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A Quantitative Analysis of the Countercyclical Capital Buffer

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Abstract

What are the quantitative macroeconomic effects of the countercyclical capital buffer (CCyB)? I study this question in a nonlinear DSGE model with occasional financial crises, which is calibrated and combined with US data to estimate sequences of structural shocks. Raising capital buffers during leverage expansions can reduce the frequency of crises by more than half. A quantitative application to the 2007-08 financial crisis shows that the CCyB in the 2.5% range (as in the Federal Reserve's current framework) could have greatly mitigated the financial panic of 2008, for a cumulative gain of 29% in aggregate consumption. The threat of raising capital requirements is effective even if this tool is not used in equilibrium.

JEL Codes: E4, E6, G2
Keywords: countercyclical capital buffer, financial crises, macroprudential policy

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1 Introduction

The 2008-2009 financial crisis and subsequent Great Recession triggered a large debate among academics and policymakers that eventually led to large-scale reforms and policy recommendations in terms of financial regulation. Of particular concern were the (previously overlooked) links between the financial sector and the macroeconomy (Gertler and Gilchrist, 2018). This discussion has sparked interest on the design and implementation of macroprudential policies: a series of policy tools aimed at preventing the buildup of fragilities in the financial system that could then trigger crises with severe macroeconomic consequences.

One of the pillars of the new global framework for financial regulation, the Third Basel Accord, is a discretionary countercyclical capital buffer (known as the CCyB) that allows regulators to raise bank capital requirements during periods of credit expansion or when the buildup of vulnerabilities is perceived. According to the basic guidelines provided by the Bank of International Settlements (BIS), as of January 2019, banks that are subject to the Basel rules are required to maintain a minimum common equity tier 1 capital ratio of 7% (of risk-weighted assets). National regulators possess the discretion to require up to an additional 2.5% (Basel Committee, 2010). The basic idea is to force financial institutions to hold more capital when vulnerabilities are detected, so as to allow them to enter any potential downturns with a sufficiently high capital buffer. This buffer increases their distance to default and prevents other institutional and market-based constraints from binding, which could trigger fire sales along with other situations that can potentially deepen downturns.

The CCyB framework was formally introduced in the U.S. in September 2016 and is set by the Board of Governors of the Federal Reserve System, who votes at least once a year on its level.[1] The Fed Board reserves the right to activate the CCyB when "[...] systemic vulnerabilities are meaningfully above normal [...]", and "[...] expects to remove or reduce the CCyB when the conditions that led to its activation abate or lessen and when the release of CCyB capital would promote financial stability."[2] So far, the Board has voted twice on the level of the CCyB — in December 2017 and March 2019 — always deciding to leave it at zero.[3] In recent times, in the face of surging financial markets, several prominent policymakers and academics have advocated an increase of the CCyB — including members of the Federal Open Market Committee[4] — but this decision ultimately rests with the members of the Fed

[1] Formally, the Fed Board sets it for banking organizations with greater than $250 billion in total assets or $10 billion in on-balance-sheet foreign exposure (Federal Reserve Board, 2016).

[2] Federal Reserve Board (2016).

[3] There were no dissenting votes in December 2017, with Governor Brainard casting the single dissenting vote in March 2019.

[4] These include the presidents of the Federal Reserve Banks of Cleveland, Boston, and Minneapolis (see https://www.ft.com/content/ec8e07ee-ab08-11e8-94bd-cba20d67390c).

Board.[5]

This paper provides a positive and quantitative analysis of the effects of the CCyB. The starting point is a New Keynesian model with an explicit financial sector: impatient borrowers take on mortgages to purchase houses. These mortgages are originated by banks and are subject to endogenous default risk. Banks are subject to a financial friction (they cannot issue equity) and to a regulatory constraint in the spirit of a Basel-capital requirement. Banks fund themselves using retained earnings and deposits that are lent by savers. Due to costly liquidation, banks are subject to runs on their deposits, as in Gertler et al. (2018). Periods of high bank leverage give rise to run equilibria on the banking sector, which can then materialize via a coordination device (such as a sunspot). The final key ingredient in the model is nominal rigidities: runs destroy the banking sector, which triggers the collapse of intermediation between borrowers and savers. As borrowers have a higher marginal propensity to consume than savers, and cannot borrow from a dilapidated financial system, their consumption falls; this fall in aggregate consumption then transmits itself to a fall in GDP due to nominal rigidities.

In the model, the CCyB can be used for two important purposes. First, they can be used as an instrument ex-post: during a run, the capital of the banking sector is depleted, leading to a large rise in spreads and contraction of credit. Regulatory constraints such as capital requirements make the problem worse due to the traditional financial accelerator effect (Bernanke et al., 1996) that is compounded by endogenous default (Faria-e-Castro, 2018). Lowering capital requirements helps relax these constraints, helping reduce spreads and increase lending during periods when these constraints bind. These are the periods when the marginal propensity to consume of borrowers is the highest (as credit is scarce). Lowering capital requirements helps then resume the intermediated flow of credit and contributes to attenuating potentially large drops in GDP.

Second, the CCyB also play a more traditional role as an ex-ante instrument: by committing to raising capital requirements when the economy approaches the run region, the regulator can keep the economy away from that region. This action reduces lending and raises spreads and net interest margins. These forces, in turn, help build bank capital and prevent the economy from even entering the run region. I show that, by committing to raising capital requirements when bank leverage is sufficiently high, the regulator can almost avoid runs altogether in this model. Additionally, not only are crises less frequent, this commitment also ensures that banks are better capitalized in the event of a crisis, therefore reducing its severity. A global stochastic solution to the model is crucial to generating the nonlinearities inherent to this result.

[5] See also Liang (2017) and Furman (2018) for notable arguments for CCyB.

I calibrate the model to the U.S. economy in the pre-2008 period and combine it with macrofinancial data to study historical policy counterfactuals. First, I use the basic model without the CCyB as a measurement device and use a particle filter to estimate sequences of structural shocks for the U.S. economy around this period. Then, I use these estimated sequences of shocks to ask the following question: what would the 2008-2009 financial crisis have looked like if the CCyB were activated? I also decompose the relative contribution of being able to *raise* the CCyB before the crisis (the ex-ante benefits) and that of being able to *lower* them during the crisis (the ex-post benefits). I find that the benefits of being able to raise capital requirements ex-ante amount to 28.8% of real aggregate consumption between 2007Q1 and 2010Q4. The benefits of being able to both raise capital requirements ex-ante and lower them ex-post are similar, 29.4% of aggregate consumption, which suggests that the bulk of these gains is due to the ex-ante component. Last but not least, my model-based estimates imply that being able to capital requirements before the crisis could have basically prevented the financial crisis altogether, but not a subsequent recession. Finally, I show that while the policy is in place, capital requirements are never effectively raised in the path of the crisis. Just the *threat* of raising them is enough to trigger precautionary motives that prevent the economy from entering the crisis in the first place.

Relation to the Literature From a modeling perspective, this paper combines the non-linear New Keynesian model with long-term risky mortgages and a constrained banking sector as in Faria-e-Castro (2018) with endogenous financial crises and bank runs as in Gertler et al. (2018). This is a model of endogenous financial crises where aggregate demand externalities are crucial for the transmission of financial shocks to real activity, consistent with the empirical findings of Mian et al. (2017).[6]

A significant body of literature on macroprudential policy and optimal capital requirements has emerged in the wake of the 2008-2009 financial crisis and is too large to be reviewed here. Admati and Hellwig (2013) provide a comprehensive overview of the post-crisis debate on bank regulation. Closer in spirit to the present paper are works that study optimal capital requirements in the context of DSGE models.

A first generation of such models studies the optimal *level* of capital requirements, typically in a setting where the regulator chooses between investment growth or the benefits of liquidity provision, over the benefits of preventing financial crises. Examples of such analyzes include Van den Heuvel (2008), Nguyen (2014), Martinez-Miera and Suarez (2014), Begenau

[6]Other treatments of endogenous financial panics include Paul (2017). Like in Gertler et al. (2018), the transmission of financial shocks to the real sector occurs mainly via the production side of the economy.

(2015), and Landvoigt and Begenau (2016).[7]

A second generation of models studies how capital requirements should *optimally change* depending on current economic and financial conditions. Karmakar (2016) studies the effects of countercyclical capital requirements in the context of a real DSGE model and finds that raising capital requirements reduces volatility and raises welfare. Davydiuk (2017) shows that countercyclical capital requirements arise as the optimal Ramsey policy in a setting where the social planner tries to curb excessive lending while ensuring liquidity provision by banks. Elenev et al. (2018) study capital requirements in a model where banks can engage in excessive lending to the corporate sector; as in other papers, they find that an increase in capital requirements curbs lending/the size of the financial sector, while reducing financial fragility. They also look at the redistributive effects of capital requirements and find that an increase in capital requirements redistributes resources away from depositors and toward bankers. Finally, they find that current levels of capital requirements seem to be close to optimal. Empirically, Jiménez et al. (2017) confirm the benefits of the CCyB for the case of Spain. Mendicino et al. (2018) study the setting of optimal capital charges in a triple-decker model of default calibrated to the euro area.

Poeschl and Zhang (2018) also study a nonlinear DSGE model with anticipated banking panics, but focus on the unintended consequences of tightening capital requirements on retail banks, which can lead to intermediation to shift to the shadow banking sector and reduce financial stability.

Contrary to the predominantly normative analyses in this literature, the focus of this paper is quantitative. The main goal is to develop a quantitative framework that can be combined with data in order to study policy counterfactuals. In this spirit, I contribute to this literature by (i) showing quantitatively that the CCyB can have both ex-ante and ex-post benefits and (ii) performing a quantitative analysis of the CCyB in the U.S. economy. In particular, I estimate the model-implied probabilities of a systemic bank run in the U.S. during the financial crisis of 2008-2009 and provide model-based estimates for the potential benefits of the CCyB both before and during financial crises.

2 Model

The model extends Faria-e-Castro (2018) to include anticipated banking panics as in Gertler et al. (2018). The model is set up in discrete and infinite time, $t = 0, 1, 2, \ldots$. The economy is populated by four types of agents: households, who can be either borrowers or savers;

[7] There are also several empirical or semi-structural studies on this topic. An example of a detailed empirical analysis is Firestone et al. (2017).

commercial banks; a corporate sector consisting of intermediate goods producers and final goods retailers; and a central bank.

The structure of the model is summarized in Figure 1: borrowers differ from savers to the extent that they derive utility from housing services and can finance housing purchases by borrowing in long-term debt contracts. Banks intermediate funds between savers and borrowers, originating long-term loans and borrowing in short-term deposits. Both borrowers and savers supply their labor to monopolistically competitive producers of intermediate goods, who in turn supply a representative retailer of final goods. Borrowers can default on their payments to the bank, and banks are potentially subject to runs on their deposits. The central bank sets the policy rate using a standard Taylor rule.

There are three exogenous shocks in the model: a total factor productivity (TFP) shock to the production function, a deposit funding shock, and a sunspot that selects the equilibrium when the economy enters a region where bank runs are possible. Markets are incomplete, and all financial contracts take the form of risky debt.

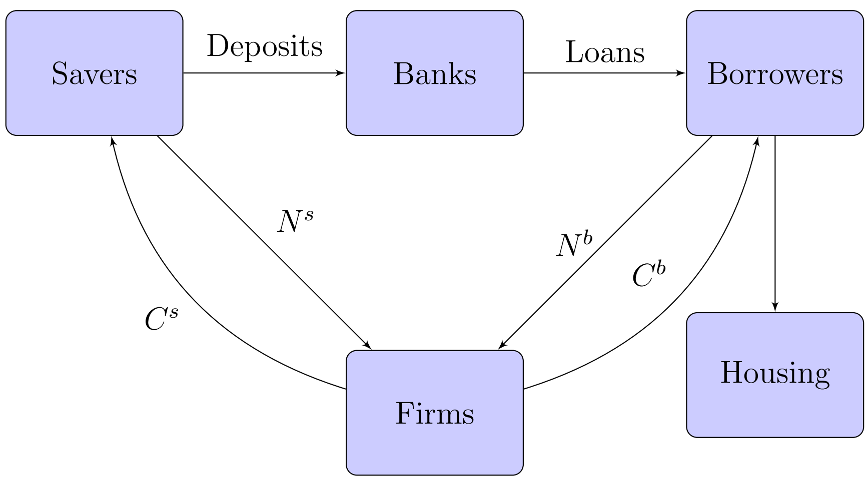


Figure 1: Structure of the model.

2.1 Environment

2.1.1 Household Preferences

There are two types of households, borrowers and savers, indexed by $i = \{b, s\}$ and in measures χ and $1 - \chi$, respectively. Households differ in terms of their preferences and the types of financial assets they have access to. Savers invest in short-term bank deposits, while borrowers can own houses and borrow in long-term debt contracts. Savers own all firms and banks in the economy.

Both borrowers and savers seek to maximize the present discounted sum of utility flows,

$$V_t^i = (1-\beta_i)u_t^i + \beta_i \mathbb{E}_t(V_{t+1}^i) \tag{1}$$

Household preferences differ in two dimensions: borrowers derive utility from houses and are more impatient, $\beta_b < \beta_s$. Instantaneous utility is defined over streams of consumption C_t^i, labor N_t^i, and housing h_t^i and is given by

$$u_t^i = \log(C_t^i) - \phi_i \frac{(N_t^i)^{1+\varphi}}{1+\varphi} + \xi^i \log(h_t^i)$$

Logarithmic preferences over consumption implicitly set the elasticity of intertemporal substitution to 1; φ is the inverse of the Frisch elasticity of labor supply, ϕ_i is a parameter governing the disutility of labor, and ξ^i is the preference parameter for housing. I assume that $\xi^b > 0 = \xi^s$, so that savers do not derive any utility from housing services. This is not a crucial assumption and is made for simplicity.[8]

2.1.2 Savers

Savers maximize utility (1) subject to a sequence of budget constraints of the type

$$P_t C_t^s + \delta_t Q_t^d P_t D_t + Q_t P_t B_t^g = P_t w_t N_t^s + Z_t^d P_{t-1} D_{t-1} + P_{t-1} B_{t-1}^g + \Gamma_t$$

where P_t is the price level, D_t are real deposits, B_t^g are risk-free bonds in zero net supply, Q_t is the inverse of the nominal interest rate, w_t is the real wage, and Γ_t are net profits and transfers from the corporate and financial sectors. Savers own all firms and banks in this economy. Z_t^d is the payoff per unit of deposits, only realized at t due to the possibility of bank failure and liquidation as explained below. δ_t is a shock that affects the relative appetite for bank deposits vis-a-vis government bonds; this is a bank funding shock that is independent of the riskiness of the banking sector. Saver first-order conditions are standard and consist of asset-pricing conditions for deposits and for government debt (the Euler equation) as well as an intratemporal labor supply condition.[9] It is useful to define the saver's stochastic discount factor for real payoffs:

$$\Lambda_{t,t+1}^s \equiv \beta \frac{C_t^s}{C_{t+1}^s} \tag{2}$$

[8] All results hold as long as the housing markets in which borrowers and savers participate are fragmented.

[9] All equilibrium conditions, including the saver's optimality conditions, are reported in Appendix A.1.

2.1.3 Borrowers

Borrowers derive utility from housing services and borrow in long-term debt contracts to finance house purchases.

Debt Contracts, Default, and Foreclosures Banks offer long-term debt contracts to borrowers: each contract has a face value of \$1 and a market price of Q_t^b. These contracts are geometrically decaying perpetuities with a coupon/decay rate of $\gamma \in [0,1]$, as in Woodford (2001). To obtain partial default in equilibrium while keeping the model environment tractable, I assume a family construct for the borrower.[10] The borrower family enters period t with an outstanding nominal debt balance $P_{t-1}\bar{B}_{t-1}^b$ and a total stock of housing h_{t-1}.[11]

At the beginning of the period, the borrower family is split into a continuum of members indexed by $i \in [0,1]$, each receiving an equal share of the debt balance and housing stock $(P_{t-1}\bar{B}_{t-1}^b, h_{t-1})$. Each of these members is then subject to two idiosyncratic shocks: first, they receive a *moving* shock with probability m, which determines whether they have to sell their house and move. After the moving shock is realized, each member i receives a *housing quality* shock $\nu_t(i)$, drawn from some distribution $F^b[0, +\infty)$ and satisfying $\mathbb{E}_t[\nu_t(i)] = 1, \forall t$.

Family members who do not move (a fraction $1 - \mathrm{m}$) simply fulfill their debt payment in the current period $\gamma \times P_{t-1}\bar{B}_{t-1}^b$. Household members that move (a fraction m) decide whether to prepay their debt balance and sell their home or to default on the mortgage and walk away from their home. The debt balance prepayment is worth $P_{t-1}\bar{B}_{t-1}^b$, and the market value of their house is $P_t p_t^h \times \nu_t(i) h_{t-1}$ given the quality adjustment. Upon default, the lender seizes the housing assets that serve as collateral; i.e., the house gets foreclosed.

Given the resale value of housing, each family member chooses to repay her maturing debt balance or default and let the bank seize her housing assets. The cost of default is the loss of housing collateral. Let $\iota(\nu) \in \{0,1\}$ denote the default choice by a member with house quality shock ν. This indicator function is equal to 1 if this member defaults on her debt repayments and zero otherwise. After default and repayment decisions are made, members reconvene in the borrower household, who then takes all relevant decisions for the current period (including the values of the states for the following period). End-of-period debt balances for the borrower family equal new borrowings L_t plus non-prepaid balances net of the current coupon:

$$P_t\bar{B}_t^b = P_t L_t + (1-\mathrm{m})(1-\gamma)P_{t-1}\bar{B}_{t-1}^b \tag{3}$$

[10] As in Landvoigt (2016) or Ferrante (2019).

[11] I use the upper bar to denote per capita variables. Since there is a mass χ of borrowers, the aggregate level of debt is $B_{t-1}^b = \bar{B}_{t-1}^b/\chi$.

Budget and Borrowing Constraints Once individual members have made their default decisions, they are regrouped in the borrower household, who chooses all control variables: consumption, labor supply, new borrowing, and new housing as well as the default rules for each individual member.[12] The budget constraint written in real terms is

$$
\begin{aligned}
&C_t^b + \frac{\bar{B}_{t-1}^b}{\Pi_t}\left[(1-\mathrm{m})\gamma + \mathrm{m}\int[1-\iota_t(\nu)]\mathrm{d}F^b\right] + p_t^h h_t^* \\
=&w_t N_t^b + Q_t^b L_t + p_t^h h_{t-1}\mathrm{m}\int \nu[1-\iota_t(\nu)]\mathrm{d}F^b
\end{aligned} \tag{4}
$$

where h_t^* are new housing purchases. New borrowing L_t is defined by (3). The law of motion for the stock of housing is

$$
h_t = h_t^* + (1-\mathrm{m})h_{t-1} \tag{5}
$$

The borrower family is subject to a loan-to-value (LTV) constraint on new borrowing; new debt balances contracted this period cannot exceed a fraction of the value of new housing purchases:

$$
L_t \leq \theta^{LTV} p_t^h h_t^* \tag{6}
$$

Optimality The borrower household chooses $(C_t^b, L_t, N_t^b, h_t, \{\iota_t(\nu)\}_{\nu\in[0,+\infty)})$ to maximize (1) subject to (3)-(6). It can be shown that the optimal default decision is static and given by a threshold rule: the borrower optimally defaults on all debt prepayments for which $\nu < \nu_t^*$, where this threshold satisfies

$$
\nu_t^* = \frac{\bar{B}_{t-1}^b}{\Pi_t p_t^h h_{t-1}} \tag{7}
$$

Basically, a "moving" member of the borrower household behaves as having limited liability when it comes the time to prepay and defaults if the remaining debt balance exceeds the market value of the house. In equilibrium, default is positive and partial and the default rate fluctuates with household leverage, which in turn depends on equilibrium objects such as the house price. Another relevant optimality condition is the asset-pricing equation for housing, which takes the form

$$
p_t^h = \frac{\frac{\xi}{h_t}C_t^b + \mathbb{E}_t\left\{\Lambda_{t,t+1}^b p_{t+1}^h[(1-\mathrm{m})(1-\lambda_{t+1}^b\theta^{LTV}) + \mathrm{m}\Psi^b(\nu_{t+1}^*)]\right\}}{1-\lambda_t^b\theta^{LTV}} \tag{8}
$$

[12] This arrangement is thus implicitly equivalent to one where borrower family members are identical agents with access to a full set of contingent claims that allow them to hedge any idiosyncratic risks within the group.

where λ_t^b is the Lagrange multiplier on the borrowing constraint (6) and $\Lambda_{t,t+1}^b$ is the borrower's stochastic discount factor for real payoffs, defined analogously to (2). $\Psi^b(\nu_{t+1}^*)$ is a partial expectation term for the house quality shock, defined as

$$\Psi^b(\nu_t^*) \equiv \int_{\nu_t^*}^{\infty} \nu \mathrm{d}F^b(\nu)$$

Condition (8) highlights that changes in borrower consumption have a first-order effect on house prices, both through the current utility dividend from housing services and through the stochastic discount factor that is applied to the continuation value.

2.1.4 Corporate Sector

The corporate sector consists of final goods retailers and intermediate goods producers. Final goods retailers are perfectly competitive and employ a continuum of intermediate goods varieties indexed by $k \in [0,1]$ to produce the final good using a Dixit-Stiglitz aggregator with constant elasticity of substitution ε:

$$Y_t = \left[\int_0^1 Y_t(k)^{\frac{\varepsilon}{\varepsilon-1}} \mathrm{d}k\right]^{\frac{\varepsilon-1}{\varepsilon}}$$

There is a continuum of intermediate goods producers, each producing a different variety k. All firms are owned by the savers and have access to a linear production technology in labor,

$$Y_t(k) = A_t N_t(k)$$

where A_t is an exogenous aggregate TFP shock. Given the constant elasticity of substitution assumption, each of these firms faces a demand schedule of the type

$$Y_t(k) = \left[\frac{P_t(k)}{P_t}\right]^{-\varepsilon} Y_t$$

I assume that firms are subject to menu costs as in Rotemberg (1982), with a standard quadratic functional form of the type

$$d\left[P_t(k), P_{t-1}(k)\right] \equiv \frac{\eta}{2} Y_t \left[\frac{P_t(k)}{P_{t-1}(k)}\Pi^{-1} - 1\right]^2$$

where Π is the inflation target set by the central bank (so that it is free to adjust prices to keep up with trend inflation) and η is the menu cost parameter. It can be shown that the first-order condition for an individual price-setting firm k combined with the assumption of a symmetric equilibrium yields a standard (nonlinear) Phillips curve that relates inflation to aggregate output:

$$\eta \frac{\Pi_t}{\Pi}\left(\frac{\Pi_t}{\Pi}-1\right)+\varepsilon\left(\frac{\varepsilon-1}{\varepsilon}-\frac{w_t}{A_t}\right)=\eta \mathbb{E}_t\left[\Lambda^s_{t,t+1}\frac{Y_{t+1}}{Y_t}\frac{\Pi_{t+1}}{\Pi}\left(\frac{\Pi_{t+1}}{\Pi}-1\right)\right] \tag{9}$$

2.1.5 Financial Sector

Banks borrow in short-term deposits and hold long-term mortgages. I assume that banks hold perfectly diversified portfolios of household debt, so that credit risk is systemic. I assume that liquidation of bank assets is costly, which potentially exposes banks to runs on their portfolios.

There is a continuum of banks indexed by $j \in [0,1]$, wholly owned by savers. Bank j enters the period with a portfolio of debt securities $b_{j,t-1}$ and deposits $d_{j,t-1}$. Each deposit entitles its owner to a unit repayment, while each debt security yields an aggregate payoff of Z^b_t. Nominal earnings at the beginning of the period are equal to

$$P_t e_{j,t} = Z^b_t P_{t-1} b_{j,t-1} - P_{t-1} d_{j,t-1} \tag{10}$$

Bank Runs and Failures If $e_{j,t} < 0$, bank j defaults and its assets are liquidated to provide pro rata payments to depositors. I assume that liquidation is costly and entails a deadweight cost equal to a fraction λ^d of the value of the assets, hence the recovery rate is equal to $1-\lambda^d$. These liquidation costs create a region of the state space where bank runs can be an equilibrium. In particular, consider the situation where

$$\begin{aligned} Z^b_t b_{j,t-1} - d_{j,t-1} &\geq 0 \\ (1-\lambda^d) Z^b_t b_{j,t-1} - d_{j,t-1} &< 0 \end{aligned}$$

This is a situation where the bank is solvent, as the market value of its assets exceeds the value of repayments on its liabilities, but illiquid: if it were to liquidate all of its assets, it would not be able to repay all of its depositors. From the point of view of an individual depositor, there is an incentive to force early liquidation if all other depositors intend to do so.[13] It is useful to define the following variables

$$u^F_{j,t} = \frac{d_{j,t-1}}{Z^b_t b_{j,t-1}}$$
$$u^R_{j,t} = \frac{d_{j,t-1}}{(1-\lambda^d) Z^b_t b_{j,t-1}}$$

Note that $u^R_{j,t} > u^F_{j,t}$ (as long as $\lambda^d > 0$). Whenever $u^R_{j,t} > 1$, bank j becomes exposed to a run equilibrium. For simplicity, I use a sunspot as a selection device for banks that are in this "run region": ω_t triggers a run whenever it takes a value of 1, which happens with probability p. With probability $1-p$, we have that $\omega_t = 0$ and no run takes place for banks with $u^R_{j,t} > 1$. Whenever $u^F_{j,t} > 1$, bank j becomes insolvent and fails with probability 1. The conditional probability of bank failure next period is therefore given by

$$\begin{aligned}
\mathbb{P}_t(\text{failure}_{j,t+1}) &= \mathbb{P}_t(u^F_{j,t+1} > 1 \vee [u^F_{j,t+1} \leq 1 \wedge u^R_{j,t+1} > 1 \wedge \omega_{t+1} = 1]) \\
&= \mathbb{P}_t(u^F_{j,t+1} > 1) + \mathbb{P}_t(u^F_{j,t+1} \leq 1 < u^R_{j,t+1} \wedge \omega_{t+1} = 1) \\
&= \mathbb{P}_t(u^F_{j,t+1} > 1) + p \times \mathbb{P}_t(u^F_{j,t+1} \leq 1 < u^R_{j,t+1}) = \mathbb{E}_t x_{j,t+1}
\end{aligned}$$

where $x_{j,t}$ is an indicator that is equal to 1 when the bank defaults. Importantly, this probability depends not only on the realizations of aggregate shocks next period, but also on endogenous decisions that are taken today (the ratio of assets to liabilities, bank leverage).

Financial Frictions I assume that due to contractual frictions that are left unmodeled, banks are forced to pay a constant fraction $1-\theta$ of their earnings as dividends every period. Thus $\theta \in [0,1]$ is the fraction of earnings that are retained. To fund their assets, banks need to use either retained earnings or new deposits. This gives rise to a flow-of-funds constraint, expressed in real terms as

$$Q^b_t b_{j,t} = \theta e_{j,t} + Q^d_t d_{j,t} \tag{11}$$

The bank also faces a leverage constraint, which constrains the market value of its assets not to exceed the ex-dividend market value of the bank. Let $V_{j,t}(e_{j,t})$ denote the value of the bank at the beginning of the period, before dividends are paid. The ex-dividend value of the bank is then given by

$$\Phi_{j,t}(e_{j,t}) \equiv V_{j,t}(e_{j,t}) - (1-\theta) e_{j,t}$$

[13] While I do not microfound the depositor problem, it is straightforward to do so using existing models of coordination-based bank runs.

The constraint imposes that the value of bank equity always exceed a fraction κ_t of the value of the bank's assets,

$$\Phi_{j,t}(e_{j,t}) \geq \kappa_t Q_t^b b_{j,t} \tag{12}$$

This constraint effectively caps the amount of lending that banks can offer every period. Banks seek to maximize the present discounted value of their dividends. The bank's problem, conditional on not having defaulted this period, is then

$$V_{j,t}(e_{j,t}) = \max_{b_{j,t},d_{j,t}} \left\{ (1-\theta)e_{j,t} + \mathbb{E}_t \frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}}(1-x_{j,t+1})V_{j,t+1}(e_{j,t+1}) \right\} \tag{13}$$

Banks solve (13) subject to the law of motion for earnings (10), the flow-of-funds constraint (11), and the capital requirement (12). A detailed derivation of the bank's problem may be found in Appendix A.2. In the appendix, I show that $\Phi_{j,t}(e_{j,t}) = \Phi_{j,t}\theta e_{j,t}$, where $\Phi_{j,t}$ can be interpreted as the marginal value of a dollar of earnings for the bank. Letting $\mu_{j,t}$ denote the Lagrange multiplier on the leverage constraint, we can write the solution to the bank's problem as

$$\mathbb{E}_t \left\{ \frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}}(1-x_{j,t+1})(1-\theta+\theta\Phi_{j,t+1}) \left[\frac{Z^b_{t+1}}{Q^b_t} - \frac{1}{Q^d_t} \right] \right\} = \kappa \mu_{j,t} \tag{14}$$

This asset-pricing condition highlights three potential sources of excess returns: current binding constraints via $\mu_{j,t}$, bank default/limited liability via $x_{j,t+1}$, and future binding constraints via $\Phi_{j,t+1}$. This last term comes from the envelope condition and is given by

$$\Phi_{j,t} = \frac{\mathbb{E}_t \left\{ \frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}}(1-x_{j,t+1})(1-\theta+\theta\Phi_{j,t+1}) \right\}}{Q^d_t(1-\mu_{j,t})} \tag{15}$$

Aggregation and Bank Entry Note that condition (15) does not depend on any bank specific variable. This means that $\Phi_{j,t} \equiv \Phi_t, \forall j$. The appendix shows that the bank's problem is homogeneous of degree 1 in the level of current earnings $e_{j,t}$. Thus all banks take decisions that are proportional to their level of current earnings. Since all banks take proportional portfolio decisions, and the sunspot is an aggregate shock that coordinates run equilibria for *all* banks, this also means that $(u^R_{j,t}, u^F_{j,t}, \mu_{j,t}) \equiv (u^R_t, u^F_t, \mu_t), \forall j$. This result allows for simple aggregation of the banking system and, in particular, allows us to focus the analysis on a representative bank whose earnings correspond to aggregate earnings for the banking system net of defaults.

Aggregate earnings $P_t E_t$ consist of retained earnings of surviving banks $P_t E^s_t$ plus earnings of new banks $P_tE_t^n$. Retained earnings for surviving banks are given by

$$P_tE_t^s = P_{t-1}(1-x_t)\theta[Z_t^b b_{j,t-1} - d_{j,t-1}]$$

I assume that, every period, savers inject an amount of equity equal to $\varpi Q_t^b P_{t-1}B_{t-1}^b$ in the banking system. During a run, this corresponds to starting equity for new banks. This implies that

$$P_tE_t^n = \varpi Q_t^b P_{t-1}B_{t-1}^b$$

and thus real aggregate bank earnings evolve as

$$E_t = (1-x_t)\theta\frac{(Z_t^b B_{t-1}^b - D_{t-1})}{\Pi_t} + \varpi Q_t^b\frac{B_{t-1}^b}{\Pi_t}$$

Asset Returns Let λ^b denote liquidation costs of default on household debt. Consider a bank that enters the period with a stock of debt securities worth B_{t-1}^b. Every period, a fraction equal to $1-\mathrm{m}$ of these mortgage holders pay their coupon γ and the remaining principal can be sold at price Q_t^b. Out of the remaining fraction m, a fraction $1-F^b(\nu_t^*)$ prepay in full. The remaining mortgages are foreclosed and liquidated by the banks (who immediately resell these houses to borrowers in the housing market). The payoff per dollar of debt securities is therefore given by

$$Z_t^b \equiv (1-\mathrm{m})[(1-\gamma)Q_t^b+\gamma] + \mathrm{m}\left[1-F^b(\nu_t^*) + (1-\lambda^b)\frac{1-\Psi^b(\nu_t^*)}{\nu_t^*}\right]$$

Similarly, for bank deposits, we define the unit return as Z_t^d, which can be written as

$$Z_t^d = 1 - x_t + \frac{x_t}{u_t^R}$$

2.1.6 Housing

I assume that the housing market is segmented: borrowers are the only agents that derive utility from housing services and the only agents that are allowed to hold housing assets intertemporally. This implies that house prices are fully determined by the borrower's stochastic discount factor. Movements in house prices are important in determining equilibrium default rates and generate pecuniary externalities through the borrowing constraint.[14] Foreclosed houses that are acquired by the banks are immediately resold back to borrow-

[14]This assumption of market segmentation has also been used by Garriga et al. (2017) and Greenwald (2016), for example.

ers. For simplicity, I also assume that the supply of housing is fixed and normalized to 1, $h_t = 1, \forall t$. This assumption, coupled with the fact that $\mathbb{E}_t(\nu) = 1 \forall t$, means that the total, quality adjusted supply of housing in the economy is equal to 1 at every point in time: $h_t \int \nu \mathrm{d}F^b(\nu) = 1 \forall t$.[15]

2.1.7 Central Bank and Monetary Policy

The central bank conducts conventional monetary policy by following a standard Taylor rule through which the policy rate Q_t^{-1} responds to deviations of GDP and inflation from their targets:

$$Q_t^{-1} = \bar{Q}^{-1} \left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\Pi} \left[\frac{GDP_t}{G\bar{D}P}\right]^{\phi_Y}$$

where $G\bar{D}P, \bar{Q}$ are the steady state values of output and the nominal interest rate. I define $GDP_t \equiv C_t + G_t$, that is, output net of resource costs.

2.2 Equilibrium

Equilibrium is defined in the standard way: it consists of allocations, prices, and policies such that (i) all agents choose allocations and optimize given prices and policies, (ii) prices clear markets given allocations and policies, and (iii) policies satisfy the government's budget constraint. A full list of the model's equilibrium conditions is provided in Appendix A.1.

For reference, the aggregate resource constraint is given by

$$C_t + G_t + \lambda_t^b \mathrm{m}\chi p_t^h[1 - \Psi^b(\nu_t^*)] + \lambda^d Z_t^b \frac{B_{t-1}^b}{\Pi_t} x_t = Y_t \left[1 - \frac{\eta}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2\right]$$

where $Y_t \equiv A_t N_t$ is gross output, $C_t \equiv \chi C_t^b + (1-\chi)C_t^s$ is aggregate consumption, and $N_t \equiv \chi N_t^b + (1-\chi)N_t^s$ are aggregate hours. Throughout, I focus on the fiscal multiplier of fiscal policies over GDP, which I define as total consumption by the private and public sectors:

$$GDP_t = C_t + G_t$$

3 Model Analysis

This section describes the calibration and characterizes the behavior of the model.

[15] This normalization is chosen to simplify algebra and the derivation of the aggregate resource constraints but is easily relaxed — the model can be easily extended to handle aggregate shocks to the average quality of housing.

3.1 Calibration

The period in the model is a quarter. Most parameters are chosen so that the model's stochastic steady state matches moments of the U.S. economy and financial system in the early 2000s, prior to the 2007 financial crisis. The model has several parameters, which I group into four broad categories. The calibration is summarized in Table 1.

Standard Macro Parameters The discount factor is set at $\beta = 0.9951$ to generate an annualized real interest rate of 2% at the deterministic steady state. The inverse Frisch elasticity of labor supply is set at $\varphi = 0.5$, which is standard in macroeconomic models. The elasticity of substitution across varieties is set at $\varepsilon = 6$, implying an average markup of 20% at the steady state. To choose the Rotemberg menu cost parameter, I set η such that the slope of a linearized Phillips curve would coincide with that of a Calvo-type model where the probability of readjusting the price every period is equal to 20%. This procedure yields $\eta = 98.06$.

I assume standard values for the Taylor rule parameters, $\phi_\Pi = 1.5$ and $\phi_Y = 0.5/4$. I assume that the central bank pursues an annualized inflation target of 2%.

Productivity and deposit funding shocks follow AR(1) processes in logs:

$$\log A_t = \rho_a \log A_{t-1} + \sigma_a \epsilon_t^a$$
$$\log \delta_t = \rho_\delta \log \delta_{t-1} + \sigma_\delta \epsilon_t^\delta$$

The shock parameters are jointly calibrated to match the persistence and volatility of aggregate consumption and bank funding costs (TED Spread) during the pre-crisis period.

Household Finance The model features a set of non-standard parameters related to household finance that I choose in order to match pre-crisis moments of the U.S. economy. Maximum LTV at origination, which determines how binding is the constraint for the borrower, is set at a standard value of 85%. The fraction of agents that move every period m is set to match an aggregate LTV of 60%. The housing preference parameter ξ is jointly chosen to generate a ratio of household debt to GDP of 70% at the stochastic steady state, the value in the early 2000s. The coupon rate $\gamma = 0.05$ is chosen to match a payment-to-income ratio of 35% for borrowers, consistent with the micro data. This rate implies an effective debt maturity of 5 years, close to the effective duration of mortgage contracts in the U.S. once prepayment risk is taken into account.

The credit risk distribution F^b is beta, with mean equal to one, and a dispersion parameter equal to σ_b. The beta assumption implies that we have closed-form expressions for the distribution function and partial expectations that appear in the equilibrium conditions:

$$F^b(\nu_t^*) = \left[\frac{\sigma^b \nu_t^*}{\sigma^b + 1}\right]^{\sigma^b}$$

$$\Psi^b(\nu_t^*) = 1 - \left[\frac{\sigma^b \nu_t^*}{\sigma^b + 1}\right]^{\sigma^b + 1}$$

Fraction of Borrowers I pick the fraction of borrowers χ to be 0.475, a middle-of-the-ground estimate that is consistent with common estimates in the literature. Broda and Parker (2014) estimate that around 40% of households in the U.S. are liquidity constrained, based on Nielsen survey data. Elenev et al. (2016) use several waves of the Survey of Consumer Finances (SCF) to estimate the fraction of the population with negative fixed income positions and arrive at 47%, a number very close to mine. It should be noted that while the fraction of borrowers is larger than the share of constrained agents that is used in the heterogeneous agents literature (Kaplan and Violante, 2014), the borrowers in this model are only *occasionally* constrained. During expansions, the constraint may not bind, in which case the aggregate marginal propensity to consume may fall. Thus, while the fraction of borrowers is constant, the fact that borrowers are only occasionally constrained implies that the aggregate MPC fluctuates with the business cycle, as in a model where the percentage of constrained agents is endogenous.

Banking Banking parameters are jointly calibrated to match a series of targets. The retained earnings parameter $\theta = 0.9224$ and set-up transfer $\varpi = 0.005$ are jointly chosen to match average net payouts at the stochastic steady state of around 3.5% (Baron, Forthcoming) and a mortgage spread of 2% annualized. These parameters also imply a leverage ratio of less than 10, which is consistent with leverage for large U.S. commercial banks. κ is set to 0.085, the standard Basel III level for capital requirements in the U.S. (which includes the baseline level of 6% plus the capital conservation buffer of 2.5%). The loss given default $\lambda^d = 0.10$ and probability of the sunspot $\Pr(\omega = 1) = 0.10$ are chosen to match an unconditional frequency of financial crises of 2.5%.

Parameter	Description	Value	Target
		Standard Parameters	
β	Discount factor	0.9951	Annualized real interest rate of 2%
φ	Frisch elasticity	0.5	Standard
ε	Elasticity of subst.	6	20% markup in SS
η	Rotemberg menu cost	98.06	Prices adjusted once every five quarters
		Policy Parameters	
$\bar{\Pi}$	Trend inflation	$1.02^{0.25}$	2% for the U.S.
ϕ_Π	Taylor rule: Inflation	1.5	Standard
ϕ_Y	Taylor rule: Output	0.5/4	Standard
		Borrower Parameters	
β_b	Borrower discount factor	0.9855	Constrained at steady state
χ	Fraction of borrowers	0.475	Response of consumption to ESA'08 in Parker et al. (2013)
θ^{LTV}	Maximum LTV at origination	0.85	Greenwald (2016)
m	Fraction of movers	0.116	Aggregate LTV of 55%
ξ	Housing preference	0.1418	Debt-to-GDP ratio of 70%
σ^b	House quality distr.	4.3513	Annual default rate of 0.5%
λ^b	Loss given default	0.30	FDIC data
γ	Maturity of debt	0.05	Payment-to-income ratio of 35%
		Banking Parameters	
θ	Retained earnings	0.9224	Net payouts of 3.5%
κ	Leverage constraint	0.085	Basel III minimum CR + CCB
ϖ	Transfer to new banks	0.005	Annual lending spread of 2%
λ^d	Liquidation costs	0.10	Frequency of financial crises of 2.5%
		Shock Parameters	
ρ_a	Persistence of TFP	0.900	Pre-crisis persistence of detrended consumption
σ_a	SD of TFP innovations	0.005	Pre-crisis volatility of detrended consumption
ρ_δ	Persistence of funding shock	0.500	Pre-crisis persistence of TED spread
σ_δ	SD of funding innovations	0.005	Pre-crisis volatility of TED spread
p	Sunspot probability	0.10	Frequency of financial crises of 2.5%

Table 1: Summary of the calibration.

3.2 Solution Method

The model features three main sources of nonlinearities: two occasional binding constraints (the capital requirement for banks and the LTV constraint for borrowers), as well as inherently nonlinear endogenous bank runs. Due to these three features, the model cannot be solved with traditional methods, such as log-linear approximations around the steady state. I solve the model using a global solution method that consists of a combination of time iteration (Judd et al., 2002), and parametrized expectations (den Haan and Marcet, 1990). The global solution method allows me to capture the nonlinearities that are inherent to the aforementioned features, as well as important precautionary motives and risk premia: bank runs in the model are akin to a "large disaster." The computational details of the solution method as well as robustness and accuracy checks regarding the numerical solution can be found in Appendix B.

The model features two endogenous and three exogenous state variables. The endogenous states are bank leverage $\text{lev}_{t-1} = \frac{D_{t-1}}{B^b_{t-1}}$ and household leverage B^b_{t-1}. The exogenous states are the TFP shocks A_t, the funding shock δ_t, and the sunspot ω_t.

Figures 12 and 13 in the Appendix plot (generalized) impulse response functions of selected endogenous variables to TFP and funding shocks, respectively.

3.3 Financial Crises

A crisis in the model is a period with a bank run: when $u^R_t \geq 1$ *and* the sunspot shock realizes $\omega_t = 1$. Note that a crisis has both an endogenous and an exogenous component: while a crisis is associated to the realization of an exogenous shock (the sunspot), a crisis can only occur if the economy endogenously moves to the "crisis region," a region of the state space where $u^R_t \geq 1$. In this sense, the endogeneity of crises is reminiscent of that that is present in standard models of sovereign default (Cole and Kehoe, 2000; Arellano, 2008).

3.3.1 Crisis Regions

Recall that

$$u^R_t = \frac{D_{t-1}}{Z^b_t B_{t-1}}$$

where D_{t-1}, B_{t-1} are pre-determined, endogenous states and Z^b_t is an equilibrium object that is a function of both endogenous and exogenous states at t. This suggests that the economy will be closer to a crisis the higher is leverage in the banking sector, defined as $\text{lev}_{t-1} = \frac{D_{t-1}}{B_{t-1}}$.[16] Figure 2 plots the different regions in the state space of the model: the horizontal axis is lev, a measure of bank leverage, and the vertical axis is B, a measure of household leverage. There are three regions in the state space: a "safe region" (blue), where $u^R_t < 1$ and no crisis occurs; a "run region" (green), where $u^R_t \geq 1, u^D_t < 1$, and the economy is subject to a crisis depending on the realization of the sunspot; and a "insolvency region" (yellow), where $u^D_t \geq 1$, and a crisis occurs with probability one. The figures shows us several things: (i) crises are more likely when bank leverage is high, which comes almost directly from the definition of u^R_t; (ii) crises are more likely when the face value of bank assets is relatively low (holding leverage constant); (iii) crises are more likely when TFP is low. All of these "comparative statics" are consistent with a large empirical literature on facts related to financial crises (Jordà et al., 2016).

[16] This definition of leverage consists of total liabilities cum interest payments divided by the "face value" of assets.

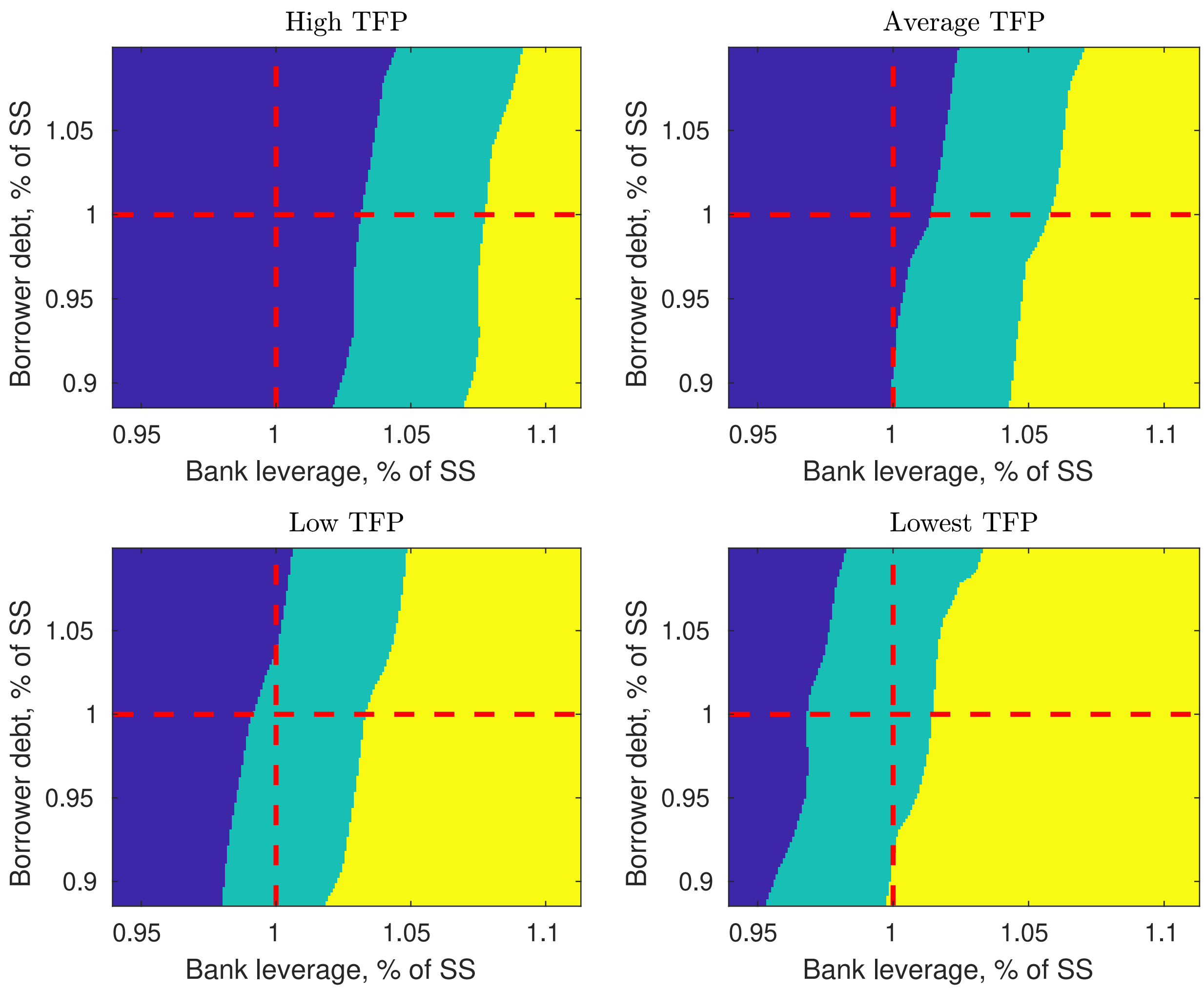


Figure 2: Model state space for different realizations of the TFP shock. The horizontal axis corresponds to lev $= \frac{D}{B^b}$, while the vertical axis is B^b. The blue area corresponds to the "safe region", where $u^R < 1$; the green area is the "illiquidity region", where $u^R \geq 1$ but $u^D < 1$; the yellow area is the "insolvency region", where $u^D \geq 1$. The dashed vertical and horizontal lines locate the stochastic steady state of the model.

Figure 2 illustrates the state space of the model but does not tell us much regarding the actual behavior of the model: it could be, for example, that agents are sufficiently risk averse such that the economy never actually exits the blue region. Figure 3 plots the distribution of states across the state space from a long simulation of the model and shows that this is not the case. The horizontal and vertical axes are the same (the endogenous states), and each point is a period in the simulation. Blue points correspond to non-crisis periods, while orange points are periods when a run has happened, $x_t = 1$.

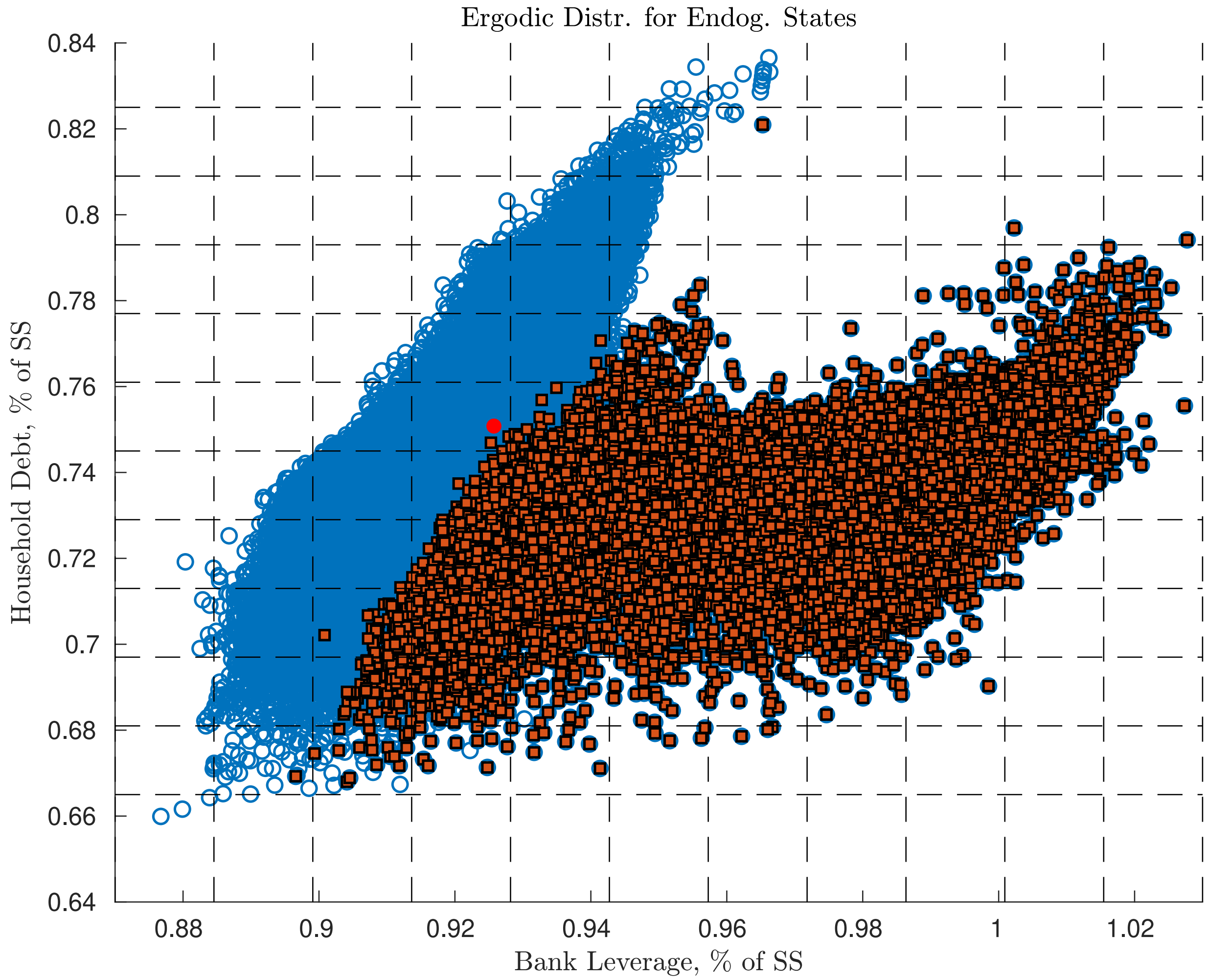


Figure 3: Joint-distribution of endogenous states from a long simulation of the model. Blue points correspond to normal periods; orange squares correspond to financial crises. The red dot is the stochastic steady state of the model.

3.3.2 The Macroeconomic Effects of a Crisis

To study the endogenous behavior of the model during a financial crisis, I simulate the model for a large number of periods and focus on the behavior of the economy when it enters into a bank run. Figure 4 plots the median behavior of GDP, borrower consumption, house prices, and credit spreads around such events, along with 95% confidence bands. Financial crises correspond to sharp contractions of GDP, consumption, and house prices, as well as large increases in credit spreads.

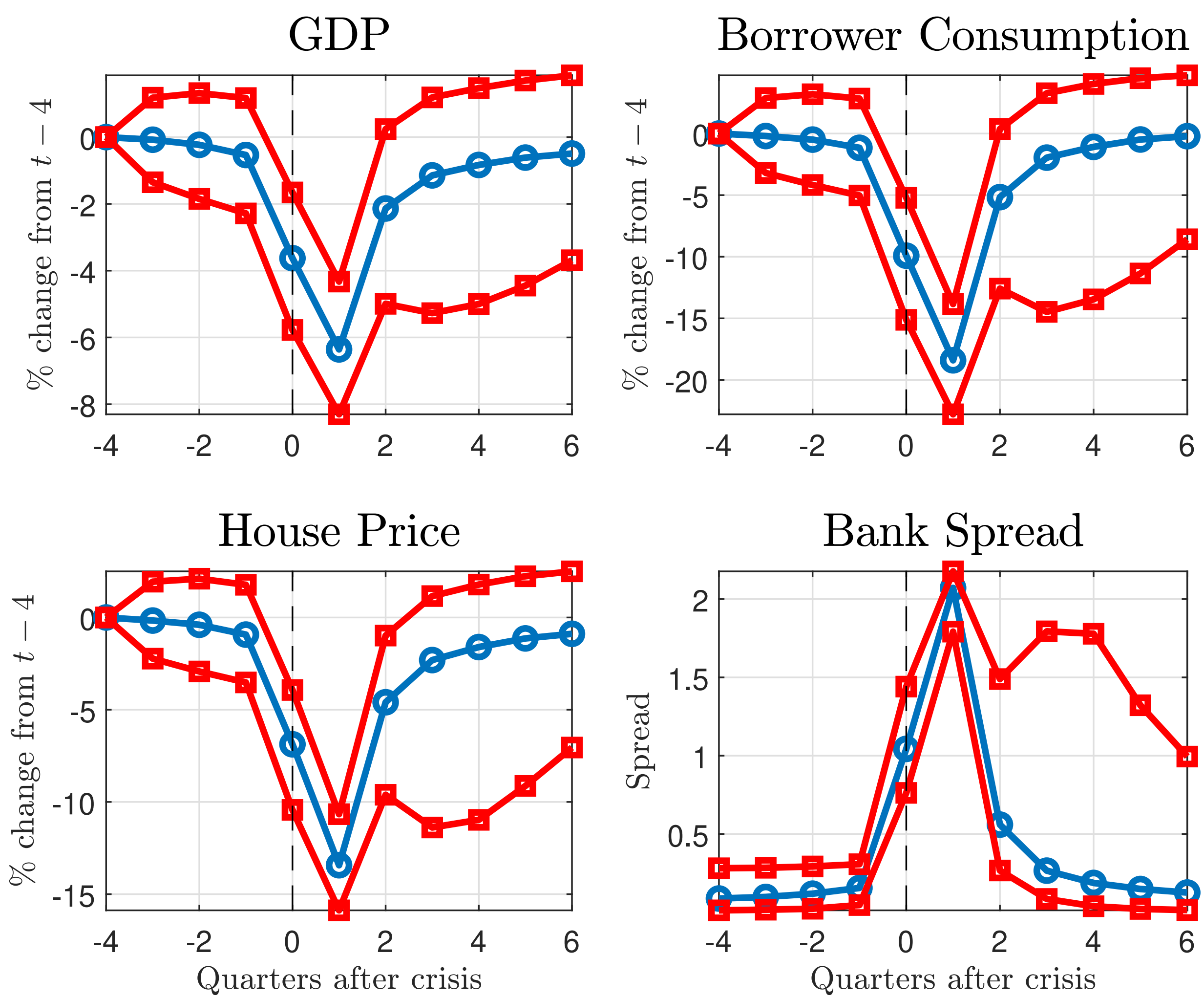


Figure 4: The blue line plots the median path of selected endogenous variables around the time the economy enters a financial crisis ($t = 0$). The red lines correspond to 95% confidence bands.

The mechanism that underlies the financial crisis is analogous to the default-collateral channel of Faria-e-Castro (2018): when a crisis starts, bank equity collapses, hampering banks' ability to intermediate. Lending to borrowers falls, and interest rates rise sharply as the banking sector struggles to satisfy capital requirements. If this drop in lending and rise in spreads is large enough, borrowers are pushed to their LTV constraint and effectively become hand-to-mouth. Since lending has fallen and interest rates have risen, disposable income falls, making borrower consumption fall almost one for one. Since borrower consumption has a first-order effect on house prices via marginal utility and the stochastic discount factor, it also causes a large collapse in house prices. This fall in house prices, in turn, raises LTV rates, which in turn endogenously lead to an increase in default rates. This further reduces bank profits, contributing to a further tightening of the constraint. This bank-borrower "doomloop" is a high-powered version of the classic financial accelerator mechanism (Bernanke et al., 1996), compounded by the endogenous default of the borrowers and the rise in deposit spreads for banks.[17] The combination of incomplete markets and demand externalities (via nominal rigidities) means that a fall in borrower consumption translates into a fall in output, throwing the economy into a recession.

3.3.3 The Macroeconomic Effects of an *Almost* Crisis

More interestingly, the probability that a crisis might occur may also trigger a recession, even if such a crisis never materializes. Figure 5 plots the median path of the economy along with 95% confidence bands for periods when the economy enters the crisis region, but manages to exit this region without a crisis ever occurring (that is, $u_t^R \geq 1$ and $\omega_t = 0$ for some periods). While the effects are more modest than those of a full blown crisis, the possibility of a crisis does cause a noticeable drop in GDP and house prices, as well as a significant rise in credit spreads. Importantly, all of these effects arise from the anticipation of a crisis: the economy is transitioning from a set of states where the probability of a crisis was zero (or at least very low) to another where the probability of a crisis rises considerably. The anticipation of a crisis can trigger a recession, even if the crisis never occurs ex-post.

4 Countercyclical Capital Buffers

I now turn to the analysis of the effects of the CCyB. I proceed in two steps: first, I show that the CCyB can be an useful instrument ex-post and that lowering them during a crisis lowers the severity of these events; second, I show that the CCyB can also be an useful ex-ante and that raising them during periods of high leverage can greatly reduce the probability of a crisis event.

4.1 Design of the CCyB

I assume that the government, via its macroprudential regulator, can steer the leverage of the banking sector via the adjustment of κ_t, the parameter in the leverage constraint. The baseline rule is a discrete rule that raises the capital buffer to its limit and lowers it back to its standard value whenever certain conditions are met. This rule is specified as follows,

[17] Depending on the constellation of endogenous states that helps trigger the crisis, the crisis can last for more than one period, in which case this mechanism is further compounded by a rise in bank deposit rates that contributes to raising bank leverage.

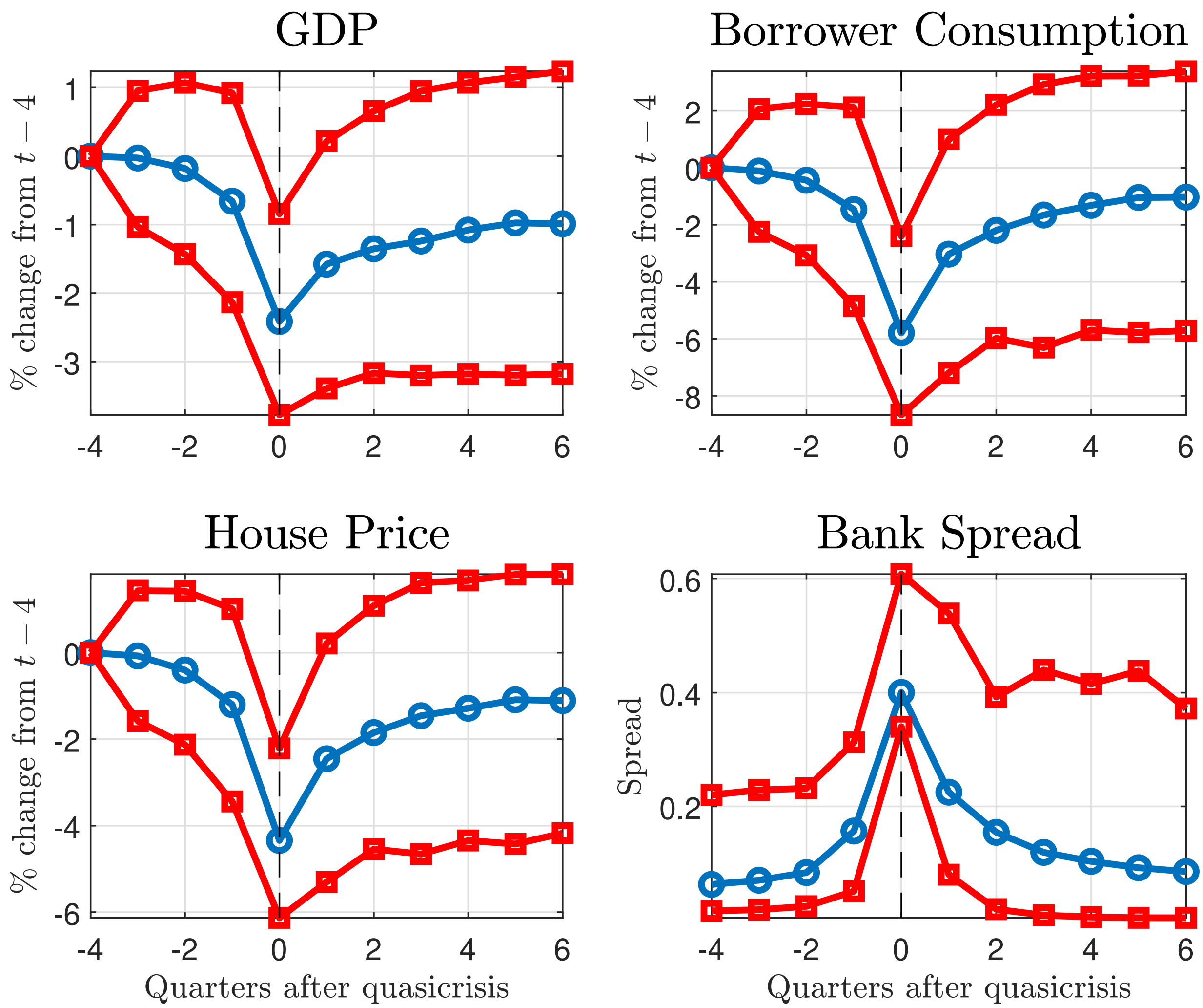


Figure 5: The blue line plots the median path of selected endogenous variables around the time the economy enters the run region ($t = 0$), but exits it with no crisis ever occurring. The red lines correspond to 95% confidence bands.

$$\kappa_t = \begin{cases} \kappa_{\text{hi}}, & \text{for } u_t^R \geq 1, \omega_t = 0 \\ \bar{\kappa}, & \text{for } u_t^R < 1 \\ \kappa_{\text{low}}, & \text{for } u_t^R \geq 1, \omega_t = 1 \end{cases} \tag{16}$$

where $\kappa_{\text{hi}} \geq \bar{\kappa} \geq \kappa_{\text{lo}}$. That is, the macroprudential regulator can set three levels of capital requirements. The baseline level, when the economy is in the safe region, is $\bar{\kappa}$. When the economy enters the run region, but no crisis has materialized, the regulator raises capital requirements in order to lower bank leverage and make the economy exit the run region. If, however, a run occurs, the regulator can lower capital requirements below their standard level in order to relax bank constraints and help break the collateral-default financial accelerator that was described in the previous section.

It should be noted that this specification for macroprudential policy is slightly different and richer than what is prescribed by Basel III. In particular, the current implementation framework for the CCyB in the U.S. is a particular case of the above framework, where $\kappa_{\text{low}} = \bar{\kappa}$. That is, the Board of Governors can raise capital requirements over and above standard levels but does not have the authority to lower them beyond standard levels during periods of distress. A potential critique of the two policy rules considered in this paper is that they requires real-time knowledge of what u_t^R is. On the other hand, this is the only variable that the regulator needs to keep track of and therefore becomes a sufficient statistic for the setting of macroprudential policy. In line with the current U.S. framework, and from a baseline level of $\bar{\kappa} = 8.5\%$, I set $\kappa_{\text{hi}} = 11\%$, and $\kappa_{\text{low}} = 6\%$.

4.2 Ex-post effects of CCyB

I first focus on an economy where $\kappa_{\text{hi}} = \kappa_{\text{med}}$. That is, the regulator can lower capital requirements during a crisis but cannot raise them ex-ante. This exercise is useful to isolate the ex-post benefits of the proposed CCyB policy. These effects are shown in Figure 6: by relaxing capital requirements in the banking sector, the regulator contains the rise in credit spreads, which means that disposable income for borrowers falls by less. This lesser fall results in consumption and GDP falling by less: although the recession is still deep, it is about one third smaller.

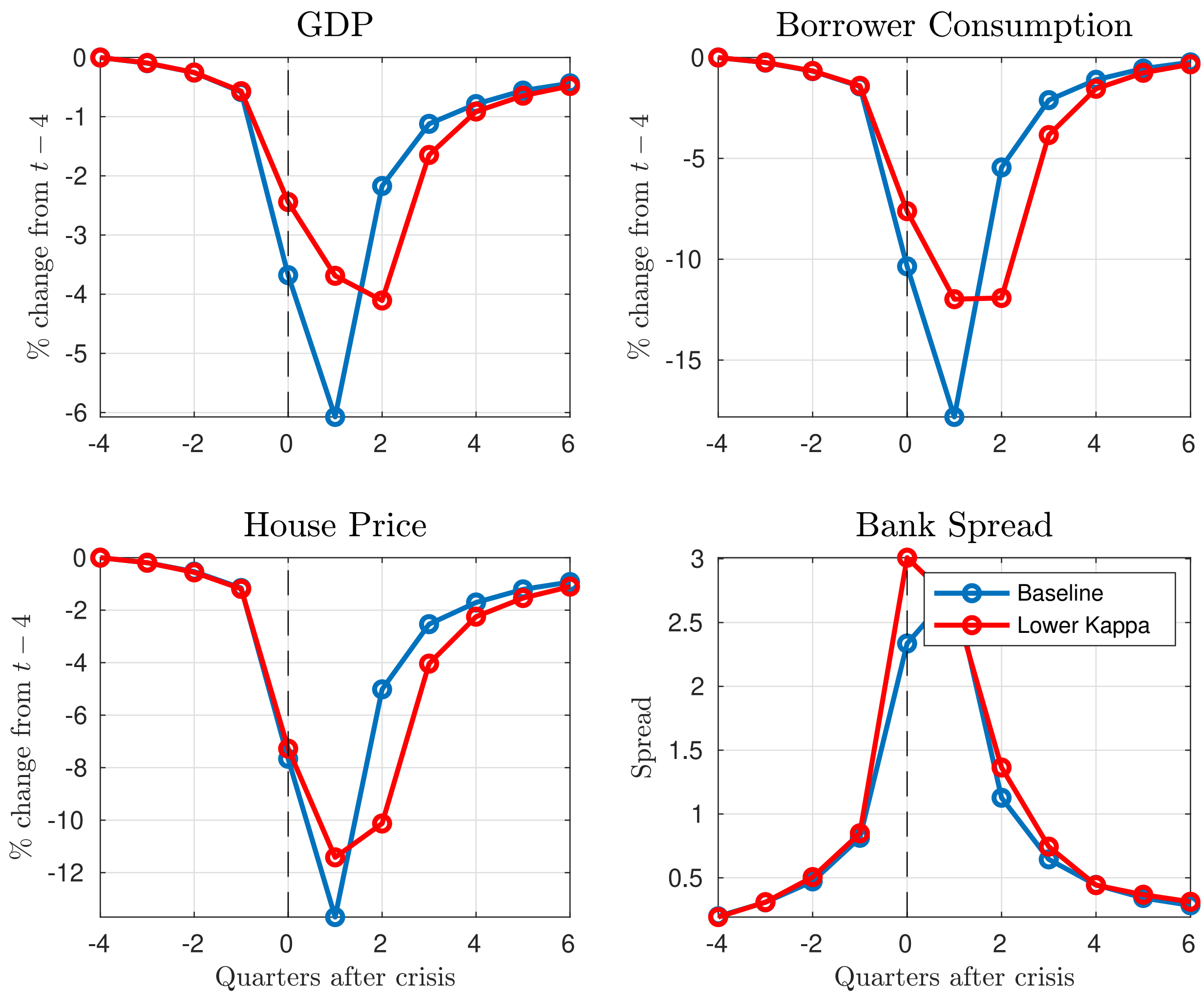


Figure 6: The blue line plots the median path of selected endogenous variables around a crisis, in the absence of macroprudential policy. The red line plots the median path of selected endogenous variables when capital requirements are lowered during a crisis.

4.3 Ex-ante effects of the CCyB

The quantitative effects of the CCyB are summarized in Table 2, where I present some moments for four different economies: (i) an economy with no CCyB, (ii) an economy where the regulator follows the "baseline" policy of raising the capital buffer when $u_t^R \geq 1$, (iii) an economy where the regulator can both raise the capital buffer, and lower it in the event of a crisis.

Variable	(i) No Policy	(ii) Baseline Policy	(iii) Raise and Lower
$100 \times \Pr(x_t = 1)$	2.54	1.05	0.92
Bank Leverage	8.11	7.94	7.95
Lagrange Multiplier Borrower	0.16	0.14	0.14
Median % Δ GDP in Crisis	-5.49	-4.31	-3.62
V_t^s	-1.99261	-1.99314	-1.99322
V_t^b	-2.06639	-2.06368	-2.06326
CEV Saver	–	-10.3%	-11.8%
CEV Borrower	–	20.6%	24.1%

Table 2: Model moments. The probability of a crisis and GDP contraction are based on model simulations. The remaining variables correspond to stochastic steady state values.

The first line of Table 2 shows that the baseline policy can reduce the probability of a crisis in more than half: from 2.54% to 1.05%. The following lines show why this is the case: both bank and borrower leverage are lower under the baseline policy. This means that the economy is, on average, further away from the run region and can therefore withstand more (and larger) shocks before entering that region. The mechanism is precautionary: banks try to stay away from the constraint. The threat of tightening the constraint if leverage is high therefore induces the banking sector to further deleverage in the first place.

The last column of Table 2 contains results for the combination of the two policies. There, we can see that a regulator who is able to both raise capital requirements when there is the risk of a run, and to lower them conditional on a run, can substantially reduce the risk of a run. Notice, however, that leverage is slightly higher in this case. The reason for this is related to the last line, where I show median GDP fall during a crisis. When the policymaker has access to both policies, the severity of the crisis is reduced; this creates moral hazard that leads to some increase in leverage at the stochastic steady state.

Welfare The last four lines of Table 2 compute welfare for borrowers and savers at the stochastic steady state, both in terms of utils and consumption-equivalent variation with respect to the no policy case. These numbers show that the introduction of the CCyB is not a Pareto improvement: borrowers are better off, but savers are worse off. Borrowers benefit from the CCyB since their consumption is the most affected during financial crises. By reducing the frequency of crises, the CCyB thus greatly reduces consumption (and labor supply) volatility for this group. Savers dislike the CCyB due to its precautionary effects on bank behavior, which reduce net payouts in equilibrium.

5 Quantitative Exercise

I now combine the calibrated model with U.S. data to perform a quantitative exercise and ask the following question: what would the Great Recession have looked like if the CCyB had been activated? To this end, I use the baseline model without the CCyB as a measurement device to estimate structural shocks for the U.S. economy and then feed the same sequences of shocks to different specifications of the model: one where the regulator can lower capital requirements during crises, one where the regulator can raise them during periods of financial fragility, and one where the regulator can do both.

5.1 Measurement and Particle Filter

Let the vector of endogenous variables in the model be denoted by $\mathbb{X}_t$, the vector of endogenous states by $\mathbb{S}_t$, and the vector of exogenous shocks by $\mathbb{Z}_t$. As this is a standard rational expectations model, we can write its solution as a set of (nonlinear) state transition equations, and a set of (nonlinear) observation equations:

$$\mathbb{S}_t = f(\mathbb{S}_{t-1}, \mathbb{Z}_t)$$
$$\mathbb{X}_t = g(\mathbb{S}_{t-1}, \mathbb{Z}_t)$$

where f, g are the state transition and observation functions, respectively. Our goal is estimate paths for $\{\mathbb{S}_t, \mathbb{Z}_t\}_{t=0}^{T}$ (where $t = 0, \ldots, T$ is the sample period). To this end, we can choose up to three data observables and back out the implied paths for states and exogenous shocks from the above system. This approach consists of, in some sense, "inverting" the model to back out model-implied estimates for the paths of the endogenous states and shocks. Since neither f nor g are necessarily invertible, this procedure can be accomplished via simulation using the particle filter as in Fernández-Villaverde and Rubio-Ramírez (2007). The details of the particle filtering procedure can be found in Appendix C.

The spirit of this exercise is to assume that the baseline model without the CCyB corresponds to the true model of the U.S. economy in the 2000-2015 period, as the CCyB policy was not in place at that time. Then, the estimated paths of the shocks can be fed to the alternative models with different CCyB specifications, to tell us what the historical effects of these policies would have looked like.

5.2 Observables

The model features three shocks: the TFP shock, the funding shock, and the sunspot shock. I estimate sequences of shocks that allow the model to match the path of two observables: detrended aggregate consumption, and a measure of bank borrowing costs (the TED spread).

Consumption Since there is no investment in the model, I focus on matching the path of aggregate consumption instead of GDP. The path of aggregate consumption is informative of the path of TFP innovations. Real aggregate consumption is the data counterpart of $C_t = \chi C_t^b + (1-\chi) C_t^s$. I use quarterly real personal consumption expenditures (PCE) from the Federal Reserve Bank of St. Louis FRED database (series code: PCECC96). I detrend this series using the approach proposed by Hamilton (2018), which involves estimating the following OLS regression:

$$\log C_{t+8} = \alpha + \sum_{i=0}^{4} \beta_i \log C_{t-i} + \epsilon_t$$

where I obtain detrended consumption as $\hat{\epsilon}_t$.

Credit Spreads The credit spread in the model is simply the difference between the price of the one-period deposit and that of a risk-free bond:

$$\text{spread}_t^d = \log Q_t - \log Q_t^d$$

Outside of financial crises, the credit risk of deposits is very low and their price mostly tracks the risk-free rate. When the financial shock hits, however, a wave of mortgage defaults can trigger large jumps in the deposit spread. For that reason, I use the data counterpart to the deposit spread — the TED spread — as the observable that allows me to identify financial shocks. The series is taken from FRED (series code: TEDRATE) and consists of the spread between the 3-month LIBOR and the yield on the 3-month Treasury bill. It is a common measure of the cost of wholesale funding for large banks.

5.3 Results

5.3.1 Filtering Results

Figure 8 shows the model-implied (median) behavior for the targeted observables, as well as the data series. Linear Gaussian state space models can match observables exactly as long as the number of shocks (plus measurement error) is at least as big as the number of

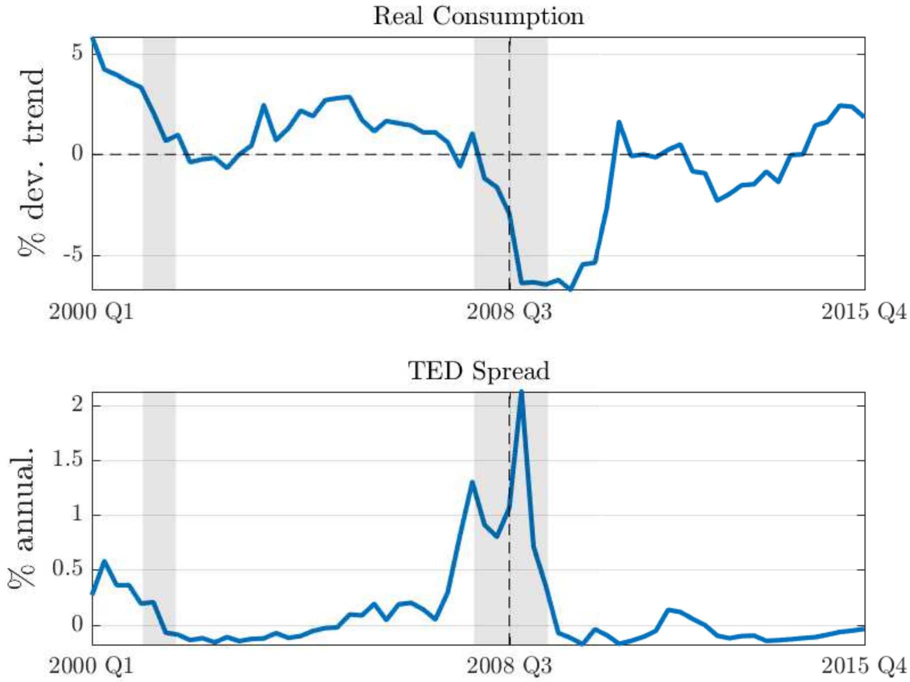


Figure 7: Detrended real consumption and annualized TED spread. Sample: 2000Q1-2015Q4. Lehman Brothers failure highlighted (2008Q3). Source: Federal Reserve Bank of St. Louis FRED.

observables. That is because the observables (and any other endogenous variables) are affine combinations of Gaussian shocks. This is no longer true when models are nonlinear and shocks are non-Gaussian: for that reason, the model is not able to exactly match the path of observables, but does a relatively good job in quantitative terms.

Figure 8: Estimated paths for observables vs. data, with 95% confidence interval.

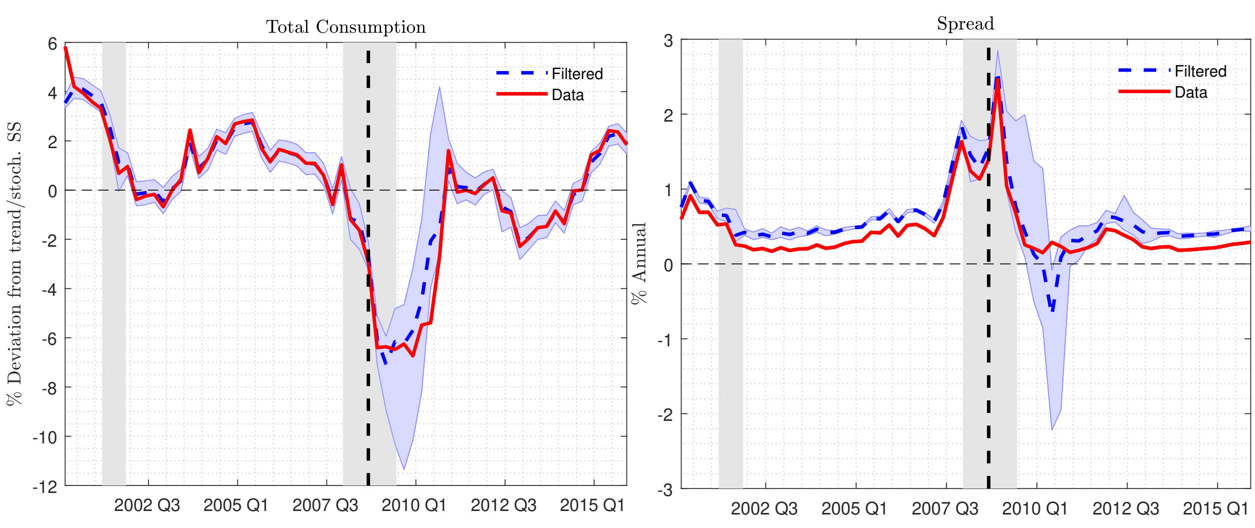
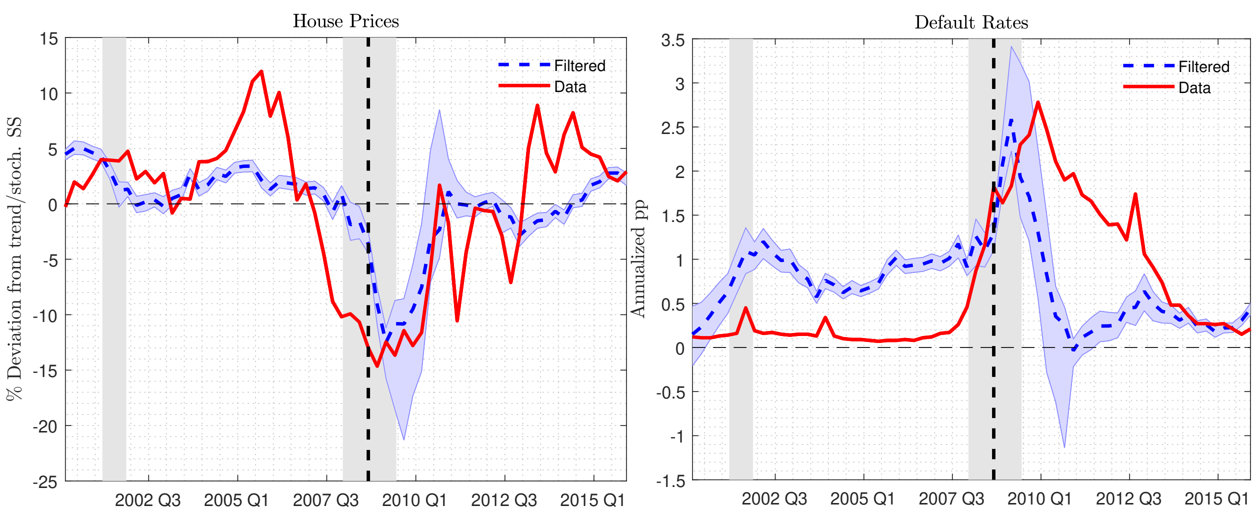


Figure 9 plots the model-implied series for other two variables vs. data: detrended house prices and mortgage default rates. While the model is unable to account for the large run-up leading to the recession, it matches the size of the fall and subsequent recovery of house prices in the U.S.. The model abstracts from certain features and frictions that have been shown to be necessary to fully account for the housing boom before the recession, such as movements in beliefs regarding future housing demand (Kaplan et al., 2017). The model also predicts a time series for default rates that broadly matches the movements in the data: the main shortcoming in this dimension is the model's difficulty in replicating the slow decrease in default rates. Models of financial frictions with occasionally binding constraints tend to have difficulty matching the persistence of negative financial shocks. Other variables are plotted in the estimation appendix; the model predicts that the U.S. economy entered a financial crisis in 2008Q3, and exited it in 2010.[18]

Figure 9: Estimated paths for house prices and default rates vs. data, with 95% confidence interval.



5.3.2 Counterfactual Experiment

Figure 10 plots the counterfactual scenario in which the regulator can raise the CCyB; Figure 11 plots the scenario in which the regulator can both raise and lower the CCyB. Figure 10 shows that raising capital requirements before the crisis could have prevented the economy from entering the crisis region altogether, thus avoiding a a large drop in consumption and a large increase in credit spreads. Eventually, the economy would have experienced a mild recession anyway, as the model estimates that the latter part of the recession is mostly attributable to other types of shocks, but the landing would have been much "softer."

[18]That is, the median value for the run indicator is equal to 1 for these dates.

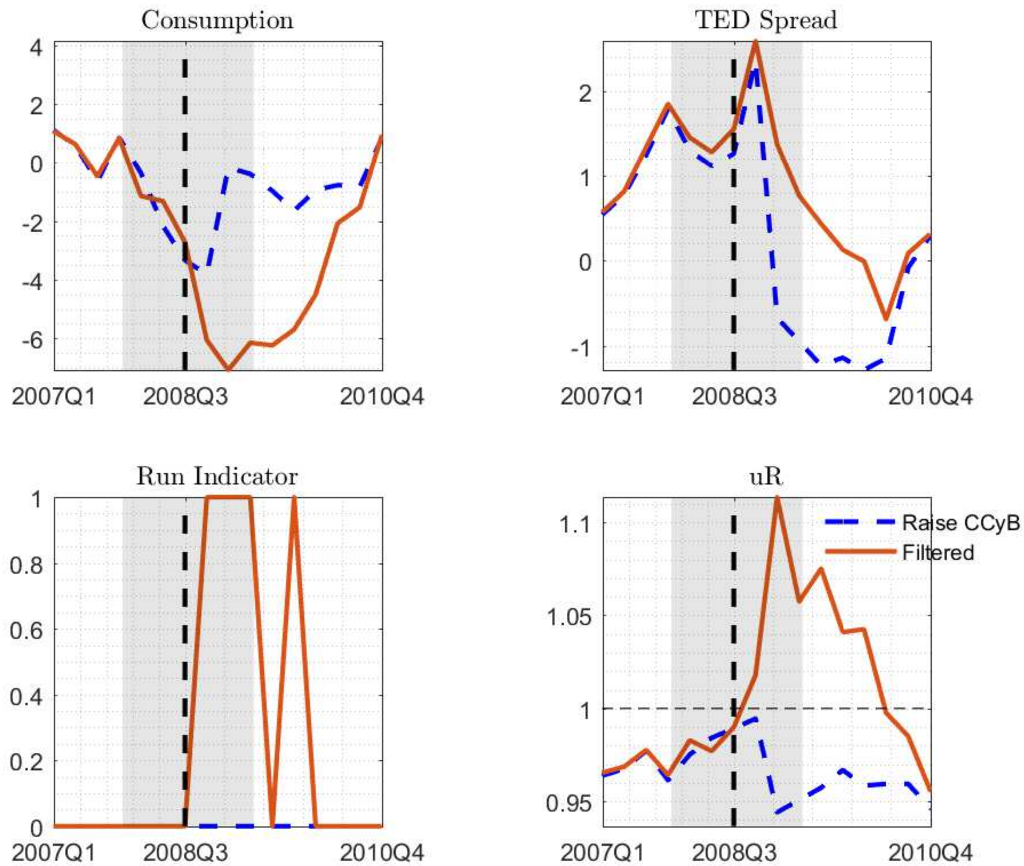


Figure 10: Estimated series (orange solid line) vs. counterfactual where regulator raises capital requirements before the crisis (dashed blue line)

To understand the magnitude of these gains, let us conduct the following simple, back-of-the-envelope calculation: real PCE consumption in 2007Q1 was about \$ 10,566.6 bn. The cumulative gain from being able to raise capital requirements (vis-à-vis no policy) is equal to 28.8%, or \$3,047.6 bn. The cumulative gain from being able to both raise and lower capital requirements is slightly larger: 29.4%, or \$3,107.9 bn. This is related to the already mentioned precautionary effect: the fact that the regulator is able to lower capital requirements during a crisis moderates the intensity of said crisis, which induces both banks and households to increase their leverage. However, since the threat of raising capital requirements implies that the economy never enters the crisis in the first place, the economy experiences the "benefits" from higher leverage, but not the costs along the equilibrium path.

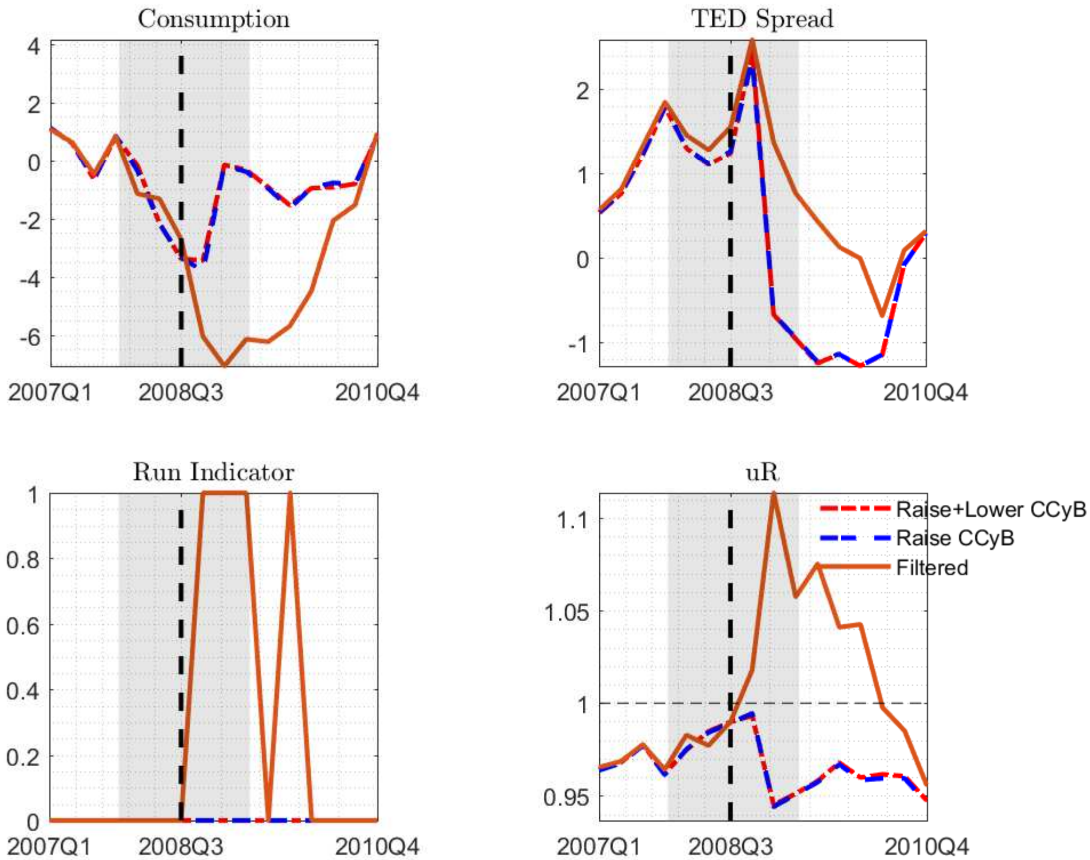


Figure 11: Estimated series (orange solid line) vs. counterfactual where regulator raises capital requirements before the crisis (dashed blue line), and counterfactual where regulator can both raise and lower capital requirements (dotted red line)

6 Conclusion

Countercyclical capital buffers were one of the pillars of post-crisis financial regulation reform. This paper investigates the effects of these regulations in the context of a nonlinear dynamic stochastic general equilibrium model where the financial sector is subject to occasional financial panics that are transmitted to real activity via aggregate demand. In the context of this model, I show that the CCyB can offer benefits ex-post, as lowering them during a crisis moderates the fall in output and consumption, as well as more traditional ex-ante benefits, as raising them during periods of leverage growth can both reduce the frequency of crises and ensure that agents are better capitalized when a crisis materializes, reducing its severity.

In a quantitative application to the 2008-09 Great Recession, I find that the benefits of being able to raise the CCyB would have been quantitatively significant, and would have allowed regulators to avoid most of the financial crisis, avoiding a cumulative fall of over 28% of aggregate consumption. In sum, I find that the benefits of this type of policy can be quantitatively very large, especially ex-ante.

In the current model, the real effects of financial panics are transmitted purely via aggregate demand. The model does not feature investment nor a link between the financial and production sectors. For this reason, and since the single component of output that fell the most was aggregate investment, it is likely that the model underestimates the true historical benefits of the CCyB. Since consumption tends to be the least volatile component of private expenditure, these numbers can be seen as a lower bound for the fall in GDP that could have been avoided. The inclusion of a more traditional investment channel could also offer potentially interesting interactions with the aggregate demand channel and is left as an avenue for future research.

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A Model Appendix

A.1 Full List of Equilibrium Conditions

Savers:

$$C_t^s (N_t^s)^{\varphi} = w_t \tag{17}$$

$$Q_t = \mathbb{E}_t \left(\frac{\Lambda_{t+1}^s}{\Pi_{t+1}} \right) \tag{18}$$

$$Q_t^d = \mathbb{E}_t \left(\frac{\Lambda_{t+1}^s}{\Pi_{t+1}} Z_{t+1}^d \delta_{t+1} \right) \tag{19}$$

$$\Lambda_{t+1}^s = \beta \frac{C_t^s}{C_{t+1}^s} \tag{20}$$

Banks:

$$E_t = \frac{(1 - x_t)}{\Pi_t} \theta \left(Z_t^b B_{t-1}^b - D_{t-1} \right) + \varpi Q_t^b \frac{B_{t-1}^b}{\Pi_t} \tag{21}$$

$$Q_t^b B_t^b = E_t + Q_t^d D_t \tag{22}$$

$$\kappa Q_t^b B_t^b \leq \Phi_t E_t \perp \mu_t \geq 0 \tag{23}$$

$$\Lambda_{t+1}^k = \frac{\Lambda_{t+1}^s}{\Pi_{t+1}} (1 - \theta + \theta \Phi_{t+1})(1 - x_{t+1}) \tag{24}$$

$$\mu_t \kappa = \mathbb{E}_t \left\{ \Lambda_{t+1}^k \left[\frac{Z_{t+1}^b}{Q_t^b} - \frac{1}{Q_t^d} \right] \right\} \tag{25}$$

$$\Phi_t = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^k \right\}}{Q_t^d (1 - \mu_t)} \tag{26}$$

$$u_t^D = \frac{D_{t-1}}{Z_t^b B_{t-1}^b} \tag{27}$$

$$u_t^R = \frac{D_{t-1}}{(1 - \lambda^d) Z_t^b B_{t-1}^b} \tag{28}$$

$$x_t = \mathbb{1}[(u_t^D \geq 1) \vee (u_t^D < 1 \wedge u_t^R \geq 1 \wedge \omega_t = 1)] \tag{29}$$

Borrowers:

$$C_t^b (N_t^b)^\varphi = w_t \tag{30}$$

$$B_t^b \leq \chi \mathrm{m} \theta^{LTV} p_t^h + B_{t-1}^b \frac{1-\gamma}{\Pi_t}(1-\mathrm{m}) \perp \lambda_t^b \geq 0 \tag{31}$$

$$\nu_t^* = \frac{B_{t-1}^b}{\chi \Pi_t p_t^h} \tag{32}$$

$$p_t^h = \frac{\xi (C_t^b)^\sigma + \mathbb{E}_t \left\{ \Lambda_{t+1}^b p_{t+1}^h \left[(1-\mathrm{m})(1-\theta^{LTV}\lambda_{t+1}^b) + \mathrm{m}\Psi^b \right] \right\}}{1-\lambda_t^b \theta^{LTV}} \tag{33}$$

$$Q_t^b - \lambda_t^b = \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}^b}{\Pi_{t+1}} \left\{ (1-\mathrm{m}) \left[(1-\gamma)(Q_{t+1}^b - \lambda_{t+1}^b) + \gamma \right] + \mathrm{m} \left[1 - F^b(\nu_{t+1}^*) \right] \right\} \right\} \tag{34}$$

$$w_t N_t^b + \frac{Q_t^b B_t^b}{\chi} = C_t^b + \frac{B_{t-1}^b}{\chi \Pi_t} \left\{ \mathrm{m}[1-F^b(\nu_t^*)] + (1-\mathrm{m})[(1-\gamma)Q_t^b + \gamma] \right\} + \mathrm{m} p_t^h [1-\Psi^b(\nu_t^*)] \tag{35}$$

$$\Lambda_{t+1}^b = \beta \frac{C_t^b}{C_{t+1}^b} \tag{36}$$

Asset payoffs:

$$Z_t^b = (1-\mathrm{m})[Q_t^b(1-\gamma)+\gamma] + \mathrm{m}\left[1 - F^b(\nu_t^*) + (1-\lambda_t^b)\frac{1-\Psi^b(\nu_t^*)}{\nu_t^*}\right] \tag{37}$$

$$Z_t^d = 1 - x_t + \frac{x_t}{u_t^R} \tag{38}$$

Phillips curve, resource constraint, and production function:

$$\eta \mathbb{E}_t \left\{ \Lambda_{t+1}^s \frac{Y_{t+1}}{Y_t} \frac{\Pi_{t+1}}{\Pi} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \right\} - \varepsilon \left(\frac{\varepsilon - 1}{\varepsilon} - \frac{w_t}{A_t} \right) = \eta \frac{\Pi_t}{\Pi} \left(\frac{\Pi_t}{\Pi} - 1 \right) \tag{39}$$

$$C_t + G_t + \lambda_t^b \mathrm{m} \chi p_t^h [1-\Psi^b(\nu_t^*)] + \lambda^d Z_t^b \frac{B_{t-1}^b}{\Pi_t}(1-x_t) = Y_t \left[1 - \frac{\eta}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 \right] \tag{40}$$

$$Y_t = A_t N_t \tag{41}$$

Monetary policy and GDP:

$$\frac{1}{Q_t} = \frac{1}{\bar{Q}} \left[\frac{\Pi_t}{\Pi} \right]^{\phi_\pi} \left(\frac{GDP_t}{G\bar{D}P} \right)^{\phi_Y} \tag{42}$$

$$GDP_t = C_t + G_t \tag{43}$$

Cumulative distribution functions and partial expectations for the risk shock:

$$F^b(\nu_t^*) = \left[\frac{\sigma^b \nu_t^*}{\sigma^b + 1}\right]^{\sigma^b} \tag{44}$$

$$\Psi^b(\nu_t^*) = 1 - \left[\frac{\sigma^b \nu_t^*}{\sigma^b + 1}\right]^{\sigma^b+1} \tag{45}$$

A.2 Bank Optimality and Aggregation

To solve the bank's problem, we start by writing the bank's franchise/continuation value as

$$\begin{aligned}
\Phi_{j,t}(e_{j,t}) &\equiv \mathbb{E}_t\left[(1 - x_{t+1})\frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}} V_{j,t+1}(e_{j,t+1})\right] \\
&= \mathbb{E}_t\left\{(1 - x_{t+1})\frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}}[(1-\theta)e_{j,t+1} + \Phi_{j,t+1}(e_{j,t+1})]\right\}
\end{aligned}$$

I assume throughout that the bank takes the possibility of a run as given. We now guess, to later verify, that the bank's franchise value is linear in current earnings,

$$\Phi_{j,t}(e_{j,t}) = \Phi_{j,t}\theta e_{j,t}$$

Under this assumption, we can reformulate the bank's problem as

$$\Phi_{j,t}\theta e_{j,t} = \max_{b_{j,t}, d_{j,t}} \mathbb{E}_t\left\{(1 - x_{t+1})\frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}}(1 - \theta + \theta\Phi_{j,t+1})e_{j,t+1}\right\}$$

subject the law of motion for earnings, the balance sheet constraint, and the leverage constraint. Replacing for the first two, we can write the bank's Lagrangian as

$$\begin{aligned}
\Phi_{j,t}\theta e_{j,t} = \max_{b_{j,t}} \mathbb{E}_t & \left\{(1 - x_{t+1})\frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}}(1 - \theta + \theta\Phi_{j,t+1})\left[\left(\frac{Z^b_{t+1}}{Q^b_t} - \frac{1}{Q^d_t}\right)Q^b_t b^b_{j,t} + \frac{\theta e_{j,t}}{Q^d_t}\right]\right\} \\
&+\mu_{j,t}\left[\Phi_{j,t}\theta e_{j,t} - \kappa_t Q^b_t b^b_{j,t}\right]
\end{aligned}$$

The first-order condition with respect to $b_{j,t}$ is then

$$\mathbb{E}_t\left\{(1 - x_{t+1})\frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}}(1 - \theta + \theta\Phi_{j,t+1})\left(\frac{Z^b_{t+1}}{Q^b_t} - \frac{1}{Q^d_t}\right)\right\} = \mu_{j,t}\kappa_t$$

Applying the envelope theorem and rewriting the Lagrangian then yields

$$\Phi_{j,t} = \frac{\mathbb{E}_t \left\{ \frac{\Lambda^s_{t,t+1}}{\Pi_{t+1}} (1 - \theta + \theta \Phi_{j,t+1})(1 - x_{t+1})] \right\}}{Q^d_t (1 - \mu_{j,t})}$$

thus confirming our conjecture that the value was linear in earnings.

A.3 GIRFs to TFP and Funding Shocks

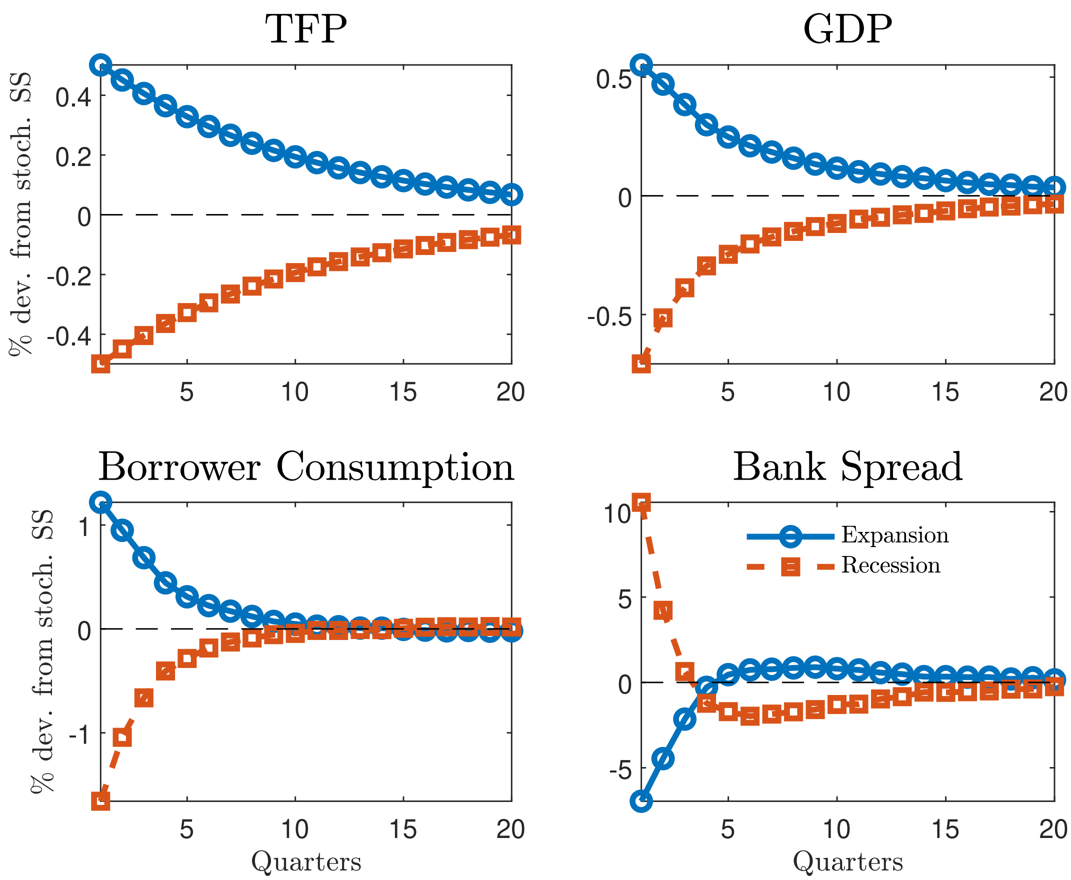

Figure 12: Response of selected variables to a one-standard deviation TFP shock.

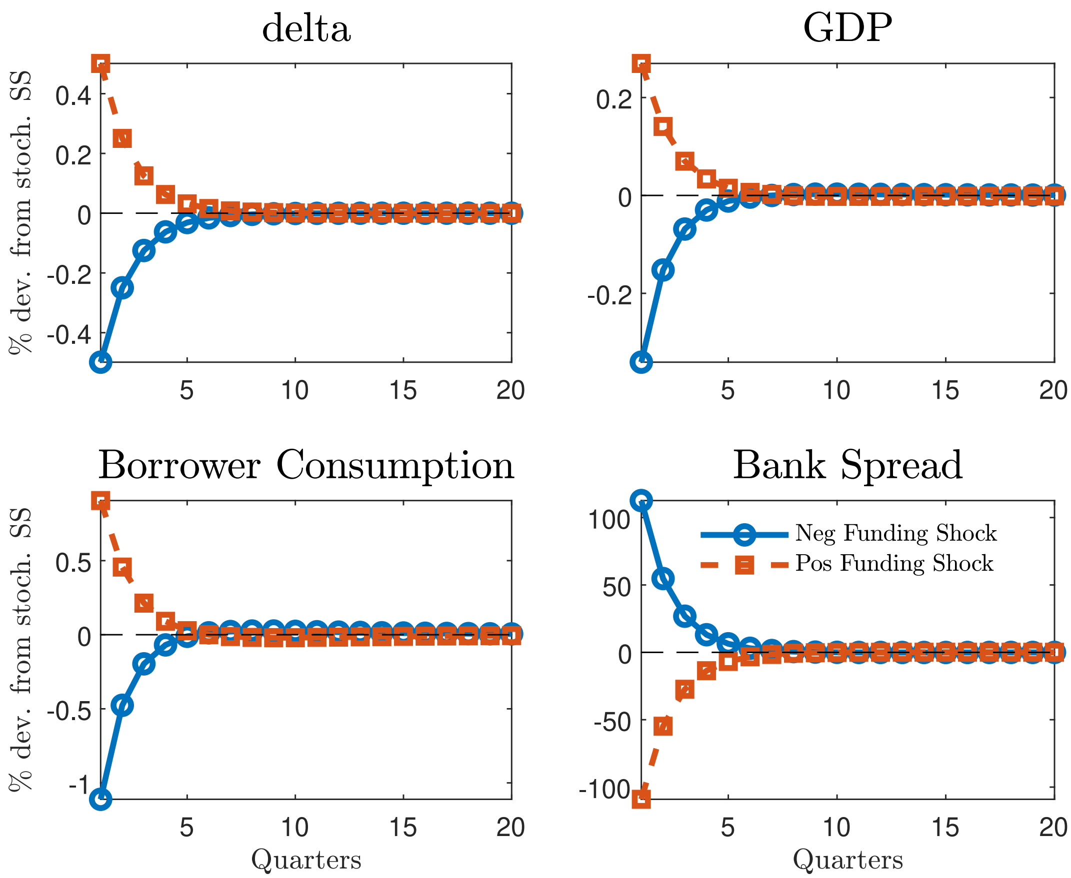

Figure 13: Response of selected variables to a one-standard deviation deposit funding shock.

B Computational Appendix

The overall methods to solve and estimate the model are taken from Faria-e-Castro (2018).

B.1 Model Solution

I adopt a global solution method that combines time iteration (Judd, 1998), parametrized expectations (den Haan and Marcet, 1990) and multilinear interpolation. Given a vector of state variables $\mathbb{S}_{t-1}$ and innovations ϵ_t, one can use the equilibrium conditions described in Appendix A to compute the values of all endogenous variables $\mathbb{Y}_t$ in the current period:

$$\mathbb{Y}_t = f(\mathbb{S}_{t-1}, \epsilon_t)$$

The procedure consists of approximating f (an infinite-dimensional object) using a finite approximation $\hat{f}$ chosen from some space of functions. The approximation is obtained by solving for $\hat{f}$ exactly at a finite number of grid points and interpolating between these when evaluating the equilibrium at points of the state space that do not belong to the grid.

In practice, it is not necessary to approximate all elements of $\mathbb{Y}_t$. Given knowledge of the current states and innovations $(\mathbb{S}_{t-1}, \epsilon_t)$, as well as of a restricted set of endogenous variables $\mathbb{X}_t \subset \mathbb{Y}_t$ ("policies"), one can use the model's static equilibrium conditions to back out the remaining elements of $\mathbb{Y}_t$. For the specific case of my model, we have that this vector of states and innovations is

$$\mathcal{S}_t \equiv (\mathbb{S}_{t-1}, \epsilon_t) = (D_{t-1}, B^b_{t-1}, A_t, \omega_t, \delta_t)$$

Policies $\mathbb{X}_t$ are typically variables that either appear inside expectation terms (and so we need to be able to evaluate them for different values of $\mathbb{S}_{t+1}$) and/or variables that cannot be determined statically without solving a nonlinear equation. Based on these criteria, I pick the following variables as the policies to solve for:

$$\mathbb{X}_t = (C^s_t, Q^b_t, p^h_t, \Pi_t, C^b_t, Q^d_t, \lambda^b_t, \mu_t)$$

I adopt some ideas from parametrized expectations algorithms: for a given $\mathcal{S}_t$, I can describe the model's equilibrium as a set of nonlinear equations of the type

$$m\left\{\mathbb{E}_t\left[h\left(\mathbb{X}_{t+1}, \mathcal{S}_{t+1}, \mathcal{S}_t\right)\right], \mathbb{X}_t, \mathcal{S}_t\right\} = 0$$

The idea is to construct a grid over the states and innovation $\mathcal{S}_t$, fix the expectations terms $\mathbb{E}_t h(\cdot)$ at each of these points, and solve a simpler system of nonlinear equations for $\mathbb{X}_t$. Since the system is relatively simple (as I am fixing the value of the expectations terms for each grid point), it is possible to compute the Jacobian analytically, which greatly improves the speed and precision of the algorithm.

The algorithm then proceeds as follows:

1. Generate a discrete grid for the state variables, $\{g_i\}_{i=1}^N = \mathbb{G} = G_D \times G_B \times G_A \times G_\omega \times G_\delta$.

2. Approximate $\mathbb{X}_t, \mathbb{E}_t h(\cdot)$ over $\mathbb{G}$ by choosing an initial guess and a functional space to define the approximant. As the initial guess, I use the model's non-stochastic steady state. This means that I can guess a value for each variable $X_t \in \mathbb{X}_t$ and each expectation term $\mathbb{E}_t h(\cdot)$ at each grid point. Call these sets of values $X^0 = \{x^0_i\}_{i=1}^N$ and $H^0 = \{h^0_i\}_{i=1}^N$. As an approximant, I use piecewise linear functions (multilinear interpolation). This approximant allows me to evaluate X^0, H^0 outside of the grid points at any combination of values for the states.

3. Given these initial guesses for the policies $\mathbb{X}^0$ and expectation terms, solve the model by using time iteration. Set $\mathbb{X}^\tau = \mathbb{X}^0$ and $H^\tau = H^0$.

   (a) For each point in the grid, g_i, solve a system of residual equations for the value of the policies at that grid point. Given our guesses for the expectation terms, this is a set of nonlinear equations of the type

   $$m\left\{h_i^\tau, \mathbb{X}^\tau, g_i\right\} = 0$$

   As mentioned, since the expectation terms are fixed at each point, this system should be simple enough so as to allow analytical computation of the Jacobian. Solving for $\mathbb{X}^\tau$ allows us to obtain a series of values for the policies at each point in the grid $\{\mathbb{X}_i^{\text{new}}\}_{i=1}^N$.

   (b) Given values for these points, compute a convergence criterion for each element of $\mathbb{X}$ as

   $$\rho_i^X = \max_i \|\mathbb{X}_i^{\text{new}} - \mathbb{X}_i^\tau\|$$

   (c) Update the guess for each point in the grid:

   $$\mathbb{X}_i^{\tau+1} = \lambda \mathbb{X}_i^{\text{new}} + (1-\lambda)\mathbb{X}_i^\tau$$

   where λ is some dampening parameter. Reevaluate (update) the policy approximant.

   (d) Use the updated policies and the model's equilibrium conditions to update the expectation terms $H^{\tau+1}$. Compute these expectations using the policy interpolants and Gauss-Hermite quadrature for the TFP process (with 15 points).

   (e) If ρ_i^X is below some pre-defined level of tolerance, stop. Otherwise, return to step (a).

Intuitively, time iteration works by guessing some functional form for the endogenous variables inside of the expectations terms and iterating backwards until today's policies are consistent with the expected future policies at each point in the state space. The innovation with respect to standard time iteration methods is that expectations are fixed at each point of the grid when solving for policies, which considerably speeds up computations. Solving models with this type of methods can be particularly challenging since very few convergence results exist (unlike, for example, value function iteration).

Occasionally Binding Constraints To deal with occasionally binding constraints, I apply the procedure described in Garcia and Zangwill (1981) and used by Judd et al. (2002). This involves rewriting inequality conditions and redefining Lagrange multipliers such that equilibrium conditions can be written as a system of equalities and standard methods for solving nonlinear systems of equations can be applied. As a concrete example, take the bank's leverage constraint and the associated Lagrange multiplier $\mu_t \geq 0$. I define an auxiliary variable $\mu_t^{\text{aux}} \in \mathbb{R}$ such that

$$\mu_t = \max(0, \mu_t^{\text{aux}})^2$$

and the inequality to which the complementarity condition $\mu_t \geq 0$ is associated reads

$$\Phi_t E_t = \kappa_t Q_t^b B_t + \max(0, -\mu_t^{\text{aux}})^2$$

Notice then that when $\mu_t^{\text{aux}} \geq 0$, the inequality holds as an equality and $\mu_t \geq 0$. On the other hand, when $\mu_t^{\text{aux}} < 0$, this variable becomes the residual for the inequality, which implies that $\Phi_t E_t > \kappa_t Q_t B_t$ and $\mu_t = 0$. Defining this auxiliary variable as the square of a max operator ensures that the system is differentiable with respect to this variable, which is helpful when using Newton-based methods to solve the nonlinear system of equilibrium conditions.

Grid Construction Grid boundaries for endogenous states are chosen to minimize extrapolation, which is important given the use of linear extrapolation. I use linear grids for all endogenous variables. In principle, it is helpful to make grids denser in regions of the state space where constraints start/stop binding. That is not easy in this model: given the large number of states, these regions can be ill behaved. Given that bank and household debt are very positively correlated, using rectangular grids is computationally costly, since it involves solving the model for many points that will never be visited during stochastic simulations. One approach to dealing with this issue is to use grid rotations based on singular value decompositions. Since my grid is constructed manually, I instead opt for redefining the state variables. In particular, I use $\text{lev}_{t-1} = \frac{D_{t-1}}{B_{t-1}^b}$ instead of D_{t-1} as a state.

B.2 Accuracy Checks

Even though the model solution is exact at the specified grid points, the simulated economy may travel to regions of the state space that do not correspond to any grid point; at these points, the equilibrium conditions are not guaranteed to hold exactly. To check accuracy of the model solution, I follow the standard procedure in the literature and evaluate the residuals at these points. To do so, I first simulate the model economy for 5,000 periods. Then, I evaluate the residual equations used to solve the model at each of the points of the state space that were "visited" in that simulation. Histograms with the decimal log of the absolute value of the residuals are shown in figure 14 for each residual equation. Most equations present average errors of order -3, which are standard in the literature for even smaller models.

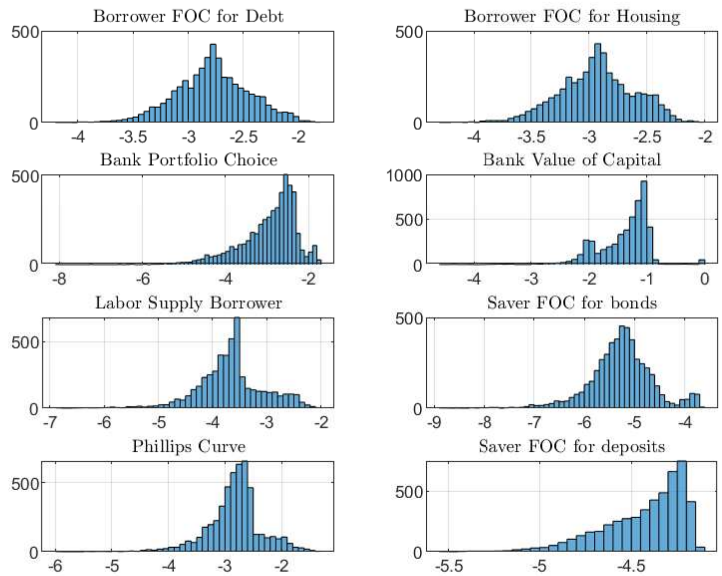

Figure 14: Residual equation errors for a 5,000 period simulation, in decimal log basis.

C Estimation Appendix

In this section, I describe the particle filter and smoother used to extract the sequences of structural shocks from the data.

Nonlinear State Space Model The first step to writing the particle filter is to write the model in nonlinear state space form. The general structure of these models is composed of two blocks: a state transition function f and an observation function g:

$$x_t = f(x_{t-1}, \epsilon_t; \gamma)$$
$$y_t = g(x_t; \gamma) + \eta_t$$

where γ is a vector of structural parameters, x_t is a vector of state variables, y_t is a vector of observable variables, ϵ_t are structural shocks, and η_t are measurement errors. The structural shocks follow some distribution with density function m, and measurement errors are assumed to be additive and Gaussian,

$$\eta_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$$

For the present model, I define

$$x_t = (\text{lev}_t, B_t^b, A_t, \omega_t, \delta_t)$$
$$y_t = (C_t, \text{spread}_t)$$

The structural shocks are the innovations to $(A_t, \omega_t, \delta_t)$, and all variables are observed with some measurement error that is Gaussian and uncorrelated across variables. For the endogenous observables, (C_t, spread_t), I set the standard deviation of the measurement error equal to 10% of the standard deviation of the data series.

Likelihood Function Given a sample of observables $y^T = \{y_t\}_{t=0}^T$, we can apply the typical factorization and write the likelihood given parameters γ as

$$\mathcal{L}(y^T; \gamma) = \prod_{t=1}^{T} p(y_t | y^{t-1}; \gamma)$$

We can further decompose the period-by-period conditional density $p(y_t|y^{t-1}; \gamma)$ as

$$\mathcal{L}(y^T; \gamma) = \prod_{t=1}^{T} \int p(y_t | x_t; \gamma) p(x_t | y^{t-1}; \gamma) \mathrm{d}x_t$$

The first term is easy to evaluate: $p(y_t|x_t; \gamma)$ is given from the observation equation and the density function for the measurement error. Given the assumption that measurement error is additive and Gaussian, $\eta_t \sim \mathcal{N}(0, \Sigma)$, we can simply write

$$p(y_t | x_t; \gamma) = \phi[y_t - g(x_t; \gamma)]$$

where ϕ is the (multivariate) standard normal density.

The harder part is to evaluate the second term, $p(x_t|y^{t-1}; \gamma)$, which is a complicated function of the states. This is where the particle filter is helpful, since it allows us to compute this conditional density by simulation.

Bootstrap Filter Our goal is to evaluate $p(x_t|y^{t-1};\gamma)$ at each t. The particle filter is a way of obtaining a sequence of state densities conditional on past observations, $\{p(x_t|y^{t-1};\gamma)\}_{t=0}^{T}$. Throughout the procedure, we have to keep track of a sequence of sampling weights, $\{\{\pi_t^i\}_{i=1}^{N}\}_{t=0}^{T}$. It proceeds as follows:

1. **Initialization**. Set $t = 1$ and initialize $\{x_0^i, \pi_0^i\}_{i=1}^{N}$ by taking N draws from the model's ergodic distribution and set $\pi_0^i = \frac{1}{N}, \forall i$.

2. **Prediction**. For each particle i, draw $x_{t|t-1}^i$ from the proposal density $h(x_t|y^t, x_{t-1}^i)$. This involves randomly drawing one vector of structural innovations ϵ_t^i and computing

$$x_{t|t-1}^i = f(x_{t-1}^i, \epsilon_t^i)$$

3. **Filtering**. Assign to each draw $x_{t|t-1}^i$ a particle weight given by

$$\pi_t^i = \frac{p(y_t|x_{t|t-1}^i;\gamma)p(x_t|x_{t|t-1}^i;\gamma)}{h(x_t|y^t, x_{t-1}^i)}$$

   Noting that

$$p(y_t|x_{t|t-1}^i;\gamma) = \phi(y_t - g(x_{t|t-1}^i;\gamma))$$

   we can compute each particle weight as

$$\pi_t^i = \frac{p(y_t|x_{t|t-1}^i;\gamma)}{\sum_{i=1}^{N} p(y_t|x_{t|t-1}^i;\gamma)}$$

   This generates a swarm of particle weights that add up to 1, $\{\pi_t^i\}_{i=1}^{N}$.

4. **Sampling**. Sample N values for the state vector with replacement, from $\{x_{t|t-1}^i\}_{i=1}^{N}$ using the weights $\{\pi_t^i\}_{i=1}^{N}$. Call this set of draws $\{x_t^i\}_{i=1}^{N}$, and set the weights back to $\pi_t^i = \frac{1}{N}, \forall i$.

These steps generate a sequence of $\{\{x_{t|t-1}^i\}_{i=1}^{N}\}_{t=0}^{T}$, which can then be used to generate $\{\{p(y_t|x_{t|t-1}^i;\gamma)\}_{i=1}^{N}\}_{t=0}^{T}$. This then allows us to evaluate the likelihood as

$$\mathcal{L}(y^T;\gamma) \simeq \prod_{t=1}^{T} \frac{1}{N} \sum_{i=1}^{N} p(y_t|x_{t|t-1}^i;\gamma)$$

Filtered States At the end of the process, we have a sequence of simulated swarms of particles for each time period $\{\{x_t^i\}_{i=1}^{N}\}_{t=0}^{T}$. These can be treated as empirical conditional densities for the state, given the observed data until t, or y^t.

Other Details I use a swarm of 100,000 particles to run the filter. To initialize the filter, I obtain the initial conditions for the states by running a long simulation of the model without financial crises and drawing $\{x_0^i\}_{i=1}^N$ by sampling uniformly from that simulation.

D Additional Figures

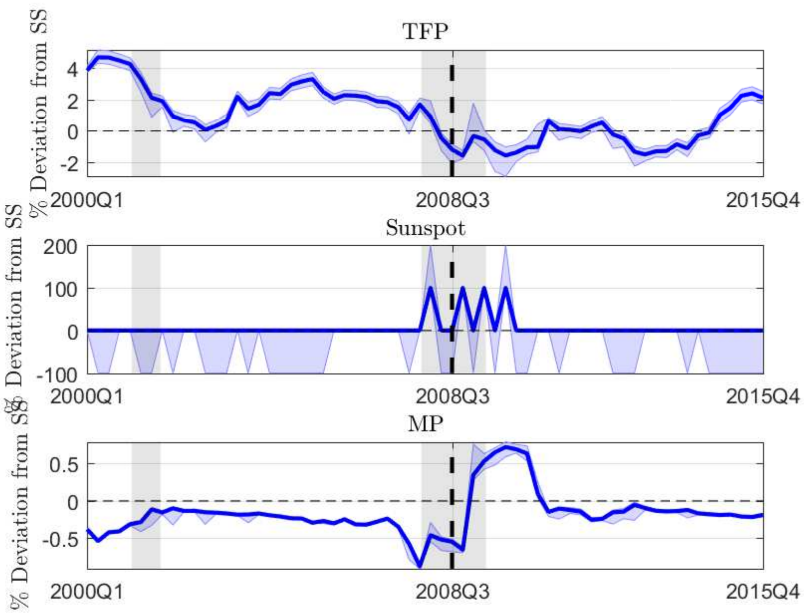

Figure 15: Estimated paths for structural shocks, with 95% confidence intervals.