Wage Setting Under Targeted Search

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Wage Setting Under Targeted Search

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Abstract

When setting initial compensation some firms set a fixed non-negotiable wage while others bargain. In this paper we propose a parsimonious search and matching model with two sided heterogeneity, where search intensity and the degree of randomness in matching are endogenous, and firms decide whether to bargain or post wages. We study the implications of heterogeneous search costs and market tightness on the choice of the wage setting mechanism, as well as the relationship between bargaining prevalence and wage level, residual wage dispersion, and labor market tightness. We find that bargaining prevalence is positively correlated with wages, residual wage dispersion, and labor market tightness, both in the model and in the data.

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1 Introduction

Evidence from both establishment and employee surveys demonstrates that, when setting initial compensation, around two thirds of firms stipulate a fixed non-negotiable wage, while the other third bargains with the employee (see Hall and Krueger (2012), Brenzel, Gartner, and Schnabel (2014), and Doniger (2015)). Why do these two wage setting protocols coexist?

In this paper, we propose a parsimonious way to model how firms decide whether to post or bargain over wages, and how workers decide where to send their job applications given available job postings. Search intensity and the degree of randomness in matching are endogenous in our model.

We study the implications of having heterogeneous search costs and varying market tightness on the choice of the wage setting mechanism on the side of firms, and explore the relationship between bargaining prevalence and wage level, residual wage dispersion, and labor market tightness.

We make three important predictions. First, a tighter labor market results in more firms opting to bargain. Second, relative costs of search play a key role in the choice of the wage setting protocol. When firms face lower search costs than workers, they can post low wages because they can identify good workers themselves and not share the surplus with them. When firms face higher search costs than workers, they set higher wages to delegate the search problem to the workers and encourage self selection. At very high search costs for the firm, if the surplus generated by a match is not high enough, the firm will not give the majority of the surplus to the worker, but will choose to bargain instead.

This relationship between relative search costs and the choice of the wage setting protocol, leads to our third prediction. Bargaining prevalence is positively correlated with the level of wages, residual wage dispersion, and labor market tightness. We validate this last prediction using data from the Survey of Consumer Expectations, and find that the data exhibit the same positive relationships.

We blend the stochastic discrete choice literature with the frictionless matching environment of Becker (1973) with two-sided heterogeneity and assume that, on both sides of the market, individual types of agents are characterized by multidimensional attributes. Even though agents know the distribution and their preferences over types,
they do not know where to find a particular type. To do so, they decide how much
effort they want to exert to locate a particular match by trading off the cost of search
with the payoff they can achieve if successful in finding their desired match.

An agent chooses whom to contact in a probabilistic way, and the strategies chosen
are discrete probability distributions over types. Each element of the distribution
represents the probability with which an agent will target (i.e., contact) each potential
match based on the agent’s expected payoff. Exerting more search effort, which
results in a higher search cost, allows agents to spot a particular type more accurately.
Given the discrete nature of the probability distributions, we model the search cost as
proportional to the distance between an uninformed—uniform—strategy, where every
type has the same probability of being contacted and the distribution that is chosen by
the agent.\(^1\)

The optimal probability distribution representing an agent’s contacting strategy
balances two motives: the productive and the strategic. The productive motive pushes
the agent to pursue the potential match that gives the agent the highest payoff. The
strategic motive pushes the agent to pursue the potential match that is more likely
to reciprocate interest. Thus, people act strategically not only when deciding whether
to form a match or wait for a better option (like in Eeckhout (1999)), but also when
choosing whom to contact.

In the model, if a firm were to bargain over the wage, search happens simultaneously,
that is, both workers and firms sort through the whole pool of agents on the other side
of the market by exerting search effort to maximize expected payoffs. On the other
hand, if a firm were to post wages, search happens sequentially, such that the workers
decide where to send their application based on the wages posted, and on a second
stage the firms exert search effort to sort through and decide whom to hire from among
the workers that applied to them. Firms decide whether to bargain or to post wages
based on the protocol that maximizes their net income flow.

Understanding the determinants of residual wage dispersion has been a long standing
question in the literature. We believe that it is natural to think that the wage setting
mechanism can affect both wage levels and wage dispersion, and as such, to validate
the implications of the model, we study the relationship between bargaining prevalence

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\(^1\)We borrow our cost specification from the literature on discrete choice under information frictions. See Cheremukhin et al. (2015) and Matejka and McKay (2015).
and the wage level, wage dispersion and market tightness in the model and in the data.

To study these relationships in the data, we use the Survey of Consumer Expectations of the Federal Reserve Bank of New York for the years 2013-2017. We follow Faberman, Mueller, Sahin and Topa (2019) to obtain a measure of residual wages by regressing log annualized real wages on job characteristics and demographic characteristics and use the residuals from the regression to compute the weighted standard deviations of residual log real wages by occupation.

To construct a measure of bargaining prevalence we add to the same regression a set of variables representing measures of search effort, such as the type of work a person is looking for (full/part time), the number applications that were sent, the number of potential employers that contacted the worker, the number of job offers received, and indicators of search methods used. We take the part of the wage variation jointly explained by these additional variables as a proxy for bargaining prevalence, and we rescale the proxy to cover the unit interval.

We find that the correlation between average wages and bargaining prevalence is 0.47 in the data, and 0.90 in the model, and that the correlation between residual wage dispersion and bargaining prevalence is 0.58 in the data and 0.49 in the model.

In terms of market tightness, and similarly to search costs, in the model a tighter labor market implies that more firms choose to bargain, hence there is higher residual wage dispersion. Morin (2019) shows that there is evidence of residual wage dispersion being procyclical, consistent with the prediction of our model.

The paper proceeds as follows. In Section 2 we lay down the wage bargaining and wage posting protocols, and set up the wage setting choice problem. Section 3 discusses the theoretical implications of the model. Section 4 compares the model implications with those in the data, and Section 5 concludes.

Related literature

In Cheremukhin, Restrepo-Echavarria, and Tutino (2020) we developed a theory of targeted search where search was simultaneous and the payoff was set through bargaining. That model we applied to the marriage market. In this paper we focus on the labor market and extend our previous set up to allow firms to choose between a bargaining protocol with simultaneous search, or a wage posting protocol where search
is sequential. Like in Cheremukhin, Restrepo-Echavarria, and Tutino (2020), our paper effectively blends two sources of randomness used in the literature. The first source is a search friction with uniformly random meetings and impatience, as in Shimer and Smith (2000). The second approach introduces unobserved characteristics as a tractable way of accounting for the deviations of the data from the stark predictions of the frictionless model, as in Choo and Siow (2006) and Galichon and Salanie (2012). We introduce a search friction into the meeting process by endogenizing agents’ choices of whom to contact. We build on the discrete choice rational inattention literature—i.e., Cheremukhin, Popova, and Tutino (2015) and Matejka and McKay (2015)—that derives multinomial logit decision rules as a consequence of cognitive constraints that capture limits to processing information. Therefore, the equilibrium matching rates in our model have a multinomial logit form similar to that in Galichon and Salanie (2012). Unlike Galichon and Salanie, the equilibrium of our model features strong interactions between agents’ contact rates driven entirely by their choices, rather than by some unobserved characteristics with fixed distributions.

The search and matching literature has seen multiple attempts to produce intermediate degrees of randomness with which agents meet their best matches. In particular, Menzio (2007) and Lester (2011) nest directed search and random matching to generate outcomes with an intermediate degree of randomness. Our paper produces equilibrium outcomes in between uniform random matching and the frictionless assignment, endogenously, without nesting these two frameworks. One recent paper considering our specification of targeted search with information costs in application to the labor market is Wu (2020).

Also note that although the directed search literature, such as Eeckhout and Kircher (2010) and Shimer (2005), technically involves a choice of whom to meet, the choice is degenerate—directed by signals from the other side. See Chade, Eeckhout and Smith (2017) for a thorough summary of this literature.

The paper is also related to a growing literature on the coexistence of various wage setting mechanisms. For instance, Barron et. al. (2006), Hall and Krueger (2012), Brenzel et. al. (2014) and Doniger (2015) estimate the prevalence of wage posting in survey data for the U.S. and Germany and find prevalence of both bargaining and wage-

\footnote{Also, see Yang’s (2013) model of “targeted” search that assumes random search within perfectly distinguishable market segments.}
posting. Several models have been developed to explain their co-existence. Michelacci and Suares (2006) argue that despite inefficiencies inherent in bargaining, it may be preferable over wage posting in the presence of large heterogeneity in unverifiable productivity, because it renders any posted wage contract incomplete. In this model, as well as in Doniger (2015) wage posting and bargaining coexist because firms that bargain can use an ex post signal on the productivity or the outside option of a worker, while a wage posting firm cannot condition on worker-specific characteristics. These two approaches follow a large literature on the micro-foundations of mechanisms including Postel-Vinay and Robin (2004), Bontemps et. al. (2000), Holzner (2011). A recent paper by Flinn and Mullins (2018) allows for a wage posting firm to condition the wage contract on worker productivity, but the heterogeneity in workers’ outside options still leads to the co-existence of different wage-setting protocols.

We endogenize the processing of information on both sides of the market (by workers about firms and by firms about workers) assuming all of the characteristics of workers and firms are observable (at a cost). We place no restrictions on heterogeneity allowing both workers and firms to differ along multiple dimensions including productivity and outside options. In our model, firms choose to post a low wage schedule when it is cheap for them to distinguish among the applicants. As it becomes more expensive to screen workers, firms incentivize workers to self-select by posting higher wage schedules. Firms facing high costs of screening would have to promise very high wages for workers to self-select, leaving only a small fraction of the surplus for the firm. When it is too costly to screen workers, firms choose wage bargaining over a posted wage schedule. Wage bargaining and wage posting coexist in our model because of large heterogeneity in search costs among firms.

2 Targeted search

Firms are looking to fill a vacancy, and workers who are either employed or unemployed are looking to find a job.

We build on the frictionless matching environment of Becker (1973), where firms and workers are heterogeneous in their type and search for a match. The premise of our targeted search model is that both workers and firms know the distribution and their preferences over types on the other side of the market, but there is noise—agents
cannot locate potential matches with certainty. However, they can pay a search cost to help locate them more accurately.

We model this by assuming that each agent chooses a probabilistic search strategy that can be interpreted as a search intensity over types, where each element of this distribution reflects the likelihood of contacting a particular agent on the other side. A more targeted search, or a probability distribution that is more concentrated on a particular group of agents (or agent) is associated with a higher cost, as the agent needs to exert more effort to locate a particular potential match more accurately.

The economy contains a large, finite number of individual agents: workers whose types are indexed by $x \in \{1, \ldots, W\}$ and firms whose types are indexed by $y \in \{1, \ldots, F\}$.

We denote by $\mu_x$ the number of workers of type $x$ and by $\mu_y$ the number of firms of type $y$. We think of workers and firms characterized by a multidimensional set of attributes. Types $x$ and $y$ are unranked indices that aggregate all attributes.

A match between any worker of type $x$ and any firm of type $y$ generates a payoff (surplus) $\Phi_{xy}$. Note that we do not place any restrictions on the shape of the payoff function. We normalize the outside option of both to zero. We denote the payoff (wage) appropriated by the worker $\omega_{xy}$ and the payoff appropriated by the firm $\eta_{xy}$ such that $\eta_{xy} = f_{xy} - \omega_{xy}$.

Agents form a match if they meet according and each agent (weakly) benefits from forming a match; i.e., each agent’s payoff is non-negative. Since a negative payoff corresponds to absence of a match, we make the following assumption on the payoffs:

**Assumption 1.** The payoffs are non-negative:

$$f_{xy} \geq \omega_{xy} \geq 0.$$

When seeking to form a match, both workers and firms are aware of the number of agents of each type and the characteristics of their preferred types on the other side of the market. They face a noisy search process where they are uncertain about how to locate their preferred match. In this environment, each agent’s action is a probability distribution over agents on the other side of the market. Since the number of potential matches is finite, the strategy of each agent is a discrete probability distribution. Let $\bar{p}_x(y)$ be the probability that a worker of type $x$ targets or sends an application to a firm of type $y$. Similarly, we denote by $\bar{q}_y(x)$ the probability that a firm of type $y$
targets or looks at the application of a worker of type $x$.

Reducing the noise to locate a potential match more accurately is costly: it involves a careful analysis of the profiles of potential matches, with considerable effort in sorting through the multifaceted attributes of each firm and candidate. When seeking to form a match, agents rationally weigh costs and benefits of targeting the type characteristics that result in a suitable match. A worker rationally chooses their strategy $\bar{p}_x(y)$ by balancing the costs and benefits of targeting a given firm. A strategy $\bar{p}_x(y)$ that is more concentrated on a particular firm of type $y$ affords them a higher probability to be matched with their preferred firm. However, it requires more effort to sort through profiles of all the firms in the market to locate their desired match and exclude the others. So locating a particular firm or worker more accurately requires exerting more search effort, so it is costlier.

We assume that agents enter the search process with a uniform prior of who to target, $\bar{p}_x(y)$ and $\bar{q}_y(x)$. Choosing a more targeted strategy implies a larger distance between the chosen strategy and the uniform prior and is associated with a higher search effort. A natural way to introduce this feature into our model is the Kullback-Leibler divergence (relative entropy),\(^3\) which provides a convenient way of quantifying the distance between any two distributions, including discrete distributions as in our model. We assume that the search effort of worker $i$ of type $x$ is defined as follows:

$$\kappa_x = \sum_{y=1}^{F} \mu_y \bar{p}_x(y) \ln \frac{\bar{p}_x(y)}{\bar{p}_x(y)}.$$  \hspace{1cm} (2.1)

We assume that the search costs $c_x(\kappa_x)$ are a function of the search effort $\kappa_x$. Note that $\kappa_x$ is increasing in the distance between a uniform distribution over firms and the chosen strategy, $\bar{p}_x(y)$. If an agent does not want to exert any search effort, she can choose a uniform distribution over types and meet firms randomly. As she chooses a more targeted strategy, the distance between the uniform distribution and her strategy $\bar{p}_x(y)$ grows, increasing search effort $\kappa_x$ and the overall cost of search. By increasing search effort, agents bring down uncertainty about locating a prospective match, which

\(^3\)In the model of information frictions used in the rational inattention literature, $\kappa_x$ represents the relative entropy between a uniform prior and the posterior strategy. This definition is a special case of Shannon’s channel capacity, where information structure is the only choice variable (See Thomas and Cover (1991), Chapter 2). See also Cheremukhin et al. (2015) for an application to stochastic discrete choice with information costs.
allows them to target their better matches more accurately.

Likewise, a firm’s cost of search $c_y(\kappa_y)$ is a function of the search effort defined as:

$$\kappa_y = \sum_{x=1}^{F} \mu_x \bar{q}_y(x) \ln \frac{\bar{q}_y(x)}{\bar{\bar{q}}_y(x)}.$$  

(2.2)

Furthermore, we assume the following:

**Assumption 2.** The search costs of agents $c_x(\kappa)$ and $c_y(\kappa)$ are strictly increasing, twice continuously differentiable and (weakly) convex functions of search effort.

As a special case, we consider a linear cost of search. Then, the total costs of search for a worker of type $x$ are given by $c_x = \theta_x \kappa_x$ and for a firm of type $y$ by $c_y = \theta_y \kappa_y$, where $\theta_x \geq 0$ and $\theta_y \geq 0$ are the marginal costs of search.

For convenience in comparing wage posting and bargaining setups, we introduce a new notation for the strategies of the workers and the firms. We define the workers’ and firms’ search intensities the ratios of their posterior and the prior: $p_x(y) = \frac{\bar{p}_x(y)}{\bar{\bar{p}}_x(y)}$ and $q_y(x) = \frac{\bar{q}_y(x)}{\bar{\bar{q}}_y(x)}$ respectively.

The meeting rate depends on the strategies of each agent, $p_x(y)$ and $q_y(x)$, and a congestion function $\phi(p_x(y), q_y(x), \mu_x, \mu_y)$, which depends in some general way on the strategies of all other agents as well as the number of agents of each type. Given this, the total number of matches formed between workers of type $x$ and firms of type $y$ is given by

$$M_{x,y} = p_x(y) q_y(x) \phi(p_x(y), q_y(x), \mu_x, \mu_y).$$

**Assumption 3.** The congestion function is twice continuously differentiable in each $p$ and $q$.

We introduce this congestion function following Shimer and Smith (2001) and Mortensen (1982), who assume a linear search technology. Note that if $\phi(...)=1$, then a match takes place if and only if there is mutual coincidence of interests; i.e., both agents draw each other out of their respective distribution of interests. By introducing this congestion function we are allowing for matches to depend in some general way on both an agent’s search intensity\footnote{Note that here, search intensity refers to how concentrated the distribution of interests of an agent is. A higher search intensity results in assigning higher probability to one or several agents within an agent’s distribution of interests.} for a specific agent ($p$ and $q$) and on the number
of agents taking part.

Note that when setting up the congestion function we implicitly assume that there are no direct inter-type congestion externalities. However, our model still features strong indirect equilibrium interactions between the strategies of agents that work akin to inter-type congestion by attracting or deterring agents.

Since each individual agent is “small” compared with the population of agents of his type, we assume the following:

**Assumption 4.** Agents take the meeting rates they face as given, disregarding the dependence of the congestion function on agents’ own search intensities.

### 2.1 Simultaneous search with bargaining

Under simultaneous search, workers are deciding to which firm to send an application at the same time as firms are looking at available workers to decide which applications to look at. The set of actions \( s \in S \) is given by the cartesian product of the sets of strategies of workers \( p_x(y) \in S_x \) and firms \( q_y(x) \in S_y \), where in general

\[
S_x = \left\{ p_x(y) \in R^M : p_x(y) \geq 0, \sum_{y=1}^{M} \mu_y p_x(y) \leq \sum_{y=1}^{M} \mu_y \right\},
\]

\[
S_y = \left\{ q_y(x) \in R^F : q_y(x) \geq 0, \sum_{x=1}^{F} \mu_x q_y(x) \leq \sum_{x=1}^{F} \mu_x \right\}.
\]

Figure 2.1 illustrates the strategies of workers and firms. The solid arrows show the probability \( p_x(y) \) that a worker of type \( x \) assigns to targeting a firm of type \( y \). Similarly, dashed arrows show the probability \( q_y(x) \) that a firm of type \( y \) assigns to targeting a worker of type \( x \). Once these are selected, both workers and firms make one draw from their respective distributions to determine where to send an application and which applications to request.

Both firms and workers choose their optimal strategies and if a firm and a worker match, the payoff is split between them. The payoff and the split generated by any potential \((x, y)\) match are exogenous and known ex-ante. Wages are set by ex post wage-bargaining with bargaining power \( \beta \) implying a wage \( \omega_{xy} = \beta f_{xy} \) which is fully anticipated ex ante.

The meeting rate faced by a firm of type \( y \) conditional on targeting a worker of type
Figure 2.1: Strategies of Firms and Workers under Simultaneous Bargaining
\( x \) takes on the form \( P_x(y) = \mu_x p_x(y) \phi_{xy} \). Similarly, the meeting rate faced by a worker of type \( x \) conditional on targeting a firm of type \( y \) is \( Q_y(x) = \mu_y q_y(x) \phi_{xy} \).

The priors are given by \( \tilde{p}_x(y) = 1/\delta_x \) and \( \tilde{q}_y(x) = 1/\delta_y \) where \( \delta_x = \sum_{y=1}^{F} \mu_y \) and \( \delta_y = \sum_{x=1}^{W} \mu_x \) are the total number of options for the worker and firm respectively.

Both firms and workers maximize the expected value of their payoffs net of the search costs. For a worker of type \( x \), the problem is

\[
Y_x = \max_{p_x(y) \in S_x} \sum_{y=1}^{F} \omega_{xy} Q_y(x) p_x(y) - c_x(\kappa_x(p_x(y)))
\]

for all \( x \in \{1, \ldots, W\} \), where \( \kappa_x = 1/\delta_x \sum_{y=1}^{F} \mu_y p_x(y) \ln p_x(y) \) and the meeting rates \( Q_y(x) \) are taken as given.

Likewise, a firm \( j \) of type \( y \) solves

\[
Y_y = \max_{q_y(x) \in S_y} \sum_{x=1}^{W} \eta_{xy} P_x(y) q_y(x) - c_y(\kappa_y(q_y(x)))
\]

for all \( y \in \{1, \ldots, F\} \), where \( \kappa_y = 1/\delta_y \sum_{x=1}^{W} \mu_x q_y(x) \ln q_y(x) \) and the meeting rates \( P_x(y) \) are taken as given.

The above expressions allow for a precise definition of the equilibrium:

**Definition.** A matching equilibrium is a set of admissible strategies for workers, \( \{p_x(y) \in S_x \}_{x \in \{1, \ldots, F\}} \), firms, \( \{q_y(x) \in S_y \}_{y \in \{1, \ldots, M\}} \); and meeting rates \( P_x(y) \) and \( Q_y(x) \), such that the strategies solve the problems in (2.3) and in (2.4) for each individual firm and worker given the meeting rates, which are consistent with the strategies of the agents.

### 2.1.1 Characterization of equilibrium

We can re-write the objective functions of the agents introducing the linear constraints on strategies via Lagrange multipliers (\( \lambda_x \) and \( \lambda_y \)). Then the first-order conditions for optimality are
\[ \frac{\partial Y_x}{\partial p_x (y)} = \omega_{xy} \mu_y q_y (x) \phi_{xy} - \theta_x \frac{1}{\delta_x} \mu_y (\ln p_x (y) + 1) - \theta_x \frac{1}{\delta_x} \lambda_x \mu_y = 0, \tag{2.5} \]

\[ \frac{\partial Y_y}{\partial q_y (x)} = \eta_{xy} \mu_x p_x (y) \phi_{xy} - \theta_y \frac{1}{\delta_y} \mu_x (\ln q_y (x) + 1) - \theta_y \frac{1}{\delta_y} \lambda_y \mu_x = 0. \tag{2.6} \]

Since the objective functions of agents are twice continuously differentiable and concave in their own strategies, first-order conditions are necessary and sufficient conditions for equilibrium. Rearranging and substituting out Lagrange multipliers, we obtain the following proposition:

**Proposition 1.** Under assumptions 1-4, a matching equilibrium satisfies

\[ p_x^* (y) = \exp \left( \frac{\omega_{xy} q_y^* (x) \phi_{xy}^*}{\theta_x / \delta_x} \right) \sum_{y' = 1}^M \mu_{y'} / \sum_{y' = 1}^M \mu_{y'} \exp \left( \frac{\omega_{xy'} q_{y'}^* (x) \phi_{xy'}^*}{\theta_x / \delta_x} \right), \tag{2.7} \]

\[ q_y^* (x) = \exp \left( \frac{\eta_{xy} p_x^* (y) \phi_{xy}^*}{\theta_y / \delta_y} \right) \sum_{x' = 1}^F \mu_{x'} / \sum_{x' = 1}^F \mu_{x'} \exp \left( \frac{\eta_{x'y} p_{x'}^* (y) \phi_{x'y}^*}{\theta_y / \delta_y} \right). \tag{2.8} \]

Each agent of either type optimally chooses whom to target based on two motives: the productive and the strategic. The productive motive leads the agent to seek out the most desirable type on the opposite side of the market based on the payoff (given by \( \omega_{xy} \) and \( \eta_{xy} \)). The strategic motive, on the other hand, leads the agent to go after someone who is more likely to reciprocate interest (given by \( q_y^* \) and \( p_x^* \)).

The congestion function, \( \phi_{xy}^* \), can scale up or down the probability of forming a match between individuals of type \( x \) and \( y \) and, as such, will affect optimal strategies.

The fact that an agent cannot locate their preferred match with accuracy, mitigates the strategic motive by increasing uncertainty about attributes of potential matches which, in turn, may increase the probability of a type with less-than-desirable attributes seeking out a match with more desirable attributes if the surplus of the match is high enough. As a result, higher search costs boost the productive motive.

Even though in the model there are no direct inter-type congestion externalities, it still features strong indirect equilibrium interactions between the strategies of agents. Because of the strategic motive, if a worker \( x \) knows that a firm \( y \) places a high probability on them, the best thing for them to do is to reciprocate by also placing a high
probability on $y$. This will in turn affect the probability a worker $x'$ places on $y$; she will make it lower.

The equilibrium of the matching model can be interpreted as a pure-strategy Nash equilibrium of a strategic form game. The following assumption and theorem establish conditions under which a matching equilibrium exists:

**Assumption 5.** $\phi_{xy} + q_x \frac{\partial \phi_{xy}}{\partial q_y} \geq 0$ and $\phi_{xy} + p_x \frac{\partial \phi_{xy}}{\partial p_x} \geq 0$ for all admissible $p_x(y), q_y(x)$ for all $x, y$.

This assumption requires that the total matching rate $M_{x,y} = \mu_x \mu_y p_x(y) q_y(x) \phi_{xy}$ is non-decreasing in each of the strategies $p_x(y)$ and $q_y(x)$. In other words, it requires that as agents exert more search effort—or increase their search intensity—the matching rate increases.

**Theorem 2.** Under assumptions 1-5, a matching equilibrium exists.

**Proof.** Since the strategy space is a simplex and, hence, non-empty, convex and compact set, sufficient conditions for existence of the equilibrium require us to check whether the payoff functions are super-modular on the whole strategy space as in Tarski (1955). Super-modularity can be proven by showing non-negativity of the off-diagonal elements of the Hessian matrix.

Let $J_{xy} = \begin{bmatrix} \frac{\partial Y}{\partial q_y} & \frac{\partial Y}{\partial p_x} \end{bmatrix}$ be the Jacobian matrix collecting the set of first-order conditions for all $y \in \{1, ..., M\}$ and all $x \in \{1, ..., F\}$ and let $H_{xy}$ be the corresponding Hessian matrix. To derive the Hessian matrix, note that under A.1, strategies of each individual agent are non-cooperative, i.e., independent of the strategies of opposite types as well as the strategies of the other agents of their own type. Note also that we have assumed no direct inter-type congestion externalities. These assumptions produce a Hessian matrix with a block-diagonal structure, which greatly simplifies the analysis.

For illustrative purposes, suppose that there are four players (two workers and two firms).\(^5\) Suitably rearranging the order of strategies for workers and firms, the Hessian matrix...

\(^5\)While the extension to the case of $M$ firm types and $F$ worker types is straightforward, the notation for the general case is cumbersome.
of this eight-action game can be written as

$$H_{xy} = \begin{bmatrix}
\frac{\partial^2 Y_{x1}}{\partial p_{11} \partial q_{11}} & \frac{\partial^2 Y_{x1}}{\partial p_{11} \partial q_{11}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\partial^2 Y_{x1}}{\partial p_{12} \partial q_{12}} & \frac{\partial^2 Y_{x1}}{\partial p_{12} \partial q_{12}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\partial^2 Y_{x2}}{\partial p_{21} \partial q_{21}} & \frac{\partial^2 Y_{x2}}{\partial p_{21} \partial q_{21}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\partial^2 Y_{x2}}{\partial p_{22} \partial q_{22}} & \frac{\partial^2 Y_{x2}}{\partial p_{22} \partial q_{22}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 Y_{x2}}{\partial q_{22} \partial q_{22}} & \frac{\partial^2 Y_{x2}}{\partial q_{22} \partial q_{22}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.$$ 

From this structure, it is clear that for the general case with \(y \in \{1, \ldots, M\}\) and \(x \in \{1, \ldots, F\}\), the off-diagonal elements of \(H_{xy}\) are non-negative if, simultaneously,

$$\frac{\partial^2 Y_x}{\partial p_x (y) \partial q_y (x)} = \omega_{xy} \left( \phi_{xy} + q_y (x) \frac{\partial \phi_{xy}}{\partial q_y (x)} \right) \geq 0,$$

and

$$\frac{\partial^2 Y_y}{\partial q_y (x) \partial p_x (y)} = \eta_{xy} \left( \phi_{xy} + p_x (y) \frac{\partial \phi_{xy}}{\partial p_x (y)} \right) \geq 0,$$

which requires non-negativity of payoffs as guaranteed by Assumption 1 and that each term in brackets is non-negative, guaranteed by Assumption 5.

**Assumption 6.** The congestion function is given by \(\phi_{xy} = p_x^{-\alpha_{xy}} q_y^{-\beta_{xy}}\).

The following theorem places additional restrictions on the congestion function and surplus split to achieve uniqueness in the decentralized economy:

**Theorem 3.** Under Assumptions 1-6, the matching equilibrium is unique if

a. the congestion function is the same for all pairs of types:

for all \(x \in \{1, \ldots, F\}, y \in \{1, \ldots, M\}\), we have \(\alpha_{xy} = \alpha, \beta_{xy} = \beta\);

b. the surplus split is the same for all pairs of types:

for all \(x, x' \in \{1, \ldots, F\}, y, y' \in \{1, \ldots, M\}\), we have

\[\frac{\varepsilon_{xy}}{\Phi_{xy}} = \frac{\varepsilon_{x'y'}}{\Phi_{x'y'}}, \quad \frac{\eta_{xy}}{\Phi_{xy}} = \frac{\eta_{x'y'}}{\Phi_{x'y'}},\]

and

c. the coefficients of the congestion function satisfy

\[\alpha > 0, \beta > 0, \alpha + \beta \geq 1, \min (\alpha, \beta) < 1.\]
The proof is in Appendix A. Note that the conditions that deliver uniqueness of the decentralized equilibrium do not impose any additional restrictions on the costs of search so long as they are positive, increasing and convex.

If the congestion function has a special functional form \(\phi_{xy}(p_x, q_y, \mu_x, \mu_y) = (p_x)^{-\alpha} (q_y)^{(1-\alpha)}\), the surplus is split proportionally as \(\frac{\varepsilon_{xy}}{\Phi_{xy}} = 1 - \alpha\), and the parameter \(\alpha\) is the same for all pairs of types \((x, y)\), then the competitive equilibrium exists, is unique and is constrained efficient for any positive search costs. Moreover, the aggregate matching function exhibits constant returns to scale.

2.2 Sequential search with wage posting

Search is sequential. First, firms post wage menus and commit to paying type dependent wages in case of matching. Second, because workers cannot perfectly distinguish which firm is of which type despite learning the wage menus of each type of firm, they choose a distribution of search intensities that determines the likelihood of contacting a particular firm and choose one firm from this distribution to send an application. Third, firms choose which worker to give a job offer among the set of workers that applied to the firm. Figure 2.2 illustrates the interactions and search strategies.

Although applications and/or job offers are not lost in the mail, there is still a coordination problem: \(\mu_x p_x(y)\) workers applied to type \(y\) firms, and firms sent \(\mu_y q_y(x)\) job offers, but they did not necessarily send all of those to different workers. Several firms might contact the same worker, and some workers may not get any offers. We assume that \(\mu_x p_x \mu_y q_y \phi_{xy}\) matches are created, where the coordination problem between type \(x\) workers and type \(y\) firms is captured by the same congestion function/meeting technology \(\phi_{xy}(p_x, q_y, \mu_x, \mu_y)\).

Let \(f_{xy}\) denote the revenue of the firm net of the outside option of worker of type \(x\), and \(\omega_{xy}\) the wage menu proposed to the workers of type \(x\) by firms of type \(y\).

Let \(p_x(y)\) and \(q_y(x)\) be the probabilistic search strategies of workers over firms and firms over workers respectively. We also refer to them as search intensities and normalize them to 1 for a strategy that does not affect the likelihood of finding a particular match which would be a non-informative strategy.

The game is sequential as in Stackelberg in that when firms post wages and choose their search effort they internalize the best response strategies of workers. Firms behave
Figure 2.2: Strategies of Workers and Firms under Sequential Wage Posting

Workers

1
2
3
4
5
W

Firms

1
2
3
F

Stage 1

Stage 2

Realized Applications

\[ p_1(W) \]

\[ p_2(W) \]

\[ p_3(W) \]
like leaders and workers behave like followers. However, consistent with the assumptions of the simultaneous model, neither the workers nor the firms internalize the effects of their strategies on the congestion function. This is because there are a large number of individuals of each type, so a change in an individual firm’s or worker’s strategy will not have a noticeable aggregate affect on the number of matches.

Let $\Psi_y(x)$ denote the probability of forming a match which workers face from type $y$ firms. The matching probability in equilibrium must satisfy $\Psi_y(x) = q_y(x) \phi_{xy}$. The worker receives a wage in case of matching, and bears a cost of search. Then, problem of type $x$ worker is the same as in the simultaneous case:

$$Y_x = \sum_{y=1}^{F} \mu_y \Psi_y(x) \omega_{xy} p_x(y) - \theta_x \frac{1}{\delta_x} \sum_{y=1}^{F} \mu_y p_x(y) \ln (p_x(y)) + \theta_x \lambda_x \frac{1}{\delta_x} \left( \delta_x - \sum_{y=1}^{F} \mu_y p_x(y) \right)$$

The optimal strategy of workers that derives from this problem is given by:

$$p^*_x(y) = \exp \left( \frac{\omega_{xy} \phi_{xy}}{\theta_x / \delta_x} q_y(x) \right) \frac{\sum_{y'=1}^{F} \mu_{y'}}{\sum_{y'=1}^{F} \mu_{y'} \exp \left( \frac{\omega_{xy'} \phi_{xy'}}{\theta_x / \delta_x} q_{y'}(x) \right)}.$$

Let $Q_x(y)$ denote the queue length of type $x$ workers applying to a type $y$ firm. Firm $y$ faces a queue which contains (in expectation) $Q_x(y) = \mu_x p_x(y) / \delta_x$ workers of type $x$ that applied to it.

The firm knows the composition of its queue in expectation, but does not know which worker is of which type. So each firm spends its search costs on distinguishing among workers in its queue. The firm’s probabilistic constraint is $\sum_{x=1}^{W} Q_x(y) q_y(x) \leq \delta_y$ where $\delta_y = \sum_{x=1}^{W} Q_x(y) = \sum_{x=1}^{W} \mu_x p_x(y) / \delta_x$. We think of $q_y(x)$ as of the firm’s search intensity. Search effort is then $\frac{1}{\theta_y} \sum_{x=1}^{W} Q_x(y) q_y(x) \ln (q_y(x))$. The firm spends no search effort when search intensity is $q_y(x) = 1$ for all $x$.

Firms choose wages and search strategies to maximize their expected match payoffs net of the costs of search. The problem of firm type $y$ is then:

$$Y_y = \sum_{x=1}^{W} \delta_x Q_x(y) \phi_{xy} (f_{xy} - \omega_{xy}) q_y(x) - \theta_y \frac{1}{\delta_y} \sum_{x=1}^{W} Q_x(y) q_y(x) \ln (q_y(x))$$

$$- \theta_y \lambda_y \frac{1}{\delta_y} \left( \delta_y - \sum_{x=1}^{W} Q_x(y) q_y(x) \right).$$
The firm internalizes the best responses of the workers (Equation 2.9). To internalize the responses, we need to take derivatives of $p_x (y)$ with respect to the wage $\omega_{xy}$ set by the firm, and with respect to the firm’s search strategy $q_y (x)$. If we introduce new notation $z_{xy} = \frac{\partial p_x (y)}{\partial \omega_{xy}} q_y (x) \left( 1 - \frac{\omega_{xy}}{\delta_x} p_x (y) \right)$, then the derivatives are given by:

$$\frac{\partial p_x (y)}{\partial q_y (x)} \frac{q_y (x)}{p_x (y)} = \omega_{xy} z_{xy} \text{ and } \frac{\partial p_x (y)}{\partial \omega_{xy}} \frac{1}{p_x (y)} = z_{xy}.$$ 

The problem can be rewritten as:

$$Y_y = \sum_{x=1}^{m} \mu_x \theta_y \left( \frac{1}{\delta_x \delta_y} p_x (y, \omega_{xy}, q_y (x)) q_y (x) \left( \delta_x \phi_{xy} \frac{f_{xy} - \omega_{xy}}{\theta_y / \delta_y} - \ln (q_y (x)) - \lambda_y \right) \right)$$

Then we can write the first order conditions of the firm with respect to intensities and wages as follows:

$$\frac{\partial Y_x}{\partial q_y} \frac{\delta_x}{\mu_x \delta_y \theta_y p_x (y)} = \left( 1 + \frac{\partial p_x (y)}{\partial q_y (x)} \frac{q_y (x)}{p_x (y)} \right) \left[ \frac{f_{xy} - \omega_{xy}}{\theta_y / \delta_y} \delta_x \phi_{xy} - \ln (q_y (x)) - \lambda_y \right] - 1 = 0$$

$$\frac{\partial Y_x}{\partial \omega_{xy}} \frac{\delta_x}{\mu_x \delta_y \theta_y p_x (y) q_y (x)} = \frac{\partial p_x (y)}{\partial \omega_{xy}} \frac{1}{p_x (y)} \left[ \frac{f_{xy} - \omega_{xy}}{\theta_y / \delta_y} \delta_x \phi_{xy} - \ln (q_y (x)) - \lambda_y \right] - \frac{\phi_{xy}}{\theta_y / \delta_y} = 0$$

We substitute derivatives that were obtained earlier to get:

$$(1 + z_{xy} \omega_{xy}) \left[ \frac{f_{xy} - \omega_{xy}}{\theta_y / \delta_y} \delta_x \phi_{xy} - \ln (q_y (x)) - \lambda_y \right] = 1$$

$$z_{xy} \left[ \frac{f_{xy} - \omega_{xy}}{\theta_y / \delta_y} \delta_x \phi_{xy} - \ln (q_y (x)) - \lambda_y \right] = \frac{\delta_x \phi_{xy}}{\theta_y / \delta_y}$$

The first equations needs to hold at an internal point always. Hence, we can rearrange the FOC with respect to derive the search intensity $q_y$:

$$\frac{f_{xy} - \omega_{xy}}{\theta_y / \delta_y} \delta_x \phi_{xy} - \ln (q_y (x)) - \lambda_y = \frac{1}{1 + z_{xy} \omega_{xy}}$$

The optimal strategy of the firm that derives from this problem is given by:

$$q_y^* (x) = \exp \left( \frac{f_{xy} - \omega_{xy}}{\theta_y / \delta_y} \delta_x \phi_{xy} - \frac{1}{1 + z_{xy} \omega_{xy}} \right) \frac{\sum_{x' = 1}^{m} \mu_{x'} P_{x'} (y) / \delta_x}{\sum_{x' = 1}^{m} \mu_{x'} P_{x'} (y) \exp \left( \frac{f_{xy} - \omega_{xy}}{\theta_y / \delta_y} \delta_x \phi_{xy} - \frac{1}{1 + z_{xy} \omega_{xy}} \right) / \delta_x}$$

We can combine the two optimality conditions to obtain a simple expression for the
wage for an interior solution $0 \leq \omega_{xy} \leq f_{xy}$:

$$\omega_{xy} = \left[ \frac{\theta_y/\delta_y}{\delta_x \phi_{xy}} - \frac{1}{z_{xy}} \right] f_{xy} 0.$$

To gain some intuition, note that for an interior wage, $q_y(x) \sim \exp \left( \delta_x \phi_{xy} f_{xy} \frac{\theta_y}{\delta_y} \right)$, and $\omega_{xy} \sim \frac{1}{\delta_x \phi_{xy}} \left( \theta_y/\delta_y - \frac{\theta_x/\delta_x}{q_y(x)} \right) \sim \theta_y/\delta_y - \exp \left( \frac{f_{xy}}{\theta_y/\delta_y} \right)$. Therefore, both probabilistic strategies and wages are increasing functions of the surplus. Wages also positively depend on the cost of search of firms and negatively on the cost of search of workers. Wages transfer part of the firms search cost onto the worker. When the firms’ costs of search increase, workers are promised a larger wage so as to incentivize them to better distinguish which firms to apply to and simplify the screening process for the firms, thus economizing their costs. Also, substituting back into the worker’s problem, $\ln p_x(y) \sim \omega_{xy} \phi_{xy} \sim \omega_{xy} z_{xy} \phi_{xy} = \omega_{xy} q_y(x) \frac{\theta_x/\delta_x}{\theta_y/\delta_y}$. Workers put their efforts in distinguishing firms that have a higher matching rate, hence, higher surplus.

1. Prove existence under wage posting
2. Prove uniqueness under wage posting

### 2.3 Wage setting mechanism: bargaining versus posting

Consider the same problem, but now the firm knows that it can either post a wage menu and commit to it, or bargain upon meeting. In the case of bargaining, worker gets $\omega_{xy}^B = \beta f_{xy}$ and firm gets the rest. The difference is that if there is bargaining, then there is no queue.

Each firm takes as given the strategies of other firms and can anticipate the response of workers to its own strategy. From the previous subsection recall that the payoff under bargaining is given by:

$$Y^B_y = \sum_{x=1}^{W} \mu_x \frac{\theta_y}{\delta_x} q_y(x) \left( p_x(y) \phi_{xy}^B \frac{(f_{xy} - \omega_{xy}^B)}{\theta_y/\delta_x} - \ln (q_y(x)) \right),$$

and similarly, the payoff under wage posting is the following:

$$Y^W_p = \sum_{x=1}^{W} \mu_x \frac{\theta_y}{\delta_x} q_y(x) \left( \frac{1}{\delta_x} p_x(y) \left( \delta_x \phi_{xy}^W \frac{(f_{xy} - \omega_{xy})}{\theta_y/\delta_x^W} - \ln (q_y(x)) \right) \right),$$
where $\delta_y^{WP} = \sum_{x=1}^{W} \frac{1}{\mu_x} p_x (y)$, $\delta_y^B = \sum_{x=1}^{W} \mu_x$.

To choose the optimal wage setting strategy, the firm compares the payoff it gets if it decides to bargain with the payoff it would get if it posts a wage, and chooses the protocol that gives the maximum payoff:

$$Y^*_y = \max \{ Y_y^B, Y_y^{WP} \}.$$  

### 3 Theoretical implications

In this section we explore the theoretical implications of our model. In particular, we are interested in exploring the effect of different search costs on the optimal wage setting strategy, as well as the effects of market tightness.

We start by calculating the equilibrium of the model for different search costs $(\theta_x, \theta_y)$ and different measures of workers and firms $(\mu_x, \mu_y)$, and evaluating the optimal wage setting strategy $Y^*_y = \max \{ Y_y^B, Y_y^{WP} \}$. To study the implications of the model for different search costs, we set parameter values for $f_{xy}, \phi_{xy}, \beta, \mu_x, \mu_x$ and compute the equilibrium for different $\theta_x$ and $\theta_y$, by making an initial guess for the strategies of the workers and firms $p_x (y)$ and $q_y (x)$, computing the equilibrium wage $\omega_{xy} = \left[ \frac{\theta_y}{\phi_{xy}} - \frac{1}{z_{xy}} \right] f_{xy}$, and checking if the optimality conditions for $p_x (y)$ and $q_y (x)$ are satisfied to find a fixed point. To study the implications of the model for different market tightness, we set parameter values for $f_{xy}, \phi_{xy}, \beta, \theta_x, \theta_x$, and follow the same strategy for different values of $\mu_x$ and $\mu_y$.

The results for the two exercises are found in Figures 3.1 and 3.2 respectively.\(^6\)

Figure 3.1 shows that when the costs of the firm are low relative to the worker’s, the firm prefers to post a wage of zero. This is because they can locate their preferred workers easily. As the cost of the firm increases relative to the workers there is a region where the firm still prefers wage posting, but wages are interior, this means that they have to give part of the surplus to the worker so that they self select and they can improve the quality of the match. When the cost of the firm increases even further such that it is very costly for them to locate a preferred match with accuracy, they prefer bargaining over wage posting. This is because they have to commit to giving a large fraction of the

\(^6\) Both plots show the case where $f_{xy}$ is horizontal, but results look very similar for the case where $f_{xy}$ is vertical.
surplus to the worker, to have a stronger self selection mechanism. It is clear from this relationship that interactions involving bargaining will involve higher search effort on the part of the workers, higher overall wages and higher overall wage dispersion, while wage posting will mostly correspond to little search effort on the part of workers, low wages and low wage dispersion. We explore these relationships in Section 4 where we compare the implications of the model with the data.

Figure 3.1: Wage setting for different search costs

![Wage setting for different search costs](image)

Figure 3.2 shows the relationship between market tightness and the optimal wage setting mechanism. Remember that $\mu_x$ represents the number of workers of type $x$ and $\mu_y$ the number of firms of type $y$, such that as market tightness increases ($\frac{\mu_x}{\mu_y} \ll$) bargaining becomes more predominant.
4 Empirical Validation

4.1 Data

In order to take the model to the data we need to construct the model counterparts. To that aim, in this section we discuss the data we use, and how we proceed to construct a bargaining prevalence proxy, as well as how we recover average wages and residual wage dispersion. We then illustrate the relationship between average wages and bargaining prevalence and the relationship between residual wage dispersion and bargaining prevalence in the data by occupation.

We use data from the Survey of Consumer Expectations carried out by the Federal Reserve Bank of New York. We use the main survey as well as the labor market section which contains variables reflecting search behavior, and we pull the data for the years
2013 up to 2017.

To compute average wages we take the mean of the log real wage, and to recover residual wage dispersion we follow Faberman, Mueller, Sahin and Topa (2019) and regress log annualized real wages on job characteristics (full-time status, tenure, occupation, firm size, employment benefits, job convenience) and demographic characteristics (race, hispanic origin, education, gender, age, age squared, marital status, co-habitation status, number of children, home-ownership). We use the residuals from the regression to compute the weighted standard deviations of residual log real wages by occupation.

To construct a measure of bargaining prevalence, we add to the same regression that we run to recover residual wages, a set of variables representing measures of search effort, such as the type of work a person is looking for, the number applications that were sent during the job search process, the number of potential employers that contacted the worker, the number of job offers received, as well as indicators of search methods used. We believe that the ability of a worker to increase their wage by putting more effort as well as varying the intensity and method of search reflects the extent to which the wage was bargained, rather than regarded as a given take-it-or-leave-it offer. Therefore, we take the fraction of the wage variation jointly explained by these additional variables as a proxy for bargaining prevalence. We rescale the proxy to cover the unit interval. Similarly to wage dispersion, we compute weighted averages of the bargaining proxy by occupation.\footnote{The survey contains an explicit question of whether bargaining happened when an offer was extended. However, the overlap between data on wages and on bargaining is very limited, therefore we could not include this variable in the regression for the proxy or use it as a standalone indicator of bargaining.}

Table 1 in Appendix A shows the detailed results of average wages, residual wage dispersion and our index for bargaining prevalence, as well as the number for observations for each occupation.\footnote{In the table we report the minimum number of observations for each occupation which normally is for the bargaining proxy. There are more observations for wage dispersion and many more for the average wage.} The results are depicted in Figure 4.1.

As can be seen from Figure 4.1 we find that occupations where workers bargain more have both a higher average wage as well as a higher residual wage dispersion. Both correlations are positive and different from zero at the 0.005 level of significance.

Note that occupations with high values of wage dispersion and bargaining represent construction and lawyers. This feature of the data goes in line with our mechanism,
and will be discussed in more detail in the next subsection.

Figure 4.1: Relationship between bargaining prevalence and residual wage dispersion and average wages, by occupation

4.2 Comparison of model predictions with the data

To compare the model predictions with those of the data we do some Monte-Carlo simulations by drawing from the two-dimensional grid of values of worker and firm costs as per the state space shown in Figures 3.1 and 3.2. We assume two types of workers and firms, five workers and firms of each type, with a fixed preference structure and we compute the equilibria.\(^9\)

Because in the data we look at the results by occupation, to define an occupation \(o\) in the model, we draw pairs of costs for workers \(\theta_x\) and firms \(\theta_y\) from a two-dimensional normal distribution \(N \left( \left[ \theta_x^o, \theta_y^o \right], \Sigma \right)\). The mean parameters \(\left[ \theta_x^o, \theta_y^o \right]\) of these distributions for each occupation \(o\) are also drawn randomly from the space of all possible parameters. For each occupation we generate artificial observations for frequencies of matching, amount of search effort by workers, bargained or posted wages, residual wages (demeaned for each pair of costs), as well as whether bargaining or wage-posting was used in the matching. For each occupation, we compute the average residual wage dispersion, average prevalence of bargaining and the average log wage (all weighted by matching frequency and cost pair frequency combined). The results are plotted on Figure 4.2. The model predicts positive relationships for both, like we see in the data.\(^9\)

\(^9\)In Figures 3.1 and 3.2 we show results for symmetric vertical preferences, but they differ only slightly for alternative specifications and do not seem to depend on the preference specification.
Similarly to search costs, in the model, with a tighter labor market, more firms choose to bargain, and as a result residual wage dispersion is higher, see Figure 4.3 for the results. This is in line with empirical results. Morin (2019) shows that residual wage dispersion is procyclical, supporting our finding that a tighter labor market is associated with more residual wage dispersion. At the same time Brenzel, Gartner, and Schnabel (2013) show that when labor markets are tight, bargaining dominates over wage posting. This is also in line with our theoretical findings.

Because we cannot directly observe bargaining prevalence in the data and we have constructed a proxy, we wanted to do some sort of validation exercise. To do so, we run artificial regressions of the residual wages on search effort to obtain an artificial measure of a bargaining proxy from our model simulations. Weighted averages of the bargaining proxy by occupation are strongly positively with true values of bargaining prevalence as shown in Figure 4.4.
Conclusion

We show that heterogeneity in search costs as well as labor market tightness are important for determining bargaining prevalence. Bargaining prevalence is positively correlated with the wage level, residual wage dispersion, and market tightness. All these predictions are supported by the data.
References


Appendix A: Wage Dispersion
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Table 1: Relationship between average wage, wage dispersion and bargaining prevalence