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Authors	Subhayu Bandyopadhyay, and Todd Sandler
Working Paper Number	2021-008A
Creation Date	June 2021
Citable Link	https://doi.org/10.20955/wp.2021.008
Suggested Citation	Bandyopadhyay, S., Sandler, T., 2021; Effects of Defensive and Proactive Measures on Competition Between Terrorist Groups, Federal Reserve Bank of St. Louis Working Paper 2021-008. URL https://doi.org/10.20955/wp.2021.008

Effects of Defensive and Proactive Measures on Competition Between Terrorist Groups

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June 2021
Word Count 10001

Abstract

A two-stage game investigates how counterterrorism measures affect within-country competition between two rival terrorist groups. Although such competition is commonplace (e.g., al-Nusra Front and Free Syria Army; Revolutionary Armed Forces of Colombia and the National Liberation Army; and al-Fatah and Hamas), there is no theoretical treatment of how proactive and defensive measures influence this interaction. Previous studies on rival terrorist groups are solely empirical concerning group survival, outbidding, and terrorism level, while ignoring the role that government countermeasures exert on the rival groups' terrorism. In a theoretical framework, alternative counterterrorism actions have diverse impacts on the level of terrorism depending on relative group sizes and government-targeting decisions. In the two-stage game, optimal counterterrorism policy rules are displayed in terms of how governments target symmetric and asymmetric terrorist groups. Comparative statics show how parameter changes affect Nash or subgame perfect equilibrium outcomes.

Keywords: competitive terrorist groups, defensive and proactive counterterrorism, two-stage game, comparative statics, outbidding between rival groups

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The views expressed are those of the authors and do not necessarily represent the official positions of the Federal Reserve Bank of St. Louis or the Federal Reserve System

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Introduction

Multiple rival terrorist groups may operate in the same country such as the Liberation Tigers of Tamil Eelam (LTTE) and the People's Liberation Army of Tamil Eelam (PLATE) in Sri Lanka (Bloom 2005), the Revolutionary Armed Forces of Colombia (FARC) and the National Liberation Army (ELN) (Phillips 2015), al-Fatah and Palestinian Islamic Jihad (PIJ) (Bloom 2004), and Shining Path and the Tupac Amaru Revolutionary Movement in Peru (Phillips 2015). Other notable rivalries include the Provisional Irish Republican Army (PIRA) and the Ulster Volunteer Force (UVF) in Northern Ireland, Hezbollah and Amal Movement in Lebanon, the Taliban and Islamic State (IS) in recent years in Afghanistan, al-Shabaab and Hizbul Islam in Somalia, and Lashkar-e-Islam and Tehrik-i-Taliban in Pakistan. Such rivalries may characterize some years and not others if groups come to an accommodation, merge, end operations, or are annihilated. Despite these ubiquitous rivalries, there is no theoretical treatment of how a government should interface with such groups when choosing its counterterrorism policies.[1] For instance, should the government rely on defensive policies that protect vulnerable targets, or should the government resort to proactive or offensive measures to weaken target groups? In the latter case, should the government attack the stronger or weaker terrorist group? Those and many other questions are addressed here.

In its most elementary form, a theoretical model must have three players – the government and two rival terrorist groups. This basic representation not only makes for a tractable analysis, but also provides a framework that can be extended to more rival groups and other considerations. In past theoretical work, one or more targeted governments confront a

[1] Siqueira (2005) analyzes the interface of rival factions – e.g., political and military wings – within, but not *between*, terrorist groups.

single terrorist group, whose objective is to create maximal damage with limited resources (Bandyopadhyay and Sandler 2011, 2014; Bandyopadhyay, Sandler, and Younas 2020; Mirza and Verdier 2014; Rosendorff and Sandler 2004; Rossi de Oliveira, Faria, and Silva 2018; Siqueira and Sandler 2006). When, however, there are two or more terrorist organizations operating in the same country, their objectives are to create terror *and* to bolster their reputation as a relatively stronger group. This latter aim may derive from the prospects of improved recruitment,[2] better external funding, and enhanced political influence. Those desires relate to the notion of *outbidding* for which competitive terrorist groups within a nation seek, through their attacks, to outshine their rivals (Bloom 2004, 2005; Nemeth 2014). If a rival terror group upstages its competitors, then the group may take the lead in pushing its political agenda on the government. Terrorist groups drawn from the same ideological base may possess vastly different aims – e.g., one group may be less hard-line than another in terms of acceptable concessions. In particular, many of the splinter groups from the Palestine Liberation Organization (PLO) – e.g., the Abu Nidal Organization, the Popular Front for the Liberation of Palestine (PFLP), and Black September – were more hard-line than the parent PLO.

In the current paper, the model augments the standard objective function for the terror group to include an outbidding goal that captures intergroup competition. A two-stage game is eventually formulated in which the government chooses its defensive and proactive measures against the two resident terrorist groups in the first stage, conditioned on the groups' terrorism choices in the ensuing stage. In the second stage, each terrorist group decides its effort or terrorism based on defensive and proactive policy levels of the government. Government's defense limits both groups' probability of successful attacks. In contrast, the government's

[2] Siqueira and Sandler (2006) focus on recruitment by a single terrorist group in its confrontation with the government.

proactive measures are individually directed at increasing the targeted terrorist group's cost of attacks. By being group-specific, proactive responses create greater asymmetry among rival groups.

The full game is solved through backward induction starting with the second stage in order to achieve a subgame perfect equilibrium. We are interested in the equilibrium changes stemming from altered key parameters – the attack success probability and the rival groups' marginal attack costs – that are affected by the governments' choices of defensive and proactive policies. The asymmetry/symmetry and the "selfishness" of the rival terror organizations are essential determinants of the comparative statics associated with altered counterterrorism parameters in the two-stage game involving the government and its adversarial terrorist groups. A selfish terror group places more weight on the damage that it inflicts as compared to the total losses arising from the joint terror campaigns of the two groups.

During the analysis, the paper defines and interprets optimal policy rules for defensive and proactive measures, leveled by the government at the rival terrorist groups. For instance, defensive counterterrorism reduces both groups' level of terrorism. With asymmetric rival groups, proactive policies are directed more aggressively at the more efficient (larger) of the terror groups when diminishing returns characterize such actions. Generally, a proactive campaign directed at one of the two groups reduces the overall level of terrorism. We also consider policies for symmetry-preserving identical changes in rival groups' marginal costs of attacks arising from circumstances that, say, favor terrorist groups' recruitment and attacks. A rise in terrorist group efficiency from reduced cost encourages a defensive response but only raises proactive measures when there are sufficient diminishing returns to defense. That is, the government choice between defensive and proactive responses hinge on how increased use of the respective countermeasures diminishes their effectiveness. An essential message is that how rival terror groups interact cannot be divorced from the mix and level of the government's defensive and proactive policies. This implies that the level of terrorism tied to within-country terror group rivalry is not necessarily positively related to the outbidding scenario. The resulting level of terrorism for rival groups depends on the interaction of the three agents so that outbidding may not always mean more terrorism as assumed by the empirical literature (Chenoweth 2010; Conrad and Greene 2015; Cunningham, Bakke, and Seymour 2012; Findley and Young 2012; Nemeth 2014).

The body of the paper consists of five additional sections. The ensuing section reviews the empirical literature on rival groups. In the following section, the terrorist groups' choice is isolated for given levels of defensive and proactive policies. As such, this denotes a one-shot game that later forms the second stage of the two-stage game model of the government and the rival groups. For this full-blown model, the subgame perfect equilibrium and some policy-related comparative statics are presented. The next section is then devoted to an analysis of policy in light of cost reduction for the terrorist groups. In the next-to-last section, some modeling extensions are presented, followed by a conclusion.

On the Empirical Literature on Terrorist Groups

The empirical literature on terrorist groups began to grow after some useful data sources recorded post-1969 variables of terrorist groups – e.g., their start dates, longevity, peak size, ideologies, base country or countries, and goals. Notable terrorist-group data sets include Cronin (2009) and Jones and Libicki (2008). The latter data are updated through 2016 by Hou, Gaibulloev, and Sandler (2020) using incident information drawn from the National Consortium for the Study of Terrorism and Responses to Terrorism (2018) Global Terrorism Database (GTD).

Based on these data sets, researchers analyze the determinants of terror group longevity and survival (Blomberg, Engel, and Sawyer 2010; Carter 2012; Cronin 2006, 2009; Jones and Libicki 2008; Gaibulloev and Sandler 2019). The pioneering study by Blomberg, Engel, and Sawyer (2010) is particularly noteworthy because it applies survival analysis to identify key influences on terrorist group longevity. An important independent variable in some subsequent group survival studies includes the presence of within-country rival terrorist groups. For instance, Phillips (2015) argues that violent rivalries among such groups can augment their longevity as civilian supporters take sides, encourage innovations, and provide resource support.[3] An opposite view is put forward by Young and Dugan (2014) that terrorist groups' rivalry and outbidding limit longevity as warring groups work at cross purposes. More recently, Gaibulloev, Hou, and Sandler (2020) find that group competition promotes longevity but inhibits success as the government is confronted with conflicting demands. Thus, outbidding actions by rival terror groups have mixed effects on terrorist groups' prospects of longevity and success.

In the literature, outbidding by rival terrorist groups is often associated with the level of terrorism (Bloom 2004, 2005; Chenoweth 2010; Jaeger et al. 2015; Nemeth 2014). For instance, Cunningham, Bakke, and Seymour (2012) suggest that this increased violence stems from "dual contests" – one between rival terrorist or rebel groups, and another between the terrorist groups and the government. Like our theoretical model, those authors recognize that terrorist groups' violence has private and public components. The group's own violence contributes to its reputation and size, resulting in group-specific or private benefits, while the group's violence also augments government-directed violence, thereby increasing pressures on the besieged government. The latter is a nonrivalrous and nonexcludable public benefit for all terrorist

[3] Another set of articles examines the effect of terrorist groups' alliances on their lethality and longevity (Asal and Rethemeyer 2008; Horowitz and Potter 2014; Phillips 2014). These alliances allegedly foster lethality and longevity by pooling resources and sharing innovations.

groups, because a more stressed government is more apt to concede to the rival groups' shared demands. At other times, outbidding is linked to the intensity of attacks in which there are more suicide attacks, reflective of the "quality" of violence (Bloom 2004, 2005; Conrad and Greene 2015; Findley and Young 2012). Empirical tests of outbidding provide mixed results for large *n* samples of countries – e.g., for 1970–2004, Findley and Young (2012) uncover scant evidence of outbidding affecting suicide terrorism and even less evidence of outbidding influencing total terrorism. In contrast, Cunningham, Bakke, and Seymour (2012) find that enhanced intergroup competition ratchets up the number of terrorist or rebel attacks. Nemeth (2014) also displays some evidence of competition-induced enhanced terrorist violence during 1970–1997, but his evidence is not particularly strong in terms of statistical significance. In a micro-level study of the Palestinian territory, Jaeger et al. (2015) show that outbidding occurred in a very modest fashion within religious group rivalry (Hamas and PIJ) and within secular group rivalry (al-Fatah and PFLP), but not between religious and secular rivalries. For specific case studies (i.e., Sri Lanka and the Palestinian territory) Bloom (2004, 2005) offers support that intergroup rivalry results in more suicide attacks.

This varied empirical evidence suggests that the relationship between outbidding and the level of violence may hinge on factors not necessarily captured by theoretical discussion of the empirical analysis. This intervening or additional determinant is suggested to be the actions of the state in Cunningham, Bakke, and Seymour (2012). In fact, Bloom (2005) states that outbidding consequences are affected by the state's counterterrorism strategies in response to terrorist violence. At the theoretical level, our two-stage game representation shows that intergroup competition may or may not lead to more violence depending on the government's mix of defensive and proactive policies. This then suggests that empirical models of outbidding must incorporate such policies. The myriad empirical outcomes involving the level of terrorism, associated with intergroup competition, is completely consistent with our theoretical model, where government counterterrorism actions exert a large influence on the resulting level of terrorism.

Terrorist Groups' Optimal Effort for Given Counterterrorism Levels

We first focus on the interactions of the rival terrorist groups for exogenously given counterterrorism actions by the government. Terrorist organizations *A* and *B* operate in the same country. Each organization wants to weaken the government by subjecting it to greater aggregate terror attacks, while caring about its own share of the total terrorism produced (i.e., the outbidding effect). Accordingly, we propose a reduced-form objective function of terrorist organization *i*, where both its share of attacks and total attacks enter positively in the group's objective function:

$$V^i = \left(\frac{T^i}{T}\right)^{\alpha} T^{1-\alpha} = \left(T^i\right)^{\alpha} T^{1-2\alpha}, \text{ where } 0<\alpha<1,\ \alpha>1/2\ ,\ i=A,B\,, \tag{1}$$

for which T^i is the terrorism created by organization *i* and $T = T^A + T^B$ is the total terrorism created by the two groups. In Eq. (1), α is the relative weight that the terrorist group places on its own share of terror attacks (constituting the group-specific private benefit) relative to the aggregate damage inflicted on the government (constituting the groups' public benefit). In the group's objective, $\alpha > 1/2$ reflects that the terrorist organization cares more for its share of terrorism than for the aggregate damages inflicted on the country. This assumption is in keeping with how the splintered PLO groups put their attacks above the aggregate campaign waged by all groups (see, e.g., Bloom 2004). In a later section, we consider the alternative case where $\alpha \le 1/2$, which would be more conducive to group cooperation and even merger. The Cobb-

Douglas form is used in Eq. (1) for the sake of analytical tractability.

The counterterrorism defensive level (e.g., hardening of targets), chosen by the besieged government, is *e*, where $\beta(e)$ represents the *fraction* of successful terror attacks derived from a given terrorist effort. In the absence of any defensive measures, all terror attacks are successful so that $\beta(e=0)=1$. Greater defensive safeguards reduce terrorist success at a diminishing rate, such that $\beta'(e)<0$ and $\beta''(e)>0$. We let l^i represent terrorist organization *i*'s effort level. Terrorism produced by terrorist organization *i* equals:

$$T^i = \beta(e) l^i, \tag{2a}$$

where aggregate terror is:

$$T = T^A + T^B = \beta(e)\left(l^A + l^B\right). \tag{2b}$$

We next consider the terrorist groups' costs associated with their attacks. We let c^i be the per-unit input cost for l^i of terrorist organization *i*. This cost may be the same for the two terrorist organizations (the symmetric case) or it may differ between them. Such differences may stem from various exogenous factors including disparities in their recruitment capabilities or their supporter base. The government targets its proactive effort, m^i, against terrorist organization *i* by attacking the groups' infrastructure and resources. Such directed proactive effort makes it harder for the targeted terrorist organization to recruit and operate in the country, leading to input cost c^i rising with m^i. In addition, we assume that the government is more effective at low levels of proactive measures in raising the cost of the terrorist organizations' operations, but this proactive effectiveness falls at higher levels of sure measures. Such diminishing returns apply to situations where the government's proactive campaign first depletes the most vulnerable terrorist assets. At greater proactive levels, the government must take on the more battle-hardened terrorists or find them in their most secure havens, thus reducing proactive effectiveness at the margin. Denoting group i's input cost at zero proactive effort as $\overline{c}^i$, we define $c^i = c^i\left(\overline{c}^i, m^i\right)$. When $\overline{c}^i$ is suppressed from the cost function, group i's input cost is $c^i\left(\overline{c}^i, m^i\right) = c^i\left(m^i\right)$, where $c^{i\prime}\left(m^i\right) > 0$ and $c^{i\prime\prime}\left(m^i\right) < 0$ reflecting our assumptions that targeted proactive effort raises group i's cost but at a diminishing rate of rise.

Substituting Eq. (2a) in Eq. (1), and suppressing the counterterrorism effort levels from the functional forms for now, we have that the net payoff of terrorist organization i is:

$$\pi^i = V^i - c^i l^i = \beta^{1-\alpha}\left(l^i\right)^{\alpha}\left(l^A + l^B\right)^{1-2\alpha} - c^i l^i \equiv \pi^i\left(l^A, l^B; \beta, c^i\right). \tag{3}$$

In Eq. (3), exogenously given resources from supporters finance the groups' cost of operations and lie outside of our analysis. The terrorist organizations are assumed to move simultaneously and choose their respective effort levels to maximize their respective payoffs, assuming the other organization's effort level as given (i.e., standard Nash assumption). When the two terrorist groups choose their effort levels to maximize their net payoffs, the first-order conditions of the two organizations are:[4]

$$\pi^A_{l^A}\left(l^A, l^B; \beta, c^A\right) = \beta^{1-\alpha}\left(l^A\right)^{\alpha-1}\left(l^A + l^B\right)^{-2\alpha}\left[(1-\alpha)l^A + \alpha l^B\right] - c^A = 0, \tag{4a}$$

and

$$\pi^B_{l^B}\left(l^A, l^B; \beta, c^B\right) = \beta^{1-\alpha}\left(l^B\right)^{\alpha-1}\left(l^A + l^B\right)^{-2\alpha}\left[(1-\alpha)l^B + \alpha l^A\right] - c^B = 0. \tag{4b}$$

Eqs. (4a) and (4b) implicitly define the Nash reaction functions of the two terrorist organizations, respectively, as:

$$l^A = l^A\left(l^B; \beta, c^A\right), \tag{5a}$$

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[4] The assumptions thus far ensure that the second-order conditions $\pi^A_{l^A l^A} < 0$ and $\pi^B_{l^B l^B} < 0$ are satisfied.

and

$$l^B = l^B\left(l^A;\beta,c^B\right), \tag{5b}$$

in terms of the other group's effort, the probability of success, and the group's own effort cost. Eqs. (5a) and (5b) jointly determine a Nash equilibrium, *N*, in the effort levels of the two terrorist organizations as:

$$l^{AN} = l^{AN}\left(\beta,c^A,c^B\right) \text{ and } l^{BN} = l^{BN}\left(\beta,c^A,c^B\right). \tag{6}$$

From Eqs. (4a) and (4b), we can easily show that group *i* chooses a positive effort level $l^{i0} = \left[\dfrac{\beta^{1-\alpha}\left(1-\alpha\right)}{c^i}\right]^{\frac{1}{\alpha}}$ when its rival chooses a zero-effort level. Given the second-order condition $\pi^A_{l^Al^A} < 0$, the slope of *A*'s reaction function, based on the implicit function rule applied to Eq. (4a), is:

$$\frac{dl^A}{dl^B} = -\frac{\pi^A_{l^Al^B}}{\pi^A_{l^Al^A}} \geq 0 \text{ if and only if } \pi^A_{l^Al^B} \geq 0. \tag{7}$$

Partially differentiating the expression given in Eq. (4a) with respect to l^B, we get:

$$\pi^A_{l^Al^B} = \alpha\left(2\alpha-1\right)\beta^{1-\alpha}\left(l^A\right)^{\alpha-1}\left(l^A+l^B\right)^{-2\alpha-1}\left(l^A-l^B\right) \geq 0 \text{ if and only if } l^A \geq l^B. \tag{8}$$

Similarly, the slope of *B*'s reaction function is obtained from Eq. (4b) as:

$$\frac{dl^B}{dl^A} = -\frac{\pi^B_{l^Bl^A}}{\pi^B_{l^Bl^B}} \geq 0 \text{ if and only if } \pi^B_{l^Bl^A} \geq 0. \tag{9}$$

Partially differentiating Eq. (4b) we get:

$$\pi^B_{l^Bl^A} = \alpha\left(2\alpha-1\right)\beta^{1-\alpha}\left(l^B\right)^{\alpha-1}\left(l^A+l^B\right)^{-2\alpha-1}\left(l^B-l^A\right) \leq 0 \text{ if and only if } l^A \geq l^B. \tag{10}$$

[Figure 1 near here]

Eqs. (7), (8), (9), and (10) allow us to depict the two groups' reaction functions, R^A and

R^B, respectively, and the Nash equilibrium for the symmetric case ($c^A = c^B = c$) in Figure 1, where group *B*'s effort is measured on the vertical axis and group *A*'s effort is measure on the horizontal axis. In Figure 1, the 45-degree line ray from the origin indicates when the two groups exert the same terrorism effort. Eqs. (7) and (8) imply that *A*'s reaction function is positively sloped below the 45-degree line in $\left(l^A, l^B\right)$ space where $l^A > l^B$, is vertical on the 45-degree line where $l^A = l^B$, and is backward bending above the 45-degree line where $l^A < l^B$. Analogously based on Eqs. (9) and (10), *B*'s reaction function is positively sloped above the 45-degree line in $\left(l^A, l^B\right)$ space, achieves zero slope on the 45-degree line, and is negatively sloped below the 45-degree line. The Nash equilibrium is at point *N* on the 45-degree line where the two reaction paths intersect. Each terrorist group exhibits strategic complementarity (strategic substitutability) (Bulow, Geanakoplos, and Klemperer 1985) when its effort level exceeds (is smaller than) its rival's effort level. However, at the Nash equilibrium, the slope of each organization's reaction function vis-à-vis its rival's effort level is zero.

We next turn our attention to the central parameters that determine the pattern of asymmetry in our model. Using Eqs. (4a) and (4b), we can obtain the relationship between the relative effort levels of the two terrorist organizations, $x = l^A / l^B$, and the relative input cost of the two organizations, $\lambda = c^A / c^B$, as:

$$\frac{(1-\alpha)x + \alpha}{1-\alpha+\alpha x} = \lambda x^{1-\alpha}. \tag{11}$$

Applying the implicit function rule to Eq. (11), we find that $dx/d\lambda < 0$. When $c^A = c^B \left(i.e., \lambda = 1\right)$, symmetry ensures that $l^A = l^B$ implies $x = 1$. Therefore, we have

$$l^A \geq l^B \text{ if and only if } c^A \leq c^B. \tag{12}$$

For $c^A < c^B$, λ is less than unity so that $dx/d\lambda < 0$ requires that $x > 1$ and $l^A > l^B$. Hence, Eq. (12) means that the relative sizes of the groups are entirely determined by the relative cost λ, in which the lower-cost group must be the larger. Based on Eqs. (8), (10), and (12), we have that:

$$\pi^A_{l^A l^B} \geq 0 \text{ and } \pi^B_{l^B l^A} \leq 0 \text{ if and only if } c^A \leq c^B. \tag{13}$$

[Figure 2 near here]

Figure 2 presents an asymmetric Nash reaction function diagram for $c^A < c^B$, where Eqs. (13) and (7) indicate that *A*'s reaction function is positively sloped at the Nash equilibrium N^0. Similarly, Eqs. (13) and (9) imply that *B*'s reaction function is negatively sloped at N^0. In Figure 2, the total terrorist effort of the two groups is found by dropping a line with slope –1 to the horizontal axis. Along this sum-preserving line, total terrorist effort at N^0 is L^0. For the rest of the paper, we assume without loss of generality that terrorist group *A* is at least as efficient as group *B*, such that $c^A \leq c^B$.

Comparative Statics of Counterterrorism Policy Changes

Here, we consider the effect of changes in c^A, c^B, and β on the effort levels of the two terrorist groups and, thus, on aggregate terrorism. A change in β stems from a change in the counterterrorism defense, whereas changes in the groups' costs could be driven both by changes in proactive measures and exogenous changes in input costs (i.e., changes in $\bar{c}^i$, $i = A, B$). The latter issue is considered later in the context of optimal counterterrorism policy. Totally differentiating Eqs. (4a) and (4b), we get, respectively,

$$\pi^A_{l^A l^A} dl^A + \pi^A_{l^A l^B} dl^B + \pi^A_{l^A \beta} d\beta - dc^A = 0 \text{, and} \tag{14a}$$

$$\pi^B_{l^B l^A} dl^A + \pi^B_{l^B l^B} dl^B + \pi^B_{l^B \beta} d\beta - dc^B = 0, \tag{14b}$$

which are solved simultaneously to obtain the effects of changes in the input costs and β on the changes of each terrorist group's effort level (i.e., dl^A, dl^B).

Increase in Group *A's Cost,* c^A

From Eqs. (14a) and (14b), we derive:

$$\frac{dl^A}{dc^A} = \frac{\pi^B_{l^B l^B}}{D} < 0, \tag{15}$$

when we set $dc^B = d\beta = 0$ and solve using Cramer's rule. In (15), $D = \pi^A_{l^A l^A}\pi^B_{l^B l^B} - \pi^A_{l^A l^B}\pi^B_{l^B l^A} > 0$, $\pi^A_{l^A l^A} < 0$, $\pi^B_{l^B l^B} < 0$, and Eqs. (8) and (10) indicate that $\pi^A_{l^A l^B}\pi^B_{l^B l^A} \le 0$. Thus, an increase in group *A*'s cost must reduce its terror effort. Turning to *B*'s terror effort and using Eq. (13), we find:

$$\frac{dl^B}{dc^A} = -\frac{\pi^B_{l^B l^A}}{D} \ge 0 \text{ as } c^A \le c^B, \tag{16}$$

which means that *B*'s effort will rise or remain unchanged in response to a small increase in c^A depending on whether at the initial equilibrium $c^A < c^B$ or $c^A = c^B$, respectively. With Eqs. (2a), (15), and (16), we derive the effect of a rise in *A*'s cost on aggregate terrorism and groups' effort:

$$\frac{dT}{dc^A} = \beta(e)\left(\frac{dl^A}{dc^A} + \frac{dl^B}{dc^A}\right) = \frac{\beta(e)\left(\pi^B_{l^B l^B} - \pi^B_{l^B l^A}\right)}{D} < 0, \tag{17}$$

where $\pi^B_{l^B l^B} - \pi^B_{l^B l^A} = -\alpha\beta^{1-\alpha}\left(l^A + l^B\right)^{-2\alpha}\left(l^B\right)^{\alpha-2}\left[\alpha l^B + (1-\alpha)l^A\right] < 0$. Eq. (17) establishes that an increase in c^A must reduce aggregate terrorist effort and total terrorism. Note that $\pi^B_{l^B l^B} - \pi^B_{l^B l^A} < 0$ can be written as $\pi^B_{l^B l^B}\left(1 - \frac{\pi^B_{l^B l^A}}{\pi^B_{l^B l^B}}\right) = \pi^B_{l^B l^B}\left(1 + \rho^B\right) < 0$, where $\rho^B = -\frac{\pi^B_{l^B l^A}}{\pi^B_{l^B l^B}} < 0$ is the slope of *B*'s reaction function. Given that $\pi^B_{l^B l^B} < 0$, we have that

$\pi^B_{l^B l^B}\left(1+\rho^B\right)<0$ implies $1+\rho^B>0$. The last inequality ensures that group *B*'s reaction function must be flatter than a negatively sloped 45-degree line (i.e., $\left|\rho^B\right|<1$) at the Nash equilibrium, displayed in Figure 2 at N^0. A small increase in c^A shifts *A*'s reaction curve to the left (not drawn), while leaving *B*'s reaction curve unaffected. Therefore, the new Nash equilibrium is at N^1 on *B*'s reaction curve to the northwest of the initial equilibrium N^0, showing a fall in l^A and a rise in l^B. We can also see the effect on aggregate terrorism from Figure 2. The new and lower aggregate effort at the Nash equilibrium N^1 is given by the point L^1 on the horizontal axis, which is less than L^0. Our analysis shows that any increase in group *B*'s effort is overwhelmed by group *A*'s decreased terror effort, therefore validating proactive measures or other actions to raise *A*'s terror cost.

Increase in Group *B's Cost,* c^B

Considering the effect of an increase in c^B, we see that Eqs. (14a) and (14b) yield:

$$\frac{dl^B}{dc^B}=\frac{\pi^A_{l^A l^A}}{D}<0, \tag{18}$$

and

$$\frac{dl^A}{dc^B}=-\frac{\pi^A_{l^A l^B}}{D}\le 0 \text{ as } c^A\le c^B, \tag{19}$$

when solved for $dc^A=d\beta=0$. Thus, a rise in c^B must reduce group *B*'s effort. A small increase in c^B will reduce group *A*'s effort or leave it unaffected depending on whether $c^A<c^B$ or $c^A=c^B$, at the initial equilibrium. Aggregate terrorist effort l^A+l^B must fall as c^B rises because *B*'s effort falls and *A*'s effort either falls or remains unchanged. Because aggregate terrorist effort falls, we have from Eq. (2b) that aggregate terrorism must fall. In terms of Figure 2, a rise

in group *B*'s cost shifts its reaction curve down (not drawn), moving the Nash equilibrium down *A*'s positively sloped reaction curve to point N^2, reducing both l^A and l^B. At the new equilibrium, the smaller aggregate terrorist effort is L^2.

Increase in β

The effect of an increase in β, arising from reduced defensive measures, on *A*'s effort level is obtained from Eqs. (14a) and (14b) (with $dc^A = dc^B = 0$) as:

$$\frac{dl^A}{d\beta} = \frac{\pi^A_{l^A l^B}\pi^B_{l^B \beta} - \pi^B_{l^B l^B}\pi^A_{l^A \beta}}{D} > 0, \tag{20}$$

where Eqs. (4a), (4b), and (13) yield $\pi^A_{l^A\beta} > 0$, $\pi^B_{l^B\beta} > 0$, $\pi^B_{l^B l^B} < 0$, and $\pi^A_{l^A l^B} \geq 0$. Thus, an increase in β must increase l^A. Recall from Eq. (11) that the groups' relative effort level, $x = l^A / l^B$, is independent of β. Thus, as an increase in β raises l^A, it must also increase l^B in the same direction and by the same proportion. The above findings lead to Proposition 1.

Proposition 1: An increase in defensive countermeasures reduces attacks by both terrorist organizations. Proactive counterterrorism policy against a terrorist group reduces that group's attacks and raises (reduces) the attacks of the smaller (larger) rival group. If, however, the groups are of the same size, then proactive measures against one group does not affect the rival group's terror effort. Proactive effort against any group must reduce aggregate terrorism.

Proof: The proofs of the propositions are gathered in the Appendix.

Increased defensive action reduces the marginal terrorism gains for both terrorist organizations by degrading the efficacy of their efforts. The resulting terrorist attack reductions are proportional because Eq. (11) ensures that the terrorists' effort ratio is independent of the government's defensive response. Proactive measures directed at a specific terrorist group raises its input cost and lowers its attacks, but the response of the other terrorist group depends on the pattern of strategic complementarity/substitutability at the Nash equilibrium. Recall from Eq. (12) that the lower-cost terrorist group is the larger group; but if $c^A = c^B$, then the terrorist groups are of the same size (i.e., symmetric). Under symmetry, there is no effect on the rival groups' efforts because of a zero or an infinite slope at the Nash equilibrium on the 45-degree ray out of the origin. However, under asymmetry, Figure 2 applies where a greater proactive response against *A* (the larger group) shifts *A*'s reaction curve leftward and moves the equilibrium from N^0 to N^1 raising *B*'s terrorism effort. In contrast, when proactive measures against *B* increase, the Nash equilibrium moves from N^0 to N^2, reducing *A*'s effort along *A*'s positively sloped reaction curve.

There is an interesting and novel public-private distinction that arises from our modelling of defensive and proactive measures in the presence of rival terrorist groups. Enhanced defensive measures limit *both* groups' effort-effective parameter, β, equally as these terror groups attack fortified targets. From the groups' and citizens' viewpoint, there is a publicness aspect to defensive measures. In contrast, proactive measures are group-specific and, thus, private between terrorist groups, which may induce the non-targeted group to increase its efforts. The publicness of defensive measures should favor them over proactive measures from the government's perspective. In the literature, however, this favoring stems from an entirely different rationale as a targeted country hardens targets at home in order to transfer attacks abroad in a multi-country and *single* group scenario (Gaibulloev and Sandler 2019). By contrast, the favoring of defense uncovered here arises from defense countermeasures limiting both groups' terrorism unlike proactive measures. As such, the bias to use defensive rather than proactive policy arises in a multi-group *and* in a multi-country setting. Only the latter is acknowledged in the literature.

Optimal Counterterrorism Policy

This section extends the analysis to consider optimal counterterrorism policy[5] in terms of a two-stage game where the government moves in stage 1 to minimize terrorism losses inclusive of counterterrorism costs. In stage 2, terrorist groups choose their respective effort or campaign levels. The subgame perfect equilibrium of this game is obtained by solving through backward induction, such that the Nash equilibrium of the previous section now serves as the second-stage solution. Accordingly, for stage-1 optimization, the government uses the terrorist groups' equilibrium effort functions previously derived. For simplicity, we assume that a unit of defensive or proactive counterterror effort has a constant marginal cost of unity. Using Eq. (2b) for aggregate terrorism, denoting the government's loss by Ω, and recalling that $l^{iN}\left[\beta(e), c^A\left(m^A\right), c^B\left(m^B\right)\right] \equiv l^i\left(e, m^A, m^B\right), i = A, B$ (see the Appendix), we have that the government's loss function is:

$$\Omega\left(e, m^A, m^B\right) = T + e + m^A + m^B$$

$$= \beta(e)\left[l^A\left(e, m^A, m^B\right) + l^B\left(e, m^A, m^B\right)\right] + e + m^A + m^B. \tag{21}$$

The first-order condition for the government's choice of the loss-minimizing defensive

[5] For clarity, we consider only an interior counterterrorism optimum in terms of $\left(e, m^A, m^B\right)$. Corner solutions are ruled out by assuming that $\beta'(e = 0)$, $c^{A\prime}\left(m^A = 0\right)$, and $c^{B\prime}\left(m^B = 0\right)$ are all sufficiently large so that there are net positive gains from terror reduction, starting from a zero level of any of the three counterterrorism instruments.

level is:[6]

$$\left|T_e\right| = 1 \Leftrightarrow -\left(l^A + l^B\right)\beta'(e) - \beta(e)\left(l_e^A + l_e^B\right) = 1 . \tag{22a}$$

At the optimal defensive level, Eq. (22a) indicates that the marginal gains from aggregate terrorism reduction equals the associated marginal cost of defensive actions. Moreover, we note that the first term on the left-hand side of the second equality in Eq. (22a) is the marginal benefit of reduced terrorism due to better defensive protection at given levels of terrorist effort. The second left-hand term is the marginal benefit of reduced terrorism due to the dampening effect of defense on the terrorists' aggregate effort. There are several factors, which are *ex ante* endogenous, that affect the desirability of defense in Eq. (22a). First, we note that aggregate terror is $T = \beta(e)\left(l^A + l^B\right)$. At any defense level *e*, this implies that greater terrorism is associated with a larger aggregate terrorist effort level, $l^A + l^B$. Since the aggregate terrorist effort level scales $\beta'(e)$ in Eq. (22a), a greater terror level amplifies the marginal benefit from greater defense. Second, as shown in the proof of Proposition 1 (see Appendix), the influence of defensive action on reducing groups' aggregate effort, captured by $l_e^A + l_e^B$, depends on several factors that affect the equilibrium effort levels of the terrorist groups. To summarize, if aggregate terror effort is highly elastic or responsive to defense increases and if the terrorism level is large, then the desirability of using defense increases.

The first-order conditions for the government's proactive policy choices m^A and m^B are, respectively,

$$\left|T_{m^A}\right| = 1 \Leftrightarrow -\beta(e)\left(l_{m^A}^A + l_{m^A}^B\right) = 1, \tag{22b}$$

---

[6] We assume that there are sufficient diminishing returns in defense, i.e., $\beta'' \geq 0$, and in proactive effort, $c^{i''}\left(m^i\right) \leq 0$, *i=A,B,* to ensure that second-order conditions are satisfied for minimizing government loss.

$$\left|T_{m^B}\right| = 1 \Leftrightarrow -\beta\left(e\right)\left(l^A_{m^B} + l^B_{m^B}\right) = 1. \tag{22c}$$

From Proposition 1 and its proof in the Appendix, aggregate terrorist effort must fall when either m^A or m^B is raised $\left(i.e.,\ l^A_{m^i} + l^B_{m^i} < 0;\ i = A, B\right)$. Thus, the left-hand sides of Eqs. (22b) and (22c) represent aggregate terrorism reduction for an increase in m^A and m^B, respectively. At the optimum, the marginal benefit from proactive measures against a terrorist group must equal the unit marginal cost of such measures. The marginal benefit from proactive counterterrorism is easier to interpret in the symmetric case. Recall that, at a symmetric equilibrium, equal cost for the two groups is consistent with equal terror efforts (i.e., $c^A = c^B \Leftrightarrow l^A = l^B = l$). Moreover, $\pi^A_{l^A l^B} = \pi^B_{l^B l^A} = 0$ implies $D = \pi^A_{l^A l^A}\pi^B_{l^B l^B}$. In this case, Eqs. (A5) and (A6) of the Appendix yield:

$$T_{m^A} = -\frac{2^{2\alpha}\beta^\alpha c^{A\prime}\left(m^A\right) l^{1+\alpha}}{\alpha} < 0 \text{ and } T_{m^B} = -\frac{2^{2\alpha}\beta^\alpha c^{B\prime}\left(m^B\right) l^{1+\alpha}}{\alpha} < 0. \tag{23}$$

Eq. (23) indicates that marginal terror reduction must increase for a larger terror effort level. This effect, *ceteris paribus*, suggests that in an environment of larger aggregate terrorism, there is a greater incentive for proactive measures. Under asymmetry, Proposition 2 throws light on the relative targeting of such measures between the two asymmetric terrorist groups.

Proposition 2: If the terrorist groups share a common cost function that is separable in proactive effort and exhibits diminishing effectiveness at larger proactive levels (i.e., the cost function is strictly concave in proactive measures), then the optimal proactive response against the larger group exceeds the optimal proactive response against the smaller group.

In view of Proposition 1, we know that a greater proactive response against the smaller terrorist group will reduce both groups' terrorism, while more proactive effort against the larger group lowers its terrorism but raises that of the smaller group. This outcome might suggest that the government may want to go more aggressively against the smaller group. However, Proposition 2 establishes that this conclusion is generally false. Indeed, for separable cost functions, the government is shown in Proposition 2 to favor more aggression against the larger group. This follows because the own contractionary terror effect on the larger group is sufficiently strong to overwhelm the cross-effect expansionary terror response of the smaller group for optimal proactive policy.

For some context, consider the Colombian government actions against the far-left rival FARC and ELN terrorist groups during the 1990s and more recent years prior to the peace agreement between the government and FARC in 2016. FARC was the much larger of the two rival groups (Hou, Gaibulloev, and Sandler 2020). The government focused its proactive campaign on FARC, which eventually brought them to the negotiation table. Following the end of FARC's operations, ELN still remains active but presents a less formidable threat that the Colombian government could then address. Our model suggests that Colombian government's initial focus on FARC was an optimal strategy.

Counterterrorism Response to Lower Terror Production Costs: The Symmetric Case

This section considers the effects of symmetry-preserving exogenous and simultaneous cost decreases of the two terrorist groups. The exercise applies to scenarios where similar size terrorist groups experience an improvement in their input procurement process. If, e.g., some in the population become radicalized by an external event that they sympathize with, this may raise the terrorist groups' volunteer pools and drive down input or recruitment cost. According to Hoffman (2012), many young Palestinians volunteered for PLO-affiliated terrorist groups following the 1972 Munich Olympics kidnapping and murder of Israeli athletes by Black

September. This enhanced allegiance occurred even though the kidnapping did not obtain any concessions and ended in the eventual capture or killing of the terrorists behind the attack. Another example is the increase in terrorism support for within-country rival groups after the US retaliatory bombing raid on Libya on April 15, 1986, in retribution for Libya's role in the La Belle discotheque bombing in West Berlin on April 5, 1986 (Enders and Sandler 1993). The retaliatory raid induced a wave of attacks aimed at the United States and the United Kingdom, which conducted or aided the raid, respectively. The current exercise is also relevant in a cross-sectional sense to two different venues of multi-group terrorism where input cost in one venue is lower than in another. Since the late 1990s, the appearance of terrorist networks – e.g., al-Qaida and ISIS – with affiliated, but rival, groups in more than one venue is an example. The civil war in Syria following the Arab Spring in 2011 reduced the cost of recruitment to al-Qaida and ISIS in that venue relative to elsewhere. Stresses on the Syrian government bolstered this cost reduction. In so doing, foreign fighters were attracted to the groups. The questions addressed here are how optimal counterterrorism policy adjusts to terror groups' cost decreases, and what is the net effect of such adjustments on the level of terrorism and, hence, on national welfare (negative of the loss function)?

Recall from the analysis of stage 2 that the input cost function of terror group *i* takes the form $c^i = c^i\left(m^i;\overline{c}^i\right)$, where $\overline{c}^i$ is the input cost at a zero proactive level. Eqs. (11) and (12) established earlier that the groups are symmetric if $c^A = c^B$. Additionally, we assume now that $\overline{c}^A = \overline{c}^B = \overline{c}$ so that the groups remain symmetric when proactive measures are zero. If proactive effort is positive and if the rival terror groups abide by a common input cost function, then $c^i = c\left(m^i;\overline{c}\right)$. Symmetry of the two terror groups requires that $c\left(m^A;\overline{c}\right) = c\left(m^B;\overline{c}\right)$ implies $m^A = m^B = m$. Thus, the two terror groups can be represented by a common cost function

$c^i = c\left(m;\overline{c}\right)$, $i = A, B$. Furthermore, to represent explicitly $\overline{c}$ in the cost function, we denote $c^{i\prime}\left(m^i\right)$ and $c^{i\prime\prime}\left(m^i\right)$ of the previous sections by $c_m\left(m,\overline{c}\right) > 0$ and $c_{mm}\left(m,\overline{c}\right) \leq 0$, respectively. A larger $\overline{c}$ is reasonably associated with a greater input cost at any given proactive level, such that $c_{\overline{c}}\left(m,\overline{c}\right) > 0$. Under symmetry, $l^A = l^B = l$ such that Eq. (4a) yields:

$$l\left(e,m;\overline{c}\right) = \frac{\beta\left(e\right)^{\frac{1-\alpha}{\alpha}} c\left(m;\overline{c}\right)^{-\frac{1}{\alpha}}}{4}. \tag{24}$$

In the symmetric case, the government's loss function, Eq. (21), reduces to:

$$\Omega\left(e,m;\overline{c}\right) = T + e + 2m = 2\beta\left(e\right)l\left(e,m;\overline{c}\right) + e + 2m. \tag{25}$$

The first-order conditions for the government's stage-1 choices of $\left(e,m\right)$ to minimize its loss are, respectively,

$$\Omega_e\left(e,m;\overline{c}\right) = 0 \Rightarrow -\frac{\beta'\left(e\right)\left[\beta\left(e\right)\right]^{\frac{1-\alpha}{\alpha}}\left[c\left(m;\overline{c}\right)\right]^{-\frac{1}{\alpha}}}{2\alpha} = 1, \tag{26a}$$

$$\Omega_m\left(e,m;\overline{c}\right) = 0 \Rightarrow \frac{\left[\beta\left(e\right)\right]^{\frac{1}{\alpha}}\left[c\left(m;\overline{c}\right)\right]^{-\frac{1+\alpha}{\alpha}} c_m\left(m;\overline{c}\right)}{2\alpha} = 2, \tag{26b}$$

where an increase in the proactive measure, m, represents a symmetry-preserving equal increase in m^A and m^B, which is reflected in a marginal cost of 2 on the right-hand-side of Eq. (26b). The second-order conditions, associated with the partials of Ω, of this stage-1 optimization play an important role in restricting the possible comparative-static outcomes, later captured by Proposition 3. Thus, we start with evaluating the second-order partials of the government's loss function:

$$\Omega_{ee} = \frac{c^{-\frac{1}{\alpha}}\beta^{\frac{1-2\alpha}{\alpha}}\left(\beta'\right)^2\left(E_\beta + \frac{1-\alpha}{\alpha}\right)}{2\alpha} > 0, \tag{27a}$$

where $E_\beta = \frac{\beta\beta''}{(\beta')^2} > 0$ is a measure of the strength of diminishing returns to defensive effort.

Similarly,

$$\Omega_{mm} = \frac{c^{-\frac{1+2\alpha}{\alpha}}\beta^{\frac{1}{\alpha}}(c_m)^2\left(E_m + \frac{1+\alpha}{\alpha}\right)}{2\alpha} > 0\,, \tag{27b}$$

where $E_m = -\frac{cc_{mm}}{(c_m)^2} \geq 0$ is a measure of diminishing returns to proactive effort. Finally, we have:

$$D^* = \Omega_{ee}\Omega_{mm} - (\Omega_{em})^2 > 0 \text{ when } E_\beta > \frac{\alpha(1+E_m) - E_m}{\alpha(1+E_m)+1}. \tag{27c}$$

Notice that $\frac{\alpha(1+E_m) - E_m}{\alpha(1+E_m)+1} < 1$ because $E_m \geq 0$. Thus, $E_\beta \geq 1$ is a sufficient but not necessary condition for $D^* > 0$. We assume that there are sufficient diminishing returns in defense such that Eq. (27c) is satisfied. The above findings with respect to the government's loss partials satisfy the second-order condition for a loss minimum, allowing us to prove Proposition 3 in the Appendix. In essence, the comparative statics follow the same procedure as earlier where we proceed after totally differentiating Eqs. (26a) and (26b) and solving via Cramer's rule.

Proposition 3*:* A sufficient condition for cost reduction of the terrorist groups (i.e., fall in $\overline{c}$) to increase optimal defensive effort is that the marginal effect of proactive measures on the input cost ($i.e., c_m$) is non-decreasing in $\overline{c}$ (i.e., $c_{m\overline{c}} \geq 0$) and that there is strict diminishing returns to proactive measures (i.e., $c_{mm} < 0$). The optimal proactive response, however, may rise or fall. This optimal response must rise if there are sufficiently strong diminishing returns to defense. When optimal defense rises, terrorism will rise only if there are sufficient diminishing returns to defense (when $E_\beta > 1$). National welfare loss must always rise with a fall in terrorists' costs.

Consider first the effects of the decrease in terror groups' costs on aggregate terror and national welfare loss (i.e., the last part of Proposition 3). If optimal defense rises and if diminishing returns to defensive measures are not sufficiently strong ($E_\beta \leq 1$), the counterterrorism response is strong enough to either neutralize any terror increase (when $E_\beta = 1$) or even reduce terror (when $E_\beta < 1$). However, under sufficiently strong diminishing returns, the government succeeds only in dampening but not eliminating the rise in terrorism. Regardless of the direction of change in terrorism, national welfare loss must rise when terrorists' costs fall. Even if total terrorism falls or remains constant, national welfare loss increases because of the rise in counterterrorism outlays. To understand the rest of Proposition 3 with respect to proactive and defensive responses, we offer two specific cost function examples.

Example 1. $c(m;\overline{c}) = \overline{c} + \phi(m)$, and $\phi(0) = 0, \phi' > 0, \phi'' < 0$. Moreover, for this additively separable cost function, the following holds: $c_{\overline{c}} = 1$, $c_m = \phi'(m) > 0$, $c_{mm} = \phi''(m) < 0$, and $c_{m\overline{c}} = 0$. Since $c_{\overline{c}} = 1$, the fall in $\overline{c}$ reduces the terrorists' input cost, which raises terrorists' effort in Eq. (24) and the marginal benefit of defensive measures in Eq. (26a). As a consequence, defensive efforts will tend to increase. However, Eq. (26b) suggests two opposing effects on the marginal benefit of proactive measures. With greater defense, the marginal benefit of proactive effort in Eq. (26b) is dampened, but a lower input cost of terrorism drives up the marginal benefit from the proactive response. In general, the effect on proactive effort is ambiguous. Using Eq. (A9) of the Appendix, we have that $\frac{dm}{d\overline{c}} < 0$ if $E_\beta > \frac{\alpha}{1+\alpha}$. Thus, with sufficient diminishing returns to defense $\left(i.e., E_\beta > \dfrac{\alpha}{1+\alpha}\right)$, the government finds it profitable to balance its counterterrorism increase between proactive and defensive measures. Furthermore, based on Eq. (A11) of the Appendix, we conclude that terrorism will rise when $E_\beta > 1$ given that $\dfrac{de}{d\overline{c}} < 0$. To summarize, under sufficient diminishing returns to defense, the proactive responses and terrorism all rise when terrorists' input costs fall.

Example 2. $c\left(m;\overline{c}\right) = \overline{c}\gamma\left(m\right)$, $\gamma\left(0\right) = 1$, and $\gamma' > 0, \gamma'' \leq 0$.

For this multiplicatively separable cost function, $c_{\overline{c}} = \gamma\left(m\right) > 0$, $c_m = \overline{c}\gamma'\left(m\right) > 0$, $c_{mm} = \overline{c}\gamma''\left(m\right) \leq 0$, and $c_{m\overline{c}} = \gamma'\left(m\right) > 0$. A drop in $\overline{c}$ reduces $c\left(m;\overline{c}\right)$ and raises the marginal benefit of defense in Eq. (26a), tending to raise defense. Eq. (A8) of the Appendix establishes that defense must increase if $c_{m\overline{c}} > 0$, valid in this example. For proactive measures, a fall in $\overline{c}$ reduces c_m because $c_{m\overline{c}} > 0$. This effect dampens the marginal benefit of the proactive response in Eq. (26b). If this effect is large, then the government may prefer to reduce proactive effort while augmenting defense to combat the effects of reduced terrorism production costs. For this example, we note that $Z = \dfrac{cc_{m\overline{c}}}{c_m c_{\overline{c}}}$, defined around Eq. (A9), equals unity. Using Eq. (A9), we get $dm/d\overline{c} < 0$ if and only if $E_\beta > 1$. In turn, this means that for all $E_\beta \leq 1$, which satisfy the second-order condition outlined in Eq. (27c), proactive effort must decline with a fall in $\overline{c}$.

The above examples illustrate that the choice between proactive and defensive measures in response to changes in terrorist groups' cost are interdependent in this two-stage model involving staged responses of the governments and the rival groups. Although this is a complicated comparative-statics problem, we see that how each counterterrorism measure's effectiveness reacts as its deployment is increased (i.e., diminishing returns to the measure) is a crucial consideration in the ideal mixture of such measures used by the government. The complexity of the comparative statics applied to a two-stage game necessitates some structure to the cost function of the terrorist antagonists.

Two Extensions

Terrorism and Counterterrorism for $0 < \alpha < 1/2$

For the base model in the third section, we assumed that the terrorist organizations care more for their share of attacks than for the aggregate terrorism inflicted. We now allow the terrorist group to attach a greater weight on the aggregate terrorism created so that $\alpha < 1/2$.[7] This case would involve groups that do not consider themselves as rivals. In that situation, groups may eventually merge as identified by Hou, Gaibulloev, and Sandler (2020) – e.g., Salafist Group for Preaching and Fighting merged with al-Qaida to form al-Qaida in the Islamic Maghreb (AQIM) in January 2007. Other mergers occurred among ally groups, but merger is rather rare compared to splintering and rivalry (Hou, Gaibulloev, and Sandler 2020).

Compared to the base model, the first qualitative difference occurs in Eq. (8), which now implies that $\pi^A_{l^A l^B} \leq 0$ when $l^A \geq l^B$ given $2\alpha < 1$. Accordingly, the slope of group *A*'s reaction function is the opposite concavity (i.e., C-shaped) of that in Figure 1. The reaction path's slope is negative below the 45-degree ray, vertical on the 45-degree ray, and positive above the 45-

[7] If $\alpha = 1/2$, the payoff function for a terror group becomes independent of its rival's action (see the third section), so there are no strategic issues involved. As such, we do not pursue it further.

degree ray. Similarly, Eq. (10) indicates that *B*'s reaction function is negatively sloped above the 45-degree ray, horizontal on the 45-degree ray, and is positively sloped below the 45-degree ray.

Next, we consider the effects of changes in the input costs and β (the terrorism effort-effectiveness parameter) on the Nash-equilibrium levels of groups' terror efforts, l^A and l^B for $\alpha < 1/2$. Starting with the effect of a change in *A*'s input cost, we have that Eq. (15) still indicates that a rise in c^A reduces l^A. However, because the reaction functions' slopes are reversed compared to base case with $\alpha > 1/2$, Eq. (16) states that $\frac{dl^B}{dc^A} \leq 0$ as $c^A \leq c^B$. The effect on total terrorism, captured in Eq. (17), is unchanged compared to the base case. Similarly, the effect of a rise in c^B on both l^B and total terrorism are unchanged from the base case, but now Eq. (19) implies that $\frac{dl^A}{dc^B} \geq 0$. Finally, using Eqs. (14a) and (14b), we have that $\frac{dl^B}{d\beta} = \frac{\pi^B_{l^B l^A}\pi^A_{l^A \beta} - \pi^A_{l^A l^A}\pi^B_{l^B \beta}}{D} > 0$, because from Eq. (10) we have $\pi^B_{l^B l^A} \geq 0$ when $\alpha < 1/2$ and $l^A \geq l^B$, and also because $\pi^A_{l^A \beta} > 0$, $\pi^B_{l^B \beta} > 0$, and $\pi^A_{l^A l^A} < 0$. Eqs. (11) and (12) are unaltered implying that the ratio of terrorist effort is still independent of β. Thus, $\frac{dl^B}{d\beta} > 0$ implies that $\frac{dl^A}{d\beta} > 0$.

Summarizing the above findings, we note that Proposition 1 is partially altered when $\alpha < 1/2$. The effect of defense on terrorist effort is qualitatively the same. The effect of proactive measures against a specific terror group is also qualitatively unaltered vis-à vis terror effort of the targeted group and the aggregate terrorism effort. However, the cross effect is qualitatively different. Greater proactive effort directed against the larger (smaller) group will now reduce (raise) its rival group's effort, indicating the desirability of going after the larger group. Since the analysis of the two-stage game given in the fourth and fifth sections is independent of the range of permissible α ($0<\alpha<1$), our earlier results hold for $\alpha<1/2$.

Multiple Terror Groups

We next explore the nature of terrorism and counterterrorism policy equilibrium when there are $n \geq 2$ terror groups. For some terrorism-plagued countries (e.g., Iraq, Nepal, India, Pakistan, Sri Lanka, Sudan, and Syria), more than two rival groups have co-existed at times (Hou, Gaibulloev, and Sandler 2020). As in the third section, we again assume that $\alpha>1/2$. For notational simplicity, we denote a terror group as i, where $i=1,2,...,n$. Terrorist group *i*'s objective function is the same as Eq. (1), and the terror production function is the same as Eq. (2a). However, with multiple terror groups, Eq. (2b) is:

$$T=\sum_{i=1}^{n} T^{i}=\beta \sum_{i=1}^{n} l^{i}=\beta L, \tag{28}$$

where $L=\sum_{i=1}^{n} l^{i}$ is the aggregate terrorism effort of the groups. Group *i*'s payoff function is:

$$\pi^{i}=V^{i}-c^{i} l^{i}=\beta^{1-\alpha}\left(l^{i}\right)^{\alpha} L^{1-2\alpha}-c^{i} l^{i} \equiv \pi^{i}\left(l^{1}, l^{2}, ..., l^{n} ; \beta, c^{i}\right). \tag{29}$$

Defining l^{i-} as the vector of all terrorist organizations' effort levels that excludes group *i* (i.e., $l^{i-}=L-l^{i}$, $i=1,2,...,n$), we get the first-order condition of organization *i* as:[8]

$$\pi_{l^{i}}^{i}=\beta^{1-\alpha}\left(l^{i}\right)^{\alpha-1} L^{-2\alpha}\left[(1-\alpha) l^{i}+\alpha l^{i-}\right]-c^{i}=0. \tag{30}$$

The n first-order conditions, contained in Eq. (30) for $i=1,2,...,n$, jointly determine the Nash equilibrium for the terrorist groups' effort levels. Given that the second-order condition for a

[8] We can show that the second-order conditions are satisfied.

terrorist group's optimization requires that $\pi^i_{l^i l^i} < 0$, the slope of group *i*'s reaction function vis-à-vis a change in any other group *j*'s ($j \neq i$) effort level is:

$$\frac{dl^i}{dl^j} = \frac{\pi^i_{l^i l^j}}{\left(-\pi^i_{l^i l^i}\right)} \leq 0 \text{ iff } \pi^i_{l^i l^j} \leq 0 \text{, where } \pi^i_{l^i l^j} = \alpha\beta^{1-\alpha}\left(l^i\right)^{\alpha-1} L^{-2\alpha-1}\left(2\alpha-1\right)\left(l^i - l^{i-}\right), \tag{31}$$

such that $\frac{dl^i}{dl^j} \leq 0$ if and only if $l^i \leq l^{i-}$, where group *i* exerts no more effort than the aggregate of the other groups. When the terrorist organizations face the same input cost $c^1 = c^2 =,\ldots,= c^n = c$, Eq. (30) defines a symmetric Nash equilibrium where $l^1 = l^2 =,\ldots,= l^n = l$. Thus, in this case $l^i = l$, and $l^{i-} = \left(n-1\right)l$. The condition $l^i \leq l^{i-}$ reduces to $l \leq \left(n-1\right)l$ being equivalent to $n \geq 2$. For $n = 2$, the analysis reduces to the base case, such that $\frac{dl^i}{dl^j} = 0$ at the symmetric Nash equilibrium. However, for $n > 2$, we have $l^i < l^{i-}$, such that Eq. (31) implies that $\frac{dl^i}{dl^j} < 0$ at a symmetric Nash equilibrium.[9]

The n first-order conditions in Eq. (30) can be expressed in relative terms by dividing each group's first-order condition with respect to the *n*th group's first-order condition to obtain relative effort levels purely as functions of relative costs independent of β. Thus, defensive countermeasures affect all terrorist groups' effort levels in the same direction. Using this observation and Eq. (30), we can show that, starting from a symmetric equilibrium, an increase in defensive measures reduces all groups' effort levels.

Proactive measures can be shown to reduce a targeted group's effort level, and, under initial symmetry, such targeted measures raise all rival groups' terror effort levels. Accordingly,

[9] Mirroring the third section and using Eq. (30), we can show that the slope of group *i*'s reaction function in terms of the collective effort of other groups (i.e., l^{i-}) is zero at the symmetric Nash equilibrium.

Proposition 1 is unchanged vis-à-vis defensive and group-specific proactive measures. However, proactive measures' influence on the rival, under initial symmetry, is different for $n>2$, with negatively sloped reaction functions suggesting an increase in the rivals' efforts. Aggregate terrorism falls just as in Proposition 1. Based on symmetry, we can substitute $l^i = l$ and $l^{i-} = (n-1)l$ in Eq. (30), and rely on $m^1 = m^2 =,...,= m^n = m$ to obtain:

$$l = \frac{(1-2\alpha+\alpha n)^{\frac{1}{\alpha}} \beta(e)^{\frac{1-\alpha}{\alpha}} c(m)^{-\frac{1}{\alpha}}}{n^2}. \tag{32}$$

By noting that aggregate terror $T = \beta L = n\beta l$, we have the government's payoff function,

$$\Omega = T + e + nm = \frac{(1-2\alpha+\alpha n)^{\frac{1}{\alpha}} \beta(e)^{\frac{1}{\alpha}} c(m)^{-\frac{1}{\alpha}}}{n} + e + nm, \tag{33}$$

from which we can obtain the optimal defensive and proactive levels as in the fifth section. The qualitative nature of the optimal choices remains similar to the $n=2$ case in that earlier section.

Differentiating the counterterrorism policy first-order conditions with respect to the number of groups, we find that an increase in the number of groups will tend to raise the optimal defensive effort while reducing optimal proactive effort levels.[10] This follows because of the public good nature of defensive action for which an enhanced hardening of a given number of targets deters *all* terrorist groups. By contrast, with an increased number of symmetric terrorist groups, optimal policy requires a proportional increase in targeted proactive efforts against all terror groups, so that cost considerations likely induce the government to rely more on defense. The rise in the number of terror groups is apt to raise aggregate terrorism if there are sufficient diminishing returns to defense ($E_\beta > 1$). If diminishing returns is weak (for $E_\beta \le 1$), defensive measures can increase sufficiently to keep aggregate terrorism constant (when $E_\beta = 1$) or reduce

[10] For the purpose of this exercise, we approximate the number of groups, *n*, as a continuous variable.

it (when $E_\beta < 1$). Regardless of whether terrorism rises or falls, the government's loss inclusive of counterterrorism costs must increase as the number of symmetric terror groups increases. Exogenous factors that cause terrorist groups to splinter and multiply is not good from the targeted government's viewpoint. By contrast if outbidding results in the demise of terrorist groups as in Sri Lanka, then this may be a favorable development because the government can resort to proactive measures to eliminate the remaining groups and not have to balance defensive and proactive policies.

Concluding Remarks

The current paper contains a two-stage game in which a targeted government chooses its defensive and proactive responses in the first stage against two rival terrorist groups that then decide their attacks in the second stage. In the base model, the rival groups' cost-constrained choice captures outbidding because they place more weight on their share of attacks relative to total terrorism produced by the two groups. Our theoretical framework provides the first formal analysis of outbidding where the influence of counterterrorism measures on rival terrorist groups' level of terrorism is considered. A number of messages derive from our study. First, the two-stage game is intricate and may contradict simple intuition that outbidding invariably results in more terrorism or that proactive measures should be necessarily directed at the smaller of rival groups. The latter might seem advisable because such measures aimed at the larger of two groups results in increased terrorism by the smaller group; however, the direct decrease in terror coming from targeting the larger group overwhelms the smaller groups' increased terrorism. Second, the effects of outbidding on the level of terrorism cannot ignore government counterterrorism action. Third, in the face of exogenous cost-reducing events that favor rival groups' efficiency, defensive responses by the government are generally favored over proactive measures unless defensive actions experience sufficient diminishing returns. Thus, the government's optimal mix between defensive and proactive measures hinge on how these measures lose their effectiveness through deployment. Fourth, empirical analyses of outbidding between rival groups must possess some counterterrorism measures. Fifth, the necessary structure needed to derive clear-cut results indicate that outbidding is a much more complex relationship than pre-supposed in the literature. Sixth, by potentially decreasing the number of terror groups, outbidding may simplify the choice between defensive and proactive countermeasures while lowering the targeted government's loss.

Appendix

1. *Proof of Proposition 1*

Recalling Eq. (6), we define $l^{iN}\left[\beta(e), c^A\left(m^A\right), c^B\left(m^B\right)\right] \equiv l^i\left(e, m^A, m^B\right),\ i = A, B.$ Using this definition, and Eqs. (14a) through (20), we have:

$$l_e^A\left(e, m^A, m^B\right) = l_\beta^A\left(\beta, c^A, c^B\right)\beta'(e) < 0, \tag{A1}$$

$$l_{m^A}^A\left(e, m^A, m^B\right) = l_{c^A}^A\left(\beta, c^A, c^B\right)c^{A\prime}\left(m^A\right) < 0, \tag{A2}$$

$$l_{m^B}^A\left(e, m^A, m^B\right) = l_{c^B}^A\left(\beta, c^A, c^B\right)c^{B\prime}\left(m^B\right) \le 0, \text{ because } c^A \le c^B. \tag{A3}$$

Similarly, we get $l_e^B = l_\beta^B\left(\beta, c^A, c^B\right)\beta'(e) < 0,\ l_{m^B}^B = l_{c^B}^B\left(\beta, c^A, c^B\right)c^{B\prime}\left(m^B\right) < 0,$ and $l_{m^A}^B = l_{c^A}^B\left(\beta, c^A, c^B\right)c^{A\prime}\left(m^A\right) \ge 0$. Thus, defensive countermeasures reduce each group's terrorism effort. Based on Eq. (2a), $\dfrac{dT^i}{de} = l^i\beta' + \beta l_e^i < 0$ follows. Thus, terror by each group is reduced because of greater defensive countermeasures at a given terrorist effort level and because of a reduction of the group's terrorist effort. Given Eq. (2b), the effect of defensive actions on aggregate terror is:

$$T_e = \beta(e)\left(l_e^A + l_e^B\right) + \beta'(e)\left(l^A + l^B\right) < 0. \tag{A4}$$

Next, we consider proactive effort m^i against group *i*. Since $l_{m^i}^i < 0$, Eq. (2a) yields $T_{m^i}^i = \beta l_{m^i}^i < 0$, so that terrorism by the targeted terrorist group must fall. In view of Eq. (A3), increased proactive measures against group *B* reduces *A*'s effort when $c^A < c^B$, implying that terrorism by both groups fall. If, however, enhanced proactive effort is directed against group *A*, then terror by group *B* increases because $l_{m^A}^B > 0$ when $c^A < c^B$. Finally, we use Eqs. (A2), (A3), and their counterparts for group *B* to yield:

$$T_{m^A} = \beta(e)\left(l^A_{m^A} + l^B_{m^A}\right) = \beta(e)\left(l^A_{c^A} + l^B_{c^A}\right)c^{A\prime}\left(m^A\right) = \left(\frac{dT}{dc^A}\right)c^{A\prime}\left(m^A\right) < 0\,, \tag{A5}$$

given Eq. (17). Similarly, using the subsection on increases in *B*'s cost and the equations above, we have:

$$T_{m^B} = \beta(e)\left(l^A_{m^B} + l^B_{m^B}\right) = \beta(e)\left(l^A_{c^B} + l^B_{c^B}\right)c^{B\prime}\left(m^B\right) = \left(\frac{dT}{dc^B}\right)c^{B\prime}\left(m^B\right) < 0\,. \tag{A6}$$

Eqs. (A4) through (A6) establish that aggregate terrorism must fall when either defensive action or proactive effort against either group is raised. **Q.E.D.**

2. *Proof of Proposition 2*

Based on Eqs. (22b), (22c), (A2), and (A3), and the latter equations' counterparts for rival group *B*, we get:

$$\frac{l^A_{m^B} + l^B_{m^B}}{l^A_{m^A} + l^B_{m^A}} = 1 \Rightarrow \frac{c^{A\prime}\left(m^A\right)}{c^{B\prime}\left(m^B\right)} = \frac{\left(l^A\right)^{\alpha-2}\left[\alpha l^A + (1-\alpha)l^B\right]}{\left(l^B\right)^{\alpha-2}\left[\alpha l^B + (1-\alpha)l^A\right]} = \frac{\alpha + \dfrac{(1-\alpha)}{x}}{x^{1-\alpha}\left[\alpha + (1-\alpha)x\right]}\,,$$

$$\Rightarrow \frac{c^{A\prime}\left(m^A\right)}{c^{B\prime}\left(m^B\right)} < 1 \text{ if } x = \frac{l^A}{l^B} > 1\,. \tag{A7}$$

We assume that the cost function of group i is separable: $c^i\left(m^i\right) = \overline{c}^i + c\left(m^i\right)$, where $\overline{c}^A < \overline{c}^B$, $c\left(m^i = 0\right) = 0$, $c'\left(m^i\right) > 0$, and $c''\left(m^i\right) < 0$. Suppose that optimal $m^A \le m^B$, so that $c^A\left(m^A\right) = \overline{c}^A + c\left(m^A\right) < \overline{c}^B + c\left(m^B\right) = c^B\left(m^B\right)$. With $c^A\left(m^A\right) < c^B\left(m^B\right)$, Eq. (12) implies that $x = \dfrac{l^A}{l^B} > 1$. Thus, from Eq. (A7), we have that $c^{A\prime}\left(m^A\right) < c^{B\prime}\left(m^B\right)$. Based on the separable functional forms, we know that $c^{A\prime}\left(m^A\right) < c^{B\prime}\left(m^B\right) \Rightarrow c'\left(m^A\right) < c'\left(m^B\right)$. Given that $c'' < 0$, this implies that $m^A > m^B$. Thus, assuming $m^A \le m^B$ yields a contradiction. In other words, at an interior optimum $m^A > m^B$, when $c^A < c^B \Leftrightarrow l^A > l^B$. **Q.E.D.**

3. *Proof of Proposition 3*

Differentiating Eqs. (26a) and (26b) and solving with Cramer's rule yield:

$$\frac{de}{d\overline{c}} = \frac{\Omega_{em}\Omega_{m\overline{c}} - \Omega_{mm}\Omega_{e\overline{c}}}{D^*} < 0 \text{ if and only if } c_{m\overline{c}} > \frac{c_{\overline{c}}c_{mm}}{c_m}. \tag{A8}$$

With $c_{\overline{c}} > 0$ and $c_m > 0$, strict concavity $c_{mm} < 0$ implies that $c_{m\overline{c}} \geq 0$ is sufficient for the last inequality in Eq. (A8) to be satisfied such that $\frac{de}{d\overline{c}} < 0$, which means that a symmetric fall in $\overline{c}$ for the two terrorist groups raises optimal defense. For the proactive response, we have:

$$\frac{dm}{d\overline{c}} = \frac{\Omega_{e\overline{c}}\Omega_{em} - \Omega_{m\overline{c}}\Omega_{ee}}{D^*} < 0 \text{ if and only if } \alpha^2\left(E_\beta + \frac{1-\alpha}{\alpha}\right)\left(\frac{1+\alpha}{\alpha} - Z\right) > 1, \tag{A9}$$

where $Z = \frac{cc_{m\overline{c}}}{c_m c_{\overline{c}}}$.

Case 1: $c_{m\overline{c}} > 0 \Rightarrow Z > 0$

A ***necessary*** condition for the inequality in Eq. (A9) to be satisfied is that $Z < \frac{1+\alpha}{\alpha}$, which is equivalent to $1 + \alpha(1-Z) > 0 \Leftrightarrow c_{m\overline{c}} < \frac{(1+\alpha)c_m c_{\overline{c}}}{\alpha c} > 0$. When the latter condition is satisfied, Eq. (A9) indicates that $\frac{dm}{d\overline{c}} < 0$ for $E_\beta > \frac{\alpha + (1-\alpha)Z}{1+\alpha(1-Z)} > 0$. Thus, for a sufficiently large E_β (i.e., sufficiently strong diminishing returns to defense), enhanced proactive measures are warrant when $\overline{c}$ falls.

Case 2: $c_{m\bar{c}} \le 0 \Rightarrow Z \le 0$

In this case, the ***necessary*** condition $\frac{1+\alpha}{\alpha} - Z > 0$ is automatically satisfied. Thus, Eq. (A9) implies that $\frac{dm}{d\bar{c}} < 0$ if $E_\beta > \frac{\alpha + (1-\alpha)Z}{1+\alpha(1-Z)}$. Given that $E_\beta > 0$, this condition is always met if $\alpha + (1-\alpha)Z \le 0$, which means that $|c_{m\bar{c}}| \ge \frac{\alpha c_{\bar{c}} c_m}{(1-\alpha)c}$. If, however, $\alpha + (1-\alpha)Z > 0$, then sufficient diminishing return to defense, $E_\beta > \frac{\alpha + (1-\alpha)Z}{1+\alpha(1-Z)} > 0$, is required for the inequality in Eq. (A9) to hold. Finally, turning to aggregate terrorism and using Eqs. (2b) and (24), we can express total terrorism as:

$$T = 2\beta l = \frac{\beta(e)^{\frac{1}{\alpha}} c(m;\bar{c})^{-\frac{1}{\alpha}}}{2}. \tag{A10}$$

Eq. (26a) can be written as:

$$\beta(e)^{\frac{1}{\alpha}} c(m;\bar{c})^{-\frac{1}{\alpha}} = -\frac{2\alpha\beta}{\beta'}. \tag{A11}$$

Substituting Eq. (A11) into (A10) gives:

$$T = -\frac{\alpha\beta(e)}{\beta'(e)}. \tag{A12}$$

A convenient feature of Eq. (A12) is that, at an interior optimum, aggregate terrorism is only a function of the optimal defense level (proactive responses are also optimally chosen but adjust in the background). Differentiating Eq. (A12), we get:

$$\frac{dT}{d\bar{c}} = \alpha(E_\beta - 1)\frac{de}{d\bar{c}} < 0 \text{ if } E_\beta > 1 \text{ and } \frac{de}{d\bar{c}} < 0. \tag{A13}$$

If the conditions outlined in Eq. (A13) are met, a fall in $\bar{c}$ must raise overall terrorism.

However, in view of Eq. (27c), we cannot rule out the possibility that $E_\beta \leq 1$, and, hence, we cannot deny the possibility that terrorism may be unchanged or fall despite a fall in terrorists' costs. In that event, a strong counterterrorism response to the increased terrorist efficiency neutralizes or reduces terrorism. Consider, for example, an exponential form for the $\beta(e)$ function. In this case, $E_\beta = 1$, the second-order conditions in Eqs. (27a) through (27c) are met, but Eq. (A13) shows that terrorism does not change. However, as we show next, national loss must increase with a fall in $\overline{c}$. This is possible with potentially unchanged aggregate terrorism because aggregate counterterrorism expenses are required to keep terrorism constant. Using Eqs. (24) and (25) and the envelope theorem, we have:

$$\Omega_{\overline{c}}\left(e,m;\overline{c}\right) = 2\beta\left(e\right) l_{\overline{c}}\left(e,m;\overline{c}\right) = -\frac{\beta^{\frac{1}{\alpha}} c^{-\frac{1+\alpha}{\alpha}} c_{\overline{c}}}{2\alpha} < 0. \tag{A14}$$

Eq. (A14) establishes that national welfare loss always increases with a decrease in terrorists' costs, even if terror remains constant or falls due to a strong counterterrorism response. This is possible because in the event that terrorism falls, national welfare loss rises because of the large (albeit optimal) counterterrorism response. **Q.E.D.**

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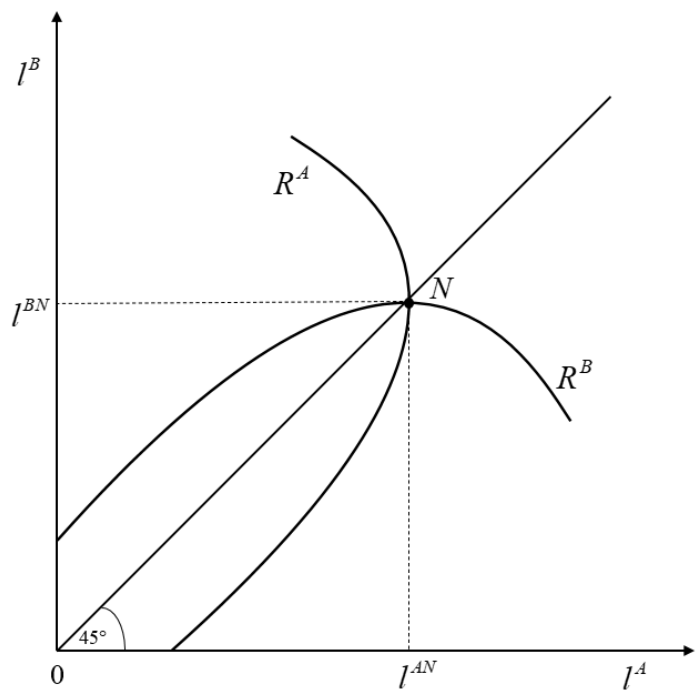

Figure 1. Nash equilibrium for two symmetric terrorist groups and fixed counterterrorism levels

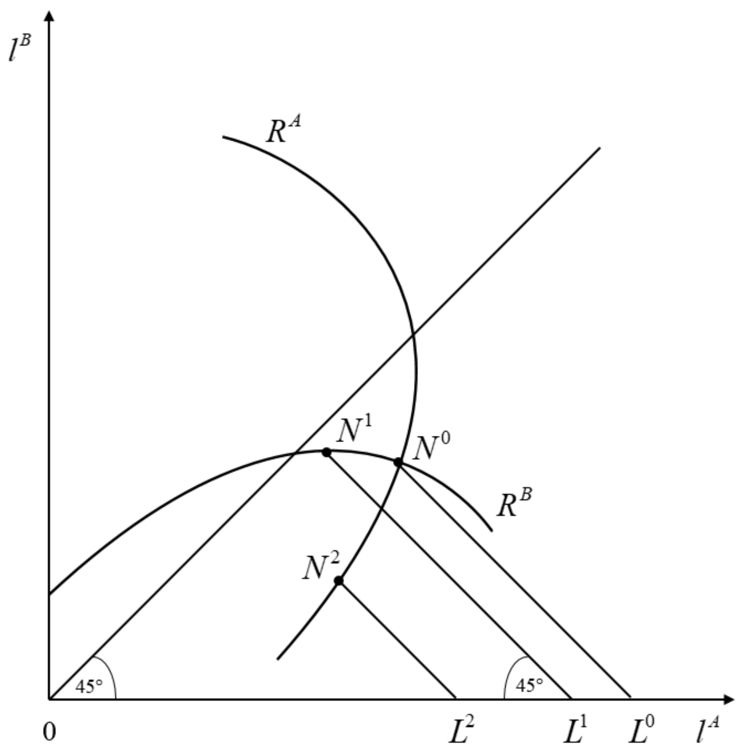

Figure 2. Nash equilibriums for two asymmetric terrorist groups and changing counterterrorism levels

Supplementary Appendix

Effects of Defensive and Proactive Measures on Competition between Terrorist Groups

1. *Deriving Eqs. (4a), (4b), and (30):*

Differentiating Eq. (29) with respect to l^i and factoring out $\left(l^i\right)^{\alpha-1} L^{-2\alpha}$ from the expression for the terrorist's marginal benefit , we get:

$$\beta^{1-\alpha}\left(l^i\right)^{\alpha-1} L^{-2\alpha}\left[\alpha L+\left(1-2\alpha\right)l^i\right]=c^i . \tag{S1}$$

Because $L=l^i+l^{i-}$, we have that $\alpha L+\left(1-2\alpha\right)l^i=\alpha l^i+\alpha l^{i-}+\left(1-2\alpha\right)l^i=\left(1-\alpha\right)l^i+\alpha l^{i-}$.

Using this fact in Eq. (S1), we get Eq. (30). When $i=A,B$, $L=l^A+l^B$, and Eq. (30) reduces to Eqs. (4a) and (4b), for $i=A$ and B, respectively.

2. *Deriving Second-Order Conditions Supporting Eqs. (4a), (4b), and (30):*

Differentiating the expression for $\pi^i_{l^i}$ in Eq. (30) with respect to l^i, we derive:

$$\pi^i_{l^il^i}=\beta^{1-\alpha}\left(l^i\right)^{\alpha-2} L^{-2\alpha-1}\left\{\left(1-\alpha\right)l^iL-2\alpha l^i\left[\left(1-\alpha\right)l^i+\alpha l^{i-}\right]-\left(1-\alpha\right)L\left[\left(1-\alpha\right)l^i+\alpha l^{i-}\right]\right\}. \tag{S2}$$

Since $L=l^i+l^{i-}$, the term in curly brackets can be reduced to:

$-\alpha\left[\left(1-\alpha\right)\left(l^i\right)^2+\left(1-\alpha\right)\left(l^{i-}\right)^2+2\alpha l^il^{i-}\right]$, which when substituted into Eq. (S2) yields

$$\pi^i_{l^il^i}=-\alpha\beta^{1-\alpha}\left(l^i\right)^{\alpha-2} L^{-2\alpha-1}\left[\left(1-\alpha\right)\left(l^i\right)^2+\left(1-\alpha\right)\left(l^{i-}\right)^2+2\alpha l^il^{i-}\right]<0, \tag{S3}$$

supporting the second-order conditions associated with Eqs. (4a), (4b) and (30).

3. *Deriving Eqs. (8), (10), and (31):*

Differentiating the expression for $\pi^i_{l^i}$ in Eq. (30) with respect to l^j, where $j\neq i$, we have:

$$\pi^i_{l^i l^j} = \alpha\beta^{1-\alpha}\left(l^i\right)^{\alpha-1} L^{-2\alpha-1}\left\{L - 2\left[\left(1-\alpha\right)l^i + \alpha l^{i-}\right]\right\}. \tag{S4}$$

Using $L = l^i + l^{i-}$ and simplifying, we have: $L - 2\left[\left(1-\alpha\right)l^i + \alpha l^{i-}\right] = \left(2\alpha - 1\right)\left(l^i - l^{i-}\right)$.

Substituting this equation into Eq. (S4) gives the expression for $\pi^i_{l^i l^j}$ in Eq. (31). Eq. (31) reduces to Eqs. (8) and (10) for $i = A$ and *B*, respectively.

4. *Deriving Eqs. (11) and (12):*

Eq. (11) is obtained by transferring the marginal cost of terrorist group *i* ($i = A, B$) to the right-hand side of Eqs. (4a) and (4b) and equating the marginal benefit ratio of the two groups with their marginal cost ratio. Eq. (11) may be expressed as:

$$\psi\left(x,\lambda\right) = 0\text{, where } \psi = \left(1-\alpha\right)x + \alpha - \lambda x^{1-\alpha}\left(1-\alpha+\alpha x\right). \tag{S5}$$

Eq. (S5) implicitly defines *x* as a function of λ, with the slope given by the implicit function rule as:

$$\frac{dx}{d\lambda} = -\frac{\psi_\lambda}{\psi_x} = \frac{\left(1-\alpha+\alpha x\right)x^{1-\alpha}}{\psi_x} < 0 \text{ iff } \psi_x < 0, \tag{S6}$$

where differentiating the expression for ψ in Eq. (S5) yields

$$\psi_x = 1 - \alpha - \lambda x^{1-\alpha}\left[\alpha + \frac{\left(1-\alpha\right)\left(1-\alpha+\alpha x\right)}{x}\right]. \tag{S7}$$

Based on Eq. (11), we substitute for $\lambda x^{1-\alpha}$ with the expression $\left[\frac{\left(1-\alpha\right)x+\alpha}{1-\alpha+\alpha x}\right]$ in Eq. (S7) and simplify to obtain:

$$\psi_x = \frac{\alpha\left(1-\alpha\right)\left(x-1\right)}{x} - \alpha\left[\frac{\left(1-\alpha\right)x+\alpha}{1-\alpha+\alpha x}\right]. \tag{S8}$$

For all $x \le 1$, Eq. (S8) establishes that $\psi_x < 0$. For $x > 1$, we have to further simplify Eq. (S8) to be able to sign ψ_x. Eq. (S8) can be reduced to:

$$\psi_x = \frac{\alpha}{x(1-\alpha+\alpha x)}\left\{(1-\alpha)\left[(1-\alpha)x(1-x)-(1-\alpha+\alpha x)\right]-\alpha x\right\}. \tag{S9}$$

For all $x > 1$, Eq. (S9) shows that $\psi_x < 0$, which proves that $\psi_x < 0$ for all possible values of *x*. Using Eq. (S6), we conclude that $dx/d\lambda < 0$.

5. Supporting Eq. (17):

Using $i = A$, $i- = B$, and $L = l^A + l^B$ in Eqs. (S2) and (31), we get:

$$\pi^A_{l^A l^A} - \pi^A_{l^A l^B}$$
$$= -\alpha\beta^{1-\alpha}\left(l^A\right)^{\alpha-2} L^{-2\alpha-1}\left\{(1-\alpha)\left[\left(l^A\right)^2+\left(l^B\right)^2\right]+2\alpha l^A l^B+(2\alpha-1)l^A\left(l^A-l^B\right)\right\}. \tag{S10}$$

In Eq. (S10), the expression within curly brackets reduces to:

$$\alpha\left(l^A\right)^2+(1-\alpha)\left(l^B\right)^2+l^A l^B = L\left[\alpha l^A+(1-\alpha)l^B\right]. \tag{S11}$$

Using (S11) in (S10), we have:

$$\pi^A_{l^A l^A} - \pi^A_{l^A l^B} = -\alpha\beta^{1-\alpha}\left(l^A\right)^{\alpha-2} L^{-2\alpha}\left[\alpha l^A+(1-\alpha)l^B\right] < 0. \tag{S12}$$

Switching *A* and *B* in Eq. (S12) and noting that $L = l^A + l^B$, we derive:

$\pi^B_{l^B l^B} - \pi^B_{l^B l^A} = -\alpha\beta^{1-\alpha}\left(l^A+l^B\right)^{-2\alpha}\left(l^B\right)^{\alpha-2}\left[\alpha l^B+(1-\alpha)l^A\right] < 0$, as claimed in Eq. (17).

6. *Deriving Eq. (A7) for Proposition 2:*

The first equation in Eq. (A7) follows immediately from Eqs. (22b) and (22c). Eqs. (A2) and (A3) imply that $\dfrac{l^A_{m^B}+l^B_{m^B}}{l^A_{m^A}+l^B_{m^A}} = \dfrac{\left(l^A_{c^B}+l^B_{c^B}\right)c^{B\prime}\left(m^B\right)}{\left(l^A_{c^A}+l^B_{c^A}\right)c^{A\prime}\left(m^A\right)}$, so that

$$\frac{l^A_{m^B} + l^B_{m^B}}{l^A_{m^A} + l^B_{m^A}} = 1 \Rightarrow \frac{c^{A\prime}\left(m^A\right)}{c^{B\prime}\left(m^B\right)} = \frac{l^A_{c^B} + l^B_{c^B}}{l^A_{c^A} + l^B_{c^A}}. \tag{S13}$$

Using Eqs. (15), (16), (18), (19), and Eq. (S12) along with its counterpart for *B*, we get:

$$\frac{l^A_{c^B} + l^B_{c^B}}{l^A_{c^A} + l^B_{c^A}} = \frac{\pi^A_{l^A l^A} - \pi^A_{l^A l^B}}{\pi^B_{l^B l^B} - \pi^B_{l^B l^A}} = \frac{\left(l^A\right)^{\alpha-2}\left[\alpha l^A + (1-\alpha)l^B\right]}{\left(l^B\right)^{\alpha-2}\left[\alpha l^B + (1-\alpha)l^A\right]}. \tag{S14}$$

Factoring out l^A and l^B from the numerator and the denominator of the last terms within square brackets, respectively, and rearranging, we derive the last equation in Eq. (A7).

7. *Deriving Eqs. (26a), (26b), and (33):*

Multiplying Eq. (32) through by $n\beta$ gives: $n\beta l = \frac{(1-2\alpha+\alpha n)^{\frac{1}{\alpha}}\beta(e)^{\frac{1}{\alpha}}c(m)^{-\frac{1}{\alpha}}}{n}$. Given that $T = n\beta l$, we immediately have Eq. (33). Using $n = 2$, and expressing $c(m)$ as $c(m,\overline{c})$, we have that Eq. (33) reduces to:

$$\Omega(e,m;\overline{c}) = \frac{\beta(e)^{\frac{1}{\alpha}}c(m;\overline{c})^{-\frac{1}{\alpha}}}{2} + e + 2m. \tag{S15}$$

Eqs. (26a) and (26b) follow from partial differentiation of Eq. (S15) with respect to *e* and *m*, respectively.

8. *Deriving Eqs. (27a), (27b), and (27c):*

Routine second-order differentiation of Eq. (S15) and rearrangement of terms yield Eqs. (27a) and (27b). Differentiating Eq. (S15) we get the following second-order cross partial:

$$\Omega_{em} = -\frac{\beta'\beta^{\frac{1}{\alpha}-1}c^{-\frac{1}{\alpha}-1}c_m}{2\alpha^2} > 0. \tag{S16}$$

Notice that $D^* = \Omega_{ee}\Omega_{mm} - \Omega_{em}{}^2 = \Omega_{ee}\Omega_{mm}\left[1-\left(\frac{\Omega_{em}}{\Omega_{ee}}\right)\left(\frac{\Omega_{em}}{\Omega_{mm}}\right)\right]$. From Eqs. (27a) and (27b), we have that $\Omega_{ee}\Omega_{mm} > 0$, so that $D^* > 0$ when $\left(\frac{\Omega_{em}}{\Omega_{ee}}\right)\left(\frac{\Omega_{em}}{\Omega_{mm}}\right) < 1$. Based on Eqs. (S16) and (27a), we can find the expression for $\left(\frac{\Omega_{em}}{\Omega_{ee}}\right)$. Similarly, the expression for $\left(\frac{\Omega_{em}}{\Omega_{mm}}\right)$ can be obtained from Eqs. (S16) and (27b). Cancelling and rearranging the resulting expressions give:

$$\left(\frac{\Omega_{em}}{\Omega_{ee}}\right)\left(\frac{\Omega_{em}}{\Omega_{mm}}\right) = \frac{\left(\beta' c_m\right)^2}{\alpha^2\left[\beta\beta'' + \frac{(1-\alpha)(\beta')^2}{\alpha}\right]\left[\frac{(1+\alpha)(c_m)^2}{\alpha} - cc_{mm}\right]}. \tag{S17}$$

Given that $E_\beta = \frac{\beta\beta''}{(\beta')^2}$ and $E_m = -\frac{cc_{mm}}{(c_m)^2}$, Eq. (S17) can be reduced to:

$$\left(\frac{\Omega_{em}}{\Omega_{ee}}\right)\left(\frac{\Omega_{em}}{\Omega_{mm}}\right) = \frac{1}{\left(\alpha E_\beta + 1 - \alpha\right)\left(\alpha E_m + 1 + \alpha\right)}, \tag{S18}$$

so that $\left(\frac{\Omega_{em}}{\Omega_{ee}}\right)\left(\frac{\Omega_{em}}{\Omega_{mm}}\right) < 1$, when $\frac{1}{\left(\alpha E_\beta + 1 - \alpha\right)\left(\alpha E_m + 1 + \alpha\right)} < 1$. The latter inequality reduces to the last inequality in Eq. (27c).

9. *Deriving Eqs. (A8) and (A9):*

The first equation in Eq. (A8) implies that:

$$\frac{de}{d\bar{c}} < 0 \Leftrightarrow \Omega_{em}\Omega_{m\bar{c}} - \Omega_{mm}\Omega_{e\bar{c}} = \Omega_{mm}\Omega_{e\bar{c}}\left[\left(\frac{\Omega_{em}}{\Omega_{mm}}\right)\left(\frac{\Omega_{m\bar{c}}}{\Omega_{e\bar{c}}}\right) - 1\right] < 0. \tag{S19}$$

Eq. (S15) then implies:

$$\Omega_{e\bar{c}} = -\frac{c^{-\frac{1}{\alpha}-1}\beta^{\frac{1}{\alpha}-1}\beta' c_{\bar{c}}}{2\alpha^2} > 0. \tag{S20}$$

Given that Ω_{mm} and $\Omega_{e\bar{c}}$ are both positive, Eq. (S19) reduces to:

$$\frac{de}{d\bar{c}} < 0 \Leftrightarrow \left(\frac{\Omega_{em}}{\Omega_{mm}}\right)\left(\frac{\Omega_{m\bar{c}}}{\Omega_{e\bar{c}}}\right) < 1. \tag{S21}$$

Using Eq. (S15), we have:

$$\Omega_{m\bar{c}} = -\frac{\beta^{\frac{1}{\alpha}} c^{-\frac{1}{\alpha}-1}}{2\alpha}\left[c_{\bar{c}m} - \frac{(1+\alpha)c_{\bar{c}}c_m}{\alpha c}\right] > 0\,. \tag{S22}$$

Substituting the expressions for Ω_{em}, Ω_{mm}, $\Omega_{e\bar{c}}$, and $\Omega_{m\bar{c}}$ obtained from Eqs. (S16), (27b), (S20), and (S22), respectively, into the second inequality of Eq. (S21), we obtain the last inequality in Eq. (A8) after some simplification. Employing a similar method, we obtain the second inequality of Eq. (A9).