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A Local-Spillover Decomposition of the Causal Effect of U.S. Defense Spending Shocks*

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Abstract

This paper decomposes the causal effect of government defense spending into: (*i*) a local (or direct) effect, and (*ii*) a spillover (or indirect) effect. Using state-level defense spending data, we show that a negative cross-state spillover effect explains the existing simultaneous findings of a low aggregate multiplier and a high local multiplier. We show that enlisting disaggregate data improves the precision of aggregate effect estimates, relative to using aggregate time series alone. Moreover, we compare two-step efficient GMM with two alternative moment weighting approaches used in existing research.

There is substantial evidence that changes in U.S. national defense spending typically have a positive and less than one-for-one effect on U.S. national output.[1] Yet, estimates of this relationship using regional cross-sectional or panel data find larger effects of a region's defense spending on that region's output.[2] The former is an aggregate effect while the latter is a local effect. The local effect of a treatment need not equal the treatment's aggregate effect in the presence of spillovers. For example, if there are negative spillovers across regions, then the local multiplier will likely provide an upwardly biased estimate of the aggregate effect of a policy.[3]

Our paper reconciles this discrepancy in a unified empirical framework. By leveraging both the cross-sectional and aggregate variation in a panel, we decompose the causal effect of govern-

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[1]See for example Hall (2009) and Ramey (2011). These papers' low aggregate defense spending multiplier finding does not condition on the state of the macroeconomy, e.g., whether the economy is at the zero-lower bound on interest rates or is in a state of "slack." Whether the aggregate defense spending multiplier is state-dependent is currently an unsettled research issue.

[2]See for example Biolsi (2019) and Nakamura and Steinsson (2014).

[3]In the causal inference literature, this is known as a failure of the stable unit value assumption (SUTVA), which requires that potential outcomes be unaffected by the treatment status of other observational units.

ment defense spending into: (*i*) a local (or direct) effect, and (*ii*) a spillover (or indirect) effect. Using state-level defense spending data, we show that negative cross-state spillovers can explain the existing simultaneous findings of a low aggregate multiplier and a high local multiplier. Moreover, our approach is such that the local and spillover responses "add up" to the aggregate response.

Typically, macroeconomists employ aggregate, time-series variation to estimate the combined effects of (*i*) and (*ii*). With exogenous variation in the aggregate treatment, this approach can estimate the treatment's aggregate effect but cannot distinguish between its local and spillover effects.

With panel or cross-sectional data, researchers often only estimate the local effect of treatment (*ii*). While this object may be of interest on its own, these papers do not typically estimate either the spillover or aggregate effects of treatment.[4]

Our setup involves a large number of moment conditions relative to parameters. Thus, the approach we propose in this paper requires that researchers take a stance on how to combine moment conditions. We compare the two-step efficient GMM with an extension of the Bartik (1991) shift-share weighting as well as a version of the "sensitivity" approach proposed in Nakamura and Steinsson (2014). We show that enlisting disaggregate data improves the precision with which we estimate the aggregate effect of defense spending, relative to using aggregate time-series variation alone.

1 The Local-Spillover Decomposition

Suppose we have data for N states and T years, indexed by i and t respectively. Let $g_{i,t}$ and $q_{i,t}$ denote state real military spending and real output, respectively. For any variable $a_{i,t}$, define: $a^s_{i,t} \equiv \sum_{j\neq i} a_{j,t}$; $a_t \equiv \sum_{i=1}^{N} a_{i,t}$; and $\bar{a}_t \equiv a_t/N$. Finally, the upper case of any lower case variable is the column vector of the i indexed lower case values, e.g., $A_t = \begin{bmatrix} a_{1,t} & \cdots & a_{N,t} \end{bmatrix}'$.

We follow Nakamura and Steinsson (2014) in focusing on two-year changes in defense spending and output. Our within-state treatment variable is the two-year change in state military spending as a share of lagged national output:

$$x_{i,t} = \frac{g_{i,t} - g_{i,t-2}}{q_{t-2}}$$

Our state-level outcome variable is the two-year change in state military spending as a share of lagged national output:

$$y_{i,t} = \frac{q_{i,t} - q_{i,t-2}}{q_{t-2}}$$

[4] Exceptions from the fiscal multiplier literature include Suárez Serrato and Wingender (2016) (spillovers among U.S. counties), and McCrory (2020) (trade spillovers between U.S. states). While there are examples in the fiscal multiplier literature that seek to estimate spillovers in the cross section, as far as we know ours is the first paper that jointly estimates the local and spillover effects so that their sum equals the aggregate effect.

We use lagged national output, q_{t-2} to normalize changes in spending and output to facilitate aggregation.

We estimate the following equation:

$$y_{i,t} = \alpha_i + \phi x_{i,t} + \frac{\beta}{N-1} x^s_{i,t} + u_{i,t} \tag{1}$$

for all i. We call ϕ the local response of the treatment $x_{i,t}$ and call β the (scaled) spillover treatment response. As motivation for this equation, see Farhi, E. and I. Werning (2016) for a presentation of a dynamic, general equilibrium currency union model in which own-region government spending and spending in the remainder of the currency union each enter as forcing variables into the linearized dynamic system.

We use an aggregate instrument z_t that satisfies the following N moment conditions:

$$E\left(z_t U_t\right) = 0 \tag{2}$$

We use the two-year change in national defense spending relative to lagged national output, x_t, as our instrument z_t. The validity of z_t as an instrument rests on the common argument that national defense spending is exogenous because it is determined by international geopolitical factors and national security concerns and thus orthogonal to local economic conditions in any state over time.[5]

Next, we apply several instructive manipulations of (1) to recover coefficients that have been estimated in recent research. First, summing across i for each t, (1) becomes:

$$y_t = \alpha + \underbrace{(\phi + \beta)}_{\mu^A} x_t + u_t \tag{3}$$

We call μ^A the aggregate multiplier because it gives the summed response of $y_{i,t}$ to a summed change in x_t.

Next, we remove time fixed effects by subtracting out the time t average of terms on the right- and left-hand sides of (1).

$$y_{i,t} - \bar{y}_t = (\alpha_i - \alpha/N) + \underbrace{\left(\phi - \frac{\beta}{N-1}\right)}_{\mu^R} (x_{i,t} - \bar{x}_t) + u_{i,t} - \bar{u}_t \tag{4}$$

We call μ^R the relative multiplier, following the nomenclature in Nakamura and Steinsson (2014).

Thus, there is a linear relationship between our local and spillover coefficients and the relative

[5]For explanations of this identification assumption as well as executions of this approach, see Barro and Redlick (2011), Hall (2009), Ramey (2011) and Ramey and Zubairy (2018).

and aggregate multipliers commonly estimated in the existing literature. The spillover multiplier equals the aggregate multiplier net of the local multiplier.

Our benchmark approach for estimating ϕ and β is two-step efficient GMM. For comparison, we also report a Bartik shift-share weighting scheme for combining moments as well as the "sensitivity" specification proposed in Nakamura and Steinsson (2014).

To ease exposition, we write our GMM estimator with state de-meaned variables, suppressing intercepts. Let Y_t be a vector stacking de-meaned $y_{i,t}$ and $\Gamma_t \equiv \begin{bmatrix} X_t & X_t^s \end{bmatrix}$ where X_t and X_t^s are vectors stacking de-meaned $x_{i,t}$ and $x_{i,t}^s$, respectively. Let $\theta_0 \equiv [\ \phi \quad \beta/(N-1)\ ]'$. We assume our time series are mixing, covariance stationary, and satisfy standard regularity conditions. Our N moment conditions can thus be written as:

$$E\left(z_t U_t\right) = E\left(z_t (Y_t - \Gamma_t \theta_0)\right) = 0. \tag{5}$$

Using a positive definite weighting matrix W, $\hat{\theta}_T^W$ is the argument minimizing the GMM criterion function $J(\theta; W)$ with the familiar quadratic form:

$$J(\theta; W) = \left[\frac{1}{T}\sum_{t=1}^{T} z_t (Y_t - \Gamma_t \theta)\right]' W \left[\frac{1}{T}\sum_{t=1}^{T} z_t (Y_t - \Gamma_t \theta)\right] \tag{6}$$

Under our assumptions $\widehat{\theta}_T^W = arg\, min_\theta\ J(\theta; W)$ will be a $\sqrt{T}$-consistent estimator of θ_0 for any positive definite W. Since our model is linear we have a closed form solution for $\widehat{\theta}_T$

$$\widehat{\theta}_T^W = \left[\left(\frac{1}{T}\sum_{t=1}^{T} z_t \Gamma_t\right)' W \left(\frac{1}{T}\sum_{t=1}^{T} z_t \Gamma_t\right)\right]^{-1} \left(\frac{1}{T}\sum_{t=1}^{T} z_t \Gamma_t\right)' W \left[\frac{1}{T}\sum_{t=1}^{T} z_t Y_t\right] \tag{7}$$

Substituting for Y_t and re-arranging,

$$\sqrt{T}\left(\hat{\theta}_T^W - \theta_0\right) = \left[\left(\frac{1}{T}\sum_{t=1}^{T} z_t \Gamma_t\right)' W \left(\frac{1}{T}\sum_{t=1}^{T} z_t \Gamma_t\right)\right]^{-1} \left(\frac{1}{T}\sum_{t=1}^{T} z_t \Gamma_t\right)' W \left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t U_t\right] \tag{8}$$

Condensing notation by using A_T to denote the matrix premultiplying the last term in the above equation we have:

$$\sqrt{T}\left(\hat{\theta}_T^W - \theta_0\right) = A_T \left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t U_t\right] \tag{9}$$

We use the conventional large sample approximation that $\frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t U_t$ is approximately distributed $N(0, \Omega)$ with

$$\Omega = lim_{T\to\infty} Var\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t U_t\right] = \sum_{k=-\infty}^{\infty} E\left[(z_t U_t)(z_{t+k} U_{t+k})'\right]. \tag{10}$$

Thus, defining $\hat{\Omega}_T$ to be a consistent estimator of Ω, in large samples $\sqrt{T}\left(\hat{\theta}_T^W - \theta_0\right)$ is approximately distributed $N(0, A_T\hat{\Omega}_T A_T')$.

Each element of Ω is of the following form:

$$\Omega_{i,j} = \sum_{k=-\infty}^{\infty} E[(z_t u_{i,t})(z_{t+k} u_{j,t+k})] \tag{11}$$

We estimate Ω using a Heteroskedasticity Auto-Correlation (HAC) covariance estimator that is a weighted sum of the sample analogs of the covariances in (11). For a given pair (i, j) and lag k

$$\widehat{C(k)}_{i,j} = \frac{1}{T}\sum_t (z_t \hat{u}_{i,t})(z_{t+k}\hat{u}_{j,t+k}) \tag{12}$$

We use two sets of weights to form our estimator of Ω_{ij} as:

$$\hat{\Omega}_{i,j} = w(i,j)_T \sum_{k=-t_0}^{t_0} \lambda(k)_T \widehat{C(k)}_{i,j} \tag{13}$$

The weights $\lambda(k)_T$ are standard time series HAC Bartlett kernel (Newey-West) weights that down weight larger lags linearly, $\lambda(k)_T = 1 - |k|/t_0$ if $k < t_0$ and zero otherwise.

We introduce an additional weight $w(i,j)_T$ in specifications where the dimension of Ω is large enough to be concerned about estimation precision given the length of our time series. We model cross-state covariances as a decreasing function of a distance measure and specify $w(i,j)_T$ to be a decreasing function of distance between states.[6] We again use a Bartlett kernel $w_{i,j} = 1 - d_{i,j}/d_0$ if $d_{i,j} < d_0$ and zero otherwise, with a distance cuttoff d_0. Our estimator of Ω, $\hat{\Omega}_T$, simply collects the $\hat{\Omega}_{i,j}$. In the special case where d_0 is taken to be arbitrarily large, so that $w(i,j)_T = 1$, $\hat{\Omega}_T$ is just the standard Bartlett (Newey-West) time series covariance matrix estimator. $\hat{\Omega}_T$ consistently estimates Ω if both weights go to one (t_0 and d_0 increase with T) so that in the limit the sample covariance for any (i, j, k) gets a weight of one, but t_0 grows slowly enough so that the variance of $\hat{\Omega}_T$ vanishes. [7]

We use geographic distance as our measure of distance between states. Specifically, we use the U.S. 2010 Census population mass centers for each state to construct distances.[8] Geographic distance between states correlates highly with variables that measure interstate interactions, such as trade flows of goods, migration between states and size of transportation costs.[9]

We obtain two step efficient GMM estimates in the conventional way: by first obtaining a

[6] Modeling cross-state covariances as a function of distances is analogous to the Conley (1999) approach for general cross sectional dependence.

[7] We note that $\hat{\Omega}_T$ remains consistent with $w(i,j)_T = 1$, this weight is used simply to improve the finite sample properties of $\hat{\Omega}_T$ by downweighting covariances that are anticipated to be small.

[8] Spherical distances between states' population centers accounting for curvature of the earth but not altitude.

[9] Analysis of the gravity model of trade, e.g. in Carrare, et.al. (2020), has established "overwhelming evidence that trade tends to fall with distance."

GMM estimate $\widehat{\theta}_T^W$ using an identity weighting matrix, $W = I_N$ and then constructing $\widehat{\Omega}_T$ whose generalized inverse is then used as W in the second step to get an efficient GMM estimate.

Comparison to Identity-Weighted GMM ("Sensitivity" Approach)

When reporting our benchmark estimate of θ below, we also provide point estimates from the first step of the efficient GMM estimation in which we set $W = I_N$. The purpose for doing this is two-fold: first, it provides a natural benchmark for interpreting our baseline results; second, it is equivalent to implementing the "sensitivity" procedure proposed in Nakamura and Steinsson (2014), applied in our setting to estimate both $\theta = [\phi, \frac{\beta}{N-1}]'$. A derivation of this equivalence appears in the Appendix.

Comparison to Bartik Estimator

Alternatively, we estimate θ via pooled instrumental variables (IV) using the so-called Bartik instruments.[10] This is a natural benchmark to consider given that in recent years a Bartik-style instrument has been employed in the fiscal multiplier literature to estimate local responses to government spending shocks (e.g., Nekarda and Ramey (2011)).[11]

We denote a local and spillover Bartik instrument as $b_i z_t$ and $b_i^s z_t$, where b_i is a weight reflecting the ratio of state defense spending to aggregate output and $b_i^s = \sum_{j \neq i} b_j$. Thus, we treat the construction of the Bartik instruments in the same manner we construct the local and spillover variables. Following the Bartik literature, we define the b_i weights from the average value of the ratio of $g_{i,t}$ and q_t in the early years of the sample:

$$b_i \equiv \frac{1}{5} \sum_{t=1}^{5} \frac{g_{i,t}}{q_t}$$

In our estimation, we treat b_i and b_i^s as fixed. Thus, the Bartik estimator is:

$$\hat{\theta}_T^B = \left(\frac{1}{NT} \sum_i \sum_t \left(\begin{bmatrix} b_i z_t \\ b_i^s z_t \end{bmatrix} \begin{bmatrix} x_{i,t} \\ x_{i,t}^s \end{bmatrix}' \right) \right)^{-1} \begin{bmatrix} \frac{1}{NT} \sum_i \sum_t b_i z_t y_{i,t} \\ \frac{1}{NT} \sum_i \sum_t b_i^s z_t y_{i,t} \end{bmatrix} \tag{14}$$

Substituting for $y_{i,t}$, scaling by $\sqrt{T}$ and rearranging yields:

$$\sqrt{T} \left(\hat{\theta}_T^B - \theta_0 \right) = \left(\frac{1}{NT} \sum_i \sum_t \left(\begin{bmatrix} b_i z_t \\ b_i^s z_t \end{bmatrix} \begin{bmatrix} x_{i,t} \\ x_{i,t}^s \end{bmatrix}' \right) \right)^{-1} \begin{bmatrix} \frac{1}{N} \sum_i b_i \left(\frac{1}{\sqrt{T}} \sum_t z_t u_{i,t} \right) \\ \frac{1}{N} \sum_i b_i^s \left(\frac{1}{\sqrt{T}} \sum_t z_t u_{i,t} \right) \end{bmatrix} \tag{15}$$

[10] For econometric investigation of this shift-share style design, see Adão, R., Kolesár, M. and E. Morales (2019), Borusyak, Hull and Jaravel (2019), and Goldsmith-Pinkham, P., Sorkin, I., and H. Swift (Forthcoming).

[11] See equation (8) in Nekarda and Ramey (2011), where the cross-sectional dimension of the panel is industry rather than region.

Rewriting makes clear this is a linear combination of $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}[z_t U_t]$:

$$\sqrt{T}\left(\hat{\theta}_T^B-\theta_0\right)=\left[\left(\frac{1}{T}\sum_i\sum_t\left(\left[\begin{array}{c} b_i z_t \\ b_i^s z_t \end{array}\right]\left[\begin{array}{c} x_{i,t} \\ x_{i,t}^s \end{array}\right]'\right)\right)^{-1}\left[\left(\begin{array}{c} (b_1,b_2,...,b_N) \\ (b_1^s,b_2^s,...,b_N^s) \end{array}\right)\right]\frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_t U_t\right]$$

Letting $\tilde{A}_T$ denote the above array in brackets:

$$\sqrt{T}\left(\hat{\theta}_T^B-\theta_0\right)=\tilde{A}_T\frac{1}{\sqrt{T}}\sum_{t=1}^{T}[z_t U_t] \tag{16}$$

Thus the large sample approximation of the distribution of the $\hat{\theta}^B$ is analogous to the GMM estimator with $\tilde{A}_T$ replacing A_T in equation (8). In general, $\hat{\theta}^B$ uses a different linear combination of the same sample moments used for GMM. Again, we use the conventional large sample distribution for inference using the same estimator of Ω.

2 Positive Local Effects and Negative Spillover Effects

We draw state GDP and military spending data directly from Nakamura and Steinsson (2014). They use state GDP data from the BEA and military spending data from various Department of Defense sources. The sample covers 1966 to 2006.

A key empirical concern is that our panel data contains the lower 48 states and our data spans only 41 years. This is particularly relevant for the finite sample properties of $\hat{\Omega}_T$ and we employ two main dimension reduction strategies to improve its performance. The first is to aggregate across some sets of states to form 33 'pseudo-states.' We aggregate within three census divisions, New England, East South Central, and Mountain, and use these along with the remaining 30 states as our pseudo-states.[12] These pseudo-states are used to define spillover variables $x_{i,t}^s$ throughout. Our other main strategy is down-weighting covariances between states as a function of their distance, for a range of cutoff d_0 in constructing $\widehat{\Omega}_T$. This in effect imposes an assumption that the magnitude of cross pseudo-state covariances declines with distance. In addition, in the Appendix, we present estimates with an alternate aggregation schemes.[13]

Table 1 reports the local, spillover and aggregate multipliers from the Bartik, identity-weighted GMM ("sensitivity"), and efficient GMM weighting schemes for the 33 pseudo-state data.[14] We estimated the above using $t_0 = 5$ and $d_0 = 2000$km.[15]

[12]The states aggregated are: New England (CT, MA, ME, NH, RI, VT), East South Central (AL, KY, MS, TN), and Mountain (AZ, CO, ID, MT, NV, NM, UT, WY).

[13]The Appendix contains the analogous results aggregating pseudo-states within 9 census divisions.

[14]The Appendix contains the analogous results using the lower 48 states; the results are across the board very similar.

[15]To give a sense of the d_0 in this context, from the perspective of New York, the sample covariance of the error with

The first row of estimates in Panel A contain estimates of equation (4) assuming $\beta = 0$; that is, an estimate of the local multiplier assuming zero spillovers and including time fixed effects. The middle column corresponds to the Nakamura and Steinsson (2014) preferred local multiplier estimate. Our estimate of 1.47 (SE=0.66) indicates that a one unit increase in defense spending in a state causes output to increase by 1.47 units in that state relative to all other states. This is essentially indistinguishable from the relative multiplier estimate of 1.5 in Nakamura and Steinsson (2014).[16] The estimate in the first column with Bartik weights is also very similar at 1.46 (SE=0.69).

Table 1: Defense spending multipliers (local, spillover and total) using three alternative ways to combine moment conditions (Bartik, identify-weighted GMM and efficient GMM), 33 pseudo-states

Panel A

	Bartik		GMM ($W = I$)		Efficient GMM	
	Local	Spillover	Local	Spillover	Local	Spillover
Zero-Spillover	1.464^{**}		1.466^{**}		1.408^{***}	
Restriction (w/ time FEs)	(0.689)		(0.655)		(0.272)	
Zero-Spillover	1.061		1.072^{*}		1.051^{***}	
Restriction	(0.678)		(0.647)		(0.304)	
Local-Spillover	1.433^{*}	-1.022	1.434^{**}	-1.024	1.235^{***}	-0.663^{***}
Decomposition	(0.759)	(0.837)	(0.722)	(0.853)	(0.308)	(0.127)

Panel B

	Bartik	GMM ($W = I$)	Efficient GMM
Sum	0.410	0.410	0.571^{*}
	(0.843)	(0.843)	(0.308)

Panel C

Aggregate Independent Variable	0.410
	(0.863)
Aggregate Dependent	0.410
	(1.381)

Notes for Panels A and B: 33 pseudo-states, T=39 and FE=fixed effects. Notes for Panel C: Obs = 1287 for "Aggregate Independent Variable" and Obs = 39 for "Aggregate Dependent" specifications. Spatial-temporal corrected standard errors ($t_0 = 5, d_0 = 2000$) reported in parentheses in all rows, except the final one, which uses the temporal correction ($t_0 = 5$). *0.10 **0.05 ***0.01.

The corresponding efficient GMM point estimate is somewhat smaller, 1.41 (SE=0.27). Note

Indiana is down-weighted by 1/2, since Indiana is approximately 1000km away, and zero with California, since it is approximately 4000km away.

[16]Our zero-spillover, time fixed effect local multiplier point estimate differs slightly from those authors because they scale both right and left-hand side variables by state GDP whereas we scale by national output. As explained previously, our national-output scaling facilitates aggregation of the state-level estimates.

that the standard error is substantially smaller under efficient GMM than the other two weighting approaches.

The next row's specification is identical to the first except that it does not include year fixed effects. The Bartik-based multiplier falls somewhat, to 1.06 (SE=0.68). The corresponding identity-weighted GMM point estimate and standard error are very similar to the Bartik estimator. The point estimate for efficient GMM is 1.05 aligns closely with the prior two estimates, albeit with a standard error half the size.

Next, the row labelled "local-spillover decomposition" estimates the full model, i.e. equation (1). Relative to the previous rows, it adds the leave-out sum of spending for each state as a regressor. For the efficient GMM weighting, the local multiplier is 1.24 (SE=0.31) and the spillover multiplier equals -0.66 (SE=0.13).[17] Each coefficient is statistically different from zero at a 1 percent level. The negative spillover multiplier implies that a state's GDP falls when, holding fixed defense spending in that state, defense spending in all other states increases. Both the Bartik approach and identity-weighted GMM estimation similarly deliver positive local and negative spillover multipliers, although with less precision.

Panel B contains the sum of the local and spillover coefficients reported in Panel A. These measure the aggregate effect of fiscal policy. The implied aggregate multiplier estimate in both the Bartik and identity-weighted GMM cases equals 0.41 (SE=0.84). The efficient GMM implied aggregate multiplier coefficient equals 0.57 (SE=0.31). Thus, although the local multiplier is greater than one, the presence of a negative spillover provides an offsetting effect. Inclusive of the spillover, there is a less than one-for-one increase on national GDP when national defense spending increases for all three weighting approaches. This is consistent with the aggregate fiscal multiplier findings described in the introduction.

Panel C contains estimates of the aggregate multiplier without using the local-spillover decomposition. The row labelled "aggregate independent variable" simply regresses $y_{i,t}$ on x_t and then multiplies that coefficient by 33, the number of 'pseudo-states' in our sample. We continue to use the spatial-temporal approach to compute standard errors. The point estimate is 0.41 (SE=0.86). Thus, it indicates a less-than-one defense spending multiplier. The final row of Table 1 reports the purely aggregate results. That is, we regress y_t on x_t. All variation used to identify the multiplier comes from the time series. By construction, the point estimate equals that from the previous specification. The standard error is much larger than when we use the dis-aggregate output measures and impose restrictions on the moment covariance matrix via our spatial correction.

The observant reader will notice that the point estimates from the first two columns of Panel B are both equal to 0.41, which is the same point estimate one acquires from estimating the aggregate regression in equation (3). This is not coincidental. In the Appendix, we prove that estimating equation (1)—either by the Bartik approach or via identity-weighted GMM—delivers

[17] Recall that the spillover multiplier, $\hat{\beta}$, is the second dimension of $\hat{\theta}_T^W$ scaled by $(N-1)$.

point estimates of the local and (appropriately scaled) spillover effects such that their summation is numerically equivalent to estimating the aggregate regression.[18]

Table 2: Defense spending multipliers (local, spillover and total) using Efficient GMM under different bandwidth choices.

	Local	Spillover	Aggregate
$t_0 = 5$, $d_0 = 1000$	1.025***	-0.494***	0.531
	(0.331)	(0.125)	(0.342)
$\mathbf{t_0 = 5}$, $\mathbf{d_0 = 2000}$	1.235***	-0.663***	0.571*
	(0.308)	(0.127)	(0.308)
$t_0 = 5$, $d_0 = 3000$	1.308***	-0.766***	0.542*
	(0.293)	(0.135)	(0.290)
$t_0 = 8$, $d_0 = 1000$	1.093***	-0.618***	0.475
	(0.309)	(0.120)	(0.312)
$t_0 = 8$, $d_0 = 2000$	1.312***	-0.783***	0.529*
	(0.288)	(0.128)	(0.283)
$t_0 = 8$, $d_0 = 3000$	1.365***	-0.862***	0.502*
	(0.272)	(0.129)	(0.268)

Notes: 33 Pseudo-states, T = 39. Estimated with pseudo-state fixed effects. *0.10 **0.05 ***0.01.

We examine the robustness of our results to differing bandwidth choices in Table 2. We present results for our benchmark $t_0 = 5$ and the larger value of $t_0 = 8$, illustrating a range of reasonable t_0 values. For each t_0 choice, we also report results for $d_0 = 1000km, 2000km, 3000km$, representing a range of reasonable cross sectional weighting choices. The results are qualitatively similar across these specifications. The local multiplier point estimates are positive, ranging in value from 1.025 to 1.365. Spillovers are systematically estimated to be negative, with point estimates ranging from -0.494 to -0.862. Despite the range in estimated local and spillover multipliers, the implied estimates of the aggregate multiplier vary less, ranging from 0.475 to 0.572.

While we regard our benchmark bandwidth choices as reasonable on a priori grounds, we recognize that others may prefer a different baseline specification. We have presented what we consider to be reasonable ranges of either t_0 and d_0. Regardless of which row in Table 2, one prefers, the conclusion is the same: the local multiplier is estimated to be greater than one, the estimated spillover multiplier is negative, and the implied aggregate multiplier estimate is less than one. Moreover beyond $d_0 = 1000km$, the local and spillover estimates are stable in magnitude.

[18] Alternative Bartik weights will attribute more or less to the local multiplier or the spillover multiplier, but their sum will still be numerically equivalent to estimating the aggregate regression.

3 Conclusion

When researchers employ panel data to estimate the causal effect of some treatment, they typically only identify a local treatment effect. In the presence of spillovers between observational units, this local effect is generally different from the treatment's aggregate (or average) effect.

However, in many contexts, particularly in macroeconomics, policymakers are interested primarily in the treatment's aggregate effect, as may be the case with government spending. In this note, we show how to augment a standard local effect regression to account for potential spillovers between observational units. By exploiting the panel structure of the data, we show how to jointly identify the local and spillover effects of the treatment in a way that allows for easy conversion to treatment's aggregate effect.

There are many potential applications of the local-spillover decomposition presented in this note. To apply our methodology in a straightforward manner, a suitable application should have three main ingredients. First, one requires sufficiently long disaggregate (individual/group) time series data on both the outcome and treatment variables of interest. This is because our methodology relies upon moment conditions that are satisfied along the time dimension.

Second, one needs some level of treatment effect similarity across individuals/groups. This is important to facilitate aggregation of estimates. A fruitful extension of our methodology would be to consider treatment effect heterogeneity in terms of both local and spillover effects.

Third, our method requires a time varying instrument. The instrument could, in the general case, vary at the individual/group level; however, as in this note's implementation, an aggregate instrument is sufficient to execute the local-spillover decomposition. As an implication, disaggregate data can be useful for uncovering and decomposing aggregate treatment effects even if identification is based primarily upon an aggregate instrument. The empirical macroeconomic literature provides some natural candidates for future research, such as oil supply shocks and identified technology shocks.

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A Appendix

A.1 Equivalance between Identity-Weighted GMM and Sensitivity Approaches

The "sensitivity" approach proposed in Nakamura and Steinsson (2014) entails estimating the relative multiplier μ_R in equation (4) by two-stage least squares with N instruments formed from the interactions of state indicators with aggregate military spending z_t. Note that because of the inclusion of time fixed effects, Nakamura and Steinsson (2014) are only able to estimate the relative multiplier rather than separately identify ϕ and $\beta/(N-1)$, the local and spillover effects of military spending.

To see the equivalence between the "sensitivity" approach and identity-weighted GMM, stack $\{Y_t, \Gamma_t\}$ into column vectors Y, Γ. The N instruments formed from state-interactions with aggregate military spending can then be compactly written as $\tilde{Z} \equiv Z \otimes I_N$, where $Z = [z_1, \ldots, z_T]'$. The two-stage least squares estimator, used in Nakamura and Steinsson (2014), is given by

$$\hat{\theta}_{NS} = \left(\Gamma' P_{\tilde{Z}} \Gamma\right)^{-1} \left(\Gamma' P_{\tilde{Z}} Y\right) \tag{17}$$

with $P_{\tilde{Z}} \equiv \tilde{Z}(\tilde{Z}'\tilde{Z})^{-1}\tilde{Z}'$. Employing standard properties of the Kronecker product yields

$$\begin{aligned}
(\tilde{Z}'\tilde{Z})^{-1} &= \left((Z \otimes I_N)'(Z \otimes I_N)\right)^{-1} \\
&= \left(Z'Z \otimes I_N\right)^{-1} \\
&= (Z'Z)^{-1} \otimes I_N
\end{aligned} \tag{18}$$

Note that the scalars $(Z'Z)^{-1}$ will cancel out in the formula for $\hat{\theta}_{2SLS}$ after plugging (18) into (17). Thus, dividing and multiplying where appropriate by $\frac{1}{T}$ delivers:

$$\hat{\theta}_{NS} = \left[\left(\frac{1}{T}\sum_{t=1} z_t \Gamma_t'\right)\left(\frac{1}{T}\sum_{t=1} z_t \Gamma_t\right)\right]^{-1}\left(\frac{1}{T}\sum_{t=1} z_t \Gamma_t'\right)\left(\frac{1}{T}\sum_{t=1} z_t Y_t\right) = \hat{\theta}_T^W \tag{19}$$

which is equivalent to (7) with $W = I_N$.

A.2 Equivalence Between Aggregate Regression and Summation of Local and Spillover Estimates

This subsection shows the numerical equivalence between aggregating the local and (appropriately scaled) spillover estimates—from either the Bartik estimator or the identity-weighted GMM estimator—and the OLS estimator from the aggregate time-series regression. We begin with the Bartik estimator. The proof for the identity-weighted GMM estimator is very similar and so some steps are skipped.

A.2.1 Bartik Estimator

Let $B \equiv [b_1, b_2, \dots, b_N]'$ be the collection of positive Bartik weights. Let $D \equiv J_N - I_N$, where J_N is the $N \times N$ matrix of ones. Then, following the description in Section 16, let B^s contain the vector of spillover Bartik weights. The spillover Bartik weights and the Bartik weights are linked by $B^s \equiv DB$. Note that this relationship holds for our main spillover coefficient as well: $X_t^s \equiv DX_t$.

With this notation, the Bartik estimator can be written in matrix form as using only the vector of changes in own-state spending X_t, changes to state output growth Y_t, and aggregate changes in spending z_t[19]:

$$\hat{\theta}_T^B \equiv \begin{bmatrix} \sum_t B'X_t z_t & \sum_t B'DX_t z_t \\ \sum_t B'DX_t z_t & \sum_t B'DDX_t z_t \end{bmatrix}^{-1} \begin{bmatrix} \sum_t B'Y_t z_t \\ \sum_t B'DY_t z_t \end{bmatrix}$$

Proposition 1. *The sum of the local effect and the appropriately scaled spillover effect estimated with the Bartik estimator is numerically equivalent the OLS estimator from the aggregate regression. That is,* $\begin{bmatrix} 1 & (N-1) \end{bmatrix} \hat{\theta}_T^B = \frac{\sum_t x_t y_t}{\sum_t x_t^2} \equiv \hat{\beta}_{agg}$.

Proof.

Simplifying the Bartik estimator, we get:

$$\hat{\theta}_T^B = \underbrace{\begin{bmatrix} \sum_t B'X_t z_t & \sum_t z_t x_t \sum_i b_i - \sum_t B'X_t z_t \\ \sum_t \sum_t z_t x_t \sum_i b_i - \sum_t B'X_t z_t & (N-2)\sum_t z_t x_t \sum_i b_i + \sum_t B'X_t z_t \end{bmatrix}^{-1}}_{\equiv (A+\tilde{B})^{-1}} \underbrace{\begin{bmatrix} \sum_t B'Y_t z_t \\ \sum_t z_t y_t \sum_i b_i - \sum_t B'Y_t z_t \end{bmatrix}}_{\equiv Z'Y}$$

where it is useful to decompose the inverse matrix in terms of matrices A and $\tilde{B}$:

$$A \equiv \begin{bmatrix} 0 & \sum_t z_t x_t \sum_i b_i \\ \sum_t z_t x_t \sum_i b_i & (N-2)\sum_t z_t x_t \sum_i b_i \end{bmatrix}, \quad \tilde{B} \equiv \begin{bmatrix} \sum_t B'X_t z_t & -\sum_t B'X_t z_t \\ -\sum_t B'X_t z_t & \sum_t B'X_t z_t \end{bmatrix}$$

Note that A^{-1} is well defined and $\tilde{B}$ has rank 1. Then, by equation (1) in Miller (1981), one can write the inverse of $(A + \tilde{B})$ as

$$(A+\tilde{B})^{-1} = A^{-1} - \frac{1}{1+g} A^{-1}\tilde{B}A^{-1}$$

where $g \equiv tr(\tilde{B}A^{-1})$. Next, calculate the two matrices in the previous expression:

$$A^{-1} = \frac{-1}{\sum_t x_t z_t \sum_i b_i} \begin{bmatrix} (N-2) & -1 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad A^{-1}\tilde{B}A^{-1} = \begin{bmatrix} (N-1)^2 & -(N-1) \\ -(N-1) & 1 \end{bmatrix}$$

Consider now

[19] This is a slight abuse of notation as B is used to refer both to the vector of Bartik weights as well as the implied weight matrix from Equation (16)

$$\begin{aligned}
\begin{bmatrix}1 & (N-1)\end{bmatrix}\frac{-1}{1+g}A^{-1}\tilde{B}A^{-1}Z'Y &= \frac{-1}{1+g}\begin{bmatrix}1 & (N-1)\end{bmatrix}\begin{bmatrix}(N-1)^2 & -(N-1)\\ -(N-1) & 1\end{bmatrix}Z'Y\\
&= \begin{bmatrix}0 & 0\end{bmatrix}Z'Y\\
&= 0
\end{aligned}$$

This implies that $\begin{bmatrix}1 & (N-1)\end{bmatrix}\hat{\theta}_T^B = \begin{bmatrix}1 & (N-1)\end{bmatrix}A^{-1}Z'Y$. Evaluating:

$$\begin{aligned}
\begin{bmatrix}1 & (N-1)\end{bmatrix}A^{-1}Z'Y &= \frac{-1}{\sum_t x_t z_t \sum_i b_i}\begin{bmatrix}1 & (N-1)\end{bmatrix}\begin{bmatrix}(N-2) & -1\\ -1 & 0\end{bmatrix}Z'Y\\
&= \frac{1}{\sum_t x_t z_t \sum_i b_i}\begin{bmatrix}1 & 1\end{bmatrix}Z'Y\\
&= \frac{1}{\sum_t x_t z_t \sum_i b_i}\sum_t x_t y_t \sum_i b_i\\
&= \frac{\sum_t x_t y_t}{\sum_t x_t^2} = \hat{\beta}_{agg}
\end{aligned}$$

where the second-to-last line uses the fact that $z_t \equiv x_t$.

□

A.2.2 Identity-Weighted GMM

Proposition 2. *The sum of the local effect and the appropriately scaled spillover effect estimated with the identity-weighted GMM estimator ($\hat{\theta}_T^I$) is numerically equivalent the OLS estimator from the aggregate regression. That is,* $\begin{bmatrix}1 & (N-1)\end{bmatrix}\hat{\theta}_T^I = \frac{\sum_t x_t y_t}{\sum_t x_t^2} \equiv \hat{\beta}_{agg}$.

Proof. Recall that the identity-weighted GMM estimator, $\hat{\theta}_T^I$, is given by

$$\sqrt{T}\left(\hat{\theta}_T - \theta_0\right) = \left[\left(\sum_{t=1}^T z_t\Gamma_t\right)'\left(\sum_{t=1}^T z_t\Gamma_t\right)\right]^{-1}\left(\sum_{t=1}^T z_t\Gamma_t\right)'\left(\sum_{t=1}^T z_t Y_t\right)$$

It is straightforward to rewrite the inverted matrix as the sum of two matrices

$$\left[\left(\frac{1}{T}\sum_{t=1}^T z_t\Gamma_t\right)'\left(\frac{1}{T}\sum_{t=1}^T z_t\Gamma_t\right)\right] = \begin{bmatrix}0 & \left(\sum_t z_t x_t\right)^2\\ \left(\sum_t z_t x_t\right)^2 & (N-2)\left(\sum_t z_t x_t\right)^2\end{bmatrix} + \begin{bmatrix}\sum_i\left(\sum_t z_t x_{it}\right)^2 & -\sum_i\left(\sum_t z_t x_{it}\right)^2\\ -\sum_i\left(\sum_t z_t x_{it}\right)^2 & \sum_i\left(\sum_t z_t x_{it}\right)^2\end{bmatrix}$$

Similarly, the second matrix can be rewritten as

$$\left(\sum_{t=1}^{T} z_t \Gamma_t\right)' \left(\sum_{t=1}^{T} z_t Y_t\right) = \begin{bmatrix} \sum_i \left(\sum_t z_t x_{it}\right)\left(\sum_t z_t y_{it}\right) \\ \left(\sum_t z_t x_t\right)\left(\sum_t z_t y_t\right) - \sum_i \left(\sum_t z_t x_{it}\right)\left(\sum_t z_t y_{it}\right) \end{bmatrix}$$

Then, following similar steps as in the proof of Proposition 1, *mutatis mutandi*, one gets to the desired result: First, one inverts the first matrix using the result from equation (1) in Miller (1981). From there, it is easy to multiply the various matrices and cancel out common scaling factors as is done above. □

A.3 Additional Tables

Table 3: Defense spending multipliers (local, spillover and total) using three different aggregation levels

Panel A

	Benchmark (33 states)		Census Divisions		48 States	
	Local	Spillover	Local	Spillover	Local	Spillover
Zero-Spillover	1.408^{***}		1.474^{***}		1.401^{***}	
Restriction (w/ time FEs)	(0.272)		(0.403)		(0.245)	
Zero-Spillover	1.051^{***}		0.694^{**}		1.139^{***}	
Restriction	(0.304)		(0.313)		(0.269)	
Local-Spillover	1.235^{***}	-0.663^{***}	1.622^{***}	-0.759^{**}	1.300^{***}	-0.736^{***}
Decomposition	(0.308)	(0.127)	(0.418)	(0.323)	(0.281)	(0.117)

Panel B

	Benchmark (33 states)	Census Divisions	48 States
Sum	0.571^{*}	0.863^{**}	0.565^{**}
	(0.308)	(0.371)	(0.279)

Notes: FE=fixed effects. T=39 for Benchmark 33 pseudo-states, 9 Census Divisions, and 48 States, respectively. Spatial-temporal corrected standard errors reported in parentheses (for "Benchmark" and "48 States" $t_0 = 5, d_0 = 2000$ and for "Census Divisions"$t_0 = 5$ and with arbitrary spatial correlation). *0.10 **0.05 ***0.01.

Table 3 presents results of efficient GMM estimation at different levels of aggregation. The columns "Benchmark" and "48 States" estimate equation (1) using their respective state panels. The "Census Divisions" panel estimates (1) subsequent to a partial aggregation of (1) from the 33 pseudo-state panel up to 9 Census divisions. For "Benchmark" and "48 States", we estimate using $t_0 = 5$ and $d_0 = 2000$. For "Census Divisions" column, likewise we estimate with $t_0 = 5$. However, $w(i, j) = 1$ for all i, j in (13) where they identify Census Divisions.