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Fiscal Dominance

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Abstract

Central banks' resolve and independence are chronically tested by fiscal authorities wishing to impose their desired policies, often leading to socially undesirable economic outcomes. I study how the fiscal and monetary authorities' disagreement over outcomes and their choice of active instruments shape the implementation of policy, dispensing with commitment or first-mover advantage. I characterize the equilibrium for various combinations of active (and correspondingly, passive) instruments, identify which sources of disagreement play a role in each case, and show whether and under what conditions time-consistency problems may disappear in the long-run. When the fiscal authority sets debt levels actively, it may be able to impose its preferences on the central bank, regardless of how monetary policy is conducted. Designing a central bank with a special concern for liquidity markets counteracts this result.

Keywords: discretion, time-consistency, government debt, deficit, inflation, institutional design, political frictions, fiscal dominance, central bank independence, actives vs. passive policies, unpleasant monetarist arithmetic.

JEL classification: E52, E58, E61, E62.

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1 Introduction

In recent decades, central banks around the world have gained independence from fiscal and political institutions. The proposition is that a disciplined monetary policy can put an effective brake on the excesses of political expediency. This is frequently achieved by endowing central banks with clear and simple goals (e.g., an inflation mandate or target), as well as sufficient control over specific policy instruments (e.g., the short-term interest rate). Ideally, goals are designed to effectively counteract fiscal biases and implement more desired outcomes; instruments are endowed so that these goals are attainable.

Despite these institutional advances, the resolve of central banks is chronically put to the test. This is most evident during deep recessions and crises, invariably triggering large and persistent fiscal expansions. These policies put pressure on outcomes of concern for central banks, such as inflation rates and the functioning of financial markets, challenging its determination to achieve stated objectives. If these pressures stem, at least in part, from differing views on preferred economic outcomes or the presence of inefficiencies, such as those resulting from time-consistency problems, then they are ever-present and hence, a perennial concern.

Our understanding of the interaction of fiscal and monetary policies, particularly the competition between authorities, is greatly influenced by the body of work following Sargent and Wallace (1981, 1987). In this tradition, the main mechanism through which the fiscal and monetary authorities interact is the consolidated government budget constraint: An unrelenting authority forces the other to accommodate its policy so that the budget constraint is satisfied. This insight lead to the concept of active and passive policies, as articulated by Leeper (1991): both authorities cannot set policy actively, i.e., one must accommodate the other. Sims (1994) goes even further by stressing that, given these considerations, inflation is fundamentally a fiscal phenomenon.

Yet, these analytical frameworks require an important degree of commitment power. This assumption is itself unpleasant as it typically leads to the implementation of time-inconsistent policies. When facing fiscal expansion, the central bank may find it *ex post* optimal to renege on its promise to be disciplined. Anticipating this reaction, the fiscal authority would not believe this promise and act knowing that its policies would eventually be accommodated by the central bank.

In this paper I dispense with commitment to understand how government policy is determined in the presence of rival authorities. As in the classic work by Sargent and Wallace a key mechanism is the (consolidated) government budget constraint, which needs to be satisfied. In contrast to the classic approach, neither authority can commit to future policies or has a first-mover advantage over the other.[1] Still, given the policy choices made by the fiscal and monetary authorities, some instruments need to adjust in order to satisfy the government budget constraint and other equilibrium conditions. Which instruments are *actively* chosen by the two competing authorities—and correspondingly, which instruments are left to adjust *passively*—affects how their preferences over outcomes are internalized and thus, what polices are implemented. This approach then shifts the focus from active/passive authorities to active/passive instruments.

The environment is a monetary economy populated by infinitely-lived agents and a government that finances the provision of a valued public good with distortionary income taxes, central bank liabilities and fiscal debt. There is imperfect record-keeping and limited commitment so that some transactions require the use of an acceptable medium of exchange, in this case, central bank liabilities (money). Financial intermediaries alleviate these frictions by

[1] First-mover advantage is a mild form of commitment and subject to a similar criticism.

channeling liquidity within the private sector appropriately. Fiscal liabilities take the form of one-period, illiquid nominal bonds.[2] There are two government authorities: a fiscal authority and a monetary authority or central bank. Each authority is endowed with its own preferences over allocations and discounting. For example, the fiscal authority could be prone to overspending or be more impatient than private agents; the central bank may be overly concerned with allocations affecting short-term liquidity markets. The preferences of government authorities are stated in very general terms to cover these and many other possibilities. A key element in the analysis is that the authorities disagree with each other and with private agents over what outcomes should be implemented through policy.

Government policy in equilibrium will depend on the authorities' preferences and on which instruments they set actively. The fiscal authority always decides on the level of public expenditure. This leaves two possible active policy instruments, one for each authority, with the remaining instruments adjusting passively in order to satisfy equilibrium conditions, which include the government budget constraint. The fiscal authority may actively set the tax rate or the level of debt (i.e., the end-of-period stock of bonds); the central bank may set the short-term interest rate (the intraperiod cost of liquidity), the long-term interest rate (the interperiod bond yield) or the growth rate of its nominal liabilities (money). I also consider the possibility that the central bank has enough power to engage in yield curve control, i.e., set both short- and long-term interest rates, in which case the fiscal authority has no active instruments beyond the level of public expenditure.

I start the analysis with the case when only the preferences of the fiscal authority matter. That is, a situation when there is fiscal dominance since the central bank is not independent, either *de jure*, due to institutional design, or *de facto*, because it shares the preferences of the fiscal authority. In this case, the lack of conflict between the authorities renders instrument choice immaterial. Thus, there is a unique implementation of policy, which is determined by various forces. First, there is motive to smooth distortions intertemporally, as in the classic treatments by Barro (1979) and Lucas and Stokey (1983), which may be further distorted by the relative impatience of the fiscal authority. Second, the interaction between debt and monetary policy leads to a time-consistency problem in debt choice, as analyzed in Martin (2009, 2011, 2013): how much debt the government inherits, affects its monetary policy since inflation reduces the real value of nominal liabilities; in turn, the anticipated response of future monetary policy affects the current demand for money and bonds, and thereby how the government today internalizes policy trade-offs. Third, the fiscal authority's preferences on allocations determine how policies distort the different intra- and inter-temporal margins. Of note, under fiscal dominance there is no time-consistency problem in steady state.[3]

Next, I study how the interaction between disagreement in preferences and choice of active instruments shape government policy. I characterize the equilibrium in each case and identify which preference disagreements play a role in policy implementation. Importantly, I focus on whether and how fiscal dominance may arise and under which conditions the time-consistency problem can be solved in steady state.

The potential for fiscal dominance arises only when the fiscal authority actively sets the debt level. In this case, the central bank's patience becomes irrelevant and fiscal dominance is achieved if the only source of disagreement concerns public spending. Thus, a central bank

[2]Some assumptions can be relaxed at the cost of a more involved analysis. One can allow for bonds to be partially liquid, i.e., be used in transactions necessitating a medium of exchange. One could also allow for long-term debt; e.g., using a decaying coupon representation, which would still imply a single long-term interest rate on bonds. Though both extensions are interesting and potentially quantitatively important, neither would meaningfully alter the theoretical results presented here.

[3]In other words, endowing the government with commitment power at the steady state has no effect on policy. See Martin (2011, 2015*a*) for further analysis and discussion.

designed with some special concerns (e.g., the functioning of liquid markets) may be able to counteract the pressure from the fiscal authority. In contrast, making the central bank overly patient is ineffective.

Time consistency problems are ever present, except when the fiscal authority actively sets the debt level. Hence, the possibility of fiscal dominance is intimately linked with the possibility of eliminating time-consistency problems in steady state. Note that this result can only be achieved when the fiscal authority discount the future at the same rate as private agents.

Using the long-term rate as the active monetary instruments allows the central bank to correct impatience from the fiscal authority. When the disagreement between the authorities is sufficiently large, the central bank may find it useful to move on to full yield curve control, i.e., actively setting short- and long-term rates. Note, however, that this move requires the fiscal authority to adopt a passive role and hence, most resembles the active monetary vs. passive fiscal policies of the classical approach.

There are some relatively recent papers which are closely related. Martin (2015*a*) and Martin (2021) study the effects on policy of having a fiscal authority prone to excessive spending. The former paper considers the effects of making the central bank independent (with its own preferences for public expenditure) but does not explore the effects of different arrangements of instrument choice or more general disagreement in preferences. The latter paper studies how fiscal rules can counteract the negative welfare effects of an expenditure bias. Niemann, Pichler and Sorger (2013) show how instrument choice can affect equilibrium policies when an impatient fiscal authority faces a benevolent central bank.[4] Finally, Barthélemy, Mengus and Plantin (2020) formalize Neil Wallace's game of chicken between the fiscal and monetary authorities and identify circumstances where there is fiscal dominance.

The paper is organized as follows. Section 2 presents the environment. Section 3 characterizes government policy when the fiscal authority is dominant and determines all government policy—alternatively, a case when the fiscal and monetary authorities share the same preferences. Section 4 characterizes government policy when the fiscal and monetary authority have competing objectives, depending on which instruments are active. Section 5 concludes.

2 Model

2.1 Environment

The economy is populated by a continuum of infinitely-lived agents, which discount the future by factor $\beta \in (0, 1)$. Each period, two competitive markets open in sequence, for expositional convenience labeled *day* and *night*. All goods produced in the economy are perishable and cannot be stored from one subperiod to the next.

At the beginning of each period, agents receive an idiosyncratic shock that determines their role in the day market. With probability $\eta \in (0, 1)$ an agent wants to consume but cannot produce the day-good x, while with probability $1 - \eta$ an agent can produce but does not want consume. A *consumer* derives utility $u(x)$, where u is twice continuously differentiable, satisfies Inada conditions and $u_{xx} < 0 < u_x$. A *producer* incurs in utility cost $\phi > 0$ per unit produced.

Agents are anonymous and lack commitment. Thus, credit arrangements are not feasible and some medium of exchange is necessary for day trade to occur, which in this economy, takes the form of central bank liabilities, which I will refer to as money. As is Berentsen et al. (2007)

[4]Other papers studying the role of policy instrument choice include Poole (1970), Canzoneri, Henderson and Rogoff (1983), Carlstrom and Fuerst (1995), Benhabib, Schmitt-Grohé and Uribe (2001), Schabert (2006), King and Wolman (2004) and Collard and Dellas (2005).

assume there exist perfectly competitive financial intermediaries that take money deposits and extend money loans. These intermediaries can costlessly record financial histories but are unable to track trading histories. Deposits and loans mature and night. Due to perfect competition and the lack of any other frictions, financial intermediaries make zero profits and the interest rates on deposits and loans are the same.

At night, all agents can produce and consume the night-good, c. The production technology is assumed to be linear in labor, such that n hours worked produce n units of output. Assuming perfect competition in factor markets, the real wage rate is equal to 1. Utility at night is given by $U(c) - \alpha n$, where U is twice continuously differentiable, $U_{cc} < 0 < U_c$ and $\alpha > 0$. Though a medium of exchange is not essential in this market, agents also trade money and bonds at night.

There is a government that supplies a valued public good g at night. Agents derive utility from the public good according to $v(g)$, where v is twice continuously differentiable, satisfies Inada conditions and $v_{gg} < 0 < v_g$. To finance its expenditure, the government may use proportional labor taxes τ, print money at rate μ and issue one-period nominal bonds, which are redeemable in money. Government policy choices for the period are announced at the beginning of each day, before agents' idiosyncratic shocks are realized. The government only actively participates in the night market, i.e., taxes are levied on hours worked at night and open-market operations are conducted in the night market. The public good is transformed one-to-one from the night-good.

All nominal variables—except for bond prices—are normalized by the aggregate money stock. Thus, today's aggregate money supply is equal to 1 and tomorrow's is $1 + \mu$. The government budget constraint can be written as

$$p_c(\tau n - g) + (1 + \mu)[1 + B'(1 + R)^{-1}] - (1 + B) \geq 0, \tag{1}$$

where B is the current aggregate bond-money ratio, p_c is the—normalized—market price of the night-good c, and $1 + R$ is the inverse of the price of a bond that earns one unit of money in the following night market). "Primes" denote variables evaluated in the following period. Thus, B' is tomorrow's aggregate bond-money ratio. In equilibrium, prices and policy variables depend on the aggregate state, B; this dependence is omitted from the notation to simplify exposition.

2.2 Problem of the agent

Let $V(m, b, B)$ be the value of entering the day market with (normalized) money balances m and bond balances b, when the aggregate state of the economy is the aggregate bond-to-money ratio, B. Upon entering the night market, the composition of an agent's nominal portfolio (money and bonds) is irrelevant, since bonds are redeemed in money at par. Thus, let $W(z, B)$ be the value of entering the night market with total (normalized) nominal balances z.

In the day market, consumers and producers exchange money for goods at (normalized) price p_x. Let x be the individual quantity consumed and κ the individual quantity produced; these quantities are generally different in equilibrium, unless there is an equal measure of consumers and producers. Let i be the interest rate earned on deposits and charged for loans. To distinguish i from R, I will refer to the former as the short-term (intraperiod) interest rate and the latter as the long-term (interperiod) interest rate.

A consumer with starting balances (m, b) and acquired loans ℓ has total liquidity $m + \ell$ to purchase day output. The problem of a consumer is

$$V^c(m, b, B) = \max_{x, \ell} \; u(x) + W(m + b - p_x x - i\ell, B)$$

subject to: $p_x x \leq m + \ell$. The problem of a producer with starting balances (m, b) is

$$V^p(m, b, B) = \max_{\kappa, d} \; -\phi\kappa + W(m + b + p_x\kappa + id, B).$$

subject to $d \leq m$. The *ex ante* value of an agent with portfolio (m, b) at the start of the period satisfies $V(m, b, B) \equiv \eta V^c(m, b, B) + (1 - \eta)V^p(m, b, B)$.

At night, the problem of an agent arriving with total nominal balances z is

$$W(z, B) = \max_{c,n,m',b'} \; U(c) - \alpha n + v(g) + \beta V(m', b', B')$$

subject to: $p_c c + (1 + \mu)[m' + b'(1 + R)^{-1}] = p_c(1 - \tau)n + z$.

2.3 Derivations

Here, we derive the conditions which characterize a monetary equilibrium. Let us start with the problem of an agent at night. Solving the budget constraint for n and replacing in the objective function, the first-order conditions imply:

$$U_c - \frac{\alpha}{1 - \tau} = 0 \tag{2}$$

$$-\frac{\alpha(1 + \mu)}{p_c(1 - \tau)} + \beta V'_m = 0 \tag{3}$$

$$-\frac{\alpha(1 + \mu)}{p_c(1 - \tau)(1 + R)} + \beta V'_b = 0 \tag{4}$$

The night-value function W is linear in z: $W_z = \frac{\alpha}{p_c(1-\tau)}$. Hence, $W(z, B) = W(0, B) + \frac{\alpha z}{p_c(1-\tau)}$, which we will use to rewrite the problem of the agent in the day. Accordingly, the problem of a consumer in the day can be rewritten as

$$V^c(m, b, B) = \max_{x,\ell} \; u(x) + W(0, B) + \frac{\alpha(m + b - p_x x - i\ell)}{p_c(1 - \tau)}$$

subject to the liquidity constraint $m + \ell - p_x x \geq 0$, with associated Lagrange multiplier ξ. The first-order conditions imply

$$1 + i = \frac{u_x p_c(1 - \tau)}{\alpha p_x} \tag{5}$$

$$\xi = \frac{u_x}{p_x} - \frac{\alpha}{p_c(1 - \tau)} \tag{6}$$

It follows that $i = 0$ if and only if $\xi^c = 0$, i.e., positive short-term interest rates are associated with consumers being liquidity-constrained.

Producers have no use for money in the day market and thus, will deposit all their money balances (without loss of generality when $i = 0$). The problem of a producer can be rewritten as

$$V^p(m, b, B) = \max_{\kappa} \; -\phi\kappa + W(0, B) + \frac{\alpha[m(1 + i) + b + p_x\kappa]}{p_c(1 - \tau)}$$

The first-order condition implies

$$\frac{\phi}{p_x} = \frac{\alpha}{p_c(1 - \tau)} \tag{7}$$

2.4 Monetary equilibrium

As shown in Lagos and Wright (2005), the assumptions on preferences imply that all agents make the same portfolio choice at night.[5] Hence, market clearing at night implies $m' = 1$ and $b' = B'$. Individual consumption at night is the same for all agents, whereas individual labor depends on whether an agent was a consumer or a producer in the day. Hence, the resource constraint at night is given by $c + g = \eta n^c + (1 - \eta) n^p$, where n^c and n^p denote night-labor by agents that were consumers or producers in the day, respectively.

Since all agents start the period with the same portfolio, $m = 1$. Clearing in the day-goods market implies $\eta(1 + \ell) = (1 - \eta) p_x \kappa$, i.e., total means of payments held by consumers (money plus loans) equals total nominal sales by producers. Clearing in financial markets implies $\eta \ell = (1 - \eta) d$, i.e., total loans equals to total deposits. Since, producers deposit all their money balances, $d = 1$ and so, $\ell = 1/\eta - 1$. The resource constraint in the day is $\eta x = (1 - \eta)\kappa$. Combining these expressions with the day-goods market clearing condition implies

$$p_x = \frac{1}{\eta x} \tag{8}$$

which is a standard condition in monetary economies: the price of the day-good p_x equals the total means of payment $1/\eta$ divided by the total quantity traded, x.

The price of the night-good p_c depends on the equilibrium quantities traded in the day and night. The relative price between day and night goods, p_x/p_c is pinned down by the first-order condition to the producer's problem, (7): a producer sells goods in the day to save on effort at night and this decision is distorted by labor taxes τ, which as shown next can be expressed a function of the night-good allocation c. Combining (2), (7) and (8) we obtain an expression for the price of the night-good

$$p_c = \frac{U_c}{\eta \phi x} \tag{9}$$

Combining (5), (7), (8) and (9) we obtain an expression for the short-term (intraperiod) interest rate,

$$i = \frac{u_x}{\phi} - 1 \tag{10}$$

When short-term interest rate is positive, the day-good consumption is below the efficient level, i.e., $u_x > \phi$. Furthermore, there is a one-to-one mapping between day-good consumption and the short-term nominal rate, so we can interchangeably refer to variations in the day-good allocation, x, and the short-term rate, i.

From (6)–(9) the Lagrange multiplier of the liquidity constraint is $\xi = \eta x(u_x - \phi)$. Since $\xi \geq 0$, this condition imposes an equilibrium restriction, $u_x - \phi \geq 0$. Equivalently, from (10), the nominal short-term interest rate cannot be negative, $i \geq 0$.

Condition (2) can be rearranged to yield:

$$\tau = 1 - \frac{\alpha}{U_c} \tag{11}$$

which states the trade-off between the marginal utility of night-good consumption and the marginal disutility of night-labor. This trade-off is distorted by the labor tax: a higher tax rate τ implies lower night-good consumption c. As with monetary policy, we can interchangeably refer to variations in the night-good allocation, c and variations in the tax rate, τ.

[5] Since V is linear in b, a non-degenerate distribution of bonds is possible in equilibrium. Here, I focus on symmetric equilibria.

Given $V(m, b, B) \equiv \eta V^c(m, b, B) + (1 - \eta)V^p(m, b, B)$ and (5)–(8) we obtain $V_m = \eta u_x x$ and $V_b = \eta \phi x$. Hence, (3), (7) and (8) imply

$$\mu = \frac{\beta u_x' x'}{\phi x} - 1 \tag{12}$$

For a given expected future day-good allocation, x', which in equilibrium is a function of debt choice, B', a higher money growth rate μ implies lower day-good consumption x.

Finally, from (4), (7), (8) and (12) we obtain

$$R = \frac{u_x'}{\phi} - 1 \tag{13}$$

The bond return reflects its liquidity premium: agents need to be compensated for the fact that bonds cannot be used to purchase day goods.[6] Note that the bond return, R, is a function of next-period's day-good allocation x', which in equilibrium depends on current debt choice, B'.

Conditions (8)–(13) map allocations into prices and policy instruments. We can use them to write the government budget constraint (1) in a monetary equilibrium as a function of allocations and debt,

$$\varepsilon(B, B', x, x', c, g) \equiv (U_c - \alpha)\, c - \alpha g + \eta \left\{\beta x'(u_x' - \phi) + \beta \phi x'(1 + B') - \phi x(1 + B)\right\} = 0. \tag{14}$$

This condition is also known as an implementability constraint, as it restricts the set of allocations that a government can implement in a monetary equilibrium.

Let $\mathbb{B} \equiv [\underline{B}, \overline{B}]$] be the set of possible debt levels, where $-1 < \underline{B} < \overline{B}$. The lower bound on $\underline{B}$ ensures that the non-negativity constraint $u_x - \phi \geq 0$ does not bind—see Martin (2011). The set $\mathbb{B}$ is assumed to be wide enough to not be binding.

3 Government policy with a consolidated government

The government can commit to policy announcements for the current period, but cannot commit to policies implemented in future periods. Policies implemented by the government in the future affect its *current* budget constraint, since future monetary policy affects the current demand for money and bonds. This is reflected by the presence of the future allocation x' in the government budget (or implementability) constraint (14). Due to limited commitment, the current government cannot directly control future policy, even though it can affect future policy through its choice of debt, B'. Future allocations depend on the policy expected to be implemented by the government, which in turn, depends on the level of debt it inherits and the exogenous aggregate state of the economy. Let $\mathcal{X}(B)$ be the day-good allocation that the current government anticipates will be implemented by future governments as a function of beginning-of-period debt; this function implies a future day-good allocation, x' for any given future state, B'. The function $\mathcal{X}$ is an equilibrium object, but the current government takes it as given.

Using the day resource constraint, we can write production in equilibrium as a function of consumption: $\kappa = \eta x/(1 - \eta)$. Thus, an agent's expected flow utility in the day is equal to $\eta[u(x) - x]$. Night output is equal to the consumption of private and public goods and so, we can use the night resource constraint to write expected night labor as $c + g$. The *ex ante*

[6] Note that, despite the linearity in the disutility of labor, the real interest rate is not exogenous, and fluctuates with variations in the tax rate. The yield on an illiquid real bond would be $\frac{U_c}{\beta U_c'} - 1$; by (11) we can see how this yield depends on taxes today and tomorrow.

period utility of an agent can thus be written in terms of the bundle (x, c, g): $\mathcal{U}(x, c, g) \equiv \eta[u(x) - \phi x] + U(c) - \alpha(c + g) + v(g)$.

The government is composed of a fiscal authority, F, and a monetary authority (or central bank), M. Each authority is endowed with a set of policy instruments, which map into allocations, as specified below. The details of which policy instruments are assigned to which authority will play a critical role in the analysis. Note, however, that the fiscal authority always chooses government expenditure, g. I will assume that neither authority has a first-mover advantage relative to the other. Hence, both will move simultaneously, albeit before private agents.[7] Each authority values the *ex ante* period utility of the agents according to a function $\mathcal{U}_k(x, c, g)$ and discounts the future by factor $\beta(1 - \delta_k)$, $k = \{F, M\}$. The following regularity assumptions will ensure that the problem of each authority is well-behaved.

Assumption 1 *For $k = \{F, M\}$:*

1. *$\mathcal{U}_k(x, c, g)$ is separable in all arguments;*
2. *there exist $\hat{x}_k > 0$, $\hat{c}_k > 0$ and $\hat{g}_k > 0$ such that $\mathcal{U}_{k,x} > 0$ for all $x \in [0, \hat{x}_k)$, $\mathcal{U}_{k,c} > 0$ for all $c \in [0, \hat{x}_k)$ and $\mathcal{U}_{k,g} > 0$ for all $g \in [0, \hat{g}_k)$;*
3. *$\mathcal{U}_{k,xx} < 0$, $\mathcal{U}_{k,cc} < 0$ and $\mathcal{U}_{k,gg} < 0$; and*
4. *$1 - \beta^{-1} < \delta_k < 1$.*

Note that Assumption 1 is satisfied for the case when an authority is benevolent, i.e., when $\mathcal{U}_k(x, c, g) = \mathcal{U}(x, c, g)$ and $\delta_k = 0$. There are two straightforward examples of deviations from benevolence that satisfy Assumption 1. The first is to put weights on some of the terms in $\mathcal{U}(x, c, g)$:

$$\mathcal{U}_k(x, c, g) \equiv \omega_{k,x}\eta[u(x) - \phi x] + \omega_{k,c}U(c) - \alpha(c + g) + \omega_{k,g}v(g)$$

One can interpret these weights as biases. For example, the central bank may be overly concerned with the functioning of liquidity markets, $\omega_{M,x} > 0$, or the fiscal authority may be prone to overspending, $\omega_{F,g} > 0$. The second straightforward deviation from benevolence is to assume that one or both government authorities are more impatient than private agents, i.e., $\delta_k \in (0, 1)$. Note that Assumption 1 also allows government authorities to be somewhat more patient than private agents.

Let us start with the case when there is no independent central bank, i.e., the monetary authority shares the preferences of the fiscal authority. The environment is equivalent to one in which there is a single government agency endowed with the preferences of the fiscal authority. The consolidated government faces an implementation problem: to maximize its utility in a monetary equilibrium, subject to its budget constraint and taking into account how future governments conduct policy. There are no concerns about instrument choice, as allocations map into prices and policy instruments, as explained above. This scenario is a useful benchmark as we can compare it to other institutional environments in which the central bank is endowed with specific preferences and instruments.

Taking as given future government policy implementing $\{\mathcal{B}, \mathcal{X}, \mathcal{C}, \mathcal{G}\}$ the problem of the current government can be written as

$$\max_{x,c,g,B'} \mathcal{U}_F(x, c, g) + \beta(1 - \delta_F)\mathcal{F}(B')$$

subject to (14) and given a continuation value consistent with expected future policy:

$$\mathcal{F}(B') = \mathcal{U}_F(\mathcal{X}(B'), \mathcal{C}(B'), \mathcal{G}(B')) + \beta(1 - \delta_F)\mathcal{F}(\mathcal{B}(B').$$

[7] There are some interesting issues that arise when government authorities move at the same time as private agents. See Ortigueira (2006) and Martin (2015*b*).

We now have the necessary elements to define an equilibrium in this economy.

Definition 1 (Consolidated government policy)
When the central bank is not independent, a Markov-Perfect Monetary Equilibrium (MPME) is a set of functions $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}, \mathcal{F}\} : \mathbb{B} \rightarrow \mathbb{R}^3_+ \times \mathbb{B} \times \mathbb{R}$*, such that for all* $B \in \mathbb{B}$*:*

$$\{\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B), \mathcal{B}(B)\} = \underset{x,c,g,B'}{\text{argmax}}\ \mathcal{U}_F(x, c, g) + \omega_F g\ + \beta(1-\delta_F)\mathcal{F}(B')$$

subject to $\varepsilon(B, B', x, \mathcal{X}(B'), c, g) = 0$ *and where*

$$\mathcal{F}(B) \equiv \mathcal{U}_F(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B)).$$

A Markov-perfect equilibrium is a fixed-point in government policy functions, so that the best response of the current government is to follow the same policies it expects to follow in the future.

With Lagrange multiplier λ_F associated with the government budget constraint, the first-order conditions of the government's problem imply:

$$\mathcal{U}_{F,x} + \lambda_F \varepsilon_x = 0 \quad (15)$$

$$\mathcal{U}_{F,c} + \lambda_F \varepsilon_c = 0 \quad (16)$$

$$\mathcal{U}_{F,g} + \lambda_F \varepsilon_g = 0 \quad (17)$$

$$\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_F] + \lambda_F \varepsilon_{x'} \mathcal{X}'_B = 0 \quad (18)$$

for all $B \in \mathbb{B}$ and where ε_i denotes the derivative of (14) with respect to variable $i = \{B, B', x, x', c, g\}$. Note that $\varepsilon_{B'} = -\beta\varepsilon'_B$, which is used to simplify (18).

A *differentiable* MPME is a set of differentiable (a.e.) functions $\{\mathcal{B}, \mathcal{X}, \mathcal{C}, \mathcal{G}, \Lambda_F\}$ that solve (14)–(18) for all B. Martin (2011) provides an extended analysis of these conditions and a characterization of the equilibrium. Below, I describe the policy trade-offs implied by these conditions.

Conditions (15)–(17) describe the static trade-offs faced by the government when choosing monetary and fiscal policies. Each policy instrument can be used to relax the government budget constraint at the cost of introducing a wedge, which lowers utility for the government (and private agents as well). For example, increasing the tax rate, τ, which by (11) lowers night-goods consumption, c, raises revenue—the term ε_c in (16) is negative—at the cost of higher distortion, i.e., lower utility—since $\mathcal{U}_{F,c} > 0$ by Assumption 1.

Equation (18), known as a Generalized Euler Equation (GEE), describes the intertemporal trade-offs faced by the government when choosing debt. The first term depends on the difference between current and future implementation costs, as reflected by the multiplier on (14), capturing the distortion-smoothing role of debt. From an *ex ante* perspective, this gap would ideally be eliminated, but this is prevented by the limited commitment friction.

The second term in (15) reflects the time-consistency problem, which consists of how current changes in debt trigger future changes in policy, which in turn, affect the current budget constraint of the government. Choosing a higher debt today implies higher distortions tomorrow (in particular, higher inflation), which affects the demand for money and bonds today. The impact on the latter is always negative: higher inflation implies higher nominal interest rates; the former depends on how the income and substitution effects determine how the current demand for money is affected by future higher inflation. When income effects dominate, the overall effect of higher debt is to relax the government budget constraint at low level of debt and to tighten it for high levels of debt.

Definition 2 (Fiscal dominance)
Fiscal dominance arises when an MPME can be characterized by (14)–(18) *for all* $B \in \mathbb{B}$.

An important point relates to time-consistency in the long run. In steady state, (18) becomes

$$\varepsilon_{B'}\delta_F + \varepsilon_{x'}\mathcal{X}_B = 0. \tag{19}$$

The presence of the derivative of the function $\mathcal{X}$ implies that a time-consistency problem is still active in steady state. However, when $\delta_F = 0$, we get $\varepsilon_{x'}\mathcal{X}_B = 0$, which given $\mathcal{X}_B < 0$ implies $\varepsilon_{x'} = 0$. In other words, there is no time-consistency problem at this steady state. Martin (2011, 2015*a*) formally show how this steady state is constrained efficient, i.e., endowing the government with commitment power at this steady state has no effect on allocations and policies.

Definition 3 (Time-consistency of steady state)
In a MPME, there is no time-consistency problem in a steady state if the steady state can be solved locally, i.e., does not depend on the derivatives of equilibrium policy functions.

4 Government policy with competing authorities

When the fiscal and monetary authorities disagree on preferences, the question is how policy is chosen and determined. Since both authorities move simultaneously (but before private agents) and the government budget constraint needs to satisfied, one policy variable cannot be chosen by either; i.e., one instrument needs to satisfy the government budget constraint, given the policy choices made by the two authorities.

As the previous analysis makes clear, having competing government authorities is only relevant when they differ in the valuation of allocations at the margin or their discounting of the future. The following assumption states that the fiscal and monetary authorities differ in at least one of these dimensions.

Assumption 2 *The fiscal and monetary authorities, F and M, respectively, are endowed with different preferences. Specifically, at least one of the following holds: $\mathcal{U}_{F,x} \neq \mathcal{U}_{M,x}$; $\mathcal{U}_{F,c} \neq \mathcal{U}_{M,c}$; $\mathcal{U}_{F,g} \neq \mathcal{U}_{M,g}$; and/or $\delta_F \neq \delta_M$.*

Since the fiscal authority represents all the government except the central bank, it will be in charge of choosing expenditure g. The remaining instruments at the government's disposal are: end-of-period debt, B'; the tax rate, τ, the short-term (intraperiod) interest rate, i; the long-term (interperiod) interest rate, R; and the money growth rate μ. Debt, taxes and expenditure are the domain of the fiscal authority, while interest rates and monetary aggregates are instruments of the central bank. Note that (13) implies B' and R are in fact the same instrument. Similarly, by (10), (12) and (13) there is overlap between i, μ and R. Hence, not all instruments can be independently chosen.

Each authority decides on policy anticipating the choices of the other authority today and those of future authorities from tomorrow onwards. These policies span continuation values for the fiscal and monetary authority, $\mathcal{F}(B)$ and $\mathcal{M}(B)$, respectively. There are various possible combinations of *active* policy instruments. In the sections below, I will consider the implications of these institutional scenarios. Recall that the fiscal authority always chooses expenditure, g.

First, the central bank could set the short-term rate, i. By (10) this is equivalent to implementing the day-good allocation, x. In this case, the fiscal authority can either choose the tax rate, τ, which by (11) is equivalent to the night-good allocation, c, or the debt level, B'. The other variable (B' or c, respectively) adjusts to satisfy the government budget constraint. Given these choices and some future implementation of the day-good allocation, $x' = \mathcal{X}(B')$, we can retrieve μ and R from (12) and (13), respectively.

Second, the central bank could determine the money growth rate, μ. By (12), μ determines the day-good allocation x, given some future day-good allocation, $\mathcal{X}(B')$. Note, however, that this case is not equivalent to setting i, as now the implementation of x depends on both the choices of μ and B'. The fiscal authority can either choose taxes, τ, or debt, B', and let the other variable adjust to satisfy the government budget constraint. We can then retrieve interest rates i and R from (10) and (13), respectively.

Third, the central bank could set the long-term interest rate, R, which by (13) is equivalent to choosing debt, B'. In this case, the fiscal authority picks the tax rate, τ, which by (11) is equivalent to implementing the night-good allocation, c. In this case, the day-good allocation x needs to adjust to satisfy the government budget constraint. By (10) and (12), this is equivalent to letting the short-term rate, i, and the money growth rate, μ, adjust passively.

Finally, the central bank could target the entire yield-curve, in which case it chooses both i and R. By (10) and (13) this is equivalent to choosing x and B'. Note that by (10) and (12) this case is equivalent to having the central bank pick i and μ. The tax rate, τ, needs to adjust to satisfy the government budget constraint. Thus, the fiscal authority is only free to determine expenditure, g.

4.1 Central bank sets the short-term interest rate

Let us begin with the way modern central banks operate in normal times, which is by setting or targeting short-term interest rates. By (10) picking i is equivalent to implementing the day-good allocation, x. Thus, we can represent the problem of the monetary authority as implementing x. The fiscal authority sets public expenditure, g, and either picks taxes, τ, or end-of-period debt, B'. One of these two policy instruments needs to adjust to satisfy the government budget constraint.

4.1.1 Fiscal authority sets the tax rate

Suppose that the fiscal authority sets the tax rate, τ. By (11) this is equivalent to implementing the night-good allocation, c. Hence, the debt level, B' adjusts to satisfy the government budget constraint.

The monetary authority then implements a day-good allocation, x, taking as given the allocations implemented by current fiscal policy, $\mathcal{C}(B)$ and $\mathcal{G}(B)$, and future policies $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}\}$ which span a continuation value $\mathcal{M}$, and understanding that debt, B', will adjust to satisfy the government budget constraint.

Similarly, the fiscal authority implements the night-good allocation, c, and public expenditure, g, taking as given the allocations implemented by current monetary policy, $\mathcal{X}(B)$, and future policies $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}\}$ which span a continuation value $\mathcal{F}$, and understanding that debt, B', will adjust to satisfy the government budget constraint.

We are now ready to formulate the problems of the fiscal and monetary authorities, and define an equilibrium in this setting.

Definition 4 (Short-term rate vs. tax rate)
When the monetary authority sets the short-term rate, i, and the fiscal authority sets the tax

rate, τ*, an MPME is a set of functions* $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}, \mathcal{F}, \mathcal{M}\} : \mathbb{B} \to \mathbb{R}^3_+ \times \mathbb{B} \times \mathbb{R}^2$*, such that for all* $B \in \mathbb{B}$*:*

$$\{\mathcal{C}(B), \mathcal{G}(B)\} = \underset{c,\ g}{\text{argmax}}\ \mathcal{U}_F(\mathcal{X}(B), c, g) + \beta(1-\delta_F)\mathcal{F}(B')$$

where $\varepsilon(B, B', \mathcal{X}(B), \mathcal{X}(B'), c, g) = 0$*;*

$$\mathcal{X}(B) = \underset{x}{\text{argmax}}\ \mathcal{U}_M(x, \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(B')$$

where $\varepsilon(B, B', x, \mathcal{X}(B'), \mathcal{C}(B), \mathcal{G}(B)) = 0$*;*

$$\begin{aligned} \mathcal{F}(B) &\equiv \mathcal{U}_F(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B)) \\ \mathcal{M}(B) &\equiv \mathcal{U}_M(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(\mathcal{B}(B)); \end{aligned}$$

and $\mathcal{B}(B)$ *solves*

$$\varepsilon(B, \mathcal{B}(B), \mathcal{X}(B), \mathcal{X}(\mathcal{B}(B)), \mathcal{C}(B), \mathcal{G}(B)) = 0.$$

We can characterize government policy by taking the first-order conditions of the problems of the fiscal and monetary authorities. For both authorities, the debt level B' adjusts to satisfy the government budget constraint, given their own actions and the anticipated policies of the other authority and both future authorities. Naturally, in equilibrium, the debt adjustment is the same as there is only one budget constraint that needs to be satisfied. We can therefore add B' as a choice variable and (14) as a constraint to each of the authorities' problems.

With Lagrange multipliers λ_M and λ_F associated with their respective constraints, the first-order conditions with respect to x for the monetary authority, and c and g for the fiscal authority are:

$$\mathcal{U}_{M,x} + \lambda_M \varepsilon_x = 0 \tag{20}$$
$$\mathcal{U}_{F,c} + \lambda_F \varepsilon_c = 0 \tag{21}$$
$$\mathcal{U}_{F,g} + \lambda_F \varepsilon_g = 0 \tag{22}$$

The last two equations, (21) and (22), are functionally identical to (16) and (17), which characterized the case with a consolidated government. The reason for this is that, in both cases, the choice of taxes and expenditure depend on the preferences of the fiscal authority. In contrast, condition (20) differs from (15) since the central bank implements the day-good allocation and may, in principle, have different preferences than the fiscal authority. Note, however, that even if there were agreement on the marginal utility of the day-good, there could still be differences due to the Lagrange multipliers being different—i.e., the authorities may disagree on the impact of debt adjustment.

As it turns out, conditions (20)–(22) also hold in all the cases considered below. The differences between scenarios will thus arise from differences in the two remaining equations characterizing the equilibrium. In this case, the two remaining conditions are the first-order conditions with respect to B' for the problems of the fiscal and monetary authorities.

In the analysis that follows, it will be convenient to save on notation by defining

$$\Delta_j \equiv \frac{\mathcal{U}_{F,j}}{\lambda_F} - \frac{\mathcal{U}_{M,j}}{\lambda_M} \tag{23}$$

for $j = \{x, c, g\}$. The term Δ_j captures the disagreement between government authorities along a particular margin.

Proposition 1 (Short-term rate vs. tax rate)
When the monetary authority sets the short-term rate, i, and the fiscal authority sets the tax rate, τ, the MPME is characterized by (14)*,* (20)*,* (21)*,* (22) *and*

$$\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda_F'] + \lambda_F \varepsilon_{x'} \mathcal{X}_B' + \beta(1-\delta_F)\lambda_F' \Delta_x' \mathcal{X}_B' = 0 \tag{24}$$
$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda_M'] + \lambda_M \varepsilon_{x'} \mathcal{X}_B' - \beta(1-\delta_M)\lambda_M' \left(\Delta_c' \mathcal{C}_B' + \Delta_g' \mathcal{G}_B'\right) = 0 \tag{25}$$

It follows that:

1. *There is no possibility of fiscal dominance.*
2. *There is always a time-consistency problem in steady state.*

4.1.2 Fiscal authority sets debt

Now suppose that the fiscal authority sets end-of-period debt, B'. Note that by (13), this choice also determines the nominal interest rate, R, for a given future policy $\mathcal{X}$. In this case, the tax rate, τ, adjusts to satisfy the government budget constraint. From (11) this is equivalent to having the night-good allocation, c, adjust.

The monetary authority then implements a day-good allocation, x, taking as given the allocations implemented by current fiscal policy, $\mathcal{B}(B)$ and $\mathcal{G}(B)$, and future policies $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}\}$ which span a continuation value $\mathcal{M}$, and understanding that the night-good allocation, c, will adjust to satisfy the government budget constraint.

Similarly, the fiscal authority sets the debt level, B', and public expenditure, g, taking as given the allocations implemented by current monetary policy, $\mathcal{X}(B)$, and future policies $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}\}$ which span a continuation value $\mathcal{F}$, and understanding that the night-good allocation, c, will adjust to satisfy the government budget constraint.

We are now ready to formulate the problems of the fiscal and monetary authorities, and define an equilibrium in this setting.

Definition 5 (Short-term rate vs. debt)
When the monetary authority sets the short-term rate, i, and the fiscal authority sets the debt, B', an MPME is a set of functions $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}, \mathcal{F}, \mathcal{M}\} : \mathbb{B} \to \mathbb{R}_+^3 \times \mathbb{B} \times \mathbb{R}^2$, such that for all $B \in \mathbb{B}$:

$$\{\mathcal{G}(B), \mathcal{B}(B)\} = \underset{g, B'}{\operatorname{argmax}} \; \mathcal{U}_F(\mathcal{X}(B), c, g) + \beta(1-\delta_F)\mathcal{F}(B')$$

where $\varepsilon(B, B', \mathcal{X}(B), \mathcal{X}(B'), c, g) = 0$;

$$\mathcal{X}(B) = \underset{x}{\operatorname{argmax}} \; \mathcal{U}_M(x, c, \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(\mathcal{B}(B))$$

where $\varepsilon(B, \mathcal{B}(B), x, \mathcal{X}(B'), c, \mathcal{G}(B)) = 0$;

$$\begin{aligned} \mathcal{F}(B) &\equiv \mathcal{U}_F(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B)) \\ \mathcal{M}(B) &\equiv \mathcal{U}_M(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(\mathcal{B}(B)) \end{aligned}$$

and $\mathcal{C}(B)$ solves

$$\varepsilon(B, \mathcal{B}(B), \mathcal{X}(B), \mathcal{X}(\mathcal{B}(B)), \mathcal{C}(B), \mathcal{G}(B)) = 0.$$

For both authorities, the night-good allocation c adjusts to satisfy the government budget constraint, given their own actions and the anticipated policies of the other authority and both future authorities. Similarly to what we did in the previous case, we can add c as a choice variable and (14) as a constraint to each of the authorities' problems. With Lagrange multipliers λ_M and λ_F associated with their respective constraints, the first-order conditions with respect to x for the monetary authority, and c and g for the fiscal authority are again given by (20), (21), (22), respectively. The two remaining conditions are the first order condition with respect to c for the monetary authority and the first-order condition with respect to B' for the fiscal authority. These two equations will differ from the last two equations in the previous scenario.

Proposition 2 (Short-term rate vs. debt)
When the monetary authority sets the short-term rate, i, and the fiscal authority sets the debt level, B', the MPME is characterized by (14), (20), (21), (22) *and*

$$\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_F] + \lambda_F \varepsilon_{x'} \mathcal{X}'_B + \beta(1-\delta_F)\lambda'_F \Delta'_x \mathcal{X}'_B = 0 \tag{26}$$

$$\Delta_c = 0 \tag{27}$$

It follows that:

1. *There is fiscal dominance when $\mathcal{U}_{M,x} = \mathcal{U}_{F,x}$ and $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$.*
2. *Central bank patience, δ_M, is irrelevant.*
3. *If $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$ then $\lambda_M = \lambda_F$.*
4. *There is no time-consistency problem in steady state when $\delta_F = 0$.*

4.2 Central bank sets the money growth rate

Consider now a central bank that determines the growth rate of its liabilities and thus, lets the short-term interest rate adjust as necessary. This is the way monetary policy was conducted in the past, before the advent of interest rate policy, but is also part of the way it is conducted now, when "quantitative easing" is invoked. By (12) we can see that μ maps into the day-good allocation today and tomorrow, i.e., x and $x' = \mathcal{X}(B')$. In order to study this case using the primal approach, we need to use (12) to define the allocation x as a function of μ and x'. Hence, let $x = \Omega(\mu, \mathcal{X}(B'))$ where

$$\Omega(\mu, \mathcal{X}(B')) \equiv \frac{\beta u'_x x'}{\phi(1+\mu)}. \tag{28}$$

Let ω_μ and $\Omega_{x'}$ denote the derivatives of the function Ω with respect to its first and second arguments, respectively. In equilibrium, $\mu = m(B)$ and so $\mathcal{X}(B) = \Omega(m(B), \mathcal{X}(B'))$. It follows that $\mathcal{X}_B = \Omega_\mu m_B + \Omega_{x'} \mathcal{X}'_B \mathcal{B}_B$, which will be used in the derivations below. Note that by (10), $x = \Omega(\mu, \mathcal{X}(B'))$ pins down the short-term rate, i.

As with the previous case, the fiscal authority sets public expenditure and either picks taxes or end-of-period debt.

4.2.1 Fiscal authority sets the tax rate

When the fiscal authority sets the tax rate, τ, (11) implies that this is equivalent to implementing the night-good allocation, c. Hence, the debt level, B' adjusts to satisfy the government budget constraint.

The monetary authority sets the money growth rate μ, which given some future allocation function $\mathcal{X}(B')$, implies a current day-good allocation by (28), $x = \Omega(\mu, \mathcal{X}(\mathcal{B}'))$. It conducts policy taking as given the allocations implemented by current fiscal policy, $\mathcal{C}(B)$ and $\mathcal{G}(B)$, and future policies $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}\}$ which span a continuation value $\mathcal{M}$, and understanding that debt, B', will adjust to satisfy the government budget constraint.

Similarly, the fiscal authority implements the night-good allocation, c, and public expenditure, g, taking as given the allocations implemented by current monetary policy, $m(B)$, and future policies $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}\}$ which span a continuation value $\mathcal{F}$, and understanding that debt, B', will adjust to satisfy the government budget constraint.

We can now formulate the problems of the fiscal and monetary authorities, and define an equilibrium in this setting.

Definition 6 (Money growth rate vs. tax rate)
When the monetary authority sets the money growth rate, μ, and the fiscal authority sets the tax rate, τ, an MPME is a set of functions $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}, m, \mathcal{F}, \mathcal{M}\} : \mathbb{B} \to \mathbb{R}^3_+ \times \mathbb{B} \times \mathbb{R}^3$, such that for all $B \in \mathbb{B}$:

$$\{\mathcal{C}(B), \mathcal{G}(B)\} = \underset{c,\ g}{\operatorname{argmax}}\ \mathcal{U}_F(\Omega(m(B), \mathcal{X}(B')), c, g) + \beta(1-\delta_F)\mathcal{F}(B')$$

where $\varepsilon(B, B', \Omega(m(B), \mathcal{X}(B')), \mathcal{X}(B'), c, g) = 0$;

$$m(B) = \underset{\mu}{\operatorname{argmax}}\ \mathcal{U}_M(\Omega(\mu, \mathcal{X}(B')), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(B')$$

where $\varepsilon(B, B', \Omega(\mu, \mathcal{X}(B')), \mathcal{X}(B'), \mathcal{C}(B), \mathcal{G}(B)) = 0$;

$$\begin{aligned} \mathcal{X}(B) &\equiv \Omega(m(B), \mathcal{X}(B')) \\ \mathcal{F}(B) &\equiv \mathcal{U}_F(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B)) \\ \mathcal{M}(B) &\equiv \mathcal{U}_M(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(\mathcal{B}(B)); \end{aligned}$$

and $\mathcal{B}(B)$ solves

$$\varepsilon(B, \mathcal{B}(B), \mathcal{X}(B), \mathcal{X}(\mathcal{B}(B)), \mathcal{C}(B), \mathcal{G}(B)) = 0.$$

Proposition 3 (Money growth rate vs. tax rate)
When the monetary authority sets the money growth rate, μ, and the fiscal authority sets the tax rate, τ, the MPME is characterized by (14), (20), (21), (22) *and*

$$\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_F] + \lambda_F\left(\varepsilon_{x'} + \Delta_x\Omega_{x'}\right)\mathcal{X}'_B + \beta(1-\delta_F)\lambda'_F\Delta'_x\left(\mathcal{X}'_B - \Omega'_{x'}\mathcal{X}''_B\mathcal{B}'_B\right) = 0 \quad (29)$$

$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda'_M] + \lambda_M\varepsilon_{x'}\mathcal{X}'_B - \beta(1-\delta_F)\lambda'_M\left(\Delta'_c\mathcal{C}'_B + \Delta'_g\mathcal{G}'_B\right) = 0 \quad (30)$$

It follows that:

1. *There is no possibility of fiscal dominance.*
2. *There is always a time-consistency problem in steady state.*

4.2.2 Fiscal authority sets debt

If the fiscal authority sets the end-of period debt level, B', then (28) establishes a one-to-one mapping between x and μ. In other words, setting the money growth rate is equivalent to implementing the day-good allocation, x. This means that this case has a representation equivalent to that when the central bank sets the short-term rate and the fiscal authority sets the debt—see Section 4.1.2.

Proposition 4 (Money growth rate vs. debt) *When the fiscal authority sets the debt, B', setting the short-term rate or the money growth rate are equivalent policies for the central bank. Definition 5 and Proposition 2 apply in both cases.*

When the fiscal authority sets the debt, the possibility of fiscal dominance arises. As shown in Proposition 4, this requires some agreement in preferences between the fiscal and monetary authorities. Furthermore, the patience of the central bank does not enter any the equations characterizing the equilibrium and therefore becomes irrelevant. More generally, the central bank loses instrument choice as a possible margin in which to counteract the fiscal authority—it does not matter whether if sets the short-term rate or the money growth rate.

4.3 Central bank targets the yield curve

If the fiscal authority is willing to let debt adjust as necessary, it opens up the possibility for the central bank to target the nominal interest rate, R. Recall that by (13), setting R is equivalent to choosing B'. In this case, the central bank can limit itself to targeting the long end of the yield curve by setting the long-term interest rate, R, and letting the short-term rate i adjust given fiscal policy. Alternatively, the central bank may want to target the whole yield curve, i.e., set both i and R. In this latter case, the fiscal authority will have to let taxes adjust to satisfy the government budget constraint and its only meaningful decision is to decide on the level of expenditure.

4.3.1 Central bank sets the long-term rate

If the central bank sets R, or equivalently, B', the fiscal authority sets τ, which by (11) implements the night-good allocation, c. In this case, the short-term rate, i, by (10) equivalently the day-good allocation, x, adjusts to satisfy the government budget constraint.

Definition 7 (Long-term rate vs. tax rate)
When the monetary authority sets the long-term rate, R, and the fiscal authority sets the tax rate, τ, an MPME is a set of functions $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}, \mathcal{F}, \mathcal{M}\} : \mathbb{B} \to \mathbb{R}^3_+ \times \mathbb{B} \times \mathbb{R}^2$, such that for all $B \in \mathbb{B}$:

$$\{\mathcal{C}(B), \mathcal{G}(B)\} = \underset{c,\ g}{\operatorname{argmax}}\ \mathcal{U}_F(x, c, g) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B))$$

where $\varepsilon(B, \mathcal{B}(B), x, \mathcal{X}(\mathcal{B}(B)), c, g) = 0$;

$$\mathcal{B}(B) = \underset{B'}{\operatorname{argmax}}\ \mathcal{U}_M(x, \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(B')$$

where $\varepsilon(B, B', x, \mathcal{X}(B'), \mathcal{C}(B), \mathcal{G}(B)) = 0$;

$$\begin{aligned} \mathcal{F}(B) &\equiv \mathcal{U}_F(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B)) \\ \mathcal{M}(B) &\equiv \mathcal{U}_M(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(\mathcal{B}(B)); \end{aligned}$$

and $\mathcal{X}(B)$ solves

$$\varepsilon(B, \mathcal{B}(B), \mathcal{X}(B), \mathcal{X}(\mathcal{B}(B)), \mathcal{C}(B), \mathcal{G}(B)) = 0.$$

Proposition 5 (Long-term rate vs. tax rate)
When the monetary authority sets the long-term rate, R, and the fiscal authority sets the tax rate, τ, the MPME is characterized by (14)*,* (20)*,* (21)*,* (22) *and*

$$\Delta_x = 0 \tag{31}$$

$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda'_M] + \lambda_M \varepsilon_{x'} \mathcal{X}'_B - \beta(1-\delta_M)\lambda'_M \left(\Delta'_c \mathcal{C}'_B + \Delta'_g \mathcal{G}'_B\right) = 0 \tag{32}$$

It follows that:

1. *There is no possibility of fiscal dominance.*

2. *Fiscal authority patience, δ_F, is irrelevant.*

3. *If $\mathcal{U}_{M,x} = \mathcal{U}_{F,x}$ then $\lambda_M = \lambda_F$.*

4. *There is no time-consistency problem in steady state when $\delta_M = 0$, $\mathcal{U}_{M,x} = \mathcal{U}_{F,x}$, $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$ and $\mathcal{U}_{M,g} = \mathcal{U}_{F,g}$.*

Proposition 5(ii) generalizes the findings of Niemann et al. (2013), which derived it for the case without taxes and when the disagreement between authorities is only about patience.

4.3.2 Central bank sets the short- and long-term rates

The last case we consider is the extreme scenario in which the central bank is able to determine the entire yield curve. This involves setting both i and R, equivalently, by (10) and (13), implement x and B'. By (12) this choices imply a money growth rate, μ. Hence, this scenario involves having the central bank picking any two out of i, R and μ. The fiscal authority sets public expenditure g and thus, lets taxes τ, or equivalently by (11) the night-good allocation, c, adjust to satisfy the government budget constraint.

Definition 8 (Yield curve control)
When the monetary authority sets both the short- and long-term rates, i and R, an MPME is a set of functions $\{\mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{B}, \mathcal{F}, \mathcal{M}\} : \mathbb{B} \to \mathbb{R}^3_+ \times \mathbb{B} \times \mathbb{R}^2$, such that for all $B \in \mathbb{B}$:

$$\mathcal{G}(B) = \underset{g}{\text{argmax}} \; \mathcal{U}_F(\mathcal{X}(B), c, g) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B))$$

where $\varepsilon(B, \mathcal{B}(B), \mathcal{X}(B), \mathcal{X}(\mathcal{B}(B)), c, g) = 0$;

$$\{\mathcal{X}(B), \mathcal{B}(B)\} = \underset{x,B'}{\text{argmax}} \; \mathcal{U}_M(x, c, \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(B')$$

where $\varepsilon(B, B', x, \mathcal{X}(B'), c, \mathcal{G}(B)) = 0$;

$$\begin{aligned} \mathcal{F}(B) &\equiv \mathcal{U}_F(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B)) \\ \mathcal{M}(B) &\equiv \mathcal{U}_M(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(\mathcal{B}(B)); \end{aligned}$$

and $\mathcal{C}(B)$ solves

$$\varepsilon(B, \mathcal{B}(B), \mathcal{X}(B), \mathcal{X}(\mathcal{B}(B)), \mathcal{C}(B), \mathcal{G}(B)) = 0.$$

Proposition 6 (Yield curve control)
When the monetary authority sets the short- and long-term rates, i and R, the MPME is characterized by (14), (20), (21), (22) *and*

$$\Delta_c = 0 \tag{33}$$

$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda_M'] + \lambda_M \varepsilon_{x'} \mathcal{X}_B' + \beta(1-\delta_M)\lambda_M' \Delta_g' \mathcal{G}_B' = 0 \tag{34}$$

It follows that:

1. *There is no possibility of fiscal dominance.*

2. *Fiscal authority patience, δ_F, is irrelevant.*

3. *If $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$ then $\lambda_M = \lambda_F$.*

4. *There is no time-consistency problem in steady state when* $\delta_M = 0$*,* $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$ *and* $\mathcal{U}_{M,g} = \mathcal{U}_{F,g}$*.*

Propositions 5(iii) and 6(iii) imply that yield curve control, as opposed to just setting the long-term rate, is only useful when there is disagreement on preferences for the day and night-good allocations. In other words, if the disagreement is limited to public expenditure and/or patience, the two cases are identical and the central bank can simply set the long-term rate to achieve the same result as with full yield curve control.

One interesting outcome in this setting is that if the disagreement between authorities is limited to the marginal utility of the day-good and the discount factor, then the preferences of the fiscal authority are rendered irrelevant. In other words, we get monetary dominance. But note that this is an extreme case: the fiscal authority is fully passive except for its choice of expenditure and the only meaningful concern is its relative impatience.

5 Conclusions

Table 1 summarizes the key results of this paper. First, the possibility of fiscal dominance arises only when the fiscal authority actively sets the debt level. As shown in Propositions 2 and 4 fiscal dominance does require some agreement in preferences. Notably, fiscal dominance occurs when the disagreement between authorities is limited to the preference for public expenditure and patience. Hence, a way for the central bank to counteract the fiscal authority in this case is to have a special concern for liquidity markets—represented here with preferences for the day-good allocation. Regardless of whether fiscal dominance is achieved, when the fiscal authority sets the debt, it renders the central bank's patience irrelevant. Thus, making the central bank overly concerned with the long-run is ineffective in this case.

Table 1: Summary of results

Active instrument	Fiscal dominance possible?	Time-consistency problem in steady state	Relevant discounting
Tax rate	No	Always	Both
Debt	Yes	Eliminated when $\delta_F = 0$	Fiscal
Long-term rate	No	Always	Monetary
Yield curve	No	Always	Monetary

Even when fiscal dominance is not achieved, setting debt actively opens up the possibility that the steady state is constrained efficient. This situations arises when the fiscal authority shares the same discount factor as private agents. In this case, the time-consistency problem is eliminated in the steady state.

When the central bank is allowed to control the long-term rate it renders fiscal patience irrelevant. As stated in Propositions 5 and 6, full yield curve control is only useful when there is sufficient disagreement between the authorities. This can be a powerful tool but it does require the fiscal authority to adopt a mostly passive role.

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A Proofs

Proof of Proposition 1.

From Definition 4 we can write the problem of the fiscal authority as:

$$\max_{c,\ g,\ B'} \mathcal{U}_F(\mathcal{X}(B), c, g) + \beta(1-\delta_F)\mathcal{F}(B')$$

subject to

$$\varepsilon(B, B', \mathcal{X}(B), \mathcal{X}(B'), c, g) = 0.$$

With λ_F as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\begin{aligned} \mathcal{U}_{F,c} + \lambda_F \varepsilon_c &= 0 & \text{(A.1)} \\ \mathcal{U}_{F,g} + \lambda_F \varepsilon_g &= 0 & \text{(A.2)} \\ \beta(1-\delta_F)\mathcal{F}'_B + \lambda_F(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B) &= 0 & \text{(A.3)} \end{aligned}$$

Conditions (A.1) and (A.2) correspond to (21) and (22) in the main text.

The problem of the monetary authority is:

$$\max_{x,\ B'} \mathcal{U}_M(x, \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(B')$$

subject to

$$\varepsilon(B, B', x, \mathcal{X}(B'), \mathcal{C}(B), \mathcal{G}(B)) = 0.$$

Using λ_M as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\begin{aligned} \mathcal{U}_{M,x} + \lambda_M \varepsilon_x &= 0 & \text{(A.4)} \\ \beta(1-\delta_M)\mathcal{M}'_B + \lambda_M(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B) &= 0 & \text{(A.5)} \end{aligned}$$

Condition (A.4) corresponds to (20) in the main text.

Now, we will derive (24) and (25) from (A.3) and (A.5). First, we need to obtain expressions for $\mathcal{F}_B$ and $\mathcal{M}_B$. From Definition 4 we have

$$\begin{aligned} \mathcal{F}(B) &= \mathcal{U}_F(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B)) \\ \mathcal{M}(B) &= \mathcal{U}_M(\mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(\mathcal{B}(B)) \end{aligned}$$

for all $B \in \mathbb{B}$. Totally differentiating both expressions with respect to B we obtain:

$$\begin{aligned} \mathcal{F}_B &= \mathcal{U}_{F,x}\mathcal{X}_B + \mathcal{U}_{F,c}\mathcal{C}_B + \mathcal{U}_{F,g}\mathcal{G}_B + \beta(1-\delta_F)\mathcal{F}'_B\mathcal{B}_B \\ \mathcal{M}_B &= \mathcal{U}_{M,x}\mathcal{X}_B + \mathcal{U}_{M,c}\mathcal{C}_B + \mathcal{U}_{M,g}\mathcal{G}_B + \beta(1-\delta_M)\mathcal{M}'_B\mathcal{B}_B \end{aligned}$$

We can use (A.3) and (A.5) to replace $\mathcal{F}'_B$ and $\mathcal{M}'_B$, respectively. We can also use (A.1), (A.2) and (A.4) to replace $\mathcal{U}_{F,c}$, $\mathcal{U}_{F,g}$ and $\mathcal{U}_{M,x}$. Then,

$$\begin{aligned} \mathcal{F}_B &= \mathcal{U}_{F,x}\mathcal{X}_B - \lambda_F\varepsilon_c\mathcal{C}_B - \lambda_F\varepsilon_g\mathcal{G}_B - \lambda_F(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B)\mathcal{B}_B & \text{(A.6)} \\ \mathcal{M}_B &= -\lambda_M\varepsilon_x\mathcal{X}_B + \mathcal{U}_{M,c}\mathcal{C}_B + \mathcal{U}_{M,g}\mathcal{G}_B - \lambda_M(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B)\mathcal{B}_B & \text{(A.7)} \end{aligned}$$

We also have that $\varepsilon(B, \mathcal{B}(B), \mathcal{X}(B), \mathcal{X}(B'), \mathcal{C}(B), \mathcal{G}(B)) = 0$ for all $B \in \mathbb{B}$. Totally differentiating this expression with respect to B yields

$$\varepsilon_B + \varepsilon_{B'}\mathcal{B}_B + \varepsilon_x\mathcal{X}_B + \varepsilon_{x'}\mathcal{X}'_B\mathcal{B}_B + \varepsilon_c\mathcal{C}_B + \varepsilon_g\mathcal{G}_B = 0.$$

which we can rearrange as

$$-(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B)\mathcal{B}_B = \varepsilon_B + \varepsilon_x\mathcal{X}_B + \varepsilon_c\mathcal{C}_B + \varepsilon_g\mathcal{G}_B$$

Replace this expression in (A.6) and (A.7) and rearrange:

$$\begin{aligned}\mathcal{F}_B &= \mathcal{U}_{F,x}\mathcal{X}_B + \lambda_F(\varepsilon_B + \varepsilon_x\mathcal{X}_B)\\ \mathcal{M}_B &= \mathcal{U}_{M,c}\mathcal{C}_B + \mathcal{U}_{M,g}\mathcal{G}_B + \lambda_M(\varepsilon_B + \varepsilon_c\mathcal{C}_B + \varepsilon_g\mathcal{G}_B)\end{aligned}$$

Now, update these expressions one period and replace in (A.3) and (A.5) to obtain:

$$\begin{aligned}\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_B] + \lambda_F\varepsilon_{x'}\mathcal{X}'_B + \beta(1-\delta_F)(\mathcal{U}'_{F,x} + \lambda'_F\varepsilon'_x)\mathcal{X}'_B &= 0\\ \varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda'_M] + \lambda_M\varepsilon_{x'}\mathcal{X}'_B + \beta(1-\delta_M)\left[(\mathcal{U}'_{M,c} + \lambda'_M\varepsilon'_c)\mathcal{C}'_B + (\mathcal{U}'_{M,g} + \lambda'_M\varepsilon'_g)\mathcal{G}'_B\right] &= 0\end{aligned}$$

where we used $\varepsilon_{B'} = -\beta\varepsilon'_B$ from (14).

Using (A.4) we can write $\varepsilon_x = -\mathcal{U}_{M,x}/\lambda_M$. Hence, $\mathcal{U}_{F,x} + \lambda_F\varepsilon_x = \mathcal{U}_{F,x} + \mathcal{U}_{F,x}(\lambda_F/\lambda_M) = \lambda_F[(\mathcal{U}_{F,x}/\lambda_F) + (\mathcal{U}_{F,x}/\lambda_M)]$. Using (23) this latter expression can be written compactly as $\lambda_F\Delta_x$. Similarly, from (A.1) and (A.2) we obtain $\mathcal{U}_{M,c} + \lambda_M\varepsilon_c = -\lambda_M\Delta_c$ and $\mathcal{U}_{M,g} + \lambda_M\varepsilon_g = -\lambda_M\Delta_g$. Hence,

$$\begin{aligned}\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_B] + \lambda_F\varepsilon_{x'}\mathcal{X}'_B + \beta(1-\delta_F)\lambda'_F\Delta'_x\mathcal{X}'_B &= 0 \quad (A.8)\\ \varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda'_M] + \lambda_M\varepsilon_{x'}\mathcal{X}'_B - \beta(1-\delta_M)\lambda'_M(\Delta'_c\mathcal{C}'_B + \Delta'_g\mathcal{G}'_B) &= 0 \quad (A.9)\end{aligned}$$

which correspond to (24) and (25), respectively.

To obtain fiscal dominance we need conditions (A.1), (A.2), (A.4), (A.8) and (A.9) to be equivalent to conditions (15)–(18). This requires $\lambda_M = \lambda_F$, $\Delta'_x = \Delta'_c = \Delta'_g = 0$ and $\delta_M = \delta_F$. These requirements violate Assumption 2. Hence, fiscal dominance is not possible.

In steady state, (A.8) and (A.9) become

$$\begin{aligned}\varepsilon_{B'}\delta_F + \varepsilon_{x'}\mathcal{X}_B + \beta(1-\delta_F)\Delta_x\mathcal{X}_B &= 0\\ \varepsilon_{B'}\delta_M + \varepsilon_{x'}\mathcal{X}_B - \beta(1-\delta_M)(\Delta_c\mathcal{C}_B + \Delta_g\mathcal{G}_B) &= 0\end{aligned}$$

If $\delta_F = \delta_M = 0$ then

$$\begin{aligned}\varepsilon_{x'} + \beta\Delta_x &= 0\\ \Delta_x\mathcal{X}_B + \Delta_c\mathcal{C}_B + \Delta_g\mathcal{G}_B &= 0\end{aligned}$$

Eliminating derivatives of policy functions in the second equation would require a violation of Assumption 2. Hence, there is always a time-consistency problem in steady state. ■

Proof Proposition 2.

From Definition 5 we can write the problem of the fiscal authority as:

$$\max_{c,\ g,\ B'} \ \mathcal{U}_F(\mathcal{X}(B), c, g) + \beta(1-\delta_F)\mathcal{F}(B')$$

subject to

$$\varepsilon(B, B', \mathcal{X}(B), \mathcal{X}(B'), c, g) = 0.$$

With λ_F as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\begin{aligned}\mathcal{U}_{F,c} + \lambda_F\varepsilon_c &= 0 \quad (A.10)\\ \mathcal{U}_{F,g} + \lambda_F\varepsilon_g &= 0 \quad (A.11)\\ \beta(1-\delta_F)\mathcal{F}'_B + \lambda_F(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B) &= 0 \quad (A.12)\end{aligned}$$

Conditions (A.10) and (A.11) are the same as (21) and (22) in the main text.

The problem of the monetary authority is:

$$\max_{x,\ c}\ \mathcal{U}_M(x, c, \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(\mathcal{B}(B))$$

subject to

$$\varepsilon(B, \mathcal{B}(B), x, \mathcal{X}(\mathcal{B}(B)), c, \mathcal{G}(B)) = 0.$$

Using λ_M as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\mathcal{U}_{M,x} + \lambda_M \varepsilon_x = 0 \tag{A.13}$$

$$\mathcal{U}_{M,c} + \lambda_M \varepsilon_c = 0 \tag{A.14}$$

Condition (A.13) is the same as (20) in the main text.

Now, we will derive (26) and (27) from (A.12) and (A.14). As in the proof of Proposition 1 we can differentiate $\mathcal{F}(B)$ with respect to B and combine with the first-order conditions to obtain

$$\mathcal{F}_B = \mathcal{U}_F \mathcal{X}_B + \lambda_B(\varepsilon_B + \varepsilon_x \mathcal{X}_B).$$

Update this expression one period and replace in (A.12) to obtain:

$$\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_B] + \lambda_F \varepsilon_{x'} \mathcal{X}'_B + \beta(1-\delta_F)(\mathcal{U}'_{F,x} + \lambda'_F \varepsilon'_x)\mathcal{X}'_B = 0$$

where we used $\varepsilon_{B'} = -\beta\varepsilon'_B$ from (14). Using (A.13) and (23) we get $\mathcal{U}_{F,x} + \lambda_F \varepsilon_x = \lambda_F \Delta_x$. Hence,

$$\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_B] + \lambda_F \varepsilon_{x'} \mathcal{X}'_B + \beta(1-\delta_F)\lambda'_F \Delta'_x \mathcal{X}'_B = 0 \tag{A.15}$$

which corresponds to (26).

Combining (A.13) and (23) we obtain

$$\Delta_c = 0 \tag{A.16}$$

which corresponds to (27).

Note that conditions (A.10), (A.11), (A.13), (A.15) and (A.16) do not depend on δ_M, so central bank patience is irrelevant.

To obtain fiscal dominance we need conditions (A.10), (A.11), (A.13), (A.15) and (A.16) to be equivalent to conditions (15)–(18). We already have that (A.10), (A.11) are functionally equivalent to (16) and (17), respectively. If $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$ then (A.16) implies $\lambda_M = \lambda_F$. If $\mathcal{U}_{M,x} = \mathcal{U}_{F,x}$, then $\lambda_M = \lambda_F$ implies $\Delta_x = 0$. Hence, (A.15) is identical to (15). $\lambda_M = \lambda_F$ also implies that (A.13) is identical to (15). We thus obtain fiscal dominance in this case.

The only condition with derivatives of policy functions is (A.15). In steady state we obtain

$$\varepsilon_{B'}\delta_F + [\varepsilon_{x'} + \beta(1-\delta_F)\Delta_x]\mathcal{X}_B = 0$$

If $\delta_F = 0$ then the expression simplifies to $\varepsilon_{x'} + \beta\Delta_x = 0$ and there is no time-consistency problem in steady state. ■

Proof of Proposition 3.

From Definition 6 we can write the problem of the fiscal authority as:

$$\max_{c,\ g,\ B'}\ \mathcal{U}_F(\Omega(m(B), \mathcal{X}(B')), c, g) + \beta(1-\delta_F)\mathcal{F}(B')$$

subject to

$$\varepsilon(B, B', \Omega(m(B), \mathcal{X}(B')), \mathcal{X}(B'), c, g) = 0.$$

With λ_F as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\mathcal{U}_{F,c} + \lambda_F \varepsilon_c = 0 \quad (A.17)$$
$$\mathcal{U}_{F,g} + \lambda_F \varepsilon_g = 0 \quad (A.18)$$
$$\mathcal{U}_{F,x}\Omega_{x'}\mathcal{X}'_B + \beta(1-\delta_F)\mathcal{F}'_B + \lambda_F(\varepsilon_{B'} + \varepsilon_x \Omega_{x'}\mathcal{X}'_B + \varepsilon_{x'}\mathcal{X}'_B) = 0 \quad (A.19)$$

Conditions (A.17) and (A.18) correspond to (21) and (22) in the main text.

The problem of the monetary authority is:

$$\underset{\mu,\ B'}{\text{argmax}}\ \mathcal{U}_M(\Omega(\mu, \mathcal{X}(B')), \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(B')$$

subject to

$$\varepsilon(B, B', \Omega(\mu, \mathcal{X}(B')), \mathcal{X}(B'), \mathcal{C}(B), \mathcal{G}(B)) = 0.$$

Using λ_M as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$(\mathcal{U}_{M,x} + \lambda_M \varepsilon_x)\Omega_\mu = 0 \quad (A.20)$$
$$\mathcal{U}_{M,x}\Omega_{x'}\mathcal{X}'_B + \beta(1-\delta_M)\mathcal{M}'_B + \lambda_M(\varepsilon_{B'} + \varepsilon_x \Omega_{x'}\mathcal{X}'_B + \varepsilon_{x'}\mathcal{X}'_B) = 0 \quad (A.21)$$

Given $\Omega_\mu < 0$ by (28), condition (A.20) corresponds to (20) in the main text.

Now, we will derive (24) and (30) from (A.19) and (A.21). We follow the same procedure as in the proof of Proposition 1: differentiate $\mathcal{F}(B)$ and $\mathcal{M}(B)$ with respect to B and combine with the first-order conditions. We obtain

$$\mathcal{F}_B = \mathcal{U}_{F,x}\Omega_\mu m_B + \lambda_F(\varepsilon_B + \varepsilon_x \Omega_\mu m_B)$$
$$\mathcal{M}_B = \mathcal{U}_{M,c}\mathcal{C}_B + \mathcal{U}_{M,g}\mathcal{G}_B + \lambda_M(\varepsilon_B + \varepsilon_c \mathcal{C}_B + \varepsilon_g \mathcal{G}_B)$$

From Definition 6, we can use $\mathcal{X}(B) = \Omega(m(B), \mathcal{X}(\mathcal{B}(B)))$ for all $B \in \mathbb{B}$ to replace m_B in the expression for $\mathcal{F}_B$. Totally differentiating both sides with respect to B we obtain $\mathcal{X}_B = \Omega_\mu m_B + \Omega_{x'}\mathcal{X}'_B\mathcal{B}_B$ and thus,

$$\Omega_\mu m_B = \mathcal{X}_B - \Omega_{x'}\mathcal{X}'_B\mathcal{B}_B$$

which implies

$$\mathcal{F}_B = (\mathcal{U}_{F,x} + \lambda_F\varepsilon_x)(\mathcal{X}_B - \Omega_{x'}\mathcal{X}'_B\mathcal{B}_B) + \lambda_F\varepsilon_B$$

Update the expressions for $\mathcal{F}_B$ and $\mathcal{M}_B$ one period and replace in (A.19) and (A.21) to obtain:

$$\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_B] + \lambda_F\varepsilon_{x'}\mathcal{X}'_B + \beta(1-\delta_F)(\mathcal{U}'_{F,x} + \lambda'_F\varepsilon'_x)(\mathcal{X}'_B - \Omega'_{x'}\mathcal{X}''_B\mathcal{B}'_B) = 0$$
$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda'_M] + \lambda_M\varepsilon_{x'}\mathcal{X}'_B + \beta(1-\delta_M)\left[(\mathcal{U}'_{M,c} + \lambda'_M\varepsilon'_c)\mathcal{C}'_B + (\mathcal{U}'_{M,g} + \lambda'_M\varepsilon'_g)\mathcal{G}'_B\right] = 0$$

where we used $\varepsilon_{B'} = -\beta\varepsilon'_B$ from (14). As in the proof of Proposition 1 combine (23) with (A.17), (A.18) and (A.20) to write $\mathcal{U}_{M,c} + \lambda_M\varepsilon_c = -\lambda_M\Delta_c$, $\mathcal{U}_{M,g} + \lambda_M\varepsilon_g = -\lambda_M\Delta_g$ and $\mathcal{U}_{F,x} + \lambda_F\varepsilon_x = \lambda_F\Delta_x$. Hence,

$$\varepsilon_{B'}[\lambda_F - (1-\delta_F)\lambda'_B] + \lambda_F\varepsilon_{x'}\mathcal{X}'_B + \beta(1-\delta_F)\lambda'_F\Delta'_x(\mathcal{X}'_B - \Omega'_{x'}\mathcal{X}''_B\mathcal{B}'_B) = 0 \quad (A.22)$$
$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda'_M] + \lambda_M\varepsilon_{x'}\mathcal{X}'_B - \beta(1-\delta_M)\lambda'_M(\Delta'_c\mathcal{C}'_B + \Delta'_g\mathcal{G}'_B) = 0 \quad (A.23)$$

which correspond to (29) and (30), respectively.

To obtain fiscal dominance we need conditions (A.17), (A.18), (A.20), (A.22) and (A.23) to be equivalent to conditions (15)–(18). This requires $\lambda_M = \lambda_F$, $\Delta'_x = \Delta'_c = \Delta'_g = 0$ and $\delta_M = \delta_F$. These requirements violate Assumption 2. Hence, fiscal dominance is not possible.

In steady state, (A.8) and (A.9) become

$$\begin{aligned} \varepsilon_{B'}\delta_F + \varepsilon_{x'}\mathcal{X}_B + \beta(1-\delta_F)\Delta_x(1-\Omega_{x'}\mathcal{B}_B)\mathcal{X}_B &= 0 \\ \varepsilon_{B'}\delta_M + \varepsilon_{x'}\mathcal{X}_B - \beta(1-\delta_M)(\Delta_c\mathcal{C}_B + \Delta_g\mathcal{G}_B) &= 0 \end{aligned}$$

If $\delta_F = \delta_M = 0$ then

$$\begin{aligned} \varepsilon_{x'} + \beta\Delta_x(1-\Omega_{x'}\mathcal{B}_B) &= 0 \\ \Delta_x(1-\Omega_{x'}\mathcal{B}_B)\mathcal{X}_B + \Delta_c\mathcal{C}_B + \Delta_g\mathcal{G}_B &= 0 \end{aligned}$$

Eliminating derivatives of policy functions in the first equation would require the non-generic $\Omega_{x'}\mathcal{B}_B = 1$; from second equation would require a violation of Assumption 2. Hence, there is always a time-consistency problem in steady state. ■

Proof of Proposition 4. See main text. ■

Proof of Proposition 5.

From Definition 7 we can write the problem of the fiscal authority as:

$$\max_{x,\ c,\ g} \ \mathcal{U}_F(x,c,g) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B))$$

subject to

$$\varepsilon(B, \mathcal{B}(B), x, \mathcal{X}(\mathcal{B}(B)), c, g) = 0.$$

With λ_F as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\mathcal{U}_{F,x} + \lambda_F\varepsilon_x = 0 \quad \text{(A.24)}$$

$$\mathcal{U}_{F,c} + \lambda_F\varepsilon_c = 0 \quad \text{(A.25)}$$

$$\mathcal{U}_{F,g} + \lambda_F\varepsilon_g = 0 \quad \text{(A.26)}$$

Conditions (A.25) and (A.26) correspond to (21) and (22) in the main text.

The problem of the monetary authority is:

$$\max_{x,\ B'} \ \mathcal{U}_M(x, \mathcal{C}(B), \mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(B')$$

subject to

$$\varepsilon(B, B', x, \mathcal{X}(B'), \mathcal{C}(B), \mathcal{G}(B)) = 0.$$

Using λ_M as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\mathcal{U}_{M,x} + \lambda_M\varepsilon_x = 0 \quad \text{(A.27)}$$

$$\beta(1-\delta_M)\mathcal{M}'_B + \lambda_M(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B) = 0 \quad \text{(A.28)}$$

Condition (A.27) corresponds to (20) in the main text.

Combining (A.24), (A.27) and (23) yields

$$\Delta_x = 0 \quad \text{(A.29)}$$

which corresponds to (31) in the main text.

Now, we will derive (32) from (A.28). As in the proof of Proposition 1 we can differentiate $\mathcal{M}(B)$ with respect to B and combine with the first-order conditions of the monetary authority to obtain

$$\mathcal{M}_B = \mathcal{U}_{M,c}\mathcal{C}_B + \mathcal{U}_{M,g}\mathcal{G}_B + \lambda_M(\varepsilon_B + \varepsilon_c\mathcal{C}_B + \varepsilon_g\mathcal{G}_B).$$

Update this expression one period and and replace in (A.28) to obtain:

$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda_M'] + \lambda_M\varepsilon_{x'}\mathcal{X}_B' - \beta(1-\delta_M)\lambda_M'(\Delta_c'\mathcal{C}_B' + \Delta_g'\mathcal{G}_B') = 0 \quad \text{(A.30)}$$

where we used used $\varepsilon_{B'} = -\beta\varepsilon_B'$ from (14), and (23) plus (A.25) and (A.26) to arrange terms, as in previous proofs. Condition (A.30) corresponds to (32) in the main text.

Note that δ_F does not appear in (A.25), (A.26), (A.24), (A.29) or (A.30). Hence, the patience of the fiscal authority is irrelevant. Also note that if $\mathcal{U}_{M,x} = \mathcal{U}_{F,x}$ then (A.29) implies $\lambda_M = \lambda_F$.

To obtain fiscal dominance we need conditions (A.25), (A.26), (A.27), (A.29) and (A.30) to be equivalent to conditions (15)–(18). This requires $\lambda_M = \lambda_F$, $\Delta_x = \Delta_c' = \Delta_g' = 0$ and $\delta_M = \delta_F$. These requirements violate Assumption 2. Hence, fiscal dominance is not possible.

The only condition with derivatives of policy functions is (A.30). In steady state we obtain

$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda_M] + \lambda_M\varepsilon_{x'}\mathcal{X}_B - \beta(1-\delta_M)\lambda_M(\Delta_c\mathcal{C}_B + \Delta_g\mathcal{G}_B) = 0$$

If $\delta_M = 0$ then the expression simplifies to

$$\varepsilon_{x'}\mathcal{X}_B - \beta(\Delta_c\mathcal{C}_B + \Delta_g\mathcal{G}_B) = 0$$

If $\mathcal{U}_{M,x} = \mathcal{U}_{F,x}$ (which implies $\lambda_M = \lambda_F$), $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$ and $\mathcal{U}_{M,g} = \mathcal{U}_{F,g}$ then $\Delta_c = \Delta_g = 0$ and the expression above simplifies to $\varepsilon_{x'} = 0$. In this case, there is no time-consistency problem in steady state even when $\delta_F > 0$. ■

Proof of Proposition 6.

From Definition 8 we can write the problem of the fiscal authority as:

$$\max_{c,\ g}\ \mathcal{U}_F(x,c,g) + \beta(1-\delta_F)\mathcal{F}(\mathcal{B}(B))$$

subject to

$$\varepsilon(B, \mathcal{B}(B), \mathcal{X}(B), \mathcal{X}(\mathcal{B}(B)), c, g) = 0.$$

With λ_F as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\mathcal{U}_{F,c} + \lambda_F\varepsilon_c = 0 \quad \text{(A.31)}$$
$$\mathcal{U}_{F,g} + \lambda_F\varepsilon_g = 0 \quad \text{(A.32)}$$

Conditions (A.31) and (A.32) correspond to (21) and (22) in the main text.

The problem of the monetary authority is:

$$\max_{x,\ c,\ B'}\ \mathcal{U}_M(x,c,\mathcal{G}(B)) + \beta(1-\delta_M)\mathcal{M}(B')$$

subject to

$$\varepsilon(B, B', x, \mathcal{X}(B'), c, \mathcal{G}(B)) = 0.$$

Using λ_M as the Lagrange multiplier associated with the constraint, the first-order conditions are:

$$\mathcal{U}_{M,x} + \lambda_M \varepsilon_x = 0 \quad \text{(A.33)}$$
$$\mathcal{U}_{M,c} + \lambda_M \varepsilon_c = 0 \quad \text{(A.34)}$$
$$\beta(1-\delta_M)\mathcal{M}'_B + \lambda_M(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B) = 0 \quad \text{(A.35)}$$

Condition (A.33) corresponds to (20) in the main text.

Combining (A.31), (A.34) and (23) yields

$$\Delta_c = 0 \quad \text{(A.36)}$$

which corresponds to (33) in the main text.

Now, we will derive (34) from (A.35). As in the proof of Proposition 1 we can differentiate $\mathcal{M}(B)$ with respect to B and combine with the first-order conditions of the monetary authority to obtain

$$\mathcal{M}_B = \mathcal{U}_{M,c}\mathcal{C}_B + \mathcal{U}_{M,g}\mathcal{G}_B + \lambda_M(\varepsilon_B + \varepsilon_g\mathcal{G}_B).$$

Update this expression one period and and replace in (A.35) to obtain:

$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda'_M] + \lambda_M\varepsilon_{x'}\mathcal{X}'_B - \beta(1-\delta_M)\lambda'_M\Delta'_g\mathcal{G}'_B = 0 \quad \text{(A.37)}$$

where we used used $\varepsilon_{B'} = -\beta\varepsilon'_B$ from (14), and (23) plus (A.31) and (A.32) to arrange terms, as in previous proofs. Condition (A.37) corresponds to (34) in the main text.

Note that δ_F does not appear in (A.31), (A.32), (A.34), (A.36) or (A.37). Hence, the patience of the fiscal authority is irrelevant. Also note that if $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$ then (A.36) implies $\lambda_M = \lambda_F$.

To obtain fiscal dominance we need conditions (A.31), (A.32), (A.33), (A.36) and (A.37) to be equivalent to conditions (15)–(18). This requires violating Assumption 2. Hence, fiscal dominance is not possible.

The only condition with derivatives of policy functions is (A.37). In steady state we obtain

$$\varepsilon_{B'}[\lambda_M - (1-\delta_M)\lambda_M] + \lambda_M\varepsilon_{x'}\mathcal{X}_B - \beta(1-\delta_M)\lambda_M\Delta_g\mathcal{G}_B = 0$$

If $\delta_M = 0$ then the expression simplifies to

$$\varepsilon_{x'}\mathcal{X}_B - \beta\Delta_g\mathcal{G}_B = 0$$

If $\mathcal{U}_{M,c} = \mathcal{U}_{F,c}$ (which implies $\lambda_M = \lambda_F$) and $\mathcal{U}_{M,g} = \mathcal{U}_{F,g}$ then $\Delta_g = 0$ and the expression above simplifies to $\varepsilon_{x'} = 0$. In this case, there is no time-consistency problem in steady state even when $\delta_F > 0$ or $\mathcal{U}_{M,x} \neq \mathcal{U}_{F,x}$. ■