because $\theta^*$ exceeds $1/2$. Thus, “Case 2” applies to the range $\theta \leq \frac{1}{2}$, such that there will always exist a critical $\alpha^0$, such that for all $\alpha$
exceeding that critical value, overall welfare is declining in \( \alpha \). Q.E.D.

4. Proof of Proposition 3

Based on Eqs. (13) and (26), government \( A \)'s marginal payoff at the first-best outcome 
\[ g_A = g_B = g_f \] consists of:

\[
\left( \Omega^A_{g_A} \right)_{\text{FB}} = -\frac{2 + T' \left( 2g_f \right)}{2} \left( s - s^c \right) + \alpha \frac{T' \left( 2g_f \right)}{2},
\]  

(A12)

where \( 0 < s^c = \frac{2 \left[ 1 + T' \left( 2g_f \right) \right]}{2 + T' \left( 2g_f \right)} < 1 \). From Eq. (13), \( g_f \) depends only on the functional forms of \( T \) and \( \phi \), and is independent of other parameters. Thus, Eq. (A12) implies that \( \left( \Omega^A_{g_A} \right)_{\text{FB}} \) is monotonically declining in \( \alpha \) because \( T' < 0 \). For a sufficiently large \( \alpha \), the second right-hand side expression of Eq. (A12) dominates, implying that \( \left( \Omega^A_{g_A} \right)_{\text{FB}} < 0 \). Moreover, as \( \alpha \to 0 \),

\[
\left( \Omega^A_{g_A} \right)_{\text{FB, } \alpha \to 0} = -\frac{2 + T'}{2} \left( s - s^c \right) > 0 \text{ if } s < s^c, \text{ where a monotonically declining \( \left( \Omega^A_{g_A} \right)_{\text{FB}} \) goes from positive as \( \alpha \to 0 \) to negative as \( \alpha \to \infty \). Given continuity, a critical \( \alpha^c \) exists for which \( \left( \Omega^A_{g_A} \right)_{\text{FB}} = 0 \), so that \( g_A = g_f \) is the best response to \( g_B = g_f \). With symmetry, \( g_B = g_f \) is the best response to \( g_A = g_f \). At \( \alpha = \alpha^c \), \( \left( g_{AP} = g_f, g_{BP} = g_f \right) \) is a symmetric political Nash equilibrium and the first-best outcome. Similarly, when \( \alpha > \alpha^c \), \( \left( \Omega^A_{g_A} \right)_{\text{FB}} < 0 \), and country \( A \) reduces its proactive measures below the first-best level. Country \( B \) faces a similar incentive. For \( \alpha > \alpha^c \), we have \( g_{AP} = g_{BP} < g_f \). If \( \alpha < \alpha^c \), then \( g_{AP} = g_{BP} > g_f \). Notice that when
$s \geq s^c$, \(\left(\Omega^A\right)_{FB} < 0\) and \(g^A\) (along with \(g^B\)) are reduced below the first-best levels. Q.E.D.

5. Proof of Corollary to Proposition 3

Let the two governments’ joint payoff be:

$$\Omega\left(g^A, g^B; \overline{\alpha}\right) = \Omega^A\left(g^A, g^B; \overline{\alpha}\right) + \Omega^B\left(g^A, g^B; \overline{\alpha}\right).$$

(A13)

Given Eqs. (17) and (18), \(\Omega^j\left(g^A, g^B\right) = H^j\left(g^A, g^B\right) + \overline{\alpha}W^j\left(g^A, g^B\right)\), where

$$H^j\left(g^A, g^B\right) = K^j + g^j - \phi\left(g^j\right) - s^j \times \left[g^j + \Theta^j T\left(g^A + g^B\right)\right] - Z^j; \ j = A, B.$$ 

The marginal effect of an increase in \(g^A\) on the joint payoff is:

$$\Omega_{g^A}\left(g^A, g^B; \overline{\alpha}\right) = H^A_{g^A}\left(g^A, g^B\right) + H^B_{g^A}\left(g^A, g^B\right) + \overline{\alpha}W^A_{g^A}\left(g^A, g^B\right) + W^B_{g^A}\left(g^A, g^B\right).$$

(A14)

We invoke the condition that \(W^A_{g^A}\left(g^A, g^B\right) + W^B_{g^A}\left(g^A, g^B\right) = 0\) to evaluate \(\Omega_{g^A}\left(g^A, g^B; \overline{\alpha}\right)\) at the first-best outcome to obtain \(\left(\Omega_{g^A}\right)_{FB} = H^A_{g^A}\left(g^A, g^B\right) + H^B_{g^A}\left(g^A, g^B\right)\). Differentiating \(H^j\) and using the first-best condition \(\phi'(g^j) = -T'(2g^j)\), we have that \(\left(\Omega_{g^A}\right)_{FB} = (1-s)(1+T') > 0\).

Assuming a cooperative lobbying optimum \(\left(g^A = g^B = g^{Coop}\right)\) exists, we have \(g^{Coop} > g^f\), which means \(W^A\left(g^{Coop}, g^{Coop}\right) < W^A\left(g^f, g^f\right) = W^A\left(g^{AP}, g^{BP}; \overline{\alpha}^c, s < s^c\right)\). The same comparison applies to country B’s welfare levels. Q.E.D.

6. Deriving Eqs. (29), (30) and the Slopes of the Defense Reaction Functions Under Symmetry

For given terrorist effort in each country, counterterror defensive actions reduce the terrorism inflicted on a targeted country such that the terrorism production functions in Eq. (A2) take the following forms:
\[ T^A = \delta(g^A)E^A \text{ and } T^B = \delta(g^B)E^B, \]  
\[ (A15) \]
where \(0 < \delta(g^j) \leq 1\), \(\delta(0) = 1\), \(\delta'(g^j) < 0\), and \(\delta''(g^j) > 0\) for \(j = A, B\). Eq. (A15) assumes that defensive measures in any country reduces terrorism at a diminishing rate.

In the absence of proactive measures, the resource constraint for the terrorist organization is \(E^A + E^B = \bar{E}\), where, without loss of generality, we set \(\bar{E} = 1\). Based on Eqs. (A1) and (A15), the organization’s constrained optimization problem yields the following terrorism levels:

\[ T^A = \frac{\delta(g^A)X(g^A, g^B)}{1 + X(g^A, g^B)} \equiv T^A(g^A, g^B) \text{ and } T^B = \frac{\delta(g^B)}{1 + X(g^A, g^B)} \equiv T^B(g^A, g^B), \]  
\[ (A16) \]
in which \(X(g^A, g^B) = \left(\frac{\psi}{1 - \psi}\right)^{1-\mu} \left[\frac{\delta(g^A)}{\delta(g^B)}\right]^{\mu} \) with \(X_{g^A} < 0\) and \(X_{g^B} > 0\) since \(\delta'(g^j) < 0\). For the rest of the appendix, we consider symmetric targeted countries, making it sufficient to focus on the own and cross partials of \(T^A(g^A, g^B)\) in Eq. (A16), which yield:

\[ T^A_{g^A} = \frac{X\delta'(g^A)}{1 + X} + \frac{\delta(g^A)X_{g^A}}{(1 + X)^2} < 0 \text{ and } T^A_{g^B} = \frac{\delta(g^A)X_{g^B}}{(1 + X)^2} > 0, \]  
\[ (A17) \]
given \(X_{g^A} < 0\), \(X_{g^B} > 0\), and \(\delta'(g^A) < 0\). Differentiating the expression for \(T^A_{g^A}\) with respect to \(g^A\) and \(g^B\) respectively, and evaluating the resulting expressions at a symmetric equilibrium, we have:

\[ T^A_{g^A} > 0 \text{ and } T^A_{g^B} < 0. \]  
\[ (A18) \]
Differentiating Eq. (8) and using Eqs. (A18) through (A18), we get \(W^A_{g^A} = -\phi'(g^A) - T^A_{g^A} < 0\) and \(W^A_{g^B} = T^A_{g^B} > 0\). Thus, the slope of country \(A\)’s reaction function, defined by Eq. (30),
satisfies \( \frac{dg_A^g}{dg_B^g} = -\frac{W_A^{g,g^g}}{W_A^{g,g^g}} > 0 \) at a symmetric equilibrium. Similarly, B’s defense reaction function is also positively sloped.

Using Eqs. (32) and (A18), we have \( \Omega_A^{g,g^g} = -(s^A + \alpha^A)T_A^{g,g^g} > 0 \). Thus, the slope of A’s politically influenced defense reaction function is: \( \left( \frac{dg_A^g}{dg_B^g} \right)_p = -\frac{\Omega_A^{g,g^g}}{\Omega_A^{g,g^g}} > 0 \). Similarly, B’s politically influenced defense reaction function is also positively sloped at the symmetric Nash political equilibrium.

7. Proof of Proposition 4:

Substituting Eq. (30) into (32), we obtain the net marginal benefit of defense of a politically influenced government at the unilateral welfare-maximizing equilibrium as:

\[
\left( \frac{\Omega_i^j}{g_i^j} \right)_{g_i^j=0} = (1-s^j)[1+T_i^j(g^A, g^B)] > 0
\]  

(A19)

because \( 1+T_i^j(g^A, g^B) = 1-\phi'(g^j) > 0 \) by Eq. (30). Thus, in the presence of lobbying, each country desires to raise its counterterror defense above the unilateral welfare-maximizing level for any defense level of the other targeted country. Similarly, we use Eqs. (30) and (31) to obtain:

\[
\left( \frac{W_i^j}{g_i^j} \right)_{g_i^j=0} = -T_i^j(g^A, g^B) < 0, \quad i, j = A, B, \quad i \neq j,
\]

(A20)

which indicates that the two targeted countries’ joint welfare can be raised by reducing each country’s counterterror defense provision below its unilateral welfare-maximizing amount. Eqs. (A19) and (A20) imply that, for symmetric countries, the Nash political equilibrium has larger defense levels than those at the Nash unilateral welfare-maximizing equilibrium. In turn, both
equilibriums have greater defense levels compared to the first-best outcome.

Suppose that \((g^A = \tilde{g}, g^B = \tilde{g})\) is the Nash unilateral welfare-maximizing equilibrium, and \((g^A = g^L, g^B = g^L)\) is the Nash political equilibrium for counterterror defense provision.

Since \(g^A = \tilde{g}\) maximizes \(A\)'s welfare at \(g^B = \tilde{g}\), strict concavity of \(A\)'s welfare function implies that \(A\)'s welfare at \((g^A = g^L > \tilde{g}, g^B = \tilde{g})\) is lower than its welfare at the unilateral welfare-maximizing equilibrium. From Eqs. (8) and (29), we note that \(W^A_T = -T^A > 0\). Thus, for \(g^A = g^L\), an increase in \(g^B\) from \(\tilde{g}\) to \(g^L\) reduces \(A\)'s welfare. Because of these own and cross effects of counterterror defense, \(A\)'s welfare at the Nash lobbying equilibrium is lower than its welfare at the Nash unilateral welfare-maximizing equilibrium. The same welfare comparison applies to country \(B\). Q.E.D.
References


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Figure 1. Nash Welfare-Maximizing Equilibrium; Nash Political Equilibrium
1. **Derivation of Eq. (8)**

Based on Eq. (7), the sum of all capitalists’ \((i = 1, 2, ..., K^j)\) utility equals:

\[
\sum_{i=1}^{K^j} u^{ij} = K^j + \pi_g^j - \frac{K^j}{N^j} T^j - \sum_{i=1}^{K^j} t^{ij} .
\]  

(M1)

Similarly, the sum of all non-capitalists’ \((i = K^j + 1, K^j + 2, ..., N')\) utility equals:

\[
\sum_{i=K^j+1}^{N'} u^{ij} = (N' - K^j) - \frac{(N' - K^j) T^j}{N^j} - \sum_{i=K^j+1}^{N'} t^{ij} .
\]  

(M2)

Using Eqs. (M1) and (M2) and noticing that \(\sum_{i=1}^{N'} t^{ij} = g^j\), the welfare sum over all individuals is:

\[
W^j = N^j + \pi_g^j - g^j - T^j .
\]  

(M3)

Substituting \(\pi_g^j - g^j = -l_g^j\) [see Eq. (6)] into Eq. (M3) yields Eq. (8) of the text.

2. **Second-Order Conditions, Second-Order Cross Partials, and Stability Conditions (for Sections 2-4):**

(a) **Section 2.1:** Given Eq. (8), the second-order partials of Section 2.1 reduce to:

\[
W_{g'g''}^j = -\phi^*(g^j) - \theta^j T^j(G) < 0, \ W_{g'g}^j = -\theta^j T^j(G) < 0, j = A, B ,
\]  

(M4)

where \(\theta^A = \theta\) and \(\theta^B = 1 - \theta\). Eq. (M4) yields:

\[
W_{g'g''}^j - W_{g'g}^j = -\phi^*(g^j) < 0 \Rightarrow |W_{g'g''}^j| > |W_{g'g}^j|, \ j = A, B .
\]  

(M5)

Eq. (M5) establishes that \(W_{g'A}^A W_{g'B}^B - W_{g'A}^B W_{g'B}^A > 0\), ensuring global stability.

(b) **Section 2.2:** Applying \(W = N^A + N^B - \phi(g^A) - \phi(g^B) - T(g^A + g^B)\), the second-order partials are:
\[ W'_{g'g''} = -\phi''(g^j) - T''(G) < 0 \quad \text{and} \quad W''_{g'g''} = -T''(G) < 0, \ j = A, B \] (M6)

implying that

\[ W'_{g'g''} - W''_{g'g''} = -\phi''(g^j) < 0 \Rightarrow \left| W'_{g'g''} \right| > \left| W''_{g'g''} \right| \Rightarrow W'_{g'g''} W''_{g'g''} - \left( W_{g'g''} \right)^2 > 0. \] (M7)

Eqs. (M6) and (M7) ensure that the second-order condition (SOC) of the first-best optimization problem is satisfied.

(c) Section 3:

In Section 3, nation B’s optimization problem is qualitatively the same as in Section 2, such that B’s SOC is still given by Eq. (M5). Using Eqs. (8), (17) and (18), we have:

\[ \Omega^A_{g'g''} = -(1 + \alpha) \phi''(g^j) - (s^j + \alpha) \theta T''(G) < 0 \quad \text{and} \]

\[ \Omega^A_{g'g''} = -(s^j + \alpha) \theta T''(G) < 0 \Rightarrow \left| \Omega^A_{g'g''} \right| > \left| \Omega^A_{g'g''} \right|. \] (M8)

Eq. (M8) ensures that the SOC for A’s lobbying-influenced optimization problem is satisfied.

Eqs. (M5) and (M8) ensure that the Nash stability condition, \[ D = \Omega^A_{g'g''} W^B_{g'g''} - \Omega^A_{g'g''} W^B_{g'g''} > 0, \] is satisfied.

(d) Section 4:

In Section 4, both nations are politically influenced for which the cross partials are qualitatively similar to those in Eq. (M8), such that the SOCs are satisfied because:

\[ \Omega^j_{g'g''} = -(1 + \alpha^j) \phi''(g^j) - (s^j + \alpha^j) \theta^j T''(G) < 0, \ j = A, B; \ \theta^A = \theta, \ \theta^B = 1 - \theta. \] (M9)

Given Eq. (M9), we get:

\[ \Omega^j_{g'g''} = -(s^j + \alpha^j) \theta^j T''(G) < 0 \Rightarrow \left| \Omega^j_{g'g''} \right| > \left| \Omega^j_{g'g''} \right|, \] (M10)

such that the stability condition \[ D^\Omega = \Omega^A_{g'g''} \Omega^B_{g'g''} - \Omega^A_{g'g''} \Omega^B_{g'g''} > 0 \] is satisfied.
3. Supporting Eq. (A18) of the Appendix:

Differentiating the expression for $T^A_{g^e}$ in Eq. (A17), we get:

$$T^A_{g^e} = \frac{X \delta^\mu(g^A) + 2 \delta^\mu(g^A)X_{g^e} + \delta(g^A) \left[ (1+X)X_{g^e} - 2 \left( X_{g^e} \right)^2 \right]}{1+X}.$$  \hspace{1cm} \text{(M11)}

Using the expression for $X$ provided after Eq. (A16) and evaluating the its derivatives at a symmetric equilibrium where $g^A = g^B = g$, $\delta(g^j) = \delta(g)$, $\delta'(g^j) = \delta'(g)$, and $\delta''(g^j) = \delta''(g)$ for $j = A,B$, we get:

$$X_{g^e} = \frac{\mu \delta^\mu}{1 - \mu} \left[ \frac{\delta^\mu + (\delta')^2}{\delta^2 (1 - \mu)} \right] \quad \text{and} \quad X_{g^e} = \frac{\mu \delta}{(1 - \mu) \delta}.$$ \hspace{1cm} \text{(M12)}

Since $X = 1$ at a symmetric equilibrium, we substitute the expressions for $X_{g^e}$ and $X_{g^e}$ from Eq. (M12) into Eq. (M11) to obtain:

$$T^A_{g^e} = \frac{\delta^\mu}{2} + \frac{\mu \delta^\mu + (\delta')^2}{4(1 - \mu) \delta} > 0.$$ \hspace{1cm} \text{(M13)}

Similarly, differentiating the expression for $T^A_{g^e}$ with respect to $g^B$ yields:

$$T^A_{g^e} = \frac{\delta' X_{g^e} + \delta \left( X_{g^e}X_{g^e} - X_{g^e}X_{g^e} \right)}{4}.$$ \hspace{1cm} \text{(M14)}

Given the expression for $X$, we evaluate the derivatives at a symmetric equilibrium using Eq. (M12) to obtain:

$$X = \frac{-\mu \delta'}{(1 - \mu) \delta} \quad \text{and} \quad X_{g^e} = \frac{-\left( \mu \delta \right)^2}{(1 - \mu)^2}.$$ \hspace{1cm} \text{(M15)}

Substituting Eq. (M15) into Eq. (M14), we get:
\[ T_{g^i g^j}^A = \frac{\delta' X_{g^i g^j}}{4} < 0 . \]  

(Eq. M16)

Eqs. (M13) and (M16) support Eq. (A18).


Using Eq. (8) and Eq. (A18), the second-order partials relevant to nation \( j \)'s welfare, evaluated at a symmetric equilibrium in Section 5, yields:

\[ W^j_{g^i g^j} = -\phi'\left(g^j\right) - T^j_{g^i g^j} < 0 \text{ and } W^j_{g^i g^j} = -T^j_{g^i g^j} > 0, \quad j = A, B . \]  

(Eq. M17)

At a symmetric equilibrium, substituting Eqs. (M13) and (M16) into Eq. (M17) gives:

\[ \left| W^j_{g^i g^j} \right| - \left| W^j_{g^i g^j} \right| = \phi'' + \frac{(2 - \mu) \delta''}{4(1 - \mu)} > 0 . \]  

(Eq. M18)

Eq. (M18) implies that the Nash stability condition, \( W^A_{g^i g^j} - W^B_{g^i g^j} > 0 \), holds.

We now turn to the first-best optimization problem. Using Eq. (8) and \( W = W^A + W^B \), the second-order partials for \( W \) are:

\[ W^i_{g^i g^j} = -\phi''\left(g^i\right) - T^i_{g^i g^j} < 0 \text{ and } W^i_{g^i g^j} = -T^i_{g^i g^j} > 0, \quad i, j = A, B, i \neq j \]  

(Eq. M19)

because \( T^j_{g^i g^j} + T^i_{g^i g^j} > 0 \), as shown below in Eq. (M21). Based on Eq. (A16), we have:

\[ T^B_{g^i g^i} = -\frac{\delta\left(g^B\right)\left[1 + X \right] X_{g^i g^i} - 2\left( X_{g^i g^i} \right)^2}{(1 + X)^3} . \]  

(Eq. M20)

At a symmetric equilibrium, the last term on the right-hand side of Eq. (M11) is the negative of the right-hand side of Eq. (M20). Thus, using Eq. (M20) in Eq. (M11) and \( X = 1 \), we get:

\[ T^A_{g^i g^i} + T^B_{g^i g^i} = -\frac{X \delta''\left(g^i\right) + 2\delta'\left(g^i\right) X_{g^i g^i}}{1 + X} = \frac{\delta''}{2} + \frac{\delta' X_{g^i g^i}}{2} , \]  

(Eq. M21)
because $\delta'$ and $X_{g^a}$ are negative, while $\delta^\sigma$ is positive.

Given Eqs. (M16), (M19), and (M21), we obtain:

$$
\left| W_{g^a} - W_{g^a} \right| = \phi^\sigma + T^A_{g^a} + T^B_{g^a} + 2T^A_{g^a} = \phi^\sigma + \frac{\delta^\sigma}{2} + \frac{\delta' X_{g^a}}{2} + \frac{\delta' X_{g^a}}{2} = \phi^\sigma + \frac{\delta^\sigma}{2} > 0,
$$

(M22)
because we know from Eqs. (M12) and (M15) that $X_{g^a} + X_{g^a} = 0$. At a symmetric equilibrium, Eq. (M22) implies that $W_{g^a} W_{g^a} - \left(W_{g^a} \right)^2 > 0$, which along with Eq. (M19) establish that the SOC for the first-best problem is satisfied.

Finally, we turn to the Nash political equilibrium of Section 5. Using Eq. (32) and (A18), the second-order partials are:

$$
\Omega_{g^a}^j \left(g^A, g^B, \alpha^j \right) = -\left(1 + \alpha^j \right) \phi^\sigma \left(g^j \right) - \left(s^j + \alpha^j \right) T^A_{g^a} < 0 \text{ and }
$$

$$
\Omega_{g^a}^j \left(g^A, g^B, \alpha^j \right) = -\left(s^j + \alpha^j \right) T^A_{g^a} > 0, \ i, j = A, B; j \neq i.
$$

(M23)

Eq. (M23) ensures that the SOC is satisfied:

$$
\left| \Omega_{g^a}^A - \Omega_{g^a}^B \right| = \left(1 + \alpha^A \right) \phi^\sigma \left(g^A \right) + \left(s^A + \alpha^A \right) \left(T^A_{g^a} + T^A_{g^a} \right) > 0,
$$

(M24)
because $T^A_{g^a} + T^A_{g^a} = \frac{\delta^\sigma \left(2 - \mu \right)}{4 \left(1 - \mu \right)},$ due to Eqs. (M13) - (M16). Eq. (M24) ensures that the Nash stability condition, $\Omega_{g^a}^A - \Omega_{g^a}^B - \Omega_{g^a}^A \Omega_{g^a}^B > 0$, is met.