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# Wage Setting Under Targeted Search

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**Abstract**

When setting initial compensation, some firms set a fixed, non-negotiable wage while others bargain. In this paper we propose a parsimonious search and matching model with two sided heterogeneity, where the choice of wage-setting protocol, wages, search intensity, and degree of randomness in matching are endogenous. We find that posting and bargaining coexist as wage-setting protocols if there is sufficient heterogeneity in match quality, search costs, or market tightness and that labor market tightness and relative costs of search play a key role in the choice of the wage-setting mechanism. We validate the model and find that bargaining prevalence is positively correlated with wages, residual wage dispersion, and labor market tightness, both in the model and in the data. We find coexistence of wage setting protocols arises from miscoordiation among firms and may lead to large welfare losses. It is precisely in these cases that a policy of mandatory wage posting tends to further reduce welfare.

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# 1 Introduction

Evidence from both establishment and employee surveys demonstrates that, when setting initial compensation, around two thirds of firms stipulate a fixed, non-negotiable wage, while the other third bargains with the employee (see Hall and Krueger (2012), Brenzel, Gartner, and Schnabel (2014), and Doniger (2015)). Why do these two wage-setting protocols coexist? What are the welfare consequences of such coexistence?

The existing literature suggests that the coexistence of the two protocols could be due to asymmetric information. If firms that bargain have an informational advantage over wage-posting firms, then better workers will self-select into bargaining firms producing a separating equilibrium. However, empirical evidence shows that bargaining seems to be important for a wide range of worker types (Caldwell and Harmon (2019)) and that both wage-setting mechanisms seem to co-exist even within narrowly defined segments of the labor market (as shown in studies of online platforms, such as Banfi and Villena-Roldan (2019) and Marinescu and Wolthoff (2020)), going against the separating equilibria generated by asymmetric information assumptions.

Motivated by these issues, in this paper, we propose a model *without* asymmetric information where the choice of wage-setting protocol, wages, search intensity, and degree of randomness in matching are all endogenous. We build on Cheremukhin, Restrepo-Echavarria, and Tutino (2020) by formulating a sequential version of the model where firms first post vacancies (which might include wage menus if they opt to post wages), workers choose where to apply in a probabilistic way, and then firms probabilistically choose among the workers that applied and make job offers. In our model, heterogeneous firms endogenously choose a wage-setting protocol — ex-post bargaining or wage posting (with commitment), and heterogeneous workers decide to which type of firm they send their job applications given available job postings. We find that even in the absence of any informational (or other) asymmetry between workers and firms, the two wage-setting protocols coexist as long as there is sufficient heterogeneity in match quality, search costs, or market tightness.

In the model, workers and firms incur search costs related to their imperfect ability to distinguish among potential partners. Even though agents know the distribution and their preferences over types, they do not know where to find a particular type. To do so, they decide how much effort they want to exert to locate a particular type of partner by trading off the cost of search with the payoff they can achieve if successful in finding their desired match. Therefore, agents choose whom to contact in a probabilistic way, and the strategies chosen are discrete probability distributions over types. Each element of the distribution represents the probability with which an agent will target (i.e., contact) each potential match based on the agent's expected payoff. Exerting more search effort, which results in a higher search cost, allows agents to spot a particular type more accurately. Given the discrete nature of the probability distributions, we model the search cost as proportional to the distance between an uninformed, uniform, strategy, where every type has the same probability of being contacted, and the distribution that is optimally chosen by the agent.[1]

We characterize theoretically and numerically the equilibrium properties of the strategies of workers and firms, the posted and bargained wages, and the frequency with which each wage-setting mechanism is used. We study the implications of different relative search costs and varying market tightness on the choice of the wage-setting mechanism, and we explore the relationship between bargaining prevalence, wage level, (residual) wage dispersion, and labor market tightness.

The reason that firms that post a wage not equal to the bargained wage is they can get workers of better skill by incentivizing them the right way. If they post a higher wage, skilled workers will self-select. If they were to deviate and post the bargained wage, then workers apply more randomly. Alternatively, if they decide to bargain, they cannot delegate effort and have to screen the workers themselves. In equilibrium with some bargaining, workers get a high posted wage with the right firm, a low posted wage with the wrong firm, and bargained wages in between. Workers prefer higher posted wages, so more workers apply to wage posting firms. However, firms can get higher profits at higher effort if they decide to bargain. Therefore, some firms will choose to bargain and some will choose to wage post. The fraction is such as to make firms indifferent.

As the cost of screening for firms increases, the spread between posted wage and bargained wage increases: a firm attracts more skilled workers by posting wages, but higher wages imply lower profit, which makes wage posting very unattractive. Therefore, most firms will choose to bargain with random candidates rather than giving all

[1]This cost specification in Cheremukhin et al. (2020) is borrowed from the literature on discrete choice under information frictions (Cheremukhin et al. (2015) and Matejka and McKay (2015)).

of the surplus to extremely skilled candidates. When workers are smart they will shift where to apply significantly in response to small changes in posted wages. Lowering the posted wage even a little would reduce the expected benefit for the firm by a lot because both the number and quality of applicants will significantly deteriorate. In equilibrium firms will not simultaneously post lower wages because each could attract a lot more and better applicants through a small increase in the posted wage.

When both workers and firms are smart, both routes are attractive: firms can incentivize workers to self-select through higher wages, or pay less and screen the workers better. The lack of coordination among firms on which mechanism to use leads to coexistence of bargaining and wage posting in equilibrium, and substantial welfare losses.

Apart from the fact that we find that two wage-setting protocols coexist if there is sufficient heterogeneity (with no need for asymmetric information), our model has several testable implications. First, labor market tightness and relative costs of search play a key role in the choice of the wage-setting protocol. When firms have lower costs than workers (or workers outnumber firms), firms can post low wages because they can identify good workers themselves and do not have to share the surplus. As firms' costs increase (or workers become scarcer), firms set higher wages to delegate the search problem to the workers and encourage self selection. At the same time, firms will find it suboptimal to give a growing fraction of the surplus to the workers, so they will gradually switch to bargaining. Bargaining will be most prevalent when firms' costs are much higher than workers' (or workers are much scarcer than job opportunities). The second set of implications is that bargaining prevalence is generally positively correlated with the level of wages, residual wage dispersion, and labor market tightness.

These implications are testable in that we can check if they are observable features of the data. So we validate our theoretical predictions using data from the Survey of Consumer Expectations of the Federal Reserve Bank of New York for 2013-2017. We follow Faberman, Mueller, Sahin, and Topa (2019) to obtain a measure of residual wages by regressing log annualized real wages on job characteristics and demographic characteristics and use the residuals from the regression to compute the weighted standard deviations of residual log real wages by occupation. To construct a measure of bargaining prevalence we add to the same regression a set of variables representing measures of search effort, such as the type of work a person is looking for (full/part time), the number of applications that were sent, the number of potential employers that contacted the worker, the number of job offers received, and indicators of search methods used. We take the part of the wage variation jointly explained by these additional variables as a proxy for bargaining prevalence, and we rescale the proxy to cover the unit interval. We find that the correlation between average wages and bargaining prevalence is 0.47 in the data and 0.60 in the model and that the correlation between residual wage dispersion and bargaining prevalence is 0.58 in the data and 0.81 in the model.

From these results we can also see that our model sheds light on the determinants of residual wage dispersion, which have been a long-standing puzzle in the literature. We believe that understanding how the choice of wage-setting mechanism affects both wage levels and wage dispersion can help explain part of this puzzle. Welfare analysis indicates that in our model, wage dispersion and labor market inefficiency (mismatch) both increase as firms choose to bargain more often.

Finally, as discussed above, the effects of labor market tightness are very similar to the effects of relative search costs. In the model, a tighter labor market implies that more firms choose to bargain, hence there is higher residual wage dispersion. Indeed, Brenzel, Gartner, and Schnabel (2014) show empirically that when labor markets are tight, bargaining dominates over wage posting. In addition, Morin (2019) shows that there is evidence of residual wage dispersion being pro-cyclical—increasing with labor market tightness—consistent with the prediction of our model.

Another important implication of our results is an evaluation of welfare consequences of labor market outcomes as well as labor market policies mandating wage posting. We find that when wage posting and bargaining coexist in equilibrium, miscoordination among firms can imply large welfare losses. While forcing firms to post wages may often be welfare improving, it is precisely in the cases of coexistence and consequent welfare losses from miscoordination that mandatory wage posting typically significantly reduces welfare further.

The paper proceeds as follows. Section 2 outlines the model; describes the sequential targeted search model, allowing for either bargaining or posting as an optimal mechanism of wage determination; and discusses the theoretical implications. Section 3 solves the model numerically and explains the driving forces behind the results. Section 4 compares model implications with the data, and Section 5 concludes.

## Related literature

In Cheremukhin, Restrepo-Echavarria, and Tutino (2020) we developed a theory of targeted search where search was simultaneous and the payoff was set through bargaining, and we analyzed it in the context of the marriage market. In this paper we focus on the labor market and extend our previous setup to a sequential search setting where we allow firms to choose either a bargaining or a wage posting mechanism. Like in Cheremukhin, Restrepo-Echavarria, and Tutino (2020), our paper effectively blends two sources of randomness used in the literature. The first source is a search friction with uniformly random meetings and impatience, as in Shimer and Smith (2000). The second approach introduces unobserved characteristics as a tractable way of accounting for the deviations of data from the stark predictions of the frictionless model, as in Choo and Siow (2006) and Galichon and Salanie (2012). We introduce a search friction into the meeting process by endogenizing agents' choices of whom to contact. We build on the discrete-choice rational-inattention literature—i.e., Cheremukhin, Popova, and Tutino (2015) and Matejka and McKay (2015)—that derives multinomial logit decision rules as a consequence of cognitive constraints that capture limits to processing information. Therefore, the equilibrium matching rates in our model have a multinomial logit form similar to that in Galichon and Salanie (2012). Unlike Galichon and Salanie, the equilibrium of our model features strong interactions between agents' contact rates driven entirely by their choices, rather than by some unobserved characteristics with fixed distributions.

The search and matching literature has seen multiple attempts to produce intermediate degrees of randomness with which agents meet their best matches. In particular, Menzio (2007) and Lester (2011) nest directed search and random matching to generate outcomes with an intermediate degree of randomness.[2] Our paper produces equilibrium outcomes in between uniform random matching and the frictionless assignment, endogenously, without nesting these two frameworks. One recent paper considering our specification of targeted search with information costs in application to the labor market is Wu (2020).

Also note that although the directed search literature, such as Eeckhout and Kircher (2010) and Shimer (2005), technically involves a choice of whom to meet, the choice is

[2] Also, see Yang's (2013) model of "targeted" search that assumes random search within perfectly distinguishable market segments.

degenerate—directed by signals from the other side. See Chade, Eeckhout, and Smith (2017) for a thorough summary of this literature.

This paper is also related to a growing literature on the coexistence of various wage-setting mechanisms. For instance, Barron, Berger, and Black (2006), Hall and Krueger (2012), Brenzel, Gartner, and Schnabel (2014) and Doniger (2015) estimate the prevalence of wage posting in survey data for the U.S. and Germany and find a prevalence of both bargaining and wage posting. Several models have been developed to explain their coexistence. Michelacci and Suarez (2006) argue that despite inefficiencies inherent in bargaining, it may be preferable over wage posting in the presence of large heterogeneity in unverifiable productivity because it renders any posted wage contract incomplete. In this model, as well as in Doniger (2015), wage posting and bargaining coexist because firms that bargain can use an ex post signal on the productivity or the outside option of a worker, while a wage-posting firm cannot condition on worker-specific characteristics. These two approaches follow a large literature on the micro-foundations of mechanisms including Postel-Vinay and Robin (2004), Bontemps, Robin, and Van Den Berg (2000), and Holzner (2011). A recent paper by Flinn and Mullins (2018) allows for a wage posting firm to condition the wage contract on worker productivity, but the heterogeneity in workers' outside options as well as ex post renegotiation under bargaining still leads to the coexistence of different wage-setting protocols. The key difference from our model is that all of these approaches assume that wage posting firms have better access to information, leading to self-selection of better workers and matching with higher-paying bargaining firms.

Our model features no such asymmetry and thus explains the coexistence of the two protocols even within narrowly defined submarkets, e.g. within individual occupations and job titles. We endogenize the processing of information on both sides of the market (by workers about firms and by firms about workers) assuming all of the characteristics of workers and firms are observable (at a cost). We place no restrictions on heterogeneity allowing both workers and firms to differ along multiple dimensions, including productivity and outside options. In our model, firms choose to post a low-wage schedule when it is cheap for them to distinguish among the applicants. As it becomes more expensive to screen workers, firms incentivize workers to self-select by posting higher-wage schedules. Firms facing high costs of screening would have to promise very high wages for workers to self-select, leaving only a small fraction of the surplus for the firm.

When it is more costly to screen workers, firms prefer wage bargaining over a posted wage schedule. Wage bargaining and wage posting coexist in our model simply because of heterogeneity among both workers and firms. We discuss the intuition and why they coexist under a wide range of parameters in Section 3.

# 2 Targeted search model with terms of trade

In this section, we present a model where firms are looking to fill a vacancy, and workers—who are either employed or unemployed—are looking to find a job. Each agent chooses a probabilistic search strategy that can be interpreted as a search intensity over types, where each element of this distribution reflects the likelihood of contacting a particular agent on the other side. A more targeted search, or a probability distribution that is more concentrated on a particular group of agents (or agent), is associated with a higher cost, as the agent needs to exert more effort to locate a particular potential match more accurately.

The economy contains a large, finite number of individual agents: workers whose types are indexed by $x \in \{1, ..., W\}$ and firms whose types are indexed by $y \in \{1, ..., F\}$. We denote by $\mu_x$ the number of workers of type $x$ and by $\mu_y$ the number of firms of type $y$. We think of workers and firms characterized by a multidimensional set of attributes. Types $x$ and $y$ are unranked indices that aggregate all attributes.

A match between any worker of type $x$ and any firm of type $y$ generates a payoff (surplus) $f_{xy}$. We do not place any restrictions on the shape of the payoff function, and we normalize the outside option of both the worker and the firm to zero. We denote the payoff (wage) appropriated by the worker $\omega_{xy}$ and the payoff appropriated by the firm $\eta_{xy}$ such that $\eta_{xy} = f_{xy} - \omega_{xy}$.

Agents form a match if they meet, and each agent (weakly) benefits from forming a match; i.e., each agent's payoff is non-negative. Since a negative payoff corresponds to absence of a match, we make the following assumption on the payoffs:

**Assumption 1.** The payoffs are non-negative:

$$f_{xy} \geq \omega_{xy} \geq 0.$$

When seeking to form a match, both workers and firms know the number of agents of each type and the characteristics of their preferred types on the other side of the market. They face a noisy search process where they are uncertain about how to locate their preferred match. In this environment, each agent's action is a probability distribution over agents on the other side of the market. Since the number of potential matches is finite, the strategy of each agent is a discrete probability distribution. Let $\bar{p}_x(y)$ be the probability that a worker of type $x$ targets or sends an application to a firm of type $y$. Similarly, we denote by $\bar{q}_y(x)$ the probability that a firm of type $y$ targets or considers the application of a worker of type $x$.

Reducing the noise to locate a potential match more accurately is costly: It involves a careful analysis of the profiles of potential matches, with considerable effort in sorting through the multifaceted attributes of each firm and candidate. When seeking to form a match, agents rationally weigh costs and benefits of targeting the type of characteristics that result in a suitable match. A worker rationally chooses their strategy $\bar{p}_x(y)$ by balancing the costs and benefits of targeting a given firm. A strategy $\bar{p}_x(y)$ that is more concentrated on a particular firm of type $y$ affords them a higher probability to be matched with their preferred firm. However, it requires more effort to sort through profiles of all the firms in the market to locate their desired match and exclude the others. So locating a particular firm or worker more accurately requires exerting more search effort, and it is costlier.

We assume that agents enter the search process with a uniform prior of whom to target, $\tilde{p}_x(y)$ and $\tilde{q}_y(x)$. Choosing a more targeted strategy implies a larger distance between the chosen strategy and the uniform prior and is associated with a higher search effort. A natural way to introduce this feature into our model is the Kullback-Leibler divergence (relative entropy),[3] which provides a convenient way of quantifying the distance between any two distributions, including discrete distributions as in our model. We assume that the search effort of worker $i$ of type $x$ is defined as follows:

$$\kappa_x = \sum_{y=1}^{F} \mu_y \bar{p}_x(y) \ln \frac{\bar{p}_x(y)}{\tilde{p}_x(y)}. \tag{2.1}$$

[3] In the model of information frictions used in the rational inattention literature, $\kappa_x$ represents the relative entropy between a uniform prior and the posterior strategy. This definition is a special case of Shannon's channel capacity, where information structure is the only choice variable (See Thomas and Cover (1991), Chapter 2). See also Cheremukhin, Popova, and Tutino (2015) for an application to stochastic discrete choice with information costs.

We assume that the search costs $c_x(\kappa_x)$ are a function of the search effort $\kappa_x$. Note that $\kappa_x$ is increasing in the distance between a uniform distribution over firms and the chosen strategy, $\bar{p}_x(y)$. If an agent does not want to exert any search effort, she can choose a uniform distribution over types and meet firms randomly. As she chooses a more targeted strategy, the distance between the uniform distribution and her strategy $\bar{p}_x(y)$ grows, increasing search effort $\kappa_x$ and the overall cost of search. By increasing the search effort, agents bring down uncertainty about locating a prospective match, which allows them to target their better matches more accurately.

Likewise, a firm's cost of search $c_y(\kappa_y)$ is a function of the search effort defined as:

$$\kappa_y = \sum_{x=1}^{F} \mu_x \bar{q}_y(x) \ln \frac{\bar{q}_y(x)}{\tilde{q}_y(x)}. \tag{2.2}$$

Furthermore, we assume the following:

**Assumption 2.** The search costs of agents $c_x(\kappa)$ and $c_y(\kappa)$ are strictly increasing, twice continuously differentiable and (weakly) convex functions of search effort.

As a special case, we consider a linear cost of search. Then, the total costs of search for a worker of type $x$ are given by $c_x = \theta_x \kappa_x$ and for a firm of type $y$ by $c_y = \theta_y \kappa_y$, where $\theta_x \geq 0$ and $\theta_y \geq 0$ are the marginal costs of search.

For convenience in comparing wage posting and bargaining setups, we introduce a new notation for the strategies of the workers and firms. We define the workers' and firms' search intensities as the ratios of their posterior and prior: $p_x(y) = \frac{\bar{p}_x(y)}{\tilde{p}_x(y)}$ and $q_y(x) = \frac{\bar{q}_y(x)}{\tilde{q}_y(x)}$, respectively.

The meeting rate depends on the strategies of each agent, $p_x(y)$ and $q_y(x)$, and a congestion function $\phi(p_x(y), q_y(x), \mu_x, \mu_y)$, which depends in some general way on the strategies of all other agents as well as the number of agents of each type. Given this, the total number of matches formed between workers of type $x$ and firms of type $y$ is given by

$$M_{x,y} = \mu_x \mu_y p_x(y) q_y(x) \phi(p_x(y), q_y(x), \mu_x, \mu_y).$$

**Assumption 3.** The congestion function is twice continuously differentiable in each $p$ and $q$.

We introduce this congestion function following Shimer and Smith (2001) and Mortensen (1982), who assume a linear search technology. Note that if $\phi(...) = 1$, then a match takes place if and only if there is mutual coincidence of interests; i.e., both agents draw each other out of their respective distribution of interests. By introducing a congestion function we are allowing for matches to depend in some general way on both an agent's search intensity[4] for a specific agent ($p$ and $q$) and on the number of agents taking part.

Note that when setting up the congestion function we implicitly assume that there are no direct inter-type congestion externalities. However, our model still features strong indirect equilibrium interactions between the strategies of agents that work akin to inter-type congestion by attracting or deterring agents.

## 2.1 Sequential targeted search

To initiate the search and matching process, firms start by posting vacancies. If the posted vacancy includes a wage menu, then the firm commits to paying a type-dependent wage in the case of matching. If vacancies do not explicitly mention wages, then wages are negotiated via Nash bargaining upon forming a match. After the vacancies are posted, and because workers cannot perfectly distinguish which firm is of which type despite learning the wage menus of each firm, they choose a distribution of search intensities that determines the likelihood of contacting a particular firm and choose one firm from this distribution to send an application. Finally, once firms have received worker's applications, each firm chooses the worker to which it will extend a job offer from the set of workers that applied to that particular firm.

When workers decide where to send their applications, they take as given the (posted or bargained) wages of firms, such that the set of strategies of workers $p_x(y) \in S_x$ is given by:

$$S_x = \left\{ p_x(y) \in R_+^F : \sum_{y=1}^{F} \frac{\mu_y}{\delta_x} p_x(y) \leq 1 \right\},$$

where $p_x(y) = \frac{\bar{p}_x(y)}{\tilde{p}_x(y)}$, and $\tilde{p}_x(y) = 1/\sum_{y=1}^{F} \mu_y = 1/\delta_x$ is the worker's uniform prior over

[4]Note that here, search intensity refers to how concentrated the distribution of interests of an agent is. A higher search intensity results in assigning higher probability to one or several agents within an agent's distribution of interests.

the whole set of firms $\left(\delta_x = \sum_{y=1}^{F} \mu_y\right)$.

The firms on the other hand will all choose a strategy $q_y(x)$, but firms that opt to bargain do not optimize over the wage, while those that opt to post wages do. As such, we define a set of strategies for when the firm bargains and another set of strategies for when the firm posts wages. The other difference between the problem of workers and firms is that firms do not sort through all the workers that are looking for a job; they only sort through those that send an application to their firm, and when doing so, firms do not know the types of the workers that applied, but they know the length and expected composition of the queue. In expectation, the queue of firm $y$ contains $\mu_x p_x(y)\,\delta_x/\mu_y$ workers of type $x$.

We define the set of strategies available for when a firm chooses to bargain, $q_y(x) \in S_y^B$, as:

$$S_y^B = \left\{ q_y(x) \in R_+^W : \sum_{x=1}^{W} a_{xy} q_y(x) \leq 1 \right\}.$$

where $q_y(x) = \frac{\bar{p}_y(x)}{\tilde{p}_y(x)}$, and $\tilde{q}_y(x) = 1/\sum_{x=1}^{W} \left(\mu_x p_x(y)\,\delta_x/\mu_y\right)$ is the firm's uniform prior over their own queue. Here we define new variables for queue weights $a_{xy} = \frac{\mu_x p_x(y)}{\Sigma_{x=1}^{W} \mu_x p_x(y)}$, and queue length $\delta_y = \Sigma_{x=1}^{W} \mu_x p_x(y)$.

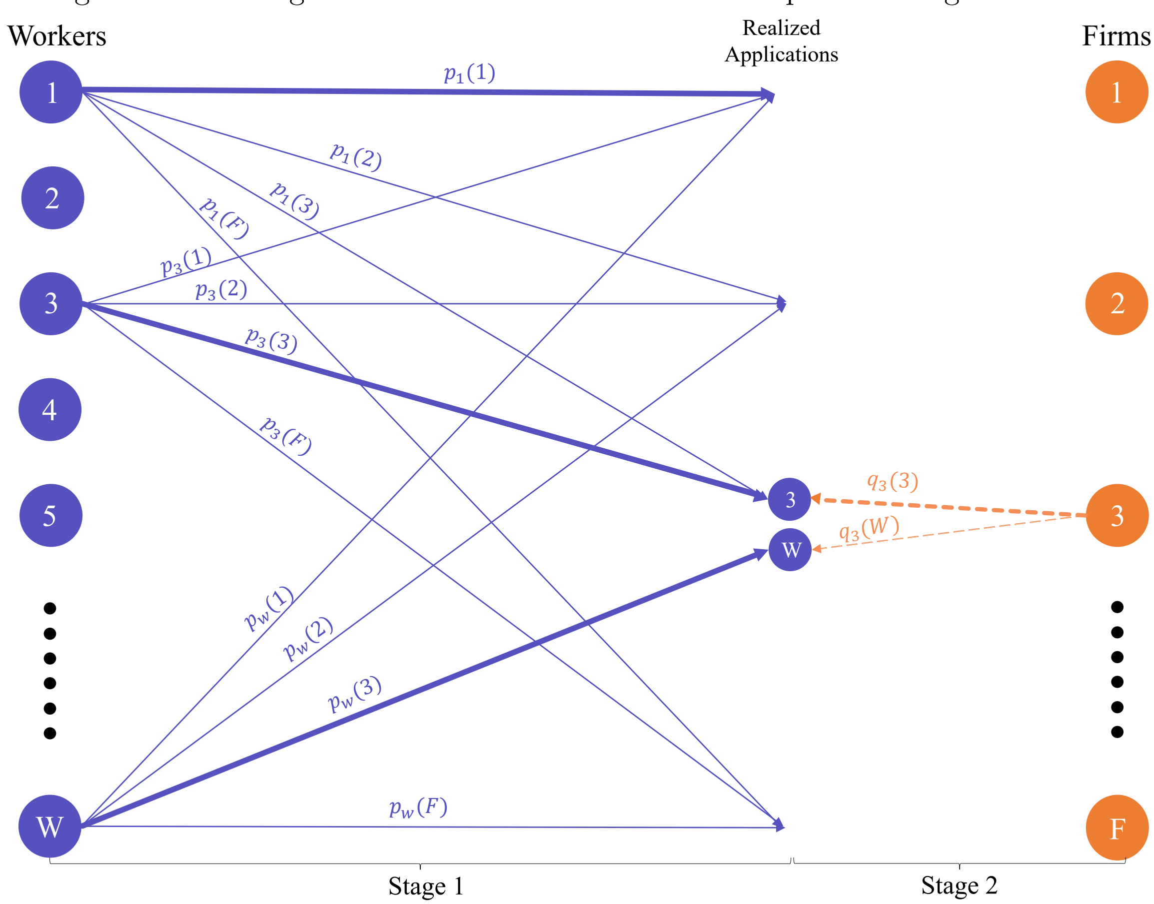
In case the firm chooses to post wages, we can augment the previous strategies as follows:

$$S_y^{WP} = \left\{ q_y(x), \omega_{xy} \in R_+^W : \sum_{x=1}^{W} a_{xy} q_y(x) \leq 1, \omega_{xy} \leq f_{xy} \right\}.$$

The set of actions $s \in S$ is given by the cartesian product of the sets of strategies of workers $s_x \in S_x$ and firms $s_y \in S_y$.

Figure 2.1 illustrates the interactions and search strategies of workers and firms. The solid arrows show the intensity $p_x(y)$ that a worker of type $x$ assigns to targeting a firm of type $y$. Similarly, dashed arrows show the intensity $q_y(x)$ that a firm of type $y$ assigns to targeting a worker of type $x$. Once these are selected, both workers and firms make one draw from their respective distributions to determine where to send an application and which applications to inspect (denoted by bold arrows).

Figure 2.1: Strategies of Workers and Firms under Sequential Targeted Search



Although applications and/or job offers are not lost in the mail, there is still a coordination problem: $\mu_x p_x(y)$ workers applied to type $y$ firms, and firms sent $\mu_y q_y(x)\,\mu_x p_x(y)$ job offers, but they did not necessarily send all of those to different workers. Several firms might contact the same worker, and some workers may not get any offers. We assume that $\mu_x p_x \mu_y q_y \phi_{xy}$ matches are created, where the coordination problem between type $x$ workers and type $y$ firms is captured by the congestion function/meeting technology $\phi_{xy}(p_x, q_y, \mu_x, \mu_y)$ described earlier.

Both firms and workers choose their optimal strategies, and if a firm and a worker match, the payoff $f_{xy}$ is split between them. As mentioned before, we consider two mechanisms in which the surplus could be split between the worker and the firm. The first mechanism is ex-post Nash bargaining with bargaining power $\beta$, implying a wage $\omega_{xy} = \beta f_{xy}$, which is fully anticipated ex ante by both sides. The second mechanism involves firms posting type-dependent wage menus in the first stage of the game.

In both cases, the game is sequential as in Stackelberg in that when firms choose their search effort (and post wages), they internalize the best response strategies of workers. Firms behave like leaders and workers behave like followers. However, consistent with the assumptions of the simultaneous model (see Cheremukhin, Restrepo-Echavarria, and Tutino (2020)), neither the workers nor the firms internalize the effects of their strategies on the congestion function. This is because there are a large number of individuals of each type, so a change in an individual firm's or worker's strategy will not have a noticeable aggregate effect on the number of matches.

**Assumption 4.** Agents take the meeting rates they face as given, disregarding the dependence of the congestion function on agents' own search intensities.

**Definition.** A *matching equilibrium* is a set of admissible strategies for workers $s_x \in S_x$, firms $s_y \in S_y$, and meeting rates, such that the strategies solve the problems for each individual firm and worker given the meeting rates, which are consistent with the strategies of the agents.

### 2.1.1 The problem of the worker

We start by describing the problem of the worker, which is unaffected by the wage-setting protocol. Workers take as given $q_y(x)\,\phi_{xy}$—the probability of forming a match with type $y$ firms. The worker receives a wage $\omega_{xy}$ in the case of matching and bears a linear cost of search $\theta_x \kappa_x (p_x (y))$. The goal of type $x$ workers is to maximize surplus subject to a constraint on strategies (with renormalized Lagrange multiplier $\lambda_x$):

$$Y_x = \Sigma_{y=1}^{F} \mu_y q_y (x) \phi_{xy} \omega_{xy} p_x (y) - \theta_x \Sigma_{y=1}^{F} \frac{\mu_y}{\delta_x} p_x (y) \ln p_x (y) + \theta_x \lambda_x \left(1 - \Sigma_{y=1}^{F} \frac{\mu_y}{\delta_x} p_x (y)\right)$$

Since the objective function of workers is twice continuously differentiable and concave in their own strategies, first-order conditions are necessary and sufficient conditions for equilibrium. Using the necessary first-order conditions we can derive a closed-form solution for the optimal strategy of workers:

$$p_x^* (y) = \frac{\exp\left(\frac{q_y(x)\phi_{xy}\omega_{xy}}{\theta_x/\delta_x}\right)}{\Sigma_{y'=1}^{F} \frac{\mu_{y'}}{\delta_x} \exp\left(\frac{q_{y'}(x)\phi_{xy'}\omega_{xy'}}{\theta_x/\delta_x}\right)}. \tag{2.3}$$

### 2.1.2 The problem of the firm

The goal of type $y$ firms is to choose search intensities over their queue of workers (and possibly wages) to maximize their expected match payoffs $f_{xy} - \omega_{xy}$, net of linear search costs $\theta_y \kappa_y (q_y (x))$ and subject to a constraint on strategies (with renormalized Lagrange multiplier $\lambda_y$):

$$\begin{aligned} Y_y &= \Sigma_{x=1}^{W} \mu_x p_x (y) \phi_{xy} q_y (x) (f_{xy} - \omega_{xy}) - \theta_y \Sigma_{x=1}^{W} \frac{\mu_x p_x (y)}{\Sigma_{x=1}^{W} \mu_x p_x (y)} q_y (x) \ln q_y (x) \\ &\quad + \theta_y \lambda_y \left(1 - \Sigma_{x=1}^{W} \frac{\mu_x p_x (y)}{\Sigma_{x=1}^{W} \mu_x p_x (y)} q_y (x)\right). \end{aligned}$$

The firm internalizes the best responses of the workers (Equation 2.3). To internalize the responses, we need to take derivatives of $p_x (y)$ with respect to the firm's search strategy $q_y (x)$ and, in the case of wage-posting, with respect to the wage $\omega_{xy}$ set by the firm. If we introduce new notation $z_{xy} = \frac{\phi_{xy} q_y(x)}{\theta_x/\delta_x}\left(1 - \frac{\mu_y}{\delta_x} p_x (y)\right)$, then the partial derivatives of (2.3) are conveniently given by: $\frac{\partial p_x(y)}{\partial q_y(x)} \frac{q_y(x)}{p_x(y)} = \omega_{xy} z_{xy}$ and $\frac{\partial p_x(y)}{\partial \omega_{xy}} \frac{1}{p_x(y)} = z_{xy}$. In addition, note that the derivatives of queue weights $a_{xy} = \frac{\mu_x p_x(y)}{\Sigma_{x=1}^{W} \mu_x p_x(y)}$ can be computed as $\frac{\partial a_{xy}}{\partial X} = a_{xy} (1 - a_{xy}) \frac{\partial p_x(y)}{\partial X} \frac{1}{p_x(y)}$.

The problem can be rewritten as:

$$Y_y = \Sigma_{x=1}^{W} \mu_x p_x (y) \phi_{xy} q_y (x) (f_{xy} - \omega_{xy}) - \theta_y \Sigma_{x=1}^{W} a_{xy} q_y (x) (\ln q_y (x) + \lambda_y) + \theta_y \lambda_y,$$

and we can write the first-order condition of the firm with respect to search intensities as follows:

$$\frac{\partial Y_y}{\partial q_y} = \mu_x p_x(y)\frac{\theta_y}{\delta_y}\begin{bmatrix} \frac{\phi_{xy}}{\theta_y/\delta_y}(f_{xy}-\omega_{xy})(1+z_{xy}\omega_{xy})-1 \\ -(\ln q_y(x)+\lambda_y)(1+(1-a_{xy})z_{xy}\omega_{xy}) \end{bmatrix} = 0.$$

Independent of whether the wage is set through wage posting or wage bargaining, it is fully anticipated by firms and workers, so in both cases the strategies of firms satisfy:

$$\ln q_y(x) + \lambda_y = \left(\frac{\phi_{xy}}{\theta_y/\delta_y}(f_{xy}-\omega_{xy})(1+z_{xy}\omega_{xy})-1\right)/\left(1+(1-a_{xy})z_{xy}\omega_{xy}\right).$$

Therefore, firms' strategies in both cases simply solve the following equation:

$$q_y^*(x) = \frac{\exp\left(\frac{\frac{\phi_{xy}}{\theta_y/\delta_y}(f_{xy}-\omega_{xy})(1+z_{xy}\omega_{xy})-1}{1+(1-a_{xy})z_{xy}\omega_{xy}}\right)}{\Sigma_{x'=1}^{W} a_{x'y}\exp\left(\frac{\frac{\phi_{x'y}}{\theta_y/\delta_y}\left(f_{x'y}-\omega_{x'y}\right)\left(1+z_{x'y}\omega_{x'y}\right)-1}{1+\left(1-a_{x'y}\right)z_{x'y}\omega_{x'y}}\right)}.$$

In the case of wage posting, the firms not only choose their distribution of search intensities, but they also optimally choose wage menus in the first stage. We can write the first-order condition with respect to wages as follows:

$$\frac{\partial Y_x}{\partial \omega_{xy}} = \mu_x p_x(y)\, q_y(x)\frac{\theta_y}{\delta_y}\begin{bmatrix} \frac{\phi_{xy}}{\theta_y/\delta_y}((f_{xy}-\omega_{xy})z_{xy}-1) \\ -(\ln q_y(x)+\lambda_y)(1-a_{xy})z_{xy} \end{bmatrix} = 0,$$

and the second-order derivatives as:

$$\frac{\partial^2 Y_x}{\partial q_{xy}^2} = -\frac{1}{q_y(x)}, \qquad \frac{\partial^2 Y_x}{\partial \omega_{xy}^2} = -\frac{\phi_{xy}}{\theta_y/\delta_y}z_{xy}.$$

Since the objective function of firms is twice continuously differentiable and strictly concave with respect to their own strategies, the first-order conditions are necessary and sufficient conditions for equilibrium for both the bargaining and wage-posting cases. Furthermore, we can combine the two optimality conditions to eliminate $q_y(x)$ and obtain a simple expression for an interior solution $0 \leq \omega_{xy} \leq f_{xy}$ for the wage:

$$\omega_{xy}^{*} = \left[ a_{xy} f_{xy} + (1 - a_x) \frac{\theta_y / \delta_y}{\phi_{xy}} - \frac{1}{z_{xy}} \right]_0^{f_{xy}}.$$

Wages stay at the limits because beyond the limits there is no match and the decision-maker is strictly worse off (as reflected in the constraints on the strategy space). In this case we can also substitute the (interior) optimal wage to obtain optimal search intensities of firms:

$$q_y^{*}(x) = \frac{\exp\left(\frac{\phi_{xy}}{\theta_y/\delta_y} f_{xy}\right)}{\Sigma_{x'=1}^{W} a_{x'y} \exp\left(\frac{\phi_{x'y}}{\theta_y/\delta_y} f_{x'y}\right)}.$$

To gain some intuition, note that for an interior wage, $q_y(x) \sim \frac{f_{xy}}{\theta_y/\delta_y}$, and $\omega_{xy} \sim f_{xy} + \frac{\theta_y}{\delta_y} - \frac{\theta_x}{\delta_x}$. Therefore, both probabilistic strategies and wages are increasing functions of the surplus. Wages also positively depend on the cost of search of firms and negatively on the cost of search of workers. Wages transfer part of the firm's search cost onto the worker. When the firms' cost of search increase, workers are promised a larger wage so as to incentivize them to better distinguish which firms to apply to and simplify the screening process for the firms, thus economizing their costs. Also, substituting wages back into the worker's problem we get $\ln p_x(y) \sim \frac{\omega_{xy}}{\theta_x/\delta_x} q_y(x) \sim q_y(x) \frac{\theta_y/\delta_y}{\theta_x/\delta_x}$. Workers put their efforts in distinguishing firms that have a higher matching rate and, hence, higher surplus.

**Proposition 1.** *Under assumptions 1- 4, there exists $\underline{\theta}$ such that for high enough costs relative to the number of agents $\left(\frac{\theta_x}{\delta_x}, \frac{\theta_y}{\delta_y}\right) > \underline{\theta}$ a matching equilibrium exists and is unique.*

*Proof.* The equilibrium of the matching model can be interpreted as a pure-strategy Nash equilibrium of a strategic form game among first-stage decisions of firms. Since the strategy space is a simplex and, hence, a non-empty, convex, compact set, sufficient conditions for the existence of the equilibrium require us to check whether the payoff functions are super-modular on the whole strategy space as in Tarski (1955). Super-modularity can be proven by showing negativity of diagonal elements and non-negativity of the off-diagonal elements of the Hessian matrix.

Let $J_y = \begin{bmatrix} \frac{\partial Y_y}{\partial q_{yx}} & \frac{\partial Y_y}{\partial \omega_{xy}} \end{bmatrix}$ be the Jacobian matrix collecting the set of first-order conditions for all firms $y \in \{1, ..., M\}$, and let $H$ be the corresponding Hessian matrix.

To derive the Hessian matrix, note that under A.1, strategies of each firm are non-cooperative, i.e., independent of the strategies of other types as well as the strategies of the other agents of their own type. Note also that we have assumed no direct inter-type congestion externalities. These assumptions produce a Hessian matrix with a block-diagonal structure, which greatly simplifies the analysis. The Hessian consists of 2x2 blocks along the diagonal of the form:

$$H_{xy} = \begin{bmatrix} \frac{\partial^2 Y_y}{\partial q_{yx}\partial q_{yx}} & \frac{\partial^2 Y_y}{\partial \omega_{xy}\partial q_{yx}} \\ \frac{\partial^2 Y_y}{\partial q_{yx}\partial \omega_{xy}} & \frac{\partial^2 Y_y}{\partial \omega_{xy}\partial \omega_{xy}} \end{bmatrix}.$$

All the remaining off-diagonal elements are zero. The derivatives of interest are quite cumbersome to compute. However, we can express the elements of the Hessian as follows (where $F$ and $G$ are some positive functions):

$$\frac{\partial^2 Y_y}{\partial q_{yx}\partial q_{yx}} = -\frac{1}{q_{xy}} + \frac{\delta_x \delta_y}{\theta_x \theta_y} F\left(f_{xy}, \omega_{xy}, q_{xy}, a_{xy}\right) \leq 0,$$

$$\frac{\partial^2 Y_y}{\partial q_{yx}\partial \omega_{xy}} = \frac{\delta_x \delta_y}{\theta_x \theta_y} G\left(f_{xy}, \omega_{xy}, q_{xy}, a_{xy}\right) \geq 0,$$

$$\frac{\partial^2 Y_y}{\partial \omega_{xy}\partial \omega_{xy}} = -\frac{\delta_x \delta_y}{\theta_x \theta_y} \phi_{xy}\phi_{xy} q_{yx} \leq 0.$$

From this structure, it is clear that if costs of search are large enough (separately or in combination) relative to the number of agents, then all of these inequalities hold, while if costs are very small (or number of agents large) the first inequality is violated. For uniqueness, we need diagonal dominance of the form:

$$\left|\frac{\partial^2 Y_y}{\partial \omega_{xy}\partial \omega_{xy}}\right| \left|\frac{\partial^2 Y_y}{\partial q_{yx}\partial q_{yx}}\right| > \left(\frac{\partial^2 Y_y}{\partial q_{yx}\partial \omega_{xy}}\right)^2.$$

If costs are large enough (or number of agents small enough), then the diagonal terms dominate the off-diagonal terms. On the contrary, when costs are small (or numbers of agents large), then diagonal dominance may well be violated; but at the same time the equilibrium may not exist, so uniqueness is of secondary interest. □

In practice, we find that the threshold $\underline{\theta}$ is quite low, allowing meaningful computations under most parameterizations of interest.

## 2.2 Choice of wage-setting mechanism: bargaining versus posting

In the previous subsection, we described the problem of the firm; how the problem varies depending on whether a firm bargains or posts wages; and how a firm chooses its strategies over workers and sets wages (when it decides to post wages.) In this subsection we describe how the firm optimally decides between the two wage setting-protocols.

To model their choice over the wage-setting mechanism, we allow firms to play probabilistic mixed strategies. We assume that each firm of type $y$ can choose probabilities $0 \leq b_y \leq 1$ of bargaining and $0 \leq c_y \leq 1$ of wage posting, such that $b_y + c_y \leq 1$. In the case of bargaining, the worker gets $\omega^b_{xy} = \beta f_{xy}$ and the firm gets the rest.

From the point of view of workers, bargaining and wage-posting firms are different and workers would like to distinguish them in the second stage of the game. Workers choose intensities $p^b_x(y)$ of targeting bargaining firms and $p^c_x(y)$ of targeting wage posting firms. Now, the workers distribute their attention between $\mu_y b_y$ bargaining firms and $\mu_y c_y$ wage posting firms of each type $y$. Therefore, their strategy space now takes the form:

$$S_x = \left\{ p^b_x(y), p^c_x(y) \in R^F_+ : \sum_{y=1}^{F} \left( \frac{\mu_y b_y}{\delta_x} p^b_x(y) + \frac{\mu_y c_y}{\delta_x} p^c_x(y) \right) \leq 1 \right\}.$$

In turn, firms that bargain will face a different queue composition compared with firms that post wages. This implies that bargaining and wage-posting firms can choose different search intensities, which we denote $q^b_y(x)$ and $q^c_y(x)$. Since the choice of wage-setting mechanism is made by firms in the first stage of the game with full commitment, workers that end up applying to a particular firm accept the chosen mechanism. This means that we can think of all the workers of type $x$ that arrive to a bargaining firm of type $y$ as coming from $p^b_x(y)$ and similarly for wage-posting. Therefore, type $y$ firms that chose to bargain encounter a proportion $\mu_x p^b_x(y)$ of workers of type $x$, and firms that wage-post encounter a proportion $\mu_x p^c_x(y)$ of workers of type $x$. Therefore, we can define queue weights as $a^b_{xy} = \frac{\mu_x p^b_x(y)}{\Sigma^W_{x=1} \mu_x p^b_x(y)}$ and $a^c_{xy} = \frac{\mu_x p^c_x(y)}{\Sigma^W_{x=1} \mu_x p^c_x(y)}$. The strategies space of firms is therefore:

$$S_y = \left\{ \begin{array}{c} q_y^b\left(x\right), q_y^c\left(x\right), \omega_{xy}^c \in R^W, b_y, c_y \in R_+ : \\ \sum_{x=1}^{W} a_{xy}^b q_y^b\left(x\right) \leq 1, \sum_{x=1}^{W} a_{xy}^c q_y^c\left(x\right) \leq 1, \omega_{xy}^c \leq f_{xy}, b_y + c_y \leq 1 \end{array} \right\}.$$

Like before, we first write down the decision problem of workers:

$$\begin{aligned} Y_x = {} & \Sigma_{y=1}^F \mu_y b_y p_x^b\left(y\right) \tfrac{\theta_x}{\delta_x} \left(q_y^b\left(x\right) \tfrac{\phi_{xy}^b}{\theta_x/\delta_x} \omega_{xy}^b - \left(\ln p_x^b\left(y\right) + \lambda_x\right)\right) \\ & + \Sigma_{y=1}^F \mu_y c_y p_x^c\left(y\right) \tfrac{\theta_x}{\delta_x} \left(q_y^c\left(x\right) \tfrac{\phi_{xy}^c}{\theta_x/\delta_x} \omega_{xy}^c - \left(\ln p_x^c\left(y\right) + \lambda_x\right)\right) + \theta_x \lambda_x. \end{aligned}$$

Since the objective function of workers is twice continuously differentiable and concave in their own strategies, first-order conditions are necessary and sufficient conditions for equilibrium. Using the first-order conditions we can derive a closed-form solution for the optimal strategy of workers:

$$p_x^b\left(y\right) = \frac{\exp\left(\frac{q_y^b(x)\phi_{xy}^b\omega_{xy}^b}{\theta_x/\delta_x}\right)}{\Sigma_{y'=1}^F \frac{\mu_{y'}}{\delta_x}\left(b_y \exp\left(\frac{q_{y'}^b(x)\phi_{xy'}^b\omega_{xy'}^b}{\theta_x/\delta_x}\right) + c_y \exp\left(\frac{q_{y'}^c(x)\phi_{xy'}^c\omega_{xy'}^c}{\theta_x/\delta_x}\right)\right)}, \tag{2.4}$$

$$p_x^c\left(y\right) = \frac{\exp\left(\frac{q_y^c(x)\phi_{xy}^c\omega_{xy}^c}{\theta_x/\delta_x}\right)}{\Sigma_{y'=1}^F \frac{\mu_{y'}}{\delta_x}\left(b_y \exp\left(\frac{q_{y'}^b(x)\phi_{xy'}^b\omega_{xy'}^b}{\theta_x/\delta_x}\right) + c_y \exp\left(\frac{q_{y'}^c(x)\phi_{xy'}^c\omega_{xy'}^c}{\theta_x/\delta_x}\right)\right)}. \tag{2.5}$$

Likewise, defining queue lengths of firms by $\delta_y^b = \Sigma_{x=1}^W \mu_x p_x^b\left(y\right)$ and $\delta_y^c = \Sigma_{x=1}^W \mu_x p_x^c\left(y\right)$, the objective function of type $y$ firms is:

$$\begin{aligned} Y_y = {} & b_y \Sigma_{x=1}^W \mu_x p_x^b\left(y\right) q_y^b\left(x\right) \tfrac{\theta_y}{\delta_y^b} \left(\tfrac{\phi_{xy}^b}{\theta_y/\delta_y^b}\left(f_{xy} - \omega_{xy}^b\right) - \left(\ln q_y^b\left(x\right) + \lambda_y^b\right)\right) + b_y\theta_y\lambda_y^b + c_y\theta_y\lambda_y^c \\ & + c_y \Sigma_{x=1}^W \mu_x p_x^c\left(y\right) q_y^c\left(x\right) \tfrac{\theta_y}{\delta_y^c} \left(\tfrac{\phi_{xy}^c}{\theta_y/\delta_y^c}\left(f_{xy} - \omega_{xy}^c\right) - \left(\ln q_y^c\left(x\right) + \lambda_y^c\right)\right) + \theta_y \varpi\left(1 - b_y - c_y\right). \end{aligned}$$

Since the game is sequential, firms need to internalize the responses of workers: $p_x^b\left(y\right)$ with respect to $q_y^b\left(x\right)$ and $b_y$ and $p_x^c\left(y\right)$ with respect to $q_y^c\left(x\right)$, $\omega_{xy}^c$ and $c_y$. If we introduce new notation $z_{xy}^b = \frac{\phi_{xy}^b q_y^b(x)}{\theta_x/\delta_x}\left(1 - \frac{\mu_y}{\delta_x} b_y p_x^b\left(y\right)\right)$ and $z_{xy}^c = \frac{\phi_{xy}^c q_y^c(x)}{\theta_x/\delta_x}\left(1 - \frac{\mu_y}{\delta_x} c_y p_x^c\left(y\right)\right)$, then the partial derivatives of (2.4) are conveniently given by: $\frac{\partial p_x^i(y)}{\partial q_y^i(x)} \frac{q_y^i(x)}{p_x^i(y)} = \omega_{xy}^i z_{xy}^i, \frac{\partial p_x^i(y)}{\partial i_y} \frac{i_y}{p_x^i(y)} = -\frac{i_y\mu_y}{\delta_x} p_x^i\left(y\right) = -g_{xy}^i$, for $i \in \{b, c\}$ and $\frac{\partial p_x^c(y)}{\partial \omega_{xy}^c} \frac{1}{p_x^c(y)} = z_{xy}^c$. In addition, note that the derivatives of queue weights $a_{xy}^i = \frac{\mu_x p_x^i(y)}{\Sigma_{x=1}^W \mu_x p_x^i(y)}$ can be computed as $\frac{\partial a_{xy}^i}{\partial X} = a_{xy}^i\left(1 - a_{xy}^i\right) \frac{\partial p_x^i(y)}{\partial X} \frac{1}{p_x^i(y)}$.

Each firm takes as given the strategies of other firms and can anticipate the response of workers to its own strategy. Here, the first-order conditions are as follows:

$$\frac{\partial Y_y}{\partial q_y^b} = b_y \mu_x p_x^b\left(y\right) \frac{\theta_y}{\delta_y^b} \left[ \begin{array}{c} \frac{\phi_{xy}^b}{\theta_y/\delta_y^b}\left(f_{xy} - \omega_{xy}^b\right)\left(1 + z_{xy}^b \omega_{xy}^b\right) - 1 \\ -\left(\ln q_y^b\left(x\right) + \lambda_y^b\right)\left(1 + \left(1 - a_{xy}^b\right) z_{xy}^b \omega_{xy}^b\right) \end{array} \right] = 0,$$

$$\frac{\partial Y_x}{\partial b_y} = \Sigma_{x=1}^W \mu_x p_x^b\left(y\right) q_y^b\left(x\right) \frac{\theta_y}{\delta_y^b} \left( \begin{array}{c} \frac{\phi_{xy}^b}{\theta_y/\delta_y^b}\left(f_{xy} - \omega_{xy}^b\right)\left(1 - g_{xy}^b\right) \\ -\left(\ln q_y^b\left(x\right) + \lambda_y^b\right)\left(1 - g_{xy}^b\left(1 - a_{xy}^b\right)\right) \end{array} \right) + \theta_y \lambda_y^b - \theta_y \varpi_y = 0,$$

$$\frac{\partial Y_y}{\partial q_y^c} = c_y \mu_x p_x^c\left(y\right) \frac{\theta_y}{\delta_y^c} \left[ \begin{array}{c} \frac{\phi_{xy}^c}{\theta_y/\delta_y^c}\left(f_{xy} - \omega_{xy}^c\right)\left(1 + z_{xy}^c \omega_{xy}^c\right) - 1 \\ -\left(\ln q_y^c\left(x\right) + \lambda_y^c\right)\left(1 + \left(1 - a_{xy}^c\right) z_{xy}^c \omega_{xy}^c\right) \end{array} \right] = 0,$$

$$\frac{\partial Y_x}{\partial c_y} = \Sigma_{x=1}^W \mu_x p_x^c\left(y\right) q_y^c\left(x\right) \frac{\theta_y}{\delta_y^c} \left( \begin{array}{c} \frac{\phi_{xy}^c}{\theta_y/\delta_y^c}\left(f_{xy} - \omega_{xy}^c\right)\left(1 - g_{xy}^c\right) \\ -\left(\ln q_y^c\left(x\right) + \lambda_y^c\right)\left(1 - g_{xy}^c\left(1 - a_{xy}^c\right)\right) \end{array} \right) + \theta_y \lambda_y^c - \theta_y \varpi_y = 0,$$

$$\frac{\partial Y_x}{\partial \omega_{xy}^c} = c_y \mu_x p_x^c\left(y\right) q_y^c\left(x\right) \frac{\theta_y}{\delta_y^c} \left[ \begin{array}{c} \frac{\phi_{xy}^c}{\theta_y/\delta_y^c}\left(\left(f_{xy} - \omega_{xy}^c\right) z_{xy}^c - 1\right) \\ -\left(\ln q_y^c\left(x\right) + \lambda_y^c\right)\left(1 - a_{xy}^c\right) z_{xy}^c \end{array} \right] = 0.$$

Like before, we can derive closed-form expressions for search intensities for bargaining and wage-posting firms:

$$q_y^i\left(x\right) = \frac{\exp\left(\frac{\frac{\phi_{xy}^i}{\theta_y/\delta_y^i}\left(f_{xy} - \omega_{xy}^i\right)\left(1 + z_{xy}^i \omega_{xy}^i\right) - 1}{1 + \left(1 - a_{xy}^i\right) z_{xy}^i \omega_{xy}^i}\right)}{\Sigma_{x'=1}^W a_{x'y} \exp\left(\frac{\frac{\phi_{x'y}^i}{\theta_y/\delta_y^i}\left(f_{x'y} - \omega_{x'y}^i\right)\left(1 + z_{x'y}^i \omega_{x'y}^i\right) - 1}{1 + \left(1 - a_{x'y}^i\right) z_{x'y}^i \omega_{x'y}^i}\right)}.$$

We can also derive the posted wages:

$$\omega_{xy}^c = \left[a_{xy}^c f_{xy} + \left(1 - a_x^c\right) \frac{\theta_y/\delta_y^c}{\phi_{xy}^c} - \frac{1}{z_{xy}^c}\right]_0^{f_{xy}}.$$

The solution for the mechanism choices $b_y$ and $c_y$ is much harder to derive in closed form. We relegate the discussion of the intuition behind the wage-posting vs bargaining decision to Section 3 where we can illustrate it with some numerical results. The matching equilibrium has similar properties to the wage-posting model, with the Hessian now having a less-tractable block-diagonal form with blocks of size 5x5. However, numerical exercises show that the main intuition for the results extends only partially, with large regions of non-existence and non-uniqueness possible depending on the congestion function. However, we find that when the congestion function has a special (Cobb-Douglas) functional form $\phi_{xy}(p_x, q_y, \mu_x, \mu_y) = (p_x)^{-\alpha}(q_y)^{-(1-\alpha)}$, the threshold $\underline{\theta}$ is still quite low, allowing computations for reasonable parameterizations.

Let us consider the forces that affect the optimally posted wage. Recall that $a^c_{xy}$ is the share of workers of type $x$ in the queue of firm of type $y$. Also note that by definition $1/z^c_{xy} \sim \frac{\theta_x/\delta_x}{\phi^c_{xy}}$. Therefore, we can rewrite the wage as:

$$\omega^c_{xy} = \left[a^c_{xy} f_{xy} + (1 - a^c_x)\frac{\theta_y/\delta^c_y}{\phi^c_{xy}} - \psi\frac{\theta_x/\delta_x}{\phi^c_{xy}}\right]_0^{f_{xy}}.$$

The first two terms of wage expression are the productivity of the match and the firm's cost of search (scaled by abundance) weighted by the importance of the type of worker for the firm. The third term is the strategically adjusted (by $\psi$) worker's cost of search (scaled by abundance). The wage is driven by the trade-off between search costs (and abundance) of the workers and firms.

Consider first the case where the firms' costs $\theta_y$ are small and the workers' costs $\theta_x$ are large. Alternatively, the same logic would work if the market is slack, meaning a large number of workers $\delta_y$ are looking for a small number of firms $\delta_x$. In the case of $\frac{\theta_y}{\delta_y} \ll \frac{\theta_x}{\delta_x}$, the wage is reduced toward the lower bound of 0 reflecting the firms' monopsony power. On the other hand, when $\frac{\theta_y}{\delta_y} \gg \frac{\theta_x}{\delta_x}$, that is the workers' costs are small and the firms' costs are large, or the labor market is very tight, the wage tends to hit the upper bound of $f_{xy}$ reflecting the workers' monopsony position. As we shall see in computations, it is more common for firms to choose bargaining instead of wage-posting when the wage they would have to post approaches the upper bound. Therefore, high bargaining prevalence in the data reflects monopsony power of workers and can arise both as a result of uneven search costs and a tight labor market. Similarly, a high prevalence of wage posting indicates strong monopsony power of firms.

## 2.3 Social planner's solution

We solve the social planner's problem for the sequential model, which allows for bargaining or wage posting or any combination of the two. In fact, as we shall see next, the wage decision disappears from the social planner's problem altogether. We can write social welfare as the sum of objective functions of all the agents in the model, as the planner takes into account all the same benefits and costs of the matching process as the agents, subject to the same constraints on search intensities as individual agents. The social welfare function is then:

$$\begin{gathered}\Omega = \Sigma_{x=1}^{W} \mu_x Y_x + \Sigma_{y=1}^{F} \mu_y Y_y = \Sigma_{x=1}^{W} \mu_x \theta_x \lambda_x + \Sigma_{y=1}^{F} \mu_y \theta_y \lambda_y \\ + \Sigma_{x=1}^{W} \Sigma_{y=1}^{F} \mu_x \mu_y p_x(y) \left( q_y(x) \phi_{xy} f_{xy} - \tfrac{\theta_x}{\delta_x} (\ln p_x(y) + \lambda_x) - \tfrac{\theta_y}{\delta_y} q_y(x) (\ln q_y(x) + \lambda_y) \right).\end{gathered}$$

The wages cancel out from the problem, and hence the decision is identical for bargaining and wage posting strategies, which we suppress for simplicity. The first-order conditions for the planner's problem can be written as follows:

$$\frac{\partial \Omega}{\partial p_x(y)} = \mu_x \mu_y \left( \begin{array}{c} q_y(x) f_{xy} \phi_{xy} (1 + \varepsilon_{\phi,p}) - \frac{\theta_x}{\delta_x} (\ln p_x(y) + \lambda_x + 1) \\ - \frac{\theta_y}{\delta_y} (1 - a_{xy}) q_y(x) (\ln q_y(x) + \lambda_y) \end{array} \right) = 0,$$

$$\frac{\partial \Omega}{\partial q_y(x)} = \mu_x \mu_y p_x(y) \left( f_{xy} \phi_{xy} (1 + \varepsilon_{\phi,q}) - \frac{\theta_y}{\delta_y} - \frac{\theta_y}{\delta_y} (\ln q_y(x) + \lambda_y) \right) = 0,$$

and we can deduce that the search intensities prescribed by the planner satisfy:

$$\frac{\theta_y}{\delta_y} (\ln q_y(x) + \lambda_y) = \phi_{xy} f_{xy} (1 + \varepsilon_{\phi,q}) - \frac{\theta_y}{\delta_y},$$

and

$$\frac{\theta_x}{\delta_x} (\ln p_x(y) + \lambda_x + 1) = q_y(x) \left( f_{xy} \phi_{xy} \left[ (1 + \varepsilon_{\phi,p}) - (1 - a_{xy}) (1 + \varepsilon_{\phi,q}) \right] + (1 - a_{xy}) \frac{\theta_y}{\delta_y} \right).$$

Now, lets compare these expressions with those of the competitive equilibrium:

$$\frac{\theta_y}{\delta_y}\left(\ln q_y\left(x\right)+\lambda_y\right)=\left(\phi_{xy}\left(f_{xy}-\omega_{xy}\right)\left(1+z_{xy}\omega_{xy}\right)-\frac{\theta_y}{\delta_y}\right)/\left(1+\left(1-a_{xy}\right)z_{xy}\omega_{xy}\right),$$

$$\frac{\theta_x}{\delta_x}\left(\ln p_x\left(y\right)+\lambda_x+1\right)=q_y\left(x\right)\phi_{xy}\omega_{xy}.$$

Comparing the conditions for the workers, to implement the strategies proposed by the social planner, workers should be promised a wage:

$$\omega_{xy}^{PO,W}=a_{xy}f_{xy}\left(1+\varepsilon_{\phi,p}\right)+\left(1-a_{xy}\right)\frac{\theta_y/\delta_y}{\phi_{xy}}.$$

And similarly, comparing the conditions for the firms, to implement the socially optimal strategies, firms should be promised a wage that satisfies:

$$\left(\phi_{xy}\left(f_{xy}-\omega_{xy}\right)\left(1+z_{xy}\omega_{xy}\right)-\frac{\theta_y}{\delta_y}\right)=\left(\phi_{xy}f_{xy}\left(1+\varepsilon_{\phi,q}\right)-\frac{\theta_y}{\delta_y}\right)\left(1+\left(1-a_{xy}\right)z_{xy}\omega_{xy}\right),$$

which boils down to a quadratic equation with respect to wages with one positive solution

$$\omega_{xy}^{PO,F}=\frac{A}{2}+\sqrt{\frac{A^2}{4}-\frac{1}{z_{xy}}f_{xy}\varepsilon_{\phi,q}},$$

where we denote $A=a_{xy}f_{xy}-\left(1-a_{xy}\right)f_{xy}\varepsilon_{\phi,q}+\frac{\theta_y/\delta_y}{\phi_{xy}}\left(1-a_{xy}\right)-\frac{1}{z_{xy}}$. Note that when $\varepsilon_{\phi,q}=0$, the socially optimal wage assigned to firms coincides with the competitive wage posted by firms:

$$\omega_{xy}^{CE}=\left[a_{xy}f_{xy}+\left(1-a_x\right)\frac{\theta_y/\delta_y}{\phi_{xy}}-\frac{1}{z_{xy}}\right]_0^{f_{xy}}.$$

Comparing the three expressions we note that, in the absence of congestion, the first two terms of the posted wage represent the wage that would properly incentivize workers, and all three terms together represent the wage that would properly incentivize firms.

More generally, when $\varepsilon_{\phi,q}\leq 0$, then $\omega_{xy}^{CE}\leq\omega_{xy}^{PO,F}$, and when $\varepsilon_{\phi,p}=0$, then $\omega_{xy}^{CE}<$

$\omega_{xy}^{PO,W}$. For $\phi = 1$ we get $\omega_{xy}^{PO,F} = \omega_{xy}^{CE} < \omega_{xy}^{PO,W}$. However, for the special (Cobb-Douglas) congestion function described earlier (implying a constant returns-to-scale matching function) it is natural to have $\omega_{xy}^{PO,W} < \omega_{xy}^{CE} < \omega_{xy}^{PO,F}$. Because of strong negative congestion externalities, both workers and firms need to be dis-incentivized from putting inefficiently high search efforts by the planner promising lower payoffs in the case of matching. Implementation of this solution looks very much like a tax scheme that can benefit both workers and firms by obtaining similar matching outcomes at a lower search cost and on top generate extra revenue for society.

# 3 Results

In this section we show the numerical results of our model and explore its theoretical implications. In particular, we explore the effects of unequal search costs and different levels of market tightness on the optimal wage-setting strategy.

We start by calculating the equilibrium of the model with a mechanism choice for different combinations of the parameters of interest $(\theta_x, \theta_y, \mu_x, \mu_y)$. We calibrate the remaining parameter values as follows: $f_{xy} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ for horizontal preferences or $f_{xy} = \begin{bmatrix} 2 & 1 \\ 1 & \frac{1}{2} \end{bmatrix}$ for vertical preferences, $\phi_{xy} = (p_x(y)\, q_y(x)\, \mu_x \mu_y)^{\frac{1}{2}}$ for the congestion function and $\beta = 0.5$ for bargaining power. For each combination $(\theta_x, \theta_y, \mu_x, \mu_y)$ on a four-dimensional equispaced exponential grid we compute the equilibrium by making an initial guess for the strategies of the workers and firms, computing the equilibrium posted wage, and then checking if the optimality conditions for the remaining strategies are satisfied. We vary the vector of strategies until we find a fixed point.

Our main results using horizontal preferences are shown in Figure 3.1.[5] The figure shows a heat map of how bargaining prevalence changes as the marginal cost of search for firms and workers varies. When the costs of the firm are low relative to the worker's, the firm chooses a high probability of wage posting $c_y = 1$ and prefers to post a wage $\omega_{xy}^{c} = 0$ (dark blue region). This is because firms can locate their preferred workers easily. As the cost of the firm increases relative to the worker's, there is a region where the firm still prefers wage posting, but wages are interior $\omega_{xy}^{c} \in (0, f_{xy})$ this means

[5]For vertical preferences the results are very similar.

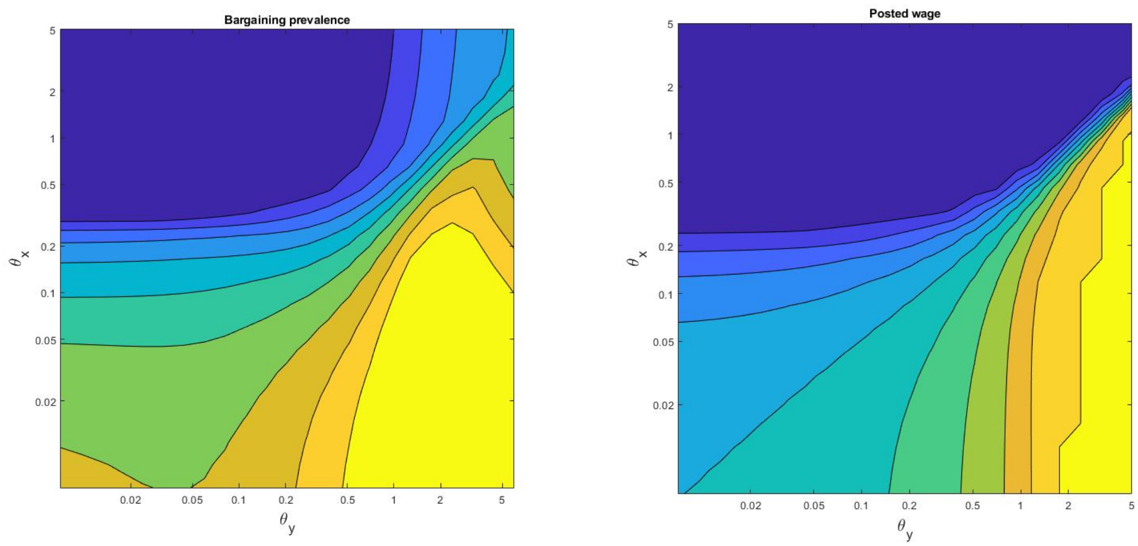
that firms have to promise part of the surplus to the worker so that the workers self select and thereby improve the quality of the match (region with lighter blues). When the cost of the firm increases even further such that it is very costly for the firm to locate a preferred match with accuracy, the bargaining option starts competing with wage posting $b_y > c_y$ (the blue turns green). This is because firms have to commit to giving workers a larger fraction of the surplus to incentivize them to self select efficiently. When the costs of firms are much higher than the costs of workers, firms predominantly choose to bargain over the wage, $b_y = 1$ (bright yellow region).

As seen in Figure 3.1 the transition from bargaining to wage posting is very smooth such that the two mechanisms coincide for a very wide band of parameter combinations. The reason for this is as follows: Along the whole interval, firms choose probabilities to remain indifferent between bargaining and wage posting. Lets start from a point where the wage a firm of type $y$ would post for workers of type $x$ is the same it would have bargained ex post. This means under both protocols the firm would screen the workers in a similar way and the workers of type $x$ are indifferent between the two protocols.

Now imagine the parameters (relative costs or relative numbers of agents) are changed such that the posted wage increases while the bargained wage remains the same. The wage posting type of firm receives a lower payoff. To keep it indifferent, the matching rate of the wage posting firm should increase and the matching rate of the bargaining firm should decrease. This naturally happens to some extent because the increase in the posted wage makes the posting firm more attractive to workers compared with the bargaining firm. But this is not enough for indifference. If the firm were to additionally increase the bargaining probability ever so slightly, this would increase the number of bargaining firms and therefore reduce the per capita interest each of them would receive from workers (who keep allocating the same total attention toward this type of firm). A small increase in the bargaining probability reduces the matching rates of bargaining types and increases the matching rates of wage-posting types, which is enough to restore indifference.

This trade-off extends continuously all the way toward the area of a minimum posted wage. As long as the posted wage is even slightly positive, workers have an incentive to actively target positive-wage firms, and hence firms need to compensate for that by slightly increasing their bargaining probability. The argument also extends towards the area of exclusive bargaining if such an area exists. The tricky part here is that depending

Figure 3.1: Wage setting (horizontal preferences)



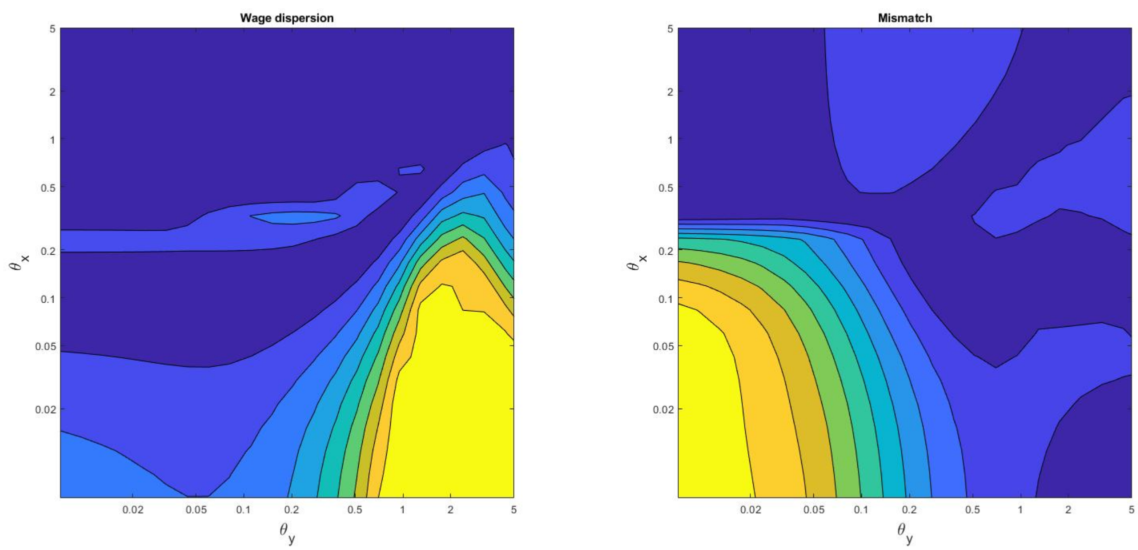

on the shape of the congestion function, the strength of the effect of a wage mechanism switching on the matching rates that firms face could behave in different ways. While under the Cobb-Douglas case presented here the equilibrium smoothly transits towards full bargaining, under the no-congestion scenario ($\phi = 1$) the matching equilibrium unravels (non-existence) before the economy reaches the area where firms would choose only bargaining.

From the expressions for strategies we can see that what affects the mechanism selection is the ratio of search costs to queue length $\theta_i/\delta_i$. Therefore, increasing the search cost of workers is equivalent to decreasing the number of firms of each type, and increasing the search costs of firms is equivalent to decreasing the number of workers of each type. Since this logic works for both workers and firms, a very similar picture shows up if the axes show the number of agents ($\mu_y$ and $\mu_x$) instead of the search costs ($\theta_y$ and $\theta_x$). Hence, when there are a lot more workers than firms, firms post a minimum wage, while as market tightness increases, the wage level and the prevalence of bargaining also increase.

Figure 3.2 shows contour plots for the average level of posted wages (bargained wages are fixed), the level of equilibrium wage dispersion,[6] and the amount of equilibrium mis-

[6] In the data this corresponds to residual wage dispersion - wage inequality observed among otherwise

Figure 3.2: Comparative statics (horizontal preferences)




match.[7] Posted wages need to be high whenever firms' costs are very high (or workers are very scarce). Firms can post minimum wages whenever they have lower costs than workers (or workers outnumber firms). Wage dispersion and mismatch both increase as firms choose to bargain more often. From these graphs it is clear that a prevalence of bargaining is almost uniformly associated with higher residual wage dispersion and higher mismatch. Both could be used as signals of labor market inefficiency.

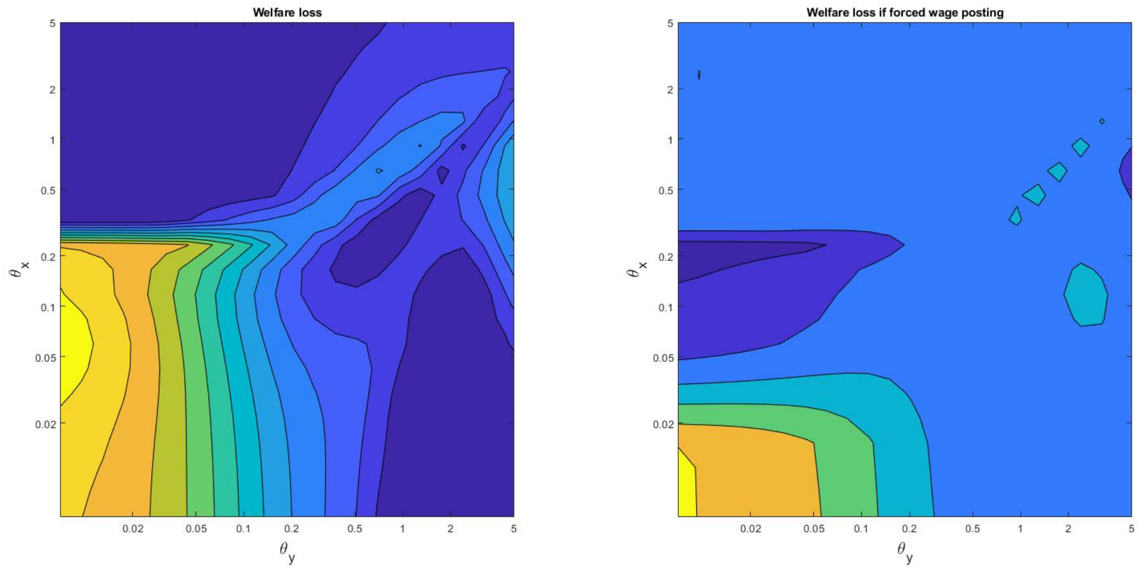
From these relationships we can see that interactions involving bargaining will involve higher search effort on the part of workers, higher overall wages and higher overall wage dispersion, while wage posting will mostly correspond to little search effort on the part of workers, low wages and low wage dispersion.

Another interesting question is the effect of policies that regulate the interaction of workers and firms. One such commonly considered policy is mandating that a range of wages be posted with each vacancy. Our model allows us to evaluate the welfare losses in a mixed competitive equilibrium and the welfare effect of forced wage posting. Figure 3.3 illustrates our findings. When wage posting dominates, or bargaining dominates,

identical workers and firms.

[7] We define mismatch as the difference between the number of matches in competitive equilibrium and those prescribed by a constrained social planner.

Figure 3.3: Welfare effects of policies




the welfare losses are relatively insignificant. The largest welfare losses in a mixed equilibrium occur when costs for workers and firms are low and comparable, so that firms are nearly evenly split between wage posting and bargaining. It is this miscoordination that leads to large welfare losses. Interestingly, it is in this same region that forced wage posting has the largest negative effect. Forced bargaining would be the preferred outcome in this case. Thus, while in many cases mandatory wage posing might improve welfare, in some important cases, especially when equilibrium losses are high, mandating wage posting may reduce welfare even more.

We further explore these relationships in Section 4 where we compare the implications of the model with the data.

# 4 Empirical Validation

## 4.1 Data

As mentioned before, our model produces some testable implications, and to compare the results of the model with the patterns observed in the data, we need to construct model counterparts to what is observable. To that aim, in this section we discuss the data we use and how we proceed to construct a bargaining prevalence proxy, as well as how we recover average wages and residual wage dispersion. We then illustrate the relationship between average wages and bargaining prevalence and the relationship between residual wage dispersion and bargaining prevalence in the data by occupation.

We use data from the Survey of Consumer Expectations carried out by the Federal Reserve Bank of New York. We use the main survey as well as the labor market section, which contains variables reflecting search behavior, and we pull the data for 2013 up to 2017.

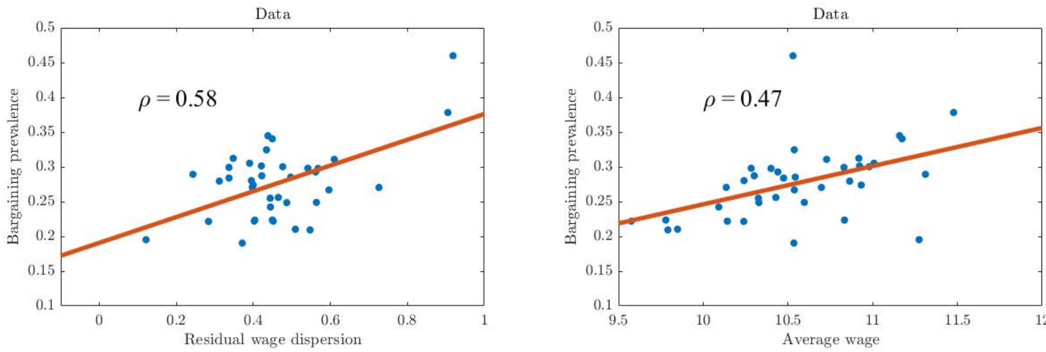
To compute average wages we take the mean of the log real wage, and to recover residual wage dispersion we follow Faberman, Mueller, Sahin, and Topa (2019) and regress log annualized real wages on job characteristics (full-time status, tenure, occupation, firm size, employment benefits, job convenience) and demographic characteristics (race, hispanic origin, education, gender, age, age squared, marital status, co-habitation status, number of children, home-ownership). We use the residuals from the regression to compute the weighted standard deviations of residual log real wages by occupation.

To construct a measure of bargaining prevalence, we add to the same regression that we run to recover residual wages; a set of variables representing measures of search effort, such as the type of work a person is looking for; the number of applications that were sent during the job search process; the number of potential employers that contacted the worker; the number of job offers received; and indicators of search methods used. We believe that the ability of a worker to increase their wage by putting in more effort as well as varying the intensity and method of search reflects the extent to which the wage was bargained, rather than the wage being a take-it-or-leave-it offer. Therefore, we take the fraction of the wage variation jointly explained by these additional variables as a proxy for bargaining prevalence. We rescale the proxy to cover the unit interval. Similarly to wage dispersion, we compute weighted averages of the bargaining proxy by occupation.[8]

Table 1 in Appendix B shows the detailed results of average wages, residual wage dispersion and our index for bargaining prevalence, as well as the number for observations

[8]The survey contains an explicit question of whether bargaining happened when an offer was extended. However, the overlap between data on wages and on bargaining is very limited; therefore, we could not include this variable in the regression for the proxy or use it as a standalone indicator of bargaining.

Figure 4.1: Relationship between bargaining prevalence and residual wage dispersion and average wages, by occupation




for each occupation.[9] The results are depicted in Figure 4.1.

As seen in Figure 4.1 we find that occupations where workers bargain more have both a higher average wage and a higher residual wage dispersion. Both correlations are positive and different from zero at the 0.005 level of significance.

Note that occupations with high values of wage dispersion and bargaining represent construction and lawyers. This feature of the data goes in line with our mechanism and will be discussed in more detail in the next subsection.

## 4.2 Comparison of model predictions with the data

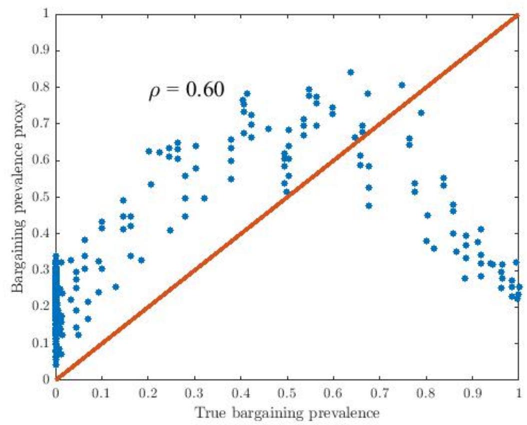
To compare the model predictions with those of the data, we do Monte-Carlo simulations by drawing from the four-dimensional grid of values of worker and firm costs as well as number of workers and firms. Earlier, we showed two-dimensional slices of that four-dimensional state space in Figures 3.1 and 3.2. We assume two types of workers and firms with a fixed preference structure and we compute the equilibria.[10]

Because we look at the results in the data by occupation, we define an occupation $o$ in the model by randomly drawing costs for workers $\theta_x$ and firms $\theta_y$ and the number of workers $\mu_x$ and firms $\mu_y$ from the space of all possible parameters. For each occupation we generate artificial observations for frequencies of matching, amount of search effort

[9] In the table we report the minimum number of observations for each occupation, which normally is for the bargaining proxy. There are more observations for wage dispersion and many more for the average wage.

[10] In Figures 4.2-4.4 we show results for symmetric horizontal preferences, but they differ only slightly for a vertical specification and do not seem to depend much on the preference specification in general.

Figure 4.2: Bargaining prevalence proxy vs truth




by workers, prevalence of bargained or posted wages, residual wages (de-meaned for each parameter combination), and the frequency of bargaining or wage-posting used in the matching. For each occupation, we compute the average log wage, the average residual wage dispersion, and the average prevalence of bargaining (all weighted by matching frequency and cost-pair frequency combined).

Because we cannot directly observe bargaining prevalence in the data but have constructed a proxy, we wanted to do some sort of validation exercise. To do so, we run artificial regressions of the residual wages on measures of search effort to obtain an artificial measure of a bargaining proxy from our model simulations. Weighted averages of the bargaining proxy by occupation are strongly positively correlated with true values of bargaining prevalence, as shown in Figure 4.2. There is a bit of non-linearity in this relationship at very high values of bargaining prevalence, which explains why there is some non-linearity in the other results that follow.

The results from the model using an artificially constructed bargaining prevalence proxy are plotted in Figure 4.3. The model predicts strongly positive (albeit somewhat non-linear) relationships between bargaining prevalence and residual wage dispersion, as well as with the average wage, consistent with what we see in the data. (The correlation with the true bargaining prevalence in the model is even stronger.)

As with search costs, with a tighter labor market more firms choose to bargain, and hence residual wage dispersion is higher, as can be seen in Figure 4.4. This is in line with empirical findings. Morin (2019) shows that residual wage dispersion is

Figure 4.3: Wage posting and Bargaining depending on parameters

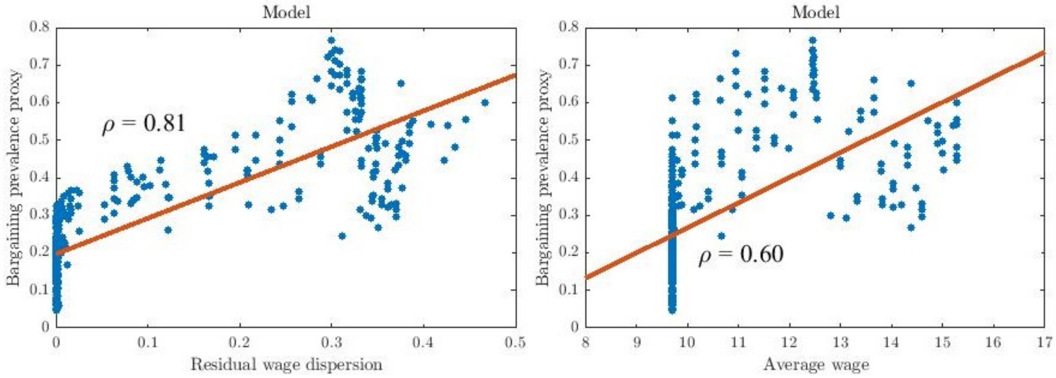
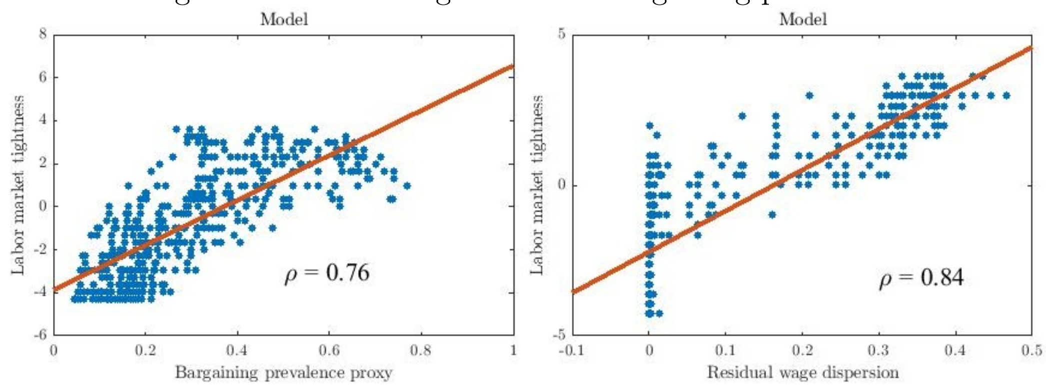



Figure 4.4: Market tightness and bargaining prevalence




pro-cyclical, supporting our finding that a tighter labor market is associated with more residual wage dispersion. At the same time, Brenzel, Gartner, and Schnabel (2014) show that when labor markets are tight, bargaining dominates over wage posting. This is also in line with our theoretical predictions.

Finally, our model provides a novel explanation for the positive dependence of observed wages on labor market tightness. This classical empirical finding can be explained by textbook models of aggregate labor market dynamics, as discussed, for example by Cahuc, Carcillo, and Zylberberg (2014). However, such dependence has also been recently documented at the within-occupation level, for instance, by Adrian and Lydon (2019), requiring an explanation. Our model explains this pattern by positing that a reduction in the relative number of workers makes it harder for firms to attract workers (fewer show up at any particular firm), which incentivizes the firms to post higher wages in order to keep the matching probability high.

# 5 Conclusion

We build a model of sequential targeted search where firms can choose between a bargaining and a wage-posting mechanism to determine equilibrium wages. We show that uneven search costs as well as labor market tightness are key determinants of bargaining prevalence. Our theory predicts that bargaining prevalence, wage level, wage dispersion, and labor market tightness are all positively correlated even within narrowly defined segments of the labor market, such as individual occupations. All these predictions are supported by the data.

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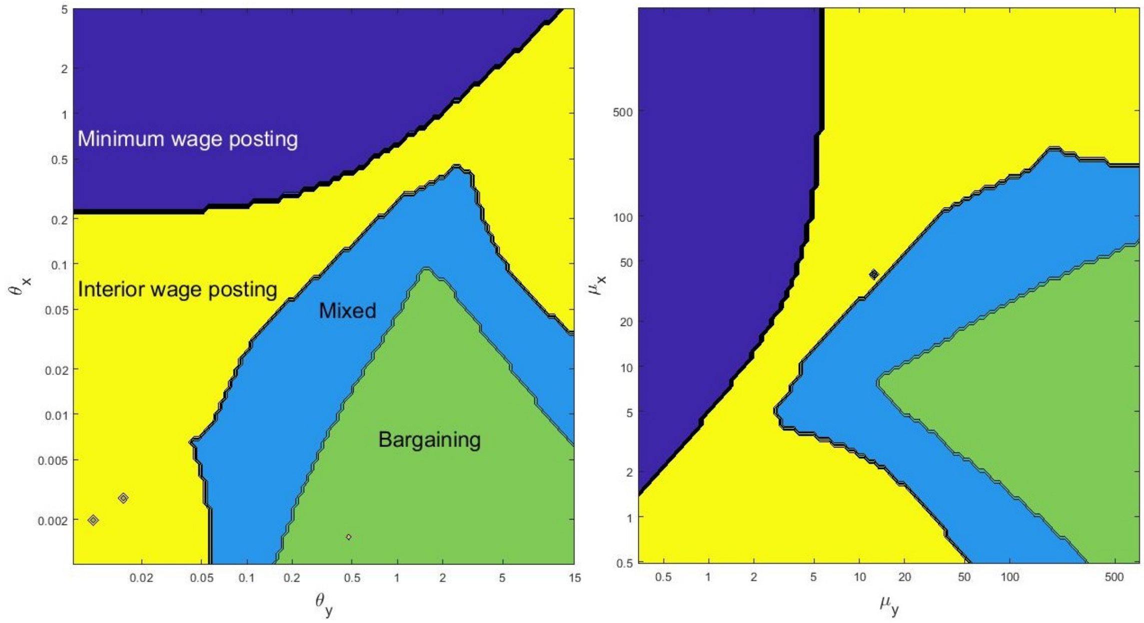
# Appendix A: Additional Graphs

Figure 5.1: Wage setting (vertical preferences)



## Appendix B: Wage Disperssion

| Occupational Description | SOC | N. Obs. | Barg.Prev. | W. disp. | Aver. W. |
|---|---|---|---|---|---|
| Management | 110 | 476 | 0.30 | 0.42 | 10.9 |
| Business Operations | 131 | 131 | 0.31 | 0.61 | 10.7 |
| Financial | 132 | 138 | 0.30 | 0.48 | 11.0 |
| Computer and Mathematical | 150 | 170 | 0.34 | 0.44 | 11.2 |
| Architecture and Engineering | 170 | 92 | 0.34 | 0.45 | 11.2 |
| Life, Physical, and Social Science | 190 | 38 | 0.31 | 0.39 | 11.0 |
| Counselors, Social Workers | 211 | 69 | 0.25 | 0.56 | 10.6 |
| Religious Workers | 212 | 18 | 0.29 | 0.56 | 10.4 |
| Lawyers, Judges | 231 | 31 | 0.38 | 0.91 | 11.5 |
| Legal Support | 232 | 28 | 0.30 | 0.57 | 10.4 |
| Educational and Library | 250 | 317 | 0.25 | 0.49 | 10.3 |
| Design, Entertainment, Sports, and Media | 270 | 89 | 0.30 | 0.54 | 10.3 |
| Healthcare Diagnosing or Treating | 291 | 101 | 0.27 | 0.40 | 10.9 |
| Health Technicians | 292 | 46 | 0.29 | 0.50 | 10.5 |
| Other Healthcare Practitioners | 299 | 7 | 0.30 | 0.34 | 10.8 |
| Healthcare Support | 310 | 67 | 0.22 | 0.45 | 10.1 |
| Supervisors of Protective Services | 331 | 6 | 0.31 | 0.35 | 10.9 |
| Firefighting and Prevention | 332 | 8 | 0.20 | 0.12 | 11.3 |
| Law Enforcement | 333 | 48 | 0.22 | 0.40 | 10.8 |
| Other Protective Services | 339 | 24 | 0.27 | 0.40 | 10.1 |
| Food Preparation and Serving | 350 | 55 | 0.22 | 0.40 | 9.6 |
| Building Cleaning and Pest Control | 372 | 20 | 0.21 | 0.55 | 9.8 |
| Grounds Maintenance Workers | 373 | 8 | 0.19 | 0.37 | 10.5 |
| Personal Care | 390 | 56 | 0.21 | 0.51 | 9.8 |
| Sales | 410 | 167 | 0.28 | 0.40 | 10.2 |
| Office and Administrative Support | 430 | 323 | 0.25 | 0.44 | 10.3 |
| Agriculture | 452 | 8 | 0.28 | 0.34 | 10.5 |
| Construction | 472 | 21 | 0.46 | 0.92 | 10.5 |
| Electrical Mechanics | 492 | 18 | 0.28 | 0.31 | 10.9 |
| Vehicle Mechanics | 493 | 14 | 0.22 | 0.28 | 10.2 |
| Installation, Maintenance, and Repair | 499 | 34 | 0.32 | 0.43 | 10.5 |
| Production | 510 | 74 | 0.26 | 0.46 | 10.4 |
| Air Transportation | 532 | 10 | 0.27 | 0.23 | 10.7 |
| Motor Vehicle Operators | 533 | 46 | 0.24 | 0.44 | 10.1 |
| Other Transportation | 536 | 12 | 0.22 | 0.45 | 9.8 |
| Material Moving | 537 | 15 | 0.29 | 0.42 | 10.3 |
| Military | 550 | 5 | 0.29 | 0.24 | 11.3 |
| Others | 990 | 135 | 0.27 | 0.60 | 10.5 |



Table 1: Relationship between average wage, wage dispersion and bargaining prevalence