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Authors	David Andolfatto
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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Maturity structure and liquidity insurance

D. Andolfatto*
david.andolfatto@stls.frb.org

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Abstract

This paper studies the optimal maturity structure for government debt when markets for liquidity insurance are incomplete or non-competitive. There is no fiscal risk. Government debt in the model solves a dynamic inefficiency. Issuing debt in short and long maturities solves a liquidity insurance problem, but optimal yield curve policy is only possible if long-duration debt is rendered illiquid. Optimal policy is implementable through treasury operations only–adjustments in the primary deficit are not necessary.

Keywords: Yield curve, maturity structure, liquidity.

JEL Codes: E4, E5

*Research Division, Federal Reserve Bank of St. Louis, Broadway and Locust, 1 Federal Reserve Bank Plaza, St. Louis, MO 63102

1 Introduction

The principles of optimal debt management are typically studied for a government subject to funding risk in an incomplete markets setting. Depending on model specifics, a variety of recommendations for optimal maturity structure emerge; see, for example, Angeletos (2002), Buera and Nicolini (2004), Lustig, Sleet, and Yeltekin (2008), Nosbusch (2008), Guibaud, Nosbusch and Vayanos (2013), and Greenwood, Hanson and Stein (2015). Funding risk plays an essential role in all these analyses. Absent risk, the optimal structure of debt–in terms of either maturity or indexation–is indeterminate (Barro, 2003).

In this paper, I ask whether there is a rationale for issuing debt of different maturities even in the absence of funding risk.[1] I use an analytical framework that combines the Samuelson (1958) model of government debt with the Diamond and Dybvig (1983) model of idiosyncratic liquidity risk. There is no aggregate uncertainty. Because the economy is dynamically inefficient, there is a welfare-enhancing role for debt. But if private risk-sharing arrangements are either unavailable or noncompetitive, then the equilibrium remains generically inefficient. I demonstrate how the slope of the yield curve can be chosen to implement the efficient allocation as an equilibrium. Yield curve control is made possible by rendering the long-duration security non-marketable, but with a treasury buy-back option set at an appropriate discount rate.[2] In this way, investors who turn out to be "impatient" have the option to redeem off-the-run securities for cash, while "patient" investors have the opportunity of earning a higher rate of return on their savings. Importantly, the optimal policy is implementable without fiscal (tax) support.

The idea that an illiquid store of value might improve risk-sharing outcomes was first explored, as far as I know, by Kocherlakota (2003).[3] His

[1] Bigio, Nuño and Passadore (2017) use a search-theoretic framework that implies a role for maturity structure even in the absence of aggregate uncertainty. Absent search frictions, however, the need for different maturities vanishes. This is in contrast to my model below where trading frictions are necessary to improve welfare.

[2] The possibility of yield curve control is not simply a theoretical assertion. From 1942-51, for example, the interest rate on 3-month U.S. Treasury securities was pegged at 3/8% and they yield on 10-year treasury securities never exceeded 2.5%; see Eichengreen and Garber (1991) and Garbade (2020).

[3] See also Shi (2008) and Andolfatto (2011). This counterintuitive result is related to the theory of the second best (Lipsey and Lancaster, 1956) which states that adding a

analysis considers two physically identical fiat tokens, labeled *cash* and *bonds*. Bonds turn into cash one period hence, but are illiquid in the sense that they cannot be used to purchase goods and services. However, in contrast to my set up, bonds are marketable–that is, they can be exchanged for cash on a financial market that opens after individuals learn their preferences. Individuals who turn out to value consumption highly sell their bonds for cash, while those willing to defer consumption take the other side of this trade. The relative illiquidity of bonds means that they must trade at a discount to cash. However, the equilibrium discount rate serves to better align the marginal utility of consumption across impatient and patient states.[4]

In the setup I study below, I consider one and two-period bonds, each of which turn into cash upon maturity. Two-period bonds with one period left to maturity are "off-the-run." If off-the-run bonds are perfectly liquid, they must compete directly with newly-issued "on-the-run" one-period bonds. That is, both instruments constitute risk-free claims to cash one period hence and so arbitrage implies that investors must then be indifferent between purchasing short-term debt and rolling it over if cash is not needed, or purchasing long-term debt and liquidating it if cash is needed. If this is the case, however, then the maturity structure of debt turns out to be inconsequential. Some friction in the secondary market for debt must be introduced for term structure manipulation to be possible. I assume that long-term debt is issued in non-marketable form. But yield curve control remains possible even with less severe trading frictions in place.[5]

trade friction need not reduce economic welfare if the initial equilibrium is inefficient.

[4]Kocherlakota (2003) does not attempt to characterize an optimal intervention. His point is simply to demonstrate how an illiquid bond can help achieve a superior risk-sharing arrangement.

[5]For example, the search frictions in Bigio, Nuño and Passadore (2017). In reality, these search frictions are relevant even for U.S. treasury securities. This is because the treasury issues securities that vary by coupon and maturity so that, over time, each becomes a unique "off-the-run" security (in the United States, each is identified by a CUSIP number) with specific duration/liquidity risk characteristics. Owing to the idiosyncratic nature of off-the-run securities, they tend to trade in relatively illiquid over-the-counter markets. On the other hand, there is nothing to prevent the central bank and treasury from operating standing facilities willing to exchange government bonds of various maturities on demand and at specified prices for central bank liabilities or relatively liquid "on-the-run" treasury securities. In short, liquidity (or the lack thereof) is a policy choice.

2 The environment

Consider an economy populated by 3-period-lived overlapping generations, which I refer to as the *young*, the *middle-aged*, and the *old*. Time is denoted by $t = 1, 2, ..., \infty$. The population is constant, so at any date t there exists an equal measure of each generation.

The young have a nonstorable endownment, ω. The middle-aged and old have no earnings. A fraction $1 - \beta$ middle-aged people want to consume early; a complementary fraction β wish to postpone consumption into old age. The young do not know whether they will be patient or impatient later in life.[6] Let c_t^m, c_t^o denote the consumption of middle-aged and old at date t, respectively. A young person at date $t \geq 1$ has preferences,

$$U_t = (1 - \beta)u(c_{t+1}^m) + \beta\alpha u(c_{t+2}^o) \tag{1}$$

There is an initial middle-aged population that lives for two periods, and an initial old generation that lives for one period. The golden rule allocation (c^m, c^o) maximizes (1) subject to the resource constraint,

$$\omega = (1 - \beta)c^m + \beta c^o \tag{2}$$

Optimality in this sense requires

$$u'(c_*^m) = \alpha u'\left(c_*^o\right) \tag{3}$$

Note that $c_*^m = c_*^o = \omega$ if $\alpha = 1$. Below I assume $\alpha > 1$ so that individuals have an *ex ante* preference to defer consumption, $c_*^m < \omega < c_*^o$.

This economy is, by construction, dynamically inefficient. Hence, there is an efficiency-enhancing role for an outside asset like government money or debt (Samuelson, 1958). However, there is also an insurance problem to be solved here. To begin, I assume the absence of any risk-sharing arrangement. My hunch is that issuing debt in different maturities may be a way to offer liquidity insurance when other risk-sharing arrangements are unavailable.

[6] A "preferred-habitat" motive could be introduced here by assuming that a subset of individuals know the preferred timing of their expenditure needs beforehand.

3 Equilibrium with government debt

In what follows, I consider a "cashless" economy where all money balances are recorded on a ledger managed by a consolidated central bank and treasury agency.

3.1 Government policy

The government treasury issues two nominal securities, both pure discount bonds representing sure claims to future cash (i.e., credits to money accounts). Let B_t^1 denote the face value of one-period securities (bills) and B_t^2 the face value of two-period securities (bonds) issued at date $t \geq 0$.[7] A bill issued at date t turns into cash at $t+1$. A bond issued at date t turns into cash at date $t+2$. Let q_t^1, q_t^2 denote the price of bills and bonds at date t, respectively. I assume that q_t^1 is the central bank policy rate; i.e., $1/q_t^1$ is the short-run nominal interest rate. Note that we might alternatively (and equivalently) imagine the treasury issuing D_t dollars of bonds at date t, with the central bank "monetizing" some fraction $0 \leq \theta_t \leq 1$ of the bond-issue with interest-bearing reserves, $B_t^1 = \theta_t D_t$.

The supply of off-the-run bonds at date t is B_{t-1}^2. If these bonds were marketable, then in a competitive and frictionless bond market, they would be priced at q_t^1. In what follows, I assume that bonds are, as a matter of policy, rendered nonmarketable.[8] I assume, however, that the treasury (or central bank) operates a facility that stands ready to purchase off-the-run bonds at an administered price $\hat{q}_t^1$. A policy of $\hat{q}_t^1 = q_t^1$ here replicates the outcome of a competitive bond market, but in general $\hat{q}_t^1 \neq q_t^1$ is possible. The price of on-the-run bonds q_t^2 will have to satisfy a no-arbitrage condition.

There is no government spending or taxes, so the government primary surplus is always equal to zero. Let $X_t \leq B_{t-1}^2$ denote the quantity of off-the-run bonds purchased by the treasury at date t at price $\hat{q}_t^1$. Then at date t, the treasury has an obligation to deliver $B_{t-1}^1 + \left[B_{t-2}^2 - X_{t-1}\right]$ dollars to those

[7] The initial debt is distributed in some manner to the initial middle-aged and initial old. As I focus on stationary equilibrium in what follows, the initial distribution of wealth can be ignored in what follows.

[8] What I have to say below will continue to hold qualitatively as long as the secondary market for bonds is not frictionless.

individuals in possession of maturing debt. In addition, it needs $\hat{q}_t^1 X_t$ dollars to finance its bond repurchases. Since tax revenue is normalized to zero here, these obligations must be met entirely with money collected through the sale of new debt, $q_t^1 B_t^1 + q_t^2 B_t^2$. The treasury's period t income and expenditure relation is given by,

$$q_t^1 B_t^1 + q_t^2 B_t^2 = B_{t-1}^1 + \hat{q}_t^1 X_t + \left[B_{t-2}^2 - X_{t-1}\right] \tag{4}$$

for $t \geq 1$ with $B_0^1, B_{-1}^2, X_0 > 0$ given. In what follows, it will be convenient to express $B_t^1 \equiv \theta_t D_t$, so that (4) may be rewritten as,

$$\left[\theta_t q_t^1 + (1-\theta_t) q_t^2\right] D_t = \theta_{t-1} D_{t-1} + \hat{q}_t^1 X_t + \left[(1-\theta_{t-2}) D_{t-2} - X_{t-1}\right] \tag{5}$$

A *government policy* consists of a sequence

$$\{q_t^1, \hat{q}_t^1, q_t^2, \theta_t, D_t, X_t\}_{t=1}^{\infty}$$

satisfying (5) for all $t \geq 1$, with given initial condition $B_0^1, B_{-1}^2, X_0 \geq 0$.

In what follows, I assume $\hat{q}_t^1 \leq q_t^1$ for all $t \geq 1$.

3.2 Individual decision making

Let p_t denote the price-level at date t. Let $b_t^1, b_t^2 \geq 0$ denote individual bill and bond holdings, respectively. The sequence of budget constraints facing a young person at date t are given by,

$$p_t \omega = q_t^1 b_t^1 + q_t^2 b_t^2 \tag{6}$$

$$p_{t+1} c_{t+1}^m = b_t^1 + \hat{q}_{t+1}^1 b_t^2 \tag{7}$$

$$p_{t+2} c_{t+2}^o = (1/q_{t+1}^1) b_t^1 + b_t^2 \tag{8}$$

Since the young do not value consumption, condition (6) anticipates that they will save their entire nominal income $p_t \omega$, here, in the form of bills and/or bonds.[9] Individuals will want to consume with probability $1-\beta$ in their middle-age. Since the "impatient" do not value future consumption, condition (7) anticipates that the impatient middle-aged will spend their entire wealth, which consists of money from maturing bills b_t^1 and money

[9] Note that the young cannot buy off-the-run bonds at price $\hat{q}_t^1$, something they would like to do if $\hat{q}_t^1 < q_t^1$.

from the sale of bonds $\hat{q}^1_{t+1}b^2_t$ back to the treasury. Patient middle-aged individuals will want to rollover their bills at interest rate $1/q^1_{t+1}$. Note that as long as $\hat{q}^1_{t+1} \leq q^1_{t+1}$, patient investors prefer not to present their bonds for early redemption. As this is the case, these latter individuals spend their accumulated wealth $(1/q^1_{t+1})b^1_t + b^2_t$ in old age.

Formally, the young solve the following problem,

$$\max_{b^1_t, b^2_t}(1-\beta)u(\left[b^1_t + \hat{q}^1_{t+1}b^2_t\right] p^{-1}_{t+1}) + \beta\alpha u(\left[(1/q^1_{t+1})b^1_t + b^2_t\right] p^{-1}_{t+2}) \tag{9}$$

subject to $\omega - [q^1_t b^1_t + q^2_t b^2_t] p^{-1}_t \geq 0$. Let $\lambda_t \geq 0$ denote the Lagrange multiplier associated with this latter constraint and define $\Pi_{t+1} \equiv p_{t+1}/p_t$. Then the first-order necessary conditions characterizing optimal behavior are given by,

$$\Pi^{-1}_{t+1}q^1_{t+1}(1-\beta)u'(c^m_{t+1}) + \Pi^{-1}_{t+1}\Pi^{-1}_{t+2}\beta\alpha u'(c^o_{t+2}) = \lambda_t q^1_t q^1_{t+1} \tag{10}$$

$$\Pi^{-1}_{t+1}\hat{q}^1_{t+1}(1-\beta)u'(c^m_{t+1}) + \Pi^{-1}_{t+1}\Pi^{-1}_{t+2}\beta\alpha u'(c^o_{t+2}) = \lambda_t q^2_t \tag{11}$$

Below I study a bills only economy ($\theta_t = 1$ for all t) followed by an economy with both bills and bonds ($0 < \theta_t < 1$ for all t). In the bills only economy, condition (11) is absent. The portfolio decision is trivial in this case,

$$b^1_t = (1/q^1_t)p_t\omega \tag{12}$$

In an economy with both bills and bonds, conditions (10)-(11) imply,

$$\frac{q^1_{t+1}(1-\beta)u'(c^m_{t+1}) + \Pi^{-1}_{t+2}\beta\alpha u'(c^o_{t+2})}{q^1_t q^1_{t+1}} = \frac{\hat{q}^1_{t+1}(1-\beta)u'(c^m_{t+1}) + \Pi^{-1}_{t+2}\beta\alpha u'(c^o_{t+2})}{q^2_t} \tag{13}$$

Note that if policy sets $\hat{q}^1_t \equiv q^1_t$ for all t, then (13) implies that if bills and bonds are to be willingly held by the young, the following condition must hold,

$$q^2_t = q^1_t q^1_{t+1} \tag{14}$$

Condition (14) is just the pure expectations hypothesis of the term structure. When this condition holds, bonds turn out to be a redundant policy instrument. However, if policy discounts off-the-run bonds ($\hat{q}^1_t < q^1_t$), then (13) implies that on-the-run bonds will have to trade at a discount ($q_t^2 < q_t^1 q_{t+1}^1$) if they are to be held. In this latter case, there is a positive term premium. The question is whether policy can exploit this term premium to improve economic outcomes. Of course, before I can answer this question, I will first need to characterize the equilibrium for a single security.

3.3 Equilibrium

Individual optimization is described either by (12) or by (13), depending on the case. As young individuals are identical, I assume they choose the same portfolios. In equilibrium,

$$b_t^1 = \theta_t D_t \text{ and } b_t^2 = (1-\theta_t) D_t \tag{15}$$

As only impatient middle-aged individuals want to dispose of their bonds, the volume of bonds repurchased by the treasury at date t equals $X_t = (1-\beta)(1-\theta_{t-1})D_{t-1}$. Combining this latter expression with the government budget constraint (5) yields,

$$\left[\theta_t q_t^1 + (1-\theta_t) q_t^2\right] D_t = \left[\theta_{t-1} + \hat{q}_t^1 (1-\beta)(1-\theta_{t-1})\right] D_{t-1} + \beta(1-\theta_{t-2}) D_{t-2} \tag{16}$$

Next, consider the budget constraint for middle-aged individuals (7) and impose the conditions (15),

$$c_{t+1}^m = \left[\theta_t + (1-\theta_t)\hat{q}_{t+1}^1\right] \Pi_{t+1}^{-1} \left(D_t / p_t\right) \tag{17}$$

From the resource constraint, we have

$$\omega = (1-\beta) c_t^m + \beta c_t^o \tag{18}$$

And finally, from the budget constraint for the young, combined with conditions (15), we have,

$$\omega = \left[\theta_t q_t^1 + (1-\theta_t) q_t^2\right] \left(D_t / p_t\right) \tag{19}$$

3.3.1 Stationary equilibrium

In a stationary equilibrium, $q_t^1, \hat{q}_t^1, q_t^2, \theta_t, c_t^m, c_t^o, \Pi_t$ are all constant. It follows from (19) that in a stationary equilibrium, the quantity of real outside assets

(D_t/p_t) is determined by,

$$(D_t/p_t) = \left[\theta q^1 + (1-\theta)q^2\right]^{-1}\omega \tag{20}$$

The right-hand-side of (20) represents the demand for real outside assets, an object that is increasing in income and decreasing in both short and long interest rates. The left-hand-side of (20) represents the real quantity of outside assets. Condition (20) implies that the equilibrium inflation rate is determined solely by the growth in nominal outside assets, $\Pi = D_t/D_{t-1}$. Given $D_0 > 0$, it follows that $D_1 = \Pi D_0$, so that the initial price-level is determined by $p_1 = [\theta q^1 + (1-\theta)q^2](D_1/\omega)$.

Using the fact that $D_t = \Pi D_{t-1} = \Pi^2 D_{t-1}$, rewrite the government budget constraint (16) as

$$\left[\theta q^1 + (1-\theta)q^2\right]\Pi^2 = \left[\theta + \hat{q}^1(1-\beta)(1-\theta)\right]\Pi + \beta(1-\theta) \tag{21}$$

This condition determines the inflation rate as a function of policy parameters and the equilibrium long rate, q^2. Imposing stationarity on condition (13) and combining with the resource constraint (18) yields,

$$q^1\Pi\left[q^2 - q^1\hat{q}^1\right](1-\beta)u'(c^m) = \left[(q^1)^2 - q^2\right]\beta\alpha u'\left(\frac{\omega-(1-\beta)c^m}{\beta}\right) \tag{22}$$

Note that (22) only holds for case (13) and under the condition $\hat{q}^1 \neq q^1$. Finally, imposing stationarity on (17) yields,

$$\Pi c^m = \left[\theta + \hat{q}^1(1-\theta)\right](D_t/p_t) \tag{23}$$

Conditions (20)-(23) characterize the equilibrium values for $c^m, \Pi, q^2, (D_t/p_t)$ as functions of the policy parameters $q^1, \hat{q}^1, \theta$.

3.4 Properties of the stationary equilibrium

3.4.1 A single security

I begin by considering the case of a single security by setting $\theta = 1$. For $q^1 = 1$, the model reduces to a standard overlapping generations model of money, with the money supply given by D_t. For $q^1 \neq 1$, money is interest-bearing. When $q^1 < 1$, money bears positive interest and when $q^1 > 1$, money bears negative interest. Imposing $\theta = 1$ on condition (21), we have an expression that determines the equilibrium inflation rate,

$$\Pi = 1/q^1 \tag{24}$$

In this economy, the equilibrium real rate of interest $r = (q^1\Pi)^{-1}$ is independent of the policy rate q^1. A lower policy interest rate $1/q^1$ implies a lower rate of inflation. Indeed, with a negative nominal interest rate, the economy experiences a deflation. Conversely, a higher policy interest induces a higher rate of inflation. The mechanism here is very straightforward: because the primary surplus is fixed, the interest expense of money can only be financed through money creation. A higher interest rate implies that the money supply must grow more rapidly.[10]

Condition (22), which characterizes the individual trade-off between holding bills and bonds is, of course, not relevant when there is only a single security. Instead, we refer to case (12). Imposing the market-clearing condition $b_t^1 = D_t$ on (12) implies condition (20) when $\theta = 1$, that is $(D_t/p_t) = \omega/q^1$. This latter condition, when combined with conditions (23) and (24), implies $c^m = \omega$. From the resource constraint (18), we have $c^o = \omega$ as well. This leads us to my first result.

Proposition 1 *In an economy with a single government security, the equilibrium allocation is given by* $(c^m, c^o) = (\omega, \omega)$ *and is invariant to the policy interest rate* $1/q^1$*. The equilibrium implements the Golden Rule if and only if* $\alpha = 1$.

That is, recall that the Golden Rule allocation satisfies (2) and (3), so that the equilibrium allocation with a single security is generically suboptimal. In particular, $u'(\omega) < \alpha u'(\omega)$, since $\alpha > 1$.

[10] While this result is consistent with the Neo-Fisherian view that interest rate policy determines inflation, the mechanism here is very different. The Neo-Fisherian view is that increasing the policy rate causes inflation expectations to rise to satisfy a no-arbitrage condition (the Fisher relation); see Williamson (2016). In the model here, inflation would rise even if individuals are myopic. Inflation rises here owing to standard "Monetarist" principles; i.e., the government is printing paper at a faster rate to finance a larger interest expense.

3.4.2 Two securities

Consider now the case of two securities $0 < \theta < 1$ but where $\hat{q}^1 = q^1$. We know from condition (14) that in this case, $q^2 = (q^1)^2$. As in the previous case, condition (22), which characterizes the individual trade-off between between holding bills and bonds, is not relevant here. This was true before because only bills were in existence. Here we have both bills and bonds, but when $\hat{q}^1 = q^1$, the two securities are effectively perfect substitutes. The condition determining the equilibrium inflation rate (21), after some algebraic manipulation, can be shown to reduce to condition (24).

Proposition 2 *In an economy with bills and bonds and policy configuration $\hat{q}^1 = q^1$, the equilibrium allocation is given by $(c^m, c^o) = (\omega, \omega)$ and is invariant to the policy interest rate $1/q^1$. The equilibrium nominal yield on bonds is determined by the pure expectations hypothesis of the term structure, i.e., $q^2 = (q^1)^2$.*

Thus, the nonmarketability of treasury securities is inconsequential if the treasury stands ready to purchase off-the-run bonds in a manner that renders them equivalent to on-the-run bills. Such a policy is exactly how a well-functioning secondary market for bonds would have priced these securities through financial arbitrage. What happens, however, if the treasury exploits the illiquidity of off-the-run bonds by discounting them relative to newly-issued bills? From condition (22), we see that setting $\hat{q}^1 < q^1$ is likely to have real consequences. Can this "wedge" be exploited to deliver superior liquidity insurance? I turn to this question next.

3.4.3 Two securities with a discount policy

For convenience, let me gather the relevant economic restrictions here. First, rearranging (22), we have

$$u'(c^m) = \Lambda \alpha u' \left(\frac{\omega - (1-\beta)c^m}{\beta} \right) \tag{25}$$

where

$$\Lambda \equiv \left[\frac{[(q^1)^2 - q^2]\,\beta}{q^1 \Pi \left[q^2 - q^1 \hat{q}^1\right](1-\beta)} \right] \tag{26}$$

Next, from the government budget constraint (21), we have

$$\left[\theta q^1 + (1-\theta)q^2\right]\Pi^2 = \left[\theta + \hat{q}^1(1-\beta)(1-\theta)\right]\Pi + \beta(1-\theta) \tag{27}$$

Finally, combine (20) and (23) to form,

$$c^m = \left[\frac{\theta + (1-\theta)\hat{q}^1}{\theta q^1 + (1-\theta)q^2}\right]\left(\frac{\omega}{\Pi}\right) \tag{28}$$

Conditions (25)-(28) characterize the equilibrium values (Π, q^2, c^m) as functions of policy parameters $(q^1, \hat{q}^1, \theta)$.

In what follows, let us fix q^1 and θ, and consider variations in the discount price $\hat{q}^1$. We know that for $\hat{q}^1 = 1$, we have $c^m = \omega$. If $\alpha > 1$, we want to manipulate $\hat{q}^1$ in a manner that reduces c^m and increases c^o. Evidently, this means adjusting $\hat{q}^1$ in a manner that sets $\Lambda = 1$, if possible. The following result simplifies the analysis that follows.

Proposition 3 *The equilibrium inflation rate associated with efficient implementation is given by* $\Pi = 1/q^1$.

The result above is not immediately obvious from inspecting (25)-(28). But the claim can be validated by "guess and verify." First, we know that efficient implementation requires $\Lambda = 1$. Then the two conditions $\Lambda = 1$ and (27) form a system of two equations describing the set of (q^2, Π) that are necessary for efficient implementation. Next, guess that $\Pi = 1/q^1$ is a solution to this system. The result is then easily verified.

Proposition 3 implies is that efficient implementation may be possible for any given monetary policy rate q^1 (and associated inflation rate). At the very least, it permits us to fix q^1 (and hence the inflation rate) and then search for values of $\hat{q}^1$ that improves risk-sharing. Since $q^1\Pi = 1$, condition (28) may be expressed as,

$$\left(\frac{c^m}{\omega}\right) = \left[\frac{\theta q^1 + (1-\theta)q^1\hat{q}^1}{\theta q^1 + (1-\theta)q^2}\right] \tag{29}$$

Clearly, $\hat{q}^1 = q^1$ implies $c^m = \omega$. We want to find a $\hat{q}^1$ that implements $c^m = c^m_*$. When $\alpha > 1$, this evidently requires discounting off-the-run bonds, $\hat{q}^1 < q^1$.

Setting $\Lambda = 1$ implies that the equilibrium discount price of bonds is given by $q^2 = (1-\beta)q^1\hat{q}^1 + \beta(q^1)^2$. Substituting this latter expression into (29) permits us to solve for the optimal discount price,

$$\hat{q}^1 = \frac{\left[\theta + (1-\theta)\beta q^1\right](c_*^m/\omega) - \theta}{(1-\theta)\beta} \tag{30}$$

This discount policy can be applied for any given interest rate (and inflation) policy q^1.

Proposition 4 *The Golden rule allocation can be implemented as an equilibrium for any given policy interest rate q^1 without the use of tax policy, if the treasury finances the government deficit with both bills and non-marketable bonds $(0 < \theta < 1)$ and stands prepared to accept off-the-run bonds at a discount determined by (30).*

4 Banking

Suppose that the treasury only issues marketable securities and that secondary markets for off-the-run bonds work perfectly well. Then, by Proposition 2, we know that the equilibrium is inefficient and that bonds are a redundant instrument. In this section, I consider the scope for private risk-sharing arrangements in this type of environment.

I assume that the realization of preference type is private information. In this case, any implementable allocation will have to be incentive-compatible. The equilibrium in Proposition 2 is incentive-compatible, since it is a market mechanism. That is, patient individuals have no incentive to misrepresent themselves as being impatient, and vice-versa.[11]

Collectively, the young of any generation can do better than what is achievable through marketable bonds. In what follows, I examine a risk-sharing arrangement implemented through a Diamond and Dybvig (1983) banking arrangement. In particular, the bank takes cash deposits $p_t\omega$, which it uses to invest in bills, against which it issues interest-bearing deposit liabilities.[12] Note that private information makes the use of demandable liabilities

[11] The equilibrium described in Proposition 4 is also incentive-compatible.

[12] I restrict attention to a bills only economy since bonds are redundant.

essential here. Formally, the Diamond and Dybvig (1983) bank solves the following problem,

$$\max(1-\beta)u(c_{t+1}^m)+\beta\alpha u(c_{t+2}^o) \tag{31}$$

subject to

$$\begin{aligned} p_t\omega &= q_t^1 b_t^1 && (32)\\ (1-\beta)p_{t+1}c_{t+1}^m &\leq b_t^1 && (33)\\ \beta p_{t+2}c_{t+2}^o &= (1/q_{t+1}^1)\left[b_t^1-(1-\beta)p_{t+1}c_{t+1}^m\right] && (34) \end{aligned}$$

Combining constraints and substituting into the objective reduces the problem to,

$$\max_{c_{t+1}^m}(1-\beta)u\left(c_{t+1}^m\right)+\beta\alpha u\left(\frac{(1/q_{t+1}^1)\left[p_t\omega/q_t^1-(1-\beta)p_{t+1}c_{t+1}^m\right]}{\beta p_{t+2}}\right) \tag{35}$$

The first-order necessary condition is given by,

$$u'(c_{t+1}^m)=(p_{t+1}/p_{t+2})(1/q_{t+1}^1)\alpha u'(c_{t+2}^o) \tag{36}$$

In a steady-state we know from arguments that should now be familiar that $\Pi=1/q^1$ and $(1-\beta)c^m+\beta c^o=\omega$ which, when combined with (36) implies,

$$u'(c^m)=\alpha u'\left(\frac{\omega-(1-\beta)c^m}{\beta}\right) \tag{37}$$

which corresponds to the Golden rule allocation.

To be clear, the optimal bank deposit contract here offers young depositors a security that resembles a term deposit with a put option that can be exercised subject to penalty. A bill offers investors a zero real rate of return, period by period (i.e., $c^m/\omega=c^o/\omega=1$). The optimal deposit contract offers depositors a superior return, but only if the deposit is held to term, i.e., $c_*^o/\omega>1$. If depositors wish to withdraw early, they earn an inferior return, i.e., $c_*^m/\omega<1$. The difference $(c_*^o-c_*^m)/\omega>0$ can be interpreted as a redemption fee retained by the bank to be disbursed to future depositors.

The question remaining is whether the optimal banking arrangement is incentive-compatible when depositors have the option of investing directly in government bills. Impatient depositors clearly have no incentive to misrepresent themselves. The question, as usual, is whether patient depositors have an incentive to misrepresent themselves as being impatient. As originally pointed out by Jacklin (1987), the availability of financial trades outside of the banking relationship may severely constrain, or even destroy, potential risk-sharing arrangements within the relationship.[13]

Risk-sharing is hampered here if patient depositors have an incentive to withdraw funds when middle-aged to pursue alternative investment opportunities. To check whether this is the case, suppose that a patient investor withdraws his money when middle-aged in period t. The maximum withdrawal permitted is $p_t c_*^m$ dollars. If the investor invests these funds in bills, the $t+1$ return is $(1/q^1)p_t c_*^m$ dollars which, when deflated by the future price-level p_{t+1}, generates $(\Pi/q^1)c_*^m$ units of consumption. Of course, since $(\Pi/q^1) = 1$, the trade is clearly unprofitable.[14]

Finally, there is the question of whether the Diamond and Dybvig (1983) bank is stable in the sense of being run-proof. Keep in mind that the bank modeled here is literally a narrow bank–its entire asset portfolio is made up of government treasury bills. Nevertheless, a bank run is not outside the realm of possibility even for a narrow bank depending on how the structure of promised returns interacts with other properties of the environment. A bank run may be possible if the bank is offering an elevated short-rate and if deposits are processed on a first-come, first-served basis. But in the model studied here, the bank is offering an elevated long-rate and there is nothing in the model environment that motivates sequential service. Moreover, if a banking arrangement did turn out to be unstable having to rely on its own reserves, access to a central bank or treasury lending facility would prevent a bank run if deposits are nominal (which, of course, they are).

4.1 A monopoly bank

Diamond and Dybvig (1983) implicitly assume that banking is competitive and costless, so that the entire surplus associated with risk-sharing accrues

[13] See also Andolfatto, Berentsen and Martin (2019) and the references cited within.

[14] It is of some interest to note that the Golden rule allocation is not implementable through a bank arrangement if $\alpha > 1$. In this latter case, we get the conventional Jacklin (1987) result that markets generically constrain banking arrangements.

to depositors. The reality, of course, seems much different than this. How does the presence of market power on the part of the banking sector influence the role of maturity structure, if it does so at all?

To study this question, I introduce a separate bank agent, one born at each date t, with risk-neutral preferences $V_t = x_{t+2}$, where x_t denotes consumption at date t. The costs of operating the bank are normalized to zero and I assume that the bank can commit to any budget-feasible promise it makes.

Let v_t denote the maximum utility associated with a young depositor's outside option, in this case, bypassing the banking system and saving directly through the treasury. Then the monopoly bank chooses an allocation (c_{t+1}^m, c_{t+2}^o) that maximizes profit

$$V_t = (1/q_{t+1}^1)\left[(p_t/p_{t+2})\omega - (1-\beta)(p_{t+1}/p_{t+2})c_{t+1}^m\right] - \beta c_{t+2}^o \tag{38}$$

subject to the participation constraint,

$$(1-\beta)u\left(c_{t+1}^m\right) + \beta\alpha u\left(c_{t+2}^o\right) \geq v_t \tag{39}$$

Let $\psi_t \geq 0$ denote the Lagrange multiplier associated with the constraint (39). An optimal deposit contract satisfies,

$$\begin{aligned} (1/q_{t+1}^1)(p_{t+1}/p_{t+2}) &= \psi_t u'\left(c_{t+1}^m\right) \\ 1 &= \psi_t \alpha u'\left(c_{t+1}^o\right) \end{aligned}$$

Invoking the steady-state equilibrium conditions, $(1/q_{t+1}^1)(p_{t+1}/p_{t+2}) = 1/(q^1\Pi) = 1$. Since the participation constraint must clearly bind ($\psi > 0$), the optimality conditions reduce to

$$u'\left(c^m\right) = \alpha u'\left(c^o\right) \tag{40}$$

$$(1-\beta)u\left(c^m\right) + \beta\alpha u\left(c^o\right) = v \tag{41}$$

From our earlier analysis, we know that $v \in [v^0, v^*]$, where

$$\begin{aligned} v^0 &= (1+\alpha)u(\omega) \\ v^* &= u(c_*^m) + \alpha u(c_*^o) \end{aligned}$$

In a bills-only economy or a bills and marketable-bonds economy, a depositor's outside option is $v^0 < v^*$. In order to extract surplus efficiently, the monopolist bank offers a deposit contract with better risk-sharing properties relative to what a depositor could achieve through the treasury market. In particular, condition (40) implies that $c^m < c^o$. But while the state-contingent consumption profile is attractive, the level of consumption is not. In more colloquial terms, the monopoly bank offers an attractive interest rate structure on deposits, but takes it all back (and more) in the form of service fees.

One way to rectify market power is to directly through bank regulations. In the context of this simple model, a simple tax on bank monopoly profits redistributed back to depositors will do the trick. In reality, of course, designing effective regulation and tax policy is not so straightforward. An alternative strategy is to impose bank discipline by offering a competing product that raises the value of outside options for bank customers. In the model here, the Golden rule allocation can be implemented through the treasury policy described in Proposition 4. A monopoly bank is compelled to offer a contract satisfying (40)-(41) subject to $v = v^*$, or risk losing its deposit funding. In equilibrium, there may be little, if any, take up of the treasury option.[15]

5 Conclusion

Discussions of maturity structure naturally focus on the problem of managing government funding risks. While this is surely the correct emphasis, it does not exclude the possibility that maturity structure may serve purposes unrelated to managing funding risk. In the application considered here, the use of non-marketable debt permits a yield curve structure that allows investors to manage their liquidity risk more efficiently, either directly, or through the competitive pressure such a policy is likely to exert on banks.

It would be of some interest to explore the use of non-marketable debt, like U.S. Saving Bonds or U.K. Certificates of Tax Deposit, for yield curve control in the presence of funding risk and macroeconomic stabilization. The analysis above suggests that encouraging secondary market liquidity for treasury securities–a goal sometimes expressed by treasury authorities–may not be conducive to attaining broader macroeconomic policy objectives.

[15] Andolfatto (2020) makes a related point in the context of central bank digital currency.

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