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Why Might Lump-sum Transfers Not Be a Good Idea?

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Abstract

We adopt an analytically tractable Aiyagari-type model to study the distinctive roles of unconditional lump-sum transfers and public debt in reducing consumption inequality due to uninsurable income risk. We show that in the absence of wealth inequality, using lump-sum transfers is not an optimal policy for reducing consumption inequality—because the Ramsey planner opts to rely solely on public debt to mitigate income risk without the need for lump-sum transfers. This result is surprising in light of the popularity of universal basic income advocated by many politicians and scholars.

JEL Classification: C61; E22; E62; H21; H30

Key Words: Lump-sum Transfers; Universal Basic Income; Ramsey Problem; Public Liquidity; Incomplete Markets; Heterogeneous-Agents.

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1 Introduction

Government transfers have played a critical role in reducing consumption inequality and mitigating idiosyncratic income risk in both developed and developing countries. For example, during the COVID-19 pandemic, large parts of the global economy have been locked down and households' income risk has risen sharply. As a response, policy makers have put forward the largest stimulus package in history—globally more than \$10 trillion dollars of fiscal stimulus programs were announced by May 2020, and close to 20% of that was intended for transfer payments. In the United States, it amounts to 2 trillion dollars, a quarter of which is earmarked for transfer payments to households.[1] However, most of these transfer programs are conditional cash payments or subsidies, which are designed to target specific groups with particular consumption needs.

But, maybe due to an insufficient social safety net in many developed countries, universal basic income (UBI) proposals are also gaining in popularity, especially in light of the increasing income inequality in recent decades and the rapid trend of productivity growth caused by automation and artificial intelligence that could significantly reduce the number of available jobs. Such a program would allow every individual to receive a fixed amount of transfer payments in every period regardless of their income levels and consumption needs, as exemplified by the 2020 presidential candidate Andrew Yang's proposal of giving every American citizen \$1,000 per month, both as a universal welfare benefit and as an insurance device to reduce consumption inequality or to mitigate idiosyncratic income risk.

Indeed, support for UBI or unconditional lump-sum transfers has been growing among policy makers globally.[2] For example, there have been multiple referendums and petitions with respect to UBI in developed countries in the past decade.[3] However, despite such popular support of UBI by many politicians and even prominent economists,[4] not much theoretical work has been devoted to justify or scrutinize the validity of such policy proposals.[5] Importantly, even the critics have also based their arguments mainly on intuitive or verbal

[1]See Cassim, Handjiski, Schubert, and Zouaoui (2020)

[2]See, e.g., Hoynes and Rothstein (2019).

[3]A list of UBI petitions, polls and referendums can be found at `https://en.wikipedia.org/wiki/Universal_basic_income` (accessed 2021-02-01).

[4]There is a long list of politicians, scholars, business entrepreneurs and others supporting UBI according to Wikipedia: `https://en.wikipedia.org/wiki/List_of_advocates_of_basic_income` (accessed 2021-02-01).

[5]One exceptional work is by Ghatak and Maniquet (2019), which we discuss in subsection 1.1.

grounds. For example, Acemoglu (2019) recently argues that "*UBI is a flawed idea, not least because it would be prohibitively expensive unless accompanied by deep cuts to the rest of the safety net. In the U.S. (population: 327 million), a UBI of just $1,000 per month would cost around $4 trillion per year, which is close to the entire federal budget in 2018.*"[6] He cautions people that "*One should always be wary of simple solutions to complex problems, and universal basic income is no exception.*"

Although it is well known that a potentially more sensible policy than UBI is a negative income tax or the so-called guaranteed minimum income program (conditional lump-sum transfers), what is less clear is when such programs are inadequate due to private information or prohibitive costs of identification. In contrast, UBI does not suffer from the private information problem and hence may offer an effective alternative for reducing consumption inequality and mitigating income risk, in addition to the standard means of consumption smoothing via buffer stock savings.

In this paper, we offer a simple model of income inequality to shed light on the controversy regarding the effectiveness or validly of UBI. Our model features unemployment risk and three policy tools available to mitigate the risk: (i) a universal lump-sum transfer, (ii) a distortionary labor tax, and (iii) a buffer-stock saving device—government bonds. We ask what the optimal Ramsey allocation would look like in an infinite-horizon Aiyagari-type model starting from time zero.

Although it is becoming generally accepted conventional wisdom that unconditional lump-sum transfers help mitigate income risk and reduce consumption inequality, it is not clear whether in an environment with borrowing constraints and incomplete insurance markets, as in the model of Aiyagari (1994), universal lump-sum transfers can indeed complement and improve the function of buffer-stock savings or individuals' self-insurance positions as a policy tool to mitigate income risk. This awkward situation is mainly due to the intractability of the Aiyagari model.

A recent work by Bhandari, Evans, Golosov, and Sargent (2017) addresses this specific issue in an Aiyagari-type model. They study the optimal quantity of public debt and optimal lump-sum transfers together with distortionary taxes in an environment with enforceable private debt contracts. But the intractability of their model makes it hard to answer our

[6] The study by Hoynes and Rothstein (2019) also shows that the UBI program is extremely expensive. Its cost is about twice as much as all exiting transfer programs combined in the United States.

question in hand regarding the distinctive roles of universal lump-sum transfers and public debt in mitigating consumption inequality due to uninsurable income risk.

Intuitively, UBI can increase welfare through the intratemporal margin by directly reducing income inequality, while public debt works through the intertemporal margin by improving individuals' self-insurance position. But, an intriguing question is how to best finance these complementary government programs? Specifically, when distortionary taxes are available to finance either the lump-sum transfer or the interest payment of public debt, what should a Ramsey allocation look like regarding the optimal quantity of public debt, the size of lump-sum transfers, and the optimal rate of distortionary taxes in order to best address the problem of consumption inequality under income shocks and borrowing constraints?

To answer these questions, we simplify the model of Bhandari, Evans, Golosov, and Sargent (2017) in a particular way so that the model becomes analytically tractable with closed-form solutions. This simplifying approach makes the Ramsey outcome transparent with clear intuition. Although the simplification comes at some costs in terms of generality, we believe that the gains are sufficiently valuable in terms of shedding light on the issue in hand.

Specifically, following the work of Lucas (1990) and Heathcote and Perri (2018), we introduce a special risk-sharing technology that enables family members within the household to pull their wealth by the end of each period such that every member starts the next period with the same level of wealth. This wealth-redistributing technology is simply a modeling strategy to gain analytical tractability by eliminating wealth heterogeneity, but it preserves income/consumption inequality as well as households' precautionary-saving motives.[7] The degenerate wealth distribution makes both the competitive equilibrium and the Ramsey problem analytically tractable and transparent. A potential limitation of our simplified model is that lump-sum transfers and public debt no longer have a wealth-redistribution effect, but the gain is that we can see clearly the distinct roles of lump-sum transfers and public debt in mitigating consumption/income inequality through the two different margins—the intratemporal and intertemporal margin along the entire transitional path.

[7]Assuming quasi-linear utility is an alternative approach to simplify the heterogenous-agents models. For example, see Wen (2009), Challe and Ragot (2011) and Wen (2015). In addition, the study by Lagos and Wright (2005) is also a well-known example in the money-search literature. Moreover, this simplified approach can be generalized further by various ways. See, e.g., the recent work by Challe, Matheron, Ragot, and Rubio-Ramirez (2017), Le Grand and Ragot (2019), and Bilbiie and Ragot (2020).

We find that, as a buffer-stock saving device, government bonds are generally more effective than lump-sum transfers in reducing consumption inequality and mitigating idiosyncratic income risk such that the optimal amount of lump-sum transfers is zero both in the Ramsey steady state and along the transition path. We demonstrate this result through several scenarios. First, we consider a benchmark case where there are no restrictions on the sign and the magnitude of government bonds and transfers—meaning that both government debt and lump-sum transfer payments can be negative. The result shows that the Ramsey planner can achieve the first-best allocation immediately starting in the first period. By amassing a large amount of government bonds to provide full self-insurance (FSI), consumption inequality is completely eliminated in the Ramsey allocation despite unemployment risk. The optimal transfer payment is negative and the optimal labor tax is zero, which means that government bonds are financed entirely by non-distortionary lump-sum taxes. This result shows that universal lump-sum transfers are not a desirable tool for mitigating consumption inequality in the absence of wealth inequality.

In the second scenario, the level of government bonds is fixed at its initial value so that the Ramsey planner cannot alter the quantity of government bonds to improve individuals' self-insurance position or buffer stock savings. In this scenario, the Ramsey outcome depends on the initial level of government bonds. We find that if the initial level of government bonds is high enough to permit FSI, then the Ramsey outcome is identical to that in the benchmark case—which is a first-best allocation. However, if the initial level of government bonds is not high enough, not only is the first-best allocation infeasible to achieve, but the Ramsey planner opts not to provide enough transfer payments to completely eliminate consumption inequality, even though it is feasible to do so. We also find that the lump-sum transfer is not always positive and that its level depends on the initial level of public debt, while the labor tax is always positive in the Ramsey steady state regardless of the sign of the lump-sum transfer. This result further highlights the critical role of public debt in reducing consumption inequality and mitigating income risk in the absence of wealth inequality.

In the third scenario, we shut down the channel of lump-sum transfers/taxes and ask the question about how much public debt the Ramsey planner is willing to supply at the expense of a distortionary labor tax. The answer is surprising: The optimal quantity of government bonds is as high as required for achieving the FSI allocation (or complete consumption equality) even at a high rate of distortionary tax on labor income—which could be very costly for the employed households. Although this surprising result is similar to the finding of Chien and Wen (2019), a crucial difference is that here we do not rely on quasi-linear preferences to achieve model tractability as in Chien and Wen (2019).[8] The intuition is that from the viewpoint of the Ramsey planner, it is always "cheaper" to borrow (by issuing government debt) whenever the risk-free rate is lower than the time discount rate. In other words, the Ramsey planner has a dominating incentive to increase the supply of government bonds until FSI (or complete consumption equality) is achieved. Notice that the gap between the interest rate and the time discount rate is a hallmark feature of Aiyagari-type models, suggesting that our result may capture a general property of infinite-horizon Aiyagari-type models with ex post heterogeneity.

Finally, we consider the fourth scenario where lump-sum transfers cannot be negative, i.e., a lump-sum tax is ruled out. This case is interesting since it places a direct competition between government bonds and lump-sum transfers in terms of their role in reducing consumption inequality and mitigating income risk. Our result indicates that, surprisingly, the option of a lump-sum transfer is never utilized—not only in the Ramsey state steady but also along the entire transition path. Therefore, the Ramsey allocation is exactly the same as that in the third scenario discussed above. Namely, public debt is always viewed by the Ramsey planner as a better tool than lump-sum transfers in improving ex-ante welfare. Opting not to use lump-sum transfers as a complementary tool to reduce consumption inequality is such a striking result in the sense that it contradicts the rationales behind UBI programs suggested by their advocates. In other words, our results provide a theoretical support for the critics of UBI such as Acemoglu (2019).[9]

In short, we find that the Ramsey planner prefers using public debt rather than lump-sum transfers to smooth individual consumption against idiosyncratic income risk in an environment with borrowing constraints and incomplete insurance markets. Specifically, in the presence of only consumption and income inequalities due to idiosyncratic risks (such as an unemployment risk), the optimal fiscal policy is to increase the supply of government bonds until every individual is satiated with buffer-stock precautionary savings such that they

[8]In a separate paper, Chien and Wen (2020b) show that this full self-insurance Ramsey outcome is time-inconsistent.

[9]We conjecture that adding wealth inequality into the model would not change our result. The intuition is that a lump-sum transfer can buffer income inequality more than wealth inequality because the former is a flow and the later is a stock variable. If our conjecture is correct, then using universal lump-sum transfer payments is unlikely to be a superior policy compared to issuing more government debt.

are no longer borrowing-constrained. When a lump-sum tax is not possible, it is optimal to tax labor to finance the public debt and have the needs of individuals' precautionary savings fully met without relying on lump-sum transfers at all. Therefore, if UBI is to have a role to play in mitigating consumption inequality in the presence of public debt, either a wealth inequality or ex ante heterogeneity, which are missing in our model setup, might be critical. Another possible justification for UBI is a constraint on the government's capacity to issue debt. We leave these important issues for future research.

1.1 Related Literature

In a static model without idiosyncratic income uncertainty and buffer-stock savings, Ghatak and Maniquet (2019) theoretically discuss the conditions that may justify UBI in several scenarios. They argue that UBI could be beneficial as a fiscal tool to alleviate poverty or to obtain social justice in developing countries with essential consumption goods. Our work focuses instead on the effectiveness of UBI as a fiscal tool in a dynamic Aiyagari-type economy to alleviate consumption inequality in the presence of idiosyncratic income risk, buffer-stock savings, and borrowing constraints. One of the questions we seek to answer is how UBI interact with the provision of public liquidity when taxes are potentially distortionary to household labor supply decisions.

Aiyagari and McGrattan (1998) show that government bonds can play an important role in providing self-insurance for households to help relax borrowing constraints. However, their analysis is based on the steady-state welfare instead of the dynamic path of expected welfare, as we base our approach in this paper. Following a similar approach, Flodn (2001) found that a lump-sum transfer is more effective than debt for improving steady-state welfare in an Aiyagari-type economy. The reason the author offers is that physical capital is crowded out by public debt and hence aggregate output is reduced. This crowding-out effect makes government debt inferior to lump-sum transfers. However, the critical differences between Floden's work and ours are the following. Like Aiyagari and McGrattan (1998), Flodn (2001) focuses on the steady-state welfare, whereas we take into account the entire dynamic path of expected future welfare at time zero. It is well documented in the existing literature that optimal policies may look dramatically different between steady-state welfare analysis and dynamic welfare analysis because the former ignores the transition path of the Ramsey allocation. For example, Domeij and Heathcote (2004) show that a capital tax could decrease steady-state welfare but improve welfare during the transition; Rohrs and Winter (2017) find that taking transitional dynamics into consideration has important implications for government debt policy; and a significant short-run macroeconomic effect of an income tax cut is also demonstrated by Heathcote (2005). More recently, Chien and Wen (2020b) show that, because of time-inconsistent Ramsey plans in Aiyagari-type models, the optimal tax and debt policies are doomed to be different between steady-state welfare analysis and dynamic welfare analysis.

Our paper also relates to the strand of literature studying conditional lump-sum transfers. Bilbiie, Monacelli, and Perotti (2013) study the macroeconomic effects of two types of fiscal policies: One considers an intratemporal redistribution, which allows an agent-specific transfer financed by lump-sum taxes; the other considers a lump-sum tax cut financed by public debt, which features intertemporal redistribution. Their results show that both polices are expansionary in a sticky-price environment. Oh and Reis (2012) also find that a conditional lump-sum transfer is expansionary if prices are sticky. Our paper complements these works by using a dynamic Ramsey approach with a particular focus on the optimality of UBI and its interaction with an optimal supply of public debt.

Our framework is in the class of models pioneered by Aiyagari (1994) in which infinitely lived agents are ex ante identical but ex post heterogenous. Our results thus may not apply to models with ex ante heterogeneity, such as the models of Judd (1985) and Werning (2007) or the overlapping generation models. For example, in a two-period ex-ante heterogeneous-agents model, Azzimonti and Yared (2017) show that the Ramsey planner optimally limits the supply of bonds such that not all households are slack in their borrowing constraints—because the Ramsey planner can borrow more cheaply and hence can relax its budget constraints by keeping some of the agents constrained. Given that the setup of such models is dramatically different from the typical Aiyagari-type models such as ours, our results suggest that more future work needs to be done to close the issue.

The rest of the paper is organized as follows. Section 2 sets up the model and defines the competitive equilibrium. Section 3 characterizes the Ramsey outcomes under different scenarios. Section 4 confirms our theoretical findings and demonstrates the transition path of the Ramsey outcome through an numerical exercise. Section 5 concludes.

2 The Model

2.1 General Environment

Time is discrete and indexed by $t = 0, 1, 2, ..., \infty$. There are three types of agents in our model economy: firms, the government, and households.

Firm. A representative firm produces output Y_t according to a linear production technology, $Y_t = N_t$, where N_t is aggregate employment. The firm hires labor from households by paying a competitive wage rate w_t. The firm's optimal condition for profit maximization in time t satisfies $w_t = 1$.

Government. In each period t, the government can issue one-period risk-free bonds B_{t+1}, collect taxes or give transfers T_t in a lump-sum fashion, and levy a flat-rate time-varying labor tax τ_t. The flow government budget constraint in time t is given by

$$\tau_t w_t N_t + \frac{B_{t+1}}{R_{t+1}} \geq B_t + T_t, \text{ for all } t \geq 0, \tag{1}$$

where R_{t+1} is the risk-free gross interest rate between time t and $t+1$, and the initial level of government bonds B_0 is exogenously given. For simplicity, we assume there is no government consumption.

Household. There is a representative household consisting of a unit measure of *ex ante* identical members (consumers), à la Lucas (1990). These individual members face idiosyncratic employment shocks in every period denoted by $\theta_t \in \{e, u\}$. The shocks are identically and independently distributed (iid) over time and across all individuals. If $\theta_t = e$, an individual can work and receive labor income; otherwise, $\theta_t = u$ and an individual is unemployed with no labor income. Let $\pi(e)$ and $\pi(u)$ denote the probabilities of employment and unemployment, respectively.

The head of the household maximizes the intertemporal welfare of all family members using a utilitarian welfare criterion (all members are equally weighted), but faces some limits to the degree of risk sharing that it can provide to family members. Specifically, there are two subperiods within each period. In the first subperiod, the members are separated from each other to work on different islands and they each receive an employment-status shock θ_t after separation. Based on the shock θ, each individual makes a consumption decision, saving decision, and labor supply decision (if employed). In the second subperiod, individuals are reunited with their family and share their asset holdings. Therefore, the idiosyncratic shocks are not fully insurable despite the family reunion, even though each individual will have the same amount of initial assets in the beginning of the next period. Hence, given that the idiosyncratic income risk is uninsurable, there are still precautionary motives to save so as to smooth the family members' consumption over time. However, the wealth distribution across family members is completely degenerate since the individuals can share their asset holdings in the end of each period.

This setup divides the entire population within the household into two groups: employed with measure $\pi(e)$ and unemployed with measure $\pi(u)$. Individuals in each group are identical since their initial wealth is the same. This setup is similar to that in Heathcote and Perri (2018), and it makes both the competitive equilibrium and the Ramsey problem analytically tractable.

The aggregate lifetime utility of the household is then given by

$$U = \sum_{t=0}^{\infty} \beta^t \left\{ \left[u(c_t^e) - v\left(n_t^e\right) \right] \pi(e) + u(c_t^u)\pi(u) \right\}, \tag{2}$$

where $\beta \in (0,1)$ is the discount factor; c_t^e and c_t^u denote consumption for the employed and unemployed individuals in time t; and n_t^e denotes the labor supply of employed individuals (note the labor supply is zero for unemployed individuals). By the law of large numbers, $\pi(e)$ represents the employed population and $\pi(u)$ the unemployed population.

The representative household's flow budget constraint is given by

$$c_t^e \pi(e) + c_t^u \pi(u) + \frac{a_{t+1}}{R_{t+1}} \leq \widehat{w}_t n_t^e \pi(e) + a_t + T_t, \tag{3}$$

where $\widehat{w}_t = (1 - \tau_t) w_t$ is the after-tax wage rate, a_{t+1} is the asset holdings determined in time t, T_t is a universal lump-sum transfer identical across all family members, and the initial asset holding a_0 is given and assumed to be the same across individuals. In addition, the budget constraint for the employed individuals is given by

$$c_t^e \leq a_t + \widehat{w}_t n_t^e + T_t, \tag{4}$$

and that for the unemployed individuals is given by

$$c_t^u \leq a_t + T_t. \tag{5}$$

2.2 Household Problem

The representative household chooses a sequence of $\{c_t^e, c_t^u, n_t^e\}_{t=0}^{\infty}$ to maximize (2) subject to (3), (4) and (5). Let $\beta^t \lambda_t^e \pi(e)$, $\beta^t \lambda_t^u \pi(u)$ and $\beta^t \mu_t$ denote the Lagrangian multipliers attached to (4), (5) and (3), respectively. The FOCs of the representative household with respect to c_t^e, c_t^u, n_t^e and a_{t+1},are given, respectively, by

$$u_{c,t}^e = \lambda_t^e + \mu_t \tag{6}$$

$$u_{c,t}^u = \lambda_t^u + \mu_t \tag{7}$$

$$v_{n,t}^e = \widehat{w}_t(\lambda_t^e + \mu_t) = (1 - \tau_t) u_{c,t}^e, \tag{8}$$

$$\beta(\lambda_{t+1}^e \pi(e) + \lambda_{t+1}^u \pi(e)) + \beta \mu_{t+1} = \frac{\mu_t}{R_{t+1}} \tag{9}$$

2.3 Competitive Equilibrium

Definition 1. *Given a_0 and B_0, a competitive equilibrium is defined as a sequence of government policies (including bonds, tax rates and lump-sum taxes/transfers) $\{B_{t+1}, \tau_t, T_t\}_{t=0}^{\infty}$, a sequence of prices $\{w_t, R_t\}_{t=0}^{\infty}$, a sequence of aggregate allocations $\{C_t, N_t\}_{t=0}^{\infty}$, and a sequence of individual allocation plans $\{c_t^e, c_t^u, n_t^e, a_{t+1}\}_{t=0}^{\infty}$ such that*

1. *Given prices, $\{c_t^e, c_t^u, n_t^e, a_{t+1}\}_{t=0}^{\infty}$ solves the household problem.*

2. *Given prices, $\{N_t\}_{t=0}^{\infty}$ solves the firm problem.*

3. *The government budget constraint in equation (1) holds for $t \geq 0$.*

4. *All markets clear for $t \geq 0$:*

$$\begin{aligned} B_{t+1} &= a_{t+1} && (10) \\ N_t &= n_t^e \pi(e), \\ N_t &= C_t = c_t^e \pi(e) + c_t^u \pi(u) && (11) \end{aligned}$$

2.4 Characterization of Competitive Equilibrium

Proposition 1. *The competitive equilibrium has the following two properties:*

1. $\lambda_t^e = 0$ *and* $c_t^e \geq c_t^u$ *for all* $t \geq 0$. *In addition,*

$$c_t^u = \begin{cases} c_t^e & \text{if } a_t + T_t \geq c_t^e, \\ a_t + T_t & \text{if } a_t + T_t < c_t^e. \end{cases} \tag{12}$$

2. *Equation (9) can be rewritten as*

$$\frac{1}{R_{t+1}} = \beta \left(\frac{u_{c,t+1}^e}{u_{c,t}^e} \pi(e) + \frac{u_{c,t+1}^u}{u_{c,t}^e} \pi(u) \right). \tag{13}$$

Proof. Please refer to Appendix A.1. □

Proposition 1 implies that if the amount of cash on hand (asset holdings plus transfers $a_t + T_t$) is sufficiently large such that the optimal consumption of an employed family member satisfies $c_t^e \leq a_t + T_t$, then all family members can obtain the same level of consumption regardless of their (un)employment status. We refer to this outcome as an FSI allocation where consumption is equalized across all family members.

Obviously, this outcome can be achieved potentially by either a sufficiently large amount of lump-sum transfers or government bonds (note $a_t = B_t$ in equilibrium). In addition, if the FSI allocation is achieved, then the risk-free rate R must be equal to the time discount rate $1/\beta$. Otherwise, we must have consumption inequality $c^u < c^e$ with $R < 1/\beta$. Notice that $R < 1/\beta$ is a hallmark feature of Aiyagari-type models, but its critical role in incentivising the Ramsey planner to amass a sufficiently large amount of government bonds to achieve the FSI allocation in a Ramsey equilibrium is seldom clearly pointed out until the recent works of Chien and Wen (2019, 2020a). We will show that it is also precisely this property in our model that makes lump-sum transfers inferior to public debt as an insurance device for reducing consumption risk.

2.5 Conditions to Support Competitive Equilibrium

Given that government policies are inside the state space of the competitive equilibrium and affect the endogenous distribution of consumption, the Ramsey problem is to pick a competitive equilibrium (through policies) that attains the maximum of the expected household lifetime utility U defined in (2). To ensure that a Ramsey plan constitutes a competitive equilibrium, however, we must show first that all possible allocations in the choice set of the Ramsey planner, $\{c_t^e, n_t^e, c_t^u, B_{t+1}, T_t\}_{t=0}^{\infty}$ (after substituting out other variables by the FOCs of the household and the firm, as well as the government budget constraint), constitute a competitive equilibrium. The following proposition states the conditions that any constructed Ramsey allocation must satisfy in order to constitute a competitive equilibrium.

Proposition 2. *Given $a_0 = B_0 > 0$, the sequence of allocations $\{c_t^e, n_t^e, c_t^u, B_{t+1}, T_t\}_{t=0}^{\infty}$ can be supported as a competitive equilibrium if and only if they satisfy the following conditions:*

1. *The resource constraint holds for all $t \geq 0$:*

$$n_t^e \pi(e) - c_t^e \pi(e) - c_t^u \pi(u) \geq 0. \tag{14}$$

2. *The implementability condition holds for all $t \geq 0$:*

$$u_{c,t}^e c_t^e \pi(e) + u_{c,t}^e c_t^u \pi(u) + \beta \left(u_{c,t+1}^e \pi(e) + u_{c,t+1}^u \pi(u)\right) B_{t+1} - v_{n,t}^e n_t^e \pi(e) - u_{c,t}^e (B_t + T_t) = 0, \tag{15}$$

where

$$c_t^u = \begin{cases} c_t^e & \text{if } B_t + T_t \geq c_t^e, \\ B_t + T_t & \text{if } B_t + T_t < c_t^e. \end{cases} \tag{16}$$

Proof. Please refer to Appendix A.2. □

3 Ramsey Outcome

To facilitate our analysis, we make the following assumptions on the utility function's relative risk aversion parameters $\{\gamma_c, \gamma_n\}$:

$$\gamma_c \equiv \frac{u_{cc} c}{u_c} < 0,$$
$$\gamma_n \equiv \frac{v_{nn} n}{v_n} > 0,$$

so the utility function exhibits constant relative risk aversion (CRRA).

Definition 2. *The first-best allocation is defined as the optimal allocation chosen by a social planner that maximizes the welfare function (2) subject only to the aggregate resource constraint (14).*

It is straightforward to verify that the first-best allocation features constant consumption and labor supply over time. In addition, consumption is completely equalized across family members and employment status, namely $c^e = c^u \equiv c^{FB}$, where c^{FB} denotes consumption in the first-best allocation.

Definition 3. *A Ramsey steady state is a long-run Ramsey allocation where aggregate and individual variables $\{N_t, C_t, B_{t+1}, c_t^e, c_t^u, n_t^e\}$ all converge to finitely positive non-zero constants.*

3.1 Ramsey Problem

Armed with Proposition 2, the Ramsey problem is given by

$$\max_{\{c_t^e, n_t^e, B_{t+1}, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \left[u(c_t^e) - v\left(n_t^e\right) \right] \pi(e) + u(c_t^u) \pi(u) \right\}$$

subject to conditions (14), (15) and (16). Condition (16) can be rewritten as

$$c_t^u = c_t^e (1 - \mathbf{1}_t) + (B_t + T_t) \mathbf{1}_t,$$

where $\mathbf{1}_t$ is an indictor function with $\mathbf{1}_t = 1$ if $c_t^e > B_t + T_t$ and $\mathbf{1}_t = 0$ otherwise.

3.2 Scenario 1: Benchmark

Recall that c^{FB} is the consumption level in the first-best allocation where individual consumption is equalized across all employment status. We first show that in scenario 1 the optimal Ramsey allocation is the first-best allocation for all periods $t \geq 0$.

Proposition 3. *Given any initial value of $B_0 > 0$, the Ramsey outcome achieves the first-best allocation where $c_t^e = c_t^u = c^{FB}$ for all $t \geq 0$. This allocation can be implemented by the following policy mix:*

1. *The distortionary labor tax $\tau_t = 0$ for all $t \geq 0$.*

2. *The government debt* $B_{t+1} = c^{FB}/\beta$ *for all* $t \geq 0$.

3. *The lump-sum transfer is* $T_0 = c^{FB} - B_0$ *in period* 0 *and is strictly negative for* $t \geq 1$*:* $T_t = c^{FB}(1 - \frac{1}{\beta}) < 0$.

Proof. Please refer to Appendix A.3. □

This Ramsey outcome suggests that it is not optimal to use lump-sum transfers to eliminate consumption inequality after $t = 1$; instead, the government should issue plenty of public debt to achieve an FSI allocation in which consumption is equalized across family members regardless of their employment status. In contrast, a lump-sum tax (negative transfer) should be used to finance the interest payments of government debt (except the initial period when bond supply is fixed) such that the distortionary labor tax is always zero for all periods $t \geq 0$.

3.3 Scenario 2: Ramsey Outcome with Fixed $B_t = B_0$

We next consider the case where the government cannot adjust its bond position such that future bond supply is determined by $B_{t+1} = B_0$ for all $t \geq 0$. In this scenario, the properties of the Ramsey allocation are characterized by the following Proposition:

Proposition 4. *Depending on the value of* B_0*, the Ramsey outcome features the following properties:*

1. *If* $B_0 \geq c^{FB}/\beta$*, then the Ramsey allocation achieves the first-best allocation in period* 0*. The distortionary labor tax* $\tau_t = 0$ *for all* $t \geq 0$*. The debt interest payment is financed solely by lump-sum taxes:* $T_t = (\beta - 1)B_0 < 0$ *for all* $t \geq 0$.

2. *If* $B_0 = 0$*, then the Ramsey allocation converges to its steady state immediately in period 0 and does not feature FSI. In addition, labor tax* $\tau_t = \tau > 0$ *and lump-sum transfers* $T_t = T > 0$*, for all* $t \geq 0$.

3. *If* $0 < B_0 < c^{FB}/\beta$*, then the Ramsey steady state does not feature FSI. Namely,* $c^e > c^u$ *and* $R < \beta^{-1}$*. In addition, the steady-state optimal labor tax* $\tau > 0$*. Optimal lump-sum transfers can be either positive or negative, depending on* B_0*: It is negative if* B_0 *is high such as being close to* c^{FB}/β*, and positive if* B_0 *is low such as being close to zero. Note that an FSI allocation is feasible but not optimal for the Ramsey planner.*

Proof. Please refer to Appendix A.4. □

Proposition 4 indicates that the Ramsey allocation has no transition path at all if the initial debt is either high enough or zero. The former case permits a first-best Ramsey allocation, which is exactly the same allocation as in Scenario 1.

In the latter case ($B_0 = 0$), the Ramsey planner has no ability to adjust the equilibrium allocation intertemporally without any government bonds. This is so because the Ramsey planner's problem becomes totally static if $B_t = 0$ for all t. Hence, the Ramsey planner can only adjust the intratemporal margin to mitigate consumption inequality by providing lump-sum transfers, which are financed by a distortionary labor tax. The trade-off facing the Ramsey planner in the case of $B_0 = 0$ is between consumption inequality and lower aggregate output due to a distortion of labor supply within a period.

In the case where $0 < B_0 < c^{FB}/\beta$, although the Ramsey planner cannot change the public debt position, it is still possible to adjust the intertemporal margin through changing the intertemporal price—the risk free interest rate R_t, which depends on the aggregate demand of bonds. The Ramsey planner can adjust the demand for bonds by affecting the distribution of consumption via lump-sum transfers.

In addition, the Ramsey planner does not use lump-sum transfers to completely eliminate consumption inequality even though it is feasible to do so—when the universal transfer payment is sufficiently high, individuals' borrowing constraints will no longer bind. This suggests that when public debt is restricted or is absent, the Ramsey planner must weight the benefit of FSI and the cost of financing it through a distortionary tax. This trade-off is different from that under flexible public debt because a lump-sum transfer smooths consumption through the intratemporal margin by directly reducing income inequality, while public debt smooths consumption through an intertemporal margin by improving individuals' self-insurance position. In the latter case, as we will show clearly in the next Scenario, from the view point of the Ramsey planner, the marginal benefit of issuing debt always dominates the present value of future marginal cost since the interest rate always lies below the time discount rate (a hallmark-feature of Aiyagari-type models), thus incentivising the Ramsey planner to keep increasing public debt until $R = 1/\beta$ and FSI is achieved. Hence, the critical difference between the intratemporal margin and the intertemporal margin is key to understand the distinct role of lump-sum transfers and public debt in improving individuals' self insurance position.[10]

3.4 Scenario 3: Ramsey Outcome with $T_t = 0$

Instead of keeping B_t constant, the third scenario restricts $T_t = 0$ for all t and allows the supply of government bonds to be flexible. In this scenario, we can see how the Ramsey planner opts to use government bonds to completely eliminate consumption inequality without the option of lump-sum taxes/transfers. It further illustrates the importance of the intertemporal margin in achieving FSI.

Proposition 5. *The long-run Ramsey outcome features the following properties:*

1. *FSI is achieved with complete consumption equality: $c^e = c^u$, and the equilibrium interest rate is equal to the time discount rate, $R = 1/\beta$.*

2. *The optimal level of government bonds is lower than that required by the first-best allocation B^{FB}. The steady-state labor income tax is strictly positive and given by $\tau = 1 - \beta$.*

3. *This Ramsey steady state is unique if $(1 + \gamma_c) \leq 0$; or alternatively, there does not exist any Ramsey steady state with inadequate self insurance such that $c^e > c^u$ and $r < \beta^{-1}$ if the CRRA parameter satisfies $(1 + \gamma_c) \leq 0$.*

Proof. Please refer to Appendix A.5. □

In this scenario the FSI allocation is optimal but not a first-best outcome, because a distortionary tax must be levied to finance public debt when a lump-sum tax is not available. This Ramsey outcome is similar to that derived in Chien and Wen (2019). But, we derive this result without relying on quasi-linear preferences as in Chien and Wen (2019). However, the intuition is the same as that provided by Chien and Wen (2019), which is that the Ramsey planner has a dominating incentive to increase the supply of government bonds when the risk-free rate is lower than the time discount rate, until FSI is achieved. More specifically,

[10] As shown recently by Chien and Wen (2020b), the Ramsey outcome based on the planner's exploitation of the intertemporal margin is not time-consistent. If the Ramsey planner had the opportunity to change its policies in the Ramsey steady state, it would do so. Since this issue is beyond the scope of this paper, it is not pursued here.

unlike individuals facing earnings risk, the Ramsey planner faces no uncertainty in allocating aggregate resources over time. Given that the planner discounts the future social welfare by β, which is lower than the inverse of the market risk-free rate R^{-1}; this implies an "arbitrage opportunity" for the planner to increase public liquidity to improve individuals' self insurance position and to front-load aggregate consumption during the transition periods. This incentive never vanishes until FSI is archived in the long run through accumulating enough public debt. This also implies that without taking into account the transitional dynamics, as in the case of maximizing only the steady state welfare, a FSI may not be optimal because the asymmetric discounting between R and β is no longer a concern in the competitive steady state (see Flodn (2001) and Chien and Wen (2020b)).

3.5 Scenario 4: Ramsey Outcome with $T_t \geq 0$

Now we rule out the possibility of a lump-sum tax by imposing the constraint $T_t \geq 0$ and allowing only the possibility of a lump-sum transfer. Recall that our benchmark scenario shows that the Ramsey planner opts to set $T_t < 0$ (using a lump-sum tax instead of a transfer) to achieve the first-best allocation and eliminate consumption inequality. Intuitively, once we add the non-negative constraint on T_t, it must be binding in a Ramsey equilibrium; otherwise the Ramsey planner would have used lump-sum transfers to help achieve the first-best allocation as in scenario 1. But a binding constraint $T_t = 0$ in the Ramsey allocation implies that as long as bonds are available it is never optimal to use lump-sum transfers to reduce consumption inequality even though it is feasible to do so. This is exactly what we show in the following Proposition:

Proposition 6. *Under the constraint $T_t \geq 0$, the Ramsey opts to set $T_t = 0$ for all $t \geq 0$, and the Ramsey outcome is identical to that in Scenario 3 where $T_t = 0$. Hence, the Ramsey planner opts to depend solely on public debt to achieve consumption equality or FSI (in the long run) without relying on lump-sum transfers in any period. Because distortionary labor taxes are used to finance the interest payments of public debt, the Ramsey outcome is not a first-best allocation.*

Proof. Please refer to Appendix A.6. □

This result indicates that, somewhat surprisingly, the option of a lump-sum transfer is never utilized by the Ramsey planner not only in the Ramsey state steady but also during the whole transition path. Therefore, the Ramsey allocation is exactly identical to that in Scenario 3. That is, the Ramsey planner always views universal lump-sum transfers as an inferior tool compared to public debt for improving ex-ante welfare in our model economy. Opting not to use lump-sum transfers as a tool to reduce consumption inequality is such a striking and maybe counter-intuitive result in light of the popularity of UBI among politicians and some economists. Consequently, our results lend a theoretical support to the critical views of Acemoglu (2019) against UBI.

However, since our result is based on a simple tractable model without wealth inequality, it may not hold when wealth-redistribution effects are introduced in the model. But we conjecture that adding wealth inequality into the model may not change our results. The intuition is that a lump-sum transfer is more effective in reducing income inequality than reducing wealth inequality because the former is a flow and the later is a stock variable. Nonetheless, to the best of our knowledge, the possibility of relying on wealth inequality to justify UBI has never been investigated by the existing literature and should thus be an important research topic in the future.

Another possibility driving our result is that FSI is possible in our model. What if it is impossible to achieve FSI because either the optimal level of public debt is infinity or there exists a debt limit on government bonds? Scenario 2 is helpful in this regard. Based on Proposition 4, if the desired level of public debt is too low, then it may be optimal to use lump-sum transfers to improve individuals' self insurance positions on top of government bonds. However, even in this case it may still be possible for the Ramsey planner to set $T_t < 0$ in order to finance the interest costs of public debt and reduce the burden on labor tax. So the answer hinges on how far away the debt limit constraint is from the optimal debt level.

4 Transition Dynamics

In what follows, we confirm our theoretical findings by numerical simulations of the model, which not only substantiate our theoretical results but also illustrate the pattern of optimal transition paths of the Ramsey allocation. Such numerical analyses are valid because the Ramsey steady state has been proved to exist under our parameter specifications.

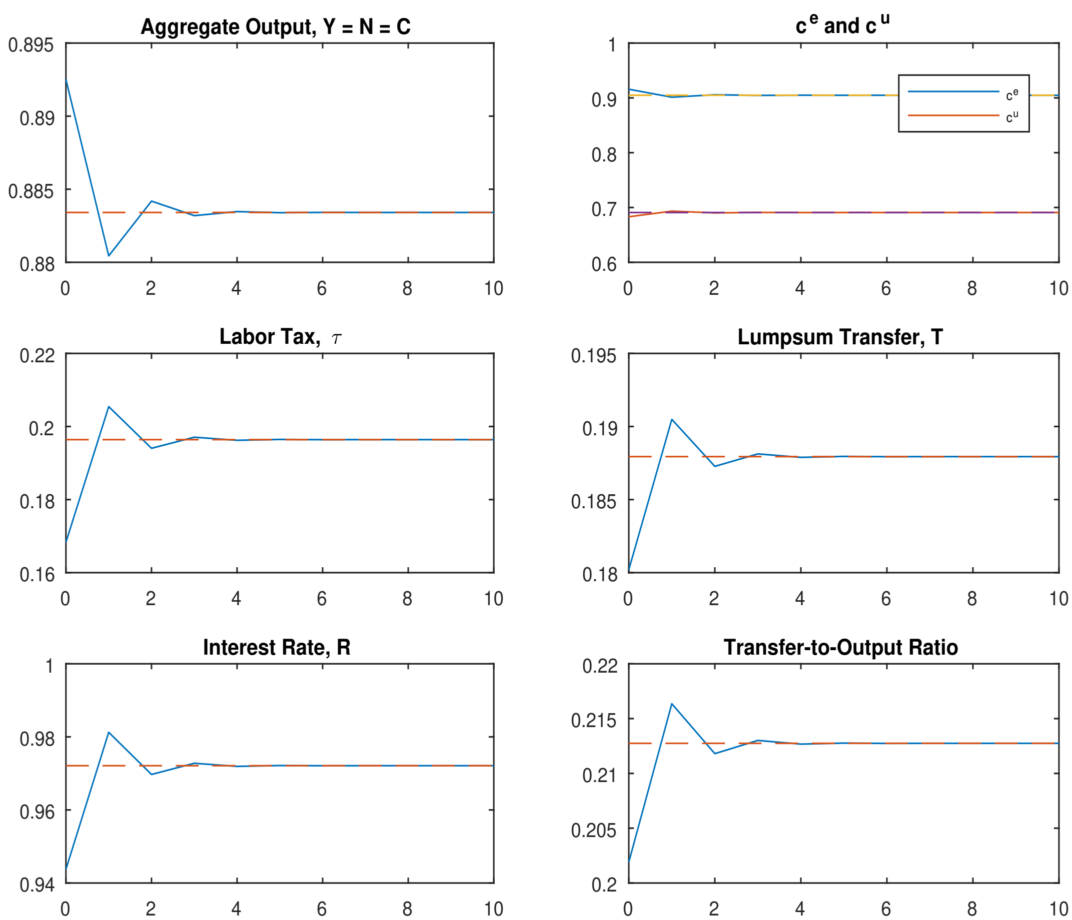
Parameter Values. First, we assume a power utility function with $u(c) = \frac{1}{1-\sigma}c^{1-\sigma}$ and $v(n) = \frac{1}{\gamma}n^{\gamma}$, where $\gamma = \sigma = 2$. Second, we set $\beta = 0.96$, and the probability distribution of being employment and unemployment is given by $\pi(e) = 0.9$ and $\pi(u) = 0.1$, respectively. The initial debt level, B_0, is set to be 50% of the optimal debt level of Scenario 1 (c^{FB}/β).

4.1 Ramsey Transition Path with Fixed $B_t = B_0$ (Scenario 2)

We first show the Ramsey transition path of Scenario 2, where B_t is fixed at its initial level. Recall that Proposition 4 shows there is no transition path at all if $B_t = 0$ for all t. This is because with zero public debt position, the Ramsey planner loses its ability to adjust the interest rate intertemporally, so the Ramsey problem becomes a static problem. However, with a fixed non-zero public debt $B_0 > 0$, the Ramsey planner can adjust the allocation intertemporally by manipulating the interest rate R_t through asset demand. As a result, there is a Ramsey transition path as shown in Figure 2.

First, the figure shows that all variables exhibit a non-monotonic zigzag convergence path toward their steady-state values. This dampening oscillating pattern of transition could be the consequence of the planner's limited ability to adjust the risk-free rate when bond supply is fixed. Second, the transition paths of all variables do not deviate greatly from their steady-state values (dashed lines) in comparison to the transition paths in Scenario 3 (shown in the next subsection). This fact also reflects the Ramsey planner's limited desire to adjust the intertemporal margin through only the asset-demand side. Third, there is a short-run consumption front-loading supported by higher work efforts (panel [1,1]) during the transition period. The interest rate R lies below the time discount rate of $1/\beta = 1.0417$ for all period (panel [3,1]). In addition, the lump-sum transfer is positive during the transition (panel [2,2]) and is financed by a distortionary labor tax (panel [2,1]). The major trade-off here is between reducing consumption inequality through a lump-sum transfer and minimizing the output wedge caused by a distortionary labor tax; however, consumption inequality is never completely eliminated in the long run even though it is feasible to do so by further increasing the amount of universal lump-sum transfers.

Figure 1: Ramsey Transition Paths of Scenario 2



Notes: Transition paths in the Ramsey economy (solid lines) and their corresponding steady states (dashed lines).

4.2 Ramsey Transition Path with $T_t = 0$ (Scenario 3)

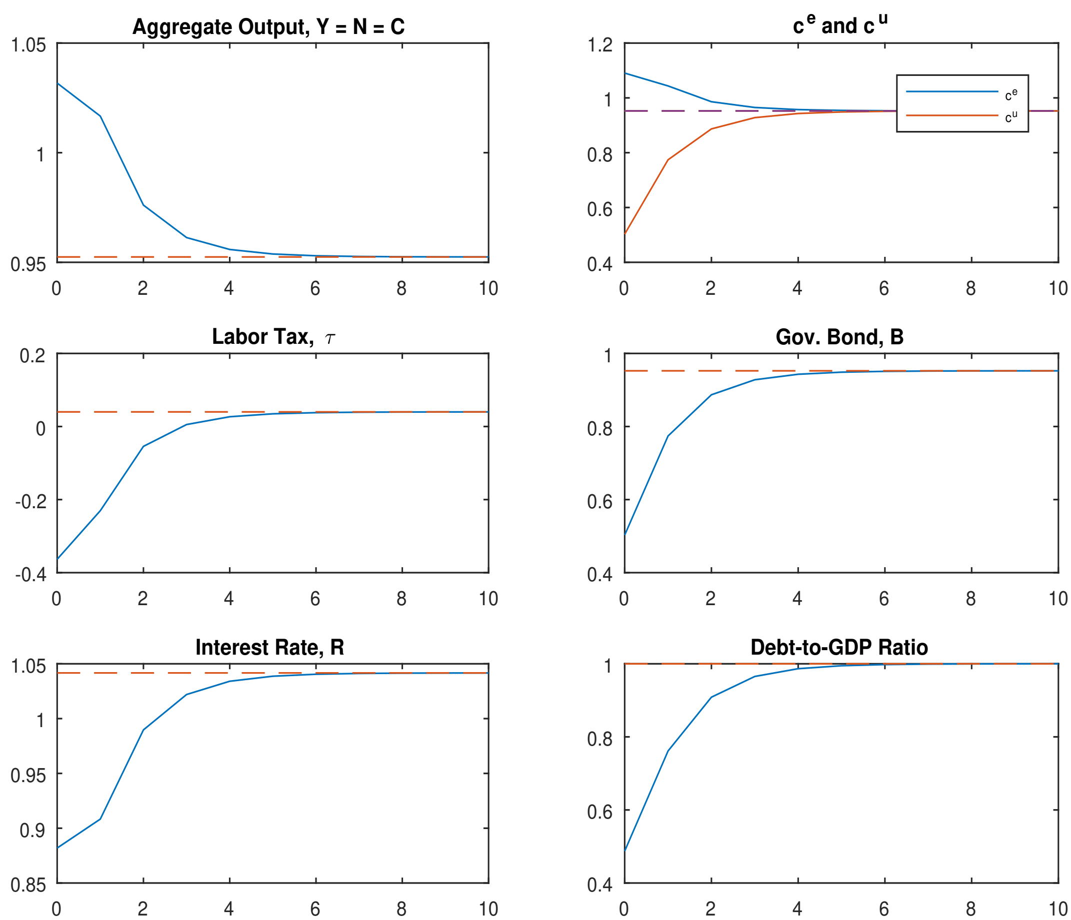
We now consider the case of Scenario 3, where neither a lump-sum tax nor a lump-sun transfer is available to the Ramsey planner, but bond supply is flexible. Figure 2 plots the transition path of the Ramsey outcome. First, the optimal debt level (panel [2,2]) and debt-to-GDP ratio (panel [3,2]) increase gradually over time so as to boost household savings and improve households' self-insurance position. As the self-insurance position improves, consumption inequality (panel [1,2]) shrinks over time. In addition, the interest rate (panel [3,1]) approaches the time discount rate from below as the borrowing constraints are relaxed over time. The paths of aggregate consumption and aggregate labor (note $C_t = N_t$ in panel [1,1]) show a clear pattern of front-loading: they both peak in the first period and then decrease back to their respective long-run steady state. The higher labor efforts are supported by a negative labor tax rate in the short run before turning positive in the longer run (panel [2,1]).

The increasing level of public debt clearly requires financing from tax revenues. The Ramsey planner opts to put the pressure of revenue collection on issuing new debt in the short run (since a lump-sum tax is unavailable) and gradually shifting the financing burden to labor taxes in the longer run. This also explains why the labor tax rate is significantly negative in the short run, so as to incentivize hard-working behavior, and gradually rises to a steady-state value of 4% in the long run (panel [2,1]).

5 Conclusion

We design a tractable heterogeneous-agents model with incomplete insurance markets to show analytically that unconditional lump-sum transfers (or UBI) may not be a good idea. The reason is similar to the permanent income hypothesis that people rationally choose to smooth consumption and buffer income shocks by precautionary savings. Given that the root cause of consumption inequality in Aiyagari-type models is the lack of sufficient self-insurance (or liquidity) to fully diversify income risk under borrowing constraints, lump-sum transfers or public debt can serve to improve welfare by providing liquidity and relaxing individuals' borrowing constraints. However, lump-sum transfers provide liquidity through the intratemporal margin, while public debt provides liquidity through the intertemporal

Figure 2: Ramsey Transition Paths of Scenario 3



Notes: Transition paths in the Ramsey economy (solid lines) and their corresponding steady states (dashed lines).

margin. Along either margin the cost of financing is identically a distortionary tax. But the intertemporal margin has an advantage over the intratemporal margin. This is because the interest cost of debt financing is cheaper when the risk-free rate of government bonds lies below the time discount rate, which is a hallmark feature of Aiyagari-type models. We show that the Ramsey planner cleverly exploits this "arbitrage" opportunity and opts to use bonds instead of lump-sum transfers to relax borrowing constraints, regardless of whether or not the lump-sum tax/transfer is feasible. If a lump-sum tax is feasible, then the Ramsey outcome is the first-best by setting labor tax to zero; if not, then it is the second-best (i.e., achieving FSI) by levying a distortionary labor tax to finance public debt. But a lump-sum transfer can achieve neither the first-best nor the second-best outcome. Hence, only when public debt is unavailable or restricted in supply can a lump-sum transfer be used to provide liquidity, but not to the degree of achieving FSI due to its diminishing benefit in front of the increasing cost of a distortionary labor tax. There is no such tradeoff in using public debt because the marginal benefit of relaxing borrowing constraints through issuing additional debt dominates the marginal cost of future taxes when the interest rate lies below the time discount rate (unless FSI is already achieved).

Of course, if lump-sum transfers can be made fully contingent on individual income status, then there is little doubt that such conditional or targeted lump-sum transfers can achieve the first-best allocation without relying on public debt. But, the curse of private information may render fully targeted or conditional lump-sum transfers infeasible or inadequate to support the first-best allocation in the real world, thus creating the role for public debt.

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A Appendix

A.1 Proof of Proposition 1

We divide the discussion into two cases: (1) $B_{t+1} > 0$, and (2) $B_{t+1} = 0$, for $t > 0$.

1. $B_{t+1} > 0$. Based on the nature of inter-period budget constraints, it must be $c_t^e \geq c_t^u$. Suppose $\lambda_t^e > 0$. Then (4) must bind. Substituting $c_t^e = a_t + \widehat{w}_t n_t^e + T_t$ into (3) gives $c_t^u = a_t + T_t - \frac{a_{t+1}}{R_{t+1}\pi(u)} < a_t + T_t$, since at the equilibrium $a_{t+1} = B_{t+1} > 0$. Therefore, $\lambda_t^u = 0$, which implies $u_{c,t}^e > u_{c,t}^u$ and therefore $c_t^e < c_t^u$. It leads to contradiction such that λ_t^e must be zero.

   Now consider two subcases: (a) $c_t^u = a_t + T_t$ and (b) $c_t^u < a_t + T_t$. In subcase (a), $\lambda_t^u \geq 0$ and hence $c_t^e \geq c_t^u = a_t + T_t$. For subcase (b), $\lambda_t^u = 0$ and hence $c_t^e = c_t^u < a_t + T_t$ according to FOCs (6) and (6). Therefore, c_t^u can be expressed as a function of c_t^e and $a_t + T_t$ as shown in equation (12).

2. $B_{t+1} = 0$. With $B_{t+1} = a_{t+1} = 0$, rewrite (3) as

   $$c_t^e \pi(e) + c_t^u \pi(u) = \widehat{w}_t n_t^e \pi(e) + a_t + T_t \geq \widehat{w}_t n_t^e \pi(e) + c_t^u$$

   where the inequality is due to $c_t^u \leq a_t + T_t$. Rearranging it leads to

   $$c_t^e - c_t^u \geq \widehat{w}_t n_t^e,$$

   which implies $c_t^e > c_t^u$. It also means that (5) must bind. Inserting $c_t^u = a_t + T_t$ into (3) gives

   $$\widehat{w}_t n_t^e \pi(e) + a_t + T_t = c_t^e \pi(e) + c_t^u \pi(u) = c_t^e \pi(e) + (a_t + T_t)\, \pi\,(u)\,,$$

   which gives

   $$\widehat{w}_t n_t^e + a_t + T_t = c_t^e.$$

   As a result, both (3) and $c_t^u = a_t + T_t$ guarantees that (4) is automatically satisfied. Consider another household problem where individuals are subject to (3) and $c_t^u =$

$a_t + T_t$. The constraint set is identical to that in the original problem. As such, we have $\lambda_t^e = 0$ as long as (3) and (5) are taken into account.

Finally, given $\lambda_t^e = 0$, the FOC (9) together with FOCs (6) and (6) can be rewritten as equation (13).

A.2 Proof of Proposition 2

The "If" Part: Given the initial B_0 and the allocation $\{c_t^e, n_t^e, c_t^u, B_{t+1}, T_t\}_{t=0}^{\infty}$, a competitive equilibrium can be constructed by using the two conditions in Proposition 2 and following the steps below that uniquely back up the sequences of the other variables:

1. a_{t+1} is chosen such that $a_{t+1} = B_{t+1}$ to clear the asset market.

2. Given c_t^e, n_t^e and $a_{t+1} = B_{t+1}$, c_t^u is chosen such that

$$c_t^u = \begin{cases} c_t^e \text{ if } c_t^e \le B_t + T_t \\ B_t + T_t \text{ if } c_t^e > B_t + T_t \end{cases}.$$

   Note that the choice of c_t^u ensures that the FOCs (6) and (7) are satisfied. In addition, equation (5) is also satisfied.

3. Aggregate C_t and N_t are chosen such that

$$\begin{aligned} N_t &= n_t^e \pi(e), \\ C_t &= c_t^e \pi(e) + c_t^u \pi(u). \end{aligned}$$

4. $w_t = 1$ so that the firm's problem is solved. τ_t is chosen such that $\widehat{w}_t = (1-\tau_t)w_t$ satisfies $\frac{v_{n,t}^e}{u_{c,t}^e} = (1-\tau_t)$

5. R_{t+1} is chosen by the household Euler equation

$$\frac{1}{R_{t+1}} = \beta \frac{u_{c,t+1}^e \pi(e) + u_{c,t+1}^u \pi(u)}{u_{c,t}^e}.$$

6. There are two conditions left. They are the resource constraint and representative household's budget constraint, which are listed in Proposition 2. The implementability

condition (derived from the representative household's budget constraint with holding equality) can be expressed as (15) by replacing τ_t, R_{t+1} and a_{t+1} by the conditions provided in above steps.

The "Only If" Part: The constraints listed in Proposition 2 are trivially satisfied because they are part of the competitive-equilibrium conditions.

A.3 Proof of Proposition 3

Let $\beta^t \mu_t$ and $\beta^t \lambda_t$ be the multipliers associated with conditions (14) and (15), respectively. The FOCs with respect to c_0^e and $\{c_t^e\}_{t=1}^{\infty}$ are given, respectively, by

$$\begin{aligned}
& u_{c,0}^e \pi(e) + u_{c,0}^u \pi(u)(1-\mathbf{1}_0) - \mu_0 \pi(e) - \mu_0 (1-\mathbf{1}_0)\pi(u) \\
& +\lambda_0 (u_{c,0}^e + u_{cc,0}^e c_0^e)\pi(e) + \lambda_0 (u_{c,0}^e (1-\mathbf{1}_0) + u_{cc,0}^e c_0^u)\pi(u) \\
& -\lambda_0 u_{cc,t}^e (B_0 + T_0) = 0,
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
& u_{c,t}^e \pi(e) + u_{c,t}^u \pi(u)(1-\mathbf{1}_t) - \mu_t \pi(e) - \mu_t (1-\mathbf{1}_t)\pi(u) \\
& +\lambda_t (u_{c,t}^e + u_{cc,t}^e c_t^e)\pi(e) + \lambda_t (u_{c,t}^e (1-\mathbf{1}_t) + u_{cc,t}^e c_t^u)\pi(u) \\
& -\lambda_t u_{cc,t}^e (B_t + T_t) + \lambda_{t-1}(u_{cc,t}^e \pi(e) + u_{cc,t}^u \pi(u)(1-\mathbf{1}_t))B_t \\
= \; & 0 \text{ for } t \geq 1.
\end{aligned} \tag{18}$$

The FOC with respect to B_{t+1}, for all $t \geq 0$, is given by

$$\begin{aligned}
& u_{c,t+1}^u \pi(u)\mathbf{1}_{t+1} - \mu_{t+1}\pi(u)\mathbf{1}_{t+1} \\
& +\lambda_t \left[(u_{c,t+1}^e \pi(e) + u_{c,t+1}^u \pi(u)) + B_{t+1} u_{cc,t+1}^u \pi(u)\mathbf{1}_{t+1} \right] \\
& -\lambda_{t+1} u_{c,t+1}^e + \lambda_{t+1} u_{c,t+1}^e \pi(u)\mathbf{1}_{t+1} = 0.
\end{aligned} \tag{19}$$

The FOCs with respect to T_t are

$$u_{c,t}^u \pi(u)\mathbf{1}_t - \mu_t \pi(u)\mathbf{1}_t + \lambda_t u_{c,t}^e \pi(u)\mathbf{1}_t - \lambda_t u_{c,t}^e + \lambda_{t-1} u_{cc,t}^u \pi(u) B_t \mathbf{1}_t = 0, \; t \geq 1 \tag{20}$$

and

$$u_{c,0}^{u}\pi(u)\mathbf{1}_{0}^{u} - \mu_{t}\pi(u)\mathbf{1}_{0}^{u} + \lambda_{0}u_{c,0}^{e}\pi(u)\mathbf{1}_{0}^{u} - \lambda_{0}u_{c,0}^{e} = 0. \tag{21}$$

The FOC with respect to n_t^e, for all $t \geq 0$, is

$$v_{n,t}^{e}\left(1 + \lambda_{t}\left(1 + \gamma_{n,t}\right)\right) = \mu_{t}. \tag{22}$$

Combining FOCs (19) and (20) implies that $\lambda_t = 0$ for all $t \geq 0$. Then, the Ramsey problem is effectively only subject to resource constraint and becomes the social planner problem. The Ramsey outcome is therefore the first-best allocation. Namely, $c_t^e = c_t^u = c^{FB}$, $R_{t+1} = 1/\beta$ for all t. In addition, let $\tau_t = 0$ and $a_{t+1} = B_{t+1} = c^{FB}/\beta$ for all $t \geq 0$. We further verify this allocation is indeed feasible to the Ramsey planner by the following steps. First, T_t is chosen such that the flow government budget constraint is satisfied. Namely, $T_0 = c^{FB} - B_0$ and $T_{t+1} = c^{FB}(1 - \frac{1}{\beta}) < 0$.

Second, this allocation satisfies household budget constraints for all period t. The first-best allocation satisfies the resource constraint by definition.

Finally, it satisfies both employed and unemployed inter-period budget constraints (4) and (5) since $c_t^e = c_t^u = B_t + T_t$ for all t. It is straightforward to verify that the household FOCs are satisfied.

A.4 Proof of Proposition 4

A.4.1 First Best Allocation if $B_0 \geq c^{FB}/\beta$

To see this, we conjecture and verify that the constraint $c_t^u \leq B_0 + T_t$ is slack resulting from the B_0 that is sufficiently high. Suppose the constraint $c_t^u \leq B_0 + T_t$ is slack for all t. That is, $\mathbf{1}_t = 0$ for all t. Then FOC with respect to T_t gives $\lambda_t = 0$ for all t. Effectively, the Ramsey problem is only subject to resource constraints and becomes the social planner problem. The Ramsey outcome is therefore the first-best allocation.

In this case, $\lambda_t = 0$ for all t. It is straightforward to verify that $c_t^e = c_t^u = c^{FB}$, $Q_{t+1} = \beta$, and $\tau_t = 0$ for all t. The government BC pins down the T_t, which is negative:

$$T_t = (\beta - 1)B_0 < 0.$$

In addition, the minimum amount of B_0 that attain first best allocation can be solved by

$$B_0^{FB} = c^{FB} - T = c - (\beta - 1)B_0^{FB} = c^{FB}/\beta.$$

Moreover, it is straightforward to verify that the constraint (16) is satisfied.

A.4.2 Ramsey Allocation if $0 < B_0 < c^{FB}/\beta$

From equation (16), we know that if $c_t^u < B_0 + T_t$ for any t, then $c_t^u = c_t^e$ and $\mathbf{1}_t = 0$. From the Ramsey FOCs with respect to T_t (equations (20) and (21)), $\mathbf{1}_t = 0$ implies $\lambda_t = 0$. Suppose $\lambda = 0$ in the Ramsey steady state, then it must be the first best allocation according to the Ramsey FOCs. However, to implement the first best allocation, B_0 is no less than c^{FB}/β, which leads to a contradiction.

Now we show that it has to be the case that $c_t^e > c_t^u$ if $\mathbf{1}_t = 1$ for all t. In this case, the FOC with respect to c_t^e are simplified as ($B_{t+1} = B_0$ and $\mathbf{1}_t = 1$)

$$u_{c,t}^e \pi(e) + \lambda_t (u_{c,t}^e + u_{cc,t}^e c_t^e)\pi(e) - \lambda_t u_{cc,t}^e c_t^u \pi(e) + \lambda_{t-1} u_{cc,t}^e B_0 \pi(e) = \mu_t \pi(e) \text{ for } t \geq 1, \quad (23)$$

and

$$u_{c,0}^e + \lambda_0 (u_{c,0}^e + u_{cc,0}^e c_0^e) - \lambda_0 u_{cc,0}^e c_0^u = \mu_0.$$

The FOC with respect to T_t is reduced into ($c_t^u = B_0 + T_t$ and $\mathbf{1}_t = 1$)

$$u_{c,t}^u \pi(u) - \lambda_t u_{c,t}^e \pi(e) + \lambda_{t-1} u_{cc,t}^u B_0 \pi(u) = \mu_t \pi(u) \text{ for } t \geq 1 \quad (24)$$

and

$$u_{c,0}^e \pi(u) - \lambda_0 u_{c,0}^e \pi(e) = \mu_0 \pi(u).$$

We then show that the Ramsey outcome does not feature FSI. To see this, the difference of FOC (23) and (24) gives

$$u_{c,t}^e + \lambda_t (u_{c,t}^e + u_{cc,t}^e c_t^e) - \lambda_t u_{cc,t}^e c_t^u + \lambda_{t-1} u_{cc,t}^e B_0 = u_{c,t}^u - \lambda_t u_{c,t}^e \frac{\pi(e)}{\pi(u)} + \lambda_{t-1} u_{cc,t}^u B_0,$$

which can be rewritten as

$$0 = u^u_{c,t} - u^e_{c,t} + \lambda_t u^e_{cc,t}(c^u_t - c^e_t) - \lambda_t u^e_{c,t}\left(\frac{\pi(e)}{\pi(u)} + 1\right) + \lambda_{t-1}B_0(u^u_{cc,t} - u^e_{cc,t}).$$

Now suppose $c^e_t = c^u_t$; then the equation above becomes

$$0 = -\lambda_t u^e_{c,t}\left(\frac{\pi(e)}{\pi(u)} + 1\right) < 0,$$

which is impossible unless $\lambda_t = 0$. Hence, the Ramsey outcomes must feature $c^e_t > c^u_t$ for all t. Since this is true for all t, then it must also be true in the Ramsey steady state.

We then show that the steady-state labor tax rate is strictly positive. According to (22), in steady state, λ_t and μ_t either both converge or move in the same direction. From (24), we know $\lambda_t = \lambda_{t-1}$ in steady state; otherwise, μ_t would turn negative. Combining (22) and (23), together with $\lambda_t = \lambda_{t-1}$, leads to

$$\begin{aligned} v^e_n\left(1 + \lambda\left(1 + \gamma_n\right)\right) &= u^e_{c,t} + \lambda(u^e_{c,t} + u^e_{cc,t}c^e_t) - \lambda u^e_{cc,t}c^u_t + \lambda u^e_{cc,t}B_0 \\ &= u^e_c\left(1 + \lambda(1 + \gamma_c) - \lambda\gamma_c\frac{T_t}{c^e_t}\right). \end{aligned}$$

From household's FOC, we know $1 - \tau = \frac{v^e_n}{u^e_c}$ such that

$$1 - \tau = \frac{1 + \lambda(1 + \gamma_c) - \lambda\gamma_c\frac{T_t}{c^e_t}}{1 + \lambda\left(1 + \gamma_n\right)}.$$

Consider two cases: (1) $T < 0$, and (2) $T > 0$.

1. $T < 0$. Given $\lambda > 0$, we have

$$\begin{aligned} &\left(1 + \lambda(1 + \gamma_c) - \lambda\gamma_c\frac{T_t}{c^e_t}\right) - \left(1 + \lambda\left(1 + \gamma_n\right)\right) \\ = \;& \lambda\left(\gamma_c - \gamma_n - \gamma_c\frac{T_t}{c^e_t}\right), \end{aligned}$$

   which is negative since $\gamma_c - \gamma_n < 0$ and $-\gamma_c\frac{T_t}{c^e_t} < 0$. It implies $\tau > 0$.

2. $T \geq 0$. From the government budget constraint, we know in steady state $\tau N = B\left(1 - \frac{1}{R}\right) + T > 0$.

As a result, τ must be greater than zero. Note that the Ramsey planner imposes a distortionary tax on labor even in the case of $T < 0$.

Finally, we prove that a steady-state FSI allocation ($c^e = c^u$) is indeed feasible to the Ramsey planner by a specific level of T, which is denoted by T^{FSI}. Consider an FSI allocation where $c^e = c^u = B_0 + T^{FSI}$, which clearly satisfies equation (16). Under this allocation, the resource constraint and implementability condition in the steady state become

$$n^e \pi(e) = c^e,$$

and

$$\beta u_c^e B_0 = v_{n,t}^e n^e \pi(e),$$

respectively. Combining the two equations above leads to

$$\beta u_c^e B_0 = v_{n,t}^e c^e,$$

which is a single equation depending on one variable T and hence can be used to solve T^{FSI}. In addition, the above equation together with (8) gives

$$\beta B_0 = \frac{v_{n,t}^e}{u_c^e} c^e = (1 - \tau) c^e, \tag{25}$$

which pins down the optimal labor tax rate $\tau = 1 - \frac{\beta B_0}{B_0 + T^{FSI}}$. Hence, this FSI allocation is feasible to the Ramsey planner since it satisfies all the constraints listed in Proposition 2 given $B_t = B_0$ for all t.

A.4.3 Ramsey Allocation if $B_0 = 0$

First, the Ramsey planner problem features no dynamics if $B_0 = 0$. Hence, the Ramsey outcome reaches it steady state in period zero. Second, following the proof in the above subsection, $c^e = c^u = c^{FB}$ is impossible given $B_0 < c^{FB}/\beta$. Hence, the Ramsey outcome reaches it steady state at period zero and must features $c_t^e > c_t^u$. Finally, in this case, according to the government budget constraint, it must be the case that $\tau_t > 0$ since $T_t = c_t^u > 0$ and $B_0 = 0$.

A.5 Proof of Proposition 5

The FOC with respect to labor supply is given by equation (22), which implies that the growth rates of μ_t and λ_t in the steady state are the same. In what follows, we show that there exists a Ramsey steady state exhibiting FSI and there is no Ramsey steady state with $R < 1/\beta$. We discuss three cases respectively below.

A.5.1 A. Proof of Existence

We show that there is a Ramsey steady state featuring FSI. Suppose in steady state, $\mathbf{1}_t = 0$ and $c^e = c^u = B$. In this case, the steady state versions of FOCs with respect to B_t, and c_t^e are simplified, respectively, as

$$-\lambda_t + \lambda_{t-1} = 0 \tag{26}$$

and

$$u_c^e + \lambda_t(u_c^e + u_{cc}^e c^e) - \lambda_t u_{cc}^e B + \lambda_{t-1} u_{cc}^e B = \mu_t, \tag{27}$$

which imply both λ_t and μ_t converge to a constant in the Ramsey steady state.

Finally, we solve for the optimal long-run tax rate $\tau > 0$ by the following steps:

1. $Q = \beta$ by equation (13).

2. The λ, μ, c^e and n^e can be solved by four equations (14), (15), (22) and (27) in steady state:

$$u_c^e + \lambda(u_c^e + u_{cc}^e c^e) = \mu,$$

$$v_n^e \left(1 + \lambda \left(1 + \gamma_n\right)\right) = \mu,$$

$$n^e = c^e / \pi(e),$$

$$\beta c^e - \frac{v_n^e n^e \pi(e)}{u_c^e} = 0.$$

3. From the equations above, we can characterize the tax and bond policy in the steady state.

   (a) The tax rate $\tau = 1 - \beta$. From the household's FOC, we know $1 - \tau = \frac{v_n^e}{u_c^e}$, which together with the last two equations above gives

$$1-\tau=\frac{v_n^e}{u_c^e}=\beta.$$

(b) The optimal level of debt B is less than B^{FB}. The B^{FB} is given by: $B^{FB}=\frac{1}{\beta}c^{FB}$. In this case, $c^e=c^u=B$ such that

$$B=c^e<\frac{1}{\beta}c^e.$$

Since $c^e<c^{FB}$, we reach the conclusion that $B<B^{FB}$.

A.5.2 B. Proof of Uniqueness

We prove that the above Ramsey steady state is unique if $(1+\gamma_c)\leq 0$. We show by contradiction that a steady-state allocation with $c^e>c^u$ is not possible if $(1+\gamma_c)\leq 0$. Suppose $c^e>c^u=B$ and hence $\mathbf{1}_t=1$. After imposing the steady-state condition, the FOCs with respect to c_t^e and B_{t+1} become

$$u_c^e+\lambda_t\left[u_c^e+u_{cc}^e c^e-u_{cc}^e c^u\right]+\lambda_{t-1}Bu_{cc}^e=\mu_t$$

$$u_c^u-\lambda_t u_c^e\frac{\pi(e)}{\pi(u)}+\lambda_{t-1}\left(u_c^e\frac{\pi(e)}{\pi(u)}+u_c^u\right)+\lambda_{t-1}Bu_{cc}^u=\mu_t.$$

Rewrite the two equations above as

$$1+\lambda_t\left[1+\gamma_c-\gamma_c\frac{c^u}{c^e}\right]+\lambda_{t-1}\gamma_c\frac{c^u}{c^e}=\frac{\mu_t}{u_c^e},$$

$$1-\lambda_t\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}+\lambda_{t-1}\left(\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}+1\right)+\lambda_{t-1}\gamma_c=\frac{\mu_t}{u_c^u}.$$

We can further rearrange them into

$$\frac{\mu_t}{u_c^e}=1-\gamma_c\frac{c^u}{c^e}\left(\lambda_t-\lambda_{t-1}\right)+\lambda_t\left(1+\gamma_c\right), \tag{28}$$

$$
\begin{aligned}
\frac{\mu_t}{u_c^u} &= 1-\lambda_t \frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}+\lambda_{t-1}\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}+\lambda_{t-1}+\lambda_{t-1}\gamma_c \\
&= 1-\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}\left(\lambda_t-\lambda_{t-1}\right)+\lambda_{t-1}\left(1+\gamma_c\right). \qquad (29)
\end{aligned}
$$

If $c^e > c^u$, then $\frac{\mu_t}{u_c^e} > \frac{\mu_t}{u_c^u}$ such that

$$
\frac{\mu_t}{u_c^e}-\frac{\mu_t}{u_c^u}=\left(\lambda_t-\lambda_{t-1}\right)\left(1+\gamma_c-\gamma_c\frac{c^u}{c^e}+\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}\right)>0,
$$

$$
\frac{1}{u_c^e}-\frac{1}{u_c^u}=\frac{\lambda_t-\lambda_{t-1}}{\mu_t}\left(1+\gamma_c-\gamma_c\frac{c^u}{c^e}+\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}\right)>0.
$$

If $1+\gamma_c-\gamma_c\frac{c_t^u}{c_t^e}+\frac{u_{c,t}^e}{u_{c,t}^u}\frac{\pi(e)}{\pi(u)}>0$, then in steady state,

$$
x \equiv \frac{\lambda_t-\lambda_{t-1}}{\mu_t}>0. \qquad (30)
$$

Therefore, (29) can be rewritten as

$$
\mu_t\left(\frac{1}{u_c^u}+\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}x\right)=1+\lambda_{t-1}\left(1+\gamma_c\right). \qquad (31)
$$

Note that from (30), we know that λ_t diverges as long as $\lim_{t\to\infty}\mu_t>0$. In addition, (22) suggests that μ_t must be strictly greater than zero. If $\gamma_c=-1$, (31) indicates that $\mu_t\left(\frac{1}{u_c^u}+\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}x\right)=1$, implying μ_t converges. It cannot be true since to satisfy $v_n^e\left(1+\lambda_t\left(1+\gamma_n\right)\right)=\mu_t$, μ_t must increase in response to the growth of λ_t. If $\gamma_c<-1$, μ_t turns negative in the long run according to (31) given that λ_{t-1} explodes. Again, it is impossible.

If $1+\gamma_c-\gamma_c\frac{c_t^u}{c_t^e}+\frac{u_{c,t}^e}{u_{c,t}^u}\frac{\pi(e)}{\pi(u)}=0$, then $\frac{1}{u_c^e}=\frac{1}{u_c^u}$ holds, which contradicts $c^e>c^u$.

If $1+\gamma_c-\gamma_c\frac{c_t^u}{c_t^e}+\frac{u_{c,t}^e}{u_{c,t}^u}\frac{\pi(e)}{\pi(u)}<0$, it must be $\frac{\lambda_t-\lambda_{t-1}}{\mu_t}<0$. Given that $\mu_t>0$, $\lambda_t<\lambda_{t-1}$ implies λ_t turns negative, which cannot be true.

As a result, when $\gamma_c\leq -1$, the only optimal policy under scenario 2 is to let $c^e=c^u$.

A.6 Proof of Proposition 6

In this case, we add one constraint to the Ramsey planner problem. The Ramsey problem of our benchmark economy is modified as:

$$\max_{\{c_t^e, n_t^e, B_{t+1}, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \left[u(c_t^e) - v\left(n_t^e\right) \right] \pi(e) + u(c_t^u)\pi(u) \right\}$$

subject to conditions (14), (15),

$$T_t \geq 0, \tag{32}$$

and

$$c_t^u = c_t^e(1 - \mathbf{1}) + (B_t + T_t)\mathbf{1},$$

where $\mathbf{1}$ is an indictor function with $\mathbf{1} = 1$ if $c_t^e > B_t + T_t$ and $\mathbf{1} = 0$ otherwise.

Let $\beta^t \kappa_t$ be the multiplier associated with the additional condition (32). The FOCs are all identical to the previous case except for the one with respect to T_t, which is modified as

$$u_{c,t}^u \pi(u)\mathbf{1}_t - \mu_t \pi(u)\mathbf{1}_t + \lambda_t u_{c,t}^e \pi(u)\mathbf{1}_t - \lambda_t u_{c,t}^e + \lambda_{t-1} u_{cc,t}^u \pi\left(u\right) B_t \mathbf{1}_t + \kappa_t = 0, \; t \geq 1. \tag{33}$$

Combining FOCs (19) and (33) implies that

$$\lambda_t (u_{c,t+1}^e \pi(e) + u_{c,t+1}^u \pi(u)) = \kappa_t > 0, \text{ for all } t$$

which suggests T_t has to be zero for all t. As a result, the Ramsey allocation is identical to the one considered in the Scenario 4.