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Optimal Monetary Policy for the Masses*

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Abstract

We study nominal GDP targeting as optimal monetary policy in a simple and stylized model with a credit market friction. The macroeconomy we study has considerable income inequality, which gives rise to a large private sector credit market. There is an important credit market friction because households participating in the credit market use non-state contingent nominal contracts (NSCNC). We extend previous results in this model by allowing for substantial intra-cohort heterogeneity. The heterogeneity is substantial enough that we can approach measured Gini coefficients for income, financial wealth, and consumption in the U.S. data. We show that nominal GDP targeting continues to characterize optimal monetary policy in this setting. Optimal monetary policy repairs the distortion caused by the credit market friction and so leaves heterogeneous households supplying their desired amount of labor, a type of "divine coincidence" result. We also further characterize monetary policy in terms of nominal interest rate adjustment.

Keywords: Optimal monetary policy, life cycle economies, heterogeneous households, credit market participation, nominal GDP targeting, non-state contingent nominal contracting, inequality, Gini coefficients.

JEL codes: E4, E5.

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1 Income inequality and monetary policy

1.1 Overview

Can monetary policy be conducted in a way that benefits all households in a world with substantial heterogeneity? Does monetary policy have an important impact on the distribution of consumption, income, and financial wealth among heterogeneous households? The spirit of many modern models of monetary policy is that such questions can be pushed into the background via an assumption of a representative household, at least if one is primarily interested in studying only the aggregate implications of monetary policy. However, recent heterogeneous agent models of monetary policy surveyed by Galì (2018, p. 102) seem to "... argue that the representative household assumption is less innocuous [than] it may appear."

In this paper we build a simple and stylized model with substantial heterogeneity in order to help provide some perspective on these questions. We follow recent work by Sheedy (2014), Koenig (2013), Azariadis, Bullard, Singh, and Suda (2018), and Bullard and Singh (2019). These papers all provide analyses of economies where the household credit market plays a key role and where that market is subject to a friction: non-state contingent nominal contracting (NSCNC). Optimal monetary policy is characterized as a version of nominal GDP targeting. The role of monetary policy is to provide a type of insurance to private sector credit markets.

1.2 What we do

In this paper, we scale up the natural household heterogeneity ordinarily present in this model to begin to approach the Gini coefficients in the U.S. data corresponding to the actual degree of income, financial wealth, and consumption inequality. We do this in a way that maintains the simple and stylized structure of the model so that the equilibrium can still be calculated with "pencil and paper" methods, even in the presence of an aggregate shock and a rich net asset-holding distribution.

More specifically, we use a version of the 241-period general equilibrium life-cycle framework of Bullard and Singh (2019) extended to include heterogeneous life-cycle productivity. Agents have homothetic preferences defined over consumption and leisure choices and are randomly assigned any one of a continuum of possible productivity profiles as they enter the economy. This creates substantial intra-cohort heterogeneity in addition to the inter-cohort heterogeneity emphasized in previous papers. We keep this increased heterogeneity manageable, indexing it to a single parameter, allowing for the complete characterization of optimal monetary policy even in this case.

1.3 Main finding

Our main finding is that nominal GDP targeting continues to characterize the optimal monetary policy in the economy with the "massive" heterogeneity as we have introduced it. The key aspect of policy continues to be countercyclical price level movements. The optimal monetary policy repairs the distortion caused by the NSCNC friction: Despite the substantial heterogeneity, all households benefit from smoothly operating credit markets. In the equilibrium we study, households with the same life-cycle productivity will consume the same amount at each date, a version of the hallmark result in this literature that credit markets under optimal policy are characterized by "equity share" contracting.[1] Equity share contracting is known to be optimal when preferences are homothetic.

An important aspect of the model equilibrium that makes results particularly transparent is that the real rate of interest in the economy under optimal monetary policy will always be exactly equal to the (stochastic) rate of growth of aggregate output. This can be interpreted as the Wicksellian natural rate of interest, and thus an important outcome from the model is that the optimal policy is in this respect very similar to the optimal policy that would arise in a representative agent New Keynesian setting.[2]

With respect to labor supply, the model predicts that under the optimal monetary policy, labor supply choices would depend on the stage of the life cycle and the household-specific productivity profile alone and not on the aggregate shock. This is a version of the result in Bullard and Singh (2019), suitably adapted to the case of heterogeneous household productivity profiles. It is a "divine coincidence" result in that the one friction in the model is completely mitigated through appropriate monetary policy, and households are therefore able to make their optimal labor supply choices. Hours worked by various households would be heterogeneous by age but insensitive to the aggregate shock in this equilibrium.

1.4 Recent related literature

Our paper is most closely related to a relatively recent literature on monetary policy with a NSCNC friction. Koenig (2013), for example, shows that a version of nominal GDP targeting would provide an optimal approach to monetary policy in a two-period model with two households, household credit and a NSCNC friction. Sheedy (2014) provides an extensive discussion of the NSCNC friction and nominal GDP targeting as a mitigant to that friction. Sheedy (2014) also provides a model that includes both a NSCNC friction as well as a sticky price friction. His calibrated economy suggests that the NSCNC friction is about nine times more important than the sticky price friction.

[1] See especially Sheedy (2014) for a discussion.

[2] Koenig (2012) explores the connections between nominal GDP targeting and conventional Taylor-rule approaches to monetary policy.

Bullard (2014) and Werning (2014) provide remarks on Sheedy (2014) and emphasize ideas about how the results may or may not apply to economies with additional heterogeneity. Azariadis, Bullard, Singh, and Suda (2018) extend the Koenig and Sheedy findings to an explicit life-cycle structure[3] and focus on issues related to the effective lower bound on nominal interest rates. They assume inelastic labor supply. Bullard and Singh (2019) study a closely related economy with elastic labor supply and find that hours worked would be heterogeneous by cohort but independent of the aggregate shock under nominal GDP targeting. The current paper adds substantial intra-cohort heterogeneity to a version of the Bullard and Singh (2019) framework.

The literature on monetary policy and heterogeneous households has been expanding in recent years. Galì (2018) summarizes many of the papers that have maintained New Keynesian features but added heterogeneous agents in the Aiyagari-Bewley tradition. The resulting equilibrium dynamics have been studied by Auclert (2019), Kaplan, Moll, and Violante (2018), and Bhandari, Evans, Golosov, and Sargent (2018). A broad theme in these papers is that it appears to be problematic to conduct an easily identifiable monetary policy (such as a Taylor-type monetary policy rule) in these settings and claim that it is optimal or close to optimal.[4] In contrast, in the present paper we maintain a high degree of heterogeneity in the sense of equilibrium Gini coefficients but limit the form of idiosyncratic risk that households face. In this setting we can argue that an easily identifiable monetary policy (nominal GDP targeting) does have a claim to optimality thanks to the equity share contracting it facilitates.

Our paper represents a departure from some of the heterogeneous agent New Keynesian literature in two dimensions. First, these papers often have a sticky price friction instead of the NSCNC friction studied in this paper. Second, a hallmark of papers in much of the heterogeneous agent monetary policy literature is that agents face uninsurable idiosyncratic labor income risk in every period. We instead include uninsurable intra-cohort heterogeneity via randomly assigned heterogeneous life-cycle productivity profiles as agents enter the model. We view these profiles as a stand-in for an exogenous, unmodeled human capital accumulation process, including schooling, parenting, and possibly other types of training that agents may be exposed to prior to entering our analysis.

Debortoli and Galì (2018) and Galì (2018) comment on two-agent New Keynesian models as a parsimonious representation of more complicated forms of heterogeneous

[3]See also Braun and Oda (2015), Eggertson, Mehrotra, and Robbins (2018), Galì (2014), Sheedy (2018), and Sterk and Tenreyro (2018) for recent studies of monetary policy in life-cycle settings. As Galì (2018) stresses, in these models the real interest rate may be importantly influenced by monetary policy in ways that are difficult to replicate in representative agent settings.

[4]Werning (2014) studies cases where incomplete-markets settings do have a clear correspondence with representative agent settings. McKay, Nakamura, and Steinsson (2016) study relatively standard "forward guidance" classes of monetary policy and find attenuated effects relative to the representative agent alternative.

agent economies with monetary policy.[5] The current paper may be viewed as somewhat intermediate between models in the Aiyagari-Bewley tradition and two-agent heterogeneous agent models: less complicated than the former, but richer than the latter.

Huggett, Ventura, and Yaron (2011) study the relative effects of the level of human capital households bring into the life-cycle model at the time they begin to make economic decisions (age 23 in their paper), along with their initial wealth, versus their ability to learn and the idiosyncratic labor income risk they experience during their lifetimes. The authors argue that most of the action is in the initial conditions as opposed to the shocks.[6] This finding helps motivate our decision to randomly assign life-cycle productivity profiles to households as they enter the model, rather than keeping track of a stochastic household productivity sequence as in Aiyagari (1994) and subsequent research. There is uninsurable labor income risk, but all of this risk is borne at the outset of the life cycle for each cohort. This decision affords us considerable tractability, allowing calculation of "paper and pencil" solutions to the model even in the face of an aggregate shock.

In the present paper there is a single asset that is traded in nominal terms.[7] While the model is quite abstract and there is no explicit housing sector, it is perhaps natural to think of this single asset as a "mortgage-backed security," because the asset is privately issued by relatively young households to move consumption earlier in the life cycle than would otherwise be possible; this consumption could be interpreted broadly as housing services. Households nearer the midpoint of the life cycle wish to own these securities as a way of saving for retirement.

There is a large literature on the macroeconomic implications of mortgage markets following the global financial crisis of 2007-2009. One of the many issues in this literature is whether fixed- or variable-rate mortgages change the nature of monetary policy transmission. For example, Garriga, Kydland, and Šustek (2017) study how the monetary transmission mechanism is altered depending on nominal contracting features in mortgage markets and find quantitatively significant impacts depending on the nature of the shock. Sheedy (2014) emphasizes that fixed- versus variable-rate mortgages would be a quantitatively important consideration in a model with a NSCNC friction. The present paper restricts attention to one-period debt contracting (each period a new contract is signed, which is effective until the following period), and so may be viewed in the spirit of variable-rate mortgages. The issue of fixed rates over longer spans of time would be an additional friction that may be an area of fruitful study in more quantitatively oriented exercises than the one in this paper.

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[5] See also Ko (2015) for an alternative way to think about how the income and wealth distribution may impinge on standard New Keynesian dynamics.

[6] Huggett, Ventura, and Yaron (2011, p. 2924) state, "We find that initial conditions (i.e., individual differences existing at age 23) are more important than are shocks received over the rest of the working lifetime as a source of variation in realized lifetime earnings, lifetime wealth, and lifetime utility."

[7] For a version with money demand included, see Azariadis, Bullard, Singh, and Suda (2018).

This paper is about optimal monetary policy, one that has probably not been followed by actual policymakers during the postwar era. We interpret Doepke and Schneider (2006) as documenting the costly nominal redistribution that may have occurred in the U.S. economy as a result. They suggest that so many assets are held and traded in nominal terms that the welfare consequences of a shock to inflation can be quantitatively large.

We abstract from issues related to the effective lower bound on nominal interest rates in this version of the model and refer interested readers to Azariadis, Bullard, Singh, and Suda (2018).

2 Environment

2.1 Background on symmetry

We impose an important set of symmetry assumptions on the model. These assumptions help to control the heterogeneity and keep the model simple and stylized. The core result of the symmetry assumptions is that we are able to guess and verify that the general equilibrium real rate of interest will be equal to the real rate of growth of the economy, even with an aggregate shock and many heterogeneous households.[8]

Why is symmetry important? In the overlapping generations framework, much depends on the relative productivity of the older cohorts versus younger cohorts and by extension on the relative demand for assets versus the relative supply of those assets. By keeping these forces in balance, we can understand the baseline equilibrium of the model as a first-best allocation of resources, and thus we will be able to illustrate how monetary policy might overcome the NSCNC friction in order to attain that allocation. It is of course also interesting to understand how departures from the symmetry assumptions will affect the general equilibrium, presumably using quantitative methods, and we leave this to future research as it is beyond the scope of the current paper.

Accordingly, we assume the following:

(1) cohorts are of equal size and total population is constant;

(2) the discount factor for all households is unity;

(3) all households have log preferences as defined below;

(4) households' life-cycle productivity profiles are hump-shaped and symmetric as defined below.

One of our goals is to keep the analysis simple and stylized in order to be able to understand the equilibrium clearly. Accordingly, we have no capital in this version of

[8] We use the terms "household" and "agent" interchangably in this paper.

the model. All loans are repaid according to contractual obligations, and so there is no default in this version. Prices are flexible. Also, we have no borrowing constraints in this version of the model. Of these, we think capital could be added without materially affecting the key results. However, default, sticky prices, and borrowing constraints would change the nature of the equilibrium and thus the characterization of optimal monetary policy, unless other policy tools were also included to address these additional frictions.

We now describe the model environment.

2.2 Cohorts

Time is discrete. At each date, a cohort enters the model consisting of a continuum of households indexed $i \in (0,1)$. Each household in this cohort will make economic decisions for $(T+1) = 241$ consecutive periods and then exit the model in such a way that the total population remains constant. We interpret this value of T to represent a quarterly model in which economic decisions are being made every 90 days. All interest rates and growth rates are therefore interpretable in quarterly terms.[9] To fix ideas, we think of each cohort as one million individuals or more, which is the type of scale that would be appropriate for a large economy similar to the size of the U.S. or the euro area. We approximate this sort of scale with a continuum of households in each cohort, and while we solve all individual household problems, we sometimes characterize behavior at the cohort level as opposed to the individual level.

The model has both real and nominal quantities. We express nearly all variables in real terms, notably real consumption c and the real wage w. The exception is net asset holding a, which is expressed in nominal terms.

Each household $i \in (0,1)$ entering the economy at date t has preferences

$$U_{t,i} = \sum_{s=0}^{T} \eta \ln c_{t,i}(t+s) + (1-\eta) \ln \ell_{t,i}(t+s), \tag{1}$$

where $\eta \in (0,1)$ controls the relative desirability of real consumption in the consumption-leisure bundle. We use subscripts to denote the date of entry into the model and parentheses to denote real time, so that $c_{t,i}(t+s)$ represents the date $t+s$ consumption of a household i who entered the economy at date t and $\ell_{t,i}(t+s) \in (0,1)$ represents the date $t+s$ leisure choice of a household i that entered the economy at date t. Preferences for the households entering the economy at date $t-1$ can be defined analogously in a time-consistent manner as

$$U_{t-1,i} = \sum_{s=0}^{T-1} \eta \ln c_{t-1,i}(t+s) + (1-\eta) \ln \ell_{t-1,i}(t+s) \tag{2}$$

[9] We stress that results are invariant for any integer value of $T \geq 2$ and that the choice of T simply reflects the time frame for economic decision making. Results also hold in continuous time, i.e., for $T \to \infty$.

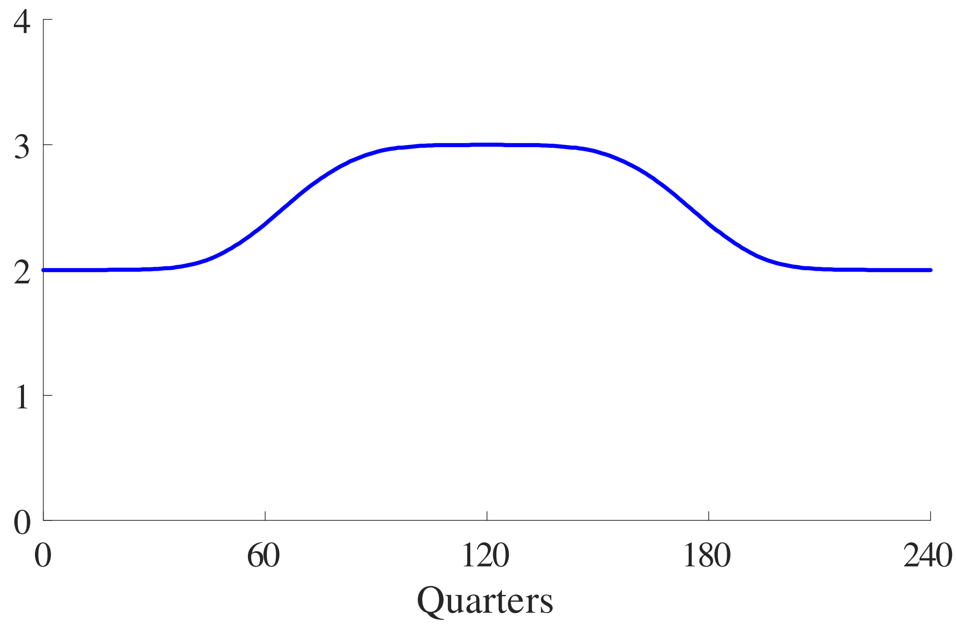

Figure 1: Baseline endowment profile.

and similarly for all households entering the economy at earlier dates.

2.3 Life-cycle productivity endowments

Each household $i \in (0,1)$ entering the economy at each date t is endowed with a known sequence of productivity (efficiency) units, "the life-cycle productivity endowment," denoted by $e_i = \{e_{s,i}\}_{s=0}^{T}$. This notation means that each household $i \in (0,1)$ entering the economy has productivity endowment $e_{0,i}$ in the first period of activity, $e_{1,i}$ in the second, and so on up to $e_{T,i}$. Households can sell the productivity units they are endowed with each period on a labor market at an economy-wide competitive real wage per effective efficiency unit and thus earn labor income at each date. We assume the entire endowment sequence is strictly positive for all households i.

As a critical aspect of the symmetry of the model economy, we assume that this productivity endowment sequence is hump-shaped and symmetric for each household i. This means that the sequence is monotonically increasing up to the middle-period endowment, $e_{120,i}$; that the sequence is hump-shaped in the sense that the middle-period endowment, $e_{120,i}$, is larger than all other endowments received by household i; that the sequence is monotonically increasing before and decreasing after the middle-period endowment; and that the sequence is symmetric defined as $e_{0,i} = e_{T,i}$, $e_{1,i} = e_{T-1,i}$, $e_{2,i} = e_{T-2,i}$, ...

We begin with the following baseline profile:

$$e_s = f(s) = 2 + \exp\left[-\left(\frac{s-120}{60}\right)^4\right]. \tag{3}$$

This is a stylized endowment profile that emphasizes that productivity near the beginning and end of the life cycle is relatively low, while productivity in the middle of the life cycle is relatively high. In short, households will have "peak earning years" for labor income. It will also turn out that households will choose to work more hours during the middle of the life cycle, and so peak labor income will be considerably higher than what is suggested by the productivity profile alone. While productivity is low at the beginning and end of the life cycle, we have chosen this profile such that households will not be tempted to supply zero labor in those circumstances (they may choose to work very few hours, but they will not choose zero hours). This means that we can restrict attention to interior solutions for the equilibria we study, even for an arbitrarily large degree of intra-cohort heterogeneity as defined in the next paragraph. The baseline endowment profile is displayed in Figure 1.[10]

We now introduce heterogeneity in the life-cycle productivity endowments. Let $\xi \geq 1$ be a within-cohort dispersion parameter. Each household $i \in (0,1)$ entering the economy at date t draws a scaling factor $x \sim \mathcal{U}\left[\xi^{-1}, \xi\right]$ that yields an endowment profile $e_{s,i} = x \cdot e_s$. Thus some households will have a relative abundance of life-cycle productivity, while other households will have a relative dearth of life-cycle productivity, but all households will face the same pattern of life-cycle productivity. If $\xi = 1$, there is no dispersion and all households are endowed with the baseline profile. Because households will never be tempted to supply zero labor no matter what scale they are on, we can choose ξ to be arbitrarily large without disturbing the equilibrium properties we describe below. However, we will show that each of the Gini coefficients of the model tends toward a limiting value as ξ increases. We will also consider an alternative model in which we draw x from a lognormal distribution instead of a uniform distribution and report results for that case below.

We motivate this productivity profile assignment as a stand-in for the end result of an unmodeled human capital development process—schooling combined with parenting and perhaps other training—that endowed the incoming agent with the given life-cycle productivity, as mentioned in the literature review section above.

The set or "mass" of heterogeneous endowment profiles is portrayed by the shaded area in Figure 2.

[10] We do not make any claims about this particular profile except that it is simple and convenient for the issues we discuss in this paper. A wide variety of profiles would satisfy the symmetry criteria and we could also consider heterogeneity among these various types of profiles. A productivity level in the middle of the life cycle that is 50 percent higher than that at the beginning or a the end of life cycle is of the same order of magnitude as much of the quantitative life-cycle literature.

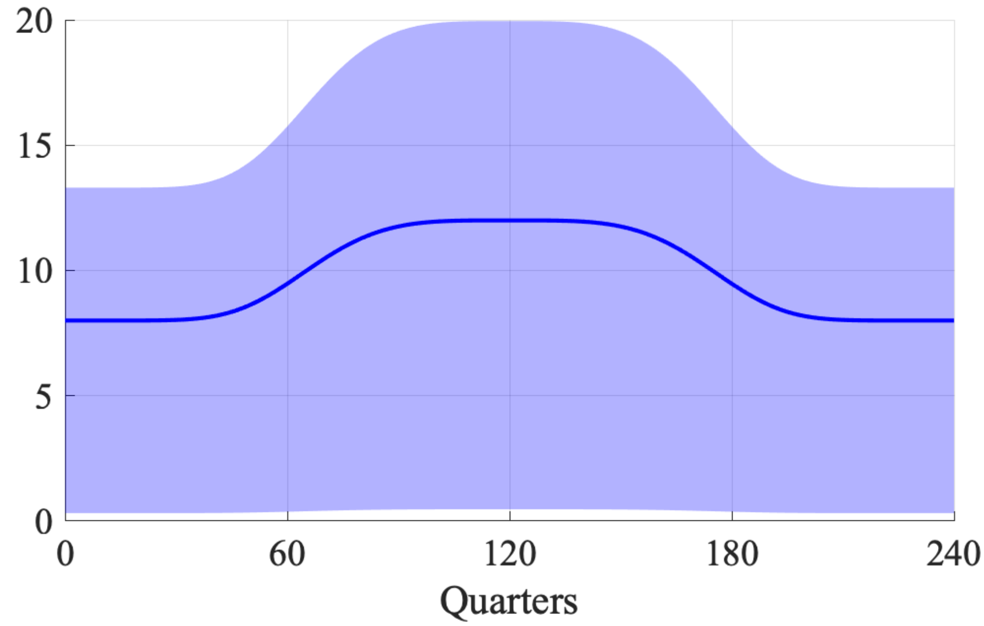

Figure 2: Endowment distribution by cohort (shaded area) and a representative endowment profile (line): $e_{s,i} \sim e_s \cdot \mathcal{U}\left(\xi^{-1}, \xi\right)$, $e_s = 2 + \exp\left[\left(-\frac{s-120}{60}\right)^4\right]$, $\xi = 6.5$.

2.4 Assets and the credit market friction

There is a single asset in the model economy, which is *privately issued* debt. This debt is a credible promise to pay a stated nominal amount in full plus agreed nominal interest, and is issued by relatively young households who wish to pull consumption forward in the life cycle. The lenders are households in their peak earning years near the middle of the life cycle who wish to accumulate assets for retirement. Given this structure, we think it is natural to motivate this abstract asset as representing mortgage-backed securities. The consumption that relatively young households wish to pull forward in the life cycle can be thought of as housing services. Thus, while the model is simple and abstract, we think it can be thought of as representing the mechanics of a quite large and important private sector credit market. The mortgage debt outstanding in the U.S. in 2017 was near $15 trillion, which is a ratio to annual GDP of about 0.80.

Households borrow in nominal terms and promise to pay off in nominal terms in a manner that does not depend on the state of the economy:

Definition 1 *Non-state contingent nominal contracting: All loan contracts are for one period, are not state contingent, and are expressed in nominal terms.*

Implicitly, there is a second asset in the model, which is currency supplied by the monetary authority. However, in this version of the model we abstract from money demand issues altogether and simply assume that the monetary policymaker controls the price level $P(t)$ directly. In the last section of the paper, we interpret the direct price level control in terms of direct control over short-term nominal interest rates. In this sense, we are making assumptions very similar to the "cashless limit" assumption in the New Keynesian literature, in which the monetary authority's control over a short-term nominal interest rate is simply asserted.[11]

There is no publicly-issued debt in this version of the model, nor is there any fiscal policy of any kind—all government expenditures and taxes are set to zero.

We assume that households that are entering the economy at date t hold no net nominal assets, which we refer to simply as "net assets." Households that entered into the economy in previous periods will generally have a non-zero net asset position at date t, which we denote by $a_{t-s,i}(t)$ for $s = 1, ..., T$ and $i \in (0, 1)$, which indicates the net asset holdings carried into the current period from date $t-1$ by each member of each cohort that entered the economy at the various dates $t-s$. There will therefore be a net asset distribution in the economy that we will have to track as part of the equilibrium. However, because all net asset positions will be linear in the real wage, it will be easy to track this distribution.

2.5 Technology

The technology is a simple extension of the endowment economy idea that "one unit of labor produces one unit of the good," but with appropriate adjustments for life-cycle productivity endowments $e_{s,i}$ and labor supply $1-\ell_{t-s,i}(t)$. We denote the level of aggregate total factor productivity as $Q(t)$, which we also call, equivalently, the level of technology or the level of labor productivity. The gross growth rate of Q follows a stochastic process. We say

$$Q(t) = \lambda(t-1, t) Q(t-1), \tag{4}$$

where $\lambda(t-1, t)$ is the growth rate of productivity between date $t-1$ and date t. The stochastic process driving the growth rate of productivity is $AR(1)$ with mean $\bar{\lambda}$:

$$\lambda(t, t+1) = (1-\rho)\bar{\lambda} + \rho\lambda(t-1, t) + \sigma\epsilon(t+1), \tag{5}$$

where $\bar{\lambda} > 1$ is the mean growth rate, $\rho \in (0, 1)$ denotes the degree of serial correlation, $\sigma > 0$ is a scale factor, and $\epsilon(t)$ is a truncated normal random variable with mean zero and bounds $\pm b$, with $b > 0$, chosen such that the zero lower bound is not encountered[12] and the level of technology Q (or other variables like the price level P)

[11] See, for instance, Woodford (2003) for a discussion of the cashless limit.

[12] The zero lower bound or effective lower bound would be encountered with a sufficiently negative shock combined with enough serial correlation to cause the expected rate of nominal GDP growth to be negative. See Azariadis, Bullard, Singh, and Suda (2018) for a discussion of this issue.

will never be negative.[13]

Aggregate output is given by

$$Y(t) = Q(t) L(t). \tag{6}$$

If we denote by $[1 - \ell_{t-s,i}(t)] \in (0,1)$ the fraction of time spent working by household i of cohort $t - s$, the labor input at date t is given by

$$L(t) = \int_0^1 \{e_{0,i}[1 - \ell_{t,i}(t)] + e_{1,i}[1 - \ell_{t-1,i}(t)] + \cdots + e_{T,i}[1 - \ell_{t-T,i}(t)]\} di. \tag{7}$$

The marginal product of labor is the real wage per effective efficiency unit, given by

$$w(t) = Q(t), \tag{8}$$

and we conclude that

$$w(t) = \lambda(t-1,t) w(t-1). \tag{9}$$

The aggregate real output growth rate is then

$$\frac{Y(t)}{Y(t-1)} = \frac{Q(t) L(t)}{Q(t-1) L(t-1)} = \lambda(t-1,t) \frac{L(t)}{L(t-1)}. \tag{10}$$

A baseline result in this model is that under the optimal monetary policy the equilibrium leisure choices ℓ are independent of the aggregate shock and hence of the real wage, so that $L(t)$ is constant in this formula. In particular, various cohorts will make the same leisure choice at the same stage of the life cycle, represented by $\ell_t(t) = \ell_{t-1}(t-1)$, $\ell_{t-1}(t) = \ell_{t-2}(t-1)$, and so on. We therefore conclude that

$$Y(t) = \lambda(t-1,t) Y(t-1) \tag{11}$$

in the equilibrium under optimal monetary policy. Along the nonstochastic balanced growth path, the gross output growth rate would be $\bar{\lambda}$, the mean rate of productivity growth. We will show below that the real interest rate equals the real output growth rate period by period in the stochastic equilibria we study.

2.6 Timing protocol

A timing protocol determines the role of information in the credit sector. We assume that nature moves first and chooses a continuum of draws defining the heterogeneous productivity profiles for the entering cohort and a value for $\epsilon(t)$, which implies a value for the productivity growth rate $\lambda(t-1,t)$ and hence a value for today's real wage $w(t)$. The monetary policymaker moves next and chooses a value for the price level $P(t)$, as described below. Households then take $w(t)$ and $P(t)$ as known and make decisions to consume and save via non-state contingent nominal contracts for the following period. These contracts carry a gross nominal interest rate denoted by $R^n(t,t+1)$. We now turn to defining these contracted values.

[13]This is just one of many possible stochastic processes that could be used.

2.7 Nominal interest rate contracts

All households meet in a competitive market for nominal loans. Households contract by fixing the nominal interest rate on consumption loans one period in advance. From the cohort t household Euler equation, the non-state contingent gross nominal interest rate in effect from period t to period $t+1$, denoted $R^n(t, t+1)$, is given by[14]

$$R^n(t, t+1)^{-1} = E_t \left[\frac{c_{t,i}(t)}{c_{t,i}(t+1)} \frac{P(t)}{P(t+1)} \right]. \tag{12}$$

We call this the contracted gross nominal interest rate, or simply the "contract rate." The E_t operator indicates that households must use information available as of the end of period t and before the realization of $\epsilon(t+1)$. This expression is the same for all households $i \in (0,1)$ in the equilibria we study. In particular, the equity share feature of the equilibrium means that all cohorts have the same expectation of their personal consumption growth rates, so that (12) suffices to determine the contract rate. Another way to say this is that there are heterogeneous households in this economy, and in particular some were born at, for instance, date $t-1$. These cohort $t-1$ households would want to contract at the nominal rate given by

$$R^n(t, t+1)^{-1} = E_t \left[\frac{c_{t-1,i}(t)}{c_{t-1,i}(t+1)} \frac{P(t)}{P(t+1)} \right]. \tag{13}$$

This would similarly be true for all other households entering the economy at earlier dates up to date $t-T$ (and across all i). However, in the equilibria we study, it will turn out that

$$\frac{c_{t,i}(t)}{c_{t,i}(t+1)} = \frac{c_{t-1,i}(t)}{c_{t-1,i}(t+1)} = \cdots = \frac{c_{t-T,i}(t)}{c_{t-T,i}(t+1)}, \tag{14}$$

$\forall i$, so that these expectations will all be the same and hence (12) suffices to determine the contract rate.

Given these considerations, individual expected consumption growth rates are equal to the expected aggregate nominal consumption growth rate and hence to the expected rate of nominal GDP growth in the equilibrium we study. This will play an important role in understanding how monetary policy works in this economy.

2.8 The monetary authority

From the discussion of assets in the subsection above, we have the assumption that the monetary authority controls $P(t)$ directly. We assume that the monetary policymaker has been asked by an enabling body exogenous to this model to achieve a gross inflation rate of π^* on average. We now assume that the monetary policymaker uses

[14] See Chari and Kehoe (1999) for more details.

the ability to set the price level at each date t to establish a fully credible policy rule $\forall t$ given by

$$P(t+1) = \frac{R^n(t,t+1)}{\lambda^r(t,t+1)} P(t). \tag{15}$$

The term $R^n(t,t+1)$ is the contract nominal interest rate effective between date t and date $t+1$, which is the expected rate of nominal GDP growth as described above. The term $\lambda^r(t,t+1)$ is the realized rate of productivity growth between date t and date $t+1$, that is, the realization of the growth rate for λ observed by the policymaker at date $t+1$. This rule delivers the exogenously given inflation rate of π^* on average. Because the realized value of productivity growth appears in the denominator, this rule calls for countercyclical price level movements. This is a hallmark of nominal GDP targeting as discussed in Koenig (2013) and Sheedy (2014).

2.9 Household budget constraints

Households have a simple sequence of budget constraints given the structure of the model and the fact that net assets are expressed in nominal terms. These budget constraints can be aggregated into a consolidated lifetime budget, which is standard. For the cohort entering the economy at date t, household i faces

$$c_t(t) + \sum_{s=1}^{T} \left(\frac{P(t+s)}{P(t)} \frac{c_{t,i}(t+s)}{\mathcal{R}^n(t,t+s)} \right) \leq$$
$$e_0 w(t)\left[1-\ell_{t,i}(t+s)\right] + \sum_{s=1}^{T} \left(\frac{P(t+s)}{P(t)} \frac{e_{s,i} w(t+s)\left(1-\ell_{t,i}(t+s)\right)}{\mathcal{R}^n(t,t+s)} \right), \tag{16}$$

where

$$\mathcal{R}^n(t,t+s) = \prod_{j=1}^{s} R^n(t+j-1,t+j). \tag{17}$$

Households entering the economy at earlier dates have a similar constraint over their remaining lifetime but also have a net asset position that they carry into date t, denoted by $a_{t-1,i}(t)$, $a_{t-2,i}(t)$,..., $a_{t-T,i}(t)$.

Now let us consider just one term in this budget constraint (16), the one applicable to date $t+1$ given by

$$\cdots \frac{P(t+1)}{P(t)} \frac{c_{t,i}(t+1)}{R^n(t,t+1)} \cdots \leq \cdots \frac{P(t+1)}{P(t)} \frac{e_1\left[1-\ell_{t,i}(t+1)\right] w(t+1)}{R^n(t,t+1)} \cdots. \tag{18}$$

The uncertainty in this expression is coming from the future real wage $w(t+1)$, which is stochastic. We can substitute the policy rule (15) directly into this expression. Noting that $w(t+1) = \lambda(t,t+1)\, w(t)$ and that ℓ choices will depend on contemporaneous consumption choices alone, the stochastic element, $\lambda(t,t+1)$, will cancel on the right-hand side and thus future income will become deterministic from the perspective of the household. This cancellation occurs for all other terms on the right-hand side of this expression, as well as for all other similar expressions for all other agents in the economy. The policymaker is providing a form of insurance to households.

More detail on the model solution is provided below and in the Appendix.

2.10 The model's simple solution

The details of the model solution are given in the Appendix, but we provide a heuristic discussion here.

We are interested in focusing on a stationary equilibrium in which time extends from the infinite past to the infinite future and where the monetary policy rule is followed credibly for all time. To obtain the solution, we begin with the problem of a single household i entering the economy at an arbitrary date t. This household has the preferences given above and faces a lifetime budget constraint expressed in nominal net asset terms. We can substitute the policymaker rule into this lifetime budget constraint to eliminate the uncertainty faced by the household and then solve the household's problem. The solution features date t consumption and leisure choices that depend solely on information available at date t and not on any future expectations. The consumption choices, as well as the net asset holding of this household, will depend linearly on the real wage, while the leisure choices will not. These same features apply to the choice problems of all other members of this cohort with different productivity profiles, as well as to all members of all cohorts entering the economy at earlier dates. The general equilibrium condition is that the net asset holding in the economy sums to zero. We guess and verify that a gross real interest rate equal to the real output growth rate satisfies this condition at each date t.

Theorem 2 *Assume symmetry as defined above. Assume the monetary authority credibly uses the price level rule given above, $\forall t$. Then the general equilibrium gross real interest rate, $R(t-1,t)$, is equal to the gross rate of productivity growth, and hence the real growth rate of the economy, $\lambda(t-1,t)$, $\forall t$.*

Corollary 3 *For any two households i and j in the model at each date t that share the same life-cycle productivity profile, consumption is equalized.*
Proof. *See the Appendix.* ■

3 Characterizing equilibrium

3.1 Overview

In this section we characterize the equilibrium using simple graphics in combination with some of the first-order necessary conditions (FONCs) from the model solution.

The model as we have presented it is too simple and stylized to provide a satisfactory match to U.S. data. In addition, we would not expect the U.S. postwar era to conform to the predictions of this model, since it is unlikely that nominal GDP targeting provides a satisfactory description of U.S. monetary policy during this era. Nevertheless, we do wish to illustrate that the model has some potential to represent a substantial degree of household heterogeneity in a manageable format, and therefore that nominal GDP targeting continues to be a promising description of an optimal approach to monetary policy even when many types of households coexist in the economy.

With this goal in mind, we present a baseline equilibrium in which Gini coefficients for income, financial wealth, and consumption are relatively close to those found in the U.S. data. We then characterize the equilibrium by looking at the schematic, cross-sectional distributions at an arbitrary date t for: (1) labor and leisure choices, (2) income according to various definitions, (3) consumption, and (4) net asset holding. These graphs illustrate key aspects of the equilibrium and show how the model can begin to confront actual household heterogeneity in the data.

The graphs we present below are static cross-sectional distributions, but the key variables in the economy other than labor supply are linear in the real wage $w(t)$, so that these distributions simply shift proportionately each period as the economy grows according to the stochastic process for λ.

We mainly describe the case where the scaling factor x is drawn from a uniform distribution. We briefly discuss how results are robust to the case where the scaling factor x is drawn from a lognormal distribution.

3.2 The labor supply distribution

We begin with the cross-sectional distribution of labor supply. The household i FONC for leisure can be written as

$$\ell_{t,i}(t+s) = (1-\eta)\frac{\bar{e}_i}{e_{s,i}} = (1-\eta)\frac{\bar{e}}{e_s}, \ \forall i, \tag{19}$$

where $\bar{e} = \sum_{s=0}^{T} e_s$ and $\bar{e}_i = \sum_{s=0}^{T} e_{s,i}$. Given that all household types receive an endowment profile that is a scaled version of the baseline profile, equation (3), they all choose the same leisure and hours worked profile over the life cycle. As illustrated in Figure 3, households work more when they are more productive, in the middle of the life cycle, and households enjoy more leisure early and late in their work life, when they are less productive.

According to Bullard and Feigenbaum (2007), in the data, the fraction of time worked is 19 percent. In our model, the average time worked over the life cycle is

$$1 - \frac{\sum_{s=0}^{T}\ell_{t,i}(t+s)}{T+1} = 1-(1-\eta)\frac{\sum_{s=0}^{T}\frac{\bar{e}}{e_s}}{T+1}. \tag{20}$$

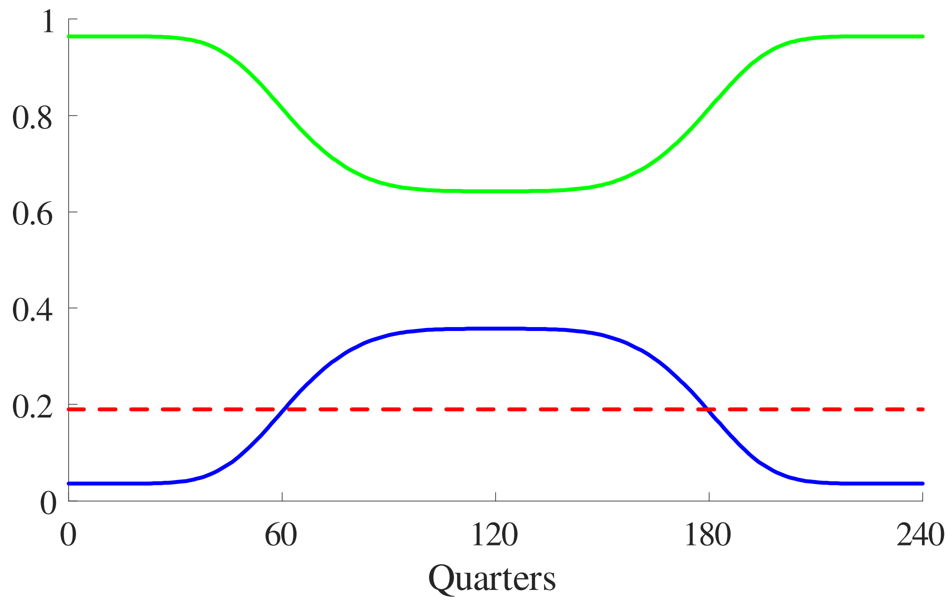

Figure 3: Leisure decisions by age (green), labor supply by age (blue) and fraction of work time in U.S. data (red), 19%.

Given the baseline income profile in (3), setting

$$\eta = \left[1 - (1-0.19)/\left(\frac{\sum_{s=0}^{T}\frac{\bar{e}}{e_s}}{T+1}\right)\right] = 0.21 \tag{21}$$

results in $1 - \frac{\sum_{s=0}^{T}\ell_t(t+s)}{T+1} = 19\%$. We have included this as a horizontal red line in Figure 3.

For hours worked, all households, rich and poor, behave in the same manner at each point in the life cycle, as the dispersion parameter ξ does not enter the FONC, equation (19). However, for other quantities, ξ will matter.

3.3 Income distributions

The period budget constraint of an household in cohort t with life-cycle productivity profile i can be written as

$$c_{t,i}(t+s) \leq e_{s,i}\left[1-\ell_{t,i}(t+s)\right]w(t+s) + R^n(t+s-1,t+s)\frac{a_{t,i}(t+s-1)}{P(t+s)} - \frac{a_{t,i}(t+s)}{P(t+s)},$$

for $s = 0, \ldots, T$, where $a_{t,i}(t-1) = a_{t,i}(T) = 0$. Equivalently, substituting for R^n from (15) and rearranging:

$$c_{t,i}(t+s) + \left[\frac{a_{t,i}(t+s)}{P(t+s)} - \frac{a_{t,i}(t+s-1)}{P(t+s-1)}\right] \\ \leq e_{s,i}\left[1-\ell_{t,i}(t+s)\right] w(t+s) + \left[\lambda(t+s,t+s-1)-1\right]\frac{a_{t,i}(t+s-1)}{P(t+s-1)}.$$

Expenditures on consumption and on acquisition of new assets have to be less than the sum of labor and capital income.

Along the non-stochastic balanced growth path with $\bar{\lambda} = 1$, there is no capital income. We have ruled out this case in the specification of the stochastic process for λ. With $\bar{\lambda} > 1$, three notions of income can be considered:

(1) labor income,

$$e_{s,i}\left[1-\ell_{t,i}(t+s)\right] w(t+s); \tag{22}$$

(2) labor income plus non-negative capital income,[15]

$$e_{s,i}\left[1-\ell_{t,i}(t+s)\right] w(t+s) + \\ + \max\left\{\left[\lambda(t+s,t+s-1)-1\right]\frac{a_{t,i}(t+s-1)}{P(t+s-1)}, 0\right\}; \tag{23}$$

(3) the non-negative component of total income,[16]

$$\max\left\{\begin{array}{c} e_{s,i}\left[1-\ell_{t,i}(t+s)\right] w(t+s) + \\ + \left[\lambda(t+s,t+s-1)-1\right]\frac{a_{t,i}(t+s-1)}{P(t+s-1)} \end{array}, 0\right\}. \tag{24}$$

Given leisure choices, labor income is linear in the real wage. The other concepts of income will also be linear in the real wage (because net asset holding is also linear in the real wage). This means that household i real income will grow at the growth rate of the aggregate economy, which is given by the stochastic process for λ.

Figure 4 portrays labor income profiles for $\xi = 6.5$ and the value of η discussed above. We discuss the other concepts of income below when we calculate Gini coefficients.

Since households work more during their peak earning years, and since different households have different levels of life-cycle productivity, the labor income distribution has a smaller range for younger and older households but is more dispersed for households closer to the midpoint of the life cycle.

[15] The idea is that typically positive capital income, e.g., from investing in stocks, is counted as a part of income. Negative capital income, e.g., interest payments on a mortgage or a student loan, are typically not considered a part of income.

[16] Households can have negative total income for some periods of their lives. In those periods, consumption is financed by going further into debt.

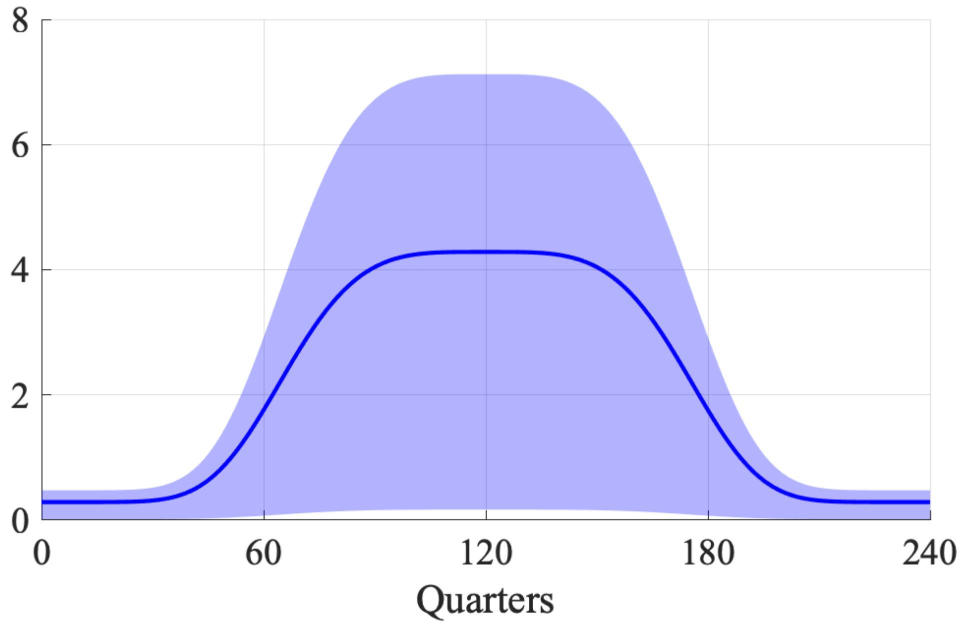

Figure 4: Distribution of labor income by cohort (shaded area) and a typical labor income profile by age (line).

3.4 The consumption distribution

The consumption FONC for households of type i is

$$c_{t,i}(t) = \eta w(t)\, \bar{e}_i. \tag{25}$$

Individual household consumption over the life cycle grows at the same rate as the economy as a whole, thanks to the linearity in $w(t)$. But individual household consumption also depends linearly on the average productivity endowment $\bar{e}_i$ over the life cycle (and by extension on the dispersion factor ξ). Households that share the same productivity endowment profile consume the same amount at each date t, regardless of where they are in their life cycle. This is the "equity share contracting" feature of the equilibrium.

Given these considerations, the distribution of consumption across all households is uniform, as portrayed in Figure 5.

3.5 The net asset-holding distribution

Net asset holding is also linear in the real wage $w(t)$ and depends on the average endowment over the life cycle $\bar{e}_i$. Households borrow to finance consumption early in the life cycle, with peak indebtedness occurring during the first half of the life cycle. Households then begin to move out of indebtedness through their peak earning

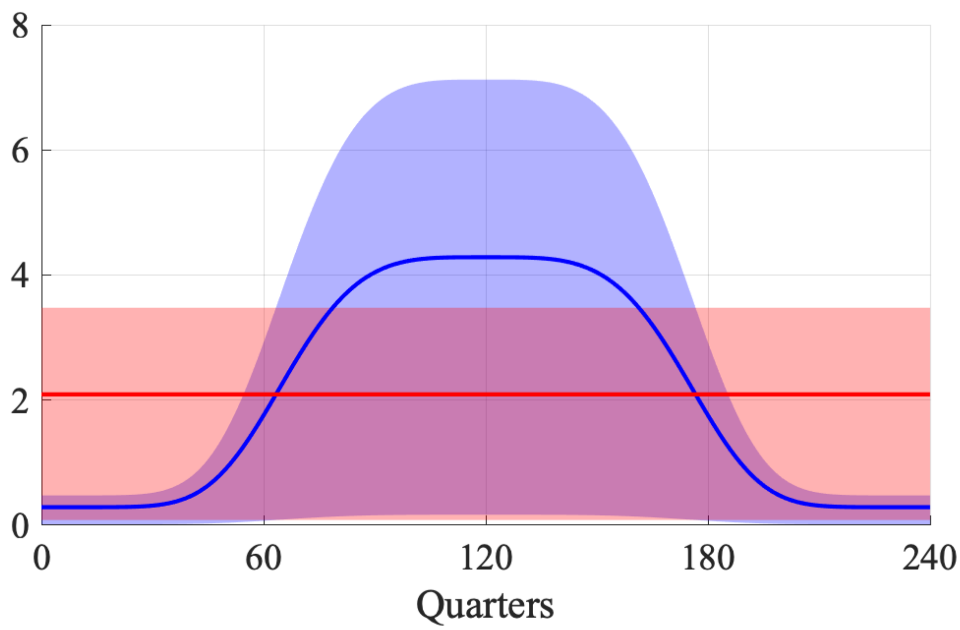

Figure 5: Distribution of labor income (blue shaded area) and of consumption (red shaded area) by cohort; A typical labor income profile by age (blue line) and the corresponding consumption profile (red line).

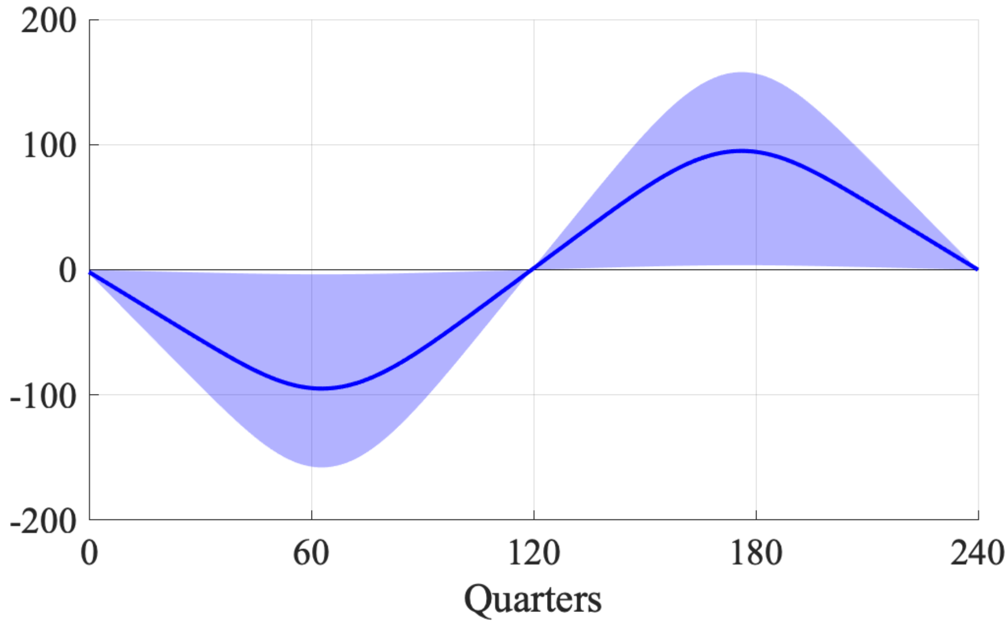

Figure 6: Distribution of net asset holding by cohort (shaded area) and a typical net asset profile by age (line).

years, reaching a period of peak net asset holdings during the second half of the life cycle. Households that have different life-cycle productivity endowments follow identical life-cycle net asset accumulation and decumulation patterns, but scaled up or down according to their value of x. Financial wealth must sum to zero as the closed economy equilibrium condition.

Figure 6 shows the net asset-holding profiles by age.

As the economy evolves, borrowing and lending both increase in proportion to increases in $w(t)$. That is, each cohort's net asset holding shifts up or down according to the value of that period's real wage $w(t)$.

3.6 Remarks on a lognormal scaling factor

We have presented a characterization of the model equilibrium based on drawing the scaling factor x each period from a uniform distribution. Drawing the scaling factor x from a lognormal distribution may be more aesthetically appealing, as it allows for arbitrarily rich and arbitrarily poor households and concentrates more households at a particular part of the life-cycle productivity profile distribution, which might be thought of as the "ordinary" life-cycle productivity profile. Our specification can accommodate this extension because agents will always choose interior solutions independently of the scale of the life-cycle productivity profile. However, we have found that the Gini coefficients implied by this type of scaling are very similar to the case where the scaling factor x is drawn from a uniform distribution. Accordingly, we do not discuss this case further in this version of the paper.

4 Gini coefficients

We now turn to a discussion of Gini coefficients based on the distributions displayed above where the scaling factor x is drawn from a uniform distribution. While this model is too simple to completely characterize the U.S. data, we hope to generate some confidence that a model in this class could begin to confront the observed level of heterogeneity in wealth, income, and consumption. In the U.S. data, it is widely believed that the consumption Gini is less than the income Gini, which is, in turn, less than the financial wealth Gini. The model naturally produces this ordering.

We begin with targets in the U.S. data. For the consumption Gini, we follow Heathcote, Perri, and Violante (2010) and set $G_{C,U.S.} = 32\%$. The income Gini is the most widely measured. We take the value reported by the Congressional Budget Office (2016), for income pre-taxes and transfers, of $G_{Y,U.S.} = 51\%$. Finally, we use the financial wealth Gini for the U.S. reported by Davies, Sandström, Shorrocks, and Wolff (2011), $G_{W,U.S.} = 80\%$.

We approximate all of the probability density functions (PDFs) from the model with Chebyshev polynomials[17] and compute the corresponding Gini coefficients as follows:

$$G_x = \frac{1}{\mu_x} \int_0^{+\infty} F_x\left(y\right)\left[1 - F_x\left(y\right)\right] dy,$$

where μ_x denotes the mean of the variable of interest, x, and F_x denotes the cumulative distribution function of x. Figure 7 portrays the PDFs for our model.

The Gini coefficients for our model are as follows:

- endowment distribution: $G_E = 33.5\%$,
- consumption distribution: $G_C = 31.8\%$,
- income distributions: $G_{Y_1} = 56.2\%$, $G_{Y_2} = 51.6\%$, $G_{Y_3} = 59.6\%$,
- wealth distribution: $G_W = 72.7\%$.

These figures suggest that it is not difficult to obtain Gini coefficients for this model that are close to those observed in the U.S. data. Not surprisingly, the more dispersion there is in endowment profiles, ξ, the higher are the Gini coefficients (see Figure 8).

The Gini coefficients of the distribution of endowments and wealth are invariant to η, because η does not affect endowments or net asset accumulation decisions. Similarly

[17] See Driscoll, Hale, and Trefethen (2014).

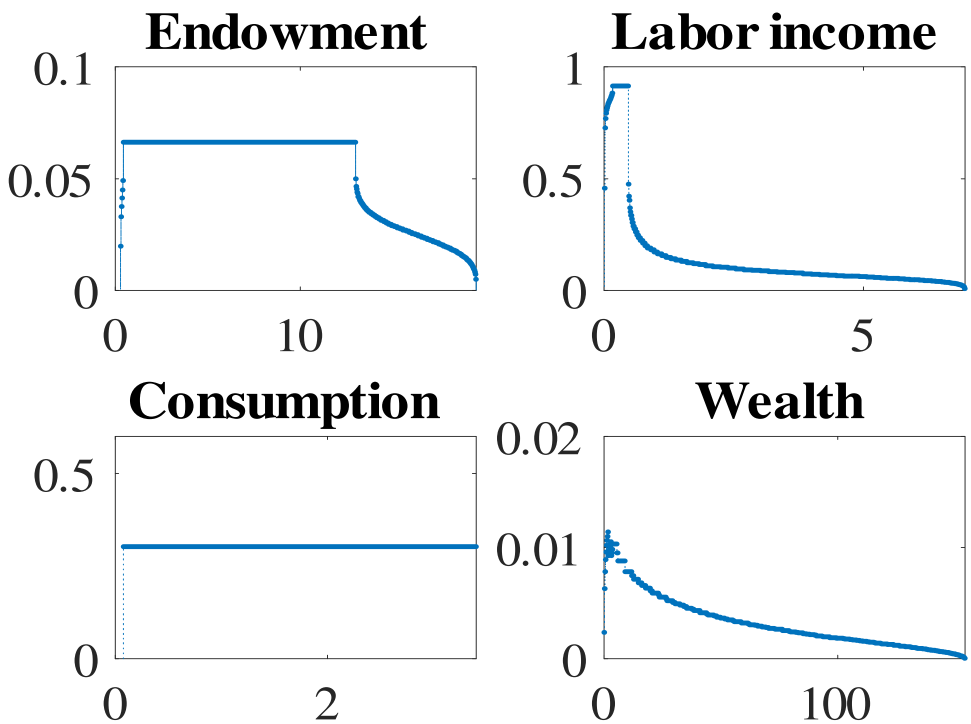

Figure 7: PDFs of endowment, labor income, consumption, and wealth. The wealth subplot omits a mass point (121/241) at 0.

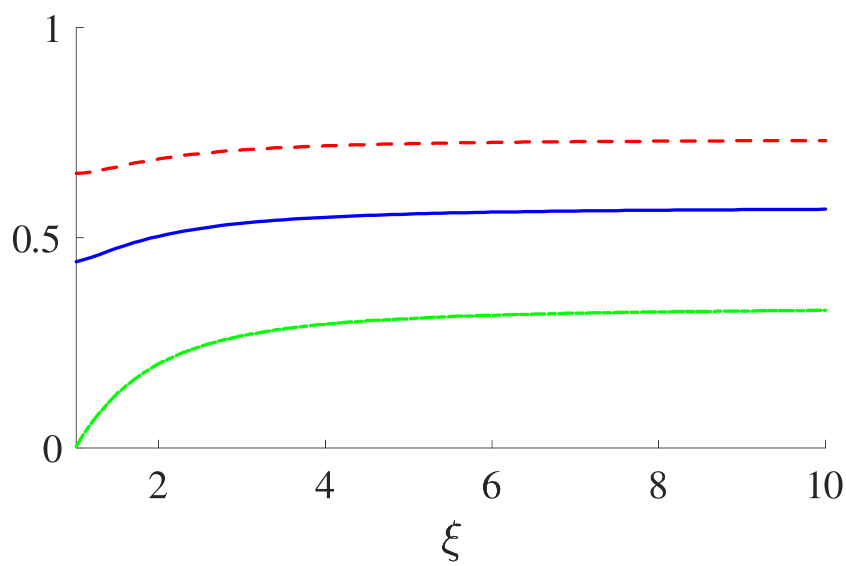

Figure 8: Gini coefficients of wealth (red), $\max(a/P, 0)$; labor income (blue), $we(1-l)$; and consumption (green) as functions of endowment dispersion, ξ.

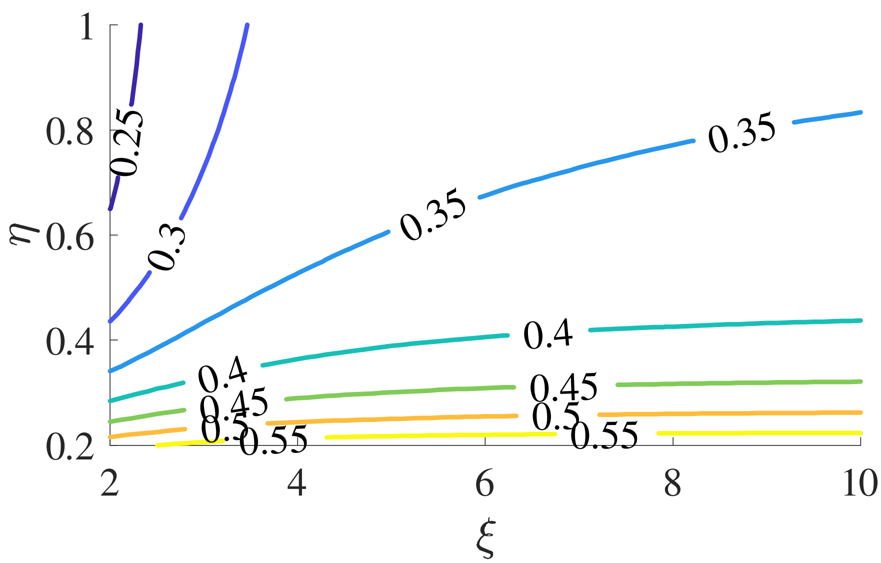

Figure 9: Gini coefficient of the distribution of labor income for different values of ξ and η.

the Gini coefficient for the consumption distribution is independent of η because η scales consumption choices in the same way for all households (see equation (25)) without affecting the distribution. As illustrated in Figure 9, the Gini coefficient of the distribution of labor income is decreasing in η.

5 Characterizing monetary policy

In this section we turn to the issue of characterizing the nature of optimal monetary policy in this model. We have said that the monetary policymaker credibly follows the price level rule (15) for all t. This rule is expressed in a compact form that can be substituted directly into the households' problems to deliver the complete markets solution of the model. The rule turns non-state contingent nominal contracts into state contingent contracts denominated in real terms. Thus the rule provides a form of insurance to credit market participants in the model, and the model equilibrium is characterized by complete markets with equity share contracting.[18]

Nevertheless, monetary policy is more commonly discussed in terms of a short-term nominal interest rate over which the monetary authority is assumed to have complete control. We now turn to an interpretation of the model along these lines.

The nominal contracts in the model are set by market participants with an understanding of the price level rule, and the price level rule is adhered to by the monetary

[18]The price level rule (15) is not the unique rule that can restore complete markets. Azariadis, Bullard, Singh, and Suda (2018) present an alternative rule that delivers the complete markets allocation.

authority with an understanding that nominal contracting is occurring. To express monetary policy in the common language of short-term nominal interest rates we can find the fixed point of this process and then illustrate the general equilibrium responses to macroeconomic shocks.

To begin, the contract nominal interest rate (12) can be understood as the expected rate of nominal GDP growth. The Wicksellian natural real rate of interest in this economy can be understood as the real rate of growth of the economy (λ). It is this growth rate that is the stochastic element of the equilibrium. The monetary policy (15) is to set the price level in a countercyclical manner such that the nominal interest rate contract, which is also the expected rate of nominal GDP growth, is always ratified ex post. This makes the real interest rate equal to the Wicksellian natural rate of interest. This aspect of policy is the same as in New Keynesian models.

It follows from this discussion that in the special case of serially uncorrelated shocks ($\rho = 0$), the expected rate of real GDP growth will never change, and hence, because of the monetary policy, the expected rate of nominal GDP growth will never change either. Thus the policy in this particular circumstance could be characterized as a nominal interest rate peg or, equivalently, perfect nominal GDP targeting. In each period, as real shocks occur, the price level is adjusted in such a way that the previous period's expectation of nominal GDP, and hence the contract nominal interest rate, turns out to be exactly correct. The economy never deviates from the nominal GDP path desired by policymakers.

In the more general case of serially correlated shocks ($\rho > 0$), this extreme result is modified. If the growth rate of the economy falls or rises persistently, the expectation of future nominal GDP growth will fall or rise persistently. This means that the contract nominal interest rate will also fall or rise persistently. The price level rule (15) still calls for countercyclical price level movements that ratify the previous period's expectations. However, these expectations are now themselves moving persistently higher or lower depending on the shocks to the real growth rate λ. We can still think of the policy as a form of nominal GDP targeting, but one that returns the economy to the desired nominal GDP path more slowly because of the persistence of real shocks.

These considerations suggest the following observations for interpreting the suggested price level rule policy in nominal interest rate terms. In general, the policymaker controlling a short-term nominal interest rate wants to follow the natural rate of real growth in the economy as part of the optimal policy. In particular. a recession is associated with lower nominal and real interest rates as part of the optimal policy. The policy also always ratifies the nominal interest rate contract, so that nominal interest rates do not react in the period of the shock but only one period later. In the period of the shock, inflation moves higher as part of the counter-cyclical price level movement associated with the optimal policy. These features are illustrated in Figure 10.

Koenig (2012) discusses the relationship between nominal GDP targeting and more familiar Taylor-type policy rule approaches to monetary policy. He argues that

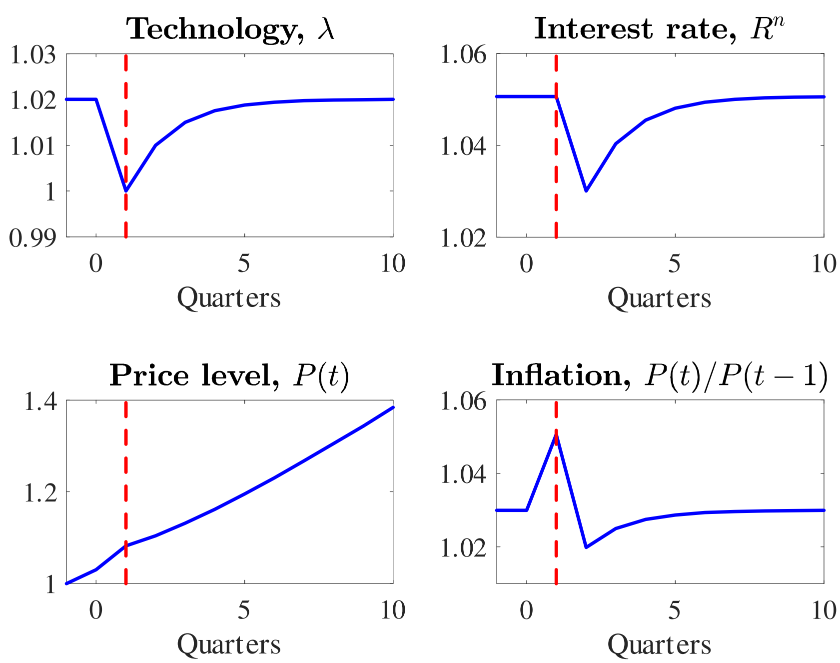

Figure 10: Monetary policy responds to a decrease in aggregate productivity growth, λ, by increasing the price level in the period of the shock. Subsequently, inflation converges to its balanced growth path value, π^*, from below. The nominal interest rate drops in the period after the shock.

the two approaches are "close cousins," but from a perspective in which sticky prices provide the key nominal friction. We hope to address this issue in the NSCNC context in future versions of this model.

6 Conclusion

In this paper we study an economy with "massive" heterogeneity and optimal monetary policy. NSCNC is the key friction in the economy. The policymaker can provide a type of insurance against aggregate shocks for all households in the model, whether they are rich or poor and whether they hold net assets or are net borrowers. This approach may provide an interesting baseline equilibrium for thinking about monetary policy in heterogeneous agent economies. The model is too simple to try to use it to aggressively confront the U.S. data on heterogeneous households, but we nevertheless characterize an illustrative benchmark equilibrium in which the Gini coefficients in the model begin to approach those in the U.S. data on dimensions of inequality with respect to income, consumption, and financial wealth.

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.

Auclert, A. (2019): "Monetary Policy and the Redistribution Channel," *American Economic Review*, forthcoming.

Azariadis, C., J. Bullard, A. Singh, and J. Suda (2018): "Incomplete Credit Markets and Monetary Policy," University of Sydney, unpublished manuscript.

Bhandari, A., D. Evans, M. Golosov, and T. Sargent (2018): "Inequality, Business Cycles and Monetary-Fiscal Policy," NBER Working Paper No. 24710.

Braun, R. A., and T. Oda (2015): "Real Balance Effects When the Nominal Interest Rate is Zero," Federal Reserve Bank of Atlanta, unpublished manuscript.

Bullard, J. (2014): "Comment on Debt and Incomplete Financial Markets: A Case for Nominal GDP Targeting," *Brookings Papers on Economic Activity*, Spring, 362–368.

Bullard, J., and J. Feigenbaum (2007): "A Leisurely Reading of the Life-Cycle Consumption Data," *Journal of Monetary Economics*, 54(8), 2305–2320.

Bullard, J., and A. Singh (2019): "Nominal GDP Targeting With Heterogeneous Labor Supply," *Journal of Money, Credit and Banking*, forthcoming.

Chari, V. V., and P. J. Kehoe (1999): "Optimal fiscal and monetary policy," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, vol. 1, Part C, chap. 26, pp. 1671–1745. North Holland, Amsterdam.

Congressional Budget Office (2016): "The Distribution of Household Income and Federal Taxes, 2013," `https://www.cbo.gov/sites/default/files/114th-congress-2015-2016/reports/51361-householdincomefedtaxes.pdf`.

Davies, J. B., S. Sandström, A. Shorrocks, and E. N. Wolff (2011): "The Level and Distribution of Global Household Wealth," *Economic Journal*, 121(551), 223–254.

Debortoli, D., and J. Galì (2018): "Monetary Policy with Heterogeneous Agents: Insights from TANK Models," CREI, unpublished manuscript.

Doepke, M., and M. Schneider (2006): "Inflation and the Redistribution of Nominal Wealth," *Journal of Political Economy*, 114(6), 1069–1097.

Driscoll, T. A., N. Hale, and L. N. Trefethen (eds.) (2014): *Chebfun Guide*. Pafnuty Publications, Oxford.

Eggertson, G. B., N. R. Mehrotra, and J. A. Robbins (2018): "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, forthcoming.

Feng, Z., and M. Hoelle (2017): "Indeterminacy in Stochastic Overlapping Generations Models: Real Effects in the Long Run," *Economic Theory*, 63(2), 559–585.

Galì, J. (2014): "Monetary Policy and Rational Asset Price Bubbles," *American Economic Review*, 104(3), 721–752.

——— (2018): "The State of New Keynesian Economics: A Partial Assessment," *Journal of Economic Perspectives*, 32(3), 87–112.

Garriga, C., F. E. Kydland, and R. Šustek (2017): "Mortgages and Monetary Policy," *Review of Financial Studies*, 30(10), 3337–3375.

Heathcote, J., F. Perri, and G. L. Violante (2010): "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006," *Review of Economic Dynamics*, 13(1), 15–51.

Huggett, M., G. Ventura, and A. Yaron (2011): "Sources of Lifetime Inequality," *American Economic Review*, 101(7), 2923–2954.

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary Policy According to HANK," *American Economic Review*, 108(3), 697–743.

Ko, D.-W. (2015): "Inequality and Optimal Monetary Policy," Rutgers University, unpublished manuscript.

Koenig, E. F. (2012): "All in the Family: The Close Connection Between Nominal-GDP Targeting and the Taylor Rule," Staff Paper No. 17, Federal Reserve Bank of Dallas.

——— (2013): "Like a Good Neighbor: Monetary Policy, Financial Stability, and the Distribution of Risk," *International Journal of Central Banking*, 9(2), 57–82.

McKay, A., E. Nakamura, and J. Steinsson (2016): "The Power of Forward Guidance Revesited," *American Economic Review*, 106(10), 3133–3158.

Sheedy, K. D. (2014): "Debt and Incomplete Financial Markets: A Case for Nominal GDP Targeting," *Brookings Papers on Economic Activity*, Spring, 301–361.

——— (2018): "Taking Away the Punch Bowl: Monetary Policy and Financial Instability," London School of Economics, unpublished manuscript.

Sterk, V., and S. Tenreyro (2018): "The Transmission of Monetary Policy Through Redistributions and Durable Purchases," *Journal of Monetary Economics*, 99, 124–137.

Werning, I. (2014): "Comment on Debt and Incomplete Financial Markets: A Case for Nominal GDP Targeting," *Brookings Papers on Economic Activity*, Spring, 368–372.

A Appendix

The model features heterogeneous households and an aggregate shock, so that the evolution of the asset-holding distribution in the economy is part of the description of the equilibrium. This would normally require numerical computation. However, symmetry, log preferences, and other simplifying assumptions allow solution by "pencil and paper" methods. In this Appendix we outline this solution in some detail. A key feature of the solution is that the asset-holding distribution will be linear in the current real wage $w(t)$, and so will simply shift up and down with changes in $w(t)$. Another key feature of the solution will be that the stochastic real rate of return on asset holdings will be equal to the stochastic real output growth rate period by period. We do not claim uniqueness of this equilibrium, but we regard the equilibrium we isolate as a natural focal point for this analysis.[19]

We guess and verify a solution given a particular price rule for P employed by the monetary authority.

(1) We first propose the state contingent policy rule for the price level P and assume that this rule is perfectly credible for all time $t \in (-\infty, +\infty)$.

(2) We then state the problem of household i entering the model at an arbitrary date t under the NSCNC friction. We substitute the proposed policy rule directly into this problem.

(3) We solve this problem and show that date t choices for $c_{t,i}(t)$ and $\ell_{t,i}(t)$ for this household depend only on date t information and not on expectations of the future stochastic evolution of wages, reflecting the insurance provided by the policymaker.

(4) We argue that suitable adaptations of this same result also apply for all households that entered the economy at dates earlier than date t with various life-cycle productivity profiles i and with non-zero net asset holdings brought into date t.

(5) We then show that the general equilibrium market clearing condition, which is that net asset holding sums to zero across the economy, is met given the derived household behavior. Thus we have identified an equilibrium of the stochastic economy in which the stochastic gross real interest rate $R(t, t+1)$ is equal to the stochastic gross rate of growth of real output in the economy, $\lambda(t, t+1)$, for all t.

Given our assumption, all household choices will be interior, meaning in particular that leisure choices will obey $0 < \ell_{t,i}(t+s) < 1$ for all s, i. We verify this aspect of the solution later.

Step 1. A household i entering the economy at date t faces uncertainty about income over its life cycle because it does not know what the real wage level is going to be in the future. The proposed state contingent policy rule provides insurance

[19] See Feng and Hoelle (2017) for a recent discussion and analysis. Typical quantitative-theoretic applications in the area of stochastic overlapping generations would be unable to address the issues brought out by the Feng and Hoelle analysis.

against this uncertainty and is given by

$$P(t+1) = \frac{R^n(t,t+1)}{\lambda^r(t,t+1)} P(t) \tag{26}$$

for all t, with $P(0) > 0$.

Step 2. We first consider households i entering the economy at date t. The problem of these agents is

$$\max_{\{c_{t,i}(t+s),\ell_{t,i}(t+s)\}_{s=0}^{T}} E_t[\sum_{s=0}^{T} [\eta \ln c_{t,i}(t+s) + (1-\eta)\ln \ell_{t,i}(t+s)] \tag{27}$$

subject to the lifetime budget constraint (which is an aggregation of the sequence of period budget constraints the agent faces) given by

$$\begin{aligned} c_{t,i}(t) + \sum_{s=1}^{T}\left(\frac{P(t+s)}{P(t)}\frac{c_{t,i}(t+s)}{\mathcal{R}^n(t,t+s)}\right) &\leq \\ e_0 w(t)[1-\ell_{t,i}(t)] + \sum_{s=1}^{T}&\left(\frac{P(t+s)}{P(t)}\frac{e_{s,i}w(t+s)(1-\ell_{t,i}(t+s))}{\mathcal{R}^n(t,t+s)}\right), \end{aligned} \tag{28}$$

where

$$\mathcal{R}^n(t,t+s) = \prod_{j=1}^{s} R^n(t+j-1,t+j). \tag{29}$$

Substitution of the policy rule into the budget constraint for these households yields a new version of the lifetime budget constraint,

$$c_{t,i}(t) + \sum_{s=1}^{T}\left(\frac{c_{t,i}(t+s)}{*(t,t+s)}\right) \leq w(t)\sum_{s=0}^{T} e_s(1-\ell_{t,i}(t+s)), \tag{30}$$

where

$$\Lambda(t,t+s) = \prod_{j=1}^{s} \lambda^r(t+j-1,t+j). \tag{31}$$

We note that by the timing protocol of this model, $w(t)$ is known by the household at the time this problem is solved. If the model had no NSCNC friction—so that asset holdings were expressed in real instead of nominal terms—and we simply replaced all gross real interest rates with gross output growth rates, we would obtain the same lifetime budget constraint expression. Therefore, if the derived behavior from this problem meets the general equilibrium condition below, we can claim that the equilibrium is that the gross real interest rate $R(t,t+1) = \lambda(t,t+1)\ \forall t$.

Step 3. The household i then solves this problem, where μ_i is the multiplier on the life-time budget constraint. The sequence of FONCs for $s = 0, 1, ..., T$ with respect to consumption is given by

$$\frac{\eta}{c_{t,i}(t)} = \mu_i, \tag{32}$$

$$\frac{\eta}{c_{t,i}(t+s)} = \frac{\mu_i}{\Lambda(t, t+s)}. \tag{33}$$

These conditions imply that the household i state contingent consumption plan for dates $t+s$, $s = 1, ..., T$, depends on the realizations of future shocks to the productivity growth rate, λ:

$$c_{t,i}(t+s) = \Lambda(t, t+s)\, c_{t,i}(t). \tag{34}$$

The sequence of FONCs for $s = 0, 1, ..., T$ with respect to ℓ is

$$\frac{1-\eta}{\ell_{t,i}(t+s)} = \mu_i w(t)\, e_{s,i}. \tag{35}$$

We combine each of these with the corresponding FONC for consumption to give the following choices for leisure for $s = 0, 1, ..., T$:

$$\ell_{t,i}(t+s) = \frac{1-\eta}{\eta} \frac{c_{t,i}(t)}{w(t)\, e_{s,i}}. \tag{36}$$

We can then substitute back for leisure into the budget constraint

$$(T+1)c_{t,i}(t) \leq w(t) \sum_{s=0}^{T} e_{s,i} \left(1 - \frac{1-\eta}{\eta} \frac{c_{t,i}(t)}{w(t)\, e_{s,i}}\right), \tag{37}$$

or

$$c_{t,i}(t) = \eta w(t)\, \bar{e}_i, \tag{38}$$

where $\bar{e}_{s,i} \equiv \sum_{s=0}^{T} e_{s,i} / (T+1)$ denotes the average endowment over the life cycle for agents with productivity profile i. We conclude that the choice for first-period consumption, $c_{t,i}(t)$, depends on the desirability of consumption versus leisure, η; the productivity profile assigned to this agent e_i; and today's wage $w(t)$ alone (and not on any future expected wages).

The amount of leisure chosen at date t and the amount chosen in the future depends on the household's position in the life cycle. These amounts are given by

$$\ell_{t,i}(t+s) = (1-\eta) \frac{\bar{e}_i}{e_{s,i}} = (1-\eta) \frac{\bar{e}}{e_s}\ \forall i, \tag{39}$$

where $\bar{e} = \sum_{s=0}^{T} e_s / (T+1)$ is the average baseline endowment. If $\eta = 1$, the household will choose no leisure. If $\eta \to 0$ and $e_{0,i} = e_{T,i}$ are small enough, then $\ell_{t,i}(t)$

and $\ell_{t,i}(t+T)$ could be larger than 1, meaning the households would supply no labor on those dates. This would violate our interior solution assumption. We assume $e_{0,i} = e_{T,i} \gg 0$ and η sufficiently large to maintain interior leisure choices.

This household will carry some nominal asset position $a_{t,i}(t)$ into the next period. The date t real value of this position is given by

$$\begin{aligned} \frac{a_{t,i}(t)}{P(t)} &= e_{0,i}\left[1-\ell_{t,i}(t)\right]w(t) - c_{t,i}(t) &(40)\\ &= e_{0,i}\left(1-(1-\eta)\frac{\bar{e}_i}{e_{0,i}}\right)w(t) - \eta w(t)\bar{e}_i &(41)\\ &= w(t)\left(e_{0,i}-\bar{e}_i\right). &(42)\end{aligned}$$

This asset position is linear in the real wage $w(t)$.

Step 4. There are also households i that entered the economy at dates $t-1, t-2, \cdots, t-T$ that would solve a similar problem. These households bring nominal asset holdings $a_{t-1,i}(t-1), a_{t-2,i}(t-1), \cdots, a_{t-T,i}(t-1)$, respectively, into the current period and have a shorter remaining horizon in their life cycle. Here we show the solution to a household problem for household i that entered the economy at date $t-1$. In particular, we show that the net asset-holding choice at date t, $a_{t-1,i}(t)$, continues to be linear in the current real wage $w(t)$. We then infer solutions for all of the other household problems for households i that entered the economy at dates $t-2, \cdots, t-T$.

Household i that entered the economy at date $t-1$ solves this problem at date t :

$$\max_{\{c_{t-1,i}(t+s),\ell_{t-1,i}(t+s)\}_{s=0}^{T-1}} E_t \sum_{s=0}^{T-1}\left[\eta \ln c_{t-1,i}(t+s) + (1-\eta)\ln \ell_{t-1,i}(t+s)\right] \tag{43}$$

subject to the lifetime budget constraint

$$\begin{aligned} c_{t-1,i}(t) + \sum_{s=1}^{T-1}\left(\frac{P(t+s)}{P(t)}\frac{c_{t-1,i}(t+s)}{\mathcal{R}^n(t,t+s)}\right) &\le e_1 w(t)\left[1-\ell_{t-1,i}(t)\right] \\ + \sum_{s=1}^{T-1}\left(\frac{P(t+s)}{P(t)}\frac{e_{s+1,i}w(t+s)\left(1-\ell_{t-1,i}(t+s)\right)}{\mathcal{R}^n(t,t+s)}\right) &+ \frac{R^n(t-1,t)\,a_{t-1,i}(t-1)}{P(t)}. \end{aligned} \tag{44}$$

In this "remaining lifetime" budget constraint, we can see from Step 2 above what the (nominal) value of $a_{t-1,i}(t-1)$ must have been from last period, namely

$$a_{t-1,i}(t-1) = P(t-1)\,w(t-1)\left(e_{0,i}-\bar{e}_i\right). \tag{45}$$

We can therefore find the value of $R^n(t-1,t)\,a_{t-1,i}(t-1)$ to be

$$R^n(t-1,t)\,a_{t-1,i}(t-1) = R^n(t-1,t)\,P(t-1)\,w(t-1)\left(e_{0,i}-\bar{e}_i\right). \tag{46}$$

We can use the policy rule $P(t) = \frac{R^n(t-1,t)}{\lambda^r(t-1,t)} P(t-1)$ and the law of motion for $w(t)$ to obtain

$$\begin{aligned} R^n(t-1,t)\, a_{t-1,i}(t-1) &= R^n(t-1,t) \frac{P(t)\,\lambda^r(t-1,t)}{R^n(t-1,t)} \frac{w(t)}{\lambda^r(t-1,t)} (e_{0,i} - \bar{e}_i) \\ &= P(t)\, w(t)\, (e_{0,i} - \bar{e}_i). \end{aligned} \tag{47}$$

Therefore, the entire real-valued term is given by

$$\frac{R^n(t-1,t)\, a_{t-1,i}(t-1)}{P(t)} = w(t)\,(e_{0,i} - \bar{e}_i). \tag{48}$$

This term is linear in $w(t)$, and since it enters the budget constraint in a lump-sum fashion, it does not affect the FONCs.

We now substitute the policy rule into the rest of the budget constraint to obtain a new version of the remaining lifetime budget constraint:

$$c_{t-1,i}(t) + \sum_{s=1}^{T-1} \left(\frac{c_{t,i}(t+s)}{\Lambda(t,t+s)} \right) \leq w(t) \sum_{s=0}^{T-1} e_{s+1} \left(1 - \ell_{t-1,i}(t+s)\right). \tag{49}$$

The FONCs for $s = 1, ..., T-1$ with respect to consumption can be expressed as

$$\frac{\eta}{c_{t-1,i}(t)} = \mu_i, \tag{50}$$

$$\frac{\eta}{c_{t-1,i}(t+s)} = \frac{\mu_i}{\Lambda(t,t+s)}, \tag{51}$$

which implies

$$c_{t-1,i}(t+s) = \Lambda(t,t+s)\, c_{t-1,i}(t). \tag{52}$$

The FONCs for $s = 0, ..., T-1$ with respect to ℓ are

$$\frac{1-\eta}{\ell_{t-1,i}(t+s)} = \mu_i w(t)\, e_{s+1,i}. \tag{53}$$

We combine each of these with the corresponding FONC for consumption to give the following choices for leisure for $s = 0, ..., T-1$,

$$\ell_{t-1,i}(t+s) = \frac{1-\eta}{\eta} \frac{c_{t-1,i}(t)}{w(t)\, e_{s+1,i}}. \tag{54}$$

We can now substitute back into the remaining life budget constraint. This can be written as

$$Tc_{t-1,i}(t) = w(t) \left[\sum_{s=1}^{T} e_{s,i} \left(1 - \frac{1-\eta}{\eta} \frac{c_{t-1,i}(t)}{w(t)\, e_{s,i}}\right) + \frac{T}{T+1} e_{0,i} - \frac{1}{T+1} \sum_{s=1}^{T} e_{s,i} \right], \tag{55}$$

or

$$c_{t-1,i}\left(t\right)=\eta w\left(t\right)\bar{e}_i, \tag{56}$$

which is linear in $w\left(t\right)$.

Equations (38) and (56) show that agents born at different dates but sharing the same productivity profile i will consume the same amount at date t. Similar logic applies for agents born at earlier dates $t-2,...,t-T$. This demonstrates the "equity share contracting" feature of the equilibrium.

Household i would then also have a desired real net asset position:

$$\begin{aligned}\frac{a_{t-1,i}\left(t\right)}{P\left(t\right)} &= e_{1,i}w\left(t\right)\left(1-\ell_{t-1,i}\left(t\right)\right)-c_{t-1,i}\left(t\right)+\frac{R^n\left(t-1,t\right)a_{t-1,i}\left(t-1\right)}{P\left(t\right)} \\ &= \left(e_{0,i}+e_{1,i}-2\bar{e}_i\right)w\left(t\right). \end{aligned} \tag{57}$$

For all other households i at date t who entered the economy at date $t-2, t-3, ..t-T$, consumption and net assets at date t will also be linear in $w(t)$.

Step 5. The general equilibrium condition is that net assets sum to zero,

$$0=\sum_{s=0}^{T-1}\frac{a_{t-s}\left(t\right)}{P\left(t\right)}, \tag{58}$$

or equivalently that

$$0=\sum_{s=0}^{T-1}a_{t-s}\left(t\right). \tag{59}$$

Suppose $T=2$. We have seen above that

$$\frac{a_{t,i}\left(t\right)}{P\left(t\right)}=w\left(t\right)\left(e_{0,i}-\bar{e}_i\right) \tag{60}$$

and that

$$\frac{a_{t-1,i}\left(t\right)}{P\left(t\right)}=\left(e_{0,i}+e_{1,i}-2\bar{e}_i\right)w\left(t\right). \tag{61}$$

The general equilibrium condition can therefore be written as

$$w\left(t\right)\left(e_{0,i}-\bar{e}_i\right)+\left(e_{0,i}+e_{1,i}-2\bar{e}_i\right)w\left(t\right) = 0 \tag{62}$$

$$w\left(t\right)\left[\left(e_{0,i}-\bar{e}_i\right)+\left(e_{0,i}+e_{1,i}-2\bar{e}_i\right)\right] = 0 \tag{63}$$

$$w\left(t\right)\left[e_{0,i}+e_{1,i}+e_{0,i}-3\bar{e}_i\right] = 0 \tag{64}$$

$$w\left(t\right)\left[e_{0,i}+e_{1,i}+e_{0,i}-\left(e_{0,i}+e_{1,i}+e_{2,i}\right)\right] = 0. \tag{65}$$

This last line will equal zero if $e_{0,i}=e_{2,i}\ \forall i$, which is the symmetry condition concerning life-cycle productivity profiles we have imposed on the model. Similar logic applies for $T>2$.

The policy rule we imposed in Step 1 modified the agents' problems in Steps 2, 3, and 4. As noted above, these modified problems were exactly the same ones that would have been generated had there been no nominal friction in the model and we had instead written the model entirely in real terms and guessed that model equilibrium would be characterized by the equality of the real interest rate and the real output growth rate at each date. The implied behavior of the households shows in Step 4 that this guess turns out to be correct. We conclude that the stochastic equilibrium is characterized by $R(t-1,t) = \lambda(t-1,t)\ \forall t$.