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# Tax Progressivity, Economic Booms, and Trickle-Up Economics\*

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## Abstract

We propose a method to decompose changes in the tax structure into orthogonal components measuring the level and progressivity of taxes. Similar to tax shocks found in the existing empirical literature, the level shock is contractionary. The tax progressivity shock is expansionary: Increasing tax progressivity raises (lowers) disposable income at the bottom (top) end of the income distribution by shifting the tax burden from the bottom to the top. If agents' marginal propensity to consume falls with income, the rise in consumption at the bottom more than compensates for the decline in consumption at the top. The resulting increase in output and consumption leads to rising capital gains for those at the high end of the income distribution that more than offsets their losses from higher income taxes. The net result is that an increasing progressivity leads to an increase in income inequality, contrary to what conventional wisdom might suggest. We interpret these results as evidence in favor of trickle up, not trickle down, economics.

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# 1 Introduction

Over the last 40 years, marginal tax rates for the top income brackets generally have fallen, resulting in a decline in income tax progressivity. The steady increase in income inequality over the same period has been well-documented [e.g., Alvaredo, Atkinson, Piketty and Saez (2013)]. Faced with increasing inequality, some suggest reversing the decrease in marginal tax rates for the highest income levels [e.g., Diamond and Saez (2011)]. At the same time, a literature has emerged to evaluate the optimal degree of progressivity of the tax system in theoretical models.<sup>1</sup> What is missing from the literature is an empirical analysis of the business cycle and inequality implications of variations in income tax progressivity. In this paper, we provide a novel measure of progressivity that we then use to undertake such an empirical analysis, using minimal restrictions from economic theory.

A key difficulty in measuring changes in progressivity is that tax legislation typically contains a combination of both level and progressivity changes. Most empirical studies that evaluate the economic effect of changes in income taxes focus on changes to the *level* of taxes. Typically, these studies identify tax shocks using narrative evidence [e.g., Romer and Romer (2010)], the unexpected changes in the tax revenue series identified by a causal ordering [e.g., Blanchard and Perotti (2002)], or sign restrictions [e.g., Mountford and Uhlig (2009)]. While estimates of the magnitude of the tax-level multiplier vary across model assumptions, the general consensus is that increasing the level of taxes is contractionary.

The contribution of this paper is to determine how variation in the tax burden across income levels affects economic outcomes. We develop a measure of income tax progressivity that is orthogonal to the level of taxes using data on the time-series variation of tax rates on different levels of wage income. Our empirical approach employs a factor-augmented VAR (FAVAR) using Bayesian methods, imposing priors from the literature on the term structure of interest rates to identify two factors: one that measures changes in the level of taxes and

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<sup>1</sup>Heathcote, Storesletten, and Violante (2017) and Feenberg, Ferriere, and Navarro (2017) both use quantitative heterogeneous agent models, concluding that the current level of U.S. progressivity is optimal. However, Feenberg, Ferriere, and Navarro (2017) suggest that, if the labor supply choice is on the extensive margin only, optimal tax progressivity is much higher. In contrast, Kindermann and Krueger (2014) find that marginal tax rates as high as 90 percent can be justified. Sachs, Tsyvinski and Werquin (2020) argue that more progressive taxes can be justified if the tax code is suboptimal.

one that measures changes in progressivity.<sup>2</sup> To evaluate the effects of shocks to the tax factors, macroeconomic variables (e.g., output, consumption, hours, deficits, tax revenue and measures of income and consumption inequality) are included in the VAR with the tax factors.

The tax level factor is identified by assuming positive factor loadings for all tax rates—increasing the tax level increases tax rates across the board—and is analogous to the level factor from the literature on the term structure of interest rates. The tax progressivity factor, on the other hand, is identified by imposing that its loadings are strictly increasing with wage income, giving it a similar flavor to the slope factor in term-structure models. Thus, an increase in progressivity can be interpreted as a counter-clockwise twist in the “term structure of taxes,” shifting tax burden from low-income agents to high-income agents. To determine their effects on macroeconomic variables, we impose the additional identifying assumption that progressivity shocks are revenue-neutral on impact. This restriction allows us to disentangle progressivity shocks from the tax level shocks.

Our two factors account for, on average, over 80 percent of the variation of tax rates for incomes over \$20,000. Consistent with the existing literature, we find that an increase in the level of taxes is contractionary. However, we also find a novel result that an increase in the progressivity of taxes is expansionary.<sup>3</sup> Expansionary tax progressivity shocks can result from heterogeneity in the workers’ marginal propensities to consume. We hypothesize that lower-income workers are credit constrained with high marginal propensities to consume relative to higher-income workers. Thus, the increase in consumption and output resulting from reducing taxes on lower-income workers is not offset by the increase in taxes on high-income workers.

Not surprisingly, we find that progressivity shocks also have distributional effects. Because increasing progressivity shifts the tax burden from low to high incomes, one might anticipate

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<sup>2</sup>While the priors influence the estimated signs and magnitudes of the loadings, they remain loose enough as to not impose the type of exact restrictions found in the literature on the term structure of interest rates [e.g., Diebold and Li (2006)].

<sup>3</sup>The result that progressivity is expansionary is consistent with empirical work in Bachmann, Bai, Lee and Zhang (2017), who study the impact of fiscal volatility in a Krusell-Smith (1998) economy with heterogeneous agents and incomplete markets. They use the Castañeda, Díaz-Giménez, and Ríos-Rull (2003) tax function and make the progressivity parameter endogenous. They find that tax progressivity must be procyclical to match the data in the context of the model economy. While this is consistent with our empirical finding, the direction of causality differs between our two papers.

income inequality falling in response to a tax progressivity shock.<sup>4</sup> To the contrary, we find evidence that income inequality rises after an increase in progressivity.<sup>5</sup> One way to reconcile these results is that tax changes have both partial and general equilibrium effects. The positive progressivity shock that would lower inequality on impact can be offset by a subsequent response at the top end of the income distribution. We posit that high-income individuals—who typically own the majority of the capital—benefit more from expansions, sufficiently enough that it offsets their higher tax burden. The key for this explanation is that the change in progressivity is on *wage* income while inequality is measured with both *wage and capital* income.<sup>6</sup> While progressivity shocks have no real impact on the deficit-to-GDP ratio, increasing progressivity eventually increases tax revenue through an eventual expansion in output.<sup>7</sup>

The literature on tax progressivity is considerably smaller than the literature on overall tax changes. Moreover, studies that estimate the time series of tax progressivity changes in the U.S. are very limited. Most of these papers [e.g., Feenberg, Ferriere, and Navarro (2017; henceforth FFN) and a few others we discuss below] estimate annual tax progressivity by assuming a parametric tax function. Mertens and Montiel Olea (2018; henceforth MMO) study the effect of changes in average marginal tax rates (AMTRs) using a narrative approach similar what Romer and Romer use for tax level changes. MMO identify a shock to the AMTR of the top one percent of the income distribution. They find that a counterfactual tax cut to this rate raises the incomes of the top one percent and temporarily raises GDP but has little

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<sup>4</sup>Progressivity’s effect on inequality is actually theoretically ambiguous. The trade-offs from changing progressivity are described in Heathcote, Storesletten, and Violante (2017), who emphasize several channels by which tax progressivity can affect the economy. A progressive tax system may provide insurance against labor market uncertainty, lowering inequality. However, it also induces distortions to labor supply that reduce returns to working and can decrease investment in human capital, raising inequality. One of our objectives is to determine which of these effects dominates at the current level of U.S. progressivity.

<sup>5</sup>This result is robust to the measure of inequality—either a Gini coefficient or the spreads between income percentiles at the top and middle of the distribution.

<sup>6</sup>As documented by Piketty, Saez and Zucman (2018), capital income is the bigger driver of inequality, and tax rates on capital income are fairly flat. We also investigate the impact on consumption inequality, but our results are inconclusive.

<sup>7</sup>This result is consistent with the findings of Sachs, Tsyvinski, and Werquin (2020), who show that increasing tax progressivity in a Mirrleesian economy calibrated to the U.S. system raises government revenue. They refer to this finding as a “trickle-up” result for government revenue. We exploit time-series variation in taxes and revenues and find evidence that increases in progressivity have historically had such a trickle-up effect on government revenue.

effect on the incomes of the bottom 99 percent. This leads to an increase in inequality. At first glance, the MMO result might seem contrary to what we find. However, their exercise is different than ours in that: (i) we condition on changes to the level of taxes; (ii) we “twist” all tax rates while they shock the tax rate for only a small slice of the income distribution; and (iii) their data isolates a only few events over the entire sample.<sup>8</sup> The second issue is important as MMO note that changes to the top marginal rate are unlikely to directly generate a large demand stimulus.

The balance of the paper is outlined as follows: Section 2 describes the data used to construct the tax factors, as well as the macroeconomic and inequality data used in the subsequent analysis. Section 3 describes the empirical model, the methods used to estimate it, and the prior used to identify the factor loadings. Section 4 first discusses the results of the estimation using aggregate data. We then present results using measures of inequality that we will use to develop a theory of the propagation of progressivity shocks in the aggregate economy. Section 5 conducts a counterfactual experiment that elucidates how changes in tax legislation manifest as changes in both level and progressivity factors. Section 6 summarizes the results and offers some takeaways.

## 2 Data

Our objective is to determine the effect of tax progressivity shocks on a variety of macroeconomic and inequality data. To construct a measure of the distributional incidence of taxes, we need a measure of the time series of tax rates as a function of income. We also require the macroeconomic data of interest, data that will facilitate identification of the tax shocks, and the aggregate inequality data.

### 2.1 Tax Data

Many tax studies use aggregate government revenue, which is a single summary statistic of total tax burden and omits the distributional effect. One possibility is to construct a measure

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<sup>8</sup>We address the latter issue of the restricted sample in a series of robustness tests.

of marginal tax rates by income brackets, which might at first appear to be straightforward. However, using tax brackets by themselves introduces a number of issues. First, the marginal tax rate is not a proper measurement of the final tax bill. Tax legislation often modifies deductions or exemptions rather than simply changing marginal rates. Second, the number of tax brackets and the incomes they are associated with changes over time. Third, the economic and demographic composition of the population can change over time.

Our data on tax rates are obtained from the TAXSIM model at the NBER for the years 1974-2015.<sup>9</sup> The TAXSIM program is essentially a tax bill computation program that uses a database of real individual-level tax returns obtained from the IRS to simulate the tax burden of the U.S. federal and state income tax systems. TAXSIM calculates tax liabilities and generates tables of aggregate statistics based on measures of income. These tax rates apply to a hypothetical taxpayer who is married, has two dependents, and no itemized personal deductions.<sup>10</sup> The tax rates are based on an increase in the taxpayer's wage for a given current income.

We use the NBER table on U.S. Federal Marginal Income Tax Rates on Wage Income Data where income levels are expressed in real-1992 dollars based on the CPI-U price index. Data are available for tax rates on wage incomes at the following levels: \$5,000; \$10,000; \$20,000; \$40,000; \$100,000; \$200,000; \$400,000; and \$1,000,000. The tax rates for the first two levels of income are largely constant at 0 or negative numbers. To produce meaningful factors we drop the \$5,000 and \$10,000 levels, leaving us with tax rates for six income levels to estimate two factors.<sup>11</sup>

The TAXSIM data contain time-series information on tax rates for a range of income levels. Our ultimate interest is in identifying tax shocks in a VAR and measuring the impact these shocks have on macroeconomic variables as well as the distribution of income and

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<sup>9</sup>These data are publicly available at <http://users.nber.org/~taxsim/conrate/>. The full dataset is available from 1960-2016. We use only data from 1974-2015 in order to match data availability for the inequality measures described in Section 2.3.

<sup>10</sup>The TAXSIM data do not solve all of the issues associated with the use of tax brackets. While the TAXSIM data do incorporate the EIC and standardized deductions for some dates, personalized deductions are omitted. Thus, the deductions associated with mortgage interest would also be omitted.

<sup>11</sup>Omitting these two income levels from the estimation is equivalent to assuming that their tax rates are constant (perhaps fixed at zero). We verified the robustness of our results to excluding the two lowest income levels. These results are included in the Robustness Appendix.

consumption. To do this, we summarize movements in the tax data by identifying two different effects.

First, we consider the overall level of taxes, similar to the exogenous tax shock evaluated in previous studies [e.g., Blanchard and Perotti (2002); Romer and Romer (2010)]. This level reflects the overall burden of taxes, where a rising level of taxes is a contractionary fiscal shock.

Second, we consider the progressivity of taxes, which determines how the tax burden is distributed across income levels. An increase in tax progressivity manifests as an increase in the taxes paid by higher-income levels and a decrease in taxes paid by lower-income levels. It is important to note that our interpretation of tax progressivity implies a shift in the tax burden across incomes that is orthogonal to changes in the level of taxes.

## 2.2 Macro Data

Our baseline case includes the two tax factors, the log of annual real GDP, and the log of annual real tax revenue for a sample period of 1974 to 2015. The revenue series is constructed as the log of current federal tax receipts converted to real terms using the GDP deflator. All of the macro data are annual and obtained from the FRED database at the Federal Reserve Bank of St. Louis.

We also estimate a number of variations of the baseline model that augment or replace real GDP as the macro variable of interest. These variations (a) replace real GDP with the log of real personal consumption, (b) include both real GDP and the log of total aggregate hours for nonfarm payrolls divided by the population 16 years and over, or (c) replace real GDP with the log of total capital expenditures.<sup>12</sup> Because changes in the level or progressivity of taxes may also affect the government surplus/deficit, we estimate an alternative model (d) that replaces federal revenue with the annual federal surplus/deficit as a share of GDP.

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<sup>12</sup>We replace GDP with consumption and investment because the two series are highly correlated. This is not a problem with hours, so, in that case, we include all five variables: the tax level, progressivity, GDP, tax revenue, and hours.



## 2.3 Inequality Data

To determine the distributional effects of progressivity shocks, we use Piketty, Saez and Zucman's (2018) data on income inequality. They combine tax return data from the IRS and data from the March Supplement of the Current Population Survey (CPS) to create a comprehensive estimate of income inequality. The CPS data is preferred to the tax data for the bottom end of the income distribution, where taxes are often not paid and returns are not filed. The tax data is preferred at the top end of the income distribution where individuals in the CPS are anonymized by top-coding.<sup>13</sup> The new inequality measure combines both tax and survey data, making it more accurate than either dataset alone.

While many studies use on the income Gini, we focus on the gaps between the median income and higher income quantiles (90, 95, 99) and the difference between the 10th percentile and the 90th percentile. As we pointed out earlier, the lower income quantiles pay little or no (and sometimes negative) taxes and would not be directly affected the progressivity shock. Thus, we focus on the differences between high incomes and median income, but include the 90-10 difference to verify that our results are robust to a broader measure of inequality.

To corroborate the mechanism for progressivity's effect on inequality, we considered five additional data series which capture the differential impact on those at the top end of the income distribution. The five series are: (i) aggregate capital gains income, (ii) capital gains income of the top 1% as a share of total income, (iii) capital gains income of the top 0.1% as a share of total income, (iv) the level of DJIA and (v) the level of the S&P500. To keep the models small (and because these series are highly correlated), we estimated models with each series separately. Because these series are correlated with output, we replaced output in the VAR with the series of interest. We use the same identifying assumptions that we used in the baseline VARs.

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<sup>13</sup> Atkinson, Piketty and Saez (2011) claim that the CPS data understate the rise in the top 1-percent of income.

### 3 Empirical Approach

To determine how tax progressivity shocks affect the national economy, we utilize a FAVAR, where the tax factors are estimated jointly with the other model parameters in a state-space framework. This section outlines the model, identification, and inference.

#### 3.1 Model

Let  $R_t = [R_{1t}, \dots, R_{Kt}]'$  be the  $(K \times 1)$  vector of period- $t$  (wage) income tax rates defined for  $K$  different income bins. Define  $\tau_t$  as the period- $t$  value of the (latent) tax level factor and  $\mu_t$  as the period- $t$  value of the (latent) progressivity factor.<sup>14</sup> The latent factors are related to the tax rates via

$$\begin{bmatrix} R_{1t} \\ R_{2t} \\ \vdots \\ R_{Kt} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_K \end{bmatrix} + \begin{bmatrix} \lambda_1^\tau & \lambda_1^\mu \\ \lambda_2^\tau & \lambda_2^\mu \\ \vdots & \vdots \\ \lambda_K^\tau & \lambda_K^\mu \end{bmatrix} \begin{bmatrix} \tau_t \\ \mu_t \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{Kt} \end{bmatrix}, \quad (1)$$

where the  $\rho_k$  are scalar intercepts,  $u_t = [u_{1t}, \dots, u_{Kt}]'$  and  $E[u_t u_t'] = \text{diag}[\sigma_1^2, \dots, \sigma_K^2]$  assumes that the only correlation in the tax rates arises from the factors. The  $\lambda_k^\tau$  and  $\lambda_k^\mu$  are factor loadings on the tax level and progressivity, respectively, that determine the relationship between the factors and the various tax rates. We discuss the importance of the factor loadings in more detail below.

Let  $X_t$  represent the  $(N \times 1)$  vector of variables of interest and define  $Y_t = [\tau_t, \mu_t, X_t']'$ . Then, the specification of the aggregate-level structural VAR is:

$$A_0 Y_t = A(L) Y_{t-1} + e_t, \quad (2)$$

where  $A_0$  is the matrix of contemporaneous effects,  $A(L)$  is a matrix polynomial in the lag

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<sup>14</sup>We also estimated a three factor model, where the third factor can be interpreted as the curvature of taxes. As opposed to the term structure literature who restrict the third factor to be concave, the tax curvature should be convex (i.e., increasing more for higher incomes). We find that the mean responses are as predicted but insignificant. These results, including the three factor model specification, are in the Robustness Appendix.

operator, and  $e_t \sim N(0, I)$  are the structural shocks. To facilitate exposition in the section outlining the structural identification below, we can rewrite (2) as

$$\begin{bmatrix} A_{0,\tau\tau} & A_{0,\tau\mu} & A_{0,\tau x} \\ A_{0,\mu\tau} & A_{0,\mu\mu} & A_{0,\mu x} \\ A_{0,x\tau} & A_{0,x\mu} & A_{0,xx} \end{bmatrix} \begin{bmatrix} \tau_t \\ \mu_t \\ X_t \end{bmatrix} = \begin{bmatrix} A_{\tau\tau}(L) & A_{\tau\mu}(L) & A_{\tau x}(L) \\ A_{\mu\tau}(L) & A_{\mu\mu}(L) & A_{\mu x}(L) \\ A_{x\tau}(L) & A_{x\mu}(L) & A_{xx}(L) \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ \mu_{t-1} \\ X_{t-1} \end{bmatrix} + \begin{bmatrix} e_t^\tau \\ e_t^\mu \\ e_t^x \end{bmatrix},$$

which isolates the effects of lags of variable  $j$  on variable  $i$  in each lag polynomial,  $A_{ij}(L)$ .<sup>15</sup>

One could imagine the relationship between tax progressivity and macroeconomic outcomes to be nonlinear. For example, the effect of a productivity shock on output may depend on the distribution of income. We view our model as a first-order approximation of this more general nonlinear model, where results are obtained under the assumption that changes in progressivity and the income distribution are local to their initial conditions (i.e., relatively small).

## 3.2 Identification

Our model presents two types of identification issues discussed below.

### 3.2.1 Factor Identification

The first issue requires imposing restrictions on the factor loadings to identify the factors. As we implied above and will discuss again in detail later, most tax legislation will be represented by a combination of both level and progressivity shocks, where the loadings determine the effect of a unit change in the corresponding tax factor on the various tax rates. As described previously, we impose that (i)  $\lambda_k^\tau > 0$  for all  $k$  and (ii)  $\lambda_m^\mu \geq \lambda_k^\mu$  for  $m > k$ . The former set of restrictions implies that raising the level of taxes increases tax rates across the board.<sup>16</sup> The second restriction implies that increasing tax progressivity raises the tax rate more as

<sup>15</sup>We incorporate a linear trend into the VAR.

<sup>16</sup>Our restriction allows the effect of tax level shocks to vary across income brackets. We also estimated a more restricted model in which the loadings on the tax level are all set to 1 in the spirit of Reis and Watson (2010). Results were qualitatively similar and are reported in the Robustness Appendix.

income increases. Note also that  $\lambda_k^\mu$  need not be strictly positive; in fact, in practice,  $\lambda_k^\mu$  will be negative for low income levels. This implies that  $\sum_k \frac{\lambda_k^\tau}{K} \tau_t$  represents the (unweighted) average level of taxes across all incomes and  $\lambda_k^\mu \mu_t$  raises (or lowers) the income-dependent tax level from the average.

These restrictions identifying the loadings differentiate our model from the parametric tax function outlined in Feldstein (1969) often used in the literature [e.g., in FFN and others].<sup>17</sup> First, the parametric tax function implies an equal increase in tax rates across all incomes for a given increase in the level, implying  $\lambda_i^\tau = \lambda_j^\tau$  for all  $i, j$ . Moreover, the parametric tax function implies the difference between tax bins is monotonically increasing with income; the restrictions on our estimated loadings imply that only the rates are monotonically increasing with income.

The use of the parametric tax function is analogous to the “functional VAR” that Inoue and Rossi (2019) propose to measure the effects of monetary policy. Inoue and Rossi identify composite shocks to the level, slope, and term structure of interest rates by examining the evolution of the parameters in Nelson-Siegel regressions. In our model environment, the Nelson-Siegel-type (1987) factors could be recovered by imposing  $\lambda_k^\tau = 1$  for all  $k$  and  $\lambda_{k+1}^\mu > \lambda_k^\mu$  for all  $k = 1, \dots, K - 1$ . Similarly, an approximation of principal components would impose  $\lambda_k^\tau \simeq 1$  for all  $k$ , but would not strictly hold to 1, and the upper block of the variance-covariance matrix of the state equation would be diagonal.

### 3.2.2 Shock Identification

The second identification issue involves identification of the structural tax level and progressivity shocks in the VAR [for an overview of these issues, see Ramey (2016)]. While many methods have been proposed to identify fiscal shocks in VARs (e.g., narrative, recursive, exclusion, sign restrictions, etc), there are no well-established methods of disentangling the tax level and progressivity shocks. In our case, the nature of the economic question and annual frequency of the data dictate our choice of the method of identification.

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<sup>17</sup>FFN construct progressivity using a nonlinear tax function based on Feldstein (1969),  $\tau(y) = 1 - \lambda y^{-\gamma}$ , where  $\tau(y)$  is the tax rate on the level of income  $y$ . The parameter  $\gamma$  in their scheme represents tax progressivity.

Two of the other empirical papers in the literature are MMO and FFN. MMO identifies their shocks using narrative methods but focus on a subset of tax rates and legislative events. FFN constructs a tax progressivity series, but they do not evaluate the effects of progressivity shocks in a VAR. Mountford and Uhlig (2009) identify fiscal shocks using the sign restriction approach of Uhlig (2005). In their case, economic theory provides unambiguous restrictions that can identify their revenue and spending shocks. Because we are interested in establishing a set of stylized facts as there exists no clear theoretical result that identifies a pure progressivity shock. Moreover, the sample and frequency of the data impose limitations on the size of the VAR, we cannot employ sign restrictions.

We identify the structural form of the VAR using *non-recursive* exclusion restrictions on the matrix of contemporaneous effects. To disentangle the two different tax shocks, the key identifying restriction—as we have discussed above—is that tax level shocks directly affect revenue contemporaneously but tax progressivity shocks do not.<sup>18</sup> Thus, the (positive) progressivity shock is a revenue-neutral (counterclockwise) twist in the “term structure” of tax rates resulting in a reallocation of the tax burden.<sup>19</sup> Our identifying assumptions imply that changing a subset of the tax rates likely results in a combination of both tax level and progressivity structural shocks. The experiment in Section 5 below highlights this.

To complete our set of identifying restrictions, we assume that the tax level and tax progressivity shocks are contemporaneously uncorrelated. While we allow both tax shocks to affect the macro variables (with the exception of our key identifying restriction), we rule out contemporaneous feedback from the macro variables to taxes.<sup>20</sup> In particular, our identifying

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<sup>18</sup>These identifying assumptions result in a third difference between our model and the FFN model that uses the tax function. A shock to the progressivity parameter in the tax function is not revenue neutral. Thus, the interpretation of the two progressivity shocks would be different.

<sup>19</sup>Our assumption is that progressivity does not affect revenue directly but, within the year, can affect revenue through a change in output.

<sup>20</sup>Our identification is consistent with block recursive ordering, where the factors are ordered first, with the additional restrictions that the progressivity shock does not affect revenue and that the two tax shocks are orthogonal. In a previous version of the paper, we produced impulse responses for shocks identified using the Cholesky identification. The results were qualitatively similar and we report them in the Robustness Appendix.

assumptions on the baseline specification impose the following structure on  $A_0$ :

$$A_0 = \begin{bmatrix} a_{\tau\tau} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & a_{\mu\mu} & \mathbf{0} & \mathbf{0} \\ a_{\tau Z} & a_{\mu Z} & a_{ZZ} & a_{TZ} \\ a_{\tau T} & \mathbf{0} & a_{ZT} & a_{TT} \end{bmatrix},$$

where  $X_t = [Z_t, T_t]$  with  $Z_t$  the log of real GDP and  $T_t$  the log of real tax revenue.

### 3.3 Inference

We estimate the FAVAR described above at the annual frequency with two lags using Bayesian techniques. Estimating parametric factors—rather than principal components—allows us to impose restrictions on the factor loadings that will produce more economically-interpretable factors.<sup>21</sup> The form of the prior we impose is relatively standard. The  $\rho_k$  are obtained by ex ante demeaning the tax series and do not require a prior; similarly, we demean the macro variables in the VAR. The variances of the tax rates are inverse gamma priors and assumed to be independent, and the factor loadings have normal priors. The prior parameters are shown in Table 1.

The FAVAR is estimated with a Gibbs sampler, a Markov-chain Monte Carlo technique. Start with an initial draw of the parameters and the factors. We then sequentially draw from one block of parameters, conditional on the last draw of the other blocks and the data. After allowing for a suitable number of draws to allow the algorithm to converge, draws from the conditional distributions reflect draws from the joint posterior density. For our sampler, we have three blocks: (1) the VAR parameters, (2) the factors, and (3) the measurement

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<sup>21</sup>In the term-structure literature, factors are often generated by methods that restrict the loadings on the factors—see, for example, Nelson and Siegel (1987). In that paper, the level factor is estimated assuming coefficients on the factor are all unitary. The yield curve then rises equally across all maturities with movements in this factor. The slope factor imposes rising coefficients across maturities, with a free parameter to govern the slope. Movements in the slope factor then have heterogeneous effects on yields. Their approach does not impose orthogonality in the two factors as would be imposed in principal components. The state-space framework will allow us to parameterize a prior that puts ex ante weight on a zero correlation between the two factors as in the principal components approach while allowing priors to push the loadings towards those in the term-structure literature.

equation parameters. Conditional on the factors, we estimate the VAR in the structural form using a sampler described in Waggoner and Zha (2003).<sup>22</sup> We then obtain the reduced-form parameters from:

$$Y_t = B(L) Y_{t-1} + \varepsilon_t, \quad (3)$$

where  $B(L) = A_0^{-1} A(L)$ ,  $\varepsilon_t \sim N(0, \Omega)$  are the reduced-form shocks, and  $\Omega = A_0^{-1} A_0^{-1'}$  is the reduced-form variance-covariance matrix. The factors are drawn from the smoothed Kalman filter posteriors using the reduced-form VAR parameters for the state equation. For this draw, we impose the additional identifying restrictions discussed above that the loadings on the first factor are positive and the loadings for the second factor are increasing across the income levels.<sup>23,24</sup> Draws from the third block are conjugate normal-inverse-gamma.

## 4 Results

### 4.1 The Tax Factors

Before we consider how different types of tax shocks affect aggregate economic outcomes, we examine some of the properties of the tax factors obtained from the FAVAR. We model changes to the tax code similarly to how one would model changes in the term structure of interest rates, thereby reducing the dimensionality of the tax rate data by summarizing their movements into the two latent factors plotted in Figure 1.

Figure 1: Tax factors from the FAVAR

Our tax factors are consistent with the tax series found in other studies. For example, the correlation between the difference of the tax level factor and the Romers' tax shock series

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<sup>22</sup>A brief overview of the prior and the sampler is provided in the Appendix.

<sup>23</sup>The factors from the state-space model depend on the VAR specification. Principal components analysis (PCA) produces similar tax factors for the baseline model. The correlation between the level (progressivity) factors obtained from these two methods is 0.97 (0.69). Despite the model dependency, we prefer the structure of the FAVAR, as it allows us to impose further identifying restrictions on the factor loadings.

<sup>24</sup>The income brackets at the high end (\$400k and \$1M) are wider than at the low end (\$20k, \$40k, and \$100k). Consequently, the prior on the progressivity loadings are slightly increasing for the top two brackets.

(for 1974-2007, dates in which the series overlap) is 0.22. The progressivity factor exhibits a sharp decline in the late 1980s followed by a steady increase, consistent with narrative evidence and with the progressivity series computed from a parametric tax function in FFN. The correlations between changes in our progressivity factor and changes in the marginal tax rates across brackets are consistent with our interpretation of progressivity. For the three lowest brackets, these correlations are negative and range from  $-0.04$  at \$20,000, to  $-0.10$  at \$40,000, to  $-0.08$  at \$100,000. Alternatively, the correlations are positive for the three highest brackets, ranging from  $0.27$  at \$200,000, to  $0.44$  at \$400,000, to  $0.40$  at \$1,000,000. Thus, while the top rate alone clearly matters for progressivity, our measure differs from this simple measure in capturing twists in the entire tax code.

Table 2: Variance Decompositions from the FAVAR

How well do these factors represent actual fluctuations in the tax rates? Table **2** shows the shares of the variance of each tax bracket explained by the two tax factors. Taken together, the level and progressivity factors explain, on average, over 70 percent of the tax data for the income brackets in our sample. These results suggest that our factor approach provides a parsimonious, yet comprehensive, characterization of movements in taxes.

Movements in the factors can also be associated with historical policy changes reflected by the vertical lines in Figure **1**. For example, in the early 1980s, the Kemp-Roth Tax Cut called for an across-the-board decline in marginal tax rates in an effort to boost the economy. This tax cut manifested more in a gradual decline in the overall level of taxes (the first factor) but a less substantial change in progressivity. On the other hand, the Reagan Tax Reform Act of 1986, which was meant to simplify the tax code by eliminating a large number of tax brackets, led to a substantial decline in both factors. The Omnibus Budget Reconciliation Act of 1990 increased the top marginal tax rate from 28 percent to 31 percent, reformed capital gains taxes, and limited high-valued itemized deductions. Subsequent to this legislation, the level of taxes do not change substantially but progressivity rises.

These historical episodes highlight the importance of modeling both changes in the level



of taxes and changes in the progressivity of taxes. While each piece of legislation affected the level of taxes, without simultaneously modeling tax progressivity, the total impact of the change in the tax law may be misinterpreted, if not mismeasured.

## 4.2 The Effect of Tax Shocks on Tax Rates

Figure 2: Factor Loadings

We now consider the static effect of a unit change in the two tax factors on the tax rates for each income level as reflected in the factor loadings. From equation (1), we can obtain the average change in the tax rates at the time of a change in either the level or progressivity.

Figure 2 plots the factor loadings for the two estimated tax factors as a function of income. Two results are immediately apparent. First, the loadings on the tax-level factor are all positive—ranging between 0.2 and 1.9 and averaging 1.1—but are not equal across income levels nor are they monotonically increasing. A unit shock to the tax-level factor results in an increase in the tax rate of about 0.4 percentage points at \$40,000 but 1.0 percentage points at \$100,000. Thus, an increase in the tax-level factor will produce an across-the-board, yet not exactly equally-distributed increase in tax rates across all income levels.<sup>25</sup>

Second, the loadings on the progressivity factor are negative for low incomes and positive for higher incomes. Thus, an increase in the progressivity factor shifts the tax burden unequally across income brackets. In particular, the twist in the tax term structure places increased tax burden only on the top three rates, with only a small increase in the lowest of these three top rates. This result is not surprising given that our prior also has increasing means across incomes and that we require a shock to the factor to be revenue neutral. However, the point values of the posteriors can differ substantially from the prior mean; for example, the prior mean at \$100,000 is 0 but the posterior mean is  $-1.2$ .

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<sup>25</sup> As a robustness check, we estimated a model in which the loadings on the level factor were restricted to be equal across incomes and fixed at 1. The resulting progressivity factor was qualitatively similar and are reported in the Robustness Appendix.

### 4.3 Macroeconomic Outcomes

Having determined how shocks to the tax factors affect the tax rates, we now turn to evaluating the effect of the tax shocks on some macroeconomic outcomes. To do this, we compute the impulse responses of the macro variables to the two tax shocks, identified by the restrictions described above.

Figure 3: Benchmark Impulse Responses

The eight panels of Figure **3** plot the impulse responses to tax level (top row) and progressivity (bottom row) shocks. Consistent with the existing literature, the tax-level shock is contractionary, increases revenue on impact but eventually lowers revenue via the economic contraction. The bottom row of Figure **3** reports the effects of a positive progressivity shock. An increase in progressivity *raises* output over a 5-year horizon—that is, shifting the tax burden from the relatively low income to the relatively high income results in an economic boom. While we assume that progressivity has no direct effect on revenue, it can indirectly have an effect within the first year through the subsequent rise in output.

Changes in the level or progressivity of taxes can, in theory, have effects on government revenue. The response of government revenue to the progressivity shock in Figure 3 is positive and significant. This result is consistent with Sachs, Tsyvinski, and Werquin (2020), who show that an increase in tax progressivity in a calibrated Mirrleesian economy raises government revenue. They refer to this finding as a “trickle-up result” for government revenue. Here, we document the empirical response of government revenue using time-series data with little economic theory. As Sachs, Tsyvinski and Werquin (2020) argue, the trickle-up result implies significant issues with the current general equilibrium approach to optimal tax theory.<sup>26</sup>

Figure 4: Alternative Measures of Economic Activity Impulse Responses

These results are consistent with several variations of the baseline model that augment

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<sup>26</sup>Results including the deficit are qualitatively similar and are discussed in the Robustness Appendix.

or replace real GDP as the macro variable of interest. Figure 4 shows the responses of three alternative measures: consumption (first column), hours (middle column), and capital expenditure (right column). These are all obtained from separate aggregate-level VARs for the sample 1974 to 2015 in which we replace output with consumption or capital expenditure or add hours to the VAR with output.<sup>27</sup> In all cases, the increase in progressivity consistently produces an economic expansion: output, consumption, hours worked, and capital expenditure all rise.

Figure 5: Benchmark and Purely Exogenous Impulse Response Functions

The expansionary effect of increasing progressivity is a striking result. Thus, it is important to verify robustness of this result to alternative identification schemes, as there are competing views on how to best identify a structural shock. For example, in the monetary literature, Coibion (2012) investigates the reasons for differences between Romer and Romer (2004) monetary shocks and monetary shocks identified with the Cholesky decomposition. In the spirit of that exercise, we construct a set of alternative progressivity shocks from our progressivity series, but we use only the observations occurring on the MMO dates. In this sense, we “filter” our full-sample shocks and employ only the exogenous events that they consider.<sup>28</sup> Figure 5 shows the result of this exercise. From the results shown in the top row, it is clear that results from the unfiltered sample and the results using only the MMO dates are qualitatively similar, although there are some differences in magnitudes. Importantly, in the second row, the response of GDP to each shock is consistent with our benchmark results.<sup>29</sup>

The expansionary effect after an increase in progressivity is a new empirical result that is uncovered by our methodology for measuring tax progressivity shocks. The result is consistent

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<sup>27</sup>For the VAR with output and hours, we retain the restrictions from the previous  $A_0$  above and impose the additional restriction that an hours shock does not affect GDP or the tax factors on impact.

<sup>28</sup>They include fifteen events in their sample from 1946-2012, with eleven of their events occurring in our sample period of 1974-2015.

<sup>29</sup>The tax-level factor falls following an increase in progressivity. To ensure that this is not responsible for the subsequent economic boom, we estimate a restricted VAR such that each factor depends only on its own lags:  $A_{\tau\mu}(L) = A_{\mu\tau}(L) = A_{\tau x}(L) = A_{\mu x}(L) = \mathbf{0}$ . Increasing the tax level remains contractionary and increasing progressivity remains expansionary. Under these restrictions, the contractionary effects of level shocks are larger and the expansionary effects of progressivity shocks are delayed. These results are available from the authors upon request.

with papers that find heterogeneous responses of consumption to changes in income [e.g., McCarthy (1995); Dynan, Skinner, and Zeldes (2004); Jappelli and Pistaferri (2010); and Parker, Souleles, Johnson, and McClelland (2013)]. Further, Gruber and Saez (2002) find that, because high earners can itemize deductions, the elasticity of taxable income varies across the income spectrum. In particular, agents at the low end of the income distribution are credit-constrained and wish to consume more. As their tax rates fall, assuming that their marginal propensity to consume is near one, they increase their consumption nearly dollar-for-dollar. Those at the higher end of the income distribution have lower marginal propensities to consume and do not reduce their consumption as tax rates rise. Additionally, changes to marginal tax rates for the wealthy appear to have little impact on work effort.<sup>30</sup> The expansion also leads to an increase in aggregate hours worked.

#### 4.4 Progressivity Shocks and Inequality

The effect of a tax progressivity shock on income inequality can be decomposed into two parts: a partial-equilibrium impact effect and the subsequent multiplier effect. Intuitively, the impact effect lowers inequality by increasing the marginal tax rate for high-income individuals and decreasing it for low-income individuals. The subsequent propagation effect is related to the economic boom set off by the increase in progressivity. If, for example, low- or middle-income wages rise as hours worked increase and labor market conditions tighten, we could see a further decline in inequality. On the other hand, if corporate profits rise in response to increased output and wages are stagnant, we could see an increase in inequality since capital ownership is concentrated at the top end of the income distribution.

Figure 6: Income Inequality IRFs

Using the Piketty, Saez and Zucman’s (2018) data on income inequality, we consider the difference between log income levels at various percentiles: 99-50, 95-50, 90-50, and 90-10. These measures provide a thorough depiction of how the income differential is changing across

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<sup>30</sup>See, for example, Zidar (2019), who considers income heterogeneity by location and finds that tax cuts for the top 10% have very small effects on employment growth in that demographic.

various levels within the overall distribution. We find that income inequality rises after the progressivity shock (see Figure 6).<sup>31</sup> This increase in income inequality, however, does not necessarily imply that those at the lower end of the distribution are less well-off in terms of welfare: The expansion increases their income and consumption, but the income of those at the top end of the distribution increases more.<sup>32</sup> Of the four inequality measures depicted in Figure 6, we see the strongest response in the changes of the 99-50 percentile and the 95-50 percentile: Those at the top gain much more than those in the middle during the expansion. Moving further into the middle of the distribution, the change for the 90-50 percentile is smaller but still significant, suggesting that the increased Gini is driven by the large gains of the upper tail. At the lower end of the distribution, the changes in the 90-10 percentile suggest an even smaller and insignificant effect on inequality on impact. This may capture stagnation in the lower-income individuals that may not experience substantial gains relative to higher-income owners of capital.

Trickle-down economics suggests that lowering tax rates on those with high incomes spurs an expansion. To the contrary, our empirical results suggest that the opposite is true: Lowering the tax burden on lower incomes sets off an expansion that also raises the incomes of those at the high end of the income distribution. This is consistent with trickle-up—not trickle-down—economics. This result can be understood by considering the fact that the change in progressivity is on wage income while income inequality is measured with both wage and capital income. Our income inequality data is from Piketty, Saez and Zucman (2018) who argue that capital income is a greater driver of income inequality than wage income. They also argue that inequality is being driven by the upper tail of the distribution, which is consistent with our interpretation of the results.

To further examine the story regarding capital incomes rising for those at the top, we consider several measures which capture this dynamic: (i) aggregate capital gains income, (ii)

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<sup>31</sup>When adding a measure of inequality to the VAR, we replace output and impose the baseline identifying restrictions on  $A_0$ .

<sup>32</sup>This result is consistent with empirical evidence that the economy disproportionately and increasingly rewards those at the top end of the distribution. It is also consistent with stagnant wages in the middle of the income distribution, which has occurred even in expansions. See for example, Saez and Zucman (2020) for recent evidence on the increasing share of income of the top one percent.

capital gains income of the top 1% as a share of total income, (iii) capital gains income of the top 0.1% as a share of total income, (iv) the level of DJIA and (v) the level of the S&P500. The anticipated response of each of these variables under our hypothesized mechanism is positive—that is, a tax progressivity shock that increases income inequality through rising capital values should increase capital gains income and increase stock prices. Figure 7 collects the impulse responses for the five variables (obtained from separate VARs) to tax level (top row) and progressivity (bottom row) shocks. We find a significant increase in all but one of the variables of interest to an increase in progressivity.<sup>33</sup> This suggests that an increase in progressivity leads to an increase in income that, in turn, leads to an increase in capital gains income and stock prices. While we do not take these results as explicit proof of our proposed mechanism, they are consistent with our argument.

Figure 7: Capital Gains, Capital Income, and Stock Market IRFs

To summarize, we find that increasing the progressivity of taxes raises GDP and income inequality. We previously hypothesized that a shock to progressivity acts initially through the consumption channel. An increase in tax progressivity raises disposable income for low-income agents and lowers disposable income for high-income agents. This, in turn, raises consumption at the low end of the income distribution and lowers consumption at the high end of the income distribution. To generate a net increase in aggregate income, consumers at the low end of the income distribution spend all or most of their newfound income, while consumers at the high end of the income distribution reduce their consumption by only a (perhaps small) portion of the decrease in disposable income.<sup>34</sup>

The textbook tax multiplier story does not imply an increase in income inequality. In the single (type) agent model, a shock to the tax rate that increases disposable income generates a multiplier effect in which the increase in consumption is repeatedly redistributed back to the

<sup>33</sup>The S&P500 does increase as well but the response is insignificant. We see an analogous, insignificant decline in this series following a tax level increase.

<sup>34</sup>This result contrasts with Kindermann and Krueger (2014), who find that aggregate consumption falls with a rise in the highest marginal tax rate. However, as they note in their paper, their results depend on assumptions about the labor productivity process and elasticities. Here, we provide a set of empirical facts independent of such assumptions.

agents. In the heterogeneous-income environment, if this multiplier income is proportionately redistributed across both high- and low-income agents, the Gini does not change.<sup>35</sup> Thus, we need to propose an alternative transmission mechanism for the multiplier in our environment.

To obtain both an increase in output and an increase in income inequality, the multiplier effect must not be equally spread over the income distribution. Suppose that low-income agents are fixed-wage laborers and high-income agents own the capital stock and that low-income agents outnumber high-income capital holders. The initial effect of the progressivity shock raises (lowers) disposable income of the low (high) income agents, with the net effect being expansionary. If the multiplier effect, however, is not spread equally across agents, low-income agents benefit only from the initial shock and not from the multiplier effect. Therefore, even though the initial shock lowers the disposable income of high-income agents, the net effect on these agents is positive via the multiplier effect. The size of the multiplier effect will depend on both the concentration of wealth and the degree of market power of the capital owners. We take no stand on the size of these, rather our empirical results rely on minimal theoretical assumptions.

On the labor side, we find that (aggregate) hours worked expanded with an increase in progressivity. In theory, increasing marginal tax rates for high-income workers should decrease the labor supply of high-income workers and decreasing marginal tax rates for low-income workers should increase the labor supply of low-income workers. However, the empirical literature has found that high-income earners adjust their tax avoidance behavior when faced with tax changes [e.g. Slemrod (1996)], not their labor supply. As Slemrod (1996) argues, “A number of studies have shown large and quick responses of reported incomes along the tax-avoidance margin at the top of the distribution, but no compelling study to date has shown substantial responses along the real-economic margin among top earners.” Our boom in hours is then likely the result of increased hours worked at the low end of the distribution and little or no response at the high end of the income distribution.

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<sup>35</sup> Moreover, a lump-sum-type redistribution would lower the Gini, as it would raise incomes at the low end of the income distribution proportionally more.

## 4.5 Robustness

In the previous sections we noted in footnotes that our results are robust to alternative identification of shocks, restrictions on factor loadings, measures of economic activity, number of tax brackets used and the number of factors estimated. Our final measure of robustness is to examine the assumption that the model parameters are stable over time. It may be the case that the relationship between taxes and income varies with structural change in the economy. To investigate this we re-estimated the model allowing for a break in the progressivity factor loading coefficient over three periods: (i) 1992, after the Reagan tax cuts and before the Clinton tax policies; (ii) 1984, ending with the start of the great moderation; (iii) 2007-2015, the great recession period. Note that we re-estimated the model allowing one break at a time.

We find that the estimated factors are very highly correlated when allowing for a break. This result is somewhat expected as both Del Negro and Otrok (2008) and Bates, Plagborg-Møller, Stock and Watson (2013) find that estimates of factors are robust to time variation in parameters or structural instability. Additionally, the impulse response functions are very similar when allowing for breaks. These results are left to the Robustness Appendix as they are very similar to the results we report in the main text.

## 5 Raising Marginal Tax Rates: An Experiment

In the previous sections, we showed that increasing the level of taxes is contractionary while increasing the progressivity of taxes is expansionary. As Figure 2 suggests, most of the tax policies enacted over the sample period are a combination of shocks to both level and progressivity. For example, we highlighted earlier that the Reagan Tax Reform Act of 1986 resulted in a decline in both factors. While the effect of the level and progressivity shocks can be evaluated in isolation in the VAR, the mapping between a real-world tax policy and the change in the level and progressivity shocks occurs in the measurement equation of the factor model. Thus, to determine the net effect of tax policies that are comprised of both a level and



progressivity shock, we must evaluate how a particular policy maps into the two factors.<sup>36</sup>

Here, we consider two examples of tax policies, both of which would be considered contractionary in an environment where the tax level was the only policy instrument. We describe how the changes in legislation map into the factors and evaluate their net effects.

At time  $T$ , the update step of the Kalman filter determines the magnitude of the change in the mean of the posterior distribution of the state vector. Thus, the effect of changing any part of the tax rate vector,  $R_T$ , on  $F_T$  would be proportional to the Kalman gain,  $G_T$ :

$$\Delta F_T = G_T^* \Delta R_T,$$

where  $G_T^*$  represents the first two rows of the Kalman gain matrix. Note that this decomposition imposes that the largest percentage of the variance of  $R_T$  is explained by the factors. The balance of the variance is remanded to the innovation in the measurement equation, which we assume does not propagate through the VAR and, thus, does not affect output.

Our first experiment considers proposed tax legislation that increases the tax rate on the top of the income distribution by 10 percentage points. In our model, this tax legislation is represented by an increase in  $R_{KT}$ , where  $K = \$1$  million and  $T$  is the time at which the tax increase is levied. Specifically, the proposed tax legislation can be modeled as

$$\Delta R_T = \left[ \mathbf{0}'_{[(K-1) \times 1]}, \Delta R_{KT} \right],$$

where  $\Delta R_{KT} = 0.1$ . Given this counterfactual change in the tax code, we can compute the effect on the factors.

Based on the means of the posterior distribution of the model parameters, the proposed tax legislation results in a change in the factors of  $\Delta F_{1T} = -0.0003$  and  $\Delta F_{2T} = 0.0002$ . Thus, the proposed legislation acts to shift down the level of taxes but increase the progressivity. Given these changes in the factors, we can compute the net effect of the tax legislation by assuming that  $\Delta F_T$  embodies a reduced-form shock. We want to isolate the effects of the policy

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<sup>36</sup> As noted in footnote 14 we also estimated a 3 factor version of the model. This in principle would allow for different set of policy options to hit a target. Since the curvature factor was not significant we do not pursue these policies here.

change via transmission through changes in tax level, progressivity, and then a composite of the two shocks. Given  $\Delta F_T = [\varepsilon_t^\tau, \varepsilon_t^\mu]'$ , we obtain the vector of structural shocks from the reduced-form effects as follows:

$$\begin{bmatrix} e_t^\tau \\ e_t^\mu \\ e_t^x \end{bmatrix} = A_0 \begin{bmatrix} \varepsilon_t^\tau \\ \varepsilon_t^\mu \\ 0 \end{bmatrix}.$$

We isolate the effects of  $e_t^\tau$  and  $e_t^\mu$  individually and then consider the net effect of a composite of the two structural shocks.

The top row of Figure 8 shows the effect of the composite shocks. The left panel shows the impulse response of output to a level shock of the appropriate size; the center panel shows the impulse response of output to a progressivity shock of the appropriate size; and the right panel shows the net effect, represented by the sum of the two components. Both a decrease in the level of taxes and an increase in tax progressivity are expansionary. Thus, the long-run net effect of the legislation shown in the right-most panel is unambiguously expansionary in our framework. In a model in which the tax level was the only instrument, the legislation would be an increase in the tax level and, thus, unambiguously contractionary.

Figure 8: Experiment 1/2

Figure 9: Graphical Depiction of the Experiments

What leads to the difference in the outcomes across the two models? While the model with just the level shock can only produce the proposed legislation with a contractionary increase in taxes, the left panel of Figure 9 shows how this legislation manifests in our environment. Each of the tax rates are the sum of a tax level component and a tax progressivity component. The increase in the highest tax rate leads to a counterclockwise rotation in the distribution of taxes—an increase in tax progressivity. The increase in tax progressivity alone would increase

the expected tax rates at incomes just below \$1 million (i.e. \$200k and \$400k). Thus, to compensate for the increase in these rates caused by the increase in progressivity, the tax level decreases.

We next consider a 10-percentage-point increase in each of the top two tax rates (at \$400k and \$1M income). This legislation produces both an increase in the level of taxes ( $\Delta F_{1T} = 3.53$ ) and the progressivity of taxes ( $\Delta F_{2T} = 0.71$ ). The right panel of Figure 9 shows why this legislation produces an increase in the level of taxes. To capture the rise in the top two rates, progressivity increases even more than in the previous counterfactual. The expected rates at the lowest incomes are now too low and the tax level must shift up to account for this.

Because one effect is contractionary and one effect is expansionary, we need to compute the net effect to determine whether the sign of response is positive or negative. The bottom row of Figure 8 shows the effect of each shock and the combined effect (right panel). Here, the level effect dominates the progressivity effect and produces a net negative effect on output in the long run.

## 6 Summary and Conclusion

The objective of this paper is to develop a set of empirical facts related to changes in the progressivity of taxes. To do so, we developed a new measure of revenue-neutral progressivity that conditions on changes in the level of taxes. The impact of our tax-level factor is consistent with the existing tax literature and most economic theory. Our progressivity factor yields novel empirical insights. In particular, we find evidence in favor of trickle-up economics. After an increase in progressivity, an economic boom is set off. Paradoxically, the boom does not lower income inequality because those in the upper tail of the distribution benefit more from economic booms. We do not interpret this as a negative result, rather it suggests that other, non-income-tax-based policies would need to be enacted to reduce inequality.

A key message of the paper in evaluating policy is that because taxes are levied on people differently and there is heterogeneity in the response to taxes across the income distribution,

one needs to evaluate the effect of any tax legislation by considering both the level of taxes and the degree of progressivity. In models where only the level shock is available, some legislation would be considered contractionary. In an environment where both level and progressivity are relevant, the effect of the same legislation might be ambiguous until one examines the relative changes in both level and progressivity.

## References

- [1] Aguiar, Mark; and Bils, Mark. 2015. “Has Consumption Inequality Mirrored Income Inequality?” *American Economic Review*, 105(9): 2725-56. <http://dx.doi.org/10.1257/aer.20120599>.
- [2] Alvaredo, Facundo; Atkinson, Anthony B.; Piketty, Thomas; and Saez, Emmanuel. 2013. “The Top 1 Percent in International and Historical Perspective.” *Journal of Economic Perspectives*, 27(3): 3-20. <http://dx.doi.org/10.1257/jep.27.3.3>.
- [3] Atkinson, Anthony B.; Piketty, Thomas; and Saez, Emmanuel. 2011. “Top Incomes in the Long Run of History.” *Journal of Economic Literature*, 49(1): 3-71. <https://doi.org/10.1257/jel.49.1.3>.
- [4] Attanasio, Orazio; and Pistaferri, Luigi. 2014. “Consumption Inequality over the Last Half Century: Some Evidence Using the New PSID Consumption Measure.” *American Economic Review*, 104(5): 122-26. <http://dx.doi.org/10.1257/aer.104.5.122>.
- [5] Bachmann, Rüdiger; Bai, Jinhui; Lee, Minjoon; and Zhang, Fudong. 2017. “The Welfare and Distributional Effects of Fiscal Volatility: A Quantitative Evaluation.” CEPR Discussion Papers 12384 [https://cepr.org/active/publications/discussion\\_papers/dp.php?dpno=12384](https://cepr.org/active/publications/discussion_papers/dp.php?dpno=12384).
- [6] Bates, Brandon J.; Plagborg-Moeller, Mikkel; Stock, James H.; Watson, Mark W.; Consistent factor estimation in dynamic factor models with structural instability, *Journal of Econometrics*, Volume 177, Issue 2, 2013, Pages 289-304, ISSN 0304-4076, <https://doi.org/10.1016/j.jeconom.2013.04.014>.
- [7] Blanchard, Olivier; and Perotti, Roberto. 2002. “An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output.” *The Quarterly Journal of Economics*, 117(4): 1329-68. <https://doi.org/10.1162/003355302320935043>

- [8] Castañeda, Ana; Díaz-Giménez, Javier; and Ríos-Rull, José-Víctor. 2003. "Accounting for the U.S. Earnings and Wealth Inequality." *Journal of Political Economy*, 111 (4): 818-857. <http://dx.doi.org/10.1086/375382>.
- [9] Coibion, Olivier. 2012. "Are the Effects of Monetary Policy Shocks Big or Small?" *American Economic Journal: Macroeconomics*, 4 (2): 1-32.
- [10] Diamond, Peter; and Saez, Emmanuel. 2011. "The Case for a Progressive Tax: From Basic Research to Policy Recommendations." *Journal of Economic Perspectives*, 25(4): 165-190. <http://dx.doi.org/10.1257/jep.25.4.165>.
- [11] Diebold, Francis X.; and Li, Canlin. 2006. "Forecasting the term structure of government bond yields." *Journal of Econometrics*, 130(2): 337-64. <https://doi.org/10.1016/j.jeconom.2005.03.005>.
- [12] Del Negro, Marco; Otrok, Christopher; 2008. "Dynamic factor models with time-varying parameters: measuring changes in international business cycles," Staff Reports 326, Federal Reserve Bank of New York.
- [13] Dynan, Karen E.; Skinner, Jonathan; Zeldes, Stephen P. 2004. "Do the Rich Save More?" *Journal of Political Economy*, 112(2): 397-444. <https://doi.org/10.1086/381475>.
- [14] Feenberg, Daniel R.; Ferriere, Axelle; and Navarro, Gastón. 2017. "Evolution of Tax Progressivity in the U.S.: New Estimates and Welfare Implications." 2017 Meeting Papers 989, Society for Economic Dynamics. <https://ideas.repec.org/p/red/sed017/989.html>.
- [15] Feldstein, Martin S. 1969. "The Effects of Taxation on Risk Taking." *Journal of Political Economy*, 77(5): 755-764. <https://doi.org/10.1086/259560>.
- [16] Fisher, Jonathan D.; Johnson, David S.; and Smeeding, Timothy M. 2013. "Measuring the Trends in Inequality of Individuals and Families: Income and Consumption." *American Economic Review*, 103(3): 184-88. <https://doi.org/10.1257/aer.103.3.184>.

- [17] Gruber, Jon; and Saez, Emmanuel. 2002. "The elasticity of taxable income: evidence and implications." *Journal of Public Economics*, 84(1): 1-32. [https://doi.org/10.1016/S0047-2727\(01\)00085-8](https://doi.org/10.1016/S0047-2727(01)00085-8).
- [18] Heathcote, Jonathan; Perri, Fabrizio; and Violante, Giovanni L. 2010. "Unequal we stand: An empirical analysis of economic inequality in the United States, 1967-2006." *Review of Economic Dynamics*, 13(1): 15-51. <https://doi.org/10.1016/j.red.2009.10.010>.
- [19] Heathcote, Jonathan; Storesletten, Kjetil; Violante, Giovanni L. 2017. "Optimal Tax Progressivity: An Analytical Framework." *The Quarterly Journal of Economics*, 132(4): 1693-1754. <https://doi.org/10.1093/qje/qjx018>.
- [20] Inoue, Atsushi; and Rossi, Barbara. 2019. "The effects of conventional and unconventional monetary policy on exchange rates." *Journal of International Economics*, 118: 419-447. <https://doi.org/10.1016/j.jinteco.2019.01.015>.
- [21] Jappelli, Tullio; and Pistaferri, Luigi. 2010. "The Consumption Response to Income Changes." *Annual Review of Economics*, 2(1): 479-506. <https://doi.org/10.1146/annurev.economics.050708.142933>.
- [22] Kindermann, Fabian; and Krueger, Dirk. 2014. "High Marginal Tax Rates on the Top 1%? Lessons from a Life Cycle Model with Idiosyncratic Income Risk." NBER Working Paper 20601. <https://doi.org/10.3386/w20601>.
- [23] Krusell, Per; and Smith, Anthony A. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy*, 106(5): 867-896. <https://doi.org/10.1086/250034>.
- [24] Litterman, R.B. 1986. "Forecasting with Bayesian Vector Autoregressions— Five Years of Experience," *Journal of Business and Economic Statistics* 4(1), pp. 35-38. <https://doi.org/10.1080/07350015.1986.10509491>.

- [25] McCarthy, Jonathan. 1995. "Imperfect insurance and differing propensities to consume across households." *Journal of Monetary Economics*, 36: 301-327. [https://doi.org/10.1016/0304-3932\(95\)01214-1](https://doi.org/10.1016/0304-3932(95)01214-1).
- [26] Mertens, Karel and Luis Montiel Olea, 2018. "Marginal Tax Rates and Income: New Time Series Evidence." *The Quarterly Journal of Economics*, 133(4): 1803–1884. <https://doi.org/10.1093/qje/qjy008>.
- [27] Mountford, Andrew; and Uhlig, Harald. 2009. "What Are the Effects of Fiscal Policy Shocks?" *Journal of Applied Econometrics*, 24: 960-992. <https://doi.org/10.1002/jae.1079>.
- [28] Nelson, Charles R.; and Siegel, Andrew F. 1987. "Parsimonious Modeling of Yield Curves." *Journal of Business*, 60(4): 473-489. <http://dx.doi.org/10.1086/296409>.
- [29] Parker, Jonathan A.; Souleles, Nicholas S.; Johnson, David S.; and McClelland, Robert. 2013. "Consumer Spending and the Economic Stimulus Payments of 2008." *American Economic Review*, 103(6): 2530-53. <https://doi.org/10.1257/aer.103.6.2530>.
- [30] Piketty, Thomas; Saez, Emmanuel; and Zucman, Gabriel. 2018. "Distributional National Accounts: Methods and Estimates for the United States." *The Quarterly Journal of Economics*, 133(2): 553-609. <https://doi.org/10.1093/qje/qjx043>.
- [31] Ramey, Valerie. 2016. "Macroeconomic Shocks and Their Propagation." In *Handbook of Macroeconomics*, eds. John B. Taylor and Harald Uhlig, 2: 71-162. Elsevier. <https://doi.org/10.1016/bs.hesmac.2016.03.003>
- [32] Reis, Ricardo and Watson, Mark W. 2010. "Relative Goods' Prices, Pure Inflation, and the Phillips Correlation." *American Economic Journal: Macroeconomics*, 2(3): 128-157. DOI: 10.1257/mac.2.3.128
- [33] Romer, Christina D., and David H. Romer. 2004. "A New Measure of Monetary Shocks: Derivation and Implications." *American Economic Review*, 94(4): 1055-1084.



- [34] Romer, Chistina D.; and Romer, David H. 2010. "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks." *American Economic Review*, 100(3): 763-801. <https://doi.org/10.1257/aer.100.3.763>.
- [35] Saez, Emmanuel and Zucman, Gabriel. 2020. "Trends In US Income and Wealth Inequality: Revising After the Revisionists," NBER Working Paper 27921.
- [36] Sachs, Dominik, Tsyvinski, Aleh, and Werquin, Nicolas. 2020. "Nonlinear Tax Incidence and Optimal Taxation in General Equilibrium" *Econometrica*, 88(2). <https://doi.org/10.3982/ECTA14681>.
- [37] Sims, C.A., and Zha, T. 1998. "Bayesian Methods for Dynamic Multivariate Models," *International Economic Review* 39(4), pp. 949-968. <http://www.jstor.org/stable/2527347>.
- [38] Slemrod, Joel B. 1996. "High Income Families and the Tax Changes of the 1980s: The Anatomy of Behavioral Response." In *Empirical Foundations of Household Taxation*, eds. Martin Feldstein and James Poterba, 169-92. Chicago, IL: University of Chicago Press. <https://doi.org/10.3386/w5218>.
- [39] Uhlig, Harald. 2005. "What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure." *Journal of Monetary Economics*, 52: 381-419. <https://doi.org/10.1016/j.jmoneco.2004.05.007>.
- [40] Waggoner D.F., and Zha, T. 2003. "A Gibbs Sampler for Structural Vector Autoregressions." *Journal of Economic Dynamics and Control* 28(2), pp. 349-366. [https://doi.org/10.1016/S0165-1889\(02\)00168-9](https://doi.org/10.1016/S0165-1889(02)00168-9).
- [41] Zidar, Owen M. 2019. "Tax Cuts for Whom? Heterogeneous Effects of Income Tax Changes on Growth and Employment." *Journal of Political Economy*, 127(3): 1437-1472. <https://doi.org/10.1086/701424>.

<b>Table 1: Priors for the Model Parameters</b>			
Parameter	Prior	Parameter	Prior
$\lambda_{20K}^\tau$	$N(1, 0.1)$	$\lambda_{20K}^\pi$	$N(-4, 1)$
$\lambda_{40K}^\tau$	$N(1, 0.1)$	$\lambda_{40K}^\pi$	$N(-2, 1)$
$\lambda_{100K}^\tau$	$N(1, 0.1)$	$\lambda_{100K}^\pi$	$N(0, 1)$
$\lambda_{200K}^\tau$	$N(1, 0.1)$	$\lambda_{200K}^\pi$	$N(2, 1)$
$\lambda_{400K}^\tau$	$N(1, 0.1)$	$\lambda_{400K}^\pi$	$N(4, 1)$
$\lambda_{1M}^\tau$	$N(1, 0.1)$	$\lambda_{1M}^\pi$	$N(8, 1)$
$\sigma_i^2$	$IG(6, 0.001)$		
<i>Structural VAR Parameters:</i>			Prior
Overall tightness of prior beliefs			$\lambda_0 = \frac{1}{\sqrt{0.001}}$
Tightens the prior around zero for coefficients on lags of other variables			$\lambda_1 = 0.2$
Controls the rate at which the prior tightens as the lag length increases			$\lambda_3 = 1$
Controls the tightness of the coefficient on the linear trend			$\lambda_4 = 10$

Table 1: Priors for the Model Parameters:  $N$  is the number of variables in the VAR and  $m$  is the number of parameters for each lag of the VAR

<b>Table 2: Variance Decompositions</b>			
Income Level	Level	Progressivity	Idiosyncratic
\$20,000	0.44	0.03	0.54
\$40,000	0.83	0.06	0.11
\$100,000	0.67	0.04	0.28
\$200,000	0.91	0.02	0.07
\$400,000	0.80	0.20	0.00
\$1,000,000	0.65	0.28	0.07

Table 2: Variance decompositions computed based on the tax level and progressivity factors produced from the FAVAR model

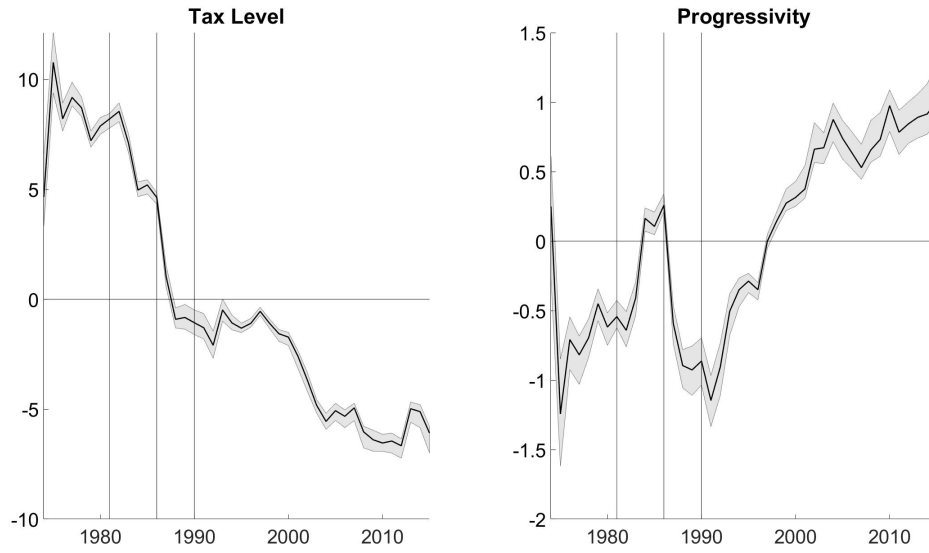


Figure 1: Posterior distribution of the tax level and progressivity factors extracted from observed tax data in the FAVAR model. The solid line and light shaded region depict the median and 68% posterior coverage of the estimated factors across Gibbs draws.

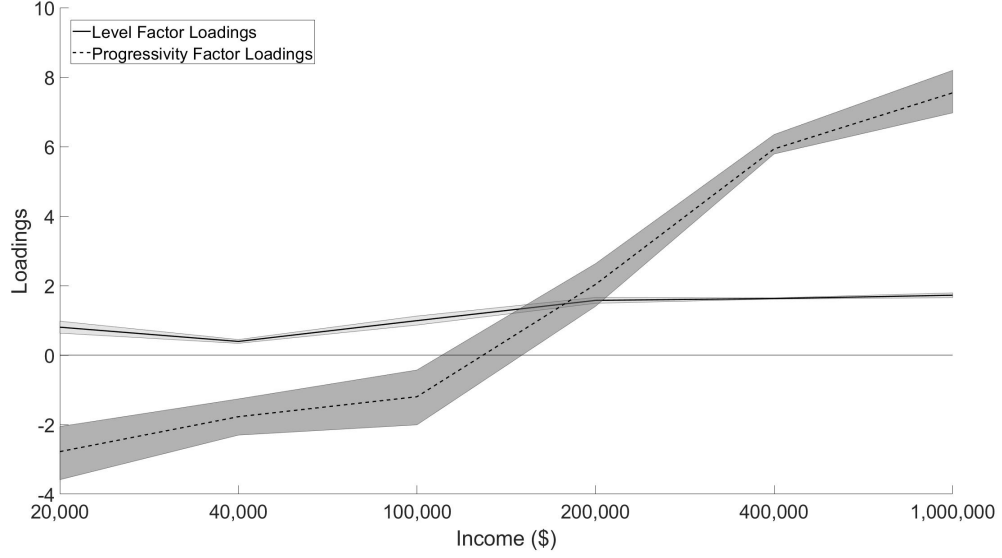


Figure 2: Posterior distribution of the factor loadings estimated from FAVAR model. The solid line and light shaded region depict the median and 68% posterior coverage of the estimated level factor loadings. The dashed line and dark shaded region depict the median and 68% posterior coverage of the estimated progressivity factor loadings.

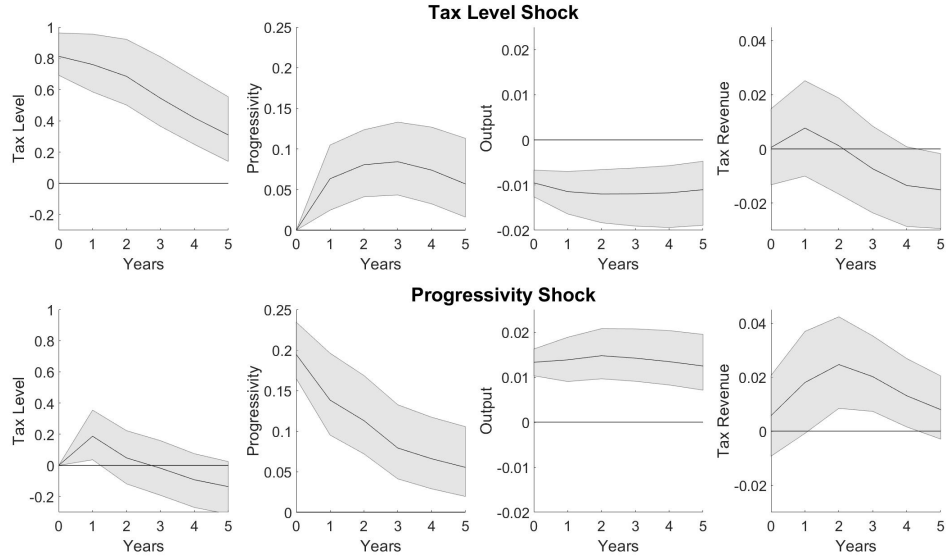


Figure 3: Impulse responses to tax shocks from the baseline FAVAR with the tax level factor, tax progressivity factor, log real GDP, and log real tax revenue. The top row depicts the responses to a tax level shock while the bottom row shows the responses to a progressivity shock. The solid line and shaded region represent the posterior median and 68% posterior coverage across all Gibbs draws.

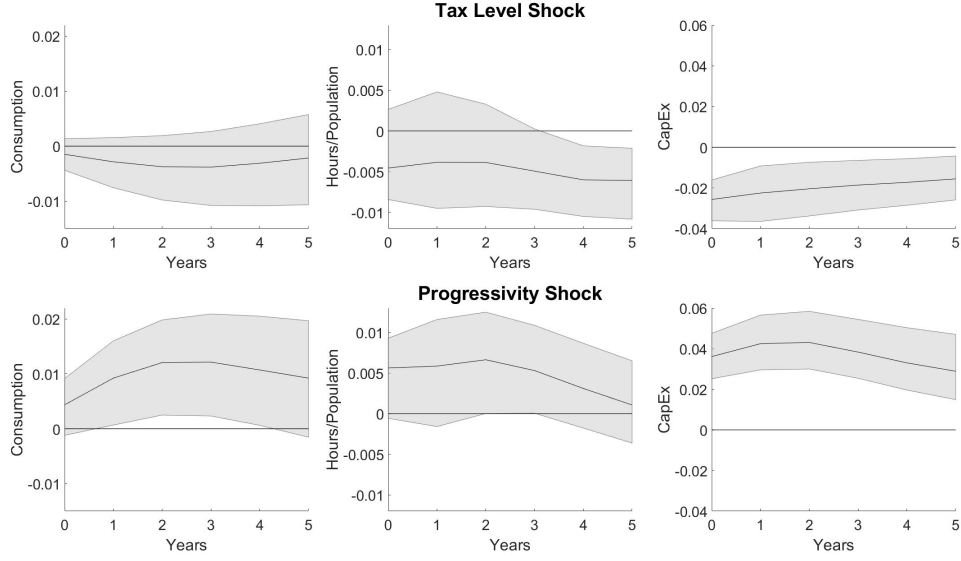


Figure 4: Impulse responses from the FAVAR with (i) the tax level factor, tax progressivity factor, log real consumption expenditure, and log real tax revenue; (ii) the tax level factor, tax progressivity factor, log real GDP, log real tax revenue, and log hours/population, and (iii) the tax level factor, tax progressivity factor, log CapEx \$, and log real tax revenue. The top row depicts the responses to a tax level shock while the bottom row shows the responses to a progressivity shock. The solid line and shaded region represent the posterior median and 68% posterior coverage across all Gibbs draws.

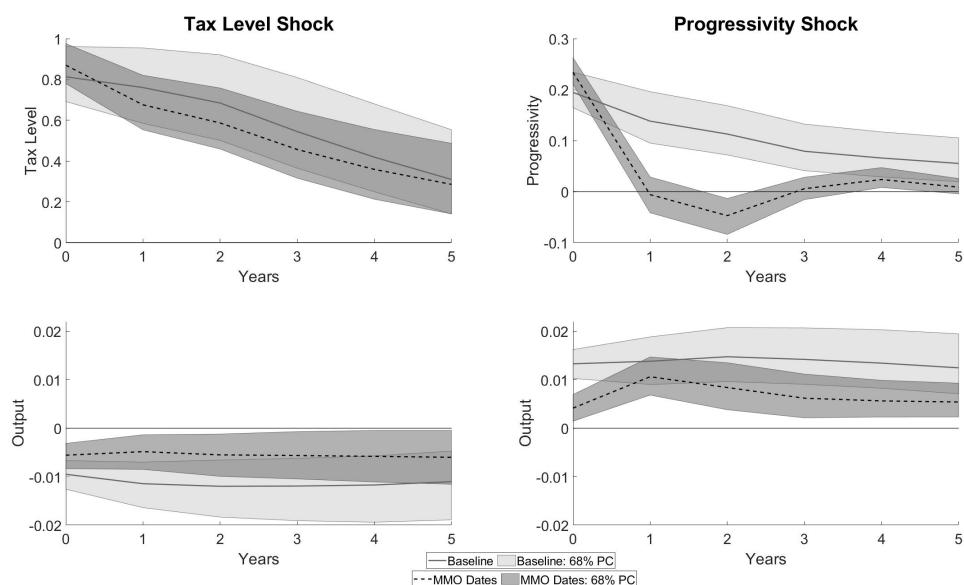


Figure 5: Impulse responses from the FAVAR with (i) the tax level factor, tax progressivity factor, log real GDP, and log real tax revenue; (ii) the tax level factor, tax progressivity factor dummied out to include only the dates identified by the narrative approach in Mertens and Montiel Olea (2018), log real GDP, and log real tax revenue. The solid line and shaded region represent the posterior median and 68% posterior coverage across all Gibbs draws.

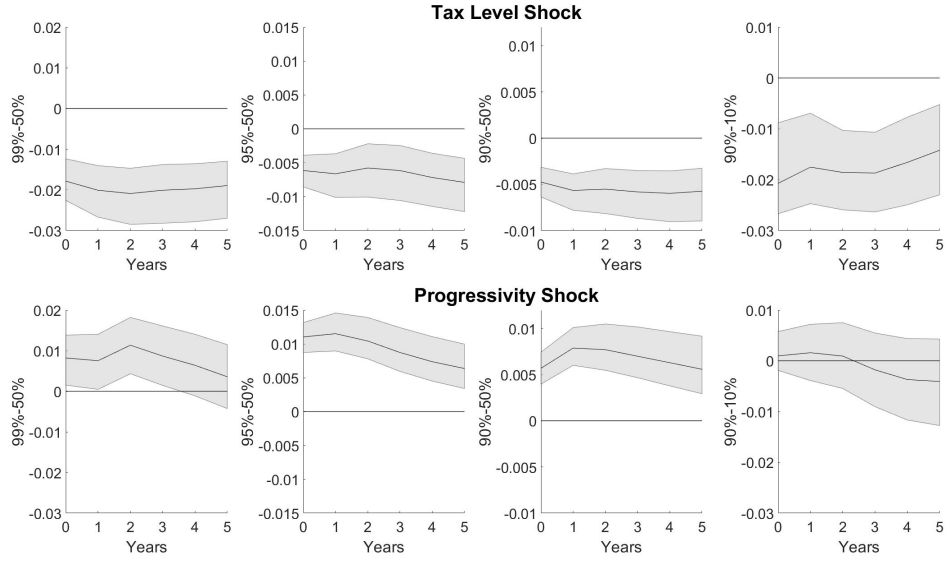


Figure 6: Impulse responses from the FAVAR with the tax level factor, tax progressivity factor, log real tax revenue, and various measures of income inequality from Piketty, Saez and Zucman (2018). The top row depicts the responses to a tax level shock while the bottom row shows the responses to a progressivity shock. The solid line and shaded region represent the posterior median and 68% posterior coverage across all Gibbs draws.

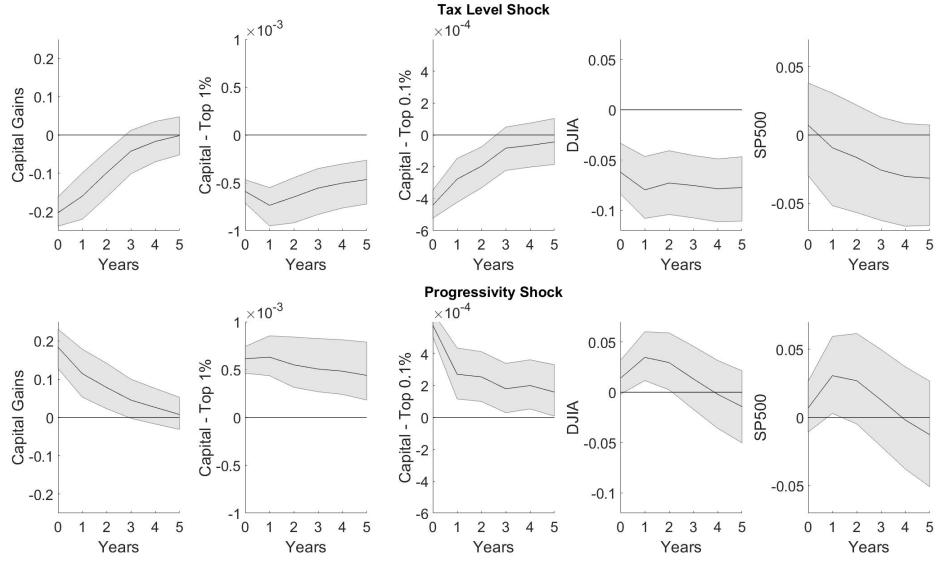


Figure 7: Responses to level and progressivity shocks from the FAVAR with the tax level factor, tax progressivity factor, log real tax revenue, and one of the following measures: (i) log real total realized capital gains; (ii) capital income share of the top 1%; (iii) capital income share of the top 0.1%; (iv) log DJIA Index; (v) log SP500. The top row depicts the responses to a tax level shock while the bottom row shows the responses to a progressivity shock. The solid line and shaded region represent the posterior median and 68% posterior coverage across all Gibbs draws.



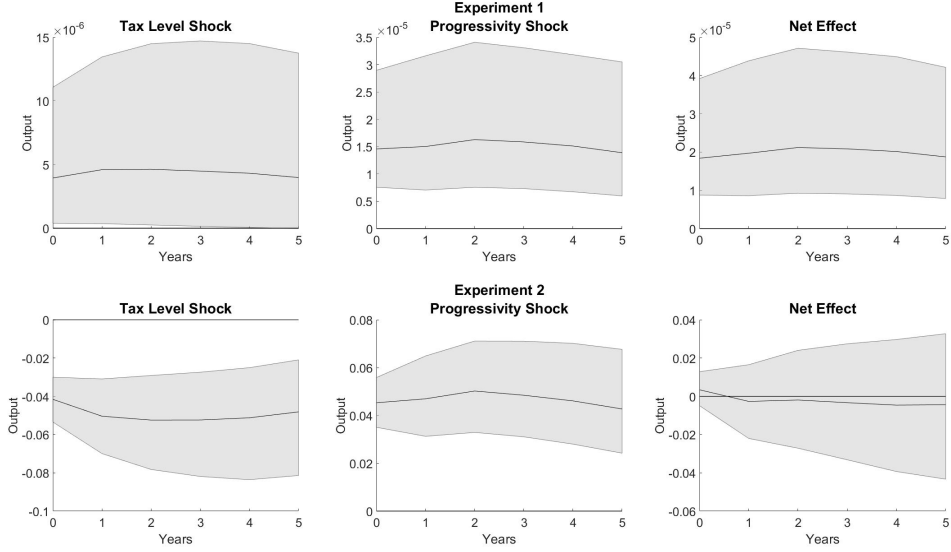


Figure 8: Impulse responses from the two policy experiments. In Experiment 1, the tax rate on the highest incomes, over \$1 million, is increased by 0.1. In Experiment 2, the tax rate on the two highest incomes, over \$400,000 and over \$1 million, are increased by 0.1. We conduct the experiment using draws of the model parameters across all Gibbs iterations and plot the median and 68% posterior coverage.

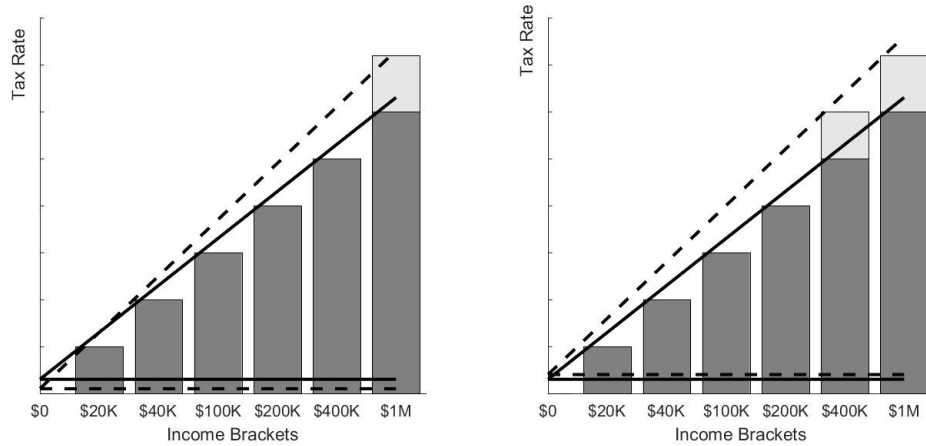


Figure 9: Graphical representation of the policy experiments and the effects on the tax level and progressivity factors. The top row shows Experiment 1 in which the tax rate on the highest incomes, over \$1 million, is increased by 0.1. The bottom row shows Experiment 2 in which the tax rate on the two highest incomes, over \$400,000 and over \$1 million, are increased by 0.1.

## A Details of the Structural VAR Estimation

### A.1 The Prior

Denote  $a_i$  and  $b_i$  as the  $i$ th columns of  $\mathbf{A}'_0$  and  $\mathbf{A}'$ , respectively. This allows one to consider each equation of the structural VAR separately, as in Waggoner and Zha (2003). We utilize a variation of the prior proposed by Sims and Zha (1998). This prior imposes independence across structural equations and thus imposes a prior on the elements of  $a_i$  to be jointly normal and mean zero. Additionally, the prior of  $b_i$  is conditional on  $a_i$ . Following the general framework of the prior described in Litterman (1986), Sims and Zha (1998) impose that the conditional mean of the coefficients in  $b_i$  on the first lag term in the VAR is equal to  $a_i$ , while the coefficients on the remaining lags have mean zero. In doing so, this assumes that the random walk is a reasonable approximation of the behavior of the variables within the VAR. In our VAR, we include the factors, log real GDP, and log real tax revenue. As discussed in Litterman (1986), if the variables within the VAR must be stationary, it may be more appropriate to specify a prior assuming less persistence. Additionally, we wish to assign some meaningful role to the factors without violating stationarity of the transition equation in the state-space system.<sup>37</sup> Furthermore, since we incorporate a linear trend component to the dynamics of the VAR, these should also be stationary. To accomplish this, we set the prior such as to impose the mean of the reduced-form coefficient in each equation on their own first lag is centered around 0. Ultimately, the prior on the VAR coefficients can be described as the following:

$$a_i \sim N(0, \overline{S}_i),$$

$$b_i|a_i \sim N(\overline{P}_i a_i, \overline{H}_i),$$

for  $i = 1, \dots, N + K$ .

Sims and Zha (1998) define a set of hyperparameters to control the tightness of various components of the prior. We adopt a subset of these hyperparameters necessary to implement the prior in our specification of the structural VAR model. These are defined in Table 2.

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<sup>37</sup>This stability is important for the Kalman filter step in which we estimate the factors.

We construct  $\bar{S}_i$  as a diagonal matrix with the standard deviation for coefficients on elements in the  $j$ th row of the VAR defined as  $\frac{\lambda_0}{\sigma_j}$ . The  $\sigma_j$  terms are scale factors that account for potential variation in the units of measure of the data. Thus,  $\sigma_j^2$  is computed as the sample variance of residuals from univariate autoregressions.

As discussed previously, we adjust the prior on the first lag to be zero. The matrix  $\bar{P}_i$  consists of  $\text{diag}[0_{1 \times K+N}]$  in the first  $N+K$  rows and zeros elsewhere, corresponding to all coefficients on lags greater than 1. We construct  $\bar{H}_i$  as a diagonal matrix with the variance of coefficients on the  $k$ th lag of variable  $j$  as  $\frac{\lambda_0^2 \lambda_1^2}{\sigma_j^2 k \lambda_3}$ .

## A.2 Drawing $\mathbf{A}_0$ and $\mathbf{A}|\mathbf{A}_0$ conditional on $\Psi_{-\mathbf{A}_0}$

To implement the draw for each of these columns, define  $c_i$  and  $d_i$  such that  $a_i = U_i c_i$  and  $b_i = V_i d_i$ , where  $U_i$  and  $V_i$  are orthonormal rotation matrices that impose the identifying restrictions placed upon  $\mathbf{A}_0$  and  $\mathbf{A}$ . Considering the normal prior for  $a_i$  and  $b_i$ , the marginal posterior pdf of  $c_i$  and  $d_i$  take the form:

$$p(c_1, \dots, c_{N+K} | \tilde{Y}_T, \tilde{X}_T, \tilde{Z}_T) \propto |\det[U_1 c_1 | \dots | U_{N+K} c_{N+K}]|^T \exp\left(-\frac{T}{2} \sum_{i=1}^{N+K} c_i' S_i^{-1} c_i\right) \quad (4)$$

$$p(d_i | c_i, \tilde{Y}_T, \tilde{X}_T, \tilde{Z}_T) = \phi(P_i c_i, H_i), \quad (5)$$

where  $H_i$ ,  $P_i$ ,  $S_i$  are the parameters of the posterior based upon the transformations described in Waggoner and Zha (2003) and  $\phi$  is the pdf of the Normal distribution with mean  $P_i c_i$  and variance  $H_i$ . The Gibbs sampler sequentially draws each  $c_i$ , conditional on the other  $c_j$ ,  $j = 1, \dots, i-1, i+1, \dots, N+K$ . Once we obtain a draw for  $c_i$ , and thus  $a_i$ , we use (5) to draw  $d_i$  and compute  $b_i$ . Upon completion of the draws for  $a_i$  and  $b_i \forall i$ , we construct the matrices  $\mathbf{A}_0$  and  $\mathbf{A}$  and convert them to the reduced-form expressions  $B = \mathbf{A}_0^{-1} \mathbf{A}$  and  $\Omega = (\mathbf{A}_0' \mathbf{A}_0)^{-1}$  for use in the Kalman filter.