How to Starve the Beast: Fiscal and Monetary Policy Rules

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Abstract

Societies often rely on simple rules to restrict the size and behavior of governments. When fiscal and monetary policies are conducted by a discretionary and profligate government, I find that revenue ceilings vastly outperform debt, deficit and monetary rules, both in effectiveness at curbing public spending and welfare for private agents. However, effective revenue ceilings induce an increase in deficit, debt and inflation. Under many scenarios, including recurrent adverse shocks, the optimal ceiling on U.S. federal revenue is about 15% of GDP, which leads to welfare gains for private agents worth about 2% of consumption. Austerity programs should be sudden instead of gradual, and focus on lowering revenue to reduce spending rather than raising revenue to lower debt.

Keywords: time-consistency, fiscal rules, discretion, government debt, inflation, deficit, institutional design, political frictions, austerity, debt sustainability.

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1 Introduction

Rulers do not often govern in the best interest of their subjects. Among other things, they may spend excessively or improperly, or succumb to political expediency and deviate from preestablished policy norms. Direct control of government actions by private agents may be difficult and costly in practice, rendering it effectively unfeasible. Aware of these issues, societies have increasingly come to rely on institutions that constrain, rather than dictate, government actions. These constraints usually take the form of simple numerical rules and have become ubiquitous in modern times. Prime examples around the world include the U.S. federal debt ceiling, the German balanced budget amendment (or “debt brake”) and the convergence criteria for joining the Euro. More broadly, policy constraints also include the fiscal consolidation and austerity programs requested by supranational entities in exchange for economic assistance.

Starting around the 1990s, there has been widespread adoption of numerical fiscal rules, in both advanced and emerging economies. According to the IMF Fiscal Rules Dataset, in 2015, 92 out of 96 surveyed countries had adopted at least one fiscal rule. According to Schaechter et al. (2012) these rules “impose a long-lasting constraint on fiscal policy through numerical limits on budgetary aggregates” and aim at “containing pressures to overspend, in particular in goods times, so as to ensure fiscal responsibility and debt sustainability.” These fiscal rules are classified according to the type of variable they attempt to constrain—e.g., deficit, revenue or debt.

This paper provides a systematic study of policy rules, taking the view that governments are naturally discretionary and prone to excessive spending. The main purpose of this exercise is to understand the effectiveness of policy rules at curbing government spending and the welfare implications for private agents. I focus on the types of policy rules we see implemented in the real world to discipline government actions: revenue ceilings, limits on the primary or total deficit, and limits on the public debt, either nominal or in terms of output. I also consider monetary policy rules, such as inflation targets, interest rate pegs and Taylor rules, though they turn out to be undesirable for the purpose of curbing public spending.

I consider a closed economy in which fiscal and monetary policies are jointly determined. The environment is a monetary economy populated by infinitely-lived agents, where a government uses distortionary taxes, fiat money and nominal bonds to finance the provision of a valued public good.1 The government is not fully benevolent, preferring higher public expenditure than private agents, and lacks the ability to commit to policy choices beyond the current period. The economy is potentially subjected to a variety of aggregate shocks to demand, productivity, public expenditure and liquidity.

Under full discretion, government policy is determined by the interaction of three main forces: distortion-smoothing, a time-consistency problem and political frictions. The incentive to smooth distortions intertemporally follows the classic arguments in Barro (1979) and Lucas and Stokey (1983). Time-consistency problems arise from the interaction between debt and monetary policy, as analyzed in Martin (2009, 2011, 2013): how much debt the government inherits, affects its monetary policy since inflation reduces the real value of nominal liabilities; in turn, the anticipated response of future monetary policy affects the current demand for money and bonds, and thereby how the government today internalizes policy trade-offs. The political friction creates an upward bias in public expenditure, which has consequences for inflation and taxation.

An important theoretical result is that in the absence of aggregate uncertainty, the dis-

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1 The model is an extension of Martin (2011, 2013), where the monetary economy is based on Lagos and Wright (2005). Most of the analysis and lessons here would carry over to economies with a cash-in-advance constraint or money-in-the-utility function, although at the cost of lower analytical tractability.
cretionary steady state is constrained-efficient. That is, endowing the government with commitment power at the discretionary steady state would not cause a change in policy. In this case, all the welfare gains from imposing policy rules would arise from correcting the political friction, by inducing a reduction in government spending or, at least, altering the means by which expenditure is financed. The key to an effective policy rule is to make the government internalize the cost of socially suboptimal policy. The addition of aggregate uncertainty does not alter this result significantly, as even discretionary governments respond to shocks with sufficient efficiency. In contrast, adopting rules far away from the steady state may significantly alter the prescription, as the time-consistency problem becomes more prominent.

Policy rules incorporate elements from both Neo-Hobbesian and Pigouvian traditions of public finance. The former advocates making the tax base inefficient to curb the *Leviathan* (the beast, i.e., the government), while the latter is concerned with how to optimally finance the provision of public goods. Policy rules are generally intended to discipline government actions, particularly, preventing it from growing too large or setting off on an unsustainable path. At the same time, rules affect how governments are financed and hence, may better align the policy-mix with the preferences of the governed.

The merits of the various policy rules are evaluated quantitatively, in an economy calibrated to the U.S. in the postwar period. I also conduct several robustness checks, including a case with a larger (less benevolent) government and one with inflation at current levels rather than the (higher) postwar average. I first study economies without aggregate uncertainty and later add fluctuations in aggregate demand, productivity, public expenditure and liquidity. Shocks are first assumed to be large and rare, but I also consider more frequent shocks. The conclusions drawn below are common across all these different specifications, unless otherwise noted.

The main lesson is that the best policy rule is a ceiling on revenue in terms of output. The optimal revenue ceiling is about 15% of GDP, three percentage points lower than with a fully discretionary government. A revenue ceiling is the only rule that significantly lowers government expenditure; all other rules are ineffective in this regard and only manage to alter the policy-mix by which expenditure is financed. Imposing the optimal revenue ceiling implies sizable welfare gains for private agents, equivalent to about 2% of consumption. Welfare gains for all other rules are at most an order of magnitude smaller. Though beneficial to private agents, revenue ceilings imply a deterioration of macroeconomic performance: debt and inflation increase significantly and output contracts. This may pose a political challenge to its effective implementation.

Budget balance (deficit) and debt rules are generally benign, even though they do not offer welfare gains as large as revenue ceilings and are ineffective at curbing government spending. Budget balance rules, limits on the primary deficit in particular, generally yield higher welfare than debt rules. This suggest that the typical focus of government reformers on debt ceilings may be misplaced. It is always more desirable, and arguably easier in practice, to aim at constraining the deficit.

Austerity programs, such as those implemented recently in several European countries, are designed to reimpose discipline on governments, often with the aim of bringing the debt burden to more sustainable levels. The optimal revenue ceiling and budget balance rules remain beneficial when debt is high, well above the steady state. Debt constraints, on the other hand, may lead to large welfare losses if imposed abruptly. Hence, austerity programs are best targeted at revenue and deficits instead of debt. In contrast to the current austerity debate (e.g., see Alesina et al., 2018), the optimal prescription here is to lower revenue to reduce expenditure (starve the beast), rather than to raise revenue to generate a surplus and thus, lower debt.

It is also instructive to explore the effects of announced reforms. In general, there is a welfare

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2See Brennan and Buchanan (1977) and Engineer (1990).
cost associated with delayed imposition of policy rules, as the benefits of constrained policy are pushed into the future. However, debt rules benefit overall from delayed implementation, as it allows debt to be adjusted gradually before the rule takes effect, which is good for distortion-smoothing purposes. Interestingly, the transition from announcement to imposition of a revenue ceiling is characterized by an improvement in government accounts: the government runs a surplus and so, decreases debt and inflation; this allows for the switch to the constrained regime to be gentler, and thus, less painful. From an outsider’s perspective, the announcement of a revenue ceiling to be imposed in the future has important, seemingly beneficial, effects today and until the rule is enforced. From a welfare perspective, however, agents would prefer to impose the rule right away.

When the economy is subjected to aggregate fluctuations, along the lines described above, most welfare gains from fiscal rules come primarily from imposing them in normal times, which also helps discipline government policy during abnormal times. The cost of suspending fiscal rules during adverse times (e.g., a recession) are minor, as long as these rules are reimposed when the economy returns to normal. In addition, the cost of implementing fiscal rules sub-optimally, e.g., picking the wrong ceiling, also carry relatively small welfare costs. Notable exceptions to these results include suspending budget balance or revenue rules when government expenditure temporarily increases (assuming agents do not value such an increase) and imposing a primary deficit ceiling when debt is well below the steady state.

To complement the analysis described above, I also consider constraints on monetary policy. These rules are not generally desirable in the present context as they yield small welfare gains and sometimes even losses relative to full discretion. More problematic is the fact that slight mis-targeting or incorrect timing can lead to large welfare losses. The reason for these negative results is that monetary policy targets hinder the ability of governments to smooth distortions across states. In effect, inflation allows for less distortionary repayments of temporary debt increases.

*Rules in practice*—Perhaps the most famous example of fiscal rules is the economic convergence criteria by prospective members of the European Economic and Monetary Union (the “Eurozone”), which arguably allowed several countries to impose discipline on their governments by targeting polices more in line with those of strong performing economies. The U.S. itself has several formal constraints on fiscal policy. The debt ceiling legislation forces the executive to seek Congressional approval when increasing federal debt beyond the pre-established limit. In addition, most states are subjected to balanced-budget rules and there have been repeated proposals to impose one at the federal level. Germany amended its constitution in 2009 to include a balanced budget rule or “debt brake” (*Schuldenbremse*), which affected both federal and local governments.

In practice, however, institutional constraints on government policy may not work as intended. Although membership to the Eurozone was granted conditional on meeting explicit convergence criteria, the reality was that many countries did not meet them (Greece being a notable example as it met none of the criteria upon entry). More recently, around 2014–2015, even core countries such as France were not satisfying European Union deficit targets. In the U.S., the debt ceiling has done very little to curtail the recent growth of public debt, which has reached levels not seen since the end of World War II. In contrast, the German amendment seems to have worked as intended, leading to a significant reduction in debt-to-GDP.

There is a natural tension between the desirability of constraining government behavior in normal and abnormal times. As wise as it may be to discipline policymakers, severe adverse shocks may require some degree of flexibility, in particular, the relaxation or outright abandonment of pre-existing rules. For example, the U.S. government arguably responded in

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3This qualification poses more of a political than an economic challenge.
a discretionary manner during the American Civil War and the two World Wars, but it would likely have been detrimental to limit its capacity to issue debt during these episodes.\footnote{See Barro (1979), Ohanian (1997), Aiyagari et al. (2002) and Martin (2012).} The German balanced budget rule allows exceptions for natural disasters and deep recessions. In the aftermath of the recent global financial crisis, some countries in the Eurozone have questioned the benefits of delegating monetary policy to a supranational entity that does not internalize regional concerns and pondered the desirability of abandoning the monetary union. In all these cases it is hard to separate the value of flexibility from the gains of political expediency.

**Rules in theory**—A perennial debate in the design of political institutions is the trade-off between commitment and flexibility, also commonly referred to as rules versus discretion. At the heart of the issue is a time-consistency problem, that is, the temptation to revise \textit{ex ante} optimal policy plans. Allowing policymakers to exercise too much discretion raises the potential for bad policy outcomes, such as, high inflation, large debt accumulation or excessive capital taxation.\footnote{See Strotz (1956), Kydland and Prescott (1977), Barro and Gordon (1983), Benhabib and Rustichini (1997), Albanesi et al. (2003), Martin (2010), among many others.} Unfortunately, forcing policymakers to implement benevolent rules is not straightforward. \textit{Ex ante} optimal policy plans are oftentimes complicated objects that cannot be easily legislated and require a great deal of foreknowledge of all possible future states of the world. There is virtue in simplicity when binding the behavior of future policymakers; simple, straightforward rules are easy to write down and make non-compliance easy to verify.

The classical approach in the literature has been to compare the outcomes under full commitment and full discretion. Here, instead, I focus on comparing full discretion with constrained discretionary policy. Related work on fiscal policy constraints includes Bohn and Inman (1996), Athey et al. (2005), Bassetto and Sargent (2006), Chari and Kehoe (2007), Niepelt (2007), Azzimonti et al. (2016), Halac and Yared (2014, 2018) and Hatchondo et al. (2017). The effect of tax cuts have been evaluated by Bohn (1991), Romer and Romer (2009, 2010), Cloyne (2013) and Fuest et al. (2018) among many others.

The rest of the paper is organized as follows. Section 2 presents the environment and derives optimal behavior by private agents, given government policy. Section 3 studies the determination of fiscal and monetary policy when the government is discretionary. Section 4 defines and analyzes policy rules that may be imposed to constraint government behavior. Section 5 analyzes the optimal implementation of each policy rule in economies without aggregate shocks. Section 6 extends the analysis to stochastic economies and adds the possibility of selectively suspending a rule, depending on the state of the economy. Section 7 covers monetary rules and shows their ineffectiveness at curbing public expenditure. Section 8 concludes.

### 2 Model

#### 2.1 Environment

Consider an economy populated by a continuum of infinitely-lived agents, which discount the future by factor $\beta \in (0, 1)$. Each period, two competitive markets open in sequence, for expositional convenience labeled \textit{day} and \textit{night}. All goods produced in the economy are perishable and cannot be stored from one subperiod to the next.

At the beginning of each period, agents receive an idiosyncratic shock that determines their role in the day market. With probability $\eta \in (0, 1)$ an agent wants to consume but cannot produce the day-good $x$, while with probability $1 - \eta$ an agent can produce but does not want consume. A consumer derives utility $u(x)$, where $u$ is twice continuously differentiable, satisfies
Inada conditions and \( u_{xx} < 0 < u_x \). A producer incurs in utility cost \( \phi > 0 \) per unit produced.

Agents are anonymous and lack commitment. Thus, credit arrangements are not feasible and some medium of exchange is necessary for day trade to occur.\(^6\) Exchange media in this economy takes the form of government-issued liabilities: cash and one-period nominal bonds. Cash is universally recognized and can be used in all transactions. Following Kiyotaki and Moore (2002), assume that agents may pledge a fraction \( \theta \in (0, 1) \) of their government bond holdings to finance day market expenditures.

At night, all agents can produce and consume the night-good, \( c \). The production technology is assumed to be linear in labor, such that \( n \) hours worked produce \( \zeta n \) units of output, where \( \zeta > 0 \). Assuming perfect competition in factor markets, the wage rate is equal to productivity \( \zeta \). Utility at night is given by \( \gamma U(c) - \alpha n \), where \( U \) is twice continuously differentiable, \( U_{xx} < 0 < U_x, \gamma > 0 \) and \( \alpha > 0 \).

There is a government that supplies a valued public good \( g \) at night. Agents derive utility from the public good according to \( v(g) \), where \( v \) is twice continuously differentiable, satisfies Inada conditions and \( v_{gg} < 0 < v_g \). To finance its expenditure, the government may use proportional labor taxes \( \tau \), print fiat money at rate \( \mu \) and issue one-period nominal bonds, which are redeemable in fiat money. Government policy choices for the period are announced at the beginning of each day, before agents’ idiosyncratic shocks are realized. The government only actively participates in the night market, i.e., taxes are levied on hours worked at night and open-market operations are conducted in the night market. The public good is transformed one-to-one from the night-good.

Let \( s \equiv \{\gamma, \zeta, \theta, \omega\} \) denote the exogenous aggregate state of the economy, which is revealed to all agents at the beginning of each period. The economy is thus subject to a variety of aggregate shocks: demand (\( \gamma \)), productivity (\( \zeta \)), liquidity (\( \theta \)) and government type (\( \omega \))—the role played by this last parameter will be explained below. The set of all possible realizations for the stochastic state is \( S \). Let \( E[s'|s] \) be the expected value of future state \( s' \) given current state \( s \).

All nominal variables—except for bond prices—are normalized by the aggregate money stock. Thus, today’s aggregate money supply is equal to 1 and tomorrow’s is \( 1 + \mu \). The government budget constraint can be written as

\[
p_c(\tau \zeta n - g) + (1 + \mu)(1 + qB') - (1 + B) = 0,
\]

where \( B \) is the current aggregate bond-money ratio, \( p_c \) is the—normalized—market price of the night-good \( c \), and \( q \) is the price of a bond that earns one unit of fiat money in the following night market. “Primes” denote variables evaluated in the following period. Thus, \( B' \) is tomorrow’s aggregate bond-money ratio. In equilibrium, prices and policy variables depend on the aggregate state \( (B, s) \); this dependence is omitted from the notation to simplify exposition.

### 2.2 Problem of the agent

Let \( V(m, b, B, s) \) be the value of entering the day market with (normalized) money balances \( m \) and bond balances \( b \), when the aggregate state of the economy is \( (B, s) \). Upon entering the night market, the composition of an agent’s nominal portfolio (money and bonds) is irrelevant, since bonds are redeemed in fiat money at par. Thus, let \( W(z, B, s) \) be the value of entering the night market with total (normalized) nominal balances \( z \).

In the day market, consumers and producers exchange money and bonds for goods at (normalized) price \( p_x \). Let \( x \) be the individual quantity consumed and \( \kappa \) the individual quantity

produced; these quantities are generally different in equilibrium, unless there is an equal measure of consumers and producers. A consumer with starting balances \((m, b)\) has total liquidity \(m + \theta b\) to purchase day output. The problem of a consumer is

\[
V^c(m, b, B, s) = \max_x u(x) + W(m + b - p_x x, B, s)
\]

subject to: \(p_x x \leq m + \theta b\). The problem of a producer is

\[
V^p(m, b, B, s) = \max_x -\phi \kappa + W(m + b + p_x \kappa, B, s).
\]

Hence, the \textit{ex ante} value of an agent with portfolio \((m, b)\) at the start of the period satisfies

\[
V(m, b, B, s) \equiv \eta V^c(m, b, B, s) + (1 - \eta) V^p(m, b, B, s).
\]

At night, the problem of an agent arriving with total nominal balances \(z\) is

\[
W(z, B, s) = \max_{c, n, m', b'} \gamma U(c) - \alpha n + v(g) + \beta E[V(m', b', B', s')|s]
\]

subject to: \(p_x c + (1 + \mu)(m' + qb') = p_c (1 - \tau)\zeta n + z\).

### 2.3 Monetary equilibrium

The resource constraints in the day and night equate total consumption to total production in each subperiod. The resource constraint in the day is \(\eta x = (1 - \eta)\kappa\). Given the assumptions on preferences, individual consumption at night is the same for all agents, whereas individual labor depends on whether an agent was a consumer or a producer in the day. Hence, the resource constraint at night is given by \(c + g = \zeta[\eta n^c + (1 - \eta)n^p]\), where \(n^c\) and \(n^p\) denote night-labor by agents that were consumers or producers in the day, respectively. As shown in Lagos and Wright (2005), the preference specification also implies that all agents make the same portfolio choice. Market clearing at night implies \(m' = 1\) and \(b' = B'\).

The literature on optimal government policy with distortionary instruments typically adopts what is known as the \textit{primal approach}, which consists of using the first-order conditions of the agent’s problem to substitute prices and policy instruments for allocations in the government budget constraint. Following this approach, the problem of a government with limited commitment can be written in terms of choosing debt and allocations. After some work (see Appendix A), we get the following conditions characterizing prices \((p_x, p_c, q)\) and policy instruments \((\mu, \tau)\) in a monetary equilibrium:

\[
p_x = \frac{(1 + \theta B)}{\phi x} \tag{2}
\]

\[
p_c = \frac{\gamma U_c (1 + \theta B)}{\phi x} \tag{3}
\]

\[
q = \frac{E[\gamma U_c (1 + \theta B)]}{E[\phi x] \bigg| s} \tag{4}
\]

\[
\mu = \frac{\beta (1 + \theta B)}{\phi x} E \left[ \frac{\gamma U_c (1 - \eta) \phi}{(1 + \theta B')} \bigg| s \right] - 1 \tag{5}
\]

\[
\tau = 1 - \frac{\alpha}{\zeta \gamma U_c} \tag{6}
\]

Condition (2) is standard in monetary economies: the (normalized) price of the day-good \(p_x\) equals the total means of payment \(1 + \theta B\) (i.e., all money plus a fraction \(\theta\) of bonds) divided by
the total quantity traded. Note that variations in $\theta B$ imply variations in the (measured) velocity of circulation of money. Condition (3) establishes the price of the night-good $p_c$, which depends on the equilibrium quantities traded in the day and night. The relative price between day and night goods, $p_x/p_c$ is pinned down by the first-order condition to the producer’s problem: a producer sells goods in the day to save on effort at night and this decision is distorted by labor taxes $\tau$, which as shown by (6) can be expressed a function of the night-good allocation $c$.

Condition (4) states the equilibrium price of government bonds as a function of next-period’s day-good allocation $x'$ and total means payment $1 + \theta' B'$. In essence, the price of a bond reflects its liquidity premium: agents need to be compensated for the fact that bonds are not as liquid as money for purchasing day goods.

Condition (5) states that, for a given expected future day-good allocation (which in equilibrium is a function of debt choice, $B'$ and the exogenous state $s'$), a higher money growth rate $\mu$ implies lower day-good consumption $x$. In other words, given current debt policy and future monetary policy, the allocation of the day-good is a function of current monetary policy. Thus, we can interchangeably refer to variations in the day-good allocation and variations in current monetary policy.

Condition (6) states the trade-off between the marginal utility of night-good consumption and the marginal disutility of night-labor. This trade-off is distorted by the labor tax: a higher tax rate $\tau$ implies lower night-good consumption $c$. As with monetary policy, we can interchangeably refer to variations in the night-good allocation and variations in the tax rate.

Using (2)–(6), we can write the government budget constraint (1) in a monetary equilibrium as a function of allocations and debt,

$$
(\gamma U_c - \alpha/\zeta) c - (\alpha/\zeta) g - \phi x(1 + B) \frac{B'}{B} + \beta E \left[ \frac{\phi x(1 + B')}{1 + \theta B} | s \right] + \beta \eta E[x'(u_x - \phi)|s] = 0 \quad (7)
$$

for all $s \in S$. Condition (7) is also known as an implementability constraint, as it restricts the set of allocations that a government can implement in a monetary equilibrium.

### 3 Discretionary Policy

#### 3.1 Problem of the government

The government can commit to policy announcements for the current period, but cannot commit to policies implemented in future periods. That is, at the beginning of the period, the current government chooses $\{B', \mu, \tau, g\}$—equivalently, implements $\{B', x, c, g\}$—taking as given expected future policy. Policies implemented by the government in the future affect its current budget constraint, since future monetary policy affects the current demand for money and bonds. This is reflected by the presence of the future allocation $x'$ in the government budget (implementability) constraint (7). Due to limited commitment, the current government cannot directly control future policy, even though it can affect future policy through its choice of debt, $B'$. Future allocations depend on the policy expected to be implemented by the government, which in turn, depends on the level of debt it inherits and the exogenous aggregate state of the economy. Let $\mathcal{X}(B, s)$ be the policy that the current government anticipates will be implemented by future governments; this function implies a future day-good allocation, $x'$ for any given future state, $(B', s')$. The function $\mathcal{X}$ is an equilibrium object, but the current government takes it as given.

From the day resource constraint, we can write production in equilibrium as a function of consumption: $\kappa = \eta x/(1 - \eta)$. Thus, an agent’s expected flow utility in the day is equal to
\[ \eta[u(x) - x] \]. Night output is equal to the consumption of private and public goods and so, we can use the night resource constraint to write expected night labor as \((c + g)/\zeta\). The ex ante period utility of an agent can be thus written in terms of the bundle \((x, c, g)\) and the aggregate state of the economy \(s\). Let \(U(x, c, g, s) \equiv \eta[u(x) - \phi x] + \gamma U(c) - (\alpha/\zeta)(c + g) + v(g)\).

As described in the introduction, the analysis presumes the government is in general not benevolent. Following Martin (2015), suppose the government values the utility of its subjects, but may value public expenditure differently: its flow utility is given by \(U(x, c, g, s) + R(g, \omega)\), where \(R\) is increasing in public expenditure, \(g\) and decreasing in the level of government benevolence, \(\omega > 0\). Let \(R(g, 1) = 0\), so that \(\omega = 1\) indicates the government is benevolent. When \(\omega \in (0, 1)\), which is the focus here, the government prefers larger public expenditure than private agents. This expenditure bias may arise from a variety of sources: a desire for empire-building, the spoils of patronage and clientelism, the existence of a self-serving public bureaucracy or the support of the sovereign’s lifestyle. Critical to the analysis below is that private agents would prefer the government to spend less, but cannot directly control nor limit this choice. Note that the presence of \(U(x, c, g, s)\) in the flow utility of the government implies it cares about how the policy-mix is perceived by private agents. In sum, the government spends too much but internalizes (imperfectly) the implementation costs for private agents.

The following assumption ensures the problem of a non-benevolent government is well-behaved. The requirement is that the problem is strictly concave in government expenditure.

**Assumption 1** Let \(\hat{g}(s)\) solve \(v_g - \alpha/\zeta + R_g = 0\) for all \(s \in S\) and let \(\hat{g} = \max_s \hat{g}(s)\). Then, \(v(g)\) and \(R(g, \omega)\) satisfy \(v_{gg} + R_{gg} < 0\) for all \(g \in [0, \hat{g}]\) and all \(\omega \in (0, 1)\).

Let \(\Gamma \equiv [\underline{B}, \overline{B}] \) be the set of possible debt levels, where \(\underline{B} < \overline{B}\) are such that they do not constrain government choices. Taking as given future government policy \(\{B, X, C, G\}\) the problem of the current government can be written as

\[
\max_{B', x, c, g} U(x, c, g, s) + R(g, \omega) + \beta E[V(B', s')|s]
\]

subject to (7) and given a continuation value consistent with expected future policy:

\[
V(B', s') \equiv U(\mathcal{X}(B', s'), C(B', s'), G(B', s'), s') + R(\mathcal{G}(B', s'), \omega') + \beta E[V(B', s', s'')|s'].
\]

We now have the necessary elements to define an equilibrium in this economy.

**Definition 1** A Markov-Perfect Monetary Equilibrium (MPME) is a set of functions \(\{B, X, C, G, V\} : \Gamma \times S \rightarrow \Gamma \times \mathbb{R}_+ \times \mathbb{R}\), such that for all \(B \in \Gamma\) and all \(s \in S\):

\[
\{B(B, s), X(B, s), C(B, s), G(B, s), V(B, s)\} = \arg\max_{B', x, c, g} U(x, c, g, s) + R(g, \omega) + \beta E[V(B', s')|s]
\]

subject to

\[
(\gamma U_c - \alpha/\zeta)c - (\alpha/\zeta)g - \phi x(1 + B)/1 + \theta B + \beta E\left[\phi x'(1 + B')/1 + \theta B'\right] |s| + \beta E[x'(u_x - \phi)]|s| = 0
\]

where \(x' \equiv X(B', s')\) and

\[
V(B, s) \equiv U(X(B, s), C(B, s), G(B, s), s) + R(G(B, s), \omega) + \beta E[V(B(B, s), s')|s].
\]

A Markov-perfect equilibrium is a fixed-point in government policy functions, so that the best response of the current government is follow the same policies it expects to follow in the future, in all states of the economy.
With Lagrange multiplier $\lambda$ associated with the government budget constraint and multiplier function $\Lambda(B, s)$ associated with future policy $\{B, X, C, G\}$, the first-order conditions of the government’s problem imply:

$$E\left[\phi x'(1 - \theta')(\lambda - \Lambda')\right] + \lambda E\left[\mathcal{B}_{X} \left\{\eta(u_x + u_xx' - \phi) + \phi(1 + B')\right\} \big| s\right] = 0 \quad (8)$$

$$\eta(u_x - \phi) - \frac{\lambda \phi(1 + B)}{1 + \theta B} = 0 \quad (9)$$

$$\gamma U_c - \alpha/\zeta + \lambda \{\gamma U_c - \alpha/\zeta + \gamma U_{cc}\} = 0 \quad (10)$$

$$v_g - \alpha/\zeta + R_g - \lambda(\alpha/\zeta) = 0 \quad (11)$$

for all $B \in \Gamma$ and all $s \in S$. A differentiable MPME is a set of differentiable (a.e.) functions $\{B, X, C, G, \Lambda\}$ that solve (7)–(11) for all $(B, s)$. Martin (2011) provides an extended analysis of these conditions and a characterization of the equilibrium, for the case of $\theta = 0$ and without aggregate shocks. Below, I describe the policy trade-offs implied by these conditions.

Conditions (9)–(11) describe the static trade-offs faced by the government when choosing the money growth rate, taxes and public expenditure. Each one of these policy instruments can be used to relax the government budget constraint at the cost of introducing a wedge, which lowers utility for the government (and private agents as well). Importantly, the incentives to inflate are increasing in debt and non-benevolence affects the amount of distortions the government is willing to impose.

Equation (8), known as a Generalized Euler Equation (GEE), describes the intertemporal trade-offs faced by the government when choosing debt. The first term depends on the difference between current and future implementation costs, as reflected by the multiplier on (7), capturing the distortion-smoothing role of debt. From an ex ante perspective, this gap would ideally be eliminated in expectation, but this is prevented by the limited commitment friction.

The second term in (8) reflects the time-consistency problem, which consists of how current changes in debt trigger future changes in policy, which in turn, affect the current budget constraint of the government. Choosing a higher debt implies higher inflation tomorrow, which affects the demand for money and bonds today. The impact on the latter is always negative: higher inflation implies higher nominal interest rates; the former depends on how the income and substitution effects determine how the current demand for money is affected by future higher inflation. When income effects dominate, the overall effect of higher debt is to relax the government budget constraint at low level of debt and to tighten it for high levels of debt.

As shown in Martin (2011, 2015) the non-stochastic version of this economy features the property that the steady state of the Markov-perfect equilibrium is constrained-efficient. Thus, endowing the government with commitment at the steady state would not affect the allocation. The result is summarized in the following proposition—details are provided in Appendix B.

**Proposition 1** Let $S = \{s^*\}$ and assume initial debt $B^* = B(B^*, s^*)$, which solves (41). Then, a government with commitment and a government without commitment will both implement the allocation $\{x^*, c^*, g^*\}$ and choose debt level $B^*$ in every period.

In the absence of aggregate fluctuations, private agents cannot be made better-off at the steady state by endowing the government with more commitment power. The only long-run inefficiency in this economy stems from the political friction, i.e., the misalignment in preferences between private agents and government. Outside the steady state or in the presence of aggregate fluctuations, government policy will exhibit inefficiencies due to both a time-consistency problem and the political friction.
4 Constrained Government Policy

Though private agents cannot dictate the government how much to spend, they may be able to regulate other components of the budget. In order to place constraints on government policy we first need to define some relevant macroeconomic variables: GDP, actual and expected inflation, the nominal interest rate, government expenditure and revenue, primary and total deficits, and debt over GDP.

4.1 Defining macroeconomic variables

Let us start by computing nominal GDP, a variable that is used to scale macroeconomic aggregates. Day and night output are equal to $\eta x_t$ and $c_t + g_t$, respectively. Then, nominal output is defined as $Y_t = p_{x,t}\eta x_t + p_{c,t}(c_t + g_t)$, which using (2) and (3) implies

$$Y_t = \frac{(1 + \theta_t B_t)[\eta \phi x_t + \gamma_t U_{c,t}(c_t + g_t)]}{\phi x_t}. \quad (12)$$

Nominal GDP, like other nominal variables, is normalized by the aggregate money stock.

Next, let us define monetary variables, like prices, inflation and interest rates. Let $\zeta_{c,t}$ and $\zeta_{c,t}$ be the day-good and night-good expenditure shares, respectively. That is, $\zeta_{c,t} = p_{x,t}(c_t + g_t)/Y_t$ and $\zeta_{c,t} = p_{c,t}(c_t + g_t)/Y_t$. Let $Y_t = \eta \phi x_t[\gamma_t U_{c,t}(c_t + g_t)]^{-1}$ and so, $\zeta_{c,t} = (1 + 1/Y_t)^{-1}$ and $\zeta_{c,t} = (1 + Y_t)^{-1}$. Fixing expenditure shares to those corresponding to the non-stochastic steady state $(\beta^*, \phi^*, \epsilon^*, g^*)$, yields $\zeta_{c,t}$ and $\zeta_{c,t}$. The price level can then be defined as: $P_t = \zeta_{c,t} p_{x,t} + \zeta_{c,t} p_{c,t}$. Using (2) and (3) we obtain

$$P_t = \frac{(1 + \theta_t B_t)(\zeta_{c,t} + \gamma_t^* U_{c,t})}{\phi x_t}. \quad (13)$$

Note that using (12) and (13) we get a formula for real GDP, $Y_t/P_t$, which is a function of $(x_t, c_t, g_t)$ and the realization of $\gamma_t$.

Next, consider fiscal variables. Government expenditure (excluding interest payments) and revenue can be both expressed in terms of GDP as $p_{c,t}g_t/Y_t$ and $\rho_t = p_{c,t} \tau_t (c_t + g_t)/Y_t$, respectively. In particular, using (3), (6) and (12) revenue can be expressed as

$$\rho_t = \frac{(\gamma_t U_{c,t} - \alpha/\zeta_t)(c_t + g_t)}{\eta \phi x_t + \gamma_t U_{c,t}(c_t + g_t)}. \quad (16)$$

The primary deficit is the difference between government expenditure before interest payments and tax revenue. The primary deficit over GDP is then defined as: $d_t = p_{c,t} [g_t - \tau_t (c_t + g_t)]/Y_t$. Using (3), (6) and (12) we obtain

$$d_t = \frac{(\alpha/\zeta_t)(c_t + g_t) - \gamma_t U_{c,t} c_t}{\eta \phi x_t + \gamma_t U_{c,t}(c_t + g_t)}. \quad (17)$$
The total fiscal deficit includes the primary deficit plus interest payments on the debt. The deficit over GDP is defined as:

\[ D_t \equiv d_t + (1+\mu_t)(1-q_t)B_{t+1}. \]

Using (4), (5), (12) and (17) we get

\[ D_t = \left( \frac{\alpha/\zeta_t}{\eta} \right) (c_t + g_t) - \gamma_t U_{c,t} c_t + \beta_t \eta B_{t+1} E_t \left[ \frac{(1-\theta_{t+1})x_{t+1}(u_{x,t+1})(1-\phi)}{(1+\theta_{t+1}B_{t+1})} \right]. \] (18)

Debt is measured at the end of the period, as in the data. Thus, debt-over-GDP is defined as

\[ (1 + \mu_t)B_{t+1}/Y_t \leq \bar{b} \]

4.2 Policy rules

The constraints on government behavior or *policy rules* studied in this paper can be categorized in three broad groups, depending on which type of policy variable they target: revenue, budget balance and debt. As mentioned above, I will also consider monetary rules in Section 7.

Revenue rules constraint tax revenue. The specific rule used here states that revenue over GDP cannot exceed a legislated ceiling: \( \rho_t \leq \bar{\rho} \), where \( \rho_t \) is given by (16). Note that revenue rules are not strictly the same as limits on tax rates. The latter would simply target \( \tau_t \) which by (6) is equivalent to determining night-good consumption, \( c_t \). However, it turns out that the quantitative performance of either type of constraint is very similar. 7

Budget balance rules are constraints to the primary or total deficit. Consider ceilings on the primary deficit \( \bar{d} \) and the total deficit \( \bar{D} \), both in terms of GDP, which take the form \( d_t \leq \bar{d} \), \( D_t \leq \bar{D} \), respectively, where \( d_t \) and \( D_t \) are as defined in (17) and (18). Importantly, budget balance rules do not directly constraint expenditure, only the revenue shortfall. In other words, the governments is free to choose how it will best achieve the required budget balance and may not do so in the way preferred by private agents.

There are two commonly-used types of debt constraints: an upper limit on debt over GDP and a ceiling on the nominal value of outstanding debt. That is, constraints of the form:

\[ (1 + \mu_t)B_{t+1}/Y_t \leq \bar{b} \]

Constraints can be imposed on all exogenous states of the world or on select ones. For example, it may be undesirable to restrict government behavior when output is low (say, requiring a surplus during a recession). However, this may be precisely the time when government behavior ought to be restricted. I will consider all these possible cases in the analysis below.

4.3 Recursive formulation

Let us now include the policy constraints in the recursive formulation of the government’s problem. The indicator function \( I_j \) states whether a constraint of type \( j = 1, \ldots, J \) is currently in effect. Hence, the exogenous aggregate state now includes the possibility of policy rules being in effect: \( s = \{ \gamma, \zeta, \theta, \omega, I_1, \ldots, I_J \} \); the state space \( S \) is correspondingly augmented. In general, whether a constraint will be in effect or not may depend on current or future states. For example, a policy rule may be implemented with some probability in future periods and remain in effect from the on, or be abandoned with some probability; or a policy rule may be in

\[ \text{Details are available upon request.} \]
effect unless there is an “adverse” shock. I will consider several specific cases in the quantitative section of the paper.

Constraints on policy can be written then as

\[ \psi_j - I_j \times \Psi \equiv \{ \bar{\rho}, \bar{d}, \bar{D}, \bar{b}, \bar{B} \} \]

(20)

The function \( \Psi \) corresponds to each of the constraints described above, given future anticipated policy function \( \mathcal{X}(B', s') \), which determines the future day-good allocation \( x' \) as a function of the aggregate state. The parameter \( \psi_j \) is the \( j \)-th element of the constraint vector \( \psi \equiv \{ \bar{\rho}, \bar{d}, \bar{D}, \bar{b}, \bar{B} \} \) and corresponds to the value by which a particular policy variable is constrained (e.g., a primary deficit ceiling, \( \bar{d} \)). Specified in this way, all constraints depend on the current policy choice \( (B', x, c, g) \), the current state \( s \) and expected future policy choices \( \mathcal{X}(B', s') \), for all \( B' \in \Gamma \) and all \( s' \in S \). However, some constraints depend trivially on some of the arguments. For example, a primary deficit ceiling only depends non-trivially on \( (x, c, g) \) and \( s \), while a nominal debt ceiling only depends non-trivially on \( B' \). Note that none of the policy constraints depend directly on the inherited level of debt, \( B \). However, they all interact with inherited debt through the budget constraint, (7).

The problem of the government can be written similarly to the unconstrained case, with the addition of (20) for all \( j = 1, \ldots, J \). Similarly, the definition of a MPME now includes (20) for all \( j = 1, \ldots, J, B \in \Gamma \) and \( s \in S \).

### 4.4 Static vs dynamic constraints

Policy rules differ in how they restrict government actions. Compare, for example, primary and total deficit ceilings: as we can see from (17) and (18), both constraints depend on \{\( x, c, g, s \)\}, but the deficit ceiling also depends on the choice of debt, \( B' \) and the expected realization of future allocations, \( \mathcal{X}(B', s') \) for all \( s' \in S \). Thus, the deficit ceiling is intrinsically dynamic while the primary deficit ceiling is static.

Primary deficit and revenue ceilings are the only rules in the set of policy constraints considered here that do not depend on \( B' \), either directly or indirectly through future policy \( \mathcal{X} \). Why is this important? Lack of dependence on \( B' \) implies that the Generalized Euler Equation (8) is left functionally intact after adding constraint (20) to the government’s problem. This is not the case for any of the other policy constraints. Condition (8) determines how the government is trading off distortion smoothing—the first term in (8)—with the time-consistency problem—the second term in (8). If (20) depends non-trivially on \( B' \), then the trade-off in (8) will be upset. In other words, the imposition of a policy constraint, though beneficial as a disciplining device, will hinder the government’s ability to smooth distortions intertemporally, which may impose a significant welfare cost to agents.

Ceilings on the primary deficit or revenue only affect equations (9)–(11). The added term in each condition is the Lagrange multiplier associated with constraint (20) times the marginal change of the constraint with respect to the relevant variable \( (x, c, \text{ or } g, \text{ respectively}) \). Hence, this type of constraint will alter the way the government views the intratemporal trade-offs in monetary policy, taxation and expenditure, without affecting the dynamic policy trade-offs in (8). Hence, these static constraints do not introduce a time-consistency problem. As such, a version of Proposition 1 holds with the addition of a primary deficit or revenue ceiling, which implies that the constrained steady state can be solved locally.
5 Optimal fiscal rules in non-stochastic economies

I first explore the optimality of policy constraints in the absence of shocks. It is important to note that the lessons learned here will carry over to the stochastic case.

5.1 Calibration

Consider the following functional forms: $u(x) = x^{1-\sigma} - 1$; $U(c) = c^{1-\sigma} - 1$; $v(g) = \ln g$; and $R(g, \omega) = (\omega^{-1} - 1)g$. The parameter $\omega > 0$ determines the degree of benevolence of the government, where $\omega = 1$ means the government is fully benevolent.

As argued in Section 3, the steady state of the non-stochastic version of the economy with a discretionary government can be solved locally, i.e., without first having to solve globally for the Markov-perfect equilibrium—see Appendix B for the relevant set of equations. This is a common property in this class of models and allows us to solve for the discretionary steady state with high numerical precision. In contrast, the economies with policy rules and/or aggregate shocks are solved globally, as detailed below.

The steady state of the economy with a discretionary government, i.e., unconstrained by policy rules, is calibrated to the post-war, pre-Great Recession U.S., 1955–2008. Government in the model corresponds to the federal government and period length is set to a fiscal year. The variables targeted in the calibration are: debt over GDP, inflation, nominal interest rate, outlays (not including interest payments) over GDP and revenues over GDP. All variables are taken from the Congressional Budget Office. Government debt is defined as debt held by the public, excluding holdings by the Federal Reserve system.

Calibrating the extent of political frictions poses a challenge. In principle, one would like to have an estimate of the socially optimal level of government expenditure. Such an estimate is of course hard, if not impossible, to come by. Instead, I use an indirect approach, exploiting the property that policy distortions (and therefore, taxes and inflation) are decreasing in benevolence—see (9)–(11). Specifically, I assume that a benevolent government would set the long-run inflation rate at 2% annual, which corresponds to the explicit target adopted by the Federal Reserve since 2012 (and implicitly before then) and by most inflation-targeting central banks around the world. Thus, the set of calibrated parameters need to hit two economies simultaneously: one targeting the actual U.S. economy and another one which shares all the same parameter values, except for $\omega = 1$, and that implements 2% inflation in steady state. Parameters values are chosen to match calibration targets exactly—see Tables 1 and 2.

Expenditure over GDP in the benevolent economy is about 3 percentage points lower than in the benchmark economy: 14.8% vs 18.0%. The size of the benevolent government would thus be similar to the actual U.S. federal government from the mid-1950s to the mid-1960s, before its permanent expansion with the introduction of the “Great Society” programs. Note also that the benevolent economy has a zero deficit in steady state, which would be an alternative benchmark for identifying the preferences of private agents that subscribe to the view that the government should “live within its means”. Table 2 presents both targeted and non-targeted moments, as a reference for the exercises conducted below.

For robustness, I also consider two alternative calibrations: one with a bigger government and one with lower inflation. The former assumes a less benevolent government and implies expenditure over GDP rises to 21%, i.e., 3 percentage points higher than in the benchmark economy. The latter redoes the calibration exercise described above, assuming inflation is 2%

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8Although there is a legislated debt ceiling, I am presuming that it has been largely irrelevant for curtailing the average amount of public debt sustained over the calibrated period.
Table 1: Benchmark calibration and alternative parameterizations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Benevolent</th>
<th>Big Government</th>
<th>Low Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>8.9790</td>
<td>8.9790</td>
<td>8.9790</td>
<td>7.4042</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9452</td>
<td>0.9452</td>
<td>0.9452</td>
<td>0.9615</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>3.7009</td>
<td>3.7009</td>
<td>3.7009</td>
<td>5.2000</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.3776</td>
<td>0.3776</td>
<td>0.3776</td>
<td>0.2432</td>
</tr>
<tr>
<td>$\phi$</td>
<td>3.7617</td>
<td>3.7617</td>
<td>3.7617</td>
<td>4.8780</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.3747</td>
<td>0.3747</td>
<td>0.3747</td>
<td>0.3289</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.3400</td>
<td>1.0000</td>
<td>0.2350</td>
<td>0.3900</td>
</tr>
</tbody>
</table>

Normalized parameters: $\gamma = \zeta = 1$.

Table 2: Non-stochastic steady state statistics for fully discretionary economies

<table>
<thead>
<tr>
<th>Variable</th>
<th>Statistic</th>
<th>Benchmark</th>
<th>Benevolent</th>
<th>Big Government</th>
<th>Low Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Targeted</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt over GDP</td>
<td>$\frac{B(1+\omega)}{\gamma}$</td>
<td>0.325</td>
<td>0.319</td>
<td>0.330</td>
<td>0.325</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>$\pi$</td>
<td>0.036</td>
<td>0.020</td>
<td>0.052</td>
<td>0.020</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>$i$</td>
<td>0.058</td>
<td>0.048</td>
<td>0.068</td>
<td>0.040</td>
</tr>
<tr>
<td>Revenue over GDP</td>
<td>$\rho$</td>
<td>0.180</td>
<td>0.152</td>
<td>0.206</td>
<td>0.180</td>
</tr>
<tr>
<td>Expenditure over GDP</td>
<td>$\frac{p_cg}{\bar{Y}}$</td>
<td>0.180</td>
<td>0.148</td>
<td>0.210</td>
<td>0.180</td>
</tr>
<tr>
<td><strong>Non-targeted</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal debt</td>
<td>$B$</td>
<td>1.733</td>
<td>1.542</td>
<td>1.945</td>
<td>1.040</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>$r$</td>
<td>0.021</td>
<td>0.028</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>Primary deficit over GDP</td>
<td>$d$</td>
<td>0.000</td>
<td>-0.004</td>
<td>0.004</td>
<td>0.000</td>
</tr>
<tr>
<td>Deficit over GDP</td>
<td>$D$</td>
<td>0.018</td>
<td>0.001</td>
<td>0.025</td>
<td>0.013</td>
</tr>
</tbody>
</table>

and 1% annual, for the U.S. and the counterfactual benevolent economy, respectively. I provide further details below.

5.2 Global numerical solution

The next step is to solve globally for the equilibria of the various economies studied here. Let $\Gamma = [B, \bar{B}]$ be the debt state space, where $B < B^* < \bar{B}$; $B^*$ is the steady state in the discretionary equilibrium and we can normally set $\bar{B} = 0$. Define a grid of $N_\Gamma$ points over $\Gamma$; I set $N_\Gamma = 10$, which appears to work best for this type of problem. Since many calculations will be conducted at the discretionary steady state, $B^*$, it is important that $B^*$ itself is a point in the grid. This will allow for more accurate welfare computations.

Next, guess the functions $B(B)$, $X(B)$, $C(B)$, and $G(B)$, all evaluated at the $N_\Gamma$ grid points. When necessary, I use cubic splines to interpolate between debt grid points and calculate the derivatives of policy functions.

To solve the Markov-perfect equilibrium when the government is discretionary, I use a non-linear equations solver to solve the system given by the non-stochastic versions of equations

Due to coexistence of differentiable and non-differentiable Markov-perfect equilibria, too many grid points may fail to find the differentiable equilibrium, which is the focus here. See Martin (2009) for further explanation.
Condition (11) can be used to solve for the multiplier function $\Lambda(B)$, so that leaves $N_F \times 4 = 40$ equations. The unknowns are the values of the policy functions $\{B, X, C, G\}$ at the grid points. In each step of the solver, the corresponding cubic splines that characterize equilibrium functions need to be updated so that the interpolated evaluations of future choices are consistent with each new guess. The precision of the solution using this algorithm is high. For the benchmark economy, the sum of square residuals of the government budget constraint, evaluated at a 1,000 grid points in $\Gamma$ is in the order of $10^{-7}$; the same calculation but performed on the total derivative of the government budget constraint with respect to $B$ (which is exactly zero in theory) is in the order of $10^{-5}$.

Computing the solution of economies with policy rules is more difficult. Beforehand, we do not know when a policy rule, which is an inequality constraint, will bind. It may be the case that a policy rule binds for all $B \in \Gamma$ or for only a specific region of the state space (or not at all). When policy rules bind only in certain regions, there may be a kink in some policy functions; this can be problematic if the kink is in $X$, whose derivative which shows up in the Generalized Euler Equation (8).

To solve economies with policy constraints, I use constrained maximization numerical algorithms, using the unconstrained (discretionary) equilibrium as the initial guess. This method is not as precise as using first-order conditions, but gets around the problem of occasionally binding policy rules. Whenever possible, as with revenue and debt ceilings, I further solve the model using the first-order conditions to the (constrained) government’s problem to increase the accuracy of the numerical solution. The algorithm is similar to the discretionary case, with the addition on one equation (the policy rule) and a corresponding Lagrange multiplier function.

5.3 Optimal policy constraints

Policy rules are imposed for all debt levels. The optimal value for a constraint is picked by evaluating welfare at the steady state of the non-stochastic fully discretionary economy and includes the full transition to the new steady state.

Welfare is expressed in terms of equivalent compensation, measured in units of night-good consumption. Formally, for each policy constraint $\Psi^j$ and associated constraint parameter $\psi_j$, $j = \{1, \ldots, J\}$, welfare is measured at each level of debt as the proportion $\Delta^j(B)$ that solves

$$\eta[u(\mathcal{X}(B)) - \phi \mathcal{X}(B)] + U(C(B)(1+\Delta^j(B, \psi_j)))+v(G(B)) - \alpha(C(B)+G(B)) + \beta V(B(B)) = \bar{V}^j(B; \psi_j)$$

where $\{B, \mathcal{X}, C, G\}$ is the fully discretionary non-stochastic Markov-perfect equilibrium, with associated agent’s value function $V(B)$, and $\bar{V}^j(B; \psi_j)$ corresponds to the agent’s value function in a Markov-perfect equilibrium associated with policy constraint $j$ and constrain parameter $\psi_j$. Given the assumptions on functional forms, the equivalent compensation has a closed-form solution:

$$\Delta^j(B, \psi_j) = \begin{cases} \frac{(1 - \sigma)[\bar{V}^j(B; \psi_j) - V(B)]}{C(B)^{1-\sigma}} + 1 \right)^{1/(1-\sigma)} - 1 \quad & \text{if } \sigma \neq 1 \\ \exp\{\bar{V}^j(B; \psi_j) - V(B)\} - 1 \quad & \text{if } \sigma = 1. \end{cases}$$

As mentioned above, the optimal value $\psi_j$ for constraint $j$ solves

$$\max_{\psi_j} \Delta^j(B^*, \psi_j).$$

Recall that the fully discretionary steady state is constrained-efficient, so that welfare gains from imposing policy constraints may come from two sources: mitigating the government’s expenditure bias, i.e., “starving the beast”; and altering the policy mix, i.e., changing which
Since we are dealing with second-best outcomes, it is not a priori obvious which of the two sources will matter the most, if at all.

As a reference, Table 3 presents the welfare gains to private agents from making the government fully benevolent, evaluated at the discretionary, non-benevolent steady state (which, of course, differs across parameterizations). The potential welfare gains from institutional reform are large in all cases considered. For the benchmark calibration, the welfare gains derived from making the government benevolent are equivalent to 10% of consumption; these gains are 76% for the “big government” case and 17% for the “low inflation” calibration.

Table 3: Welfare gains from government becoming benevolent

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Big Government</th>
<th>Low Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
<td>76%</td>
<td>17%</td>
</tr>
</tbody>
</table>

Table 4 presents the effects of policy rules, each evaluated at their corresponding optimal value. There are three sets of information on display. The first set shows the optimal values for each policy constraint. The second set shows the associated welfare gains, measured at $B^*$; these gains are further decomposed into “steady state” gains, i.e., assuming an immediate jump into the new steady state, and “transition” gains, which measure the complement. The third set shows steady state statistics under each policy constraint. Figure 1 complements this information by providing the welfare gains, $\Delta_j(B^*, \psi_j)$, for each constraint $j$, associated with various values for the constraint parameter $\psi_j$, starting below the optimal value and stopping where the rule would no longer bind (i.e., at the value equal to the discretionary steady state).

All types of constraints can be effective at increasing private agents’ welfare: gains span from a maximum of 2% of consumption for the case of a revenue ceiling to a minimum of 0.02% for the case of a debt over GDP limit. The optimal revenue ceiling is about 15% of GDP, i.e., three percentage points lower than in the unconstrained economy. The next-best constraint, a ceiling on the primary deficit of about half a percent of GDP, yields welfare gains an order of magnitude lower than the optimal constraint on revenue. As analyzed in section 4.4, limits on the primary deficit and revenue differ from other constraints, in that they do not interact with the future level of debt; essentially, they do not distort dynamic incentives, i.e., the ability of the government to smooth distortions over time. Unlike other constraints, the costs of primary deficit and revenue ceilings are all along intratemporal margins.

Figure 1 shows that policy rules are still beneficial for large intervals around their optimal value. For example, any revenue ceiling above 12% and below 18% of GDP yields positive welfare gains for private agents. However, constraints can be detrimental when they are set too tight, potentially leading to large welfare losses. Debt constraints are special in that they have multiple local maxima. Lower debt forces lower distortions and policies more aligned with the preferences of private agents, but they imply larger welfare losses due to the sudden adjustment. This trade-off is not monotone in debt.

Table 4 also shows the steady state statistics of constrained economies, which can be compared to the benchmark economy (see Table 2). First, only the revenue ceiling manages to curb government spending, and then somewhat modestly: steady state $g$ drops by about 5%.

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10 Although the phrase “starving the beast” may more commonly be used for limiting spending by cutting taxes, I use it here in a more general sense to include any mechanism that constraints government actions to curb spending.

11 Notably, almost identical gains can be obtained by placing a suitable constraint on government expenditure over GDP. The working assumption is that it is not feasible (or it is too costly) to make the government benevolent or to impose constraints on expenditure.
## Table 4: Optimal fiscal rules and steady state statistics—benchmark calibration

<table>
<thead>
<tr>
<th>Variable / Constraint</th>
<th>Revenue ceiling $\bar{\rho}$</th>
<th>P. deficit ceiling $\bar{d}$</th>
<th>Deficit ceiling $\bar{D}$</th>
<th>Debt over GDP limit $\bar{b}$</th>
<th>Debt ceiling $\bar{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal constraint</td>
<td>0.151</td>
<td>-0.007</td>
<td>0.005</td>
<td>0.272</td>
<td>1.212</td>
</tr>
<tr>
<td>Welfare gains</td>
<td>2.00%</td>
<td>0.11%</td>
<td>0.07%</td>
<td>0.02%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Steady State</td>
<td>1.29%</td>
<td>0.26%</td>
<td>0.73%</td>
<td>0.28%</td>
<td>0.36%</td>
</tr>
<tr>
<td>Transition</td>
<td>0.71%</td>
<td>-0.15%</td>
<td>-0.66%</td>
<td>-0.26%</td>
<td>-0.33%</td>
</tr>
</tbody>
</table>

### Steady state statistics

<table>
<thead>
<tr>
<th></th>
<th>Debt over GDP</th>
<th>Inflation rate</th>
<th>Nominal interest rate</th>
<th>Revenue over GDP</th>
<th>Expenditure over GDP</th>
<th>Nominal debt</th>
<th>Real interest rate</th>
<th>Primary deficit over GDP</th>
<th>Deficit over GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt over GDP</td>
<td>0.477</td>
<td>0.119</td>
<td>0.108</td>
<td>0.151</td>
<td>0.172</td>
<td>3.089</td>
<td>-0.010</td>
<td>0.021</td>
<td>0.067</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.009</td>
<td>0.041</td>
<td>0.043</td>
<td>0.186</td>
<td>0.180</td>
<td>1.414</td>
<td>0.032</td>
<td>-0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.012</td>
<td>0.043</td>
<td>0.054</td>
<td>0.182</td>
<td>0.180</td>
<td>0.681</td>
<td>0.031</td>
<td>-0.002</td>
<td>0.005</td>
</tr>
<tr>
<td>Revenue over GDP</td>
<td>0.268</td>
<td>0.030</td>
<td>0.054</td>
<td>0.180</td>
<td>0.180</td>
<td>1.288</td>
<td>0.024</td>
<td>0.000</td>
<td>0.014</td>
</tr>
<tr>
<td>Expenditure over GDP</td>
<td>0.247</td>
<td>0.027</td>
<td>0.053</td>
<td>0.180</td>
<td>0.180</td>
<td>1.144</td>
<td>0.025</td>
<td>0.000</td>
<td>0.012</td>
</tr>
</tbody>
</table>

### Figure 1: Welfare gains over full discretion, $\Delta^j(B^*, \psi_j)$

or almost one percentage point in terms of GDP. The next effective rule in this regard, the primary deficit ceiling, lowers steady state $g$ by only 0.03%; just 2 basis points when measured in terms of GDP.\(^{12}\) The superior welfare performance of the revenue ceiling is explained by its effectiveness in lowering spending. About two-thirds of the gain stem from better steady state

\(^{12}\)Since public expenditure in the model does not vary significantly with debt, these steady state comparisons are a good approximation to what is going on globally.
policy and one-third from gains along the transition to the new steady state. Note, however, that the new steady state with lower revenue features higher debt and higher inflation; this is the way that the government avoids lowering expenditure too much, given the revenue limit.

For all constraints other than the revenue ceiling, the welfare gains from imposing policy rules derive entirely from affecting the policy-mix and not at all from mitigating the expenditure bias. In all these other cases, steady state gains are traded-off with transition losses. In contrast to the revenue ceiling, all other rules yield lower inflation and debt. For example, the primary deficit ceiling forces tax rates to go up (lower $c$), which allows inflation to go down (higher $x$); the change in policy-mix results in lower steady state debt. Some constraints may lead to dramatic changes in long-run policy. For example, a deficit ceiling of half a percent of output leads to a severe contraction in debt over GDP, about half of the benchmark economy. Also note that the optimal debt over GDP limit and debt ceiling are not binding at the steady state—though the steady states themselves are affected by the fact that debt constraints would be binding if debt were increased.

5.4 Effectiveness and sustainability

Why does constraining revenue work while other policy rules fail? First, consider how the government can respond when revenue is lowered: it can raise current inflation, raise future inflation (by issuing debt) or lower spending. Table 4 (and the transitions analyzed below) show that the government chooses to do a little bit of each. Inflation however can only go so far in financing deficits before it becomes too costly. Lowering spending is a critical component of the government’s response and drives the resulting welfare gains. Second, consider the next-best rule, a primary deficit ceiling. In this case, the government can satisfy the constraint by raising revenue or lowering spending. It chooses the former, which in turn allows it to lower inflation to mitigate the costs of this action. Similarly, lowering spending is not necessary to satisfy the constraints imposed by other policy rules, which makes them ineffective in this regard.

The results derived above provide a useful roadmap for countries wishing to achieve certain policy goals, such as lower inflation or debt. For example, lower long-run debt can be achieved directly by placing a debt over GDP limit or a debt ceiling, but it is more desirable for private agents to achieve this goal through budget balance rules. The revenue ceiling offers the most effective way to curb excessive government spending. However, the government response to a revenue rule implies a deterioration of some relevant macroeconomic variables, such as elevated levels of public debt and inflation. This may pose political obstacles in its successful implementation.

The rise in debt and inflation due to the revenue ceiling can be cause for other concerns. Though in the model equilibrium levels of debt are always sustainable (i.e., the government can always repay its accumulated debt, one way or the other), the level of inflation necessary to sustain them may be interpreted as how sustainable the debt is. Higher inflation means a larger proportion of the debt gets “defaulted” on through inflation. One may also interpret higher inflation as a higher probability of outright default, particularly if the central bank is unwilling to accommodate the larger deficit.

5.5 Transitions

Figure 2 tracks the evolution of macroeconomic and policy variables following the imposition of the constraints analyzed above. Convergence to the new steady state is quite fast for debt rules, while slower for budget balance and revenue rules. In all cases, government expenditure over GDP has a flat time-profile after rules are imposed.
Figure 2: Transitional dynamics
The optimal revenue ceiling induces the government to run a large primary deficit. This implies a gradual accumulation of debt and an associated increase in inflation and interest rates. The bulk of the decline in spending happens on impact, the transition implying a slight further reduction. Real output declines gradually but permanently, due to the reduction in government spending. Note, however, that the reduction in output is lower than the reduction in spending (spending over output declines) since policy distortions are lower.

The effects of a ceiling on the primary deficit are felt immediately, with all variables jumping close to the new steady state after the rule is imposed. In contrast, a ceiling on the total deficit implies a protracted transition. On impact, the government runs a large primary surplus, gradually reducing it as debt declines. In both cases, inflation and interest rates are much lower than in the discretionary equilibrium.

Debt rules imply an immediate adjustment to the new limit by running primary surpluses and suffering a loss in output. The transition takes only a few periods, with the vast majority of adjustment occurring in the period of the reform.

5.6 Robustness I: timing of reform and austerity programs

The calculations for optimal constraints relied on them being implemented at the steady state. A potential concern is the fact that constraints could be imposed at inappropriate times. What happens when constraints are placed far from this state? In particular, how does the welfare derived from imposing the optimal values for each policy constraint depend on the level of debt at the moment of introduction? Figure 3 provides an answer to this question.

The optimal revenue ceiling remains significantly more beneficial than all other constraints, at all levels of initial debt. Gains increase for lower debt levels, but are still quite sizable at high initial debt levels. As such, a revenue ceiling in not only the most effective type of constraint, but also does not require that the imposition of the rule be implemented close to the steady state.

The optimal primary deficit target typically leads to fairly consistent welfare gains, even when initial debt is fairly high. The exception is when initial debt is low, as a cap on the primary deficit severely limits debt accumulation and thus, impinges on distortion-smoothing. On the other hand, the optimal deficit ceiling offers consistent welfare gains for all levels of debt. The difference stems from the fact that at low levels of debt, the constrained government

Figure 3: Welfare gains of optimal steady state constraints by initial debt level
can now run a primary deficit, since the interest paid on debt is low. Hence, a deficit ceiling, as opposed to a primary deficit ceiling, might be a better idea for governments with low initial debt.

In contrast to revenue and budget balance rules, both optimal debt constraints can lead to substantial welfare losses when initial debt is high. The reason for this is simple: the debt ceiling forces a sudden adjustment of debt, which goes against the desirability to smooth distortions. This results suggest that it is a bad idea to impose debt limits after events that required deficit-financing.

Austerity programs, such as those implemented by several European countries in recent times, are designed to reimpose discipline on governments, often with the aim of bringing the debt burden to more “sustainable” levels. We can use the results derived above to interpret the desirability of different austerity measures. Revenue and budget balance rules are beneficial when initial debt is (much) higher than the steady state and their imposition would be preferred to letting the economy slowly converge back to normal. The analysis suggests that targeting revenue is far more effective than targeting the deficit. Note that, in contrast to common practice, the prescription is not to raise revenue to generate a surplus and thus lower debt, but rather, to lower revenue to reduce expenditure, i.e., starve the beast.

5.7 Robustness II: big government

Consider now the case of an economy with an even less benevolent government. Table 2 shows the steady state statistics of an economy with \( \omega = 0.235 \). In this case, public expenditure over GDP is 21%, i.e., 3 percentage points higher than the calibrated economy and 6 percentage higher than the benevolent economy. As a result, inflation, deficits and debt are all higher.

Table 5: Optimal constraints—Alternative calibrations

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Benchmark</th>
<th>Big Government</th>
<th>Low Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ceiling</td>
<td>0.151</td>
<td>2.00%</td>
<td>0.130</td>
</tr>
<tr>
<td>Primary deficit ceiling</td>
<td>-0.007</td>
<td>0.11%</td>
<td>-0.014</td>
</tr>
<tr>
<td>Deficit ceiling</td>
<td>0.005</td>
<td>0.07%</td>
<td>-0.007</td>
</tr>
<tr>
<td>Debt over GDP limit</td>
<td>0.272</td>
<td>0.02%</td>
<td>0.275</td>
</tr>
<tr>
<td>Debt ceiling</td>
<td>1.212</td>
<td>0.03%</td>
<td>1.286</td>
</tr>
</tbody>
</table>

Table 5 presents the optimal constraints for this economy. When compared to the benchmark economy, the optimal constraints are all stricter when facing a less benevolent government, i.e., limits/ceiling are all lower. Since the potential gains are larger the less benevolent the government, the costs imposed on it through constraints are optimally larger. As a result, welfare gains for all types of constraints increase by about an order of magnitude. Notably, however, the welfare ranking of constraints remains the same; the best prescription is still a revenue ceiling, about 13% of GDP in this case, which is even lower than revenue in the benevolent economy (about 15% of GDP). Welfare gains due to imposing the optimal revenue ceiling are worth about 11% of consumption. Recall from Table 3 that potential gains for this calibration are 76% of consumption; thus, the gains from the optimal revenue ceiling, though larger in absolute terms, are relatively lower than in the benchmark case.
5.8 Robustness III: low inflation

Arguably, inflation is the one policy variable that in the last two decades looks significantly different from the postwar average. In order to account for this fact and thus deliver recommendations more applicable to the current economy, I consider an alternative calibration that delivers a steady state inflation of 2% annual. Parameters values and steady state statistics are presented in Table 2, along with the other economies studied above. The degree of government benevolence, $\omega$, is set so that with the “low inflation” parameterization, a benevolent government (one with $\omega = 1$) would have the same expenditure over GDP as with the benchmark parameterization, 14.8% of GDP. As we can see in Table 2, inflation and the nominal interest rate are lower, consistent with the new targets, but fiscal variables are the same as in the benchmark economy. The government in the low inflation economy is slightly more benevolent than in the benchmark economy, but not dramatically so.

The optimal constraints for the low inflation economy are presented in the last two columns of Table 5. The lessons from the benchmark economy apply to the low inflation economy: the best constraint is to impose a revenue ceiling (also 15.1% of GDP in this case). One notable difference between the benchmark and low inflation economies is that the optimal debt constraints are not as tight in the latter. This is due to the lower gains associated with enforcing a much lower debt level when inflation distortions are already smaller. In fact, the two peaks in welfare gains for debt constraints (see Figure 1) are nearly identical in the low inflation economy, with the global maximum shifting to the one closer to the discretionary steady state.

5.9 Expected reforms and implementation failure

So far, I have assumed that the imposition of policy rules is unanticipated and successful. Oftentimes, however, institutional reforms are planned in advance and not always successfully implemented. Let us then consider the effects of anticipation and failure.

The first exercise consists of assuming that, under discretion in the benchmark economy, there is a 10% probability each period that a given policy rule is implemented. After the reform is enacted, the policy rule remains in effect forever. In other words, agents expect a specific (and permanent) policy rule to be introduced in about 10 years. Table 6 summarizes the results of this exercise, labeled “expected”, assuming that the economy starts at the discretionary steady state. The results thus capture the welfare gains for private agents when the reform is announced.

It is immediately apparent that the values for the optimal constraints do not vary significantly between the “benchmark” and “expected” scenarios. That is, optimal policy rules are not altered by whether they are imposed now or at some point in the future. Welfare gains do vary. For the cases of revenue and budget balance (deficit) constraints, welfare gains for private agents drop significantly. For the case of the revenue ceiling, the loss associated with implementing the rule in the future instead of now is about one-third. This is because, in expectation, the economy remains under discretion for 10 more years and so, the benefits of rules are diluted. In contrast, debt rules benefit from not being imposed all of a sudden. This is due to distortion-smoothing: unanticipated debt rules suffer from a sudden adjustment of taxes, which severely limit their benefits; postponing their imposition allows for a smoother transition and thus, higher welfare gains.

A revenue ceiling remains the best type of policy constraint. If anything, the ratio between the benefits of a revenue ceiling and the next-best constraint increases when reforms are

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13Debt is currently also far from the postwar average, but did not look significantly different right before the most recent recession, which is not included in the target period for the calibration.
Table 6: Optimal constraints—The effects of anticipation

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Benchmark</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal Welfare</td>
<td>Optimal Welfare</td>
</tr>
<tr>
<td></td>
<td>Constraint Gain</td>
<td>Constraint Gain</td>
</tr>
<tr>
<td>Revenue ceiling</td>
<td>0.151 2.00%</td>
<td>0.147 1.32%</td>
</tr>
<tr>
<td>Primary deficit ceiling</td>
<td>−0.007 0.11%</td>
<td>−0.006 0.04%</td>
</tr>
<tr>
<td>Deficit ceiling</td>
<td>0.005 0.07%</td>
<td>0.006 0.04%</td>
</tr>
<tr>
<td>Debt over GDP limit</td>
<td>0.272 0.02%</td>
<td>0.272 0.03%</td>
</tr>
<tr>
<td>Debt ceiling</td>
<td>1.212 0.03%</td>
<td>1.180 0.04%</td>
</tr>
</tbody>
</table>

Note: “Benchmark” assumes the imposition of policy rules is unanticipated. “Expected” assumes policy rules are introduced with a 10% probability each period and remain in effect once enacted.

I next analyze the transitions associated with an expected reform. To this effect, I modify the previous exercise slightly: a reform is still expected to be attempted with 10% each period, but now it may only succeed with 50% chance. If the reform succeeds, the policy rule remains in effect afterwards; if it fails, the economy reverts to the discretionary equilibrium. Given the results above, I focus on the revenue ceiling.

Figure 4 shows several possible scenarios. In all cases, I use the optimal revenue ceiling from the benchmark economy and start the economy at the discretionary steady state. The first line shows the discretionary equilibrium for reference; the second line is the transition of the benchmark economy, with an unanticipated reform, similar to Figure 2; the third line is the transition associated with an expected reform that ultimately succeeds; the fourth line corresponds to an expected reform that ultimately fails. The reform is attempted in year 10 after the announcement. All agents understand that it has an equal chance of success and failure.

The transition charts indicate that the government prepares ahead of the potential imposition of a revenue ceiling by raising taxes and lowering debt. This also allows for a decline of inflation and interest rates. The lower debt level allows the governments to better deal with the event of being imposed a revenue ceiling; this can be seen by the fact that inflation does not jump as much as in the case of a sudden imposition of the rule. Thus, the transition towards an environment with a revenue ceiling features an improvement of government accounts: primary surpluses, lower inflation and lower debt. However, government expenditure over GDP does not change significantly (expenditure levels do drop slightly) and the economy suffers a mild recession due to the higher tax rates. Note that GDP always drops on announcement, regardless of whether the revenue ceiling is imposed at that moment or afterwards.

Clearly, the lower welfare gains associated with delaying the implementation of the revenue ceiling stem from the fact that public expenditure does not budge until the rule is actually imposed. However, as mentioned above, the government looks more disciplined before the rule is imposed since it is forearming for the possibility of a successful implementation. This provides a cautionary tale about interpreting the effectiveness of convergence criteria. Government accounts appear to improve as we move towards the imposition of rules and seem to deteriorate afterwards. However, it is only the successful implementation of the rule which provides with the actual benefit to private agents, the curbing of government expenditure.
6 Rules vs discretion in stochastic economies

The lessons derived above apply to economies without aggregate fluctuations. In this section, I will study the role of constraints on discretionary governments when the economy is subjected to a variety of (expected) aggregate shocks. Notably, the prescriptions from the non-stochastic case carry over to stochastic economies. However, studying aggregate fluctuations also allows us to derive new lessons; e.g., whether certain constraints should be suspended occasionally. I will thus focus these exercises on infrequent, big shocks, i.e., those when the temptation to abandon rules and apply discretion are arguably the greatest.

6.1 Parameterization of stochastic economies

The exogenous state of the economy is given by the values of parameters \( \{\gamma, \zeta, \theta, \omega\} \). To keep the analysis as transparent as possible and draw useful lessons, I consider economies with one type of shock at a time. Each economy has three exogenous states, \( S = \{s_1, s_2, s_3\} \). Let \( \pi_{ij} \) be the probability of going from state \( s_i \) today to state \( s_j \) tomorrow. I will interpret \( s_2 \) as “normal” times, similar to where the economy lies in the non-stochastic version of the economy. The state \( s_1 \) corresponds to “bad” times; \( s_3 \), or “good” times, is included for symmetry and so that the stochastic economy fluctuates around the calibrated non-stochastic steady state. The label “bad” refers to states of the world that feature what are generally deemed undesirable macroeconomic outcomes: low aggregate demand, high public expenditure, low average productivity and low real interest rate.

The transition matrix is characterized by two values \( \omega \) and \( \omega^* \) such that: \( \omega_{1,1} = \omega_{3,3} = \omega \); \( \omega_{1,2} = \omega_{3,1} = 1 - \omega \); \( \omega_{1,3} = \omega_{3,1} = 0 \); \( \omega_{2,2} = \omega^* \); and \( \omega_{2,1} = \omega_{2,3} = (1 - \omega^*)/2 \). In other words, \( \omega^* \) is the probability of remaining in the normal state of the world, with an equal chance of transitioning to bad times (\( s_1 \)) or good times (\( s_3 \)). During bad (good) times there is a chance \( 1 - \omega \) of transitioning back to normal times and it is not possible to immediately transition to
the good (bad) state.

For the numerical simulations, I will assume $\varpi^* = 0.98$ and $\varpi = 0.90$. That is, normal times last on average 50 years and bad (good) times have an expected duration of 10 years. This parameterization is meant to capture events such as protracted and deep recessions ($\gamma$), productivity slowdowns ($\zeta$), financial crises ($\theta$) and wars ($\omega$). That is, infrequent but painful events that strain the will to maintain rules and instead favor the adoption of more politically expedient (discretionary) policies. Such events are a good laboratories for testing whether it is a good idea to temporally suspend normally benign rules. I will also consider more frequent abnormal times, to verify the robustness of the results. For each economy, the corresponding parameter in states $s_1$ and $s_3$ is a multiple of the parameter in state $s_2$, which is equal to the calibrated parameter from Table 1. The parameterization is shown in Table 7.

Table 7: Stochastic economy parameterization

<table>
<thead>
<tr>
<th>Economy</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand shock</td>
<td>$\gamma(1 - \varrho_\gamma)$</td>
<td>$\gamma$</td>
<td>$\gamma(1 + \varrho_\gamma)$</td>
</tr>
<tr>
<td>Productivity shock</td>
<td>$\zeta(1 - \varrho_\zeta)$</td>
<td>$\zeta$</td>
<td>$\zeta(1 + \varrho_\zeta)$</td>
</tr>
<tr>
<td>Liquidity shock</td>
<td>$\theta(1 + \varrho_\theta)$</td>
<td>$\theta$</td>
<td>$\theta(1 - \varrho_\theta)$</td>
</tr>
<tr>
<td>Expenditure shock</td>
<td>$\omega(1 - \varrho_\omega)$</td>
<td>$\omega$</td>
<td>$\omega(1 + \varrho_\omega)$</td>
</tr>
</tbody>
</table>

For each type of shock and each type of constraint, I evaluate four scenarios: (i) constraints apply to all states of the world; (ii) constraints are suspended in the bad state $s_1$, and so only imposed in states $s_2$ and $s_3$; (iii) constraints are only imposed during normal times, i.e., state $s_2$; and (iv) constraints are suspended in the good state $s_3$, and so only imposed in states $s_1$ and $s_2$. For each case, the optimal constraints are calculated.

In all cases, welfare is evaluated as the equivalent compensation, in terms of night consumption, at the initial state ($B^*, s_2$), relative to the discretionary outcome. Welfare gains over full discretion are defined as the difference between welfare in a particular stochastic constrained case and the fully discretionary stochastic equilibrium.

6.2 Optimal policy constraints for demand shocks

As a benchmark case, consider an economy subjected to fluctuations in aggregate demand, i.e., with shocks to $\gamma$. To maintain focus, I will study this case exhaustively and afterwards, verify that the main results obtained for demand shocks also apply to other types of shocks. As a reference, Figure 5 shows the response of a fully discretionary government to a typical demand shock. The drop in output implies a drop in revenue; the government responds by running a primary deficit while output is below normal. This implies debt accumulation, which in turn implies an increase in inflation to partially pay for it. Inflation remains elevated after output returns to normal in order to smoothly bring back debt to pre-shock levels.

Table 8 summarizes the welfare effects of imposing constraints on policy in an economy facing demand shocks. The four right-most columns show the welfare effects of imposing, respectively, policy constraints: (i) always; (ii) in normal and good times (suspended in bad times); (iii) in normal times only; and (iv) in bad and normal times (suspended in good times). The best case is shown in bold. For each type of policy constraint, the column labeled “optimal value” shows the value that corresponds to the best case (the best values for the remaining cases are omitted to simplify exposition but can be seen on Figure 6).
Figure 5: Response of fully discretionary government to typical demand shock

There are several important observations coming out of Table 8. First, placing an upper limit on revenue improves welfare the most. The optimal value is to have a cap on revenue equal to 15.1% of GDP. Notably, this is the same result we obtained in the non-stochastic case. Second, for all types of constraints, the optimal values and welfare gains are remarkably similar to the non-stochastic benchmark economy. Part of the reason is the low recurrence of shocks. However, the shocks are assumed to be large and persistent; it does not follow a priori that infrequent shocks would matter so little for the optimal institutional prescription, both qualitatively and quantitatively. Third, for all types of constraints, most of the welfare gains come from imposing constraints in normal times and suspending constraints during abnormal (both bad and good) implies only small differences in welfare, sometimes positive other times negative. This suggests that welfare gains primordially stem from the reduction in government size and change in the long-run policy mix accomplished with the imposition of constraints, and not from inefficiencies due to how discretionary governments respond to shocks. In this case, trend considerations trump cyclical ones.

Figure 6 expands on the results summarized in Table 8. For each case, the figure plots the welfare gains associated with imposing a particular policy constraint at specific times. One property that pops up immediately is that, for each type of constraint, the optimal value is similar whether we allow the constraint to be sometimes suspended or not. As mentioned above, the welfare changes of temporarily suspending a constraint is minor relative to the overall welfare gains of imposing them in the first place. Both these results are significant for implementation, as there may be other reasons (say, political) for wanting to suspend constraint on government actions at certain times. Note, however, that these conclusions rely on the fact that constraints are to be reimposed when normal times come back.

All rules, with the exception of the debt over GDP limit (and then, only by an insignificant margin) are best imposed at all times. Fiscal rules do not hinder the government ability to
Table 8: Welfare gains over full discretion—demand shocks

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Optimal Value</th>
<th>Always in good times</th>
<th>Suspended in bad times</th>
<th>Only in normal times</th>
<th>Suspended in good times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ceiling</td>
<td>0.151</td>
<td>1.98%</td>
<td>1.80%</td>
<td>1.61%</td>
<td>1.79%</td>
</tr>
<tr>
<td>Primary deficit</td>
<td>0.006</td>
<td>0.11%</td>
<td>0.09%</td>
<td>0.08%</td>
<td>0.09%</td>
</tr>
<tr>
<td>Deficit ceiling</td>
<td>0.005</td>
<td>0.07%</td>
<td>0.07%</td>
<td>0.06%</td>
<td>0.06%</td>
</tr>
<tr>
<td>Debt over GPD limit</td>
<td>0.272</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Debt ceiling</td>
<td>1.212</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

Note: For each type of policy constraint, “optimal value” corresponds to the best scenario (bolded).

Figure 6: Welfare gains over full discretion

smooth out distortions intertemporally by employing monetary policy. In fact, as analyzed above, a limit on the primary deficit or revenue have no impact on the intertemporal trade-offs faced by the government.

Is it costly to set the wrong value for a constraint? As Figure 6 illustrates, the answer is typically negative. Revenue and budget balance (deficit) rules are benign for a significant range around the optimum. For example, revenue ceilings and primary surpluses are beneficial for a larger range around the optimum, so getting the exact value for the constraint right is not critical, limiting the costs of improper implementation.

Debt constraints are good as long as they are not too tight, as they interact with the ability of the government to smooth distortions. As in the non-stochastic case, a limit on debt over GDP, like the one imposed on Eurozone countries, has the peculiar property of two local maxima. Again, the stricter limit yields higher welfare in the benchmark calibration. The welfare gains of a debt ceiling, as the one implemented nominally in the U.S., are single peaked, but note that they rapidly convert into losses when set too high. The reason for this result is
that too-high debt ceilings do not provide the full benefits of constrained policy, but still hinder
tax-smoothing.

6.3 Robustness I: frequent abnormal times

Next we consider increasing the frequency of abnormal times or, equivalently, reducing the
duration of normal times. Let \( \varpi^* = \varpi = 0.9; \) that is, all states have now a duration of 10 years.
Table 9 presents the results. As we can see, the results obtained for the benchmark case still
apply. Even the optimal values for constraint are very close. The only significant difference is
a slight decrease in welfare gains.

Table 9: Welfare gains over full discretion when abnormal times are frequent

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Optimal Value</th>
<th>Always in good times</th>
<th>Suspended in bad times</th>
<th>Only in normal times</th>
<th>Suspended in good times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ceiling</td>
<td>0.151</td>
<td>0.94%</td>
<td>0.84%</td>
<td>1.34%</td>
<td></td>
</tr>
<tr>
<td>Primary deficit ceiling</td>
<td>-0.007</td>
<td>0.10%</td>
<td>0.04%</td>
<td>0.06%</td>
<td></td>
</tr>
<tr>
<td>Deficit ceiling</td>
<td>0.005</td>
<td>0.06%</td>
<td>0.04%</td>
<td>0.05%</td>
<td></td>
</tr>
<tr>
<td>Debt over GDP limit</td>
<td>0.272</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.01%</td>
<td></td>
</tr>
<tr>
<td>Debt ceiling</td>
<td>1.211</td>
<td>0.03%</td>
<td>0.02%</td>
<td>0.03%</td>
<td></td>
</tr>
</tbody>
</table>

Note: For each type of policy constraint, “optimal value” corresponds to the best scenario (bolded).

6.4 Robustness II: productivity, liquidity and expenditure shocks

We now verify that the main results derived for aggregate demand shocks also apply to other
types of shocks. Table 10 summarizes the welfare effects of imposing constraints on policy in
economies facing productivity, liquidity and government expenditure shocks.

Although each case presents its own idiosyncracies, the similarities across economies are
notable. For each type of shock the best prescription remains a revenue ceiling of about 15% of
GDP. It is always best not to suspend this constraint. Again, even if the constraint is imposed
only during normal times, due to the distortion-smoothing motive, it is disciplining government
behavior during abnormal times. Except for the case with expenditure shocks, the welfare loss
of suspending budget balance and revenue rules during abnormal times (good, bad or both)
is very small. The exception arises since spending shocks stem from variations in government
e benevolence; thus, it becomes important to constraint the government during bad times.

As for the other policy constraints, balance budget rules dominate debt rules. Among these
rules, requiring a small primary surplus, slightly more than half a percent of GDP, is the best
prescription. In most cases, it is best to impose constraints at all times, with an insignificant
welfare loss when this it is optimal to suspend occasionally a rule.

6.5 Robustness III: correlated shocks

One possible concern is that government spending may become more valuable to private agents
during adverse times. For instance, during recessions a series of policies are implemented (some-
times automatically) to help distribute the burden across agents; extensions to unemployment
insurance are a prime example. Though the present environment is not rich enough to model
this effects, we can capture some of this mechanism by assuming that the value of public goods
Table 10: Welfare gains over full discretion

<table>
<thead>
<tr>
<th>Productivity shocks</th>
<th>Constraint</th>
<th>Optimal Value</th>
<th>Always</th>
<th>Suspended in bad times</th>
<th>Only in normal times</th>
<th>Suspended in good times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ceiling</td>
<td>0.151</td>
<td>2.01%</td>
<td>1.84%</td>
<td>1.62%</td>
<td>1.77%</td>
<td></td>
</tr>
<tr>
<td>Primary deficit ceiling</td>
<td>−0.007</td>
<td>0.11%</td>
<td>0.10%</td>
<td>0.08%</td>
<td>0.09%</td>
<td></td>
</tr>
<tr>
<td>Deficit ceiling</td>
<td>0.005</td>
<td>0.07%</td>
<td>0.07%</td>
<td>0.06%</td>
<td>0.06%</td>
<td></td>
</tr>
<tr>
<td>Debt over GPD limit</td>
<td>0.272</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
<td></td>
</tr>
<tr>
<td>Debt ceiling</td>
<td>1.216</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.03%</td>
<td>0.03%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liquidity shocks</th>
<th>Constraint</th>
<th>Optimal Value</th>
<th>Always</th>
<th>Suspended in bad times</th>
<th>Only in normal times</th>
<th>Suspended in good times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ceiling</td>
<td>0.151</td>
<td>2.01%</td>
<td>1.84%</td>
<td>1.60%</td>
<td>1.77%</td>
<td></td>
</tr>
<tr>
<td>Primary deficit ceiling</td>
<td>−0.006</td>
<td>0.10%</td>
<td>0.10%</td>
<td>0.09%</td>
<td>0.09%</td>
<td></td>
</tr>
<tr>
<td>Deficit ceiling</td>
<td>0.005</td>
<td>0.07%</td>
<td>0.07%</td>
<td>0.06%</td>
<td>0.06%</td>
<td></td>
</tr>
<tr>
<td>Debt over GPD limit</td>
<td>0.325</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.01%</td>
<td>0.02%</td>
<td></td>
</tr>
<tr>
<td>Debt ceiling</td>
<td>1.214</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.03%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Government expenditure shocks</th>
<th>Constraint</th>
<th>Optimal Value</th>
<th>Always</th>
<th>Suspended in bad times</th>
<th>Only in normal times</th>
<th>Suspended in good times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ceiling</td>
<td>0.151</td>
<td>2.28%</td>
<td>1.76%</td>
<td>1.64%</td>
<td>2.14%</td>
<td></td>
</tr>
<tr>
<td>Primary deficit ceiling</td>
<td>−0.007</td>
<td>0.14%</td>
<td>0.08%</td>
<td>0.08%</td>
<td>0.13%</td>
<td></td>
</tr>
<tr>
<td>Deficit ceiling</td>
<td>0.004</td>
<td>0.09%</td>
<td>0.06%</td>
<td>0.06%</td>
<td>0.09%</td>
<td></td>
</tr>
<tr>
<td>Debt over GPD limit</td>
<td>0.325</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
<td></td>
</tr>
<tr>
<td>Debt ceiling</td>
<td>1.211</td>
<td>0.04%</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.04%</td>
<td></td>
</tr>
</tbody>
</table>

Note: For each type of policy constraint, “optimal value” corresponds to the best scenario (bolded).

increases when the demand for private goods decreases. That is, model demand and (valued) expenditure shocks as being perfectly correlated. Now, whenever there is a demand shock, there is a simultaneous change in the valuation of public goods. Let \( v(g) = \psi \ln g \), where \( \psi \) takes the value 1 in normal times (as in the benchmark calibration), 1.1 in bad times and 0.9 in good times. Table 10 presents the results for the case of correlated demand and expenditure shocks. As we can see, the results do not vary significantly from having only demand shocks.

7 Monetary policy rules

Having evaluated the effectiveness and desirability of fiscal rules, one could naturally wonder about the effects of constraints on monetary policy. Many countries adopt them. For example, Australia, Canada, New Zealand, Sweden and the U.K., among many others, have adopted inflation-targeting regimes. Although the specific implementation varies somewhat across countries, there is widespread agreement that inflation targets have been successful in keeping in-
Table 11: Welfare gains over full discretion—correlated demand and public expenditure shocks

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Optimal Value</th>
<th>Always in bad</th>
<th>Suspend in good</th>
<th>Always in normal</th>
<th>Suspend in normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ceiling</td>
<td>0.151</td>
<td>1.94%</td>
<td>1.76%</td>
<td>1.61%</td>
<td>1.80%</td>
</tr>
<tr>
<td>Primary deficit ceiling</td>
<td>-0.007</td>
<td>0.10%</td>
<td>0.09%</td>
<td>0.08%</td>
<td>0.09%</td>
</tr>
<tr>
<td>Deficit ceiling</td>
<td>0.004</td>
<td>0.06%</td>
<td>0.06%</td>
<td>0.06%</td>
<td>0.06%</td>
</tr>
<tr>
<td>Debt over GDP limit</td>
<td>0.272</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Debt ceiling</td>
<td>1.213</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

Note: For each type of policy constraint, “optimal value” corresponds to the best scenario (bolded).

flation low and stable.\textsuperscript{14} The convergence criteria for Eurozone membership included explicit inflation and interest rate goals, which were arguably effective.\textsuperscript{15} Perhaps more applicable to developing countries, currency substitution is a simple and effective way to adopt the monetary policy of a more disciplined country.\textsuperscript{16}

Just like fiscal rules are not always effective, monetary policy rules are not always successful at constraining central banks. For example, in the U.K., inflation was allowed to grow above its target band as a response to the deep recession and elevated unemployment levels that followed the 2007-08 financial crisis.

### 7.1 Monetary rules

An inflation target restricts a government to implement policy so that expected inflation is within a given interval, that is, $\pi^{e}_{t+1} \in [\bar{\pi}, \bar{\pi}]$, where $\pi^{e}_{t+1}$ is defined in (14). Similarly, an interest rate rule restricts policy to be consistent with the nominal interest rate fluctuating within a given interval, that is, $i_t \in [\underline{i}, \bar{i}]$, where $i_t$ is defined in (15). For the purpose of the exercises in this paper, I will focus on strict rules: an inflation target, $\bar{\pi} = \bar{\pi}$, an interest rate peg, $\underline{i} = \bar{i}$, and a Taylor rule, as described below.\textsuperscript{17}

Taylor rules, as first proposed by Taylor (1993), are often perceived as describing actual central bank behavior or as benchmarks of how it should behave. There has even been a recent push in the U.S. to legislate such a rule. In the present context, we can think of a Taylor rule as another monetary policy rule, disciplining the behavior of the government. Consider the following forward-looking variant of the Taylor rule:

$$1 + i_t = (1 + r^T)(1 + \pi^{e}_{t+1})^\varphi(1 + \pi^T)^{1-\varphi}$$

where $i_t$ is the nominal interest rate given by (15), $\pi^{e}_{t+1}$ is the expected inflation rate given by (14), $\pi^T$ is the desired target for inflation, $r^T$ is the real interest rate in a non-stochastic steady state associated with $\pi^T$ (standing-in here for the “natural real rate”) and $\varphi > 0$.\textsuperscript{18} By the

\textsuperscript{14}See Mishkin (1999) and Svensson (1999) for analyses of the international experience with inflation targeting and its comparison to other, less formally institutionalized, monetary policy regimes. See also Martin (2015).

\textsuperscript{15}See Martin and Waller (2012).

\textsuperscript{16}At the moment, there are several countries exclusively using foreign currency; e.g., Ecuador, El Salvador and Panama all use the U.S. dollar. A currency board, such as the one adopted by Argentina (1991-2002), Hong-Kong (since 1983) and Bulgaria (since 1997), is a weaker version of this type of constraint. There are also examples of countries allowing the legal circulation of both domestic and foreign currencies.

\textsuperscript{17}Allowing for small intervals around a monetary target did not seem to have any measurable impact on the results. Ceilings or floors proved to be worse than strict rules and hence omitted from the exercises presented here.

\textsuperscript{18}Given that prices here are flexible, I omit a response of the interest rate to the output gap, but this feature can be easily incorporated.
Fisher equation, \(1 + i_t \equiv (1 + \pi_{t+1}^e)(1 + r_t^e)\). Using (14) and (15) and imposing a steady state, we obtain
\[
1 + r^T = \frac{1}{\theta(1 + \pi^T) + (1 - \theta)\beta}.
\] (22)

Note that if government bonds are illiquid, \(\theta = 0\), then \(1 + r^T = \beta^{-1}\). If monetary policy follows a Taylor rule, then the government is constrained to implement expected inflation and the nominal interest rate in a way consistent with (21). If we take the parameter \(\varphi\) as given (say, at the standard value of 1.5 used in the New-Keynesian literature), then the choice of the optimal Taylor-rule constraint involves a choice of \(\pi^T\).

### 7.2 Alternative implementations

As with fiscal rules, constraints on monetary policy are imposed in terms of observable policy variables. Alternatively, one could restrict government actions by placing constraints on allocations. For example, in an economy without aggregate shocks, a specific inflation rate could be achieved by imposing a particular day-good allocation \(x\). Suppose we restrict the government to conduct policy such that it implements \(x = \bar{x}\). Since now \(\lambda = 0\), the non-stochastic version of conditions (7)–(11) (see Appendix B) implies \(B = B'\) (since now \(\lambda = \Lambda'\)—see Martin, 2015 for derivation details) and hence, from (14) we get \(\pi_{t+1} \equiv \beta(\eta u_x/\phi + 1 - \eta) - 1 = \bar{\pi}\).

In other words, there is a one-to-one mapping between \(\bar{x}\) and \(\bar{\pi}\). A constraint \(x = \bar{x}\) and the assumption that future governments also set \(x' = \bar{x}\) is a very different object than the constraint considered here, \(\pi_{t+1}^e = \bar{\pi}\), which in the non-stochastic case takes the form: \(\pi_{t+1} \equiv \beta(\eta u_x/\phi + 1 - \eta) - 1 = \bar{\pi}\). This constraint depends on the current choice for \(c_t\), but also on \(x_{t+1}\) and \(c_{t+1}\) which the current government cannot directly control. Since in equilibrium, \(x_{t+1} = \mathcal{X}(B_{t+1})\) and \(c_{t+1} = C(B_{t+1})\), the way a government facing constraint (14) can comply is by appropriately choosing \(B_{t+1}\) and \(c_t\) (i.e., debt and taxes), rather than directly through the implementation of a day-good allocation \(x_t\).

Note that both of the approaches described above would successfully implement the same inflation target. The sources of the different implementation are the limited commitment friction and the presence of debt, which is an endogenous state variable. The current government is best-responding to future policy. In turn, future policy may itself be restricted, due to a policy constraint; but the shape of the policy function is an equilibrium outcome and not a constraint on the current government.

The above example has important implications. In general, similar policy outcomes could in principle be achieved by placing restrictions on allocations. This approach could even be desirable from a welfare perspective.\(^{19}\) Consistent with the objective of evaluating real-world constraints, I here take the stand that monetary rules take the form of constraints on observed policy variables instead of allocations. The fact that we can write these problems in terms of allocations rather than policy variables is solely for analytical convenience.

### 7.3 Lessons

Tables 12 and 13 present the results to exercises similar to those conducted for fiscal rules. The bottom line is that monetary policy rules are not generally desirable mechanisms to discipline profligate governments. Both in economies with and without aggregate fluctuations, monetary policy targets yield small welfare gains and even losses relative to full discretion. More problematic is the fact that slight mis-targeting or incorrect timing can lead to large welfare losses.

\(^{19}\)One can show numerically that this is indeed the case for an inflation target.
The reason for these negative results is that monetary policy targets hinder the ability of governments to smooth distortions across states. In effect, inflation allows for less distortionary repayments of temporary debt increases. This does not mean that monetary rules are generally undesirable (they could be beneficial for “stabilization” purposes), but rather, that they should not be used as tool to curb public spending.

Table 12: Optimal monetary policy rules in non-stochastic economies

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Benchmark</th>
<th>Big Government</th>
<th>Low Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation target</td>
<td>0.040</td>
<td>0.00%</td>
<td>0.060</td>
</tr>
<tr>
<td>Interest rate peg</td>
<td>0.055</td>
<td>0.02%</td>
<td>0.060</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>0.051</td>
<td>0.01%</td>
<td>0.088</td>
</tr>
</tbody>
</table>

Table 13: Welfare gains over full discretion—demand shocks

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Optimal Value</th>
<th>Always Suspended in bad times</th>
<th>Only in normal times</th>
<th>Suspended in good times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation target</td>
<td>0.040</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Interest rate peg</td>
<td>0.055</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>0.052</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>

Note: For each type of policy constraint, “optimal value” corresponds to the best scenario (bolded).

8 General Lessons and Conclusions

The exercises presented in this paper evaluate the effectiveness and desirability of policy rules aimed at containing government spending. First and foremost, imposing a ceiling on revenue is always the best policy rule and it is never optimal to suspend this constraint. For an economy calibrated to the U.S. and subject to a variety of shocks, the optimal revenue ceiling is usually about 15% of GDP. Welfare gains to private agents of imposing the optimal revenue ceiling are sizable, about 2% of consumption. Welfare gains for other rules are at most an order of magnitude lower.

Second, revenue ceilings are the only rule that effectively induces governments to lower spending. The reduction in government size is modest, but significant enough to explain the superior welfare gains derived from imposing revenue ceilings over all other types of constraints. However, the reduction in government size carries a cost in terms of macroeconomic aggregates: higher debt and inflation and lower output. Hence, there is a meaningful trade-off in the implementation of revenue ceilings, which could impede their successful implementation. The work by Romer and Romer (2009, 2010) suggests this to be the case.

Third, most welfare gains come primarily from imposing constraints in normal times, which also helps discipline government policy during abnormal times. Key to this result is a commitment to reimpose rules when the state of the economy is back to normal. But this suggests that the issue is political rather than economical and should be addressed as such. The welfare costs of implementing fiscal rules sub-optimally, e.g., picking the wrong ceiling or suspending constraints during abnormal times, are relatively small. Notable exceptions include suspending
budget balance or revenue rules when government expenditure temporarily increases (assuming agents do not value such an increase) and imposing a primary deficit ceiling with very low debt.

Fourth, debt limits are generally effective, but always dominated by budget balance rules, even when these latter constraints are not implemented at their best (e.g., because society allows them to be suspended in bad times). This suggest that the typical focus of government reformers on debt ceilings may be misplaced. It is always more desirable, and arguably easier in practice, to aim at constraining the deficit.

Fifth, the socially effective way to combat inefficiently high public expenditure, is not more pre-commitment to government actions, but rather commitment to rules that constrain government action. The difference is that the former does not address the political friction, while the latter makes the government internalize the cost of socially suboptimal policy.
References


Appendix

A Derivation of monetary equilibrium conditions (2)–(6)

Here, we derive conditions (2)–(6) which characterize a monetary equilibrium. Let us start with
the problem of an agent at night,

\[
W(z, B, s) = \max_{c,n,m',b'} \gamma U(c) - \alpha n + v(g) + \beta E[V(m', b', B', s')|s]
\]

subject to the budget constraint: \( p_c c + (1 + \mu)(m' + q b') = p_c (1 - \tau) \zeta n + z \). Solving the budget constraint for \( n \) and replacing in the objective function, the first-order conditions imply:

\[
\gamma U_c - \frac{\alpha}{(1 - \tau) \zeta} = 0 \quad (23)
\]

\[
-\frac{\alpha(1 + \mu)}{p_c (1 - \tau) \zeta} + \beta E[V_m'|s] = 0 \quad (24)
\]

\[
-\frac{\alpha(1 + \mu)q}{p_c (1 - \tau) \zeta} + \beta E[V_b'|s] = 0 \quad (25)
\]

The night-value function \( W \) is linear in \( z \), \( W_z = \frac{\alpha}{p_c (1 - \tau) \zeta} \). Hence, \( W(z, B, s) = W(0, B, s) + \frac{\alpha z}{p_c (1 - \tau) \zeta} \), which we will use to rewrite the problem of the agent in the day. Accordingly, the problem of a consumer in the day can be rewritten as

\[
V^c(m, b, B, s) = \max_x u(x) + W(0, B, s) + \frac{\alpha(m + b - p_x x)}{p_c (1 - \tau) \zeta}
\]

subject to the liquidity constraint \( p_x x \leq m + \theta b \), with associated Lagrange multiplier \( \xi \). The first-order condition is

\[
u_x - \frac{\alpha p_x}{p_c (1 - \tau) \zeta} - \xi p_x = 0 \quad (26)
\]

Similarly, the problem of a producer can be rewritten as

\[
V^p(m, b, B, s) = \max_\kappa - \phi \kappa + W(0, B, s) + \frac{\alpha(m + b + p_x \kappa)}{p_c (1 - \tau) \zeta}
\]

The first-order condition implies

\[
-\phi + \frac{\alpha p_x}{p_c (1 - \tau) \zeta} = 0 \quad (27)
\]

Given \( V(m, b, B, s) \equiv \eta V^c(m, b, B, s) + (1 - \eta) V^p(m, b, B, s) \) and using (27) we obtain:

\[
V_m = \frac{\phi}{p_x + \eta \xi}
\]

\[
V_b = \frac{\phi}{p_x + \eta \theta \xi}
\]

Using these expressions, together with (27), we can rewrite (24) and (25) as

\[
1 + \mu = \frac{\beta p_x E[\phi/p_x' + \eta \xi'|s]}{\phi} \quad (28)
\]

\[
q = \frac{E[\phi/p_x' + \eta \theta \xi'|s]}{E[\phi/p_x' + \eta \xi'|s]} \quad (29)
\]
In equilibrium, we have \( m' = 1 \) and \( b' = B' \). Furthermore, the day and night resource constraints imply \( \kappa = \eta/(1 - \eta)x \) and \( n = c + g \), respectively. The liquidity constraint of consumers in the day holds with equality (wlog if it does not bind); thus,

\[
p_x = \frac{1 + \theta B}{x}
\]

which gives us (2).

Next, notice that (23) can be rearranged to yield (6):

\[
\tau = 1 - \frac{\alpha}{\zeta \gamma U_c}
\]

Plugging (30) and (31) into (27) yields (3):

\[
p_c = \frac{\gamma U_c (1 + \theta B)}{\phi x}
\]

Given (30)–(32) we can solve for the Lagrange multiplier of the liquidity constraint:

\[
\xi = \frac{(u_x - \phi)x}{(1 + \theta B)}
\]

Hence, (28) and (29) imply, respectively, (5) and (4), i.e.,

\[
\mu = \frac{\beta (1 + \theta B)}{\phi x} E \left[ \frac{x'(\eta u'_x + (1 - \eta) \phi)}{(1 + \theta B')} | s \right] - 1
\]

### B Discretionary government in non-stochastic economies

Consider a non-stochastic economy, i.e., when parameters \( \{\gamma, \zeta, \theta, \omega\} \) are constant over time. Conditions (7)–(11) become:

\[
(\gamma U_c - \alpha/\zeta) c - (\alpha/\zeta) g - \phi x (1 + B) + \beta \phi x'(1 + B') + \beta \eta x'(u'_x - \phi) = 0
\]

\[
\phi x'(1 - \theta)(\lambda - \lambda') \frac{1}{(1 + \theta B')^2} + \lambda \lambda' \left[ \eta(u'_x + u'_{xx} x' - \phi) + \phi(1 + B') \right] = 0
\]

\[
\eta(u_x - \phi) - \frac{\lambda \phi (1 + B)}{1 + \theta B} = 0
\]

\[
\gamma U_c - \alpha/\zeta + \lambda \{\gamma U_c - \alpha/\zeta + \gamma U_{cc}\} = 0
\]

\[
v_g - \alpha/\zeta + R_g - \lambda (\alpha/\zeta) = 0
\]

Note that we can normalize \( \gamma = \zeta = 1 \). It will be useful to define \( 1 + \hat{B} \equiv (1 + B)/(1 + \theta B) \) and write the Markov-perfect equilibrium as a function of \( \hat{B} \) instead of \( B \). This implies a set of equilibrium functions \( \{\hat{B}, \hat{X}, \hat{G}, \hat{\Lambda}\} \). Note that \( \hat{X}_B = \hat{X}_B (1 + \theta B)^2 / (1 - \theta) \). We thus get,

\[
(U_c - \alpha) c - \alpha g - \phi x (1 + \hat{B}) + \beta \phi x'(1 + \hat{B'}) + \beta \eta x'(u'_x - \phi) = 0
\]

\[
\phi x'(\lambda - \lambda') + \lambda \hat{X}_B' \left[ \eta(u'_x + u'_{xx} x' - \phi) + \phi(1 + \hat{B}') \right] = 0
\]

\[
\eta(u_x - \phi) - \lambda \phi (1 + \hat{B}) = 0
\]

\[
U_c - \alpha + \lambda \{U_c - \alpha + U_{cc}\} = 0
\]

\[
v_g - \alpha/\zeta + R_g - \lambda \alpha = 0
\]
When characterized in terms of \( \tilde{B} \), the equilibrium is independent of \( \theta \). Hence, the equilibrium is equivalent to the one studied in Martin (2011), which assumed \( \theta = 0 \), and we can apply the results derived there—the addition here of \( R(g, \omega) \) is immaterial for the equivalence result since, from Assumption 1, it can be viewed as being subsumed in the function \( v(g) \).

Two results apply. First, the proof of Proposition 1 here follows from Proposition 5 in Martin (2011). Second, when restricting the debt state space to be above \(-1\), there is a unique steady state \( B^* \). Proposition 2 in Martin (2011) shows that \( \tilde{B}^* > -1 \), \( \Lambda(\tilde{B}^*) > 0 \) and \( \chi_B(\tilde{B}^*) < 0 \); Propositions 3 and 4 establish further properties, including stability under certain conditions. Despite the variable transformation, the actual steady state bond-to-money ratio has the same property as that would essentially resolve the political friction at (virtually) no cost.

Steady states in non-stochastic economies with a discretionary government can thus be solved locally by finding the solution to the system above.

\[ B^* = -\frac{1 + \eta \Phi(x^*)}{1 + \theta \eta \Phi(x^*)} \]

where \( \Phi(x^*) \equiv \left( \frac{u^*_x + u^*_x x^*}{\phi} \right) - 1 \) and \( \{x^*, c^*, g^*\} \) jointly solve

\[
\begin{align*}
(U^*_c - \alpha) c^* - \alpha g^* + \eta x^*[u^*_x - \phi + (1 - \beta)u^*_x x^*] &= 0 \\
\alpha (u^*_x - \phi) + (v^*_g + R^*_g - \alpha)(u^*_x + u^*_x x^* - \phi) &= 0 \\
(v^*_g + R^*_g)(U^*_c - \alpha) + (v^*_c + R^*_c - \alpha)U^*_c c^* &= 0
\end{align*}
\]

Steady states in non-stochastic economies with a discretionary government can thus be solved locally by finding the solution to the system above.

C Constraints on allocations

The exercises conducted in the main body of the paper consist of placing constraints on policy variables, e.g., a revenue ceiling or an inflation target. Alternatively, one could imagine placing constraints on allocations. Constraining allocations, such as day-good and night good consumption, is simple to implement, but sacrifices realism and hence, the usefulness of the resulting policy prescriptions. Constraining public expenditure \( g \) directly was assumed to be infeasible as that would essentially resolve the political friction at (virtually) no cost.

It is instructive, however, to consider optimal constraints on allocations and compare them to the policy rules studied in the paper. Moreover, there is a mapping between allocations and policy variables. To keep things simple, consider the case without aggregate shocks. First, targeting \( x \) is equivalent to targeting current inflation in the day market. To see this, note that inflation in the day market is \( \pi_t^x \equiv p_{x,t}(1 + \mu_{t-1})/p_{x,t-1} - 1 \), which using (2) and (5) implies \( \pi_t^x = \beta(\eta u_{x,t}/\phi + 1 - \eta) - 1 \). Second, from (6), targeting the night-good \( c \) is equivalent to targeting the tax rate \( \tau \).

Consistent with the exercises in the paper, I here consider three types of constraints: a day-good inflation target (\( x = x^T \) or, equivalently \( \pi^x = \pi^T \)); a ceiling on the tax rate (\( c \leq \tilde{c} \) or, equivalently, \( \tau \leq \tilde{\tau} \)); and a ceiling on public expenditure (\( g \leq \tilde{g} \)). Table 14 presents the results, which should be compared to Tables 4 and 12. As we can see, the day-good inflation target performs much better than other monetary policy rules and yields welfare gains similar to the primary deficit ceiling. Hence, this inflation target would be preferred to other monetary policy rules, though it would involve constructing the proper price index. The tax rate ceiling performs similarly to the revenue ceiling, in terms of macroeconomic outcomes and welfare, but

\[(\text{There is an infinite number of steady states below } -1, \text{ including one which implements the first-best allocation.})\]
offers lower welfare. Thus, a revenue ceiling is still preferred. As anticipated, an expenditure ceiling essentially resolves the political frictions and hence, yields welfare gains almost identical to making the government fully benevolent. As a final remark, placing a ceiling on public expenditure over GDP slightly out-performs a ceiling on \( g \).

Table 14: Optimal allocation constraints and steady state statistics—benchmark calibration

<table>
<thead>
<tr>
<th>Variable / Constraint</th>
<th>Day-market inflation target ( x = x^T )</th>
<th>Tax rate ceiling ( c \leq \bar{c} )</th>
<th>Expenditure ceiling ( g \leq \bar{g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal constraint</td>
<td>0.017</td>
<td>0.170</td>
<td>0.103</td>
</tr>
<tr>
<td>Welfare gains</td>
<td>0.09%</td>
<td>1.82%</td>
<td>9.93%</td>
</tr>
<tr>
<td>Steady State</td>
<td>0.16%</td>
<td>1.09%</td>
<td>10.08%</td>
</tr>
<tr>
<td>Transition</td>
<td>−0.07%</td>
<td>0.74%</td>
<td>−0.15%</td>
</tr>
</tbody>
</table>

**Steady state statistics**

<table>
<thead>
<tr>
<th></th>
<th>Day-market inflation target ( x = x^T )</th>
<th>Tax rate ceiling ( c \leq \bar{c} )</th>
<th>Expenditure ceiling ( g \leq \bar{g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt over GDP</td>
<td>0.305</td>
<td>0.480</td>
<td>0.318</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.017</td>
<td>0.120</td>
<td>0.018</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.046</td>
<td>0.108</td>
<td>0.047</td>
</tr>
<tr>
<td>Revenue over GDP</td>
<td>0.185</td>
<td>0.151</td>
<td>0.149</td>
</tr>
<tr>
<td>Expenditure over GDP</td>
<td>0.180</td>
<td>0.172</td>
<td>0.144</td>
</tr>
<tr>
<td>Nominal debt</td>
<td>1.593</td>
<td>3.139</td>
<td>1.516</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>0.029</td>
<td>−0.011</td>
<td>0.028</td>
</tr>
<tr>
<td>Primary deficit over GDP</td>
<td>−0.005</td>
<td>0.021</td>
<td>−0.005</td>
</tr>
<tr>
<td>Deficit over GDP</td>
<td>0.008</td>
<td>0.068</td>
<td>0.009</td>
</tr>
</tbody>
</table>