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Are Unconditional Lump-sum Transfers a Good Idea?

Are Unconditional Lump-sum Transfers a Good Idea?

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Abstract

The role of unconditional lump-sum transfers in improving social welfare in heterogenous agent models has not been thoroughly understood in the literature. We adopt an analytically tractable Aiyagari-type model to study the distinctive role of unconditional lump-sum transfers in reducing consumption inequality due to ex-post uninsurable income risk under borrowing constraints. Our results show that in the presence of ex-post heterogeneity and in the absence of wealth inequality, unconditional lump-sum transfers are not a desirable tool for reducing consumption inequality—the Ramsey planner opts to rely solely on public debt and a linear labor tax (in the absence of a lump-sum tax) to mitigate income risk without the need for lump-sum transfers, in contrast to the result obtained by Werning (2007), Azzimonti and Yared (2017), and Bhandari, Evans, Golosov, and Sargent (2017) in models with ex-ante heterogeneity.

1 Introduction

Unconditional lump-sum transfers, such as universal basic income (UBI) proposals, are gaining in popularity. For example, there have been multiple referendums and petitions with respect to UBI in developed countries in the past decade.[1] However, despite such popular support of UBI by many politicians and even prominent economists,[2] not much theoretical work has been devoted to justify or scrutinize the validity of such policy proposals. Importantly, even the critics have based their arguments mainly on intuitive or verbal grounds.[3]

One exceptional theoretical analysis in this regard is Werning (2007), who studies Ramsey fiscal policies in an environment where households are heterogenous due to ex-ante differences in their productivity—which is fixed and time-invariant. Despite the availability of public debt and a linear labor tax, optimal unconditional lump-sum transfers could be positive if the degree of ex-ante heterogeneity is high enough. Hence, UBI could improve social welfare through a redistributional effect.

In contrast, this paper studies the role of unconditional lump-sum transfers in an alternative and popular framework where households are heterogeneous ex-post, as in the model of Aiyagari (1994). To the best of our knowledge the role of universal lump-sum transfers in a Ramsey allocation is not well understood in Aiyagari-type models. This awkward situation is due mainly to the intractability of the Aiyagari model, which creates a formidable challenge to analyzing optimal fiscal policies.

The main goal of our paper is to clarify the role of lump-sum transfers in Aiyagari-type models when the Ramsey planner can simultaneously issue public debt and levy a linear labor tax. To this end, we simplify the model of Bhandari, Evans, Golosov, and Sargent (2017) in a particular way so that their model becomes analytically tractable with closed-

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[1]A list of UBI petitions, polls and referendums can be found at `https://en.wikipedia.org/wiki/Universal_basic_income` (accessed 2021-02-01). In addition, support for UBI has been growing among policymakers globally as shown by Hoynes and Rothstein (2019). A UBI program would allow every individual to receive a fixed amount of transfer payments in every period regardless of their income levels and consumption needs, as exemplified by the 2020 presidential candidate Andrew Yang's proposal of giving every American citizen $1,000 per month.

[2]There is a long list of politicians, scholars, business entrepreneurs and others supporting UBI according to Wikipedia: `https://en.wikipedia.org/wiki/List_of_advocates_of_basic_income` (accessed 2021-02-01).

[3]For example, Acemoglu (2019) recently argues that *"UBI is a flawed idea, not least because it would be prohibitively expensive unless accompanied by deep cuts to the rest of the safety net.* He cautions people that *"One should always be wary of simple solutions to complex problems, and universal basic income is no exception."*

form solutions. This simplifying approach makes the Ramsey outcome transparent with clear intuition. Although the simplification might come at some costs in terms of generality, we believe that the gains are sufficiently valuable to shed new light on the issue at stake.

More specifically, our model features idiosyncratic income risk with borrowing constraints, an incomplete insurance market, and three policy tools available to mitigate the risk: (i) a non-negative universal lump-sum transfer, (ii) a distortionary linear labor tax, and (iii) a buffer-stock saving device—government bonds. We ask what the optimal Ramsey allocation would be in such an infinite-horizon Aiyagari-type model starting from time zero.

The intriguing question we seek to answer is how to best finance these complementary government programs to reduce consumption inequality caused by idiosyncratic income shocks under borrowing constraints. Specifically, when unemployment risk is prolonged and distortionary taxes are available to finance either the lump-sum transfer or the interest payment of public debt, what should a Ramsey allocation look like both in transition and in the long run regarding the optimal quantity of public debt, the size of the lump-sum transfers, and the optimal rate of distortionary taxes?

We find that government bonds—as a buffer-stock saving device—are generally more effective than unconditional lump-sum transfers (or UBI) in alleviating income risk and reducing consumption inequality due to unemployment shocks, such that the optimal amount of lump-sum transfers is zero both in the Ramsey steady state and along the transitional path. In addition, the long-run optimal quantity of government bonds is as high as required for achieving a full self-insurance (FSI) allocation (or complete consumption equality) even at the cost of a high distortionary tax rate levied on labor income. Our result indicates that, surprisingly, public debt is always viewed by the Ramsey planner as a better tool than universal lump-sum transfers in improving time-zero expected lifetime welfare in our model with only ex-post heterogeneity.[4]

Intuitively, unconditional lump-sum transfers can increase welfare through the *intra-temporal* margin by directly reducing consumption inequality via the income channel, while public debt works through the *inter-temporal* margin by reducing consumption inequality via improving individuals' self-insurance position. The difference in these two margins explains exactly why in ex-post heterogeneous-agent models the Ramsey planner prefers using public

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[4]In this paper we use "universal lump-sum transfers," "unconditional lump-sum transfers," and UBI interchangeably.

debt rather than lump-sum transfers to smooth individual consumption against idiosyncratic income risk; and it also explains perfectly why lump-sum transfers could potentially play an important redistributional role in ex-ante heterogeneous-agents models, as shown by Werning (2007).

Our model framework is in the class of models pioneered by Aiyagari (1994) in which infinitely lived agents are ex-ante identical but ex-post heterogenous. Our result thus may not apply to models featuring ex-ante heterogeneity in addition to ex-post heterogeneity, such as Bhandari, Evans, Golosov, and Sargent (2017), Judd (1985) or the overlapping generation models. For example, in a two-period ex-ante heterogeneous-agent model, Azzimonti and Yared (2017) show that the Ramsey planner optimally limits the supply of bond such that not all households are slack in their borrowing constraints because the Ramsey planner can borrow more cheaply and hence relax government budget constraints by keeping some of the agents borrowing-constrained. Our paper thus complements nicely to this literature by clarifying the distinct roles of public debt and lump-sum transfers in improving welfare in ex-ante or ex-post heterogenous-agent models. Therefore, if UBI has a role to play in mitigating consumption inequality in the real world, it is more likely to be associated with its wealth-redistribution effect or ex-ante heterogeneity that are missing in our model by design.

Our result complements the findings of Bhandari, Evans, Golosov, and Sargent (2017) and Azzimonti and Yared (2017). We show that the optimal debt level under (only) ex-post heterogeneity should approach full self-insurance as long as the interest rate lies below the time discount rate, whereas in this literature the Ramsey planner opts to provide only partial insurance to the rich agents and use a low interest rate tactic to redistribute income from the rich to the poor. Specifically, in the model of Bhandari, Evans, Golosov, and Sargent (2017), the Ramsey planner opts to exercise monopoly power to keep the interest rate sufficiently low so that it can borrow more cheaply from the rich agents and subsidize the poor agents. Similarly, in the model of Azzimonti and Yared (2017), there is no ex-post uncertainty and thus there is no insurance role for government debt. Consequently, the Ramsey planner chooses not to issue sufficient amount of debt to completely relax the borrowing constraints for all households and instead opts to monopolize the interest rate by borrowing more cheaply from the rich and subsidize the poor through redistribution.

The rest of the paper is organized as follows. Section 2 sets up the model and defines the

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competitive equilibrium. Section 3 characterizes the Ramsey outcomes. Section 4 confirms our theoretical findings and demonstrates the transition path of the Ramsey outcome through a numerical exercise. Section 5 concludes.

2   The Model

2.1   General Environment

Time is discrete and indexed by $t = 0, 1, 2, ..., \infty$. There are three types of agents in our model economy: firms, government, and households.

Firm. A representative firm produces output Y_t according to a linear production technology, $Y_t = N_t$, where N_t is aggregate employment. The firm hires labor from households by paying a competitive wage rate w_t. The firm's optimal condition for profit maximization in time t satisfies $w_t = 1$.

Government. In each period t, the government can issue one-period, risk-free bonds B_{t+1}, give transfers $T_t \geq 0$ in a lump-sum fashion, and levy a flat-rate, time-varying labor tax τ_t. The government flow budget constraint in time t is given by

$$\tau_t w_t N_t + \frac{B_{t+1}}{R_{t+1}} \geq B_t + T_t, \text{ for all } t \geq 0, \tag{1}$$

where R_{t+1} is the risk-free gross interest rate between time t and $t+1$, and the initial level of government bonds B_0 is exogenously given. For simplicity, we assume there is no government consumption.

Household. There is a representative household consisting of a unit measure of *ex-ante* identical members (consumers), à la Lucas (1990). These individual members face idiosyncratic employment shocks in every period denoted by $\theta_t \in \{e, u\}$. The shocks are identically and independently distributed (iid) over time and across all individuals. For example, if $\theta_t = e$, then an individual can work and receive labor income; otherwise, $\theta_t = u$, and an individual is unemployed with no labor income. Let $\pi(e)$ and $\pi(u)$ denote the probabilities of employment and unemployment, respectively.

The head of household maximizes the intertemporal welfare of all family members using a utilitarian welfare criterion (all members are equally weighted), but faces some limits to

the degree of risk sharing that it can provide to family members. Specifically, there are two subperiods within each period. In the first subperiod, the members are separated from each other and they each receive an employment-status shock θ_t after separation. Based on the shock θ, each individual makes a consumption decision, saving decision, and labor supply decision (if employed). In the second subperiod, individuals are reunited with their family and share their asset holdings. Therefore, the idiosyncratic shocks are not fully insurable despite the family reunion, even though each individual will have the same amount of initial assets in the beginning of the next period. Hence, given that the idiosyncratic income risk is uninsurable, there are still precautionary motives to save so as to smooth each family member's consumption over time. However, the wealth distribution across family members is completely degenerate by design since the individuals can share their asset holdings in the end of each period.

This setup divides the entire population within the household into two groups: employed with measure $\pi(e)$ and unemployed with measure $\pi(u)$. Individuals in each group are identical since their initial wealth is the same. This setup is similar to that in Heathcote and Perri (2018), and it makes both the competitive equilibrium and the Ramsey problem analytically tractable.[5] Finally, as in Werning (2007), we simply assume that any agent-specific targeted transfer is ruled out in our model, which can be easily motivated by an assumption of private information.

The aggregate lifetime utility of the household is then given by

$$U = \sum_{t=0}^{\infty} \beta^t \left\{ [u(c_t^e) - v(n_t^e)] \pi(e) + u(c_t^u)\pi(u) \right\}, \tag{2}$$

where $\beta \in (0,1)$ is the discount factor; c_t^e and c_t^u denote, respectively, the consumption of the employed and unemployed individuals in time t; and n_t^e denotes the labor supply of employed individuals (note the labor supply is zero for unemployed individuals). By the law of large numbers, $\pi(e)$ represents the employed population and $\pi(u)$ the unemployed population.

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[5] Assuming quasi-linear utility is an alternative approach to simplify the heterogenous-agents models. For example, see Wen (2009), Challe and Ragot (2011) and Wen (2015). In addition, the study by Lagos and Wright (2005) is also a well-known example in the money-search literature. Moreover, this simplified approach can be generalized further by various ways. See, e.g., the recent work by Le Grand and Ragot (2019), and Bilbiie and Ragot (2020).

The representative household's flow budget constraint is thus given by

$$c_t^e \pi(e) + c_t^u \pi(u) + \frac{a_{t+1}}{R_{t+1}} \leq \widehat{w}_t n_t^e \pi(e) + a_t + T_t, \tag{3}$$

where $\widehat{w}_t = (1 - \tau_t)w_t$ is the after-tax wage rate, a_{t+1} is the asset holdings determined in time t, T_t is a universal lump-sum transfer identical across all family members, and the initial asset holding a_0 is given and assumed to be the same across individuals. In addition, the budget constraint for the employed individuals is given by

$$c_t^e \leq a_t + \widehat{w}_t n_t^e + T_t, \tag{4}$$

and that for the unemployed individuals is given by

$$c_t^u \leq a_t + T_t. \tag{5}$$

2.2 Household Problem

The household head chooses a sequence of $\{c_t^e, c_t^u, n_t^e\}_{t=0}^{\infty}$ to maximize (2) subject to (3), (4) and (5). Let $\beta^t \lambda_t^e \pi(e)$, $\beta^t \lambda_t^u \pi(u)$, and $\beta^t \mu_t$ denote the Lagrangian multipliers associated with constraints (4), (5), and (3), respectively. The FOCs with respect to c_t^e, c_t^u, n_t^e, and a_{t+1} are given, respectively, by

$$u_{c,t}^e = \lambda_t^e + \mu_t, \tag{6}$$

$$u_{c,t}^u = \lambda_t^u + \mu_t, \tag{7}$$

$$v_{n,t}^e = \widehat{w}_t(\lambda_t^e + \mu_t) = (1 - \tau_t)u_{c,t}^e, \tag{8}$$

$$\beta(\lambda_{t+1}^e \pi(e) + \lambda_{t+1}^u \pi(e)) + \beta \mu_{t+1} = \frac{\mu_t}{R_{t+1}}. \tag{9}$$

2.3 Competitive Equilibrium

Definition 1. *Given a_0 and B_0, a competitive equilibrium is defined as a sequence of government policies $\{B_{t+1}, \tau_t, T_t\}_{t=0}^{\infty}$, a sequence of prices $\{w_t, R_t\}_{t=0}^{\infty}$, a sequence of aggregate allocation $\{C_t, N_t\}_{t=0}^{\infty}$, and a sequence of individual allocation plans $\{c_t^e, c_t^u, n_t^e, a_{t+1}\}_{t=0}^{\infty}$ such that*

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1. *given prices, the sequence $\{c_t^e, c_t^u, n_t^e, a_{t+1}\}_{t=0}^{\infty}$ solves the household problem;*

2. *given prices, the sequence $\{N_t\}_{t=0}^{\infty}$ solves the firm problem;*

3. *the government budget constraint in equation (1) holds for $t \geq 0$; and*

4. *all markets clear for $t \geq 0$:*

$$B_{t+1} = a_{t+1}, \tag{10}$$
$$N_t = n_t^e \pi(e),$$
$$N_t = C_t = c_t^e \pi(e) + c_t^u \pi(u), \tag{11}$$

where the first equation is the asset market-clearing condition, the second equation is the labor market-clearing condition, and the third equation is the consumption goods market-clearing condition.

2.4 Characterization of Competitive Equilibrium

Proposition 1. *A competitive equilibrium has the following two properties:*

1. *$\lambda_t^e = 0$ and $c_t^e \geq c_t^u$ for all $t \geq 0$; namely, the consumption level of the employed cannot be lower than that of the unemployed. In addition,*

$$c_t^u = \begin{cases} c_t^e & \text{if } a_t + T_t \geq c_t^e, \\ a_t + T_t & \text{if } a_t + T_t < c_t^e. \end{cases} \tag{12}$$

2. *The Euler equation (9) for optimal household saving can be rewritten as*

$$\frac{1}{R_{t+1}} = \beta \left(\frac{u_{c,t+1}^e}{u_{c,t}^e} \pi(e) + \frac{u_{c,t+1}^u}{u_{c,t}^e} \pi(u) \right). \tag{13}$$

Proof. Please refer to Appendix A.1. $\qquad\square$

Proposition 1 implies that if the amount of cash on hand (asset holdings plus transfers $a_t + T_t$) is sufficiently large such that the optimal consumption of an employed family member satisfies $c_t^e \leq a_t + T_t$, then all family members can obtain the same level of consumption

regardless of their (un)employment status. We refer to this outcome as an FSI allocation where consumption is equalized across all family members.

Obviously, this outcome can be achieved potentially in a competitive equilibrium by either a sufficiently large amount of lump-sum transfers or a sufficiently large amount of government bonds (note $a_t = B_t$ in equilibrium). In addition, if the FSI allocation is achieved in the steady state, then the risk-free rate R must be equal to the time discount rate $1/\beta$.[6] Otherwise, we must have consumption inequality $c^u < c^e$ with $R < 1/\beta$ in the steady state. Notice that $R < 1/\beta$ is a hallmark feature of Aiyagari-type models, but its critical role in incentivising the Ramsey planner to amass a sufficiently large amount of government bonds to achieve the FSI allocation in a Ramsey equilibrium is seldom acknowledged or clearly pointed out until the recent works of Chien and Wen (2019, 2021). We will show that it is precisely this property that makes lump-sum transfers inferior to public debt as an insurance device for reducing consumption inequality or income risk.

2.5   Conditions to Support Competitive Equilibrium

Given that government policies are inside the state space of the competitive equilibrium and affect the endogenous distribution of consumption, the Ramsey problem is to pick a particular competitive equilibrium (through policies) that attains the maximum of the time-zero expected lifetime utility U defined in (2). The outcome is called a Ramsey allocation.

To ensure that a Ramsey allocation constitutes a competitive equilibrium, however, we must show first that all possible allocations in the choice set of the Ramsey planner, $\{c_t^e, n_t^e, c_t^u, B_{t+1}, T_t\}_{t=0}^{\infty}$, constitute a competitive equilibrium (after substituting out prices and policy variables by using the FOCs of the competitive equilibrium and the government budget constraint). The following proposition states the conditions under which a Ramsey allocation must be satisfied in order to constitute a competitive equilibrium.

Proposition 2. *Given $a_0 = B_0 > 0$, the sequence of allocations $\{c_t^e, n_t^e, c_t^u, B_{t+1}, T_t\}_{t=0}^{\infty}$ can be supported as a competitive equilibrium if and only if they satisfy the following conditions:*

1. The resource constraint holds for all $t \geq 0$:

$$n_t^e \pi(e) - c_t^e \pi(e) - c_t^u \pi(u) \geq 0. \tag{14}$$

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[6]In this paper, any variable without subscript t is referred to as its steady-state value.

2. The implementability condition holds for all $t \geq 0$:

$$u_{c,t}^e c_t^e \pi(e) + u_{c,t}^e c_t^u \pi(u) + \beta \left(u_{c,t+1}^e \pi(e) + u_{c,t+1}^u \pi(u) \right) B_{t+1} - v_{n,t}^e n_t^e \pi(e) - u_{c,t}^e (B_t + T_t) = 0,$$

(15)

where

$$c_t^u = \begin{cases} c_t^e & \text{if } B_t + T_t \geq c_t^e \\ B_t + T_t & \text{if } B_t + T_t < c_t^e \end{cases}.$$

(16)

Proof. Please refer to Appendix A.2. □

3 Ramsey Allocation

To facilitate our analysis, we make the following assumptions on the utility function's relative risk aversion parameters $\{\gamma_c, \gamma_n\}$:

$$\begin{aligned} \gamma_c &\equiv \frac{u_{cc}c}{u_c} < 0, \\ \gamma_n &\equiv \frac{v_{nn}n}{v_n} > 0, \end{aligned}$$

so the utility function exhibits constant relative risk aversion (CRRA).

In addition, we assume that the initial bond supply B_0 is sufficiently low so that the competitive equilibrium does not feature FSI or achieve perfect consumption equality.

Definition 2. *A interior Ramsey steady state is a long-run Ramsey allocation where aggregate and individual variables $\{N_t, C_t, B_{t+1}, c_t^e, c_t^u, n_t^e\}$ all converge to finitely positive non-zero constants.*

3.1 Ramsey Problem

Armed with Proposition 2, the Ramsey problem is to solve

$$\max_{\{c_t^e, n_t^e, B_{t+1}, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ [u(c_t^e) - v(n_t^e)] \pi(e) + u(c_t^u)\pi(u) \right\}$$

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subject to conditions (14), (15), and (16). Notice that condition (16) can be rewritten as

$$c_t^u = c_t^e(1 - \mathbf{1}_t) + (B_t + T_t)\mathbf{1}_t,$$

where $\mathbf{1}_t$ is an indictor function with $\mathbf{1}_t = 1$ if $c_t^e > B_t + T_t$ and $\mathbf{1}_t = 0$ otherwise. In addition, the Ramsey planner is also subject to a non-negative constraint on lump-sum transfers:

$$T_t \geq 0, \tag{17}$$

which implies that a lump-sum tax $(T_t < 0)$ is not allowed in our analysis.[7]

3.2 Ramsey Allocation

Proposition 3. *The Ramsey allocation has the following properties:*

1. *The optimal amount of a lump-sum transfer is zero for all periods except in the first period $t = 0$ when bond supply is fixed in that period.*

2. *In the long run there exists an interior Ramsey steady state featuring*

   (a) *equality between the interest rate and the time discount rate $(R = 1/\beta)$;*

   (b) *FSI with complete consumption equality, $c^e = c^u$; and*

   (c) *a positive labor income tax rate at $\tau = 1 - \beta$.*

3. *The interior Ramsey steady state is unique if $(1 + \gamma_c) \leq 0$; in other words, if the CRRA parameter satisfies $(1 + \gamma_c) \leq 0$, then there does not exist any other Ramsey steady state with inadequate self insurance such that $c^u < c^e$ and $R < \beta^{-1}$.[8]*

Proof. Please refer to Appendix A.3. □

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[7]Allowing a lump-sum tax is trivial because it can be used as a substitute for a distortionary labor tax to finance government debt. See our working paper, Chen, Chien, Wen, and Yang (2021), for the case of a lump-sum tax.

[8]In a separate paper, Chien and Wen (2021), we show that if $(1 + \gamma_c) > 0$, then in addition to the full self-insurance steady state in the Ramsey problem, it may also exist other types of steady states where the Lagrangian multipliers of the Ramsey problem diverge to infinity. Since we are unable to determine which steady state yields higher social welfare and constitutes the Ramsey allocation, we must leave it to future research regarding what happens to the Ramsey allocation under the condition $(1 + \gamma_c) > 0$.

This Proposition indicates, somewhat surprisingly, that the option of a lump-sum transfer is never used by the Ramsey planner—not only in the state steady but also during the whole transition path except the initial period $t = 0$. The reason the initial period lump-sum transfer is non-zero is that the Ramsey planner cannot adjust the initial bond level B_0, so it must rely on lump-sum transfers to mitigate income risk and reduce consumption inequality in period 0. But afterward the Ramsey planner sets lump-sum transfers to zero once adjusting the bond supply is possible. Hence, the Ramsey planner always views universal lump-sum transfers as an inferior tool compared to public debt in improving ex-ante welfare in our model economy.

Opting not to use lump-sum transfers as a policy tool to reduce consumption inequality is such a striking result and maybe counter-intuitive, especially in light of the conventional wisdom that lump-sum transfers are viewed as non-distortionary and hence able to address the problem of income inequality better than public debt under incomplete markets. But the key point to realize is that regardless of lump-sum transfers or public debt, they both need to be financed by distortionary labor taxes. Thus the issue at stake is which policy tool is more beneficial—it turns out that the answer hinges critically on the difference between the intratemporal margin and the intertemporal margin.

The insight behind Proposition 3 is that the Ramsey planner has a dominating incentive to increase the supply of government bonds (instead of lump-sum transfers) when the risk-free rate is lower than the time discount rate. More specifically, given that the planner discounts the future by β, which is lower than the inverse of the market risk-free rate R^{-1}, there is an "arbitrage opportunity" for the planner to exploit to improve individuals' self insurance positions and to front-load aggregate consumption during the transition period. This arbitrage opportunity exists as long as $R < \frac{1}{\beta}$ and will never vanish until FSI is archived in the long run where $R = \frac{1}{\beta}$ and consumers have accumulated enough government bonds. This also implies that without taking into account the transitional dynamics, as in the case of maximizing only the steady state welfare, a FSI may be sub-optimal because the asymmetric discounting between R^{-1} and β is no longer present (see Floden (2001) and Chien and Wen (2020)); hence, lump-sum transfers in such a case may not be dominated by public debt in mitigating idiosyncratic income risks.

However, since our results in Proposition 3 are based on a simple tractable model without wealth inequality, they may not hold when wealth-redistribution effects are introduced into

the model. But we conjecture that adding wealth inequality into the model may not change our results. The intuition is that a lump-sum transfer is more effective in reducing income inequality than reducing wealth inequality because the former is a flow and the later a stock variable. Nonetheless, to the best of our knowledge, the possibility of relying on wealth inequality to justify UBI has never been investigated by the existing literature and should thus be an important research topic in the future.

4   Transition Dynamics

In what follows, we confirm our theoretical findings by numerical simulations of the model, which not only substantiate our theoretical results but also illustrate the pattern of optimal transition paths of the Ramsey allocation. Such numerical analyses are valid because the Ramsey steady state has been proved to exist under our parameter specifications.

Parameter Values. First, we assume a power utility function with $u(c) = \frac{1}{1-\sigma} c^{1-\sigma}$ and $v(n) = \frac{1}{\gamma} n^{\gamma}$, where $\gamma = \sigma = 2$. Second, we set $\beta = 0.96$, and the probability distribution of employment and unemployment is given by $\pi(e) = 0.9$ and $\pi(u) = 0.1$, respectively. The initial debt level, B_0, is set at 90% of its level in the corresponding Ramsey steady state.

Figure 1 plots the transition path of the Ramsey outcome. First, the optimal debt level (panel [2,2]—row 2 and column 2) shows a jump in the second period and then gradually increases toward the Ramsey steady state. The sharp increase of government debts boosts household savings and improves households' self-insurance position. As the self-insurance position improves, consumption inequality (panel [1,2]) shrinks over time. In addition, the interest rate (panel [3,1]) approaches the time discount rate from below as the borrowing constraints are relaxed over time. The paths of aggregate consumption and aggregate labor (note $C_t = N_t$ in panel [1,1]) show a clear pattern of front-loading: They both peak in the first period and then decrease back to their respective long-run steady state. The higher labor efforts are supported by a low labor tax rate in the short run before reaching the long-run level (panel [2,1]).

The increasing level of public debt clearly requires financing from tax revenues. The Ramsey planner opts to put the pressure of revenue collection on issuing new debt in the short run and gradually shifting the financing burden to labor taxes in the longer run. This also explains why the labor tax rate is significantly lower in the short run, so as to incentive

Figure 1: Ramsey Transition Path
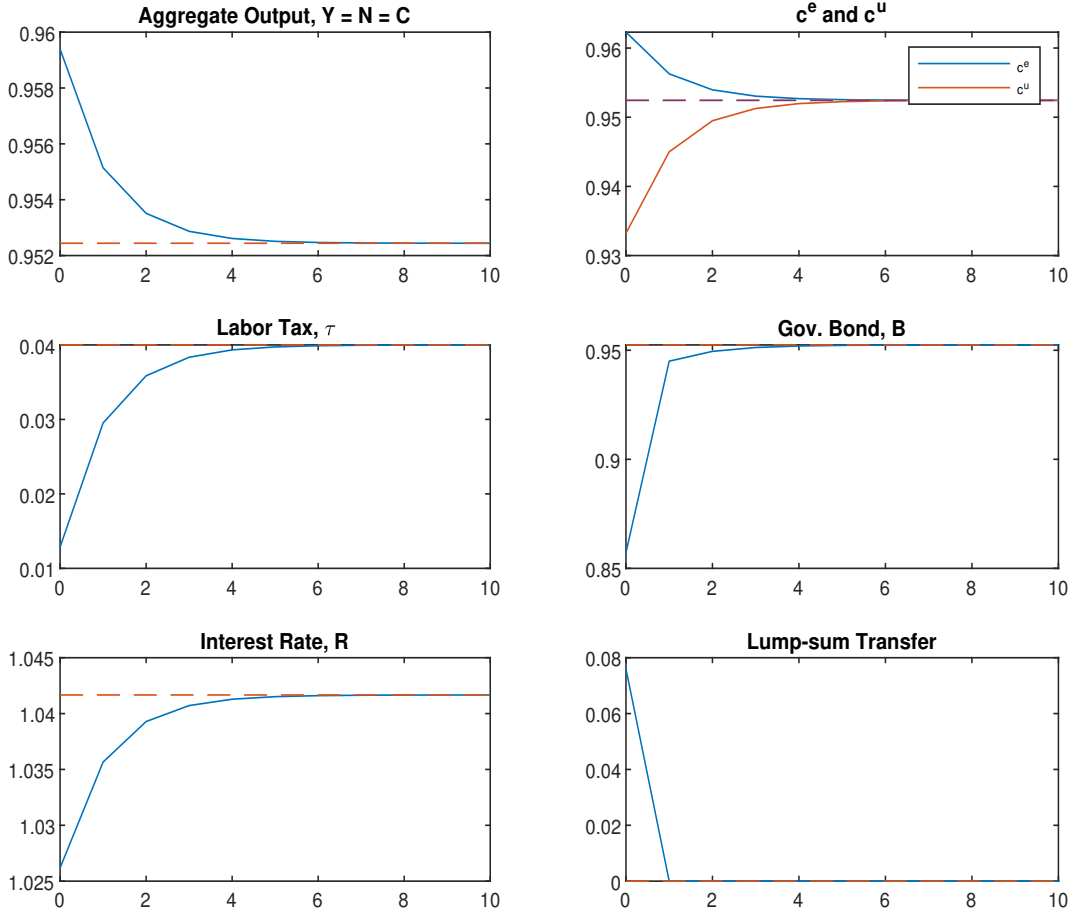
Notes: Transition paths in the Ramsey economy (solid lines) and their corresponding steady states (dashed lines).

hard-working behavior, and gradually rises to a steady-state value of 4% in the long run (panel [2,1]). Finally, the lump-sum transfer is positive only at the initial period and jumps to zero immediately at period 1 (panel [3,2]), which confirms our theoretical finding that the lump-sum transfer is never utilized whenever government debt is available.

5 Conclusion

We find that when heterogeneity is ex-post instead of ex-ante, public debt is always viewed by the Ramsey planner as a better tool than lump-sum transfers in improving social welfare

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under incomplete insurance markets with borrowing constraints. Given that the root cause of consumption inequality in Aiyagari-type models is the lack of sufficient self-insurance (or liquidity) to fully diversify income risk under borrowing constraints, lump-sum transfers and public debt can both serve to improve welfare by providing liquidity and relaxing individuals' borrowing constraints. However, lump-sum transfers provide liquidity through the intra-temporal margin, while public debt provides liquidity through the inter-temporal margin. Along either margin the cost of financing is identically a distortionary tax. But the intertemporal margin has an advantage over the intratemporal margin because the interest cost of debt financing is cheaper when the risk-free rate of government bonds lies below the time discount rate, which is a hallmark feature of Aiyagari-type models. We show that the Ramsey planner cleverly exploits this "arbitrage" opportunity and opts to use bonds instead of lump-sum transfers to relax borrowing constraints.

Of course, if lump-sum transfers can be made fully conditional on individual income status, then there is little doubt that such targeted lump-sum transfers can achieve the first-best allocation without relying on public debt. But, the curse of private information may render fully targeted or conditional lump-sum transfers infeasible or inadequate to support the first-best allocation in the real world, thus creating the role for public debt in mitigating income risk and reducing consumption inequality.

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A Appendix

A.1 Proof of Proposition 1

We divide the discussion into two cases: (1) $B_{t+1} > 0$, and (2) $B_{t+1} = 0$, for $t > 0$.

1. $B_{t+1} > 0$. Based on the nature of inter-period budget constraints, it must be $c_t^e \geq c_t^u$. Suppose $\lambda_t^e > 0$. Then (4) must bind. Substituting $c_t^e = a_t + \widehat{w}_t n_t^e + T_t$ into (3) gives $c_t^u = a_t + T_t - \frac{a_{t+1}}{R_{t+1}\pi(u)} < a_t + T_t$, since at the equilibrium $a_{t+1} = B_{t+1} > 0$. Therefore, $\lambda_t^u = 0$, which implies $u_{c,t}^e > u_{c,t}^u$ and therefore $c_t^e < c_t^u$. It leads to contradiction such that λ_t^e must be zero.

   Now consider two subcases: (a) $c_t^u = a_t + T_t$ and (b) $c_t^u < a_t + T_t$. In subcase (a), $\lambda_t^u \geq 0$ and hence $c_t^e \geq c_t^u = a_t + T_t$. For subcase (b), $\lambda_t^u = 0$ and hence $c_t^e = c_t^u < a_t + T_t$ according to FOCs (6) and (6). Therefore, c_t^u can be expressed as a function of c_t^e and $a_t + T_t$ as shown in equation (12).

2. $B_{t+1} = 0$. With $B_{t+1} = a_{t+1} = 0$, rewrite (3) as

$$c_t^e \pi(e) + c_t^u \pi(u) = \widehat{w}_t n_t^e \pi(e) + a_t + T_t \geq \widehat{w}_t n_t^e \pi(e) + c_t^u$$

   where the inequality is due to $c_t^u \leq a_t + T_t$. Rearranging it leads to

$$c_t^e - c_t^u \geq \widehat{w}_t n_t^e,$$

   which implies $c_t^e > c_t^u$. It also means that (5) must bind. Inserting $c_t^u = a_t + T_t$ into (3) gives

$$\widehat{w}_t n_t^e \pi(e) + a_t + T_t = c_t^e \pi(e) + c_t^u \pi(u) = c_t^e \pi(e) + (a_t + T_t) \pi(u),$$

   which gives

$$\widehat{w}_t n_t^e + a_t + T_t = c_t^e.$$

   As a result, both (3) and $c_t^u = a_t + T_t$ guarantee that (4) is automatically satisfied. Consider another household problem where individuals are subject to (3) and $c_t^u =$

$a_t + T_t$. The constraint set is identical to that in the original problem. As such, we have $\lambda_t^e = 0$ as long as (3) and (5) are taken into account.

Finally, given $\lambda_t^e = 0$, the FOC (9) together with FOCs (6) and (7) can be rewritten as equation (13).

A.2 Proof of Proposition 2

The "If" Part: Given the initial B_0 and the allocation $\{c_t^e, n_t^e, c_t^u, B_{t+1}, T_t\}_{t=0}^\infty$, a competitive equilibrium can be constructed by using the two conditions in Proposition 2 and following the steps below that uniquely back up the sequences of the other variables:

1. a_{t+1} is chosen such that $a_{t+1} = B_{t+1}$ to clear the asset market.

2. Given c_t^e, n_t^e and $a_{t+1} = B_{t+1}$, c_t^u is chosen such that

$$
c_t^u = \begin{cases} c_t^e \text{ if } c_t^e \leq B_t + T_t \\ B_t + T_t \text{ if } c_t^e > B_t + T_t \end{cases}.
$$

   Note that the choice of c_t^u ensures that the FOCs (6) and (7) are satisfied. In addition, equation (5) is also satisfied.

3. Aggregates C_t and N_t are chosen such that

$$
\begin{aligned}
N_t &= n_t^e \pi(e), \\
C_t &= c_t^e \pi(e) + c_t^u \pi(u).
\end{aligned}
$$

4. $w_t = 1$ so that the firm's problem is solved. τ_t is chosen such that $\widehat{w}_t = (1 - \tau_t)w_t$ satisfies $\frac{v_{n,t}^e}{u_{c,t}^e} = (1 - \tau_t)$

5. R_{t+1} is chosen by the household Euler equation

$$
\frac{1}{R_{t+1}} = \beta \frac{u_{c,t+1}^e \pi(e) + u_{c,t+1}^u \pi(u)}{u_{c,t}^e}.
$$

6. There are two conditions left. They are the resource constraint and the representative household's budget constraint, which are listed in Proposition 2. The implementability

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condition (derived from the representative household's budget constraint with holding equality) can be expressed as (15) by replacing τ_t, R_{t+1} and a_{t+1} by the conditions provided in the above steps.

The "Only If" Part: The constraints listed in Proposition 2 are trivially satisfied because they are part of the competitive-equilibrium conditions.

A.3   Proof of Proposition 3

A.3.1   Ramsey FOCs

Let $\beta^t \mu_t$, $\beta^t \lambda_t$ and $\beta^t \kappa_t$ be the multipliers associated with constraints (14), (15) and (17), respectively. The FOCs with respect to c_0^e and $\{c_t^e\}_{t=1}^\infty$ are given, respectively, by

$$u_{c,0}^e \pi(e) + u_{c,0}^u \pi(u)(1 - \mathbf{1}_0) - \mu_0 \pi(e) - \mu_0(1 - \mathbf{1}_0)\pi(u) \tag{18}$$
$$+ \lambda_0(u_{c,0}^e + u_{cc,0}^e c_0^e)\pi(e) + \lambda_0(u_{c,0}^e(1 - \mathbf{1}_0) + u_{cc,0}^e c_0^u)\pi(u)$$
$$- \lambda_0 u_{cc,0}^e(B_0 + T_0) = 0,$$

and

$$u_{c,t}^e \pi(e) + u_{c,t}^u \pi(u)(1 - \mathbf{1}_t) - \mu_t \pi(e) - \mu_t(1 - \mathbf{1}_t)\pi(u) \tag{19}$$
$$+ \lambda_t(u_{c,t}^e + u_{cc,t}^e c_t^e)\pi(e) + \lambda_t(u_{c,t}^e(1 - \mathbf{1}_t) + u_{cc,t}^e c_t^u)\pi(u)$$
$$- \lambda_t u_{cc,t}^e(B_t + T_t) + \lambda_{t-1}(u_{cc,t}^e \pi(e) + u_{cc,t}^u \pi(u)(1 - \mathbf{1}_t))B_t$$
$$= \quad 0 \text{ for } t \geq 1.$$

The FOC with respect to B_{t+1}, for all $t \geq 0$, is given by

$$u_{c,t+1}^u \pi(u)\mathbf{1}_{t+1} - \mu_{t+1}\pi(u)\mathbf{1}_{t+1} \tag{20}$$
$$+ \lambda_t \left[(u_{c,t+1}^e \pi(e) + u_{c,t+1}^u \pi(u)) + B_{t+1} u_{cc,t+1}^u \pi(u)\mathbf{1}_{t+1} \right]$$
$$- \lambda_{t+1} u_{c,t+1}^e + \lambda_{t+1} u_{c,t+1}^e \pi(u)\mathbf{1}_{t+1} = 0.$$

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The FOCs with respect to T_t are

$$u_{c,t}^u \pi(u)\mathbf{1}_t - \mu_t \pi(u)\mathbf{1}_t + \lambda_t u_{c,t}^e \pi(u)\mathbf{1}_t - \lambda_t u_{c,t}^e + \lambda_{t-1} u_{cc,t}^u \pi(u) B_t \mathbf{1}_t + \kappa_t = 0, \ t \geq 1. \quad (21)$$

and

$$u_{c,0}^u \pi(u)\mathbf{1}_0 - \mu_0 \pi(u)\mathbf{1}_0 + \lambda_0 u_{c,0}^e \pi(u)\mathbf{1}_0 - \lambda_0 u_{c,0}^e + \kappa_0 = 0. \quad (22)$$

The FOC with respect to n_t^e, for all $t \geq 0$, is

$$v_{n,t}^e (1 + \lambda_t (1 + \gamma_{n,t})) = \mu_t. \quad (23)$$

A.3.2 Zero Lump-sum Transfers

We first show that $T_t = 0$ for all $t \geq 1$ period. Combining FOCs (20) and (21) implies that

$$\lambda_t (u_{c,t+1}^e \pi(e) + u_{c,t+1}^u \pi(u)) = \kappa_t > 0, \ \text{for all } t \geq 1,$$

which suggests constraint (17) is strictly binding, and hence T_t has to be zero for all $t \geq 1$.

A.3.3 FSI Ramsey Steady State

In what follows, we first show that there exists a Ramsey steady state exhibiting FSI and then show that there is no Ramsey steady state with $R < 1/\beta$.

A. Proof of Existence We show that there is a Ramsey steady state featuring FSI. Notice that the FOC with respect to labor supply is given by equation (23), which implies that the growth rates of μ_t and λ_t in the steady state are the same. Suppose that in the steady state, $\mathbf{1}_t = 0$ and $c^e = c^u = B$. In this case, the steady state versions of FOCs with respect to B_t, and c_t^e are simplified, respectively, as

$$-\lambda_t + \lambda_{t-1} = 0 \quad (24)$$

and

$$u_c^e + \lambda_t(u_c^e + u_{cc}^e c^e) - \lambda_t u_{cc}^e B + \lambda_{t-1} u_{cc}^e B = \mu_t, \quad (25)$$

which imply both λ_t and μ_t converge to a constant in the Ramsey steady state.

Finally, we solve for the optimal long-run tax rate $\tau > 0$ by the following steps:

1. $R = 1/\beta$ by equation (13).

2. The λ, μ, c^e and n^e can be solved by the four equations (14), (15), (23) and (25) in the steady state:

$$u_c^e + \lambda(u_c^e + u_{cc}^e c^e) = \mu,$$

$$v_n^e(1 + \lambda(1 + \gamma_n)) = \mu,$$

$$n^e = c^e/\pi(e),$$

$$\beta c^e - \frac{v_n^e n^e \pi(e)}{u_c^e} = 0.$$

3. From the equations above, we can characterize the tax and bond policy in the steady state.

   (a) The tax rate $\tau = 1 - \beta$. From the household's FOC, we know $1 - \tau = \frac{v_n^e}{u_c^e}$, which together with the last two equations above gives

   $$1 - \tau = \frac{v_n^e}{u_c^e} = \beta.$$

   (b) The optimal level of debt B is less than B^{FB}. The B^{FB} is given by: $B^{FB} = \frac{1}{\beta}c^{FB}$. In this case, $c^e = c^u = B$ such that

   $$B = c^e < \frac{1}{\beta}c^e.$$

   Since $c^e < c^{FB}$, we reach the conclusion that $B < B^{FB}$.

B. Proof of Uniqueness We prove that the above Ramsey steady state is unique if $(1 + \gamma_c) \leq 0$. We show by contradiction that a steady-state allocation with $c^e > c^u$ is not possible if $(1 + \gamma_c) \leq 0$. Suppose $c^e > c^u = B$ and hence $\mathbf{1}_t = 1$. After imposing the steady-state condition, the FOCs with respect to c_t^e and B_{t+1} become

$$u_c^e + \lambda_t[u_c^e + u_{cc}^e c^e - u_{cc}^e c^u] + \lambda_{t-1} B u_{cc}^e = \mu_t$$

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$$u_c^u - \lambda_t u_c^e \frac{\pi(e)}{\pi(u)} + \lambda_{t-1}\left(u_c^e \frac{\pi(e)}{\pi(u)} + u_c^u\right) + \lambda_{t-1}Bu_{cc}^u = \mu_t.$$

Rewrite the two equations above as

$$1 + \lambda_t\left[1 + \gamma_c - \gamma_c\frac{c^u}{c^e}\right] + \lambda_{t-1}\gamma_c\frac{c^u}{c^e} = \frac{\mu_t}{u_c^e},$$

$$1 - \lambda_t\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)} + \lambda_{t-1}\left(\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)} + 1\right) + \lambda_{t-1}\gamma_c = \frac{\mu_t}{u_c^u}.$$

We can further rearrange them into

$$\frac{\mu_t}{u_c^e} = 1 - \gamma_c\frac{c^u}{c^e}(\lambda_t - \lambda_{t-1}) + \lambda_t(1 + \gamma_c), \tag{26}$$

$$\begin{aligned}
\frac{\mu_t}{u_c^u} &= 1 - \lambda_t\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)} + \lambda_{t-1}\frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)} + \lambda_{t-1} + \lambda_{t-1}\gamma_c \\
&= 1 - \frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}(\lambda_t - \lambda_{t-1}) + \lambda_{t-1}(1 + \gamma_c). \tag{27}
\end{aligned}$$

If $c^e > c^u$, then $\frac{\mu_t}{u_c^e} > \frac{\mu_t}{u_c^u}$ such that

$$\frac{\mu_t}{u_c^e} - \frac{\mu_t}{u_c^u} = (\lambda_t - \lambda_{t-1})\left(1 + \gamma_c - \gamma_c\frac{c^u}{c^e} + \frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}\right) > 0,$$

$$\frac{1}{u_c^e} - \frac{1}{u_c^u} = \frac{\lambda_t - \lambda_{t-1}}{\mu_t}\left(1 + \gamma_c - \gamma_c\frac{c^u}{c^e} + \frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}\right) > 0.$$

If $1 + \gamma_c - \gamma_c\frac{c_t^u}{c_t^e} + \frac{u_{c,t}^e}{u_{c,t}^u}\frac{\pi(e)}{\pi(u)} > 0$, then in steady state,

$$x \equiv \frac{\lambda_t - \lambda_{t-1}}{\mu_t} > 0. \tag{28}$$

Therefore, (27) can be rewritten as

$$\mu_t\left(\frac{1}{u_c^u} + \frac{u_c^e}{u_c^u}\frac{\pi(e)}{\pi(u)}x\right) = 1 + \lambda_{t-1}(1 + \gamma_c). \tag{29}$$

Note that from (28), we know that λ_t diverges as long as $\lim_{t\to\infty}\mu_t > 0$. In addition, (23) suggests that μ_t must be strictly greater than zero. If $\gamma_c = -1$, (29) indicates

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that $\mu_t \left(\frac{1}{u_c^u} + \frac{u_c^e}{u_c^u} \frac{\pi(e)}{\pi(u)} x \right) = 1$, implying μ_t converges. It cannot be true since to satisfy $v_n^e \left(1 + \lambda_t \left(1 + \gamma_n \right) \right) = \mu_t$, μ_t must increase in response to the growth of λ_t. If $\gamma_c < -1$, μ_t turns negative in the long run according to (29) given that λ_{t-1} explodes. Again, it is impossible.

If $1 + \gamma_c - \gamma_c \frac{c_t^u}{c_t^e} + \frac{u_{c,t}^e}{u_{c,t}^u} \frac{\pi(e)}{\pi(u)} = 0$, then $\frac{1}{u_c^e} = \frac{1}{u_c^u}$ holds, which contradicts $c^e > c^u$.

If $1 + \gamma_c - \gamma_c \frac{c_t^u}{c_t^e} + \frac{u_{c,t}^e}{u_{c,t}^u} \frac{\pi(e)}{\pi(u)} < 0$, it must be $\frac{\lambda_t - \lambda_{t-1}}{\mu_t} < 0$. Given that $\mu_t > 0$, $\lambda_t < \lambda_{t-1}$ implies λ_t turns negative, which cannot be true.

As a result, when $\gamma_c \leq -1$, the only optimal policy is to let $c^e = c^u$.