Counterterrorism Policy: Spillovers, Regime Stability, and Corner Solutions

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Counterterrorism Policy: Spillovers, Regime Stability, and Corner Solutions

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Abstract

This paper takes a unique approach to the scenario where a resident terrorist group in a (fragile) developing nation poses a terrorism threat at home and abroad. The host developing nation’s proactive countermeasures against the resident terrorist group not only limits terrorism at home and abroad, but also bolsters regime stability at home. A two-stage game is presented in which the developed country takes a leadership role to institute a tax-subsidy combination to discourage (encourage) proactive measures at home (abroad) in stage 1. Stage 2 involves both nations’ counterterrorism choices under alternative stage-1 public-policy packages. Unlike the extant literature, we explore corner and interior solutions in both stages based on the terrorists’ targeting preferences and the host nation’s regime-stability preferences. Surprisingly, the developed nation may profit from policy packages that reduce global counterterrorism while raising global terrorism. This outcome and others involve engineered counterterrorism burden shifting.

**Keywords:** counterterrorism policy, tax-subsidy redistributive schemes, externalities, terrorist-targeting preferences, publicness and corner solutions

**JEL Codes:** D74, H23, H41

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1. Introduction

Terrorism is the premeditated use or threat to use violence by individuals or subnational groups to obtain a political or social objective through the intimidation of a large audience beyond that of the immediate victim (Enders and Sandler 2012, 4). The intent of the terrorists is to generate enough audience cost through their terrorist campaign that the government is compelled to concede to some of the terrorists’ demands. To limit terrorism and its associated audience cost, governments deploy two forms of counterterrorism: defensive and proactive measures. Defensive or passive actions dissuade terrorists by limiting their anticipated net gains through target hardening, greater operating cost, enhanced border security, augmented first responses, or greater penalties. By contrast, proactive or offensive measures attack the terrorists, their resources, or their support base directly in order to reduce their capabilities, thus protecting all potential targets at home or abroad.

Since the landmark study of Landes (1978) on US skyjackings, social scientists apply theoretical and empirical tools to investigate counterterrorism tools and their effectiveness (e.g., Bandyopadhyay and Sandler 2011; Bier, Oliveros, and Samuelson 2007; Hausken, Bier, and Zhuang 2009; Jacobson and Kaplan 2007). Many studies, like the current one, employ game theory to capture the strategic interactions among terrorists, their host country, and a targeted country abroad (Bapat 2011; Kunreuther and Heal, 2003; Sandler and Lapan 1988; Schneider, Brück, and Meierriecks 2015). An important scenario that underlies the current analysis is where a terrorist group resides in a regime-fragile developing country while posing a terrorist risk to a foreign developed country (Bandyopadhyay, Sandler, and Younas 2011; Bapat 2011; George 2018). For example, Bapat (2011) investigates the moral-hazard problem associated with a
developed country providing military (proactive) aid to the host country to eliminate its resident terrorist threat. The moral hazard arises because the recipient country would lose its aid if it were to eliminate the terrorist threat as intended. Given the resident terrorist threat to the host and developed countries, either country’s counterterror efforts yield public good spillovers at home or abroad.

The main purpose of the current paper is to cast this two-country counterterrorism scenario in a different light in which the host developing country’s proactive measures against the resident terrorists promote global security while fostering its regime stability, so that joint products of varying degrees of publicness are derived. These complementary joint products change the way the host country responds to the developed country’s own proactive measures and its actions to subsidy counterterrorism abroad. Unlike the extant literature, our two-stage game allows the developed country to lead by deploying a tax-subsidy package in the first stage to curb either its own proactive measures or to stimulate the developing country’s proactive measures, or both in the ensuing second stage, where both countries simultaneously decide their counterterrorism responses. The leader developed country’s public-policy package allows it to profit from exploiting the developing country’s proactive response. We relate the profit opportunities to key elasticity and diminishing returns tied to promoting regime stability or counterterrorism. In contrast to the literature, we explore corner and interior solutions in both stages that hinge on the terrorists’ targeting preferences and the host nation’s regime-stability preferences. Some surprising results ensue including the case where the developed nation gains by engineering its tax-subsidy package to reduce global counterterrorism while augmenting global terrorism. This follows because the developing country’s leadership role allows it to gain

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1 With relatively small changes in the model, we can allow for defensive measures or both defensive and proactive measures – see Bandyopadhyay and Sandler (2011) and Sandler and Siqueira (2006).
from a reduced share of counterterrorism through clever burden-shifting policies. Despite the common terrorist threat, the targeted countries are not necessarily in the fight together owing to strategic considerations. We show that these and other asymmetries between the developed and host developing countries’ proactive responses depend on the strength of diminishing returns to counterterrorism or diminishing returns to fostering regime stability. A stronger preference for regime stability or greater efficiency in its counterterror production raises the host developing nation’s share of global proactive effort.

The remainder of the paper contains five sections. Section 2 contains the basic model, followed by comparative statics in Section 3 involving terrorists’ targeting preference and regime stability parameters. The two-stage policy-counterterrorism game is contained in Section 4 with its focus on interior and corner solutions in both stages. Section 5 presents generalizations to the basic models. Concluding remarks follow in the final section.

2. The Model

We envision a world with developed nation $A$ and developing nation $B$, both of which are targets of terrorist attacks originating from a terrorist group hosted in $B$. Accordingly, attacks in nations $A$ and $B$ are incidents of transnational and domestic terrorism, respectively. In recent years, this is a common scenario (Gaibulloev and Sandler 2019). An instance may be an authoritarian or a democratically elected regime in a developing nation that suffers from terrorism threats from a resident extremist organization with Western-based grievances. For example, al-Qaida in Afghanistan launched four terrorist skyjackings on American soil on September 11, 2001 (henceforth, 9/11). Another example is Morocco whose resident terrorists targeted Madrid’s

\[^2\text{For domestic terrorism, victims and perpetrators are from the venue country, while for transnational terrorism, at least one victim or perpetrator is not from the venue country (Enders, Sandler, and Gaibulloev 2011).}\]
commuter trains on March 11, 2004. Moreover, we assume that nation $B$ possesses weak political institutions and suffers from internal strife leading to regime instability, so that counterterrorism efforts not only address terrorism at home but also may bolster regime stability. Within this scenario, we initially set up a simultaneous-move Nash counterterrorism-choice game between the two countries in light of the common terrorist threat in $B$.

We denote the incidence of terrorism in nation $i$ as $T^i$ and assume that both nations consume a private numeraire good $x^i$. Nation $i$’s endowment is $\omega^i$ of the private good. $A$’s utility function is

$$U^A = U^A(x^A, T^A),$$
where $U^A_{x^A} > 0$ and $U^A_{T^A} < 0$,

so that utility increases with the private good and decreases with terrorism. In addition to terrorism, nation $B$ also suffers from regime instability, where $Z$ denotes $B$’s regime stability. Hence, $B$’s utility is:

$$U^B = U^B(x^B, T^B, Z),$$
where $U^B_{x^B} > 0$, $U^B_{T^B} < 0$, and $U^B_Z > 0$.

The global terrorism level equals $T = T^A + T^B$. $B$’s resident terrorist group assigns its terrorist attacks between the two targeted country as follows: $T^A = (1-\theta)T$ and $T^B = \theta T$, where $0 < \theta < 1$ indicates the group’s preference for attacking nation $B$ and thus nation $A$. Proactive or offensive measures by the two targeted countries against the terrorist organization are indicated by $g^A$ and $g^B$, respectively, in which the sum of global proactive efforts, $G = g^A + g^B$, reduces the terrorist organization’s total resources, thereby limiting global terrorism. In addition, we assume non-increasing marginal returns from counterterrorism effort, such that global terrorism is represented by

$$T = T(G), T'(G) < 0, T''(G) \geq 0 \iff \frac{d[T'(G)]}{dG} \leq 0.$$
Along with terror reduction, $g^B$ also provides greater internal security for nation $B$ by strengthening the existing regime. Instances include the current Afghan, Iraq, or Yemen governments. In Afghanistan, actions that weaken Islamic State in Iraq and Syria (ISIS) or the Taliban not only reduces terrorist attacks at home and abroad, but also fosters the legitimacy of the Afghan government. Since $g^B$ produces two goods – terrorism reduction and regime stability, we follow the joint-product model of Sandler and Hartley (2001) and cast $B$’s production of regime stability as:

$$Z = \alpha g^B \text{ with } \alpha > 0.$$  \hfill (4)

### 2.1 Unilateral Utility-Maximizing Choices of the Targeted Governments

Nation $A$’s budget constraint is $x^A + g^A = \omega^A$, with unit prices for the private good and counterterrorism. Using Eqs. (1) and (3) along with $T^A = (1 - \theta) T$, we get:

$$U^A = U^A \left[ \omega^A - g^A, (1 - \theta) T \left( g^A + g^B \right) \right].$$  \hfill (5)

Under the standard simultaneous-move Nash assumption, nation $A$ chooses $g^A$ to maximize $U^A$ taking $g^B$ as given. The first-order condition (FOC), allowing for a corner solution, is:\footnote{The reasonable assumption of increasing disutility from terrorism allows second-order conditions to be satisfied for both nations.}

$$U^A_{g^A} (\cdot) = -U^A_{x^A} (\cdot) + (1 - \theta) U^A_{T^A} (\cdot) T'' (G) \leq 0, \quad g^A U^A_{g^A} (\cdot) = 0.$$  \hfill (6a)

When $g^A > 0$, the first relationship in Eq. (6a) holds as an equality and implicitly defines the Nash reaction function of nation $A$ as

$$g^A = g^A \left( g^B \right),$$ \hfill (6b)

otherwise $g^A = 0$. Using Eqs. (2)-(4) and $T^B = \theta T$, we have:
\[ U^B = U^B \left[ \omega^B - g^B, \theta T \left( g^A + g^B \right), \alpha g^B \right]. \] (7)

The Nash FOC for B, allowing for a corner solution, is:
\[ U^B_{g^B} (\cdot) = -U^A_{x^A} (\cdot) + \theta U^B_{T^B} (\cdot) T'(G) + \alpha U^B_Z (\cdot) \leq 0, \quad g^B U^B_{g^B} (\cdot) = 0. \] (8a)

If \( g^B > 0 \), the first relationship in Eq. (8a) holds as an equality and again implicitly defines B’s Nash reaction function as
\[ g^B = g^B \left( g^A \right), \] (8b)
otherwise \( g^B = 0 \). Eqs. (6b) and (8b) jointly determine the respective Nash equilibrium level of proactive efforts of the two nations. To display a clearer picture of the Nash equilibrium and the effects of various parameters on this equilibrium, we turn to the case of quasi-linear and separable utility functions.

2.2 Quasi-Linear and Separable Utility Functions

To simplify the analysis, we assume that A’s utility function takes the form:
\[ U^A = U^A \left( x^A, T^A \right) \equiv x^A + u \left( T^A \right), \quad u(0) = 0, \] (9)

where \( u' \left( T^A \right) < 0 \) and \( u'' \left( T^A \right) < 0 \iff \frac{d \left| u' \left( T^A \right) \right|}{dT^A} > 0 \). The negative first derivative indicates that \( T^A \) is a bad, while the negative second derivative means that the marginal disutility of terrorism \( u' \left( T^A \right) \) is increasing in \( T^A \). Similarly, B’s utility function is:
\[ U^B = U^B \left( x^B, T^B, Z \right) \equiv x^B + u \left( T^B \right) + f \left( Z \right), \] (10)

where \( u \) has the same functional form as that for nation A, so that \( u(0) = 0 \), \( u' \left( T^B \right) < 0 \), and \( u'' \left( T^B \right) < 0 \). The function \( f \left( Z \right) \) represents B’s utility from regime stability, such that marginal
utility from enhanced regime stability is \( f'(Z) > 0 \) but with diminishing marginal utility so that \( f''(Z) < 0 \).

### 2.3 Reaction Functions and an Interior Nash Equilibrium

Based on Eqs. (5) and (9), we have \( U^A = \omega^A - g^A + u[(1-\theta)T(G)] \). At an interior solution, nation \( A \)'s Nash FOC for its choice of \( g^A \) is:

\[
\frac{\partial U^A}{\partial g^A} = -1 + (1-\theta)u'
\left[(1-\theta)T(g^A + g^B)\right]T'(G) = 0.
\]  

(11a)

The second-order condition (SOC) is \( U''_{g^A} = (1-\theta)\left[u''\left(T^A\right)T'^{''}(1-\theta) + (T'^{''})^2 u''\left(T^A\right)(1-\theta)\right] < 0 \), which holds given earlier assumptions. Because \( U^A_{g^A}(\cdot) \) is solely a function of the aggregate proactive provision \( G = g^A + g^B \), we have that \( U^A_{g^A} = U^A_{g^B} \). Accordingly, Eq. (11a) implicitly defines \( A \)'s Nash reaction function, \( g^A(g^B) \), with slope:

\[
\frac{dg^A}{dg^B} = -\frac{U^A_{g^A}g^B}{U^A_{g^B}g^A} = -1.
\]

(11b)

For some of the later analyses and drawing \( A \)'s reaction path, \( R^A \), we note that Eq. (11a) defines \( A \)'s desired amount of \( G \) at an interior solution as

\[
G = G^{\dagger^A}(\theta) = \frac{dG}{d\theta} = \frac{T'(G)\left[u'(T^A) + T^A u''(T^A)\right]}{(1-\theta)\left[u'(T^A)T'^{''}(G) + (1-\theta)(T'^{''})^2 u''(T^A)\right]} < 0
\]

(11c)

by the implicit function rule. At an interior Nash equilibrium for nation \( A \), Eq. (11c) shows that aggregate counterterrorism provision (and, hence, global terrorism) is determined entirely by the

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4 Throughout the paper, detailed proofs of complex expressions are displayed in the online appendix.
terrorists’ targeting preference, $\theta$, regardless of shifts in nation $B$’s reaction function.

$B$’s Nash reaction function accounts for joint products owing to regime stability concerns. Using Eqs. (7) and (10), we have $U^B = \omega^B - g^B + u[\theta T(G)] + f(\alpha g^B)$. At an interior optimum, $B$’s FOC for its choice of $g^B$ is:

$$\frac{\partial U^B}{\partial g^B} = -1 + \theta u'[\theta T(G)]T'(G) + \alpha f'(\alpha g^B) = 0,$$  \hspace{1cm} (12)

with SOC, $U^B_{g^B} = \theta[u'(T^B)T^*(G) + (T')^2 u''(T^B)\theta] + \alpha^2 f''(\alpha g^B) < 0$. Eq. (12) implicitly defines $B$’s Nash reaction function, $g^B = g^B(g^A)$, with slope,

$$\frac{dg^B}{dg^A} = -\frac{U^B_{g^B}}{U^A_{g^A}} = -\frac{\theta[u'(T^B)T^*(G) + (T')^2 u''(T^B)\theta]}{\theta[u'(T^B)T^*(G) + (T')^2 u''(T^B)\theta] + \alpha^2 f''(\alpha g^B)} < 0,$$  \hspace{1cm} (13)

where $\left|\frac{dg^B}{dg^A}\right| < 1$, given the second negative term in the denominator. Thus, $B$’s reaction path is downward sloping and flatter at any point in $(g^A, g^B)$ space than $A$’s reaction path whose slope is $-1$. This observation rules out multiple interior Nash equilibria and assures the stability of the Nash equilibrium.

[Figure 1 near here]

Figure 1 presents an interior Nash equilibrium, $N$, with $A$’s and $B$’s levels of proactive efforts equal to $(g^A_N, g^B_N)$, where reaction paths $R^A$ and $R^B$ intersect. The vertical (horizontal) axis measure $B$’s ($A$’s) counterterrorism provision. Given the negative 1 slope of $A$’s Nash reaction path, this path makes a 45-degree angle with the horizontal axis, where $G^N(\theta)$ denotes the total Nash counterterrorism amount for the terrorist group’s target preferences. Figure 1 underscores the public nature of proactive measures despite the asymmetries between the two
countries arising from B’s counterterrorism-derived regime stability.

3. Comparative Statics

3.1 Changes in Relative Targeting of the Two Nations by the Terrorists

Exogenous factors – e.g., post-9/11 security enhancements or political developments – can cause a resident terrorist group to redistribute its attacks between targets at home and abroad as captured by changes in \( \theta \). We begin by analysing the comparative statics of \( \theta \) at an interior equilibrium before turning to the analysis at a corner solution. Eqs. (11a) through (11c) implies:

\[
\left( \frac{\partial g^A}{\partial \theta} \right)_{k^g} = G^{dA} (\theta) < 0 ,
\]

(14)

so that a rise in \( \theta \), indicative of terrorists aiming more attacks against host nation B, shifts developed nation A’s reaction path closer to the origin in Figure 2. Based on B’s FOC in Eq. (12), we get:

\[
\left( \frac{dg^B}{d\theta} \right)_{k^g} = -\frac{T'(G)[u'(T^B)+T^B u''(T^B)]}{U_{g^B g^B}^B} > 0 ,
\]

(15)

which shows that a rise in \( \theta \) shifts nation B’s reaction path upward.

[Figure 2 near here]

Figure 2 displays the comparative statics of an increase in \( \theta \) in terms of the shifts of the two reaction paths, where the new equilibrium, \( N^2 \), must lie northwest of the initial equilibrium, \( N^1 \), implying a decrease in the Nash equilibrium level of \( g^A \) and an increase in that of \( g^B \) as displayed. Given Eq. (11c), we know that the aggregate level of counterterrorism, \( G = G^{dA} (\theta) \), must fall from \( G^1 \) to \( G^2 \) in Figure 2 as \( \theta \) rises. Even though B contributes more
counterterrorism as terrorists direct a greater share of attacks to B, A’s proactive effort falls sufficiently to reduce aggregate counterterrorism, resulting in increased global terrorism.

**Proposition 1**

For an interior counterterrorism equilibrium, an enhanced preference by terrorists for attacking the host developing nation results in global counterterrorism efforts falling and global terrorism rising. Terrorism must rise in the source developing nation; however, terrorism in the developed nation only falls if there are sufficiently strong diminishing returns to counterterrorism.

**Proof:**

Eqs. (14)-(15) indicate how a rise in $\theta$ must reduce global counterterrorism efforts $G$ and raise global terrorism $T(G)$, which immediately implies that B’s terrorism $(T^B = \theta T)$ must increase.

Next, consider how A’s terrorism, $T^A = (1-\theta)T(G)$, responds to larger $\theta$:

$$\frac{dT^A}{d\theta} = -T(G) + (1-\theta)T'(G)G^{dA'}(\theta).$$

(16a)

Substituting Eq. (11c) into Eq. (16a), we derive:

$$\frac{dT^A}{d\theta} = \frac{u'(T^A)(T''(G)^2(1-\varepsilon_G))}{u'(T^A)T''(G) + (1-\theta)(T'G)^2u'(T^A)} < 0,$$

iff $\varepsilon_G = \frac{d \ln[T'(G)]}{d \ln[T(G)]} = \frac{TT'^*}{(T')}^2 > 1$,

(16b)

where $\varepsilon_G \geq 0$ is an elasticity that measures the percentage rate of increase of the marginal productivity of counterterrorism in response to a percentage increase in global terrorism (due to a lowering of $G$). When diminishing returns are strong, lower levels of $G$ are associated with high returns to counterterrorism and $\varepsilon_G$ is large. If these diminishing returns are sufficiently
large, then the inequality in Eq. (16b) is satisfied and $A$’s terrorism must fall with an increase in $\theta$. This inequality cannot be satisfied when there is constant marginal return to counterterrorism, so that $T$ is linear in $G$ and $T^* = 0 = \varepsilon_G < 1$. Thus, diminishing marginal returns to counterterrorism are necessary for $A$’s terrorism to fall as $\theta$ rises. Q.E.D.

When the developing source nation is targeted more intensively by the terrorist organization, terrorism there must increase both because of the enhanced targeting effect and reduced global counterterrorism efforts. However, for the developed nation, the reduced targeting effect opposes the enhanced terrorism arising from reduced global counterterrorism. The former effect dominates if there are enough diminishing returns to these efforts. With strong diminishing returns, a lowering of $G$ sharply raises marginal terrorism reduction $|T'(G)|$ from counterterrorism, which dampens nation $A$’s incentive to significantly cut back on its proactive provision (i.e., $A$’s reaction path’s leftward shift is relatively small). Accordingly, for a relatively small $|G^{d_d} (\theta)|$, the targeting effect captured by the first term on the right-hand side of Eq. (16a) dominates. This whole enhanced $\theta$ scenario is relevant after 9/11 and the greater difficulty that terrorists found in traveling abroad due to bolstered border security (Gaibulloev, Sandler, and Santifort 2012; Gaibulloev and Sandler 2019; Schneider, Brück, and Meierrieks 2015). Therefore, there ensued more domestic terrorist attacks by resident terrorist groups in developing countries, and a decline in transnational attacks on developed countries’ soil (Enders and Sandler 2006, 2012; Hou, Gaibulloev, and Sandler 2020). Host countries’ examples include Iraq and Afghanistan, both of which have experienced lots of recent domestic terrorism.

3.2 Relative Targeting and Corner Solutions
Consider $\theta = \theta^N$ for an interior Nash equilibrium $\left(g_A^{4N} > 0, g_B^{BN} > 0\right)$. If $\theta$ is reduced just a little below $\theta^N$, nation $A$’s reaction path shifts rightward and nation $B$’s reaction path shifts down as the terrorists reapportion more terrorism to $A$. The Nash equilibrium moves southeast of the initial point in $\left(g_A^4, g_B^B\right)$ space. With continual reductions of $\theta$, assuming a corner solution exists (where $g_B^B = 0$), the Nash equilibrium eventually moves to a corner at $\theta = \theta^1$ depicted as point $N^1$ in Figure 3.\textsuperscript{5} At this point, the FOC of nation $B$ is just met, such that $g_B^B = 0$ is $B$’s best response to $g_A^A = G^{dA}(\theta^1)$.

[Figure 3 near here]

In Figure 3, the following equation implicitly defines the critical corner-producing share, $\theta^1$:

\[
\left(\frac{\partial U_B}{\partial g_B^B}\right)_{g^B = 0} = -1 + \theta u' \left[\theta T[G^{dA}(\theta)]\right] T'[G^{dA}(\theta)] + \alpha f''(0) = 0 \Rightarrow \theta^1 = \theta^1(\alpha). \tag{17}
\]

A reduction of $\theta$ below $\theta^1$ shifts $B$’s reaction path further inward and $A$’s reaction path further outward, such that the new Nash equilibrium is now a corner solution at $N^{10}$ in Figure 3 for which $U_A^{4A} = 0$, $g_A^A = G^{dA}(\theta^0) > 0$, $U_B^{g_B} < 0$, and $g_B^B = 0$. If, similarly, starting from $\theta = \theta^N$ we keep raising the terrorists’ preference $\theta$ for attacking the developing nation, then the Nash equilibrium keeps moving northwest in $\left(g_A^4, g_B^B\right)$ space. If a corner solution for nation $A$ is possible, then we eventually reach $\theta = \theta^2$ where $A$’s FOC is just met at $g_A^A = 0$ and

\[G = g_B^B = G^{dA}(\theta).\]

Substituting this outcome into $B$’s FOC gives:

\[\theta^1(\alpha).
\]

\textsuperscript{5} In our context, a corner solution may be ruled out for nation $B$ if $f'(Z) \to \infty$ as $Z \to 0$. An example of such a functional form is $f(Z) = Z^\mu$, $0 < \mu < 1$, which is reasonable when regime stability assumes paramount importance for more fragile regimes. Recent experiences of the Arab Spring or the Syrian Civil War highlight the importance of considering such government objective functions in terror-afflicted nations.
\[
\left( \frac{\partial U^B}{\partial g^B} \right)_{g^t=0, g^A=\alpha G^{dA}(\theta)} = -1 + \theta u'\left( \theta T\left[ G^{dA}(\theta) \right] \right) T'\left[ G^{dA}(\theta) \right] + \alpha f'\left[ \alpha G^{dA}(\theta) \right] = 0,
\]

\[
\Rightarrow \theta^c_2 = \theta^2(\alpha).
\]

(18)

Analogous to the discussion above, raising \( \theta \) above \( \theta^c_2 \) moves the Nash equilibrium to a corner where \( U^A_g < 0, g^A = 0 \) (not displayed in Figure 3), where the aggregate \( G \) is no longer tied down by \( A \)’s FOC, so that \( G = g^B > G^{dA}(\theta) \). Thus, if corner solutions on both axes are possible, an interior equilibrium with proactive measures in both nations obtains for \( \theta^c_1 < \theta < \theta^c_2 \). When, however, \( \theta \leq \theta^c_1 \) or \( \theta \geq \theta^c_2 \), only nation \( A \) or \( B \) contributes to proactive measures in the Nash equilibrium. In the latter case, the Nash equilibrium is defined by \( B \)'s reaction function \( G = g^B \left( g^A = 0 \right) \).

Given the preceding discussion, \( \theta < \theta^c_2 \), \( G = G^{dA}(\theta) \), and \( G^{dA}(\theta) < 0 \) reflect a decline in global counterterrorism effort in response to increased targeting of \( B \). Global terrorism, \( T \), and terrorism in developing nation \( B \), \( T^B = \theta T \), must increase due to a larger \( \theta \); but there are conflicting targeting and counterterrorism effects on \( T^d \). Eqs. (16a) and (16b) apply with \( A \)'s terrorism falling if \( \varepsilon_g > 1 \). However, for \( \theta > \theta^c_2 \), nation \( A \) free rides on nation \( B \) and Eqs. (16a) and (16b) no longer apply. The effects of changes in \( \theta \) for \( \theta > \theta^c_2 \) are summarized in Proposition 2 below.

**Proposition 2**

Global terrorism and \( A \)'s terrorism must fall with a rise in \( \theta \) for \( \theta > \theta^c_2 \), while source nation \( B \)'s terrorism increases for sufficiently strong diminishing returns such that \( \varepsilon_g \geq 1 \). For \( \varepsilon_g < 1 \), \( B \)'s terrorism is more apt to increase with a rise in \( \theta \) if (a) regime stability is relatively efficiently
produced compared to counterterrorism; (b) there are strong diminishing returns to both regime stability production and counterterrorism; and (c) $B$’s marginal disutility of terrorism is relatively large.

Proof:

For $\theta > \theta^{-2}$, nation $B$ is the sole provider of counterterrorism and Eq. (12) defines the equilibrium level of $G(=g^B)$. In turn, Eq. (15) defines the change in global counterterrorism effort with respect to $\theta$. A rise in $\theta$ raises $G$ and reduces global terrorism $T$ along with terrorism in the developed nation $T^A=(1-\theta)T$. However, $T^B=\theta T$ may rise or fall depending on the strengths of increased targeting and global terrorism reduction effects. Using Eq. (15) in the online appendix, we get:

$$
\left(\frac{dT^B}{d\theta}\right)_{\theta^*=0} = T + \theta T'(G)\left(\frac{dg^B}{d\theta}\right)_{\theta^*=0} > 0 \text{ if and only if } \left(\frac{\alpha}{|T|}\right)^2 |f^*(\alpha g^B)| > \theta(1-\varepsilon_G)\frac{u'(T^B)}{T(G)} , \text{ where } G = g^B. 
$$

(19)

If $\varepsilon_G \geq 1$, the right-hand side of the last inequality in Eq. (19) is nonpositive while the left-hand side is strictly positive. Thus, $\frac{dT^B}{d\theta} > 0$ when $\varepsilon_G \geq 1$. For $\varepsilon_G < 1$, $\frac{dT^B}{d\theta} > 0$ when, ceteris paribus, the following are true: (a) $\alpha/|T|$ is large, suggesting that $B$’s regime stability effort is relatively more productive than that of its counterterrorism; (b) $|f^*(\alpha g^B)|$ and $\varepsilon_G$ are both large suggesting strong diminishing returns to both regime stability production and counterterrorism; and (c) $B$’s marginal disutility of terrorism $|u'(T^B)|$ per unit of $T^B$ is relatively small. Q.E.D.
Since the United States secured its borders following 9/11, there are relatively few transnational terrorist attacks on US soil with most attacks staying in a host developing country (e.g., Afghanistan) as domestic terrorism (Gaibulloev, Sandler, and Santifort 2012). Thus, the corner solutions involving $\theta > \theta^2$ assumes a heightened relevance. At such corners, developing nation B’s proactive choices are all that matter. The increase in global counterterrorism efforts when $\theta > \theta^2$ is reminiscent of Bergstrom, Blume, and Varian’s (1986) Theorem 4 for which public good provision increases beyond a corner solution involving only a subset of contributors, which in the current case is a single targeted host country. Proactive countermeasures possess publicness properties (nonexcludable and nonrival benefits) by weakening a common terrorist threat. Reliance on the host country to rid itself of its resident terrorists can have a real payback for the developed nation.

3.3 Changes in Regime Stability Parameters

We now allow B’s utility from regime stability to take the form mentioned in footnote 5, namely $f(Z) = Z^\mu$ with $0 < \mu < 1$. The marginal utility from regime stability is $f'(Z) = \mu Z^{\mu-1}$. Larger values of $\mu$ indicate greater marginal utility for the same $Z$, which suggests that we can use $\mu$ as a measure of nation B’s preference for regime stability. Another relevant regime stability parameter is $\alpha$, which, as described in Eq. (4), is the efficiency with which regime stability is produced through B’s counterterrorism effort.

Proposition 3

At an interior counterterrorism equilibrium, stronger preference ($\mu$) for regime stability or greater efficiency in the production of regime stability ($\alpha$) raises the developing nation’s share
of global counterterrorism effort without changing global or national terrorism levels. If nation
B is the only counterterrorism provider, a rise in either $\alpha$ or $\mu$ raises global effort and reduces
terrorism in each nation.

Proof:

Substituting $f(Z) = Z^\mu$ in Eq. (12) gives:

$$\left( \frac{dg^B}{d\mu} \right)^{\scriptscriptstyle(k)'} = -\frac{\left( g^B \right)^{\scriptscriptstyle(u-1)} \alpha^\mu \left[ 1 + \mu \ln \left( \alpha g^B \right) \right]}{\theta \left[ u'(T^B)T''(G) + (T')^2 u''(T^B)\theta \right] + \alpha^2 f''(\alpha g^B)} > 0,$$

(20)

establishing that an increased marginal utility of regime stability shifts out $B$’s reaction path.

Based on Eqs. (11a) through (11c), we note that regime stability has no effect on $A$’s reaction
function. Thus, at an interior Nash equilibrium, aggregate counterterrorism $G = G^{\scriptscriptstyle(dd)}(\theta)$ is
unaffected by an increase in $\mu$. Accordingly, terrorism levels in each nation are unaffected but
developing nation $B$’s share of counterterrorism expense, $g^B / G$, rises. At a corner solution,
where $g^A = 0$ and $G = g^B$, an increase in $\mu$ raises aggregate counterterrorism and reduces
terrorism in both nations. The analysis for a change in $\alpha$ is qualitatively similar and therefore
not presented. Q.E.D.

Regime stability concerns create a nation-specific incentive for the host developing
nation $B$ to raise its own counterterrorism $g^B$. However, that increase does not affect overall $G$, because $B$’s counterterrorism provision limits developed nation $A$’s incentive to provide $g^A$.

Along $A$’s reaction path, the reduction in $g^A$ precisely matches the increase in $g^B$, thus keeping
global provision $G$ unchanged. For an interior counterterrorism Nash equilibrium, greater
care concern about regime stability in $B$ merely redistributes proactive measures between the two
targeted countries. This is reminiscent of the neutrality theorem for pure public goods (Warr 1982), which is often not relevant when there are joint products (Cornes and Sandler 1984, 1994, 1996). For our model, joint products do not undo neutrality given the special nature of $A$’s utility function with its absence of income effects, which imparts the $-1$ slope to $A$’s reaction path. However, when $g^A = 0$, any rise of $\alpha$ or $\mu$ from this corner solution raises overall counterterrorism $g^B = G$, thereby reducing terrorism in both nations. This shows that regime stability concerns in the country hosting the terrorist group can ameliorate terrorism when there is a corner solution, which is more relevant in a post-9/11 world with a hosting state having to assume much of the responsibility for its domestic terrorism threats. Regime stability concerns in a host country is of current concern as terrorists take up residency in fragile states to operate with impunity (George 2018).

4. Policy Options for the Developed Nation

We now consider strategic policies that the developed nation $A$ can institute to move the counterterrorism equilibrium in its favor. For example, a first-mover central authority in $A$ (e.g., the legislative branch) chooses tax/subsidy policies to support counterterrorism efforts chosen by the two nations’ counterterrorism authorities (e.g., defense departments). Formally, we present a 2-stage game, where proactive levels $g^A$ and $g^B$ are chosen simultaneously by the two nations in stage 2. In stage 1, $A$’s central authority chooses a tax $t$ (or subsidy if $t < 0$) per unit of $g^A$ and a subsidy $s \geq 0$ per unit of $g^B$ for nation $B$. In stage 1, $A$ also chooses a lump-sum tax $L$ to balance its budget, such that $L = sg^B - tg^A$.

We solve the two-stage game by backward induction by analyzing stage 2 first where the

---

6 We assume that $s \geq 0$ because a sovereign nation $B$ cannot be forced to accept a tax on its counterterror choice.
objective function of nation \( A \)'s counterterrorism authority is:

\[
U^A \left( g^A, g^B, t \right) = \omega^A - (1 + t)g^A + u\left[ (1 - \theta)T(G) \right] - L. \tag{21}
\]

Because \((t, s, L)\) are given in stage 2, the simultaneous-move Nash FOC for maximizing \( U^A \) with respect to \( g^A \) is: \(^7\)

\[
\frac{\partial U^A}{\partial g^A} = -\left(1 + t\right) + (1 - \theta)u'\left[ (1 - \theta)T(G) \right]T'(G) \leq 0, \quad g^A \frac{\partial U^A}{\partial g^A} = 0. \tag{22a}
\]

When \( g^A > 0 \), the first relationship in Eq. (22a) holds as an equality and implicitly defines

\[
g^A = g^A \left( g^B, t \right), \tag{22b}
\]

where, as in Eq. (11b), the slope of \( A \)'s reaction path is \(-1\). Analogous to Eq. (11c), \( A \)'s desired level of \( G \) for a given \( t \) is:

\[
G = G^{dA} (t); \quad G^{dA} (t) = \frac{1}{\left(1 - \theta\right)u'(T^*)T'(G) + (1 - \theta)(T)^2 u''(T^*)} < 0. \tag{22c}
\]

Given its subsidy, nation \( B \)'s objective function is:

\[
U^B = \omega^B + (s - 1)g^B + u\left[ \theta T(G) \right] + f(\alpha g^B) \tag{23}
\]

with the following FOC for its choice of \( g^B \):

\[
\frac{\partial U^B}{\partial g^B} = s - 1 + \theta u'\left[ \theta T(G) \right]T'(G) + \alpha f'(\alpha g^B) \leq 0, \quad g^B \frac{\partial U^B}{\partial g^B} = 0. \tag{24a}
\]

For an interior solution, the first part of Eq. (24a) is an equality and implicitly defines

\[
g^B = g^B \left( g^A, s \right). \tag{24b}
\]

\(^7\) The structure of this stage-2 game is identical to that described in Section 2.3 except for the presence of the policy parameters \((s, t)\). The second own and cross partials involving only \((g^A, g^B)\) do not feature \((s, t)\), so that their expressions are identical to those in Section 2.3. This implies that all the relevant second-order and stability conditions are satisfied.
4.1 Interior Counterterrorism Equilibrium \((g^A > 0, g^B > 0)\)

This subsection investigates an interior stage-2 equilibrium but allows for a corner first-stage solution for the tax-subsidy choice. Section 4.2 considers corner solutions for stage 2 and discusses appropriate policy in that context. Eqs. (22b) and (24b) jointly determine the interior stage-2 equilibrium as \(\left[ g^A (t, s), g^B (t, s) \right] \). By differentiating Eqs. (22a) and (24a) and applying Cramer’s rule, we have:

\[
g^A_t (t, s) = \frac{U^B_{g^A g^B}}{D} < 0, \tag{25a}
\]

where \(D = U^A_{g^A g^B} - U^B_{g^A g^B} > 0\). The latter ensures stability of the Nash equilibrium, while \(U^B_{g^A g^B} < 0\) satisfies the SOC for \(B\)’s proactive choice. Similarly, we have:

\[
g^A_s (t, s) = \frac{U^A_{g^A g^B}}{D} < 0, \tag{25b}
\]

where \(U^A_{g^A g^B} = (1 - \theta)[u'(T^A)T^* + (1 - \theta)(T')^2 u''(T^A)] = U^A_{g^A g^B} < 0\). Moreover, the following holds:

\[
g^B_t (t, s) = -\frac{U^B_{g^A g^B}}{D} > 0, \quad U^B_{g^A g^B} = \theta \left[ u'(T^B)T''(G) + (T')^2 u''(T^B) \right] < 0, \tag{25c}
\]

and

\[
g^B_s (t, s) = -\frac{U^A_{g^A g^B}}{D} = -g^A_s (t, s) > 0. \tag{25d}
\]

Given the balanced-budget constraint \(L = sg^B - tg^A\) in Eq. (21), \(A\)’s first-stage objective function becomes:

\[
U^A (t, s) = \omega^A - g^A (t, s) + u \left( (1 - \theta)T \left[ G^{dd}(t) \right] \right) - sg^B (t, s). \tag{26}
\]
Because stage-2 equilibrium \( G = G^{dd} (t) \) is independent of \( s \), we have \( G^{dd}(t) = g_i^A + g_i^B \).

Moreover, Eq. (22a) implies that \((1-\theta)u'(T^A)T' = 1 + t\) where \( t \) can be negative. Using these findings, we see that Eq. (26) yields optimal stage-1 policy choices as:

\[
U_i^A(t,s) = -g_i^A + (1-\theta)u'(T^A)T'G^{dd}(t) - sg_i^B = tg_i^A + g_i^B (1 + t - s) = 0 ,
\]

(27a)

and

\[
U_i^B(t,s) = -g_i^B - sg_i^B - g_i^B = 0 \leq 0 , \ sU_i^A = 0 .
\]

(27b)

**Proposition 4**

At an interior counterterrorism Nash equilibrium, there is a strictly positive tax on the developed nation’s counterterrorism effort, which may be complemented by a subsidy for the developing nation’s counterterrorism effort. Global counterterrorism is lower and national terrorism levels are higher at this tax-subsidy equilibrium compared to the equilibrium with no taxes and subsidies.

**Proof:**

First consider an interior optimum where both nations’ counterterrorism levels are positive.

For notational convenience, let we denote the slope of \( B \)’s reaction function in Eq. (24b) as

\[
\rho^B = \frac{\partial g_i^B (g_i^A, s)}{\partial g_i^A} .
\]

This slope has the same functional form as previously outlined in Eq. (13) with \( \rho^B < 0 \) and \( \rho^B + 1 > 0 \). For stage 1, we first consider an interior policy equilibrium \((t > 0,s > 0)\), such that the FOCs in Eqs. (27a) and (27b) hold as equalities. In this case, we can derive the optimal policy combination \((t^*,s^*)\) as:
\[ t^* = -\frac{g^B \rho^B}{(1+\rho^B)g^B_s(t,s)} > 0, \quad (28a) \]

and

\[ s^* = 1 - \frac{g^B}{g^B_s(t,s)}, \text{ where } 0 < s^* < 1. \quad (28b) \]

Since \( t \) can be either positive or negative, we do not need to consider a corner solution for \( t \) provided an optimal \( t \) exists. If, however, \( U_s^A < 0 \) at the policy optimum described in Eqs. (27a) and (27b), then the optimum policy combination is:

\[ t^* = -\frac{\rho^B}{1+\rho^B} \geq 0, \text{ and } s^* = 0. \quad (29) \]

At an interior stage-2 equilibrium, Eq. (22c) indicates that global counterterrorism provision depends only on the tax \( t \) for which it declines monotonically. Thus, with a positive tax as described in Eq. (28a) or Eq. (29), \( G \) must be lower at the policy equilibrium than at \( t = 0 \). Thus, global and national terrorism levels are higher at the tax-subsidy equilibrium compared to the equilibrium that arises at \( (t = 0, s = 0) \). \textbf{Q.E.D.}

Consider the three terms on the right side of the first equation in Eq. (27a). The first term reflects the savings of \( A \)'s counterterrorism expenditure \( g^A \) owing to the tax, while the second term captures \( A \)'s utility loss due to increased terrorism (caused by reduced global counterterrorism). The last term indicates increased subsidy costs \( [i.e., \partial(s^B g^B)/\partial t] \) incurred by nation \( A \) at a given \( s \geq 0 \), because nation \( B \) reacts to a reduction of \( g^A \) by raising \( g^B \). Using stage-2 FOCs, we can summarize those costs and benefits in an easy-to-interpret form for the left-hand side of the last equation in Eq. (27a). The first term is \( tg^A_s \), which measures the
distortion loss arising from government intervention on $g^A$ through the tax. The second term captures the externality gains through the induced increase in $g^B$. At nonintervention $t = s = 0$, the distortion loss vanishes, but the potential for the externality gain remains because $U^A_t(t, s) = g^B_t > 0$. At the optimum, the distortion loss $t g^A_t$ has to be weighed against the externality gain $g^B_t(1 + t - s)$ to obtain the optimal tax $t^* > 0$ as indicated in Eq. (28a).

The positive tax arises out of $A$’s incentive to move the second-stage counterterrorism equilibrium to a point that resembles a Stackelberg leadership point in $(g^A, g^B)$ space so as to shift a greater counterterrorism burden onto the hosting nation (Sandler and Siqueira 2006). As a tax reduces $g^A$, $A$’s stage-2 reaction path shifts closer to the origin. The stage-2 equilibrium moves to a point on $B$’s reaction path where $g^B$ is higher as a response to a tax-lowered $g^A$. Along $B$’s reaction path, the increase in $g^B$ is less than in a one-to-one relationship to the reduction in $g^A$, so that global counterterrorism falls with terrorism increasing in both nations. As shown above, with the right combination of trade-offs, $A$ can benefit from its leadership.

We now focus on the effect of the subsidy. At a given $t$, the subsidy shifts out $B$’s reaction path but does not affect $A$’s reaction path. With the one-to-one reduction in $g^A$ for an increase in $g^B$ along $A$’s reaction path, $G$ and terrorism levels cannot change, leaving terrorism unchanged everywhere. Nevertheless, $A$ is incentivized to use the subsidy despite this matching substitution of $g^B$ for $g^A$ as long as the marginal cost of such subsidization is balanced by the savings from reduced provision of $g^A$. This is indeed the case when

$$\frac{d(s g^B)}{ds} = \frac{dg^A}{ds} \iff g^B + sg^B_s = g^A_s, \text{ which is the expression one gets from reorganizing the terms in Eq. (28b).}$$

If, however, the net gain is negative, then the optimal subsidy is zero. Even if the
optimal subsidy is zero, the optimal tax is still positive because, as discussed earlier at \( t = s = 0 \), \( U_t^A = g^B > 0 \), thus justifying a tax.

Our analysis of such public policy on the part of a targeted developed country is novel and somewhat surprising. We see that the developed country is in a position from which to exploit the source country’s regime fragility through a tax-subsidy combination. To gain more perspective on this interesting situation, we analyze the corner solutions for this tax-subsidy arrangement.

### 4.2 Corner Solutions for Counterterrorism

**Case 1:** \( g^A (t = 0, s = 0) = 0, g^B (t = 0, s = 0) > 0 \).

In this scenario, there is no possibility to implement a positive tax on \( g^A \). Does it make sense instead to choose a subsidy (i.e., \( t < 0 \)) on \( g^A \) to get to an interior equilibrium in stage 2? If we consider the use of some policy combination \((t, s)\) that moves the second-stage Nash solution to an interior equilibrium \([g^A (t, s) > 0, g^B (t, s) > 0]\), then Eq. (27a) indicates that

\[
U^A_t (t, s) = t g^A_t + g^B_t (1 + t - s) \Rightarrow \left[U^A_t (t, s)\right]_{t=0} = g^B_t (1 - s) > 0,
\]

for \( 0 \leq s < 1 \). Eq. (30) means that, starting from \( t = 0 \), a small subsidy on \( g^A \) (i.e., a small reduction of \( t \) below zero) is welfare reducing, so that a subsidy on \( g^A \) cannot be optimal; hence, \( g^A = 0 \) and \( g^B = G \). Using this fact in Eq. (21), we have:

\[
U^A (g^A = 0, g^B = G, t = 0) = \omega^A + u \left[(1 - \theta)T\left(g^B = G\right)\right] - sg^B.
\]

For this corner solution, Eq. (24a) implicitly defines \( g^B = g^B (s) \), where the implicit function
The optimal subsidy is

\[ s^* = 0 \text{ if } \left( \frac{dU^A}{ds} \right)_{s=0} < 0 \text{ or } s^* = (1 - \theta) T'(G) u'(T^A) - \frac{g^B}{(ds)} \geq 0 \text{ if } \frac{dU^A}{ds} = 0. \]  

(33)

The optimal subsidy is

\[ s^* = 0 \text{ if } \left( \frac{dU^A}{ds} \right)_{s=0} < 0 \text{ or } s^* = (1 - \theta) T'(G) u'(T^A) - \frac{g^B}{(ds)} \geq 0 \text{ if } \frac{dU^A}{ds} = 0. \]  

(33)

Case 2: \( g^A(t = 0, s = 0) > 0, g^B(t = 0, s = 0) = 0 \).

Without a tax/subsidy intervention by developed nation \( A \), source developing nation \( B \) has no incentive to engage in counterterrorism and free rides on \( A \). Proposition 5 summarizes the findings of Case 1 and presents the results of Case 2.

**Proposition 5**

If \( A \) is the free rider in a nonintervention \( (t = s = 0) \) counterterrorism equilibrium, the optimal policy for \( A \) is a policy combination \( (t = 0, s^*) \), where \( s^* \) is positive when there are strong counterterrorism gains for \( A \) from \( B \)'s counterterrorism efforts. If, however, \( B \) is the free rider in
a non-intervention equilibrium, then A’s optimal policy is to subsidize B sufficiently (above a critical $s^r$) to move the second-stage Nash equilibrium to an interior point. The optimal policy is a combination of a strictly positive subsidy and tax $(s^* > 0, t^* > 0)$.

**Proof:**

An abbreviated proof is provided in the appendix with further details in the online appendix.

If developed nation A finds that it is not worth pursuing its own proactive countermeasures against B-based terrorists given host B’s positive proactive provision, then a tax on $g^d$ cannot raise A’s welfare. However, A can still seek to increase global proactive efforts by encouraging B’s countermeasures through a subsidy. If, instead, the source nation of terrorism tends to free ride on the developed nation’s response, the latter has an interest to engage in a tax-subsidy policy to get the source nation to engage in proactive measures. While a subsidy encourages the source developing nation to choose a positive proactive effort through suitable reduction of its net marginal cost of effort $(1 - s)$, a tax on A’s own proactive effort is also used to benefit from nation B’s reaction (as in Section 4.1 above). This result, which shows that the developed nation’s optimal policy must move the developing nation from a free riding to an interior counterterror equilibrium, is novel to this literature. The result also relates well to US policy in countries like Afghanistan, Kenya, Iraq, Pakistan, the Philippines, Somalia, and Yemen where the United States has either used source nations’ military resources or sought to encourage greater use of such resources through training and direct funding, while trying to limit (not necessarily eliminate) its own involvement. Thus, Proposition 5 provides a theoretical foundation for the US-proxy War on Terror after 9/11 that continues to the present.

In a much different context, Bandyopadhyay, Sandler, and Younas (2011) investigate the
use of foreign aid to induce a source developing country to augment its counterterrorism efforts at home. Their model focuses on alternative forms of aid (tied and untied) and a mix of defensive and proactive countermeasures. The current model gets much cleaner results by limiting A’s policy choices to proactive measures at home and the use of a tax at home and a subsidy abroad to address the foreign terrorist threat and take advantage of substitutability of proactive efforts between the two countries owing to a common terrorist threat. In the current analysis, counterterrorism in the source country not only curbs terrorism worldwide but also fosters regime stability. In Bandyopadhyay, Sandler, and Younas (2011), host’s counterterrorism efforts actually worsen regime stability through a backlash. Unlike this earlier analysis, the corner solution in the source country is now ruled out through an optimal policy intervention – see Proposition 5 – constituting a novel finding. In the current analysis, we are interested in the role of terrorists’ targeting parameters and diminishing returns in counterterrorism and regime stability in affecting corner outcomes for the two countries.

5. Alternate Model Specifications

In this section, we relax three of our assumptions to show that key results generally hold. We used quasi-linear preferences for most of the preceding analysis to promote tractability of a fairly complex problem that involves strategic behavior, joint products, and externalities. Although quasi-linearity is widely used in public economics (e.g., in preference-revelation studies), the associated sanitizing of income effects can potentially lead to some qualitative differences from earlier findings. Accordingly, we present a model that allows for income effects to investigate

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8In a related article, Garcia-Alonso, Levine, and Smith (2016) look at a developed country intervening directly militarily or subsidizing counterterrorism in the source country. Military intervention results in regime instability in the source country from backlash. Not only do we play down direct military intervention but our analysis here is more focused on the use of tax-subsidy at home and abroad to address corner solutions and account for the joint-product regime stability in the source country. Also, we are interested in changes in the targeting parameters.
how our central results may or may not be affected.

5.1 Separable Utility Functions

Let $A$’s utility function take the form:

$$U^A(x^A, T^A) \equiv \tilde{u}(x^A) + u(T^A), \quad \tilde{u}(0) = 0, u(0) = 0,$$

(34)

where $\tilde{u}'(x^A) > 0$ and $\tilde{u}''(x^A) \leq 0$. The second derivative is weakly negative, so that quasi-linearity remains a degenerate case. Similarly, $B$’s utility function is:

$$U^B(x^B, T^B, Z) \equiv \tilde{u}(x^B) + u(T^B) + f(Z),$$

(35)

where $\tilde{u}$ has the same functional form as that for nation $A$. Given that $x^A = \omega^A - g^A$, $A$’s Nash FOC for an interior choice of $g^A$ is:

$$U^A_{g^A}(g^A, g^B, \theta) = -\tilde{u}'(\omega^A - g^A) + (1 - \theta)u'[\theta T(g^A + g^B)]T'(G) = 0,$$

which implicitly defines $A$’s Nash reaction function. A critical difference between Eq. (36a) and its earlier counterpart, Eq. (11a), is that the former no longer entirely determines overall counterterror, $G$. The slope of $A$’s reaction path is:

$$\rho^A = -\frac{U^A_{g^B g^A}}{U^A_{g^A g^A}} = -\frac{N^A}{\tilde{u}''(x^A) + N^A} < 0,$$

(36b)

where $U^A_{g^B g^A} = N^A = (1 - \theta)[u'(T^A)T^A + (1 - \theta)(T')^2 u''(T^A)] < 0$ and $U^A_{g^A g^A} = \tilde{u}''(x^A) + N^A < 0$.

Assuming invertibility, the absolute value of the slope of $A$’s reaction path in $(g^A, g^B)$ space is

$$\frac{1}{|\rho^A|} = \frac{\tilde{u}''(x^A) + N^A}{N^A} > 1.$$  Thus, $A$’s reaction path is downward sloping and steeper than $A$’s

---

9 We note that all second order and stability conditions for Section 5 are satisfied.
reaction path previously shown in Figure 1. Given B’s income constraint, \( x^B = o^B - g^B \), B’s FOC is:

\[
U^B_{g^B, \theta}(g^A, g^B, \theta) = -\tilde{u}^A(\phi^B - g^B) + \theta u^A \left[ \theta T \left( g^A + g^B \right) \right] T'(G) + \alpha f'(\alpha g^B) = 0, \quad (37a)
\]

which implicitly defines B’s reaction function with slope,

\[
\rho^B = -\frac{U^B_{g^B, \theta}}{U^B_{g^B, \theta}} = -\frac{N^B}{\bar{u}^*(x^B) + \alpha^2 f^*(\cdot) + N^B} < 0, \quad (37b)
\]

where \( N^B = U^B_{g^B, \theta} = \theta \left[ u^B \left( T^B \right) T'' + \theta \left( T' \right)^2 u^B \left( T^B \right) \right] < 0 \) and \( U^B_{g^B, \theta} < 0 \). Because

\[
\left| \rho^B \right| = \frac{N^B}{\bar{u}^*(x^B) + \alpha^2 f^*(\cdot) + N^B} < 1, \quad B\text{'s reaction path is also downward sloping and flatter than } A\text{'s reaction path in } (g^A, g^B) \text{ space. If an interior Nash equilibrium exists, it is unique and stable.}
\]

Differentiating Eqs. (36a) and (37a) and solving via Cramer’s rule, we derive:

\[
\frac{d g^A}{d \theta} = \frac{U^A_{g^B, \theta} U^B_{g^B, \theta} - U^B_{g^A, \theta} U^A_{g^B, \theta}}{\bar{D}} < 0, \quad \bar{D} = U^A_{g^B, \theta} U^B_{g^A, \theta} - U^A_{g^A, \theta} U^B_{g^B, \theta} > 0, \quad (38a)
\]

where \( U^A_{g^B, \theta} = -T' \left[ u^A \left( T^A \right) + T^A u^A \left( T^A \right) \right] < 0 \) and \( U^B_{g^A, \theta} = T' \left[ u^B \left( T^B \right) + T^B u^B \left( T^B \right) \right] > 0 \). Similarly, we have:

\[
\frac{d g^B}{d \theta} = \frac{U^A_{g^B, \theta} U^B_{g^A, \theta} - U^A_{g^B, \theta} U^B_{g^B, \theta}}{\bar{D}} > 0. \quad (38b)
\]

Eqs. (38a) and (38b) establish that increased relative targeting of \( B \) reduces \( A \)’s counterterrorism provision and raises that of \( B \). Assuming that \( \tilde{u}^A(\cdot) = u^A(\cdot) = 0 \) and \( \tilde{u}^B(\cdot) = \tilde{u}^A(\cdot) = \tilde{u}^B(\cdot) = \tilde{u}^B(\cdot) = 0 \),

where \( \tilde{u}^B(\cdot) = 0 \) is a constant. Eqs. (38a) and (38b) yield:

\[
\frac{d G}{d \theta} = \frac{d g^A}{d \theta} + \frac{d g^B}{d \theta} = -\frac{\left( U^A_{g^B, \theta} + U^B_{g^A, \theta} \right) \tilde{u}^B(\cdot)}{\bar{D}}. \quad (39)
\]
Given our assumption that \( u'''(\cdot) = 0 \), \( u''(T^A) = u''(T^B) = u''(T^0) \leq 0 \) is a constant. Using the above expressions for \( U_{g^A}^A \) and \( U_{g^B}^B \), we get:

\[
(0)(0)(0) = T'(\cdot)[u'(T^B) - u'(T^A) + (T^B - T^A)u''(T^0)].
\]

When \( \theta \leq 1/2 \) so that \( B \)-based terrorists direct more attacks against \( A \), \( T^A \geq T^B \) and \( (T^B - T^A)u''(T^0) \geq 0 \). Furthermore, increasing marginal disutility of terrorism implies that

\[
\left| u'(T^A) \right| \geq \left| u'(T^B) \right| \iff u'(T^A) \leq u'(T^B) \iff u'(T^B) - u'(T^A) \geq 0.
\]

Thus, given \( T'(\cdot) < 0 \),

\[
U_{g^A}^A + U_{g^B}^B \leq 0 \text{ if } \theta \leq 1/2.
\]

Since \( U_{g^A}^A < 0 \) and \( \tilde{u''} \leq 0 \), Eq. (39) establishes that \( \frac{dG}{d\theta} < 0 \) when \( \theta \leq 1/2 \), so that global counterterror effort falls and \( T(G) \) rises with a rise in \( \theta \). \( B \)'s terrorism must rise with \( \theta \) starting from \( \theta \leq 1/2 \). Targeting and counterterrorism effects oppose each other in nation \( A \), leaving the effect on \( T^A \) unclear \textit{ex ante}. However, starting from equal terror levels in the two countries (i.e., \( \theta = 1/2 \)), we can show that \( A \)'s terrorism falls with increased targeting of \( B \) under the same conditions as in Proposition 1. We note that when \( T^A = T^B \),

\[
U_{g^A}^A + U_{g^B}^B = 0.
\]

Using this fact in Eq. (39), we find that

\[
\frac{dT^A}{d\theta} < 0 \text{ if } \theta = \frac{1}{2} \text{ and if } \varepsilon_G \geq 1,
\]

(40)

where \( \varepsilon_G \) is defined in Eq. (16b). Thus, all the central findings of Proposition 1 continue to hold with a more general utility function allowing for income effects and some reasonable parameters.

\[5.2\] Two Source Nations of Terrorism

The second extension lets there be two developing nations, \( B \) and \( C \), each of which hosts a terrorist organization and has regime stability objectives denoted by \( Z^B = \alpha g^B \) and \( Z^C = \alpha g^C \), respectively. To maintain tractability, we return to quasi-linear utility functions. This extension
captures many real-world scenarios, e.g., al-Qaida is located in Afghanistan and ISIS is located in Iraq and both groups represent a US mainland threat. More elaborate extensions could allow for even more terrorist groups and host countries, but the current extension identifies the main considerations. Each terrorist organization has two targets: their resident nation and developed nation A. Total terrorism produced by B’s resident terrorist group is $T(G^B)$, where $G^B$ is aggregate counterterror effort aimed at terrorists in B and $T(\cdot)$ shares the functional form outlined in Eq. (3). Let $g^{AB}$ be A’s counterterror effort aimed at B’s terrorists so that $G^B = g^{AB} + g^B$, where $g^B$ is B’s own counterterror effort directed at its resident terrorists. Domestic terror in B is $T^B = \theta^B T(G^B)$ where $\theta^B$ is the domestic share of $T(G^B)$. Nation A endures $(1-\theta^B)T(G^B)$ terrorism originating from B. Analogously, total terrorism produced by C’s resident terrorists adheres to the same functional form as in Eq. (3), denoted by $T(G^C)$ where $G^C$ is aggregate counterterror effort directed at C’s terrorists. Let $g^{AC}$ be A’s counterterror effort aimed at C’s terrorists such that $G^C = g^{AC} + g^C$, where $g^C$ is C’s own counterterror effort against its resident terrorist group. Domestic terrorism in C is $T^C = \theta^C T(G^C)$ with $\theta^C$ being the domestic share of $T(G^C)$. Accordingly, terrorism suffered by A from C’s resident terrorists is $(1-\theta^C)T(G^C)$. Total terror in A coming from the two source nations is: $T^A = (1-\theta^B)T(G^B)+(1-\theta^C)T(G^C)\equiv T^A(G^B,G^C,\theta^B,\theta^C)$. Since A contributes to counterterrorism in both B and C, its budget constraint is $\omega^A = x^A + g^{AB} + g^{AC}$ so that A’s income-constrained utility function is:

---

\(^{10}\)If the same named terrorist group, e.g., ISIS, is located in two or more countries, the groups act autonomously even though they share the same trademark.
\[ U^A(x^A, T^A) = x^A + u(T^A) = \omega^A - g^{AB} - g^{AC} + u\left[T^A(G^B, G^C, \theta^B, \theta^C)\right]. \]  

(41)

Nation B’s utility function is:

\[ U^B(x^B, T^B, Z^B) = x^B + u(T^B) + f(Z^B) = \omega^B - g^B + u\left[\theta^B T(G^B)\right] + f(\alpha g^B). \]  

(42)

Similarly, because C’s endowment is \( \omega^C = x^C + g^C \), C’s utility function is:

\[ U^C(x^C, T^C, Z^C) = x^C + u(T^C) + f(Z^C) = \omega^C - g^C + u\left[\theta^C T(G^C)\right] + f(\alpha g^C). \]  

(43)

At an interior optimum, A’s Nash FOC for its choice of \( g^{AB} \) and \( g^{AC} \) are, respectively,

\[ U_{g^{AB}}^A(g^{AB}, g^{AC}, g^B, g^C, \theta^B, \theta^C) = -1 + (1 - \theta^B)u'[T^A(\cdot)]T'(G^B) = 0, \]  

(44a)

and

\[ U_{g^{AC}}^A(g^{AB}, g^{AC}, g^B, g^C, \theta^B, \theta^C) = -1 + (1 - \theta^C)u'[T^A(\cdot)]T'(G^C) = 0. \]  

(44b)

These two FOCs jointly determine the aggregate counterterror levels as:

\[ G^B = G^B(\theta^B, \theta^C), \text{ and } G^C = G^C(\theta^B, \theta^C). \]  

(45)

Eq. (45) corresponds to Eq. (11c) and shows that aggregate counterterror provisions \( (G^B, G^C) \) are determined entirely by nation A’s FOCs independent of counterterror choices of nations B and C. Given that \( G^B = g^{AB} + g^B \) and \( G^C = g^{AC} + g^C \), we can use Eq. (45) to write A’s reaction function with respect to \( g^B \) and \( g^C \), respectively, as

\[ g^{AB} = G^B(\theta^B, \theta^C) - g^B \text{ and } g^{AC} = G^C(\theta^B, \theta^C) - g^C. \]  

(46)

The first equation in Eq. (46) is A’s reaction path with respect to B’s counterterrorism choice in \( (g^{AB}, g^B) \) space and has a slope of \(-1\). Similarly, A’s reaction path with respect to C’s counterterrorism choice has slope of \(-1\) in \( (g^{AC}, g^C) \) space. Nation B’s FOC is:

\[ U_{g^B}^B(g^{AB}, g^{AC}, g^B, g^C, \theta^B, \theta^C) = -1 + \theta^B u'[\theta^B T(G^B)]T'(G^B) + \alpha f'(\alpha g^B) = 0, \]  

(47)
which implicitly defines $B$’s Nash reaction function and is structurally identical to Eq. (12). The Nash equilibrium in $\left( g^{AB}, g^B \right)$ space, defined by the first equation in Eq. (46) and the right-hand expression in Eq. (47), is similar to Figure 1. Analogously, $C$’s FOC is:

$$U^C_g \left( g^{AB}, g^{AC}, g^B, g^C, \theta^B, \theta^C \right) = -1 + \theta^C u' \left[ \theta^C T \left( G^C \right) \right] T' \left( G^C \right) + \alpha f' \left( \alpha g^C \right) = 0, \quad (48)$$

which implicitly gives $C$’s reaction function. The second equation in Eq. (46) and right-hand expression in Eq. (48) jointly determine the Nash equilibrium in $\left( g^{AC}, g^C \right)$ space, which also looks like Figure 1.

We now consider a change in targeting by $B$’s terrorists (i.e., $\theta^B$) while keeping $\theta^C$ unchanged. Suppressing $\theta^C$ from the functional forms, we can write Eqs. (44a) and (44b) as:

$$U^A_{g,AB} \left( G^B, G^C, \theta^B \right) = -1 + \left( 1 - \theta^B \right) u' \left[ T^A \left( G^B, G^C, \theta^B \right) \right] T' \left( G^B \right) = 0, \quad (49a)$$

and

$$U^A_{g,AC} \left( G^B, G^C, \theta^B \right) = -1 + \left( 1 - \theta^C \right) u' \left[ T^A \left( G^B, G^C, \theta^B \right) \right] T' \left( G^C \right) = 0. \quad (49b)$$

Differentiating these equations and applying Cramer’s rule, we can show that

$$\frac{dG^B}{d\theta^B} < 0; \quad \frac{dG^C}{d\theta^B} < 0 \text{ iff } \varepsilon^B_g = \frac{T \left( G^B \right) T'' \left( G^B \right)}{\left[ T' \left( G^B \right) \right]^2} \geq 1. \quad (50)$$

Eq. (50) establishes that aggregate counterterror effort directed at $B$’s resident terrorists must fall with increased targeting of $B$, which, in turn, augments global and domestic terrorism originating from $B$. However, aggregate counterterror effort directed at $C$ falls only if there are sufficient diminishing returns to counterterrorism in $B$, owing to similar factors driving Proposition 1.

Diminishing returns limit the reduction in $G^B$, thus allowing $T^A$ to fall (shown below) due to reduced targeting of $A$. At a lower $T^A$, the reduced marginal disutility from terrorism
incentivizes A to reduce its counterterror provision in C. When \( G^C \) falls, domestic terrorism rises in C along with A’s terrorism originating from C (despite the overall reduction in \( T^A \)). The rise in C’s terrorism is a negative externality, stemming from external factors that C suffers involving the other two nations.

As in Proposition 1, we now establish that overall terrorism in A must fall with an increase in \( \theta^B \) in the presence of strong diminishing returns to terrorism reduction in B. Notice that Eq. (49b) may be written as \( (1 - \theta^C)u'(T^A)T'(G^C) = 1 \). Given \( \theta^C \), this last equation implicitly defines \( T^A \) as a function of \( G^C \), where

\[
\left( \frac{dT^A}{dG^C} \right)_{\theta^C} = \frac{T''(G^C)u'(T^A)}{T'(G^C)u''(T^A)} > 0.
\]  

(51)

Eqs. (50) and (51) imply that:

\[
\frac{dT^A}{d\theta^B} = \left( \frac{dT^A}{dG^C} \right)_{\theta^C} \left( \frac{dG^C}{d\theta^B} \right) < 0 \text{ iff } \varepsilon^B_g \geq 1.
\]  

(52)

In summary, the findings of this subsection with two host resident terrorist countries preserve the essential messages of Proposition 1 while bringing some novel externalities into play.

5.3 Foreign Interests of the Developed Nation

We return to the two-country baseline model and allow developed nation A to have vulnerable interests in nation B. A developed nation like the United States not only cares about its homeland interests but also about its citizens, financial interests, or security assets abroad. In addition to its own terrorism incidence, B’s terrorism is also likely to cause disutility for A within B. Similar considerations are rarely relevant for the developing nation – i.e., Afghanistan has few or no assets in the United States. To represent these facts, we keep B’s utility function
unaltered, while proposing the following quasi-linear utility function for $A$:

$$U^A = x^A + u\left(T^A\right) + v\left(T^B\right),$$

(53)

where $v(\cdot)$ is $A$’s disutility from terrorism in $B$. We assume that $v'(\cdot) < 0$ and $v''(\cdot) < 0$ such that terrorism in $B$ raises $A$’s disutility at an increasing rate. Using $x^A = \omega^A - g^A$ and noting that $T^A = (1 - \theta)T$ and $T^B = \theta T$, $A$’s Nash FOC for an interior solution is:

$$U^A_g\left(g^A, g^B, \theta\right) = -1 + (1 - \theta)u'[\left(1 - \theta\right)T(G)]T'(G) + \theta v'[\theta T(G)]T''(G) = 0.$$  (54)

Eq. (54) is similar to Eq. (11a) except for the presence of some additional foreign-terrorism-related disutility terms, which incentivize $A$ to raise its proactive countermeasure, $g^A$.

Following the steps used in Eqs. (11b) and (11c), we know that $A$’s reaction path defined by Eq. (54) is linear with slope of $-1$ in $(g^A, g^B)$ space. $B$’s reaction function is identical to Eq. (12) and is therefore not repeated. The two nations’ reaction paths and Nash equilibrium resembles Figure 1. Using Eq. (54) and the implicit function rule, we get:

$$\frac{dG}{d\theta} = \frac{T'(\cdot)\left[u'(\cdot) + (1 - \theta)u''(\cdot)T\left[v'(\cdot) + \theta v''(\cdot)T(\cdot)\right]\right]}{\left[(1 - \theta)u'(\cdot) + \theta v'(\cdot)\right]T'' + (T')^2\left[\theta^2 v''(\cdot) + (1 - \theta)^2 u''(\cdot)\right]} < 0,$$

iff

$$\left[u'(T^A) + T^A u''(T^A)\right] > \left[v'(T^B) + T^B v''(T^B)\right].$$

(55)

The last inequality in Eq. (55) is satisfied when $A$’s homeland terrorism concerns, reflected by the left-hand side, outweigh its foreign interests represented by the right-hand side. This is a reasonable scenario since developed countries have much more to lose in terms of assets and political prestige from a terrorist attack on its interests at home than abroad.

Assuming that homeland concerns dominate, we have that a rise in $\theta$ must reduce $G$ and augment global terrorism as well as terrorism in $B$. As in Proposition 1, we can further show that
terrorist attacks in $A$ fall if $\varepsilon_g \geq 1$.

6. Concluding Remarks

The last two decades of modern terrorism witness terrorist groups taking up residency in developing or regime-challenged countries—e.g., al-Qaeda in Afghanistan, al-Nusra in Syria, ISIS in Iraq, and al-Qaeda in the Arabian Peninsula (AQAP) in Yemen. The resident terrorist group can direct their attacks at home or abroad; the latter is illustrated by the four 9/11 hijackings by al-Qaida terrorists on US soil. To capture such familiar scenarios, we initially put forward a two-country game where the proactive choices of developed nation $A$ and regime-challenged developing nation $B$ are interrelated, where $B$ hosts the resident terrorist group.

Our theoretical exercise differs from the extant literature in a number of ways. First, we find that if the terrorist group’s preference for attacking the host developing country increases, then global counterterror efforts fall, and overall terrorism rises but with attacks declining in the developed country, given strong diminishing returns to its proactive measures. Second, unlike the literature, we show the relevancy of counterterror corner solutions involving the host developing country, which is motivated in part by its interest in maintaining regime stability. Third, host nation $B$’s enhanced desire for promoting regime stability and its greater efficiency in producing this stability serve to augment its share of global counterterror effort without altering global or national terrorism levels. Fourth, we examine developed nation $A$’s motivation to institute a tax-subsidy scheme in stage 1 to get developing nation $B$, hosting the terrorist group, to assume a larger share of the proactive burden in stage 2. This burden-shifting exercise involves $A$ taxing its own counterterror efforts and subsidizing those of host country $B$. Even though global terrorism increases with this public policy, $A$ profits sufficiently in some scenarios from its reduced counterterror expenditure. Our approach differs from the literature by
emphasizing asymmetries between target countries and how corner solutions can favor one
country over another in unexpected ways. When we allow for two host countries, we can
resurrect Proposition 1 under some reasonable conditions. If Proposition 1 still holds, then other
propositions could be shown to follow. Relaxing the quasi-linearity of the utility function to
allow for income effects can still provide a result similar to Proposition 1. The same is true if we
allow the developed nation to sustain damage from terrorism in the host country provided that
terrorism losses at home dominate those abroad.
Math Appendix: Abbreviated Proof of Proposition 5

The first part of the proposition has been proven in the text. For Case 2, if \( g^B(t = 0, s = 0) = 0 \), then \( g^A = G^{dA}(t = 0) = G^0 > 0 \). Eq.(24a) implies that

\[
\left( \frac{\partial U^B}{\partial g^B} \right)_x = s - 1 + \theta u \left[ \theta T \left( G^0 \right) \right] T' \left( G^0 \right) + \alpha f^\prime \left( 0 \right) \leq 0, \tag{A1}
\]

One can increase \( s \) from zero, while holding the tax at zero, without affecting either nation’s utility until the subsidy reaches a critical value \( s^c \) (assuming it exists for values less than unity) where \( B \)’s FOC is just met at \( g^B = 0 \), such that

\[
\left( \frac{\partial U^B}{\partial g^B} \right)_{s = c} = s^c - 1 + \theta u \left[ \theta T \left( G^0 \right) \right] T' \left( G^0 \right) + \alpha f^\prime \left( 0 \right) = 0,
\]

\[
\Rightarrow s^c = 1 - \theta u \left[ \theta T \left( G^0 \right) \right] T' \left( G^0 \right) - \alpha f^\prime \left( 0 \right) < 1. \tag{A2}
\]

When \( s > s^c \), but \( s \to s^c \) [i.e., \( s \to \left( s^c \right)^+ \)], we have \( g^B \left( t = 0, s \right) \to 0^+ \). We now have an interior stage-2 equilibrium for which the analysis of Section 4.1 applies. From Eq.(25d), we know that \( g^A_s = -g^B_s \), which, when combined with Eq.(27b), gives:

\[
\left( \frac{dU^A}{ds} \right)_{s = 0, r} = -g^A_s - sg^B_s - g^B = \left( 1 - s \right) g^B_s - g^B. \tag{A3}
\]

Given Eqs. (24a) and (25d), Eq. (A3) reduces to:

\[
\left( \frac{dU^A}{ds} \right)_{s = 0, g^B \to 0^+} = \frac{1 - s}{\alpha^2 \left| f^\prime \left( 0 \right) \right|} > 0, \text{ where } s < 1. \tag{A4}
\]

When \( g^B \) is near zero, the first equation on the right in Eq. (A3) shows that there are only two marginal effects from an increase in \( s \); namely, the savings in provision cost of \( g^A \) and the
increased subsidy payments to nation B. Since Eq. (25d) shows that $|g_s^A| = |g_s^B|$, the savings in the provision of $g^A$ dominates the excess payments in subsidies $sg_s^B$ as long as $s < 1$. Thus, developed nation $A$’s welfare is increasing in $s$ for $s > s^c$ and $t = 0$. Therefore, the optimal subsidy has to be strictly positive because $s^* > s^c > 0$. Moreover, given Eq. (27a), we have $\left(U_t^A\right)_{t=0} = (1 - s)g_t^B > 0$, thereby supporting a positive tax on $g^A$ for any interior stage-2 equilibrium when $s < 1$. The optimal tax and subsidy are both strictly positive, given by Eqs. (28a) and (28b) of Section 4.1. Q.E.D.
References


Figure 1. Nash equilibrium counterterrorism game
Figure 2. Comparative statics for targeting: interior solution
Figure 3. Comparative statics for targeting: corner solution