Seigniorage and Sovereign Default: The Response of Emerging Markets to COVID-19

Emilio Espino, Julian Kozlowski, Fernando M. Martín and Juan M. Sánchez

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FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
P.O. Box 442
St. Louis, MO 63166

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Seigniorage and Sovereign Default:
The Response of Emerging Markets to COVID-19∗

Emilio Espino
UTDT

Julian Kozlowski
FRB of St. Louis

Fernando M. Martin
FRB of St. Louis

Juan M. Sánchez
FRB of St. Louis

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Abstract

Monetary policy affects the tradeoffs faced by governments in sovereign default models. In the absence of lump-sum taxation, governments rely on both distortionary taxes and seigniorage to finance expenditure. Furthermore, monetary policy adds a time-consistency problem in debt choice, which may mitigate or exacerbate the incentives to accumulate debt. A deterioration of the terms-of-trade leads to an increase in sovereign-default risk and inflation, and a reduction in growth, which are consistent with the empirical evidence for emerging economies. An unanticipated shock resembling the COVID-19 pandemic generates a significant currency depreciation, increased inflation and distress in government finances.

JEL Classification: E52, E62, F34, F41, G15.

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1 Introduction

Emerging markets are prone to experience sovereign debt crises, often leading to default. A now large literature, spanned by the work of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008), explains this fact by contending that emerging countries under-insure against negative shocks by overborrowing during booms. It is also widely understood, though largely ignored by this literature, that domestic policy frameworks affect the resilience of economies to shocks—e.g., see Caballero (2003). In particular, a heavy reliance on seigniorage (i.e., inflation tax) to finance chronic deficits is often an important source of fragility. The goal of this paper is to better understand the link between domestic policies and sovereign debt, with a special emphasis on the role played by monetary policy. To this effect, we extend the standard sovereign default model to include distortionary taxes, fiat money and an equilibrium nominal exchange rate.

We find that the presence of distortionary taxation leads the government to also rely on seigniorage (i.e. inflationary tax) to finance its expenditure. In turn, this implies that there is a time-consistency problem in debt choice, which we explain below. In a quantitative version of the model, calibrated to match long-term averages of emerging Latin-American countries, we find that sovereign debt spreads, inflation and output growth react in an empirically plausible way to shocks to the terms of trade and world interest rates. As an application of our model, we study the policy response in emerging markets to the COVID-19 pandemic. We find that an important component of the response is a substantial monetization of the fiscal deficit, which contributes to a large depreciation of the domestic currency and a rise in inflation. We also find a significant increase of distress in sovereign debt markets, mostly due to the perceived duration of the shock.

We study a tradable-nontradable (TNT) small open economy (as in Uribe and Schmitt-Grohé, 2017, §8), extended to include production, money and sovereign default. Firms produce both non-tradable goods and export goods; agents consume non-tradable goods and imported goods. Consumers need money to finance their purchases of non-tradable goods, which gives rise to a demand for fiat money. The government provides a valued public good and makes transfers to individuals, which it finances with labor taxes, money and external debt. Government debt is issued in foreign currency to foreign risk-neutral investors. In the event of default, the government enjoys a cut on its external liabilities but suffers temporary exclusion from financial

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1We further show that if the government had access to unconstrained lump-sum taxes, then it would run the Friedman rule, which involves managing the money supply so that the opportunity cost of holding money is zero. In a steady state, this policy implies contracting the money supply at the households’ discount rate.
markets and a productivity loss. We further assume that the government’s inability to commit extends to all future policy actions.

The government in our model conducts policy taking into consideration its own budget constraint and the balance of payments. These two restrictions are naturally interconnected when trade with the rest of the world is not balanced. In this framework there is a further connection given that current debt issuance affects future government policies, which in turn, affects the current domestic demand for money. Therefore, even in the absence of a default option, the government faces a time-consistency problem. Note that this problem is not driven by the \textit{ex post} incentives to inflate the debt away, since all debt is issued in foreign currency. Instead, it arises from the interaction between the dynamic problems of agents and government, through the government budget constraint.

We are able to characterize necessary conditions of the optimal government policy using first-order conditions. In particular, the decision of how much debt to issue depends on three factors. The first involves distortion-smoothing: debt allows the government to trade-off intertemporally how severely the balance of payments restricts its policy. If the domestic economy is present biased (i.e., impatient relative to the rest of the world) then this factor provides incentives to accumulate debt. The second factor reflects the negative impact of more debt, which leads to a higher default premium and thus, mitigates the desire to accumulate debt. The third factor is the time-consistency problem, which arises through the government budget constraint: higher debt tomorrow leads to larger future distortions when repaying and a larger default probability, both of which affect the demand for money today. This factor may be positive or negative, depending on agents’ preferences, and thus, may reinforce or mitigate the incentives to issue debt.

The evaluation of the model follows the empirical strategy in Drechsel and Tenreyro (2017). They look at the reaction of spreads to shocks to terms of trade in Argentina using a commodity price index, which is arguably exogenous. We extend their analysis by considering more countries (adding Brazil, Colombia, and Mexico), analyzing the reaction of more variables (adding inflation and growth), and more shocks (adding shocks to the world interest rate). In the model, as in the data, we find large and negative effects on spreads and inflation to shocks to the terms of trade. These shocks have a positive effect on growth, slightly larger in the model than in the data.

Then, we model the COVID-19 shock as a combination of severe, unpredictable shocks—a “perfect storm” as described by Hevia and Neumeyer (2020). In particular, the economy
experiences a drop in productivity, an increase in agents’ cost of going to work, a decline in the price of exported goods (commodities) and an increase in government transfers. Furthermore, there is uncertainty about how quickly (and to what extent) the economy returns back to normal. We calibrate the shocks to productivity and labor disutility to reproduce the drop in real GDP and employment, and feed the observed drop in commodity prices and increase in government transfers. In our quantitative analysis, we find that the government response to the COVID-19 shock involves: increased fiscal deficit, as tax rates decline slightly and government expenditure increases; a substantial monetization of the fiscal deficit, which contributes to a large depreciation of the currency and an increase in inflation; and a significant increase of distress in sovereign debt markets. When decomposing the shock we find that the deterioration of the terms-of-trade and the possibility of the shock lasting more than one year account for a large share of the increase in sovereign spreads, while the productivity and labor shocks explain the contraction in economic activity, but not the debt crisis. We finally argue that more conservative fiscal and monetary policies would help mitigating the rise of inflation, depreciation and debt spreads, but at the cost of a more severe recession.

The literature on sovereign default models has evolved from the framework developed by Eaton and Gersovitz (1981) to quantitative models that account for stylized facts about emerging-market business cycles (Aguiar and Gopinath, 2006; Arellano, 2008). Although new models have added realistic features like long-term debt (Hatchondo and Martinez, 2008; Hatchondo, Martinez, and Sosa-Padilla, 2016) and sovereign debt restructuring (Yue, 2010; Dvorkin, Sánchez, Sapriza, and Yurdagul, forthcoming), there are only a few papers about the role of fiscal policy and almost no work on monetary policy. We aim to bridge that gap by developing a sovereign default model that incorporates exchange rates, inflation, and domestic policy (fiscal and monetary).

In terms of the analysis of fiscal policy, Cuadra, Sánchez, and Sapriza (2010) show how a desirable counter-cyclical fiscal policy is reversed by including debt with a risk of default. Their analysis is related because the government cannot commit to future policies—it chooses taxes every period to maximize lifetime utility. The critical difference is that we add money, which significantly extends the scope of the analysis. This element also complicates the environment by adding an intertemporal optimization problem for households. Bianchi, Ottonello, and Presno (2019) also analyze the pro-cyclicality of fiscal policy in emerging markets. They argue, similar to Cuadra, Sánchez, and Sapriza (2010), that pro-cyclical fiscal policy is a feature of countries with a high risk of default. Their crucial contribution is to show that this is true even in a model with nominal rigidities and significant Keynesian stabilization gains. More recently,
Anzoategui (2019) studies the effect of alternative fiscal rules. The main finding is that more pro-cyclical fiscal policies (austerity during bad times) did not help to reduce sovereign spreads or debt-to-GDP ratios, but harmed economic activity. This result is in line with our findings in the last section of the paper, when we analyze more conservative fiscal policies.\footnote{Fiscal rules are also analyzed by Hatchondo, Roch, and Martinez (2012), although they focus on the best rule to restrict policy to avoid overborrowing.}

Concerning monetary policy, there is recent work studying the currency composition of debt and inflation (Sunder-Plassmann, 2018; Ottonello and Perez, 2019). These papers show that debt denominated in local currency raises incentives to dilute debt repayment through inflation. Closer to our work, Arellano et al. (2019) analyzes the interaction of sovereign default risk with a monetary policy rule in a cashless economy. They argue that the model rationalizes the positive co-movements of sovereign spreads with domestic nominal rates and inflation in Brazil. Their work complements ours since they study the case in which central bankers in emerging markets can commit to a Taylor rule. In contrast, we assume that both fiscal and monetary authorities cannot commit to future policies. In the last section of the paper, we consider how our results change when the government can commit to a constant monetary or fiscal rule. Also, because our model has money, we study how seigniorage is used to finance the government deficit.

Another essential feature of our model is that the role of exchange rates. In that sense, it connects with the work Na, Schmitt-Groh, Uribe, and Yue (2018), which points to the link between devaluations and default. In a model with downward nominal wage rigidity, they show that an optimal exchange rate devaluation occurs in periods of default, lowering the real value of wages to reduce unemployment. Proposition 2 in Na, Schmitt-Groh, Uribe, and Yue (2018) and in our work are also related. Both show how to recover a “real” economy as in Eaton and Gersovitz (1981). In their case, the key in an optimal devaluation to undo wage rigidity, while in our model, it is unconstrained lump-sum taxation.

The most recent paper related to our work is Arellano, Bai, and Mihalache (2020), which also study the effects of the COVID-19 shock on emerging economies. They argue that default risk may limit the response to the epidemic and, as a consequence, find substantial gains from debt relief. In contrast, we take as given the main element of the fiscal relief, namely transfers to households, and analyze how emerging market cope with it by adjusting external debt and domestic fiscal and monetary policies.

Finally, our paper is also related to work in closed economies, e.g., Díaz-Giménez, Giovannetti, Marimón, and Teles (2008) and Martin (2009, 2011), among others. These papers study government policy without commitment in monetary economies. Unlike our work, they do not
consider the role of sovereign default risk. However, these papers share important similarities in the nature of the time-consistency problem.

2 Model

2.1 Environment

We study a small open economy populated by a large number of identical infinitely-lived agents, with measure 1. Time is discrete. Throughout the paper, we make use of recursive notation, denoting next-period variables with a prime.

Preferences, endowments and technology

There are three private goods and one public good in the economy. First, there is a non-tradable good that is consumed and produced domestically, their quantities being denoted $c^N$ and $y^N$, respectively. Second, there is tradable imported good, that is consumed domestically but not produced. Let $c^T$ denote the consumption of this imported good. Third, there is a tradable export good, that is not consumed domestically, and is only produced to be exported. Let $y^T$ denote the production of this export good. Finally, the government can transform non-tradable output $y^N$ one-to-one into a public good, $g$.

The representative household is endowed with one unit of time each period, which can be either consumed as leisure, $\ell$ or supplied in the labor market, $h$. Thus, $\ell + h = 1$.

Preferences are represented by a time-separable, expected discounted utility. Let period utility be given by

$$u(c^N, c^T) + v(\ell) + \vartheta(g),$$

where $u$, $v$ and $\vartheta$ are strictly increasing, strictly concave, $C^2$ and satisfy standard boundary conditions. Let $\beta \in (0, 1)$ denote the discount factor.

There is an aggregate production technology that transforms hours worked, $h$, into non-tradable output, $y^N$ and exportable goods, $y^T$. This technology is represented by a cost function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$, which is strictly increasing, strictly convex and homogeneous of degree one. Given $h$, feasible levels of $(y^N, y^T)$ must satisfy

$$F(y^N, y^T) - h \leq 0. \quad (1)$$

Market structure
Agents can exchange both tradable and non-tradable goods, as well as domestic currency (fiat money), while trading of other financial assets will be restricted to the government. Let $M^d$ denote individual money holdings. Prices are denominated in domestic currency (i.e., pesos) and given by $P^X$, $P^M$ and $P^N$ for exports, imports and non-tradable goods, respectively. Let $W$ denote the nominal wage in units of domestic currency.

The nominal exchange rate $E$ is defined as the units of domestic currency necessary to purchase one unit of foreign currency (i.e., pesos per dollar). We assume that the law of one price holds for tradable goods and so $P^X = E p^T$ and $P^M = E$, where $p^T$ is the international price of export goods and the international price of importable goods has been normalized to 1. Thus, $p^T$ also stands for the terms of trade.

In order to study a stationary environment, we normalize nominal variables by the stock of the money supply, $M$. Let $\mu$ denote the growth rate of the money supply and so, $M' = (1+\mu)M$ denotes its law of motion. We define the corresponding normalized variables as $p^N = P^N/M$, $w = W/M$, $e = E/M$ and $m = M^d/M$.

To motivate a role for fiat money, we assume that households face a cash-in-advance constraint when purchasing non-tradable goods,

$$p^N c^N \leq m. \quad (2)$$

That is, (normalized) expenditure on non-tradable goods, $p^N c^N$ cannot exceed (normalized) money balances available at the beginning of the period, $m$.

**Government and the Balance of Payments**

The government provides a public good, $g$, which is transformed one-to-one from non-tradable output. It may also make lump-sum transfers to households. Let $\gamma$ be the real value (in units of non-tradable output) of government transfers. We assume that transfers are exogenous, non-negative, and represent non-discretionary redistributive policy. To finance its expenditure, the government may tax labor income $wh$ at rate $\tau$, increase the money supply at rate $\mu$, and issue debt in international credit markets. Debt takes the form of one-period discount bonds that pay one unit of foreign currency and trade for $q$, also denominated in foreign currency. Let $B$ denote the value of maturing debt and $qB'$ the funds collected from issuing new debt $B'$, both expressed in foreign currency units.

We consolidate the fiscal and monetary authority and write the government budget con-
straint in (normalized) units of domestic currency as follows\(^3\)

\[ p^N(g + \gamma) + eB \leq \tau wh + \mu + eqB'. \]  

(3)

The balance of payments, expressed in units of foreign currency, implies

\[ p^T y^T - c^T = B - qB', \]  

(4)

where the left-hand side of the expression above is the trade balance, while the right-hand side is the change in the country’s net asset position plus implicit debt interest payments.

Combining (3) and (4) we can express the government budget constraint as relationship between the external sector (the trade balance) and the public sector (the primary surplus plus seigniorage):

\[ \tau wh - p^N(g + \gamma) + \mu - e(p^T y^T - c^T) \geq 0. \]  

(5)

2.2 The problem of the representative firm

Local firms produce non-tradable and tradable goods by hiring labor according to the technology represented by \( F \). Constant returns to scale imply that we can assume that the industry behaves as if there were a representative firm that solves the static problem

\[
\max_{y^N, y^T, h} \quad p^N y^N + ep^T y^T - wh
\]

subject to (1). The necessary and sufficient first-order conditions imply expressions for the wage and exchange rate as functions of \((y^N, y^T, p^N, p^T)\) as follows

\[
w = \frac{p^N}{F_N},
\]

(6)

\[
e = \frac{p^N F_T}{p^T F_N},
\]

(7)

where \( F_j \) is the partial derivative of \( F \) with respect to \( y^j \), \( j = \{N, T\} \).

\(^3\)As we argue in Section 3.6, when lump-sum taxes are available, as in the standard Eaton-Gersovitz model, the government sets distortionary taxes equal to zero and follows the Friedman rule. In this case, the model becomes Ricardian: the government budget constraint solves for lump-sum taxes and places no further restrictions on government policy, i.e., is not a constraint in the government’s problem.
2.3 The problem of the representative household

The endogenous state of the economy consists of the amount of maturing foreign debt, $B$, and an indicator function $I$, which specifies whether the government is in default ($I = D$) or not ($I = P$). As we shall explain below, the default state may last several periods, while the country is excluded from international credit markets. Agents know the government’s default state before making any decisions at the beginning of every period. The exogenous state of the economy is summarized by $s$, known at the beginning of each period. The state $s$ may include any variable that evolves stochastically over time, e.g., the terms of trade, $p_T$. The set of all possible realizations for the stochastic state is $S$. Note that we are allowing for the possibility of exogenous state variables to depend on the default state.

Agents know the laws of motion of all aggregate state variables. All prices and government policies are perceived by agents as being functions of the aggregate state. This dependence is omitted to simplify notation. The period budget constraint of the household is

$$p^N c^N + ec^T + m'(1 + \mu) \leq (1 - \tau)wh + m + p^N \gamma,$$

where, recall, $p^N$, $w$, $e$, $m$ are all normalized by the aggregate money supply at the beginning of the period. In addition, as mentioned above, trading in the non-tradable good market is subject to the cash-in-advance constraint, (2).

The individual state variable is the household’s (normalized) money balances at the beginning of the period, $m$. Let $V(m, B, I, s)$ denote the agent’s value function as a function of individual and aggregate state variables. Let $\mathbb{E}[V(m', B', I', s')|B, I, s]$ be the expected value of the agent’s value function in the next period, given current aggregate state $(B, I, s)$.

The problem of the representative household is

$$V(m, B, I, s) = \max_{(c^N, c^T, m')} u(c^N, c^T) + v(1 - h) + \beta \mathbb{E} [V(m', B', I', s')|B, I, s]$$

subject to (2) and (8). As derived in Appendix A, the solution to this problem is characterized by

$$\left(\frac{1 - \tau}{e} \right) \frac{wu_T}{u} = v_t,$$

$$\left(\frac{1 + \mu}{e} \right) \frac{wu_T}{u} = \beta \mathbb{E} \left[ \frac{u'_N}{p^N} \bigg| B, I, s \right],$$

plus constraints (2) and (8). Here, $u_j$ denotes the partial derivative of $u$ with respect to the
consumption good $c^j$, with $j = \{N, T\}$ and $v$ is the derivative of $v$ with respect to $\ell = 1 - h$.

Conditions (9) and (10) show how government policy distort households’ choices. The tax rate introduces a wedge between the marginal utilities of consumption of tradable goods and leisure, while the money growth rate distorts the substitution between current consumption of tradable goods and future consumption of non-tradable goods.

2.4 Monetary equilibrium

Since all agents are identical, $c^N$, $c^T$ and $h$ should be interpreted as referring to aggregate quantities from now on.

The resource constraint in the non-tradable sector is

$$c^N + g = y^N.$$  \hfill (11)

From (1) and (11), labor is a function of non-tradable (private) consumption, public expenditures and the production of tradables, i.e.,

$$h = F(c^N + g, y^T).$$  \hfill (12)

All agents enter the period with the same money balances, $m$. Market clearing implies that $m = m' = 1$. The Lagrange multiplier associated with the cash-in-advance constraint must be non-negative and this implies the following equilibrium condition

$$u_N - \frac{u_T F_N p_T}{F_T} \geq 0.$$  \hfill (13)

If (13) is positive then the cash-in-advance constraint (2) binds; if is equal to zero then the cash-in-advance constraint is slack. Condition (13) reflects an inefficiency wedge, as the marginal rate of substitution between tradable and non-tradable goods is lower than their relative price (i.e., agents would like to consume relatively more non-tradable goods).

Without loss of generality, the cash-in-advance constraint (2) is satisfied with equality.\footnote{If the cash-in-advance constraint is slack, then the price level $p^N$ is, in general, indeterminate. A standard assumption is to take the limiting case, when the constraint is satisfied with equality.}

Then,

$$p^N = \frac{1}{c^N}.$$  \hfill (14)
The equilibrium wage can be derived by combining (6) and (14):

\[ w = \frac{1}{c^N F_N}. \] (15)

Similarly, the equilibrium exchange rate follows from (7) and (14)

\[ e = \frac{1}{c^N p T} \frac{F_T}{F_N}. \] (16)

3 Government policy

3.1 Government budget constraint in a monetary equilibrium

Below, we formulate the problem of the government following the *primal approach*. That is, we solve for allocation and debt choices that are implementable in a monetary equilibrium. In order to proceed, we need to use the equilibrium conditions derived above, to replace prices \((p^N, w, e)\) and policies \((\mu, \tau)\) in the government budget constraint (5).

To obtain an expression for the tax rate, combine (9), (15) and (16)

\[ \tau = 1 - \frac{v_T F_T}{u_T p^T}. \] (17)

Similarly, the money growth rate can be written by combining (10), (14) and (16)

\[ \mu = \frac{\beta E \left[ u'_N c^N | B, I, s \right]}{u_T c^N p^T (F_N / F_T)} - 1. \] (18)

Using (14)–(18) we obtain the government budget constraint in a monetary equilibrium

\[ u_T [c^T - \gamma p^T (F_N / F_T)] - v_T F(c^N + g, y^T) + \beta E \left[ u'_N c^N | B, I, s \right] \geq 0, \] (19)

which depends on \((c^N, c^T, y^T, g, c^{Nt}, c^{Tt})\).

3.2 Repayment and default

Suppose the government is currently not excluded from international credit markets. At the beginning of any such period, the government decides between repaying (P) and defaulting (D)

\[ u_T c^T + u_N c^N \beta E \left[ \frac{u'_N c^{Nt}}{u^{Nt}} | s \right] = v_T h + u_N \gamma, \] i.e., the (adjusted) value of consumption must equal the value of its cost, in utility units.

\[ u_T c^T + u_N c^N \beta E \left[ \frac{u'_N c^{Nt}}{u^{Nt}} | s \right] = v_T h + u_N \gamma, \] i.e., the (adjusted) value of consumption must equal the value of its cost, in utility units.
on its debt. If it decides to default, then debt is set to zero. Define

$$V(\hat{B}, s, \varepsilon^P, \varepsilon^D) = \max\{V^P(B, s) + \varepsilon^P, V^D(s) + \varepsilon^D\},$$

where $V^P(B, s)$ and $V^D(s)$ denote the value of repayment and default, respectively, which are defined in detail below. Notice that this decision is also influenced by a random additive shock to utility. Next, we explain how these shocks are used to obtain some useful expressions under particular assumptions on their distribution.

Assume that each $\varepsilon^j$ is independently, identically distributed extreme value (Gumbel or type I extreme value). The difference between these two shocks will affect the default decision. This difference has mean zero and is distributed logistic; i.e., $\varepsilon = \varepsilon^P - \varepsilon^D$ follows

$$F(\varepsilon) = \frac{\exp[\varepsilon/\kappa]}{1 + \exp[\varepsilon/\kappa]},$$

where $\kappa > 0$ is the scale parameter of the distribution, which will be useful to control the variance of the $\varepsilon$ shocks.

Let $P(B, s)$ be the probability of repayment, for any given $(B, s)$, which can be expressed as

$$P(B, s) = \Pr(V^P(B, s) - V^D(s) \geq -\varepsilon).$$

This probability has a simple expression given the assumptions on $\varepsilon^i$. Following McFadden (1974), this integral results in a closed-form expression

$$P(B, s) = \frac{\exp[V^P(B, s)/\kappa]}{\exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa]},$$

which, in turn, implies

$$\frac{\partial P(B, s)}{\partial B} = \frac{\partial V^P(B, s)}{\partial B} \frac{P(B, s)(1 - P(B, s))}{\kappa}. \quad (21)$$

Next, we can derive a closed-form expression for the expectation of the value function with respect to the utility shocks

$$\mathbb{E}_\varepsilon[V(B, s)] = \mathbb{E}_\varepsilon[V(B, s, \varepsilon^P, \varepsilon^D)] = \kappa \ln \left\{ \exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa] \right\}.$$
Using this expression, we can easily see that
\[
\frac{\partial V(B, s)}{\partial B} = P(B, s) \frac{\partial V^P(B, s)}{\partial B}.
\] (22)

Expressions (21) and (22) will be useful to characterize the choice of debt, as long as \( V^P \) is differentiable.

### 3.2.1 The price of debt

In equilibrium, zero-expected profits by risk-neutral international lenders implies that
\[
Q(B', s) = \frac{E[P(B', s')|s]}{1 + r}.
\] (23)

Taking the derivative with respect to \( B' \) and combining with (21), implies
\[
\frac{\partial Q(B', s)}{\partial B'} = E\left[ \frac{\partial V^P(B', s')}{\partial B'} \frac{P(B', s')(1 - P(B', s'))}{\kappa} \right] \frac{1}{1 + r}.
\] (24)

### 3.3 Problem of the government

Every period, the government first decides on whether to repay or default on its debt. After that, it implements policy for the period, taking into account the response of private domestic agents and, when appropriate, international lenders and government policies it expects to implement in the future. A period policy consists of choices on the amount of future debt, the money growth rate, the tax rate, and government expenditure. If the government decided to default, then its debt is set to zero, and the country is excluded from international credit markets. When in default, the government regains access to international credit markets at the beginning of the period with probability \( \delta \). Hence, \( 1/\delta \) is the expected duration of exclusion from international credit markets.

If the government is currently repaying, the probability that it will remain in repayment status tomorrow is given by \( P(B', s') \), for any given \((B', s')\), as derived above. On the other hand, if the government is currently in default, the probability that it will transition to repayment status tomorrow is given by \( \delta P(0, s') \), for any given \( s' \). Recall that to compute these probabilities we need to know the value functions \( V^P(B, s) \) and \( V^D(s) \), which we will derive below.

As explained above, we follow the primal approach to formulate the government’s problem. Hence, we used equilibrium conditions to express domestic prices, the money growth rate, and
the tax rate as functions of current and future allocations. Every period, the government then chooses a debt level (when repaying) and domestic policies that implement the allocation \((c^N, c^T, y^T, g)\). These choices need to satisfy the balance of payment, (4), the government budget constraint, (19), and the non-negativity constraint, (13).

When the government is in repayment status, \(I = P\), its policies are a function of the state \((B, s)\); let the relevant policy functions be denoted by \(\{B, C^N, C^T, Y^T, G\}\). When the government is in default, \(I = D\), its policies are a function of the state \(s\); let the relevant policy functions be denoted by \(\{\bar{C}^N, \bar{C}^T, \bar{Y}^T, \bar{G}\}\). Given these policy functions, we can define the value functions \(V^P(B, s)\) and \(V^D(s)\) as follows,

\[
V^P(B, s) = u(C^N(B, s), C^T(B, s)) + v(1 - F(C^N(B, s) + G(B, s), Y^T(B, s))) + \vartheta(G(B, s)) + \beta E[V(B'(s), s')|s],
\]

\[
V^D(s) = u(\bar{C}^N(s), \bar{C}^T(s)) + v(1 - F(\bar{C}^N(s) + \bar{G}(s), \bar{Y}^T(s))) + \vartheta(\bar{G}(s)) + \beta E[\delta V(0, s') + (1 - \delta)V^D(s')|s],
\]

\[
V(B, s) = \kappa \ln \{\exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa]\},
\]

for all \((B, s)\). Using (20) and (23), these functions imply expressions for \(P(B, s)\) and \(Q(B', s)\).

### 3.3.1 Repayment

The problem of the government in the repayment state is

\[
\max_{(B', c^N, c^T, y^T, g)} u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta E[V(B', s')|s] \quad \text{(PP)}
\]

subject to

\[
p^T y^T - c^T + Q(B', s)B' - B = 0, \quad (25)
\]

\[
u_T c^T - \gamma u_T p^T (F_N/F_T) - v_T F(c^N + g, y^T) + \beta \mathbb{E} \left[ u_N c^N | P, s \right] = 0, \quad (26)
\]

\[
u_N - u_T p^T (F_N/F_T) \geq 0. \quad (27)
\]

The constraints in the government’s problem correspond to the balance of payment, (4), the government budget constraint, (19), and the non-negativity constraint, (13). Note that the expectation term in the government budget constraint is conditioned on the current state being repay \((I = P)\); hence, the relevant transition probabilities are \(P(B', s')\) for repay and \(1 - P(B', s')\) for default, for all \((B', s')\).
3.3.2 Default

The problem of the government in the default state is

$$\max_{(c^N, c^T, y^T, g)} u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta \mathbb{E}[\delta V(0, s') + (1 - \delta) V^D(s') | s]$$  \hspace{1cm} (DP)

subject to

$$p^T y^T - c^T = 0, \hspace{1cm} (28)$$
$$u_T c^T - \gamma u_T p^T \left( \frac{F_N}{F_T} \right) - v_l F(c^N + g, y^T) + \beta \mathbb{E}\left[ u'_N c^N | D, s \right] = 0, \hspace{1cm} (29)$$
$$u_N - u_T p^T \left( \frac{F_N}{F_T} \right) \geq 0. \hspace{1cm} (30)$$

In this case, note that the balance of payments is simply the trade balance, as the government is excluded from international credit markets. The expectation term in the government budget constraint is conditioned on the current state being default ($I = D$); hence, the relevant transition probabilities are $\delta P(0, s')$ for repay and $1 - \delta P(0, s')$ for default, for all $s'$.

3.4 Domestic policy

We begin by characterizing domestic policy, which involves choices for the allocations $(c^N, c^T, y^T, g)$, given state $(B, s)$, repayment status $I = \{P, D\}$ and a debt policy $B(B)$. In what follows, we assume that $u(c^N, c^T)$ is separable in its arguments.

**Assumption 1.** $u_{NT} = 0$.

To simplify exposition, let $\Gamma(c^N, c^T, y^T, g; s) \equiv u_T p^T \left( \frac{F_N}{F_T} \right)$, which is an expression that shows up in the government budget and non-negativity constraints. Note that $\Gamma_T = d\Gamma / dc^T = \Gamma \times (u_{TT} / u_T) < 0$, while the convexity of $F$ implies that $\Gamma_N = \Gamma_g = \Gamma \times \left( F_{NN} / F_N - F_{NT} / F_T \right) > 0$ and $\Gamma_y = \Gamma \times \left( F_{NT} / F_N - F_{TT} / F_T \right) < 0$. Also define $\Phi \equiv -v_l + v_{lT} F(c^N + g, y^T) < 0$.

We first consider the government’s problem when repaying, (PP)–(27). Let $\xi$, $\lambda$ and $\zeta$ be the Lagrange multipliers associated with the constraints (25), (26) and (27), respectively. The
necessary first-order conditions with respect to \((c^N, c^T, y^T, g)\) are:

\[
\begin{align*}
        u_N - v_{\ell}F_N + \lambda(F_N^T \Phi - \gamma \Gamma_N) + \zeta(u_{NN} - \Gamma_N) &= 0, \\
u_T - \xi + \lambda(u_T + u_{TT}c^T - \gamma \Gamma_T) - \zeta \Gamma_T &= 0, \\
        -v_{\ell}F_T + \xi p^T + \lambda(F_T^T \Phi - \gamma \Gamma_y) - \zeta \Gamma_y &= 0, \\
        -v_{\ell}F_N + \vartheta_g + \lambda(F_N^T \Phi - \gamma \Gamma_g) - \zeta \Gamma_g &= 0.
\end{align*}
\] (31) (32) (33) (34)

Suppose that the non-negativity constraint (27) does not bind, i.e., \(\zeta = 0\). Given \(\Gamma_N = \Gamma_g\), (31) and (34) imply

\[
u_N = \vartheta_g.
\] (35)

Using (31) and (33) to solve for the Lagrange multipliers we obtain:

\[
\begin{align*}
\lambda &= \frac{v_{\ell}F_N - u_N}{F_N^T \Phi - \gamma \Gamma_N}, \\
\xi &= \frac{u_N F_T^T \Phi - v_{\ell}F_T^T \Gamma_N + (v_{\ell}F_N - u_N)\gamma \Gamma_y}{p^T(F_N^T \Phi - \gamma \Gamma_N)},
\end{align*}
\]

and so, (31) implies

\[
F_T^T(u_N - \Gamma) + \gamma \left[ (u_T p^T - v_{\ell}F_T)^T \Gamma_N + (v_{\ell}F_N - u_N)\gamma \Gamma_y \right] = p^T(v_{\ell}F_N - u_N)(u_T + u_{TT}c^T - \gamma \Gamma_T),
\] (36)

where we used the definition of \(\Gamma\) to simplify the expression. We now verify under which conditions \(\zeta = 0\) holds.

**Proposition 1.** Suppose that \(\gamma = 0\) (zero transfers). Then, \(\zeta = 0\) and so (27) does not bind if and only if

\[u_T + u_{TT}c^T \geq 0.\]

**Proof.** See Appendix B. \(\quad \Box\)

When transfers are zero, whether (27) binds or not depends on preferences. If \(u_T + u_{TT}c^T > 0\), then (27) is satisfied with strict inequality; when \(u_T + u_{TT}c^T = 0\), then (27) is satisfied with strict equality but still slack. In both of these cases, \(\zeta = 0\). In contrast, when \(u_T + u_{TT}c^T < 0\), then (27) binds and so, \(\zeta > 0\).

When transfers are positive, the analysis is more involved. It is easy to show that when \(F\) is close to linear, so that \(\Gamma_N \approx \Gamma_g \approx 0\), the left-hand side of (36) is non-positive, while the right-hand side is also non-positive when \(u_T + u_{TT}c^T \geq 0\), but also possibly when \(u_T + u_{TT}c^T < 0\).
Our numerical exercises show that \( \zeta = 0 \) when \( u_T + u_T^T c_T^T \geq 0 \), or \( u_T + u_T^T c_T^T < 0 \) and \( \gamma \) is sufficiently large. The intuition for this result is that transfers increase the distortions that need to be financed and hence, provide more incentives for the government to also rely on seigniorage.

To sum up, when preferences are such that (27) does not bind and so \( \zeta = 0 \), the solution to \((c^N, c^T, y^T, g)\) when repaying is given by (25), (26), (35) and (36). The first-order conditions to the government’s problem in default, (DP)–(30), are functionally identical to the ones derives under repayment. Hence, the solution for allocations when in default is characterized by (28), (29), (35) and (36).

### 3.5 Debt policy

We now characterize debt choice in the event the government decides to repay its inherited debt. The necessary first-order condition of problem (PP) with respect to \( B' \) is

\[
\beta \frac{\partial \mathbb{E}[V(B', s')|s]}{\partial B'} + \xi \left[ Q(B', s) + \frac{\partial Q(B', s)}{\partial B'} B' \right] + \lambda \beta \frac{\partial \mathbb{E}[u'_N c^N|P, s]}{\partial B'} = 0. \tag{37}
\]

As reflected by the three terms in (37), debt choice affects the continuation value for the government and how tightly the balance of payment and government budget constraints bind.

We can further characterize this equation, using some of the expressions derived above. The envelope condition of problem (PP) implies \( \frac{\partial \mathbb{E}[V(B', s')]}{\partial B'} = -\xi \). Hence, using (21)–(24), we obtain

\[
\frac{\partial \mathbb{E}[V(B', s')|s]}{\partial B'} = -\mathbb{E}[\mathcal{P}(B', s')\xi'|s] \\
Q(B', s) = \frac{\mathbb{E}[\mathcal{P}(B', s')|s]}{1+r} \\
\frac{\partial Q(B', s)}{\partial B'} = -\frac{\mathbb{E}[\mathcal{P}(B', s')(1 - \mathcal{P}(B', s'))\xi'|s]}{\kappa(1+r)}.
\]

The last term on (37) requires a bit more work. Given that \( \mathcal{P}(B', s') \) is the probability of transitioning from \( \mathcal{I} = P \) to \( \mathcal{I}' = P \), for all \((B', s')\), we can write

\[
\mathbb{E}[u'_N c^N|P, s] = \mathbb{E}[\mathcal{P}(B', s')u'_N c^N + (1 - \mathcal{P}(B', s'))\bar{u}'_N \bar{c}^N|s],
\]

where \( u'_N c^N \) corresponds to the repay state tomorrow, \( \mathcal{I}' = P \), and \( \bar{u}'_N \bar{c}^N \) corresponds to the default state tomorrow, \( \mathcal{I}' = D \). Note that the expectation on the right-hand side (only conditional on \( s \)) is taken with respect to \( s' \). We can take the derivative of the expression above
with respect to $B'$ to obtain
\[
\frac{\partial E\left[u'_{N'C^N|P,s}\right]}{\partial B'} = E\left[\mathcal{P}(B',s')(u'_N + u'_{N'C^N})C^N_B + (u'_NC^N - \bar{u}'_NC^N)\mathcal{P}'_B|s\right],
\]
where $C^N_B$ and $\mathcal{P}'_B$ denote the derivatives of $C^N(B',s')$ and $\mathcal{P}(B',s')$ with respect to $B'$. Recall that, when in default, allocations are not a function of $B$, i.e., $\bar{C}^N(s)$ and so, $\bar{C}^N_B = 0$. Recall that from (21), we have an analytical expression for $\mathcal{P}'_B$. In addition, using the envelope condition, \(\frac{\partial v^*(B,s)}{\partial B} = -\xi\), we obtain
\[
\frac{\partial E\left[u'_{N'C^N|P,s}\right]}{\partial B'} = E\left\{\mathcal{P}(B',s') \left[(u'_N + u'_{N'C^N})C^N_B - \frac{(u'_NC^N - \bar{u}'_NC^N)(1 - \mathcal{P}(B',s'))\xi'}{\kappa}\right] | s \right\}.
\]
We can now write the equation characterizing debt choice as,
\[
E\left\{\mathcal{P}(B',s') \left[\left(\frac{\xi}{1+r} - \beta\xi'\right) - \frac{\xi B'(1 - \mathcal{P}(B',s'))\xi'}{\kappa(1+r)}\right] | s \right\} + \lambda\beta\mathbb{E}\left\{\mathcal{P}(B',s') \left[(u'_N + u'_{N'C^N})C^N_B - \frac{(u'_NC^N - \bar{u}'_NC^N)(1 - \mathcal{P}(B',s'))\xi'}{\kappa}\right] | s \right\} = 0. \quad (38)
\]
Note that if there are no aggregate shocks (other than the extreme value shocks, \(\varepsilon\)) then $\mathcal{P}(B',s')$ only depends on $B'$ and is always positive; hence, the term $\mathcal{P}(B',s')$ multiplying all the expression in square brackets in (38) can be eliminated.

The Generalized Euler Equation (38) highlights three components in the government’s debt decision. The first component, $E\{\mathcal{P}(B',s')[(1 + r)^{-1}\xi - \beta\xi']|s\}$, corresponds to distortion-smoothing: debt allows the government to trade-off intertemporally how tightly the balance of payments binds. The distortion-smoothing term has an intrinsic present bias since the government is relatively impatient as $\beta(1+r) < 1$. In other words, this term would not be zero if the government kept expected distortions constant over time, i.e., set $\xi = E[\xi'|s]$. This present bias is the channel that motivates debt accumulation in the sovereign default literature.

The second component, $-\kappa(1+r)^{-1}\xi B'E\{\mathcal{P}(B',s')|(1 - \mathcal{P}(B',s'))\xi'|s\}$, captures the default premium: more debt leads to a higher probability of default and hence, a higher interest rate. The default premium term reflects the added financial cost due to default risk. In the sovereign default literature, this term moderates debt accumulation.

The third component, $\lambda\beta\mathbb{E}\{\mathcal{P}(B',s')[(u'_N + u'_{N'C^N})C^N_B - (u'_NC^N - \bar{u}'_NC^N)(1 - \mathcal{P}(B',s'))(\xi'/\kappa)]|s\}$, reflects a time-consistency problem in debt choice, which operates through the government budget constraint: higher debt leads to a change in policies tomorrow, which affects the demand for money today and hence, how tightly the current government budget constraint binds. This
channel is absent in standard sovereign default models. Below, we analyze this term in more detail.

### 3.5.1 The time-consistency problem

There are two parts in the time-consistency term in (38) and we will analyze each in turn.

We follow Martin (2011) to interpret the expression $P(B', s')(u'_N + u'_NC^N)C^N_{B'}$. The envelope condition from the household’s problem implies $V_m = u_N/p^N$ (see derivation in Appendix A). When using equilibrium condition (14) we obtain, $V_m = u_N C^N$, which states the equilibrium value of arriving to the period with an additional unit of domestic currency. We can further establish how this value changes with debt: $dV_m/dB = (u_N + u_N C^N)C^N_{B'}$. Hence, first part of the time-consistency term in (38) reflects how a change in debt directly affects the demand for money and therefore, how tightly the government budget constraint binds. Note that this first part of the time-consistency channel is multiplied by $P(B', s')$, implying that it only operates within repayment states today and tomorrow (recall that $\bar{C}_B^N = 0$).

The second part of the time-consistency term, is the expression $(u'_N C^N - \bar{u}'_N C^{N'})P'_{B'}$, where $P'_{B'} = -P(B', s')(1 - P(B', s'))(\xi'/\kappa) < 0$. As explained above, $V_m = u_N C^N$; hence, this term reflects the impact of debt choice on the current money demand, through the change in the repayment probability. As the government issues more debt, it lowers the probability of repayment, $P'_{B'} < 0$. This matters to domestic households since the value of an extra unit of money depends on whether a government repays or defaults, as policies (and distortions) are different in each case. Again, the sign of $u'_N C^N - \bar{u}'_N C^{N'}$ depends on the curvature of $u$. Assuming that $C^{N'} > \bar{C}^{N'}$ (an assumption we verify numerically), this difference is positive (negative) if the substitution (income) effect dominates.

So, how does the time-consistency channel operate? First, issuing more debt today alters
future fiscal and monetary policies in the repayment state, as well as the probability of repayment. Second, since domestic policy instruments are distortionary, this anticipated change in future policy when repaying, alters the marginal value of money tomorrow and hence, households’ current money holdings decisions. Third, the change in future repayment probability also alters the expected marginal value of money tomorrow, as policies differ if the government repays or defaults. Fourth, these changes in the current demand for money affect the real value of domestic government liabilities and hence, its budget constraint. Importantly, this effect is not internalized by the government tomorrow, which results in a time-consistency problem as the governments today and tomorrow value current debt issuance differently.

The time-consistency channel alters how the other two components in (38) are traded off when the government decides how much debt to issue. The time-consistency problem may be positive, zero, or negative, depending on the assumptions on preferences, thus altering the standard trade-off in sovereign debt choice. For example, if the utility is logarithmic and separable, then \( u_N + u_{NN}C_N = u_N'C_{Nt} - \bar{u}_N'C_{Nt} = 0 \) and so, there is no time-consistency problem due to interplay of debt policy and the money demand. In this case, the government would trade-off its present bias with the default risk premium, i.e., the desire to accumulate debt is moderated by the extra financial cost of supporting it due to the higher default probability.

Suppose instead that \( u_N + u_{NN}C_N < 0 \), which also implies \( u_N'C_{Nt} - \bar{u}_N'C_{Nt} < 0 \), as argued above. Given that \( C_B^N < 0 \) (since higher debt implies larger distortions and hence, lower consumption) and \( P_B^N < 0 \) (as shown above), the time-consistency term would now be positive, countering the default premium term and reinforcing the present bias term. When the income effect dominates the substitution effect in the preference for the non-tradable good (and the demand for money), the time-consistency channel provides an added incentive to accumulate debt. In the opposite case, \( u_N + u_{NN}C_N > 0 \), which implies \( u_N'C_{Nt} - \bar{u}_N'C_{Nt} > 0 \), the time-consistency channel mitigates the incentive to accumulate debt.

3.6 Comparison to real models of sovereign default

Here we show that our setting encompasses the celebrated Eaton and Gersovitz (1981) economy. Let \((\hat{B}', \hat{c}_N', \hat{c}_T', \hat{g}T', \hat{g})\) denote the solution to problem (PP) ignoring constraints (26) and (27), and associated with the value of default, \( \hat{V}^D(s) \) that solves (DP) without constraints (29) and (30). This will be referred to as the *EG real allocation*.

**Proposition 2.** If lump-sum, unconstrained taxes are available, real models of sovereign default in the tradition of Eaton and Gersovitz (1981) can be interpreted as the centralized version of
the decentralized economy under certain policies. That is, the solution to the problem (PP) and the EG real allocation coincide under an optimal monetary and fiscal policy.

Proof. See Appendix B. \(\square\)

When lump-sum taxes are available, the government finds it optimal to (i) set the distortionary tax rate \(\tau\) equal to zero, and (ii) conduct monetary policy \(\mu\) so that the cash-in-advance constraint in the household problem does not bind. Lump-sum taxes adjust so that, under these policies, the government budget constraint is satisfied with no need for distortions. Hence, in this version of the model, the government budget constraint is no longer a restriction in the government’s problem. In effect, the policy regime becomes Ricardian.

We can write the analog of the Generalized Euler Equation (38) for the case with unconstrained lump-sum taxes as follows:

\[
E \left\{ P'(B', s') \left[ \frac{u_T}{1+r} - \beta u'_T - \frac{u_T B'(1 - P(B', s')) u'_T}{\kappa (1+r)} \right] \right\} = 0. \tag{39}
\]

In this case, the multiplier of the balance of payment constraint (\(\xi\)) is equal to \(u_T\).\(^6\) We can see that, with lump-sum taxes, the government trades off distortion-smoothing (with present bias) and the default premium. The time-consistency term is absent since the government budget constraint is automatically satisfied with lump-sum taxes and so it is no longer a restriction to policy implementation.

**Proposition 3.** If lump-sum, unconstrained taxes are not available, there is no feasible monetary policy and fiscal policy that decentralizes the EG real allocation.

Proof. See Appendix B. \(\square\)

To grasp the idea behind this result, observe that in order to implement the EG real allocation, the government budget constraint must be

\[
\hat{e} [\hat{Q}(B', s) B' - B] = \hat{p}^N (\hat{g} + \gamma) - \hat{\mu}.
\]

Since \(\hat{g} + \gamma \geq 0\) and implementing the EG real allocation requires \(\hat{\mu} < 0\), the government would need to run a Ponzi-scheme, with \(B' > B\). This would lead to \(\hat{Q}(B', s) = 0\) in finite time and thus, a contradiction.

\(^6\)Note that with distortionary taxes, \(\xi\) is not equal to \(u_T\) in general; this would require the government budget constraint to be slack, i.e., \(\lambda = 0\).
4 Calibration

In this section, we describe the functional forms adopted for the quantitative analysis, discuss the sources of the parameters set externally, and explain how we set the remaining parameters’ values to match some relevant statistics.

4.1 Choice of functional forms

The utility functions for consumption and leisure are

\[ u(c^N, c^T) = \alpha^N \left( \frac{c^N}{1 - \sigma^N} \right)^{1-\sigma^N} + \frac{\alpha^T (c^T)^{1-\sigma^T}}{1 - \sigma^T}, \]
\[ v(\ell) = \alpha^H \frac{\ell^{1-\varphi}}{1-\varphi}, \]

We will set \( \sigma^N = \sigma^T = \sigma \) what implies that \( 1/\sigma \) will be both the intra-temporal elasticity of substitution between \( c^N \) and \( c^T \), and the inter-temporal elasticity of substitution.

The utility associated with the public good is

\[ \vartheta(g) = \alpha^G \log(g), \]

which is a standard representation in the optimal taxation literature and close to empirical estimates.\(^7\)

The function describing the labor requirement for production is

\[ F(y^N, y^T) = \left[ \frac{(y^N)^\rho + (y^T)^\rho}{A} \right]^{1/\rho}, \]

where \( A \) is a measure of labor productivity and \( \frac{1}{\rho} \) is an elasticity of substitution, determining how costly it is to change the composition of \( y^N \) and \( y^T \) that is produced, in terms of labor units.

Finally, we assume that the economy experiences a drop in productivity when the government is in default. Following the work of Arellano (2008) we allow this penalty to vary with

\(^7\)A more general representation with constant relative risk aversion, \( \alpha^G (g^{1 - \nu} - 1)/(1 - \nu) \), converges to log-utility as \( \nu \) approaches 1. Nieh and Ho (2006) estimates values of \( \nu \) around 0.8. Azzimonti et al. (2016), among others, uses log-utility for the public good. See also the discussion in Debortoli and Nunes (2013).
the state of the economy. Productivity when in default, takes the following form,

$$A^{def} = A - [\lambda_1 + \lambda_2 \times \text{gap}]$$  \hspace{1cm} (40)$$

where $\lambda_1 > 0$ is the intercept and $\lambda_2 > 0$ makes the default cost an increasing function of the output gap. Note that the gap is exogenous and depends only on parameters. To compute this relation, we use a Taylor expansion,

$$\text{gap} = \sum_i \frac{\partial y^{US}}{\partial s_i} \Delta s_i,$$  \hspace{1cm} (41)$$

where the sum is over all the exogenous state variables $s_i$, the derivative is the change in output measure in dollars with respect to each exogenous state variable $s_i$ taken at the steady state (defined below) and $\Delta s_i$ is the change in the exogenous state variable $s_i$ with respect to its value in steady state. Thus, for example, if there are shocks to productivity $A$ and the price of exports, $p^T$, the gap would be

$$\text{gap} = \frac{\partial y^{US}}{\partial A} (A' - A) + \frac{\partial y^{US}}{\partial p^T} (p^{T'} - p^T).$$  \hspace{1cm} (42)$$

This specification allows the cost of default to depend on shocks to many variables in a parsimonious manner, since there are always only two parameters to calibrate, $\lambda_1$ and $\lambda_2$. The gap is measured in terms of output in foreign currency (dollars) since this captures the country’s capacity to repay its debt. However, our approach is flexible and the gap could alternatively be specified using exports or real GDP.

### 4.2 Parameters

We calibrate period length to a year. Table 1 shows the values of parameters which are set externally.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Basis</th>
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</thead>
<tbody>
<tr>
<td>$r$</td>
<td>risk-free rate</td>
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<td>long-run average</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>curvature of leisure</td>
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<td>Frisch elasticity</td>
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<td>$\delta$</td>
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<td>exclusion duration</td>
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<tr>
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<td>preference share for $c^T$</td>
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<td>normalization</td>
</tr>
<tr>
<td>$\sigma^N$</td>
<td>curvature of $c^N$</td>
<td>0.50</td>
<td>see appendix F</td>
</tr>
<tr>
<td>$\sigma^T$</td>
<td>curvature of $c^T$</td>
<td>0.50</td>
<td>see appendix F</td>
</tr>
<tr>
<td>$\rho$</td>
<td>elasticity of substitution between $y^N$ and $y^T$</td>
<td>1.50</td>
<td>see appendix F</td>
</tr>
</tbody>
</table>
The risk-free interest rate is set at 3% annual in line with the average real interest rate of the world since 1985 in King and Low (2014). We calibrate the value of $\varphi$ to 1.50 so that the Frisch elasticity is one-half on average.\(^8\) Considering the duration of a default episode from Das et al. (2012) and the length of exclusion after restructuring from Cruces and Trebesch (2013) we choose an expected period of exclusion after a default of 6 years, which implies $\delta = 1/6$.\(^9\)

We set $\sigma_N = \sigma_T = 0.5$. As shown in the previous section, $\sigma_T < 1$ is a sufficient condition for the non-negativity constraint in the government’s problem to be satisfied with strict inequality (it is also necessary when transfers $\gamma$ are zero). Note that $\sigma_N < 1$ implies the time-consistency problem has a negative sign, mitigating the incentives to accumulate debt. This choice implies that importable goods are gross substitutes for non-tradable goods as in the estimates of Ostry and Reinhart (1992).\(^10\)

The value of $\rho$, which determines the elasticity of substitution between $y^N$ and $y^T$ in the cost function, is set to 1.5. A number larger than one guarantees that $F$ is convex and so it ensures that the production possibilities frontier is concave. Appendix F also provides an analysis of the choice of $\rho$.

The remaining parameters are calibrated jointly to match a set of long-run averages. We use data from Brazil, Colombia, and Mexico for the period 1991–2017 because of data availability. For the model, we use the steady-state that would obtain if the economy remained in the debt-repay state indefinitely, and there were no shocks (other than the extreme value shocks). We will mention the critical parameter that reproduces each moment as we explain the choice of targets for exposition. Table 8 in the appendix shows the marginal reaction of moments to parameters. The corresponding expressions for each target are presented in Appendix C.

We set $A$ so that the steady-state real GDP is equal to 1, making some statistics easier to read. The value of the discount factor $\beta$ helps the model reproduce a rate of inflation equal to 6.3% annual. The parameter $\gamma$ matches the ratio of transfers to GDP, which in the data average is 13.1%. The value of $\alpha^H$ will be critical to making the model replicate the long-run average for the employment-to-population ratio of 0.59. The weight in the utility of the government consumption good, $\alpha^G$, delivers government consumption over GDP of 14.9%. The

\(^8\)We can calibrate this parameter externally because we target the value of $h$.

\(^9\)Here the duration of exclusion is exogenous. The country reenters with no debt after exclusion. See Dvorkin et al. (forthcoming) for a model of endogenous restructuring.

\(^10\)However, the estimates in Ostry and Reinhart (1992) are in the range of 1.22-1.27, and our calibration implies an elasticity equal to 2. We analyze in Appendix F how our results change setting $\sigma^N = \sigma^T = 1.5$, which implies that the goods are complements with an elasticity of 0.66. We find that many results are robust to this choice but, comparing the reaction of spreads, inflation, and exports to shocks in $\dot{p}^T$ and $r$, we find some reasons to favor our benchmark calibration.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Statistic</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1.4748</td>
<td>Real GDP</td>
<td>1.000</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.8206</td>
<td>Inflation</td>
<td>0.063</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1216</td>
<td>Transfers/GDP</td>
<td>0.131</td>
</tr>
<tr>
<td>$\alpha^N$</td>
<td>3.3113</td>
<td>Exports/GDP</td>
<td>0.181</td>
</tr>
<tr>
<td>$\alpha^H$</td>
<td>1.0104</td>
<td>Employment/Population</td>
<td>0.589</td>
</tr>
<tr>
<td>$\alpha^G$</td>
<td>0.5808</td>
<td>Gov. Consumption/GDP</td>
<td>0.149</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.1086</td>
<td>Debt/GDP</td>
<td>0.290</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0947</td>
<td>Default</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Parameter $\alpha^N$ takes a value that helps the model reproduce the ratio of exports to GDP, which is 18% in the data. The value of $\lambda_1$ is critical to reproduce the cost of default in the steady-state and, therefore, takes a value that helps the model reproduce the external debt to GDP ratio. Finally, the scale parameter in the distribution of taste shocks, $\kappa$, determines the risk of sovereign default, and it is calibrated to reproduce a default rate of 1% annual. We choose a default rate target that is lower than the 1.7% calculated by Tomz and Wright (2013) because: (i) during our sample period the three countries we selected did not default; and (ii) we calibrate to a version of the model in which default occurs only due to the extreme value shocks.\(^{11}\)

The last parameter to calibrate is $\lambda_2$. Recall from equation (40) that this parameter determines how the cost of default falls as the economy deteriorates. Consequently, this parameter plays no role in the steady-state but is critical to determine the reaction of spreads to shocks. We calibrated $\lambda_2 = 0.0606$ so that the model replicates the increase in spreads during the recent COVID-19 pandemic—see Section 6. The results are shown in Figure 1. In steady-state, the cost of default, which is given exclusively by $\lambda_1$, is close to 7.4 percent. We find that this cost varies slightly with the output gap, reaching a 7 percent loss when the output gap is at -10 percent.

### 5 Quantitative evaluation

In this section, we analyze how spreads, inflation and growth respond to shocks to the price of exports, $p^T$. We interpret the comparison of the model with the empirical estimates as a validation check of our model. In particular, comparing the response of spreads is a test of our structural assumption for the functional form of the cost of default. Similarly, comparing the

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\(^{11}\) Alternatively, we could target the average EMBI for these countries. Matching the average for this period of above 300 basis points would require a higher value of $\kappa$ and a smaller value of $\beta$. That calibration, which we have also experimented with, yield similar results.
reaction of inflation will be useful to test our assumptions for the inclusion of money in the model. Finally, the impact on growth helps validate our choices for the curvature of the utility function $u(c^N, c^T)$ and the elasticity of substitution between $y^T$ and $y^N$ in the production cost function. The regressions will control only for country fixed-effects since we want to capture the overall effect of the shock. In Appendix E we conduct a similar analysis using shocks to the risk-free rate, $r$. The conclusions are similar.

5.1 The impact of shocks to the price of exports

It is well-known that debt crises in developing countries are often associated with drops in commodity prices. In a recent paper, Drechsel and Tenreyro (2017) test this hypothesis by analyzing the endogenous reaction of interest rate spreads to shocks in commodity prices. They argue that using commodity prices is ideal because this variable is exogenous to the country. Therefore, the direction of causality goes from commodity prices to the state of the economy. They also show that commodity prices correlate very highly with the terms of trade for Argentina.

To measure interest rates spreads, Drechsel and Tenreyro (2017) use alternative variables for which they can obtain a long time series. We choose to use the EMBI spread since it is closer to the measure of the spread in sovereign debt in our model, as we explain below. The trade-off is that the EMBI spread is only available since 1997. Since our sample is shorter than in Drechsel and Tenreyro (2017), in addition to Argentina, we incorporated the three countries used for the calibration (Brazil, Colombia, and Mexico) to perform a panel data analysis. In addition to spreads, we analyze the impact of terms of trade for two critical variables in our

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12 Additional endogenous controls or controls that are intermediate outcomes would bias our estimate of the overall effect. This is what Angrist and Pischke (2009) call bad controls, i.e., “variables that are themselves outcome variables in the notional experiment at hand.”

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analysis: inflation and growth.

The series used by Drechsel and Tenreyro (2017) for commodity prices do not correlate well with the terms of trade for Brazil and Mexico. Hence, we also run another regression, using the terms of trade for each country. The estimates presented in Table 3 are, for the most part, very similar when using commodity prices or terms of trade. Thus, either measure would be a good comparison with shocks to $p_T$ in our model.

Table 3: Estimated response to changes in export prices

<table>
<thead>
<tr>
<th>Variable</th>
<th>EMBI</th>
<th>Inflation</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terms of trade</td>
<td>-0.159</td>
<td>-0.365</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td>(0.162)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Commodity prices</td>
<td>-0.138</td>
<td>-0.303</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.347)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Observations</td>
<td>88</td>
<td>76</td>
<td>152</td>
</tr>
<tr>
<td></td>
<td></td>
<td>163</td>
<td>156</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>184</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.095</td>
<td>0.114</td>
<td>0.0166</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.021</td>
</tr>
</tbody>
</table>

Note: We use data from 1970 to 2018 for Argentina, Brazil, Colombia, and Mexico. The series for EMBI starts in 1997. Standard errors clustered by country are shown in parentheses. The regression for inflation is a quantile regression with country dummies to deal with periods of hyperinflation in Argentina and Brazil.

All the coefficients presented in Table 3 are semi-elasticities since the series for terms of trade and commodities prices are in log deviations from the mean. The main finding in Drechsel and Tenreyro (2017) is that “a 10 percent deviation of commodity prices from their long-run mean can move Argentina’s real interest spread by almost two percentage points.” Our estimates are very similar to theirs, as they indicate (first row) that a 10% deviation of terms of trade below their long-run mean moves these countries EMBI spreads by 1.6 percentage points. The estimate for commodity prices is similar: A 10% deviation of commodity prices below their long-run mean increases these countries’ EMBI spreads by 1.4 percentage points.

The impact on inflation can be understood following a similar logic. The results suggest that a 10% fall in the terms of trade relative to their long-run mean implies an increase in inflation of 3.65%. The estimate using commodity prices is quite similar, but the standard errors are quite large.

The last variable we study is real GDP growth. We know from Kehoe and Ruhl (2008) that in a multi-sector model, the first-order effect of changes in the terms of trade on real GDP is zero. In the data, however, the effect is small but significantly different from zero: A 10% fall in the terms of trade relative to their long-run mean implies an increase in inflation of 3.65%. Since Argentina and Brazil experienced hyperinflations during the sample period, we used quantile regressions to avoid the impact of very high inflation rates (in some cases, higher than 1,000% annual). The median regression used minimizes the sum of absolute residuals instead of the squared of residual, reducing the impact of extreme values. Similar results are obtained if we do a panel regression but include dummies for periods of annual inflation greater than 100%.

---

13Since Argentina and Brazil experienced hyperinflations during the sample period, we used quantile regressions to avoid the impact of very high inflation rates (in some cases, higher than 1,000% annual). The median regression used minimizes the sum of absolute residuals instead of the squared of residual, reducing the impact of extreme values. Similar results are obtained if we do a panel regression but include dummies for periods of annual inflation greater than 100%.
deviation of the terms of trade below their long-run mean decreases real GDP growth by 0.22 percentage points. Instead, our estimate using commodity prices is slightly larger; a shock of equal magnitude would generate a decline in real GDP growth of 0.33 percentage points.

5.2 Model response to shocks in $p^T$

Now, we show the impact of a shock to $p^T$ in our model. To compute it, we start the economy at the steady-state and track the response of a one-time unanticipated shock to $p^T$. Since the model response is nonlinear, we need first to select a size for the shock. We consider three alternative sizes to show how the reaction in the model depends on the shock size. First, we compute the mean of the log deviations of commodity prices from their long-run mean, which is 6.3%. That value is a reference to compare it with the coefficient in the regressions. Then, we considered a smaller shock, 4%, and a larger one, 8%. Next, we need to pick the persistence of the shock, i.e., the probability that it continues into the following period. To this effect, we estimate an autoregressive process for commodity prices and obtain a coefficient of 0.8164 and a standard deviation of 0.0999. These two values imply that a 6.3% shock takes, on average, 7.65 years to get back to the steady-state. Thus, we use a probability of the shock continuing into the next period equal to 0.8693.

Table 4 presents the results. The first column shows the value of the variables before the shock, at $t-1$. Then, for each shock we compute, we show the value in the period of the shock, $t$, and the “regression” coefficient, which is a semi-elasticity.

The measure of spreads we use in the model is also referred to as the country risk premium. It uses the price at which the government bond would sell in a secondary market that operates in the period after the bond is issued, once the exogenous shock is realized, but before the government makes any decision in that period. Hence, we define the spread as $[1 - P(B, s)]/P(B, s)$. This measure is the closest in our model to the EMBI spread in the data.

<table>
<thead>
<tr>
<th>Table 4: Endogenous response to a change in $p^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous</td>
</tr>
<tr>
<td>period, $t-1$</td>
</tr>
<tr>
<td>$p^T$</td>
</tr>
<tr>
<td>Spread</td>
</tr>
<tr>
<td>Inflation</td>
</tr>
<tr>
<td>Growth</td>
</tr>
</tbody>
</table>

Note: The size of the shock is measured in log deviations from the steady state value.

The coefficient for spreads is -0.12 for the smaller shock and increases to -0.20 for the larger
shock. Recall that we calibrated the model to match the increase in spreads during the COVID-19 pandemic. These numbers are quite close to the estimates obtained in the regression analysis presented in Table 3. For the terms of trade, it was -0.159 with standard errors of 0.08, so the model is at most one standard error from the point estimate. Something similar occurs for the coefficient estimated using commodity prices. Thus, the coefficients’ similarity suggests that the reaction of spreads during the pandemic is in line with historical evidence, which is captured by the regressions.

Now we focus on the comparison of the reaction of inflation in the model and the data. The coefficient in the data is -0.365 for terms of trade shocks, with standard errors of 0.16 (recall that the estimate for commodity prices is not very precise). The same coefficient in the model ranges from -0.24 to -0.30, depending on the shock size. The similarity is reassuring, given that adding inflation is one of the critical contributions of this paper. This estimate implies that inflation increases significantly after a decline in the price of export goods.

Finally, we analyze the impact of a change in $p^T$ on real GDP growth. Note that growth is zero at the steady-state. When $p^T$ falls, growth declines slightly, implying a positive coefficient equal to 0.05. This coefficient is larger but still in line with the empirical estimates.  

We also computed all these semi-elasticities for a calibration that assumes $\sigma^N = \sigma^T = 1.5$, which we present in the appendix. As discussed above, in this case, the time-consistency problem in debt choice goes in the opposite direction compared to the benchmark calibration, which assumed $\sigma^N = \sigma^T = 0.5$. We find that the semi-elasticities for inflation and spreads have all of the same sign as in the benchmark, but are about one-fourth larger. Though of similar magnitude as in the benchmark, the effect on growth has the opposite sign, which is counterfactual. This finding suggests that the benchmark calibration with $\sigma^N = \sigma^T = 0.5$ better captures the effect on output growth of shocks to the price of exports.

6 COVID-19: policy response in emerging markets

As the COVID-19 pandemic spreads across countries, several emerging economies have faced difficulties meeting external debt obligations while addressing the health emergency. In this section, we analyze our model’s predictions for the policy response to the COVID-19 shock, decompose the different components of the shock, and analyze the consequences of alternative

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14 For example, an 10% deviation in the terms of trade below their long-run mean would decrease real GDP growth by 0.22–0.33 percentage points in the data and 0.52 percentage points in the model. This finding is reassuring, given that we chose an arbitrary value of $\rho$ in the cost function. The results suggest that this is the right choice and, if anything, we should compare our results with a lower value of $\rho$.
6.1 The response of key variables to COVID-19

We model the COVID-19 shock as an unanticipated event that combines shocks to productivity, marginal utility of leisure, terms of trade, and transfers. The calibration of each shock is as follows:

(i) Decrease productivity, $A$ by 2.6% to reproduce a drop in real GDP of 8.1%; this target is obtained by combining the prediction for 2020 in a recent study by CEPAL (2020) with data from Cadena et al. (2017) about the expected trend in output growth in Latin America.

(ii) Increase the utility weight of leisure, $\alpha^H$ by 11% to reproduce a drop in employment relative to the trend of 5.7%, obtained from the same sources as in (i).

(iii) Decrease the price of exports, $p^T$ by 17%, to match a 17% drop in commodity prices as reported by IMF (2020).

(iv) Increase real transfers $\gamma$ by 19%, to match an increase equal to 2.5 percentage points of pre-shock GDP, which is reported in the policy tracker developed by Cavallo and Cai (2020).

The COVID-19 shock is modeled as an unexpected shock to all these variables. Two additional parameters determine its severity. We calibrate the probability that the shock continues into the next period, as $\theta = 1/3$, to match an optimistic forecast for the expected time until a vaccine is available of 1.5 years (Thompson, 2020). We also allow for a parameter, $\eta$ that states the share of the shock that remains in the long run. In the benchmark calibration, we set this “scarring effect” to zero. Finally, recall that $\lambda_2$, which governs the change in the cost of default as the output gap changes, is set so that the model replicates a 300 basis points increase in the EMBI spreads.

Figure 2 shows the response of critical variables to the shock. In this simulation, we assume that the COVID-19 shock lasts for one year. The blue line in each panel represents the benchmark calibration for each of the shocks comprising the COVID-19 shock, while the dashed lines assume that each shock is either 25 percent larger or smaller than the benchmark calibration.

The top panels show that government revenue over GDP (panel a) decreases by almost four percentage points, while government expenditure over GDP (panel b) increases over four
Figure 2: A Crisis in Emerging Markets

(a) Tax revenue / GDP

(b) Government Expenditure / GDP

(c) Inflation

(d) Currency Depreciation

(e) Debt / GDP

(f) Country Risk Premium

Note: The solid blue line represents the response to the benchmark calibration of shocks to productivity, utility weight of leisure, price of exports, and real transfers. The dashed lines represent shocks that are 25% larger and smaller than the benchmark calibration.
percentage points. This change is an unprecedented shock to government finances that increases the government’s primary fiscal by roughly eight percentage points of GDP. Consequently, the government increases the money growth rate, which contributes to an increase in inflation (panel c), from about 6% to 20% annually. At the same time, the exchange rate depreciates by more than 40% (panel d).

The deterioration of the government finances is also reflected in the rise of the ratio of debt to GDP (panel e) from 29% to 38%. This rise is mostly due to the sharp decline in GDP measured in dollars, about 24%; the debt stock remains roughly constant. The increase in credit spreads from 1% to 4% annual (panel f) implies that the government must endure a substantially higher financial cost to sustain the same level of debt.

Figure 3 can be used to understand the behavior of debt and spreads during the crisis. Panel (a) shows how the resources that can be raised in credit markets change with the amount of debt issued. The blue and red lines are constructed using bond prices at issuance, before and after the COVID-19 shock. Both lines first increase and then decrease since, for sufficiently high debt, the decline in bond prices (i.e., the rise in the interest rate) dominates. Therefore, there is a maximum amount of resources that can be collected from credit markets, and the government never finds it optimal to issue more debt than this threshold. Note that after the realization of the COVID-19 shock, bond prices fall, and thus fewer resources can be obtained when promising to repay the same amount of debt in the following period. Panel (b) displays the interest rate spreads at the time of issuance associated with different amounts of debt issued.

Figure 3: Spreads at issuance, Debt, and the COVID-19 shock

We identified some illustrative points in Figure 3. The blue and red dots correspond to the debt chosen in the periods before the shock and when the shock hits, respectively. Note that the red dot is to the left of the blue dot, indicating that the optimal amount of debt promised
for the following period falls in response to the shock, i.e., \( B' < B \). As panel (a) shows, this reduction means that the debt is not fully rolled-over when the shock hits, i.e., \( B'q < B \). If the government wanted to roll over all its outstanding debt obligations, it would need to choose a significantly larger \( B' \) (the horizontal dotted line), which, in this case, coincides with the maximum amount of resources it could raise in credit markets (the purple dot). However, the government does not choose the purple dot because, as panel (b) shows, it would imply paying almost 800 basis points in the spread at issuance. Finally, notice that if the government preferred to prevent the financial crisis and keep spreads at issuance constant, at 100 basis points, it would need to reduce debt to the yellow dot, which amounts to repaying debt in the amount of three percentage points of the previous year’s GDP.

Figure 4 shows the country risk premia and exchange rates in Colombia, Mexico, and Brazil. We target an increase in spreads of 300 basis points, so credit spreads behave in the model as in the data by construction. Exchange rates depreciated between 30 and 40 percent in the data. The model successfully predicts a depreciation of 40%, which is a non-targeted moment. Figure 4 also includes Argentina, where the economy was very close to default even before the pandemic (spreads were already 2,000 basis points in January) and is currently in the process of debt restructuring. The depreciation of the exchange rate during the pandemic was close to 50 percent.

The model predicts that the primary deficit over GDP increases by 8.2 percentage points, which is in line with the fiscal policies implemented in Argentina, Brazil, Colombia, and Mexico to combat the economic effects of the COVID-19 pandemic (see Cavallo and Cai, 2020, for details). Argentina increased transfers to needy families and continued providing utility services to households in arrears for 180 days. Brazil also provided transfers for vulnerable households and extended credit to small and medium-sized enterprises (SMEs) to finance salaries. The programs have an estimated impact of 4.8 percent of GDP on the primary deficit. The “Fiscal Rule Consultative Committee” of Colombia allowed for a 2020 fiscal deficit of 6.1 percent, given the expected economic impact of COVID-19. The response included transfers to low-income families, payroll subsidies, and suspension of pension contributions. Finally, Mexico set up a Health Emergency Funds to request additional resources to Congress of about 3 percent of GDP.

In terms of monetary policy, Argentina increased the monetary base significantly and lowered reserve requirements on bank lending to households and SMEs. The exchange rate depreciated close to 40 percent in the informal exchange rate market. In Brazil, the central bank decreased the policy rate to its historical low, reduced reserve requirements, allowed banks to extend
loans without loan-loss provisions, and created a facility to lend to financial institutions using corporate debt as collateral. The depreciation between the beginning of the year and mid-May was 30 percent. Colombia also cut the policy rate, added facilities to buy purchase security from credit institutions, and lowered reserve requirements. The depreciation of the exchange rate during this period was 16 percent. In Mexico, the central bank reduced the policy rate and opened financing facilities for commercial and development banks to channel resources to households and SMEs.

6.2 Understanding the components of the COVID-19 shock

We modeled the COVID-19 shock as a combination of factors impacting the economy negatively. In this section, we analyze each component’s contribution separately by considering their effects one at a time, and the role played by the persistence of the overall shock.
The results of the decomposition exercise are presented in Table 5. First, the shock to productivity, $A$ by itself generates slightly more than one-third of the total decline in real GDP. This shock is also critical for the rise in inflation, as it generates an increase equal to one-fourth of the total change.

Table 5: Identifying the role of each component of the COVID-19 shock

<table>
<thead>
<tr>
<th></th>
<th>All shocks</th>
<th>Productivity</th>
<th>Only shocks to</th>
<th>Export prices</th>
<th>Transfers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$ Real GDP, percent</td>
<td>-8.10</td>
<td>-2.94</td>
<td>-3.58</td>
<td>-0.96</td>
<td>-0.74</td>
</tr>
<tr>
<td>$\Delta$ Employment, percent</td>
<td>-5.70</td>
<td>-0.34</td>
<td>-3.64</td>
<td>-0.77</td>
<td>-0.86</td>
</tr>
<tr>
<td>$\Delta$ Debt / GDP, pp</td>
<td>9.01</td>
<td>0.97</td>
<td>1.19</td>
<td>5.56</td>
<td>0.75</td>
</tr>
<tr>
<td>$\Delta$ Country risk premium, pp</td>
<td>3.00</td>
<td>0.98</td>
<td>0.57</td>
<td>1.14</td>
<td>0.00</td>
</tr>
<tr>
<td>$\Delta$ Expenditure, percent</td>
<td>6.40</td>
<td>-0.88</td>
<td>-1.17</td>
<td>0.11</td>
<td>8.48</td>
</tr>
<tr>
<td>$\Delta$ Tax rate, pp</td>
<td>-3.82</td>
<td>-0.46</td>
<td>-1.27</td>
<td>-0.98</td>
<td>-0.75</td>
</tr>
<tr>
<td>$\Delta$ Inflation, pp</td>
<td>14.3</td>
<td>3.60</td>
<td>4.99</td>
<td>1.89</td>
<td>2.13</td>
</tr>
<tr>
<td>$\Delta$ Depreciation, pp</td>
<td>36.3</td>
<td>3.68</td>
<td>5.90</td>
<td>17.1</td>
<td>3.94</td>
</tr>
</tbody>
</table>

Note: pp stand for percentage points.

Second, the “stay-at-home” aspect of the COVID-19 shock, which we modeled as an increase in $\alpha^H$, has an even more significant impact on real GDP and inflation. As expected, it generates most of the drop in employment.

Third, the decline in export prices, $p^T$ has a relatively small effect on economic activity but accounts for more than 60% of the increase in the debt-to-GDP ratio and almost 40% of the increase in the country risk premium. These effects are significant and a consequence of the sizeable nominal depreciation of the currency (about 17%) it implies.

Fourth, the increase in transfers, $\gamma$ more than accounts for the increase in real government expenditure—other factors mitigate this effect by inducing a drop in government consumption, $g$. The increase in transfers, by itself, does not seem to create financial stress.

Table 6 shows the role played by the persistence of the combined shock. The column labeled “higher persistence” shows the effect of increasing the probability that the COVID-19 state continues into the next period, $\theta$, from 0.33 to 0.5. This change increases the expected duration of the shock by 50%, from one-and-a-half to two years. The comparison with the benchmark (first column) reveals that, although the effect of a longer expected duration on economic activity is minimal, it is essential for the severity of the financial crisis. As the economy is expected to remain in bad shape longer, the country finds it more difficult to borrow from international markets to smooth out the shock’s economic costs. Hence, a higher persistence implies a smaller increase in debt-to-GDP, a more significant rise in sovereign debt spreads, and more substantial increases in inflation and currency depreciation.

The column labeled “scarring” assumes that all the affected variables (e.g., productivity)
do not return to their original value; instead, 10% of the effect of COVID-19 shock remains indefinitely. For example, productivity declines by 2.6% during the COVID shock, so it settles at 0.26% below the pre-shock value once the shock is over. This scarring aspect of the shock has a significant effect on the severity of the sovereign debt crisis, without further affecting economic activity. Real GDP now declines by 8.2% instead of 8.1%, but debt spreads increase by 749 instead of 300 basis points. The scarring effect of the shock lowers the ability of the country to repay its debt; hence, international credit markets need to be compensated for the added risk of default.

Finally, the last column of Table 6 has both higher persistence and scarring. The results are similar to those presented above, only compounded: despite a small change in economic activity, debt spreads, inflation, and currency depreciation all increase significantly. Overall, the message is that if the COVID-19 shock is expected to last longer and/or to have lasting adverse effects, the country will endure a more severe crisis, despite similar observable economic fundamentals.

Figure 5 further explains the effects of the persistence of the shock on debt and spreads. Higher $\theta$ (the probability that the shock continues into the next period) or $\eta$ (the share of the shock that remains in the long run) raise the level of spreads the government must pay at issuance for the same size of the promised amount for the next period, $B'$. For example, if the government keeps $B' = B$, the spreads increase as displayed by panel (b) vertical line. The colored dots in panel (b) of Figure 5 represent spreads at issuance of the level of debt for the next period, $B'$. Note that to mitigate the increase in spreads, the government reduces $B'$ as persistence increases. The spreads associated with issuances at the times of COVID-19 are in the range of 200 basis points. The higher spreads and the reduction in $B'$ imply that fewer resources are raised from credit markets. As shown in panel (a), between our original

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### Table 6: Identifying the role of the persistence of the COVID-19 shock

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Higher persistence</th>
<th>Scarring</th>
<th>Higher persistence and scarring</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$, prob. COVID stays in t+1</td>
<td>0.33</td>
<td>0.50</td>
<td>0.33</td>
<td>0.50</td>
</tr>
<tr>
<td>$\eta$, share of COVID stays in the SS</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$\Delta$ Real GDP, percent</td>
<td>-8.10</td>
<td>-8.28</td>
<td>-8.22</td>
<td>-8.34</td>
</tr>
<tr>
<td>$\Delta$ Employment, percent</td>
<td>-5.70</td>
<td>-6.09</td>
<td>-6.04</td>
<td>-6.38</td>
</tr>
<tr>
<td>$\Delta$ Debt / GDP, pp</td>
<td>9.01</td>
<td>8.27</td>
<td>8.23</td>
<td>7.53</td>
</tr>
<tr>
<td>$\Delta$ Country risk premium, pp</td>
<td>3.00</td>
<td>5.95</td>
<td>7.49</td>
<td>13.04</td>
</tr>
<tr>
<td>$\Delta$ Expenditure, percent</td>
<td>6.40</td>
<td>5.95</td>
<td>5.95</td>
<td>5.53</td>
</tr>
<tr>
<td>$\Delta$ Tax rate, pp</td>
<td>-3.82</td>
<td>-3.04</td>
<td>-3.04</td>
<td>-2.36</td>
</tr>
<tr>
<td>$\Delta$ Inflation, pp</td>
<td>14.29</td>
<td>17.06</td>
<td>17.08</td>
<td>19.71</td>
</tr>
<tr>
<td>$\Delta$ Depreciation, pp</td>
<td>36.31</td>
<td>43.46</td>
<td>43.63</td>
<td>50.40</td>
</tr>
</tbody>
</table>

Note: pp stands for percentage points.
calibration of the COVID-19 shock and the case with higher persistence and scarring effects, the resources raised from international credit markets decrease by almost five percentage points of the previous year’s GDP.

Figure 5: Spreads at issuance and the Persistence of COVID-19 shock

(a) Resources raised in debt market
(b) Sovereign spreads at issuance

6.3 The cost of counterfactual policy responses

How important is it to use the right policy mix? Or asked another way: What would be the impact of having a more conservative central banker or fiscal authority? This section answers these questions by analyzing counterfactual economies with policies fixed at alternative values during the COVID-19 shock period.

The second and third columns of Table 7 show the effects of tighter monetary and fiscal policy, respectively. In the case labeled “constant $\mu$”, the money growth rate is kept at its pre-COVID level, while in the column labeled “constant $\tau$”, the tax rate is held at the pre-shock level. Note that, when the shock hits, the government would like to do counter-cyclical policy, reducing taxes and increasing the money growth rate. Holding these instruments constant, or moving them in the opposite direction, are policies that have been recommended by the International Monetary Fund during previous financial crises.\(^{15}\)

The results in Table 7 show that adopting either a tighter monetary or fiscal policy during the COVID-19 shock worsens economic activity, as both real GDP and employment fall more than in the benchmark case. However, policy fundamentals improve: tax revenue now increases and

\(^{15}\)For example, the agreement between the IMF and Argentina in 2018 asked the government to set $\mu = 0$. In particular, it said: “To decisively reduce inflation, the Central Bank will shift toward a stronger, simpler, and more verifiable monetary policy regime, temporarily replacing the inflation targeting regime with a monetary base target. At the center of the new framework is a commitment to cap the growth of money to zero percent per month (calculated as the change in the monthly average) until June 2019.” (IMF, 2018).
Table 7: Identifying the role of the policy mix during COVID-19 shock

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Constant $\mu$</th>
<th>Constant $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ money growth rate</td>
<td>0.20</td>
<td>0.06</td>
<td>0.15</td>
</tr>
<tr>
<td>$\tau$ taxes</td>
<td>0.21</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>$\Delta$ Real GDP, percent</td>
<td>-8.10</td>
<td>-9.50</td>
<td>-8.67</td>
</tr>
<tr>
<td>$\Delta$ Employment, percent</td>
<td>-5.70</td>
<td>-6.84</td>
<td>-6.16</td>
</tr>
<tr>
<td>$\Delta$ Debt / GDP, pp</td>
<td>9.01</td>
<td>7.91</td>
<td>8.54</td>
</tr>
<tr>
<td>$\Delta$ Country risk premium, pp</td>
<td>3.00</td>
<td>2.89</td>
<td>2.95</td>
</tr>
<tr>
<td>$\Delta$ Expenditure, percent</td>
<td>6.40</td>
<td>5.02</td>
<td>6.48</td>
</tr>
<tr>
<td>$\Delta$ Tax rate, pp</td>
<td>-3.82</td>
<td>5.34</td>
<td>0.00</td>
</tr>
<tr>
<td>$\Delta$ Inflation, pp</td>
<td>14.3</td>
<td>11.5</td>
<td>13.5</td>
</tr>
<tr>
<td>$\Delta$ Depreciation, pp</td>
<td>36.3</td>
<td>26.6</td>
<td>32.8</td>
</tr>
</tbody>
</table>

Note: pp stand for percentage points.

by more than expenditure, thus reducing the primary deficit; inflation and currency depreciation increase by less than in the benchmark; and spreads do not rise as much. In sum, a tighter monetary or fiscal policy improves the countries fiscal outlook, making debt repayment more likely, despite the deeper recession. As such, it is not surprising that international organisms would recommend this type of policy stance.

An essential lesson from Table 7 is that the predicted inflation during the COVID-19 shock is only partially driven by policy. As the case with a constant $\mu$ suggests, even if monetary aggregates are kept constant, annual inflation would increase significantly by 11.5 percentage points in our simulation. The external shock to the terms of trade drives most of the inflation. With a constant money supply, the inflation rate in non-tradable goods increases by only 2.3 percentage points when the COVID-19 shock hits.

7 Conclusions

We build an equilibrium model of sovereign debt for emerging markets that incorporates money, nominal exchange rates, and optimal fiscal and monetary policy. Including money adds a time-consistency problem to the standard sovereign default models that may reinforce or mitigate incentives to issue debt.

A version of this model calibrated to match critical long-run averages and the rise in spreads during the COVID-19 shock can also reproduce the response of spreads, inflation, and growth to terms of trade shocks. This quantitative version of the model is then used to analyze policy responses to the unexpected economic challenges brought by the COVID-19 pandemic. The model predicts a substantial nominal depreciation, small tax cuts, increased government expenditure, and rising inflation.
References


Appendix

A Derivations

A.1 Monetary equilibrium

In order to characterize the solution to the household problem, let $\chi$ and $\psi$ denote the Lagrange multipliers associated with constraints (8) and (2), respectively. The necessary first-order conditions with respect to $(c^N, c^T, h, m')$ for an interior solution are

\[ u_N - p^N(\chi + \psi) = 0, \]
\[ u_T - e\chi = 0, \]
\[ -v^e + \chi(1 - \tau)w = 0, \]
\[ \beta\mathbb{E}[V'_m|I, s] - \chi(1 + \mu) = 0, \]

where $V_m$ denotes the partial derivative of $V$ with respect to the individual state variable, $m$. The corresponding envelope condition implies that $V_m = \chi + \psi$. From (43) and (44) we can solve for the Lagrange multipliers

\[ \chi = \frac{u_T}{e}, \]
\[ \psi = \frac{u_N}{p^N} - \frac{u_T}{e}. \]

Replacing these expressions in (45) and (46) yields (9) and (10). Using (7) to replace $e$ in (48) and imposing $\xi \geq 0$ yields (13).

A.2 Government budget constraint

Take the government budget constraint (5), multiply both sides by $F_N c^N$ and use (12), (14), (15) and (16) to obtain

\[ \tau F(y^N, y^T) + F_N (\mu c^N - g - \gamma) - (F_T/p^T)(p^T y^T - c^T) \geq 0. \]

Next, replace the tax rate, $\tau$ using (17) and the money growth rate, $\mu$ using (18) to obtain the government budget constraint in a competitive equilibrium

\[ [1-(F_T/p^T)(v_l/u_T)] F(c^N + g, y^T) - F_N (c^N + g + \gamma) + \beta(F_T/p^T)\mathbb{E}[u'_N c^N|I, s] / u_T - (F_T/p^T)(p^T y^T - c_T) \geq 0. \]
Since $F(y^N, y^T) = F_N y^N + F_T y^T = F_N (c^N + g) + F_T y^T$ we obtain

$$(F_T/p^T) \{ c^T - (v_L/u_T) F(c^N + g, y^T) + \beta \mathbb{E} [u'_N c^N | I, s] / u_T \} - \gamma F_N \geq 0.$$ 

which, after multiplying both sides by $u_T (p^T/F_T)$, implies (19).

### B Proofs

**Proof of Proposition 1.** Suppose $\zeta = 0$ and let $\gamma = 0$. Then, (36) simplifies to

$$\Phi F_T (u_N - \Gamma) = p^T (v_L F_N - u_N) (u_T + u_T T c^T). \tag{49}$$

Note that (27) implies $u_N - \Gamma \geq 0$. Since $\Phi < 0$, the left-hand side of (49) is non-positive. Next, since $\gamma = 0$ we get $\lambda = (v_L F_N - u_N) / (F_N \Phi)$; thus, $\lambda > 0$ and $\Phi < 0$ imply $v_L F_N - u_N < 0$. Hence, $u_N - \Gamma > 0$ if and only if $u_T + u_T T c^T > 0$, while $u_N - \Gamma = 0$ if and only if $u_T + u_T T c^T = 0$. If preferences are such that $u_T + u_T T c^T < 0$, then (49) cannot be satisfied, a contradiction. In this case, $\zeta > 0$ and therefore, (27) binds. \hfill \Box

**Proof of Proposition 2.** Suppose lump sum, unconstrained taxes $T$ are available. The government budget constraint then becomes

$$T = p^N (g + \gamma) - \tau w h - \mu + e (p^T y^T - c^T). \tag{50}$$

We argue that, in this case, the government budget constraint does not restrict the allocative decisions taken by the government. In order to do that, we proceed as follows. First, we solve problem (PP) ignoring the government budget constraint (26)—equivalently, (50)—as well as the non-negativity constraint (27) to characterize the necessary conditions that the EG real allocation must satisfy. Then, we construct the lump-sum taxes $\hat{T}$, the corresponding monetary policy and taxes ($\hat{\mu}, \hat{\tau}$) as well as prices ($\hat{p}^N, \hat{c}, \hat{w}$) that support the EG real allocation ($\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g}$) as an equilibrium in our setting for which the government budget constraint (50) replaces (26) in the original problem (PP)–(27).
The necessary first-order conditions characterizing the EG real allocation are
\begin{align*}
\hat{u}_N &= \hat{v}_t \hat{F}_N, \quad (51) \\
\hat{v}_t \hat{F}_T &= p^T \hat{u}_T, \quad (52) \\
\hat{\vartheta}_g &= \hat{v}_t \hat{F}_N, \quad (53)
\end{align*}
which imply \( \frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{v}_T \hat{F}_N \), i.e., the non-negative constraint (27), which we ignored to derive the EG real allocation, is satisfied with equality. The balance of payment implies
\begin{equation}
p^T \hat{y}^T - \hat{e}^T + \hat{Q}(B', s) \hat{B}' - \hat{B} = 0. \quad (54)
\end{equation}

We now construct the policies and prices that support the EG real allocation in our setting when lump-sum transfers are available. The price of non-tradable goods and wages are determined by
\begin{align*}
\hat{p}^N &= \frac{1}{\hat{c}^N}, \\
\hat{w} &= \frac{\hat{p}^N}{\hat{F}_N},
\end{align*}
while the exchange rate is determined by
\begin{equation}
\hat{e} = \frac{\hat{p}^N \hat{F}_T}{p^T \hat{F}_N},
\end{equation}
The monetary policy has to be tailored so that
\begin{equation}
\hat{\mu} = \frac{\beta \mathbb{E} \left[ \hat{u}'N \hat{c}^N | B, \mathcal{I}, s \right]}{\hat{u}_T \hat{c}^N p^T (F_N / \hat{F}_T)} - 1,
\end{equation}
as it has to decentralize money holdings such that \( m' = m = 1 \). Since \( \frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{v}_T \hat{F}_N \) we obtain
\begin{equation}
\hat{\mu} = \frac{\beta \mathbb{E} \left[ \hat{u}'N \hat{c}^N | B, \mathcal{I}, s \right]}{\hat{u}_N \hat{c}^N} - 1. \quad (55)
\end{equation}

Importantly, this means that the Euler equation in the household problem is satisfied as
\begin{equation}
\frac{\hat{u}_N}{\hat{p}^N} = \beta \mathbb{E} \left[ \frac{\hat{u}'_N}{\hat{p}^N} | B, \mathcal{I}, s \right],
\end{equation}
i.e., the Friedman rule.
On the other hand, taxes are given by

\[
\hat{\tau} = 1 - \frac{\hat{v}_T F}{\hat{u}_T p_T} = 0.
\]

Finally, lump-sum transfers are designed to make the budget constraint of the government (50) hold,

\[
\hat{T} = \hat{p}^N (\hat{g} + \gamma) - \hat{\mu} + \hat{\epsilon} (p_T y_T - \hat{c}_T).
\]

Proof of Proposition 3. As shown in Proposition 2, the real EG allocation implies zero taxes. Thus, combining the balance of payment with the government budget constraint when lump-sum taxes are not available implies

\[
\hat{\epsilon} [\hat{Q}(\hat{B}', s) \hat{B}' - B] = \hat{p}^N (\hat{g} + \gamma) - \hat{\mu},
\]

i.e., a non-linear first-order difference equation in \( B \).

First, consider a steady state. Since \( \hat{g} + \gamma \geq 0 \) and \( \hat{\mu} < 0 \) (see (55) above), \( \hat{\epsilon} > 0 \) and \( \hat{Q}(\hat{B}', s) < 1 \), it follows that any steady state would require \( B < 0 \); i.e., the government must accumulate a sufficiently large amount of assets to finance its expenditures. This asset position would never be reached as (56) implies that the amount of \( B \) is increasing and the initial stock debt is positive.

Since \( \hat{Q}(\hat{B}', s) \) is smaller than 1 and decreasing in \( \hat{B}' \), the debt process is a submartingale. Hence, the so-called Doob decomposition implies that every submartingale can be decomposed into the sum of a martingale and a predictable increasing process. This would mean that domestic government must finance its expenditures by means of a Ponzi scheme, which contradicts that \( \hat{Q}(\hat{B}', s) \) is always positive in finite time.

C Definition of key variables

- Nominal GDP (in pesos)
  \[
  Y_t = \epsilon_t p_t^T y_t^T + p_t^N y_t^N
  \]
- GDP in usd
  \[
  Y_{t, USD} = p_t^T y_t^T + \frac{1}{\epsilon_t} p_t^N y_t^N
  \]
• The GDP deflator is
\[ P_t^y = \left( \frac{e_t y^T T}{Y} \right) e_t p_t^T + \left( \frac{p_t^N Y^N}{Y} \right) p_t^N \]

• Real GDP
\[ Y_t^R = \frac{Y_t}{P_t^y} \]

• Consumption expenditures (in pesos)
\[ C_t = e_t^T c_t + p_t^N c_t^N \]

• Consumption price index
\[ P_t^c = \left( \frac{e_t c_t^T}{C} \right) e_t + \left( \frac{p_t^N c_t^N}{C} \right) p_t^N \]

• Inflation, measured as the change in the consumption price index, is
\[ \pi_t = \frac{P_t^c}{P_{t-1}^c} (1 + \mu_{t-1}) - 1 \]

• Currency depreciation is given by
\[ \Delta_t = \frac{e_t}{e_{t-1}} (1 + \mu_{t-1}) - 1 \]

Note that inflation and currency depreciation are corrected by the money growth rate, since prices are normalized by the money stock.

D Identification

To provide a heuristic proof of identification, we computed how each parameter would change if we change one target at the time by 10 percent. The results, presented in Table 8, justify the link between parameters and targets mentioned in the calibration section. The first column shows how each parameter change we target a default 10 percent larger, i.e., 1.1 instead of 1 percent. Note that the more significant change is \( \kappa \). By increasing \( \kappa \) 9.38 percent (from 0.095 to 0.104) and adjusting all the parameters very slightly, the model can replicate all the targets perfectly. Thus, we selected \( \kappa \) as the critical parameter to get the default rate.

In the second column of Table 8, we present the percent change in each parameter that would allow the model to replicate a debt to GDP ratio 10 percent larger. In addition to the
change in $\kappa$, which we already show is key to replicating the default rate, the most substantial change is in $\lambda_1$. It is reasonable that $\lambda_1$ is useful to replicate indebtedness since it determines the size of the default punishment.

Continuing with the same logic, we connected each parameter with a moment and the critical change in the parameters highlighted in bold in Table 8.

Table 8: Percent change in each parameter when a target is increased by 10 percent

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default</th>
<th>Debt</th>
<th>G</th>
<th>Hours</th>
<th>Exports</th>
<th>Inflation</th>
<th>Transfers</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>9.38</td>
<td>8.44</td>
<td>-11.01</td>
<td>55.16</td>
<td>7.52</td>
<td>8.10</td>
<td>-21.72</td>
<td>4.80</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>-0.27</td>
<td>-0.65</td>
<td>0.45</td>
<td>-1.92</td>
<td>-0.59</td>
<td>-0.27</td>
<td>0.92</td>
<td>0.02</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>0.15</td>
<td>0.61</td>
<td>12.85</td>
<td>-6.64</td>
<td>-9.04</td>
<td>-0.65</td>
<td>5.66</td>
<td>4.89</td>
</tr>
<tr>
<td>$\alpha_H$</td>
<td>0.05</td>
<td>0.18</td>
<td>-2.11</td>
<td>-27.92</td>
<td>-7.04</td>
<td>0.56</td>
<td>-1.74</td>
<td>4.88</td>
</tr>
<tr>
<td>$\alpha_N$</td>
<td>0.15</td>
<td>0.62</td>
<td>1.44</td>
<td>-6.64</td>
<td>-10.17</td>
<td>-0.65</td>
<td>5.66</td>
<td>0.00</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.06</td>
<td>-0.28</td>
<td>-2.52</td>
<td>7.13</td>
<td>2.36</td>
<td>1.25</td>
<td>-5.36</td>
<td>0.00</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.25</td>
<td>0.00</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>$A$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-9.09</td>
<td>-0.99</td>
<td>0.00</td>
<td>0.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>

Note: Each number represents the percentage change in the parameter when the target is increased by 10 percent.

E  The impact of shocks to the risk-free rate

Another feature of sovereign debt markets is the response to shocks to the world interest rate (Uribe and Yue, 2006). In this section, we analyze the response to these shocks in the data and the model. An advantage of using world interest rates is that they do not respond to the state of the economy in emerging markets so that we can interpret their variations as exogenous to the countries of interest.

Methodologically, we proceed along the lines of the previous section. We use two alternative measures for the world interest rate. The first is the 1-Year Treasury Constant Maturity Rate for the US. This rate is highly correlated with the US policy rate, more so since the early ’80s. The second measure is from Drechsel and Tenreyro (2017) and consists of the UK nominal interest rate, published by the Bank of England, minus the UK inflation rate provided by the UK Office for National Statistics (ONS). As in the case of commodity prices, it is reassuring that our results, presented in Table 9, are similar in both cases. Below, we will compare changes in these variables with a change in $r$ in our model.

The estimates for the semi-elasticity of the EMBI spread to world interest rates are both significant. Naturally, as a country must pay more in interest payments, the risk of default
Table 9: Estimated response to changes in world interest rates

<table>
<thead>
<tr>
<th>Variables</th>
<th>EMBI</th>
<th>Inflation</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.008</td>
<td>0.063</td>
<td>0.003</td>
</tr>
<tr>
<td>Treasury rate</td>
<td>(0.002)</td>
<td>(0.031)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>UK real rate</td>
<td>0.011</td>
<td>0.066</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.144)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Observations</td>
<td>89</td>
<td>52</td>
<td>173</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.013</td>
<td>0.0042</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0059</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.028</td>
</tr>
</tbody>
</table>

Note: We use data from 1970 to 2018 for Argentina, Brazil, Colombia, and Mexico. The series for EMBI starts in 1997. Standard errors clustered by country are shown in parentheses. The regression for inflation is a quantile regression with country dummies to deal with periods of hyperinflation in Argentina and Brazil.

captured by the EMBI spread increases. The estimated coefficients imply that when the world interest rate increases by 50%, the EMBI spread increases by 40-55 basis points. For example, this means that when the US rates increase from 2% to 3% annually, the EMBI spread for Colombia goes from 300 to 340-355 basis points.

The results for inflation are very similar for both measures of the world interest rate. However, they are significant only for the Treasury rate.

16 The estimated coefficient implies that when the world interest rate increases by 50%, inflation increases by 3.2 percentage points. This effect is significant but notice that the standard error size also indicates that there is less precision in its estimation.

Finally, the coefficient for growth is not significantly different from zero for the Treasury rate and is significant for the UK rate. This estimate implies that a 50% increase in the world interest rate induces a reduction in growth of 0.4 percentage points. For example, this coefficient means that as the US rate increases from 2% to 3% annually, growth in Argentina declines from 3% to 2.6%.

Now, we estimate the counterpart of these numbers in the model. The risk-free rate, \( r \), gives the world interest rate. We follow the same strategy as in the previous section and evaluate a one-time, unanticipated change in \( r \). For the size of the shock, we use the mean log deviation of the UK rate from its long-run mean, which is 70%.17 We also consider a smaller shock, 50%, and a larger one, 85%. Estimating an autoregressive process for \( r \), we found that the probability of this shock persisting in the next period is 0.9255. In the model, an increase in \( r \) raises the cost of debt and leads to higher distortions and lower output. Table 10 shows these results.

As the risk-free rate increases, the model implies that spreads and inflation increase, while

---

16 As in the previous section, we use quantile regression to avoid estimates that are profoundly affected by periods of hyperinflation in Argentina and Brazil.

17 Using the Treasury rate yields similar results.
Table 10: Endogenous response of the economy to a change in $r$

<table>
<thead>
<tr>
<th></th>
<th>Previous period, $t-1$</th>
<th>Shock 50%</th>
<th>Shock 70%</th>
<th>Shock 85%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.030</td>
<td>0.050</td>
<td>0.061</td>
<td>0.070</td>
</tr>
<tr>
<td>Spread</td>
<td>0.010</td>
<td>0.019</td>
<td>0.029</td>
<td>0.039</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.063</td>
<td>0.082</td>
<td>0.039</td>
<td>0.094</td>
</tr>
<tr>
<td>Growth</td>
<td>0.000</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
</tbody>
</table>

Note: The size of the shock is measured in log deviations from the steady state value.

The semi-elasticities in the model vary from 0.019 to 0.034, which means the model overestimates the reaction of spreads to $r$. However, the difference is not very large. The estimated semi-elasticity using the UK rate is 0.011, with a standard error of 0.002. In the model, with a 50% shock to $r$, the semi-elasticity is within two standard errors of the data estimate.

The semi-elasticities of inflation to a shock in $r$ vary from 0.039 to 0.048, depending on the size of the shock. In the data, the same elasticity is a bit higher, 0.063. Still, the data’s standard errors are large, implying that the response in the model is within the confidence interval of the data estimates.

Finally, output decreases as $r$ increases since distortions need to increase to pay for the higher cost of financing. In the data, one estimate is not significantly different from zero, and the other yields a semi-elasticity of -0.008 with a standard error of 0.003. In the model, the semi-elasticity is $-0.001$, independently of the size of the shock. Although the model underestimates the effects, it is reassuring that they go in the same direction as in the data.

As we did in the case of $p^T$, we also computed these semi-elasticities for the recalibration with $\sigma^N = \sigma^T = 1.5$. The semi-elasticities for spreads are about one-fourth smaller. In the case of inflation, they are about 10 percent larger. For economic growth, the semi-elasticities have a positive sign between 0.007 and 0.009, depending on the shock size. This result suggests that the benchmark calibration with $\sigma^N = \sigma^T = 0.5$ should be preferred to $\sigma^T = \sigma^N = 1.5$, given its ability to replicate the effect of world interest rates on growth.

### F The choice of $\rho$ and $\sigma$

This section discusses the choice of $\sigma^N = \sigma^T = 0.5$ and $\rho = 1.5$ by comparing the results for alternative parameters. In particular, we consider $\sigma^N = \sigma^T = 1.5$ and $\rho = 2$. Recall that we set
\( \sigma_N = \sigma_T = 0.5 \) because \( \sigma_T < 1 \) is sufficient for the non-negativity constraint in the government’s problem to be satisfied with strict inequality (it is also necessary when transfers \( \gamma \) are zero). The value of \( \rho \) determines the elasticity of substitution between \( y^N \) and \( y^T \) in the cost function and is set to 1.5. A number larger than one ensures that the production possibilities frontier is concave.

First, we repeat the analysis of the semi-elasticities concerning changes in the price of exports for the alternative values of \( \sigma_N, \sigma_T, \) and \( \rho \). Table 11 shows that the semi-elasticity for spreads is similar but slightly smaller for the three sizes of the shock considered. In contrast, the semi-elasticity for inflation and growth is larger than in the case of \( \sigma_N = \sigma_T = 0.5 \). The larger difference is in the effect on growth because the sign of the effect is different. For example, for large shocks, it is 0.052 in the case \( \sigma_N = \sigma_T = 0.5 \), 0.022-0.033 in the data, and -0.06 for \( \sigma_N = \sigma_T = 1.5 \). This result strongly suggests that our choice in the benchmark calibration is the appropriate one.

Table 11: Endogenous response to a change in \( p^T \): \( \sigma = 1.5 \)

<table>
<thead>
<tr>
<th>Period, ( t )</th>
<th>Previous</th>
<th>Shock -4.0%</th>
<th>Shock -6.3%</th>
<th>Shock -8.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^T )</td>
<td>1.000</td>
<td>0.961</td>
<td>0.939</td>
<td>0.923</td>
</tr>
<tr>
<td>Spread</td>
<td>0.010</td>
<td>0.014</td>
<td>-0.105</td>
<td>-0.134</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.063</td>
<td>0.0746</td>
<td>-0.286</td>
<td>0.083</td>
</tr>
<tr>
<td>Growth</td>
<td>0.000</td>
<td>0.002</td>
<td>-0.053</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Note: The size of the shock is measured in log deviations from the steady state value.

A similar comparison can be made for shocks to the world interest rate. Table 12 presents the results. Again, the effect on spreads and inflation have the same sign and similar magnitude, and the impact on growth has the opposite sign. The findings reinforce the idea that our choice of \( \sigma_N = \sigma_T = 0.5 \) is appropriate.

Table 12: Endogenous response of the economy to a change in \( r \): \( \sigma = 1.5 \)

<table>
<thead>
<tr>
<th>Period, ( t )</th>
<th>Previous</th>
<th>Shock 50%</th>
<th>Shock 70%</th>
<th>Shock 85%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>0.030</td>
<td>0.050</td>
<td>0.061</td>
<td>0.070</td>
</tr>
<tr>
<td>Spread</td>
<td>0.010</td>
<td>0.022</td>
<td>0.024</td>
<td>0.035</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.063</td>
<td>0.081</td>
<td>0.035</td>
<td>0.092</td>
</tr>
<tr>
<td>Growth</td>
<td>0.000</td>
<td>0.003</td>
<td>0.006</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Note: The size of the shock is measured in log deviations from the steady state value.

In terms of the differences for \( \rho = 2 \), Table 13 shows semi-elasticities that are similar to those for \( \rho = 1.5 \). The larger difference is the effect on inflation, which is smaller in the case of \( \rho = 1.5 \). For example, for an intermediate size of the shock, the value of the semi-elasticity
is -0.173 for $\rho = 2$ and -0.276 for $\rho = 1.5$. The same semi-elasticity is slightly above -0.3 in the data, so our benchmark calibration with $\rho = 1.5$ is preferred in this dimension. However, the standard errors in the estimation are large to assign too much weight to this comparison.

Table 13: Endogenous response to a change in $p^T$: $\rho = 2$

<table>
<thead>
<tr>
<th></th>
<th>Previous period, $t - 1$</th>
<th>Shock -4.0%</th>
<th></th>
<th></th>
<th>Shock -6.3%</th>
<th></th>
<th></th>
<th>Shock -8.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^T$</td>
<td>1.000</td>
<td>0.961</td>
<td>$t$</td>
<td>0.939</td>
<td>$t$</td>
<td>0.923</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>0.010</td>
<td>0.014</td>
<td>-0.113</td>
<td>0.019</td>
<td>-0.155</td>
<td>0.026</td>
<td>-0.197</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.063</td>
<td>0.069</td>
<td>-0.155</td>
<td>0.074</td>
<td>-0.173</td>
<td>0.078</td>
<td>-0.187</td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>0.000</td>
<td>-0.002</td>
<td>0.050</td>
<td>-0.003</td>
<td>0.049</td>
<td>-0.004</td>
<td>0.048</td>
<td></td>
</tr>
</tbody>
</table>

Note: The size of the shock is measured in log deviations from the steady state value.

Table 14 present the semi-elasticities for shocks to the world interest rate with $\rho = 2$. While the effect on inflation is similar to the one with $\rho = 1.5$, the impacts on spreads and growth are quite different. The impact on the spread is much larger and counterfactual with $\rho = 2$. For example, for the largest shock, the semi-elasticity is 0.424 instead of 0.034. The number we estimated in the data is 0.008-0.011. Finally, the impact on growth has the opposite sign. These results are reassuring of our choice of $\rho$.

Table 14: Endogenous response of the economy to a change in $r$: $\rho = 2$

<table>
<thead>
<tr>
<th></th>
<th>Previous period, $t - 1$</th>
<th>Shock 50%</th>
<th></th>
<th></th>
<th>Shock 70%</th>
<th></th>
<th></th>
<th>Shock 85%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.030</td>
<td>0.050</td>
<td>$t$</td>
<td>0.061</td>
<td>$t$</td>
<td>0.070</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>0.010</td>
<td>0.053</td>
<td>0.087</td>
<td>0.158</td>
<td>0.212</td>
<td>0.370</td>
<td>0.424</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.063</td>
<td>0.081</td>
<td>0.037</td>
<td>0.094</td>
<td>0.044</td>
<td>0.105</td>
<td>0.049</td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
<td></td>
</tr>
</tbody>
</table>

Note: The size of the shock is measured in log deviations from the steady state value.

In Figure 6, we replicated the analysis in Figure 2 but including the benchmark calibration and also the results for these two alternative calibrations. Since we re-calibrated all the parameters, the initial steady state is the same in the three cases. The country risk premium effect is also the same because we re-calibrated $\lambda_2$ as well. The changes are all of the same sign and similar magnitude. The most notorious difference is in the reduction in the tax rate. In the case with $\rho = 2$, this reduction is larger by three percentage points, while in the case with $\sigma_N = \sigma_T = 1.5$, this reduction is smaller by about three percentage points. Another difference is that the changes in inflation, depreciation, and debt/GDP are slightly smaller in the benchmark than in these two alternative calibrations.
Figure 6: A Crisis in Emerging Markets

(a) Tax revenue / GDP  
(b) Government Expenditure / GDP

(c) Inflation  
(d) Currency Depreciation

(e) Debt / GDP  
(f) Country Risk Premium

Years after the shock