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Authors	Michael T. Owyang, Jeremy M. Piger, and Daniel Soques
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Contagious Switching[*]

Michael T. Owyang[†], Jeremy Piger[‡], Daniel Soques[§]

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Abstract

We analyze the propagation of recessions across countries. We construct a model that allows for multiple qualitative state variables in a vector autoregression (VAR) setting. The VAR structure allows us to include country-level variables to determine whether policy also propagates across countries. We consider two different versions of the model. One version assumes the discrete state of the economy (expansion or recession) is observed. The other assumes that the state of the economy is unobserved and must be inferred from movements in economic growth. We apply the model to Canada, Mexico, and the United States to test if spillover effects were similar before and after the North American Free Trade Agreement (NAFTA). We find that trade liberalization has increased the degree of business cycle propagation across the three countries.

JEL Codes: C32; E32

Keywords: time varying transition probabilities, NAFTA, business cycle synchronization

1 Introduction

The study of trade liberalization's effect on business cycle synchronization offers two competing theories with opposite implications.[1] On the one hand, trade liberalization can be synchronizing if the spillover from domestic shocks is greater for trading partners than non-trading partners [Frankel and Rose (1998); Baxter and Kouparitsas (2005); and Kose and Yi (2006) to name just a few]. On the other hand, trade liberalization can spur industrial specialization, which may prevent or mitigate such spillovers [Imbs (2004)]. The empirical literature studying changes in synchronization over time remains inconclusive as to the direction of the effect of trade liberalization on synchronization. Stock and Watson (2005) and Kose, Otrok and Whiteman (2008) estimate factor models and show that the importance of the global factor has increased over time, in line with growth in global trade. However, despite significant trade liberalization, Doyle and Faust (2005) conclude that the correlation between GDP growth rates in Canada and the United States has remained unchanged since the 1960s, while Heathcote and Perri (2003) argue the United States is less correlated with Europe, Canada, and Japan over the same period.

Measuring synchronization is another unsettled issue. A common approach, typified by Frankel and Rose (1998), is to measure synchronization using bivariate contemporaneous correlations between measures of output growth for each country. These correlations, while computationally simple, may not take into account all of the information available to the econometrician. For example, such correlations do not measure dynamic relationships between the business cycles of two countries, an important omission if there are lags in the propagation of shocks across countries. A number of other papers have considered alternative methods of measuring the interaction between business cycles. Kose, Otrok, and Whiteman (2003, 2008) estimate factor models with global and regional factors, where an increase in the variance share explained by the global or regional factors suggests higher synchronization. Hamilton and Owyang (2012) collect similar cycles into clusters (regions) that move together. This reduces the dimensionality of the problem

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[1]Business cycle synchronization is distinct from growth rate convergence. The former measures the correlation between cycles (and possibly their leads and lags) and the timing of countries' turning points. The latter studies potential declines in the differences between countries' growth rates.

but forces the cycles of series in the same cluster to be essentially identical. Leiva-Leon (2017) considers pairwise series of binaries where a third binary switches the interaction from fully synchronized to fully unsynchronized.

In this paper, we continue the study of business cycle synchronization and propagation, proposing a model in which the state of the business cycle in one country can affect the current and future state of business cycles in other countries. Unlike much of the existing literature, which explores business cycle synchronization and propagation using linear models of output growth, here, we focus explicitly on synchronization and propagation of business cycle phases, namely recessions and expansions. This allows us to explore how recessions—which are persistent, large, deviations from trend growth—propagate across countries, without such analysis being confounded by higher frequency, and generally smaller, fluctuations. To capture switching between expansion and recession regimes we utilize a Markov-switching process, which has been employed extensively and successfully for modeling business cycles and other persistent regimes in economics data.[2]

Perhaps the papers most closely related to ours use multivariate Markov-switching models with transition probabilities that depend on the states for the other series. Kaufmann (2010) proposes a Markov-switching model in which series are endogenously grouped as either leading or coincident, with each group governed by a separate Markov-switching process. The two Markov states are dynamically linked by allowing the transition probability for each state to depend on the lagged value of the other state. Billio, Casarin, Ravazzolo and van Dijk (2016) extend the multi-country panel VAR of Canova and Ciccarelli (2004, 2009) to include Markov-switching in parameters designed to capture expansions and recessions for different countries. In their model, the time-varying transition probabilities for the business cycle state in each country are determined via a logit process and depend on weighted sums of the lagged value of the business cycle states in other

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[2] A short list of early contributions to the literature using Markov-processes to model business cycles includes Hamilton (1989), Chauvet (1998) and Kim and Nelson (1998). Markov-switching models have also been applied in many other areas, including to capture mean and persistence shifts in measures of inflation and interest rates [Evans and Wachtel (1993); Garcia and Perron (1996); Ang and Bekaert (2002)], volatility regimes in equity returns [Turner, Startz and Nelson (1989); Hamilton and Susmel (1994); Hamilton and Lin (1996); Dueker (1997)], shifts in Federal Reserve policy [Sims and Zha (2006)], and asymmetry in the effects of monetary policy shocks [Garcia and Schaller (2002); Kaufmann (2002); Ravn and Sola (2004); Lo and Piger (2005)].

countries. Agudze, Billio, Casarin and Ravazzolo (2018) implement a dynamic panel with series-specific Markov-switching processes, where the interactions of the Markov states are modeled via a network. Specifically, these authors allow the time-varying transition probability for each state to depend on summary aggregated measures of the lagged values of other states that are "local" and "global" to the state in question, where local and global states are endogenously determined through a network interaction framework.

In this paper, we describe and measure the alternation between expansion and recession regimes in different countries using a multivariate Markov-switching framework, where the Markov-switching process has time-varying transition probabilities as in Filardo (1996). Our model extends and differs from the existing literature on such models on two levels. First, we allow for the dynamic interaction of country business cycle phases, which allows us to ask whether one country's business cycle phase propagates to other countries over time. This is similar in spirit to the models presented in Billio et al. (2016) and Agudze et al. (2018), except our model permits the business cycle state for each country to directly and differentially impact the business cycle state of each other country. This is achieved by modeling the underlying unobserved latent variables driving the Markov-switching process with a VAR. Second, our model structure produces a VAR for a system containing the observed macroeconomic variables and the unobserved latent variables, which allows us to, among other things, identify shocks to policy variables and determine their effect on the probability of changing future country-level business cycle phases.

We demonstrate how to estimate the model in both the case where business cycle phases are observed and the case where they are not. In the former case, one could use outside measures of the business cycle—e.g., the U.S. business cycle dates produced by the National Bureau of Economic Research (NBER) Business Cycle Dating Committee. The latter case can be particularly useful when the model is applied to sub-national cycles (regions or industries) or for countries (e.g., emerging markets) where "official" business cycle dates are unknown. In this latter case, we estimate both discrete latents and continuous latents that depend on each other. We propose a Metropolis step that

allows us to form a joint proposal that combines steps from a standard Kalman filter and a Bayesian modification of the Hamilton (1989) filter.

We apply our model to the U.S., Canada, and Mexico to determine whether the North American Free Trade Agreement (NAFTA) altered the propagation of business cycles across these countries. For the full sample, we find that an increase in the probability of recession in Canada or the United States leads to a statistically significant increase in the recession probabilities in its neighbors. An adverse shock to Mexico, on the other hand, has a subsequent but statistically insignificant increase in the recession probability for the United States, while Canada remains unaffected.

In subsample analysis, we find a relatively low degree of recession spillovers prior to the introduction of NAFTA. However, since NAFTA was adopted in 1994, we find that recession shocks originating from the United States or Canada lead to a significantly higher recession probability in the other two nations. Additionally, we find that shocks from Mexico propagate to the United States during the NAFTA period. Therefore, our paper adds to the evidence that trade liberalization increases the degree of business cycle synchronization across countries.

The balance of the paper is laid out as follows. Section 2 describes the model with both observed and unobserved states. Section 3 outlines the Bayesian estimation of the model. We describe in detail the sampler block required to obtain the joint draw of the discrete and continuous latent states. This section also describes the data and VAR identification. Section 4.1 presents the empirical results for the observed states. We also present the computation of the dynamic marginal effects. Section 4.2 presents the results with unobserved states. Section 5 introduces a break at the implementation of NAFTA and re-estimates the model for the pre- and post-break periods. Section 6 offers some conclusions.

2   Empirical Setup

Consider the interaction between the business cycles of $n = 1, ..., N$ countries over $t = 1, ..., T$ periods. Let $S_{nt} = \{0, 1\}$ represent the discrete business cycle phase for country n at time t, where $S_{nt} = 0$ represents an expansionary phase and $S_{nt} = 1$ represents a recessionary phase.[3] Collect the business cycle phases into a vector $S_t = [S_{1t}, ..., S_{Nt}]'$.

2.1   Observed Regimes

To model the interdependence of business cycle phases across countries, we must specify how S_{nt} affects S_{mt}, $m \neq n$. Assume, initially, that each S_{nt} is observed. Further, suppose that the business cycle phases propagate across countries with a lag. Let z_{nt} represent a continuous latent variable related to the binary observed variable S_{nt} through the deterministic relationship:

$$
S_{nt} = \begin{cases} 1 & \text{if} \quad z_{nt} \geq 0 \\ 0 & \text{otherwise} \end{cases}.
$$

Through the latent variable z_{nt}, we can study how other variables—both macro variables and the business cycle phases of other countries—affect the future business cycle phase of country n. Let $y_t = [y_{1t}, ..., y_{Jt}]'$ represent a $(J \times 1)$ vector of macro variables that could include policy variables or other economic indicators which can be country-specific or global variables. Let $z_t = [z_{1t}, ..., z_{Nt}]'$ collect the continuous latent business cycle indicators.

Define $Y_t = [z_t', y_t']'$, where the relationship between the contemporaneous Y_t and its lags follows a vector autoregression (VAR):

$$
Y_t = B_0 + B(L) Y_{t-1} + u_t, \tag{1}
$$

---

[3]The two-regime framework is common in regime-swtiching models of the business cycle [Hamilton (1989), Chauvet (1998), Chauvet and Piger (2008), Camacho, Perez-Quiros, and Poncela (2015), among others]. One could consider a larger number of regimes in our framework as in Kim and Piger (2002) and Billio, Casarin, Ravazzolo, and van Dijk (2016), among others. However, we choose to focus on the two-regime case due to our application to observed recession dates and for parsimony.

where $u_t = [u_{1t}^z, ..., u_{Nt}^z, u_{1t}^y, ..., u_{Jt}^y]'$ and $E_t[u_t u_t'] = \Sigma$. For exposition, we write (1) in a more detailed form:

$$
\begin{bmatrix} z_t \\ y_t \end{bmatrix} = \begin{bmatrix} B_0^z \\ B_0^y \end{bmatrix} + \begin{bmatrix} B^{zz}(L) & B^{zy}(L) \\ B^{yz}(L) & B^{yy}(L) \end{bmatrix} \begin{bmatrix} z_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t^z \\ u_t^y \end{bmatrix},
$$

where $B^{ij}(L)$ represents the lagged effect of j on i. Because the z_t are latent, we make scale assumptions by restricting their variances for identification. In particular, we assume that

$$
\Sigma_{zz} = E_t[u_t^z u_t^{z\prime}]
$$

has unit diagonal elements. In subsequent sections, we can impose additional restrictions on the decomposition of the VAR variance-covariance matrix that identify the structural form of the VAR from its reduced form.

The current model has a form similar to a multiple-binary-variable extension of Dueker's (2005) Qual-VAR. In that paper, a single binary variable indicates the state of the economy and can be affected—and, importantly, can affect—a vector of macro-economic variables at lags. This version of our model with observed S_t collapses to the Qual-VAR when S_t is a scalar.[4] Because of the assumption that the reduced-form VAR errors are multivariate normal, the z_t equations in the VAR resemble a multivariate extension of the dynamic probit outlined in Eichengreen, Watson, and Grossman (1985). Chib and Greenberg (1998) develop methods of estimating the static multivariate probit, which is equivalent to the z_t equations in the VAR in our model with observed S_t with the additional assumption that $B^{zz}(L) = 0$. This observed-S_t version of our model is perhaps most similar to the multivariate dynamic probit of Candelon, Dumitrescu, Hurlin, and Palm (2013) in which we add a propagation mechanism for the covariates that allows the latents to affect macro variables at lags.

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[4]The Prob-VAR outlined in Fornari and Lemke (2010) is a more restricted version of the Qual-VAR, where they assume $B^{zz}(L)$ and $B^{yz}(L)$ are both identically zero. Their model, then, is essentially a VAR in the macro variables y and a probit where lags of y determine a scalar z. Their application is to forecasting S using iterative multistep methods. The VAR forms forecasts for y, which in turn informs the forecast of S at longer horizons.

Two key features differentiate our model from a set of independent time-varying transition probability switching models. First, there is a lagged cross-regime effect that is embedded in the off-diagonal elements of $B^{zz}(L)$. The lagged effect represents the contagious switching, where a regime change in one country can spill over into the regimes of its neighbors. Further notice that the regime cross-series dependence is a function of the continuous latent rather than the binary latent. This means that z_{nt} may be thought of as representing the strength of the business cycle phase. Second, there is a contemporaneous cross-regime effect that is embedded in the tetrachoric correlation term in Σ. The tetrachoric effect can represent either simultaneity of shocks that cross country borders or within-quarter contagion effects. The model allows us to test for the presence of cyclical contagion, the speed at which it acts, and the degree to which countries affect each other. In addition, countercyclical or prophylactic policy can be included in the y_j's to determine whether, say, changes in fiscal or monetary policy can reduce the probability of recessions.

2.2 Unobserved Regimes

While we previously assumed that the S_t are observed, we can relax this assumption by including a vector of economic indicators whose means depend on the discrete regimes. Unobserved regimes can be relevant for a number of reasons. For example, one simply might not have the data available as all countries do not construct or announce business cycle turning points. On the other hand, some countries have more than one set of turning point dates, suggesting some uncertainty over the timing of the events. In the U.S., the NBER Business Cycle Dating Committee dates are widely accepted as the "official" business cycle turning points. However, these dates are not revised even in the presence of new or revised data. Moreover, other measures such as the OECD Recession Indicators may vary slightly from the NBER in the timing and definition of the turning points. In some of these cases, it may be advantageous to estimate the regime changes directly from the data.

Suppose, then, that each of the N countries can be characterized by a period$-t$

business cycle indicator, x_{nt}. While x_{nt} could be any scalar or vector of contemporaneous indicator(s) of the cycle, for the purposes of exposition, we refer to x_{nt} as the output growth rate. Collect the period$-t$ output growth rates into a vector $x_t = [x_{1t}, ..., x_{Nt}]'$. We assume that output growth is a stochastic sampling from a mixture of normals, where μ_{n0} and μ_{n1} are the means of the two normal distributions and we impose $\mu_{n0} > \mu_{n1}$ for identification. Note that the mixtures can be potentially different for each country, as evidenced by the index n. The interpretation of our assumption is that each country's economy moves between two business cycle phases, a relatively high mean (expansion) and a relatively low mean (recession). Note that we do not impose that the mean recession growth rate is negative, but it must be less than the mean growth rate in expansion.[5]

During each period, a country n's business cycle phase is represented by the latent variable S_{nt} that determines which of the two distributions x_{nt} is drawn from that period. The process can be summarized by

$$x_{nt} = \mu_{n0} + \Delta\mu_n S_{nt} + \phi_n (L) x_{n,t-1} + \varepsilon_{nt}, \qquad (2)$$

where we can define $\Delta\mu_n = \mu_{n1} - \mu_{n0}$, $\Delta\mu_n < -\mu_{n0}$, as implied by our identifying assumption, and $\varepsilon_{nt} \sim N(0, \sigma_n^2)$. We impose that the output volatility is time invariant and that the output shocks are uncorrelated across countries, serially uncorrelated, and uncorrelated with the shocks to the variables in the VAR. In the current application, we suppress the autoregressive dynamics, $\phi_n (L) = 0$.[6]

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[5]Alternatively, we could restrict the mean growth rate in expansion to be strictly positive and the mean growth rate in recession to be strictly negative. Allowing the recessionary growth rate to be postive, but less than the expansionary growth rate, provides the flexibility to match recession characteristics of developing countries, such as Mexico.

[6]These assumptions are made for expositional clarity and are generally consistent with those in the business cycle identification literature (see, for example, Owyang, Piger and Wall, 2005). They are straightforward to relax.

3 Estimation and Data

3.1 The Sampler

We estimate the model using the Gibbs sampler, a Markov-Chain Monte Carlo algorithm that draws a block of the model parameters—including the underlying continuous states–conditional on the remaining parameters and the data. Let Ω_t represent the data available at time t. We specify a standard set of priors for the model with observed regimes. The parameters in B are multivariate normal and we assume a standard Minnesota prior. We assume similar priors to Chan and Jeliazkov (2009) on the parameters λ in the diagonal matrix D and the parameters a in the lower triangular matrix L of the decomposition $\Sigma = L'^{-1}DL^{-1}$.[7] For the case with unobserved regimes, we also need to set priors for the intercepts, the AR terms, and the innovation variances in the x_t equation. We assume that the parameters in the x_t equation have a Normal-inverse Gamma prior. Table 1 contains the parameterization of the prior for the more general model with unobserved regimes; the model with observed regimes has the same priors without the parameters governing the process for x (i.e., μ and σ).

Table 1: Prior Specifications for Estimation

Parameter	Prior Distribution	Hyperparameters	
$b = vec(B)$	$N\left(\bar{b}_0, \bar{B}_0\right)$	Minnesota Prior (See Appendix)	
a_k	$N\left(\boldsymbol{a}_{k0}, \boldsymbol{A}_{k0}\right)$	$\boldsymbol{a}_{k0} = \boldsymbol{0}$, $\boldsymbol{A}_{k0} = (0.15^2) * \boldsymbol{I}$	$\forall k$
λ_k^{-1}	$\Gamma\left(\frac{\nu_{k0}}{2}, \frac{\delta_{k0}}{2}\right)$	See Appendix	
$\mu_n \mid \sigma_n^{-2}$	$N\left(m_{n0}, \sigma_n^2 M_{n0}\right)$	$m_{n0} = [1, -1]'$, $M_{n0} = 2 * \boldsymbol{I}_2$	$\forall n$
σ_n^{-2}	$\Gamma\left(\frac{v_{n0}}{2}, \frac{\tau_{n0}}{2}\right)$	$v_{n0} = 1$, $\tau_{n0} = 1$	$\forall n$

We divide the exposition of the sampler into two parts. In the first part, we outline the sampler for the case where S_t is observed. In this case, there are three blocks for estimation: (1) the coefficient matrices for the VAR, $B = \{B_0, ..., B_P\}$; (2) the VAR variance-covariance matrix, Σ; and (3) the latent states, $\{z_t\}_{t=1}^T$. The first block is conjugate normal. Because of the restrictions on the latent variances, the second block requires a Metropolis step, which is a modification of the algorithm outlined in Chan and Jeliazkov

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[7]See the Technical Appendix for the full parameterization of the variance-covariance matrix Σ.

(2009). The third block is executed by drawing the continuous latent state variable recursively from smoothed Kalman posterior distributions.[8] The Technical Appendix outlines the state space of the model and each of the draws.

Aside from two additional blocks to sample the additional parameters in the x_t equation, the case of unobserved regimes adds a wrinkle that warrants more explanation. Because the sign of z_{nt} is determined by the value of S_{nt} and the past z_t determine the transition probabilities for S_{nt}, these two values must be sampled simultaneously. Thus, the sampler for the unobserved state case has five blocks: (1) the coefficient matrices for the VAR, $B = \{B_0, ..., B_P\}$; (2) the VAR variance-covariance matrix, Σ; (3) the coefficients for the measurement equation, $\Psi = \{\mu_0', \mu_1', \phi'\}$; (4) the measurement innovation variances, $\{\sigma_n^2\}_{n=1}^N$; and (5) the latent states, $\{z_t, \}_{t=1}^T$ and $\{S_t\}_{t=1}^T$. The two additional blocks (3) and (4) yield conjugate posterior distributions. We outline the filter used to obtain draws of block (5) below; other draws are detailed in the Technical Appendix. We run the Gibbs sampler for 10,000 iterations after an initial burn-in period of 20,000 iterations; convergence diagnostics are available from the authors upon request.

3.1.1   Drawing $\{z_t\}_{t=1}^T$, $\{S_t\}_{t=1}^T$ conditional on B, Σ, $\{\sigma_n^2\}_{n=1}^N$, Ψ

Unfortunately, when $\{S_t\}_{t=1}^T$ is unknown, we cannot draw the sequences of the two latent variables in separate blocks. The value of S_{nt} is directly related to the sign of z_{nt}. One might posit a draw in which the full sequence $\{S_\tau\}_{\tau=1}^T$ is drawn, conditional on the past iteration of $\{z_\tau\}_{\tau=1}^T$; then, a draw of the full sequence of $\{z_\tau\}_{\tau=1}^T$, conditional on the new draw of $\{S_\tau\}_{\tau=1}^T$, where each S_{nt} determines the direction of the truncation of z_{nt}. However, any draw that changes S_{nt} across Gibbs iteration invalidates the last draw of z_{nt}, as the truncation would be improper. Drawing the full sequence $\{z_\tau\}_{\tau=1}^T$ first also would be invalid. While we can obtain a Kalman posterior for z_{nt}, the exact conditional distribution will be truncated. Simply drawing z_{nt} from the Kalman posterior and then assigning S_{nt} based on the sign of z_{nt} would ignore information in the xs that inform S_{nt}.

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[8]This differs from Dueker's original sampler. In this sampler, Dueker used exact conditional distributions for the interior $T - 2P$ periods. The first P periods were drawn using Metropolis-Hastings. The last P periods were drawn by iterating forward the mean of the exact conditional distribution for the $T - P - 1$ period.

We adopt an alternative approach that takes advantage of both the Kalman filter and Hamilton's Markov switching filter to draw candidates for a Metropolis-in-Gibbs step. Because we need to use the draws of lagged z_t to form the transition probabilities for the Hamilton filter, we cannot draw the candidates using smoothed probabilities. Instead, for each t, we draw a candidate S_t^*, conditional on lags of z_t, using the forward component of the Hamilton filter. We then draw a candidate z_t^* from the posterior obtained by the forward component of the Kalman filter.

Specifically, start with a set of initialization probabilities, $\Pr[S_{n0}]$, which could be the steady state regime probability, and initialize the vector of latents, z_0, and the state covariance matrix, $P_{0|0}^z$. The goal is to obtain (jointly) a candidate pair of vectors (z_t^*, S_t^*) for each $t = 1, ..., T$. We can form the joint proposal density as

$$p\left(z_t^*, S_t^*|\Omega_t\right) = p\left(z_t^*|\Omega_t, S_{nt}^*, \{z_\tau\}_{\tau=1}^{t-1}\right) \prod_{n=1}^{N} p\left(S_{nt}^*|\Omega_t, \{z_\tau\}_{\tau=1}^{t-1}\right).$$

We draw the candidate S_{nt}^* from

$$\Pr\left[S_{nt}^* = 1|\Omega_t\right] = \frac{\sum_{S_{n,t-1}} \ell\left(S_{nt}^* = 1, S_{n,t-1}|\Omega_t, \{z_\tau\}_{\tau=1}^{t-1}\right) \Pr\left[S_{nt}^* = 1|S_{n,t-1}, \{z_\tau\}_{\tau=1}^{t-1}\right] \Pr\left[S_{n,t-1}|\Omega_{t-1}, \{z_\tau\}_{\tau=1}^{t-1}\right]}{\sum_{S_{nt}} \sum_{S_{n,t-1}} \ell\left(S_{nt}^*, S_{n,t-1}|\Omega_t, \{z_\tau\}_{\tau=1}^{t-1}\right) \Pr\left[S_{n,t}^*|S_{n,t-1}, \{z_\tau\}_{\tau=1}^{t-1}\right] \Pr\left[S_{n,t-1}|\Omega_{t-1}, \{z_\tau\}_{\tau=1}^{t-1}\right]},$$

where $\ell\left(.,.|.\right)$ is the likelihood and

$$\Pr\left[S_{nt}^* = 1|S_{n,t-1}, \{z_\tau\}_{\tau=1}^{t-1}\right] = \Pr\left[z_{nt} > 0|z_{t-1}\right]$$

are the transition probabilities that depend on the lagged continuous latent variable for all n.

The conditional distributions for the z_{nt}'s can be obtained by the forward component of the Kalman filter. Based on the state equation, (1), the Kalman filter obtains the forecast density for the vector z_t, conditional on its lags. Then, the filter updates the forecast density using information from the current realization of y_t to obtain

$$p\left(z_t|\Omega_t\right) \sim N\left(\widehat{z}_{t|t}, P_{t|t}^z\right),$$

where $\widehat{z}_{t|t}$ is the mean of the conditional distribution and $P_{t|t}^z$ is the covariance matrix.[9] Then, conditional on S_t^*, we can draw the candidate from the truncated normal, where the truncation direction depends on S_t^*:

$$p\left(z_t | \Omega_t, S_t\right) \sim TN\left(\hat{z}_{t|t}, P_{t|t}^z, S_t\right).$$

Finally, we validate the candidate (z_t^*, S_t^*)—drawn jointly for all n—using standard MH acceptance probabilities. The candidate (z_t^*, S_t^*) is accepted with probability α, where

$$\alpha = \min\left[1, \frac{\pi\left(S_t^*, z_t^*\right) f(x_t, y_t | S_t^*, z_t^*) q\left(S_t^{[i-1]} | z^{[i-1]_t}\right)}{\pi\left(S_t^{[i-1]}, z_t^{[i-1]}\right) f(x_t, y_t | S_t^{[i-1]}, z_t^{[i-1]}) q\left(S_t^*, z_t^*\right)}\right],$$

where $\pi\left(.,.\right)$ is the prior, $f(x, y | ., .)$ is the joint likelihood, and $q\left(. | .\right)$ are the move probabilities. Because we have an independence chain, the ratio of the move probabilities collapses to 1. Using the fact that y does not depend on S and the identity $P(S_t | z_t) = 1$, the posterior likelihood is

$$\pi\left(S_t, z_t\right) f(x_t, y_t | S_t, z_t) = f(y_t, z_t | S_t) \prod_{n=1}^{N} f(x_{nt} | S_{nt}),$$

where

$$f(y_t, z_t | S_t) = \frac{1}{|2\pi\Sigma|^{1/2}} \exp\left\{-\frac{1}{2} u_t' \Sigma^{-1} u_t\right\}$$

and

$$f(x_{nt} | S_{nt}) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left\{-\frac{1}{2\sigma_n^2} \varepsilon_{nt}' \varepsilon_{nt}\right\}.$$

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[9]Each of these quantities will be a subvector and submatrix, respectively, of the output of the Kalman filter.

3.2 Data

We apply the model to the NAFTA member countries (Canada, Mexico, and the United States). We estimate two versions of the model: First, we consider the model with observed recessions that requires two sets of data: (1) the recession indicators, S_t, and (2) the macroeconomic variables in y_t. For the recession indicators, we use NBER dates for the United States, recession dates from the C.D. Howe Institute for Canada, and recession dates obtained from the quarterly application of the Bry-Boschan (BBQ) method for Mexico.[10] For the macroeconomic variables, we use the U.S. effective federal funds rate prior to 2009Q1 and after 2015Q4, and the Wu and Xia (2016) shadow short rate during the period from 2009Q1 to 2015Q4, over which the federal funds rate was at the zero lower bound. The effective federal funds rate comes from the Federal Reserve Bank of St. Louis' FRED database and the shadow short rate is available from the Federal Reserve Bank of Atlanta. In order to properly identify shocks to monetary policy, we add two additional series. The first is the inflation rate, measured as the difference in the log of the PCE price level. The second is the change in log commodity prices obtained from the Commodity Research Board. Both of these series were obtained from the FRED database.

Next, we consider the model with unobserved regimes. This model also requires two sets of data: (1) x_t, the variable that informs S_t, and (2) the macroeconomic variables in y_t. For the latter, we use the same macroeconomic variables as in the previous experiment For x_t, we use the first principal component across four series for each country, including real GDP growth, employment growth, industrial production growth, and retail trade growth. The real GDP growth data comes from the OECD Quarterly National Accounts, employment growth comes from the OECD Short-Term Labour Market Statistics, and industrial production growth and retail trade growth come from the OECD Monthly Economic Indicators.[11]

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[10] The BBQ algorithm is described in detail in Harding and Pagan (2002).

[11] Alternatively, we could use factor analysis to estimate a common factor x_t from the business cycle indicators. This approach would synthesize a factor-augmented VAR model with the QualVAR of Dueker (2005). Such a framework accounts for the uncertainty in x but adds complexity. We leave this for future research.

All of the data are available for the United States and Canada for the period 1980Q1-2018Q1. For Mexico, real GDP growth and industrial production growth are available from 1980Q1; retail trade growth is available starting in 1986Q1; and employment growth is available starting in 2005Q1. To deal with the unbalanced panel of data for Mexico, we extract the first principal component using probabilistic principal component analysis [see Tipping and Bishop (1999)].

3.3 Identifying the VAR

We identify four structural shocks from the VAR, one being a U.S. monetary policy shock, identified as a shock to the federal funds rate, and the others being the shocks to the business cycle indicator of each of the three countries. We identify these structural shocks using the Cholesky decomposition applied using a specific ordering restriction on the VAR. Specifically, the ordering of the variables is:

$$Y_t = [z_{US,t}, z_{CA,t}, z_{MX,t}, PCE_t, FFR_t, PCOM_t]',$$

which implies that the federal funds rate responds contemporaneously to inflation and the business cycle indicators but not vice versa. Moreover, the causal ordering assumes that shocks to the business cycle variable in the United States affect the business cycle indicator in Canada and Mexico contemporaneously but not vice versa. Finally, the causal ordering assumes that shocks to the Canadian business cycle variable affect the business cycle indicator in Mexico contemporaneously but not vice versa. These identification schemes are consistent with an existing literature studying spillover effects of U.S. monetary policy shocks using VARs, e.g. Kim (2001).[12]

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[12]In results not reported here, we considered an alternative identification of structural shocks based on a mixture of sign and exclusion restrictions. The results are broadly similar to those under the standard recursive identification.

4   Empirical Application

In this section we describe the application of our multivariate Markov-switching model to study the propagation of North American business cycles. Others have previously studied the transmission of U.S. shocks to other countries [see, among many, Kim (2001) and Feldkircher and Huber (2016), who consider the international transmission of U.S. monetary shocks (and others) in VARs]. Here we examine how a shock to the business cycle indicator of each North American country propagates to the probability of a recession in other North American countries. We also study how U.S. monetary policy shocks affect the probability of a recession in Canada and Mexico. We set the lag order of the VAR to $P = 1$.

4.1 S_t Observed Results

We first consider the version of the model with observed recessions. Again, for this experiment, we take recession values from sources external to the model and treat these as given. Figure **1** shows the posterior median and 68% highest posterior density (HPD) interval for the latent continuous recession variables z_{nt}, along with the values of the observed recession indicators shaded in gray. Because the timings of the recessions are taken as data, the signs of the z_{nt}s are deterministic (there are no false positives, etc.); however, the dynamics of the z_{nt}s are produced by the dynamics of the model.

4.1.1   Impulse Responses

Figure **2** shows the responses of each latent business cycle variable to a shock in each of the other business cycle variables. The columns show, respectively, the effects of one-standard-deviation contractionary shocks to the U.S., Canadian, and Mexican latent business cycle indicators. Recall that the shock—a one-standard-deviation increase in z_{nt}—is an adverse shock, pushing the economy closer to or into recession. The rows show the shocks' effects on the U.S., Canadian, and Mexican latent business cycle indicator, respectively.
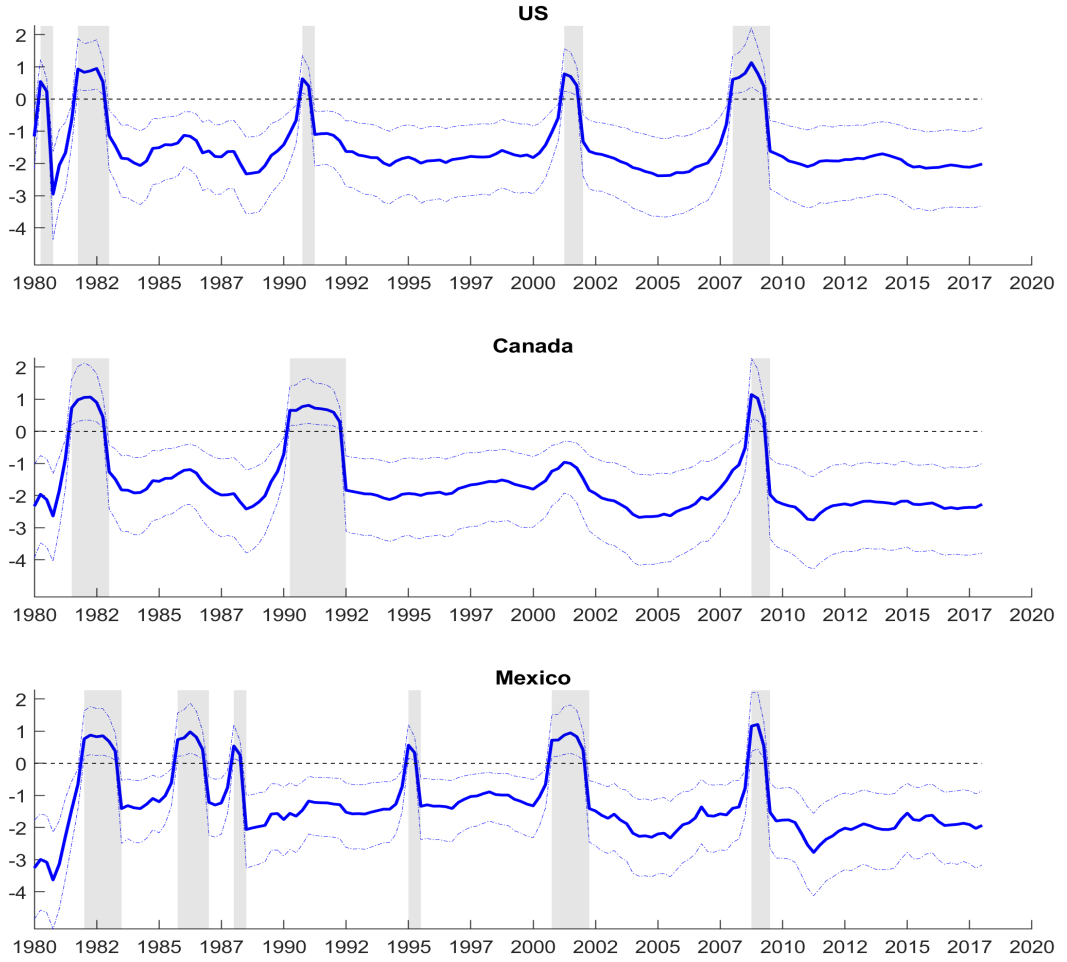
Figure 1: *Continuous Recession Variables.* This figure shows the posterior median (solid line) and the 68% HPD interval (dashed line) of the continuous recession variable z_{nt} when the discrete regime S_{nt} is observed. Gray shading reflects recession dating for each country (U.S. dates from NBER; Canada dates from C.D. Howe Institute; Mexico dates from BBQ estimation).
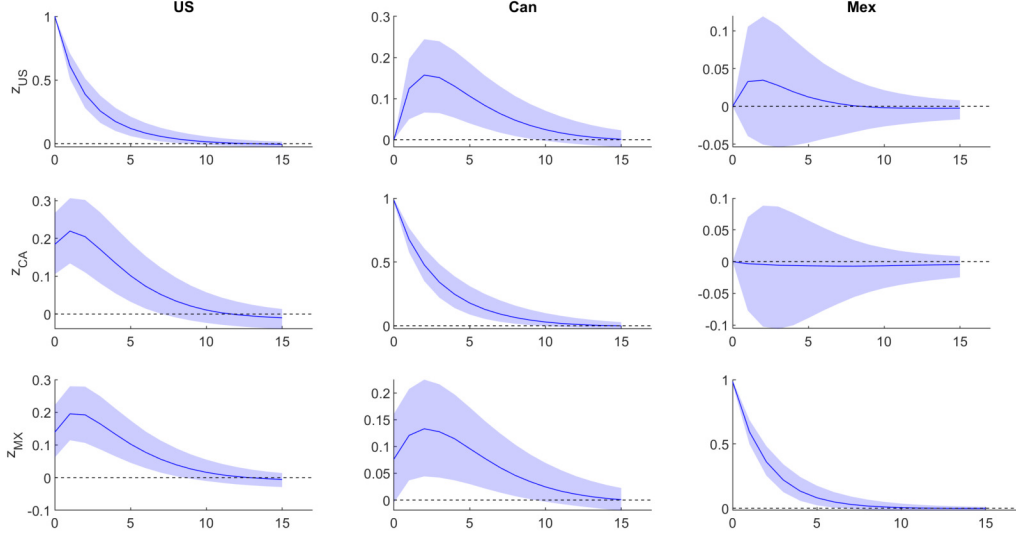
Figure 2: *Impulse Response Functions of z_{mt} to shocks to z_{nt}.* This figure shows the response of each country's response to a shock to the continuous recession variable. The first column shows the response of each of the three countries to a recessionary shock to the U.S. The second and third columns show the individual country responses to Canada and Mexico, respectively. The solid line shows the posterior median response and the shaded region shows the 68% HPD interval.

Overall, a shock to the business cycle indicator produces the expected response: An increase in z_{nt} pushes the domestic economy toward or into recession. Similar results could be obtained from a univariate Markov-switching model; however, our multivariate model also allows us to investigate the cross-country effects of a domestic recession shock. Adverse shocks in the United States have statistically important effects on both the Canadian and Mexican business cycles, raising the likelihood of recessions in both countries. Similarly, an adverse shock in Canada significantly increases the probability of a recession in both the United States and Mexico. However, an adverse economic shock in Mexico does not affect the business cycles of its neighbors in a statistically relevant way—the uncertainty bands contain zero throughout the response horizon.[13]

These results, taken as a whole, are consistent with the literature (e.g., Feldkircher and Huber, 2016) that finds a significant role for trade in transmitting shocks internationally.

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[13]In results not shown here, a shock to the U.S. latent business cycle indicator reduces inflation, the fed funds rate, and commodity price inflation. A shock to Canada's business cycle indicator has qualitatively similar results to the U.S. shock. A shock to Mexico's business cycle variable decreases U.S. inflation but does not affect the federal funds rate or commodity price inflation. These results are available upon request.
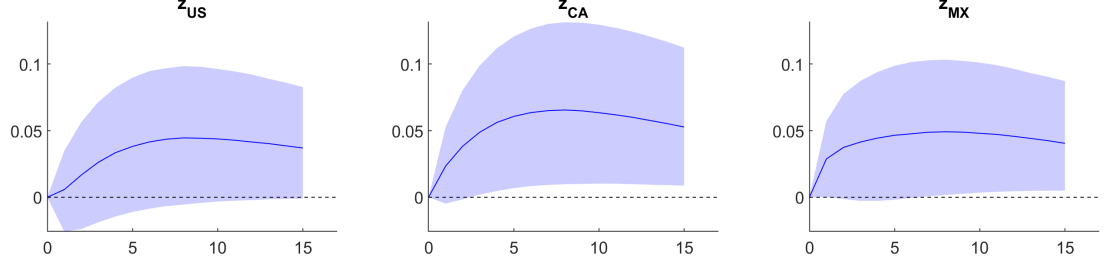
Figure 3: *Impulse Response Functions of z_{mt} to shocks to FFR_t*. This figure shows the response of each country's recession variable to a shock to the shadow rate. The solid line shows the posterior median response and the dashed lines show the 68% HPD interval.

Perhaps not surprisingly, over the sample period trade between Canada and Mexico is relatively small compared to trade between the United States and Canada.[14] Also, trade between the United States and Mexico is a small fraction of U.S. GDP but a large fraction of Mexican GDP.[15]

We find that U.S. business cycle conditions spill over to the U.S.'s neighbors. A next logical question is whether contractionary U.S. policy (i) affects U.S. business cycle conditions and (ii) spills over into its neighbor's business cycle conditions. Figure **3** shows the responses of business cycle conditions, the z_{nt}s, to a one-standard-deviation increase in the federal funds rate.[16] As expected, the U.S. recession variable increases as the policy rate rises, however the effect is not significant since zero is in the relatively wide HPD interval. Additionally, we find that the U.S. monetary policy shocks spill over to both the Canadian and Mexican economies, increasing each of their recession variables in a statistically significant manner.[17]

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[14] Across our sample period 1980 - 2018, the average total trade between Canada and Mexico is 0.13% of Candian GDP whereas total trade between Canada and the United States averages 9.15% of Canadian GDP.

[15] The average total trade between the United States and Mexico over the period 1980 - 2018 was 0.46% of U.S. GDP compared to 4.01% of Mexican GDP.

[16] The interest rate equation has elements of the Taylor rule, including both a measure of economic activity z_{nt} and the inflation rate.

[17] This finding is related to the large literature on the international spillovers of monetary policy. See Claessens, Stracca, and Warnock (2016) for an extensive overview of both the theoretical and empirical research on this topic.

4.1.2 Quantifying the Spillover Effect

We have demonstrated that adverse business cycle shocks spill over across borders in the NAFTA region. Our model also allows us to quantify this response in terms of the change in the probability of a future turning point. However, unlike standard impulse responses, computing these marginal effects requires knowing the conditions at the time of the shock. Moreover, our model differs from the typical probit model because the marginal effects from our model are dynamic. Thus, we cannot simply choose the initial conditions at the time of the shock; we need to account for how sequences of shocks could alter the recession probabilities.

To compute the dynamic marginal effects, we use a technique similar to a generalized impulse response [Koop, Pesaran, and Potter (1996)]. Let $\Omega_t = \{Y_t, \Omega_{t-1}\}$ represent the observed history at time t. The objective is to obtain $p(z_{n,t+h}|\Omega_{t-1})$ and, from this, a conditional probability $\Pr[S_{n,t+h}|\Omega_t]$. On the other hand, a δ-standard-deviation shock to z_{mt} implies increasing Y_t by δb_m (where b_m is the mth column of the Cholesky factorization of Σ), which yields an alternative initial condition, $\Omega_t^*(\delta) = \{Y_t + \delta b_n, \Omega_{t-1}\}$. The dynamic marginal effect is $\Pr[S_{n,t+h}|\Omega_t^*(\delta)] - \Pr[S_{n,t+h}|\Omega_t]$ integrated over a specified set of histories, Ω_t. Because the conditional probabilities are nonlinear functions of the future shocks, we must simulate sets of these future (reduced-form) shocks and apply them to both $\Pr[S_{n,t+h}|\Omega_t^*(\delta)]$ and $\Pr[S_{n,t+h}|\Omega_t]$. The dynamic marginal effect is then $\Pr[S_{n,t+h}|\Omega_t^*(\delta)] - \Pr[S_{n,t+h}|\Omega_t]$, subject to sets of future shocks, averaged over the histories of interest and draws from the parameters. Appendix A.7 outlines this calculation in detail.

In principle, one could integrate over the entire set of histories to get the unconditional dynamic marginal effect. However, we are interested in a particular representative Ω_t of economic interest. We compute the dynamics marginal effects of country m experiencing a large idiosyncratic, adverse shock when all countries are in expansion. That is, we consider the change in the recession probability produced by a three-standard-deviation increase in z_{mt} (i.e., $\delta = 3$) when all three countries are initially in expansion. In our

case, we consider histories Ω_t when all three countries are in expansion at time $t - 1$.[18] We consider very large shocks because we are interested in assessing the probability of changes in the business cycle phase; smaller shocks are unlikely to produce such nonlinear responses.

Because the dynamic marginal effects have similar shapes as the linear impulse responses, we do not illustrate them here; however, they can have asymmetric magnitudes depending on the starting conditions. Because the starting values are set to the expansion average, the initial probability of recession is relatively low for each country when they start in expansion. Thus, the scenario we consider starts with countries securely in expansion and subjects them to large adverse shocks that essentially guarantee a subsequent recession in the domestic economy. We then assess how this shock affects the probability of a recession in the other countries. We are more interested in whether a foreign recession is likely than whether it is more likely to occur in a particular period. Thus, we compute the change in the probability of a recession over the next four periods.

Table 2 presents the posterior medians for the marginal effects. A three-standard-deviation shock to z_{US} increases the probability of a U.S. recession in that quarter by 83.78 percentage points. In that same quarter, the probability of recession rises 8.11 and 9.91 percentage points for Canada and Mexico, respectively. However, the shock to z_{US} propagates across future horizons. The probability of a recession over the next year rises by 16.59 percentage points for Canada and 19.11 percentage points for Mexico.

We find similar effects for an increase in Canada's recession variable. A three-standard-deviation shock to z_{CAN} increases the probability of a recession over the next four quarters by 11.73 percentage points for the United States and 12.87 percentage points for Mexico. A shock to Mexico's recession variable propagates relatively less than shocks to z_{US} and z_{CAN}. A three-standard-deviation shock to z_{MEX} leads to a 3.23 percentage point increase in the probability of recession over the next year for the United States and virtually no effect on the probability of recession in Canada.

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[18]We also considered an alternative scenario where all three countries are in recession and country m experiences an expansionary shock (negative shock to z_{mt}). These results are available upon request.

	Immediate Effect	Effect Over Next 4Q
Shock to z_{US}		
U.S.	83.78 [73.42,90.99]	65.99 [60.74,70.27]
Canada	8.11 [3.60,15.32]	16.59 [8.58,24.25]
Mexico	9.91 [3.60,17.12]	19.11 [11.92,27.68]
Shock to z_{CAN}		
U.S.	–	11.73 [3.33,19.72]
Canada	75.68 [59.91,85.59]	61.79 [53.92,68.84]
Mexico	4.50 [0,9.91]	12.87 [4.55,21.04]
Shock to z_{MEX}		
U.S.	–	3.23 [−2.52,8.94]
Canada	–	−0.05 [−4.71,5.92]
Mexico	83.78 [75.23,89.64]	61.03 [56.70,65.29]

Table 2: *Marginal Effects*: This table shows the posterior median for the marginal effect of a three-standard-deviation shock to z_{nt} on the probability of recession for each of the three countries. The second column shows the marginal increase in the probability of recession in the same quarter as the shock, and the third column lists the marginal effect four quarters after the shock. The brackets underneath the posterior medians provide the 68% HPD interval.

4.2 S_t Unobserved Results

In this section, we report the results of the estimation with the S_{nt}s unobserved. Following Diebold and Rudebusch (1996), we define x_{nt} as the first principal component of real GDP growth, industrial production growth, employment growth, and retail trade growth.[19] Many of the underlying results are similar to those obtained with the S_{nt}s observed. For example, the impulse responses are, as expected, qualitatively similar in both cases. In the interest of brevity, we do not report these results, but they are available upon request from the authors.

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[19]These are the four primary variables that the NBER Business Cycle Dating Committee highlights when determining U.S. business cycle turning points (https://www.nber.org/cycles/recessions.html). Diebold and Rudebusch (1996) show that a Markov-switching model applied to the first principal component of these four series produces an accurate replication of the NBER business cycle dates. See also Chauvet (1998) and Kim and Nelson (1998).
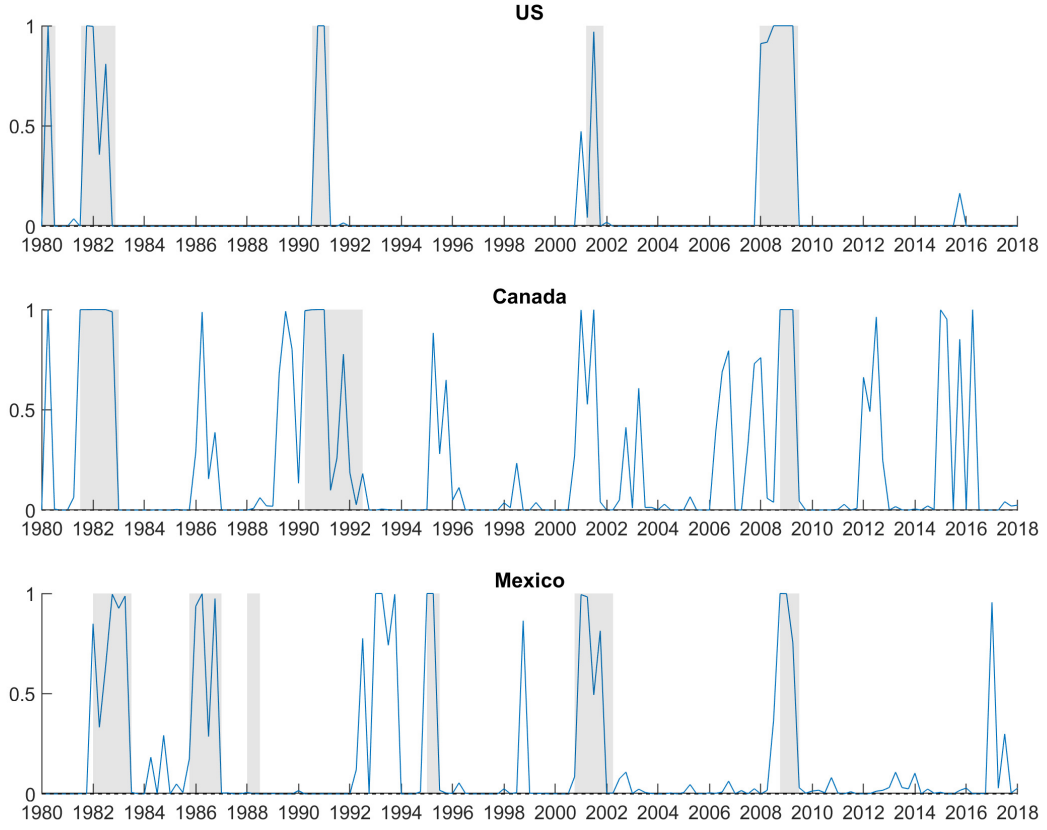
Figure 4: *Posterior Probability of Recession*. This figure shows the posterior probability of recession for each country. Gray shading reflects recession dating for each country (U.S. dates from NBER; Canada dates from C.D. Howe Institute; Mexico dates from BBQ estimation).

The central issue for S_{nt} unobserved is how well the estimated states compare to the observed states. Obviously, in many applications, this comparison would not be available. However, for our application, we do have an objective measure of the states to compare, keeping in mind that the methods and variables used to identify the external recession dates may differ substantially from ours.

Figure 4 shows the mean posterior probability of recession for the three countries along with the shaded recession dates for each country.[20] Because the states are not predetermined, the filter picks up a fair number of false positives and a few false negatives in the middle of recessions. One potential explanation for this result is the relatively higher volatility of the z_{nt} variable in the S_{nt} unobserved case compared with the observed

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[20] We use the same recession dates for each country from the application to when S_t is observed.

S_{nt}.[21]

Another way to evaluate the regime inference in the unobserved S_{nt} model is to compute the area under the receiver operator characteristic curve (AUROC), which measures accuracy by weighing both false positives and false negatives.[22] For reference, a pure coin flip would have an AUROC of 0.5 and larger AUROC suggests more accurate regime inference. Comparing the unobserved S_{nt} model with the observed S_{nt} model is uninformative; however, we can compare our model with a simple univariate, constant transition probability Markov-switching model. Table 3 displays the AUROC for both our model (OPS) and the univariate Markov-switching model (MS). For each country, the contagious switching model correctly identifies a large proportion of the business cycle dates, represented by an AUROC of greater than 0.90. While the univariate model does marginally better than our model for the United States and Canada, our model does better for Mexico. This suggests that accounting for information about U.S. and Canadian recessions that may propagate to Mexico helps identify recessions south of the border.

Table 3: AUROC

Country	OPS	MS
U.S.	0.955	0.975
Canada	0.926	0.944
Mexico	0.910	0.889

5 The Effect of NAFTA

Armed with a model of cross-country business cycle propagation, we investigate whether trade liberalization altered the transmission of business cycles across borders [see also Kose, Prasad, and Terrones (2003)]. A number of studies [for example, Burfisher, Robinson, and Thierfelder (2001), Miles and Vijverberg (2011), among others] have attempted

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[21] One solution to these problems could be to allow the intercept term in the VAR to switch as a function of S_{nt}. While this would introduce a number of complications, it would allow the persistence of the recession and expansion regimes to be different.

[22] The receiver operator characteristic curve plots the true positive rate against the false positive rate. Because the model output is a posterior recession probability, the ROC curve varies the threshold at which the probability is classified as a positive outcome. See Berge and Jordà (2011) for more details.
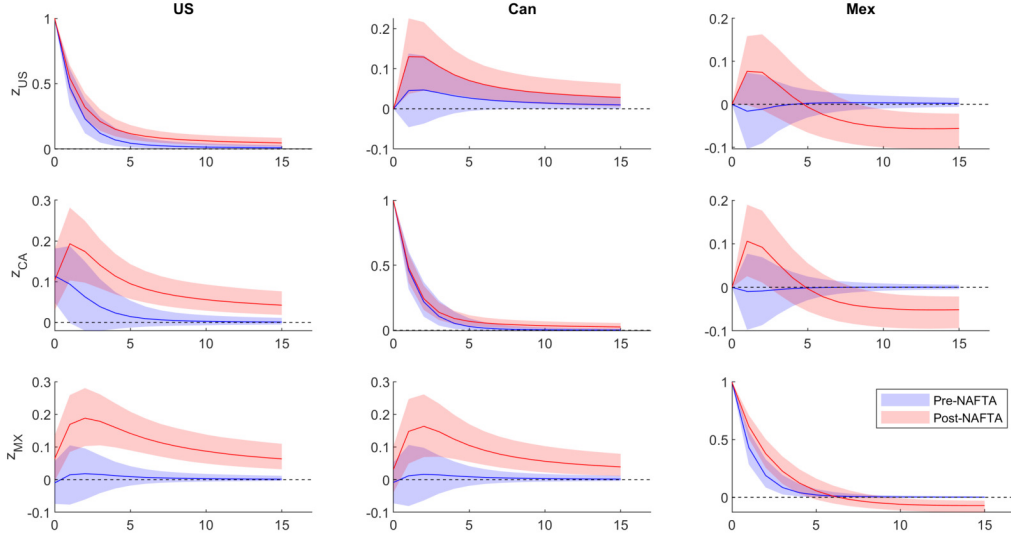
Figure 5: *Impulse Response Functions of z_{mt} to shocks to z_{nt} Using Pre- and Post-NAFTA Samples.* This figure shows the response of each country's response to a shock to the continuous recession variable based on pre-NAFTA sample (1980Q1 - 1993Q4) and post-NAFTA sample (1994Q1 - 2018Q1). The solid line shows the posterior median response and the shaded regions show the 68% HPD interval.

to evaluate the effects of NAFTA, which liberalized trade between the U.S., Canada, and Mexico. From 1990 (before NAFTA) to 2015 (well after NAFTA was enacted), total trade volume between the three countries rose from $333 billion to $2.137 trillion. In 1990 before NAFTA, the correlations between the United States and Canada, the United States and Mexico, and Mexico and Canada GDP growth rates were 0.87, -0.02, and 0.12, respectively; in 2015, those correlations were 0.78, 0.63, and 0.54, respectively.

To account for NAFTA, we re-estimate the model imposing a break in 1994, the time of NAFTA's implementation. That is, we estimate the model using two separate subsamples: (i) Pre-NAFTA 1980Q1 - 1993Q4 and (ii) NAFTA 1994Q1 - 2018Q1.[23] We present results from the model with an unobserved business cycle state; results with S_t is observed are qualitatively similar but with sharper inference because of reduced uncertainty and larger spillovers.

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[23]In addition to NAFTA, other changes, particularly to the U.S. economy, occurred with roughly similar timing. For example, the Great Moderation started in 1984 (McConnell and Perez-Quiros, 2000); monetary policy transparency increased in 1994 (Swanson, 2006); and labor productivity surged in the late 1990s (Fernald, 2015). Thus, our result that NAFTA changed business cycle spillovers is suggestive rather than conclusive.

Similar to Figure 2, each row of Figure 5 shows the response of a country's business cycle indicator to a one-standard-deviation increase in the business cycle indicator for the country indicated by the column. The figure shows the responses for both the pre-NAFTA and NAFTA periods in blue and red, respectively.[24] Before the trade agreement, the only significant spillover effect comes from the United States to Canada.

After the trade agreement, we find that recessions spread across all three nations. The magnitude of the recession pass-through from the United States to Canada in the two subsamples is similar, but becomes statistically significant for a longer horizon only after NAFTA is enacted. NAFTA's effect on the pass-through of recession from the United States to Mexico is substantially larger. The differences in the changes in magnitudes for the transmission of U.S. shocks to Canada and Mexico are consistent with the effect of trade liberalization. While NAFTA increased the trade volume between the United States and Canada, its effects on U.S.-Mexico trade volume was on the order of three times larger over the same period. After the enactment of NAFTA, shocks originating in Canada and Mexico are more effectively transmitted to the United States. Similarly to shocks originating in the United States, the differences are more substantial for the U.S.-Mexico relationship than for the U.S.-Canada relationship.

In most of the other cases, the median response of z_{mt} to a shock to z_{nt} is larger after NAFTA went into effect. In particular, we find that Canadian shocks significantly transmit to Mexico and some evidence that Mexican shocks spread (albeit not significantly) to Canada in the NAFTA period. While NAFTA did substantially increase the trade volume between the two countries, the total trade volume between Canada and Mexico is still only a small fraction of the trade between the United States and its neighbors.

In order to interpret this change in the degree of spillovers pre-NAFTA and NAFTA, we contextualize them by calculating the dynamic marginal effects as we did in Section 4.1.2. Table 4 displays the marginal effects of a three-standard-deviation shock to z_n

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[24]In Figure 5, the NAFTA period includes the Great Recession. Previous studies have suggested that the Great Recession substantially increased business cycle synchronization. We considered a NAFTA period that excluded the Great Recession and found qualitatively similar results for the relationship between the United States and Mexico. However, excluding the Great Recession results in no recessions for Canada during the NAFTA period. These results are available upon request.

	Pre-NAFTA	Post-NAFTA
Shock to z_{US}		
Canada	9.89 [3.12,17.86]	18.95 [11.70,25.46]
Mexico	1.58 [−5.11,6.53]	18.90 [10.90,27.75]
Shock to z_{CAN}		
U.S.	5.70 [−3.44,15.13]	14.19 [4.33,20.63]
Mexico	0.81 [−5.29,6.39]	14.49 [6.27,24.46]
Shock to z_{MEX}		
U.S.	−0.73 [−9.05,6.95]	7.67 [−0.22,16.45]
Canada	−1.70 [−8.73,5.56]	8.78 [0.54,16.69]

Table 4: *Marginal Effects Pre- and Post-NAFTA*: This table shows the posterior medians for the marginal effect of a three-standard-deviation shock to z_{nt} on the probability of recession over the next four quarters for each country. The second column shows the estimated effects using the pre-NAFTA sample (1980Q1 - 1993Q4), and the third column shows the effects under the post-NAFTA sample (1994Q1 - 2018Q1). The brackets underneath the posterior medians provide the 68 % HPD interval.

on the probability of recession over the next four quarters for the other two countries. Before NAFTA, the cross-country spillovers of a U.S. recessionary shock are relatively small, with only a 9.89 percentage point increase for Canada and a 1.58 percentage point increase for Mexico. A negative shock to Canada has similar effects on the other two countries. Shocks to Mexico's recessionary variable have essentially no marginal effect on the one-year probability of recession for the United States or Canada.

After NAFTA went into effect, we find a substantial change in the degree of propagation. A three-standard-deviation shock to z_{US} increases the probability of recession over the next year in Canada and Mexico by 18.95 percentage points and 18.90 percentage points, respectively. Similarly, adverse shocks to the Canadian economy have similar spillover effects on the probability of recession in the United States or Mexico. Lastly, a shock to z_{MEX} affects the recession probabilities for both the United States and Canada after the trade agreement.

Consistent with a number of previous studies, we conclude that trade liberalization

has affected how business cycles transmit across borders. However, our results suggest that the transmission between the United States, Canada, and Mexico does not occur contemporaneously. Thus, computing simple correlations between measures of GDP may not tell the whole story behind business cycle synchronization. Moreover, the effect is bilateral, suggesting that transmission is not influenced only by the size of the U.S. economy.

Our results may have important implications when analyzing the costs and benefits of free trade agreements. In particular, Caliendo and Parro (2015) estimate the overall welfare gains of NAFTA's tariff reductions were relatively small on the member countries. Similarly, Auer, Bonadio, and Levchenko (2020) find that revoking NAFTA would lead to a 0.2 percent reduction in U.S. welfare and a considerably larger effect on Canada and Mexico. Policymakers should properly weigh the welfare benefits of free trade agreements with the cost of potentially increased volatility from foreign-country spillovers implied by our model's findings.

6   Conclusions

In this paper, we have developed a multivariate time-varying transition probability Markov-switching model in which the state of the business cycle in one country can affect the current and future state of the business cycle in other countries. The model structure nests a VAR, which allows us to evaluate the within- and cross-country effects of macroeconomic shocks. We show how the model can be estimated using both observed or unobserved business cycle phases. We apply the model to the United States, Canada, and Mexico and find that there is a propensity for cycles to propagate across borders. Additionally, we find that U.S. monetary policy shocks affect the recession probabilities of both Canada and Mexico.

We then consider whether trade liberalization affected the propagation of business cycles. We estimate the model for subsamples before and after a predetermined NAFTA break and find that recessions did not propagate pre-NAFTA. However, after NAFTA,

adverse shocks originating in any of the three nations spread to the other two with the exception that shocks to Mexico do not transmit to Canada. This provides suggestive evidence that trade liberalization increases the degree of business cycle synchronization.

Finally, it is worth noting that our model, in principle, can be used to track the propagation of any set of binary outcomes. For example, a version of the model could be developed to track contagion effects in bank failures, or other instances of financial crises.

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A Technical Appendix

The following appendix describes in detail the draws for the parameters and the calculation of the dynamic marginal effects. We first outline the state space representation of the VAR. We then describe the two draws that are invariant to whether we observe the regime. These draws condition only on the continuous latent state, z_t. We then describe the draw for the continuous latent variable z_t when S_t is observed. Th next subsection describe the draws for the parameters in the measurement equation that relates the discrete regime to the growth variable, x_t. Finally, we outline how to calculate the dynamic marginal effects.

A.1 The State Space Representation

Recall that z_t is $(N \times 1)$ and y_t is $(J \times 1)$ and let $K = N + J$. Define $Y_t = [z_t', y_t']'$ and $\varsigma_t = \left[Y_t', Y_{t-1}', \cdots, Y_{t-P+1}' \right]'$ as the state in the state-space representation of the model with measurement equation:

$$y_t = H\varsigma_t,$$

and transition equation:

$$\varsigma_t = M + F\varsigma_{t-1} + e_t,$$

where

$$e_t \sim N(\mathbf{0}_{KP \times 1}, Q).$$

The parameters of the state space are defined as follows:

$$Q = \begin{bmatrix} \Sigma & \mathbf{0}_{K \times K(P-1)} \\ \mathbf{0}_{K(P-1) \times K(P)} \end{bmatrix},$$

$$H = \begin{bmatrix} \mathbf{0}_{J \times N} & \mathbf{I}_J & \mathbf{0}_{J \times K(P-1)} \end{bmatrix},$$

$$M = \begin{bmatrix} B_0 \\ \mathbf{0}_{K(P-1)\times 1} \end{bmatrix},$$

and

$$F = \begin{bmatrix} B_1 & B_2 & \cdots & B_{P-1} & B_P \\ \boldsymbol{I}_K & \mathbf{0}_{K\times K} & \cdots & \mathbf{0}_{K\times K} & \mathbf{0}_{K\times K} \\ \mathbf{0}_{K\times K} & \boldsymbol{I}_K & \cdots & \mathbf{0}_{K\times K} & \mathbf{0}_{K\times K} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0}_{K\times K} & \mathbf{0}_{K\times K} & & \boldsymbol{I}_K & \mathbf{0}_{K\times K} \end{bmatrix}.$$

Notice that the measurement equation is deterministic.

A.2 Drawing B conditional on Σ, $\{z_\tau\}_{\tau=1}^{t-1}$

Conditional on Σ, the VAR parameters B are conjugate Normal. Define $\varsigma_t = \left[Y_t', Y_{t-1}', \cdots, Y_{t-P+1}'\right]'$. Then, the VAR can be written as:

$$Y_t = Bx_t + u_t,$$

where $x_t = \left[1, Y_{t-1}', ..., Y_{t-P}'\right]'$. Stacking the observations, we get:

$$Y = XB' + U$$

where $Y = [Y_{P+1}, Y_{P+2}, ..., Y_T]'$ and $X = [X_{P+1}, X_{P+2}, ..., X_T]'$. Let $\hat{B}' = (X'X')^{-1}(X'Y)$ be the OLS estimates for B'.

We assume a Minnesota prior for $B = [B_0, B_1, ..., B_P]$. The prior distribution for $b = vec(B')$ is $b \sim N(\bar{b}_0, \bar{B}_0)$ where $\bar{b}_0 = \mathbf{0}_{K(P+1)\times 1}$ and the diagonal elements of $\bar{B}_0$ are

set according to:

$$var\left(B_{l,ij}\right) = \begin{cases} \left(\frac{\omega_1}{p^{\lambda_3}}\right)^2 & \text{if } l > 0 \text{ and } i = j \\ \left(\frac{\hat{s}_i \omega_1 \omega_2}{\hat{s}_j p^{\omega_3}}\right)^2 & \text{if } l > 0 \text{ and } i \neq j \\ \left(\hat{s}_i \omega_4\right)^2 & \text{if } l = 0 \end{cases}.$$

where $\hat{s}_j$ is the standard deviation of residuals from a univariate, first-order autoregression of y_{jt} and we set the hyperparameters to $\omega_1 = 0.2$, $\omega_2 = 0.5$, $\omega_3 = 2$, and $\omega_4 = 0.20$. For the model where S is unobserved we set a slightly tighter prior of $\omega_4 = 0.05$. Define $\hat{b} = vec(\hat{B}')$. The posterior distribution for b is:

$$b \sim N(\bar{b}_1, \bar{B}_1),$$

where

$$b_1 = B_1^{-1}(\bar{B}_0^{-1}\bar{b}_0 + \Sigma^{-1} \otimes X'X\hat{b}),$$

$$B_1 = (\bar{B}_0^{-1} + \Sigma^{-1} \otimes X'X)^{-1}.$$

We redraw B if the usual stationarity condition is violated.

A.3 Drawing Σ conditional on B, $\{z_t\}_{t=1}^{T}$

The draw of Σ is nonstandard due to the restrictions placed on the variance parameters of the latent variables z_{nt} for $n = 1, ... N$. Specifically we restrict the variance of each z_{nt} to 1. We adopt the algorithm outlined by Chan and Jeliazkov (2009) for drawing restricted covariance matrices. This method takes advantage of the decomposition $\Sigma = L'^{-1}DL^{-1}$ where

$$D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{N+J} \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ a_{21} & 1 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{N+J1} & a_{N+J2} & \cdots & \cdots & 1 \end{bmatrix}.$$

As noted by Chan and Jeliazkov (2009), the first N diagonal parameters $\lambda_1, ..., \lambda_N$ are deterministic functions of the a_{ij} parameters of the lower triangular matrix due to the identification restrictions placed on z_{nt}. Therefore $\lambda_1, ..., \lambda_N$ and the a parameters must be drawn jointly via a Metropolis-Hastings step. However the remaining diagonal parameters $\lambda_{N+1}, ..., \lambda_{N+J}$ can be drawn unrestricted via a Gibbs sampling step.

We first outline the prior distributions. For λ_k for $k = N + 1, ..., N + J$, we assume the following prior:

$$\lambda_k \sim IG\left(\frac{v_{k0}}{2}, \frac{\delta_{k0}}{2}\right),$$

where $v_{k0} = 20$ and δ_{k0} is set so that the prior mean for λ_k equals $\hat{s}_j^2$, which implies $\frac{\delta_{k0}}{2} = \hat{s}_k^2\left(\frac{v_{k0}}{2} - 1\right)$. For $\mathbf{a}_k = [a_{k1}, ..., a_{k,k-1}]'$ for $k = 2, ..., N + J$, we assume the prior:

$$\mathbf{a}_k \sim N\left(\mathbf{a}_{k0}, \mathbf{A}_{k0}\right).$$

The algorithm for drawing all of the parameters governing Σ is:

Step 1: Draw $\lambda_{N+1}, ..., \lambda_{N+J}$ from equation (2.5) from Chan and Jeliazkov (2009):

$$\lambda_k \sim IG\left(\frac{v_{k0} + T}{2}, \frac{\delta_{k0} + s_k}{2}\right),$$

where s_k is the (k, k)-element of $\sum_{t=1}^{T} w_t w_t'$ and $w_t = Lu_t$. In our application, $u_t = Y_t - Bx_t$ where $x_t = [1, Y_{t-1}', ..., Y_{t-P}']'$.

Step 2: We draw a candidate $\mathbf{a}_k^c$ from the multivariate t-distribution outlined in equation (3.7) of Chan and Jeliazkov (2009):

$$\mathbf{a}_k^c \sim MVT\left(\mathbf{a}_k, \tau\mathbf{A}_k, \kappa\right),$$

where $\mathbf{A}_k = \left(A_{k0}^{-1} + \lambda_k^{-1} X_k' X_k\right)^{-1}$, $\mathbf{a}_k = \mathbf{A}_k \left(A_{k0}^{-1} a_{k0} - \lambda_k^{-1} X_k' z_k\right)$, $z_k = [u_{1k}, ..., u_{Tk}]'$, $X_k = [z_1, ..., z_{k-1}]$, u_{tk} is the k-th element of u_t, τ is a tuning parameter, and κ represents the degrees of freedom. Based on the candidate draw $\mathbf{a}_k^c$ for $k = 2, ..., N+J$, we compute the associated candidates for the diagonal parameters $\lambda_1^c, ..., \lambda_N^c$ using (3.3) and (3.4) from Chan and Jeliazkov (2009):

$$\lambda_1^c = 1,$$

$$\lambda_k^c = 1 - \sum_{j=1}^{k-1} (a^{kj})^2 \lambda_j,$$

where a^{kj} is the (k, j)-element of the lower triangular matrix L^{-1}. The candidate draw is accepted with probability:

$$\alpha = \min \left\{ 1, \frac{l\left(Y \mid \Sigma^c\right) \prod_{k=2}^{N+J} f_T\left(\mathbf{a}_k^c \mid \mathbf{a}_{k0}, \tau \mathbf{A}_{k0}, \kappa\right) f_N\left(\mathbf{a}_k^{i-1} \mid \mathbf{a}_k, \mathbf{A}_k\right)}{l\left(Y \mid \Sigma^{i-1}\right) \prod_{k=2}^{N+J} f_T\left(\mathbf{a}_k^{i-1} \mid \mathbf{a}_{k0}, \tau \mathbf{A}_{k0}, \kappa\right) f_N\left(\mathbf{a}_k^c \mid \mathbf{a}_k, \mathbf{A}_k\right)} \right\}$$

where $f_T(\cdot)$ is the multivariate t density. If the candidate is accepted, we use $\lambda_1^c, ..., \lambda_N^c$ and $\mathbf{a}_2^c, ..., \mathbf{a}_{N+J}^c$ with the draw from Step 1 for $\lambda_{N+1}, ..., \lambda_{N+J}$ to calculate the new covariance matrix $\Sigma = L'^{-1} D L^{-1}$. Otherwise, we use the previous draw $\lambda_1^{i-1}, ..., \lambda_N^{i-1}$ and $\mathbf{a}_2^{i-1}, ..., \mathbf{a}_{N+J}^{i-1}$ to calculate the covariance matrix.

A.4 Drawing z_t conditional on B, Σ, and observed $\{S_\tau\}_{\tau=1}^T$

We implement the Kalman filter with smoothing to draw the vector z_t given the state vector $S_t = [S_{1t}, ..., S_{Nt}]'$. If the sign of the draw for z_t does not match the state implied by S_t, we redraw until the condition is met. Since Q is singular, we use the modification outlined by Kim and Nelson (1999) that simplifies the backwards smoother to only the relevant conditioning factors.

A.5   Drawing $\Psi = \{\mu_0', \mu_1', \phi'\}$ conditional on $\{S_\tau\}_{\tau=1}^T$ and $\{\sigma_n^2\}_{n=1}^N$

A.5.1   Drawing $\mu_n = [\mu_{n0}, \mu_{n1}]'$ given S_n, σ_n^2, and ϕ_n

We first define:

$$\tilde{x}_{nt} = \frac{x_{nt} - \phi_n(L)x_{nt-1}}{\sigma_n},$$

$$\tilde{S}_{nt} = \left[\frac{1 - S_{nt}}{\sigma_n}, \frac{S_{nt}}{\sigma_n}\right],$$

$$\tilde{S}_n = \left[\tilde{S}_{n1}^{\,\backprime}, ..., \tilde{S}_{nT}^{\,\backprime}\right]^{\backprime}.$$

Assuming a normal prior distribution $\mu_n \sim N\left(m_{n0}, M_{n0}\right)$, we draw the regime-specific growth parameters from

$$\mu_n \sim N\left(m_{n1}, M_{n1}\right),$$

where

$$M_{n1} = \left(M_{n0}^{-1} + \tilde{S}_n'\tilde{S}_n\right)^{-1},$$

$$m_{n1} = M_{n1}\left(M_{n0}^{-1}m_{n0} + \tilde{S}_n'x_n\right).$$

A.5.2   Drawing ϕ_n given S_n, σ_n^2, and μ_n

Similar to the draw for μ_n, we define:

$$\breve{x}_{nt} = \frac{x_{nt}}{\sigma_n} - \tilde{S}_{nt}\mu_n,$$

$$\breve{x}_n^T = [\breve{x}_{n,p+1}, .., \breve{x}_{n,T}],$$

and $\breve{X}_n^T$ as the $[p \times (T-p)]$ matrix containing the p lags of $\breve{x}_n^T$. Then assuming the prior distribution $\phi_n \sim N\left(p_{n0}, P_{n0}\right)$ and the roots of $1 - \phi_n(L)$ fall outside the unit circle, we have the following posterior distribution:

$$\phi_n \sim N\left(p_{n1}, P_{n1}\right),$$

where

$$P_{n1} = \left(P_{n0}^{-1} + \breve{X}_n^{T\prime} \breve{X}_n^T \right)^{-1},$$

$$p_{n1} = P_{n1} \left(P_{n0}^{-1} p_{n0} + \breve{X}_n^{T\prime} \breve{x}_{nt} \right).$$

A.6   Drawing $\left\{ \sigma_n^2 \right\}_{n=1}^N$ conditional on $\{S_t\}_{\tau=1}^T$ and $\Psi = \{\mu_0', \mu_1', \phi'\}$

The error variance for the business cycle process is drawn from the following posterior distribution:

$$\sigma_n^{-2} \sim \Gamma \left(\frac{\upsilon_{n0} + T}{2}, \frac{\tau_{n0} + \sum_{t=1}^T \hat{\varepsilon}_{nt}^2}{2} \right),$$

where

$$\hat{\varepsilon}_{nt} = x_{nt} - \mu_{n0} - \Delta\mu_n S_{nt} - \phi(L)x_{nt-1}.$$

A.7   Calculating the Dynamic Marginal Effects

Our goal is to measure the increase in the probability of recession h periods ahead caused by a structural shock today. Specifically, we want the dynamic marginal effect:

$$DME_{nh}(\delta, \Omega_{t-1}, \Theta) = Pr(S_{n,t+h} = 1 | v_t = \delta, \Omega_t, \Theta) - Pr(S_{n,t+h} = 1 | v_t = \mathbf{0}, \Omega_t, \Theta), \quad (3)$$

where δ is the structural shock of interest, Ω_t is a particular history $\{Y_t, \Omega_{t-1}\}$, and Θ is a particular Gibbs draw of the parameters and latents. In order to estimate this dynamic marginal effect, we use the following algorithm which is similar to the calculation of generalized impulse responses from Koop, Pesaran, and Potter (1996):

1. Given a history Ω_t and parameter draw Θ, compute:

$$\hat{Y}_t^\delta = Y_t + A_0 v_t,$$

$$\hat{Y}_t^0 = Y_t,$$

where A_0 represents the Cholesky factorization of Σ.

2. Iterate forward for each horizon $h = 1, ..., H$ using $q = 1, ..., Q$ different paths of

future reduced-form shocks $\boldsymbol{u}^q = \{u_{t+1}^q, ..., u_{t+h}^q\}$:

$$\hat{Y}_{t+h}^{\delta,q} = B_0 + B_1 \hat{Y}_{t+h}^{\delta,q} + u_{t+h}^q$$

$$\hat{Y}_{t+h}^{0,q} = B_0 + B_1 \hat{Y}_{t+h}^{0,q} + u_{t+h}^q$$

where $u_\tau \sim N(0, \Sigma)$, $\hat{Y}_t^{\delta,q} = \hat{Y}_t^\delta$ for all q, and $\hat{Y}_t^{0,q} = \hat{Y}_t^0$ for all q. This step provides us with Q future paths of $\hat{z}_{n,t+h}$ under two scenarios: the structural shock at time t is either $v_t = \delta$ or $v_t = \mathbf{0}$.

3. For each scenario, integrate over the various shock paths to get:

$$Pr(S_{n,t+h} = 1 | v_t, \Omega_t, \Theta) = \frac{1}{Q} \sum_{q=1}^{Q} I(\hat{z}_{n,t+h} \geq 0 | v_t, \Omega_t, \Theta, \boldsymbol{u}^q)$$

where $I(\cdot)$ is the indicator function. Plug this into (3) to get $DME_{nh}(\delta_m, \Omega_t, \Theta)$.

4. Average over the K relevant histories $\{\Omega_t^1, ..., \Omega_t^K\}$ to get:

$$DME_{nh}(\delta, \Theta) = \frac{1}{K} \sum_{k=1}^{K} DME_{nh}(\delta, \Omega_t^k, \Theta)$$

5. Repeat steps 1 - 4 for each Gibbs draw Θ to get the posterior distribution of $DME_{nh}(\delta)$.