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Working Paper 2018-035A
https://doi.org/10.20955/wp.2018.035

November 2018
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November 16, 2018

Abstract

We consider the effects of uncertainty shocks in a nonlinear VAR that allows uncertainty to have amplification effects. When uncertainty is relatively low, fluctuations in uncertainty have small, linear effects. In periods of high uncertainty, the effect of a further increase in uncertainty is magnified. We find that uncertainty shocks in this environment have a more pronounced effect on real economic variables. We also conduct counterfactual experiments to determine the channels through which uncertainty acts. Uncertainty propagates through both the household consumption channel and through businesses delaying investment, providing substantial contributions to the decline in GDP observed after uncertainty shocks. Finally, we find evidence of the ability of systematic monetary policy to mitigate the adverse effects of uncertainty shocks. [JEL: C34, E2, E32]

Keywords: uncertainty, time-varying threshold VAR, monetary policy, generalized impulse response functions

*The authors benefitted from conversations with Joshua Chan, Ana Galvao, Mike McCracken, Alessia Paccagnini and from comments by participants at the Applied Time Series Workshop at the St. Louis Fed. The authors thank Brian Levine and Hannah G. Shell for research assistance. The views expressed herein reflect those of the authors and should not be taken to reflect those of the Federal Reserve Bank of St. Louis, the Federal Reserve Board of Governors, or the Federal Reserve System.

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1 Introduction

Since the onset of the financial crisis, research into the measurement and effects of uncertainty has proceeded at a feverish pace. A rise in uncertainty is widely believed to have detrimental effects on macro, micro, and financial market outcomes and induce responses from monetary, fiscal, and regulatory policy. Theoretical models suggest that increasing uncertainty can have effects through a number of economic channels. For example, firms may delay investment and hiring during periods of high uncertainty [Bernanke (1983); Dixit and Pindyck (1994)]. Households may exercise precautionary reductions in spending [Basu and Bundick (2017)]. Financing costs may rise [Gilchrist, et al. (2014); Pastor and Veronesi (2012)]. Uncertainty about policy, in particular, can have detrimental economic effects [Friedman (1968); Rodrik (1991); Higgs (1997); Hassett and Metcalf (1999)]. Despite the relatively large theoretical literature, there is much less empirical evidence on the channels through which uncertainty affects the economy.

The majority of this evidence on the effect of uncertainty shocks on key economic variables such as employment, industrial production, real GDP growth, and inflation has been produced in a linear environment using VARs [see Bloom (2009); Baker, Bloom, and Davis (2016); Jurado, Ludvigson, and Ng (2015); Rossi and Sekhposyan (2015); and Leduc and Liu (2016)].\(^1\) In short, most researchers, regardless of the econometric approach, find that uncertainty shocks reduce economic activity (e.g., IP or real GDP growth), raise unemployment, and lower inflation for several months after the shock. This finding is consistent with the earlier literature by Bernanke (1983) and Dixit and Pindyck (1994) who found real option effects on fixed investment—that is,

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\(^1\)Scotti (2016), asking a somewhat different question, finds that uncertainty measures that relate to the state of the real economy—as opposed to financial market volatility or forecast disagreement—produce more modest effects on economic activity. Strobel (2015) found that uncertainty measures based on realized variables like Baker, Bloom, and Davis (2016) are more volatile than measures based on forecasts.
delaying expenditures on irreversible investment projects—during periods of increased uncertainty.

Linear models do not account for the possibility that the level of uncertainty can also affect how shocks propagate. While linear models are more common in the uncertainty literature, some nonlinear models have been estimated. Caggiano, et. al. (2017) use a nonlinear VAR to show that uncertainty shocks are larger during periods when the zero lower bound is binding on the FOMC’s federal funds target rate. Carriero, et. al. (forthcoming) employ a linear VAR with stochastic volatility which is driven by aggregate macroeconomic and financial uncertainty. They find that macroeconomic uncertainty shocks have large effects primarily on real activity while financial uncertainty shocks transmit to macroeconomic conditions via their impact directly on financial variables. Jones and Enders (2016) estimate a logistic smooth transition autoregressive process and allow uncertainty to drive the transition between different environments. They find that rising uncertainty has greater effects than falling uncertainty and the linear model underestimates the effects of uncertainty during the global financial crisis. Mumtaz and Theodoridis (2016) use a time-varying-parameter VAR with stochastic volatility and find that the response of output to uncertainty shocks has declined over time. Finally, Shin and Zhong (2018) use sign restrictions in the VAR, allowing for stochastic volatility, and define an uncertainty shock as that which increases the variance of the economic shocks. Thus, in their model, an uncertainty shock can simultaneously affect the volatility and the mean of the VAR. For the U.S., the authors find stronger evidence suggesting financial uncertainty shocks reduce output and prompt a monetary easing in comparison to the effects of shocks to macro uncertainty.

In this paper, we develop a model that is easy to estimate but also incorporates nonlineairities through which the level of uncertainty can affect how shocks propagate. Our model is a time-varying threshold VAR in which shocks that lower uncertainty
have limited linear effects but shocks that raise uncertainty above the threshold can have amplification effects. Our model is, in part, based on the asymmetric models used in the oil shock literature [see Hamilton (1996)] that use the maximum over a previous window as the threshold.

We find that, in the nonlinear framework, uncertainty shocks have larger effects than what is typically found in linear models. Moreover, compared to our linear analogue which has a persistent response after declining on impact, our nonlinear model exhibits a deep contraction, and gradual recovery, in real variables following shocks that raise uncertainty above the threshold. An important component of our model is that the threshold for the nonlinearity is time-varying. Thus, our framework accommodates agent indifference to sustained levels of uncertainty, even if uncertainty is high relative to historical standards. We find that contractions in investment and consumption contribute substantially to the decline in GDP observed after uncertainty shocks. In particular, business fixed investment and durables consumption exhibit deep, persistent contractions in uncertain environments, thus supporting the view that firms and households delay expenditure when faced with spikes in uncertainty. Finally, we conduct counterfactual experiments by shutting down various channels through which uncertainty shocks can propagate to the broader economy. We find evidence of the ability of systematic monetary policy to mitigate the adverse effects of uncertainty shocks.

The balance of the paper is laid out as follows. Section 2 presents the max VAR and compares it to the linear and threshold VARs. Section 3 provides the details of the Bayesian estimation of the model and the computation of the impulse responses.

Baker, Bloom, and Davis (2016) find that an increase in their Economic Policy Uncertainty index from its 2005-2007 average to its 2011-2012 average (around 90 index points) results in a drop in industrial production of 1.1% and declining employment by 0.35%. While we don’t study IP, we find a median response of a roughly 1.15% decline in employment by 8 quarters after an uncertainty shock around one-third the magnitude of the shock in Baker, Bloom, and Davis (2016). Alternatively, Caggiano, et.al. (2017) use the VIX and find that, when not at the zero lower bound, uncertainty shocks trigger a 0.25% decline in real GDP and consumption but a decline of around 2% in investment after two quarters. We find considerably larger effects of uncertainty shocks for all three variables.
Section 4 presents the empirical results. Section 5 considers the channels of the effects of uncertainty. Section 6 discusses a series of robustness checks and Section 7 offers some conclusions.

2 Empirical Model

The workhorse model used to evaluate the effects of uncertainty and the channels in which they act is the VAR. We describe two model environments: (i) a linear VAR with off-the-shelf uncertainty shocks and (ii) a nonlinear VAR with our max uncertainty shock. We then compare our non-linear environment to the threshold VAR, a commonly used nonlinear model that can capture differences in the phases of the business cycle.

2.1 The Linear VAR

One of the standard methods for evaluating the effects of uncertainty is to compute the impulse responses from a VAR. Let $X_t$ reflect a vector of macro variables and $Z_t$ reflect the measure of uncertainty. A conventional reduced-form VAR has the form

$$
\begin{bmatrix}
Z_t \\
X_t
\end{bmatrix} = \begin{bmatrix}
b^{zz} (L) & b^{zx} (L) \\
b^{xz} (L) & b^{xx} (L)
\end{bmatrix} \begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} + \begin{bmatrix}
\varepsilon^z_t \\
\varepsilon^x_t
\end{bmatrix},
$$

(1)

where the $b^{ij} (L)$ are lag polynomials reflecting $j$’s effect on $i$ and $\varepsilon_t = [\varepsilon^z_t, \varepsilon^x_t]' \sim N (\mathbf{0}, \Omega)$ are the reduced-form errors.\(^3\)

The structural form of the VAR can be obtained in the usual way, either through

\(^3\)We utilize a constant volatility model for the balance of the paper. A number of other studies have investigated the use of stochastic volatility to both identify and measure the effects of uncertainty. In these models, uncertainty has both linear effects and affects the variance of shocks in the VAR. We estimated a model that included common stochastic volatility (see Carriero, Clark, and Marcellino, forthcoming) but found no important differences in the shapes of the responses to uncertainty shocks. During periods of high uncertainty, when the variances are large, the common volatility parameter scales the responses but the shapes are unchanged. These results are available upon request.
sign or exclusion restrictions. In our case, we obtain the structural form by computing the Cholesky decomposition assuming that uncertainty is ordered first in the VAR. We identify the structural uncertainty shocks consistent with the extant literature through causal ordering restrictions on the contemporaneous effects matrix. In particular, Baker, Bloom, and Davis (2016) order the economic policy uncertainty variable first in the VAR. Thus, shocks to the macro variables do not contemporaneously affect uncertainty but shocks to uncertainty do contemporaneously affect macro variables.\footnote{Baker, Bloom, and Davis (2016) verify that the responses to uncertainty are robust to changes in the causal ordering.}

Notice that the conventional VAR implies a linear effect of a shock to uncertainty on the macro variables. In particular, the effect of shocks in the VAR are (1) independent of the history of the variables, (2) symmetric with respect to the direction of the shock, and (3) scaled by the magnitude of the shock. Thus, a small change in uncertainty has a correspondingly small effect on the macro variables and decreases in uncertainty have the same magnitude effect on the macro variables (albeit in the opposite direction). Moreover, the level of uncertainty at the time of the shock does not matter in the linear VAR: the effect of uncertainty shocks in times of low uncertainty have the same effect as a similar magnitude shock in times of high uncertainty.

### 2.2 The Max Uncertainty VAR

Our model is based on the conjecture that the effects of uncertainty may depend on the level of uncertainty. One way to account for potential nonlinear effects of uncertainty on macro variables is to construct a new variable, \( \hat{Z}_t \), that reflects the percentage increase in uncertainty over the previous maximum within the last \( m \) periods:

\[
\hat{Z}_t = \max \left\{ 0, \frac{Z_t - \max \{Z_{t-1}, \ldots, Z_{t-m}\}}{\max \{Z_{t-1}, \ldots, Z_{t-m}\}} \right\}.
\]  

(2)
We consider the maximum value of uncertainty over the previous \( m = 4 \) quarters.

Our construction is similar to Hamilton’s (1996) max oil variable, defined with monthly data as the percentage increase in the price of oil over its maximum during the last \( m = 12 \) months. Because Hamilton assumes that oil prices are essentially exogenous, he can estimate the effect of a max oil shock using only the equations for the macroeconomic variables in the VAR, (1):

\[
X_t = b^{xx} (L) X_{t-1} + b^{xz} (L) \widehat{O}_{t-1} + \varepsilon^x_t,
\]

where \( O_t \) is the period–t price of oil and \( \widehat{O}_{t-1} \) is defined similarly to (2). Notice that there is no feedback from \( X_{t-1} \) into \( \widehat{O}_t \) and no linear effect of \( O_t \).

For our application, we want to allow for feedback from the macroeconomic variables to uncertainty. We cannot, however, insert max uncertainty, \( \widehat{Z}_t \), directly into the VAR as it would imply a counterfactual linear relationship between \( X_{t-1} \) and \( \widehat{Z}_t \). Instead, we posit the following model:

\[
\begin{bmatrix}
Z_t \\
X_t
\end{bmatrix} =
\begin{bmatrix}
b^{zz} (L) & b^{zx} (L) \\
b^{xz} (L) & b^{xx} (L)
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} +
\begin{bmatrix}
0 \\
\widehat{b}^{xz} (L)
\end{bmatrix} \widehat{Z}_{t-1} +
\begin{bmatrix}
\varepsilon^z_t \\
\varepsilon^x_t
\end{bmatrix},
\]

where again \( \varepsilon_t = [\varepsilon^x_t, \varepsilon^z_t]' \sim N (0, \Omega) \) are the reduced-form errors. The model (3) has a number of characteristics: (i) it preserves the linearity between uncertainty and its own lags through the lag polynomial \( b^{zz} (L) \), (ii) it allows lagged macro variables to (linearly) affect uncertainty through the lag polynomial \( b^{zx} (L) \), (iii) it allows uncertainty to affect macro variables linearly through the lag polynomial \( b^{xz} (L) \) in low uncertainty times, but (iv) it introduces a nonlinearity in the effects of uncertainty on macro variables around a threshold determined by the history of uncertainty. When uncertainty exceeds
peak levels—i.e., $Z_t > \max \{Z_{t-1}, ..., Z_{t-m}\}$, its effect on macroeconomic variables is amplified, switching from $b^{xx} (L) Z_{t-1}$ to $b^{xx} (L) Z_{t-1} + \tilde{b}^{xz} (L) \tilde{Z}_{t-1}$.

The model partitions the space of the relevant history of uncertainty into two sub-sets: One in which uncertainty is sufficiently below its past max that a one-standard-deviation shock will not change the dynamics and one in which uncertainty is close enough to its (recent) historical max that a positive shock can produce nonlinear effects. Notice that, in addition to the nonlinear effects around the threshold, the model produces directionally asymmetric effects. A negative shock to uncertainty can produce some effects via the lag polynomial $b^{xz} (L)$ whether in times of high or low uncertainty. On the other hand, only a positive shock to uncertainty can trigger the additional effect through $\tilde{b}^{xz} (L)$ only if the new level of uncertainty is sufficiently high. In addition, we explicitly assume that uncertainty does not have nonlinear effects on itself, reflected by the assumption $\tilde{b}^{xz} (L) = 0$ imposed in the second term of eq. (3). This assumption prevents uncertainty amplification—that is, when in a high uncertainty state, the uncertainty shock does not have a larger effect on itself.

2.3 The Case for the Max Uncertainty VAR vs. a Fixed Threshold

Our model introduces a nonlinearity when uncertainty reaches a local peak but nests a standard, linear VAR in times of relatively low uncertainty. Given this setup, one might ask why our model is preferable to other nonlinear models—e.g., Markov-switching VAR, STVAR, or threshold (TVAR) models. Constant transition probability Markov-switching models (e.g., Hamilton, 1989) impose that movement between regimes is independent of the level of the variables in the model. Smooth-transition VARs and threshold VARs allow for interaction between the model variables and the regime but generally impose a fixed threshold.
Our setup is most comparable to the threshold model with a time-varying threshold. To see this, consider the threshold VAR of the form:

\[
\begin{bmatrix}
Z_t \\
X_t
\end{bmatrix} = \begin{bmatrix}
b^{xz} (L) & b^{zx} (L) \\
b^{zx} (L) & b^{xx} (L)
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} + I_{[Z_{t-1}>Z^*]} \begin{bmatrix}
0 & 0 \\
\Delta b^{xz} (L) & 0
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} + \begin{bmatrix}
\varepsilon_t^z \\
\varepsilon_t^x
\end{bmatrix},
\]

where \( I_{[Z_{t-1}>Z^*]} \) is an indicator variable that takes on a value of 1 when \( Z_{t-1} > Z^* \) and 0 otherwise that imposes a similar change in dynamic to our model. In (4), the threshold value \( Z^* \) is constant. The first term on the RHS is the standard linear VAR and the second term triggers an amplification effect in the channel from uncertainty to the macro variables. When uncertainty rises above the threshold value, its effect on the macro variables changes to \( b^{xz} (L) + \Delta b^{xz} (L) \); at values below the threshold, uncertainty only affects the macro variables linearly through \( b^{xz} (L) \).

We can write our model similarly:

\[
\begin{bmatrix}
Z_t \\
X_t
\end{bmatrix} = \begin{bmatrix}
b^{xz} (L) & b^{zx} (L) \\
b^{zx} (L) & b^{xx} (L)
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} + I_{[Z_{t-1}>Z_{t-1}^*]} \begin{bmatrix}
0 & 0 \\
\hat{b}^{xz} (L) & 0
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} + \begin{bmatrix}
f(Z_{t-1}) \\
\varepsilon_t^x
\end{bmatrix},
\]

where \( Z_{t-1}^* \) is now time-varying. Note also that the effect of uncertainty on macro variables during uncertain times is determined by a function \( f (Z_{t-1}) \) determined by eq. (2), which scales the effect of uncertainty on macro variables by the percentage that uncertainty rises above its local max.

Our model has some advantages over conventional nonlinear models. Compared with the standard constant transition probability Markov-switching models, the max uncertainty VAR allows the level of uncertainty to determine how uncertainty affects macro variables. Compared with conventional smooth transition VARs and time-varying transition probability models, we add the flexibility of a time-varying, history-dependent threshold. Moreover, our setup implies that only positive shocks to uncertainty—events that make uncertainty rise—propagate nonlinearly to the macro variables. Similar mod-
els with time-varying thresholds—e.g., Dueker, Owyang, and Sola (2012), use autoregressive processes to filter the threshold. However, these models are hard to estimate and, generally, use two-sided filters to obtain the threshold value. These techniques heighten uncertainty at the end of the sample, making prediction from these models difficult. Moreover, the policymaker never really knows how close the economy is to the tipping point in real time.

On the other hand, our model does not allow the same interaction between uncertainty levels and other shocks as a full regime-switching model. This would be important if the recession was caused by rising levels of uncertainty or if a different shock caused the recession, which then triggered the rise in uncertainty, as conjectured in Bloom (2014). Stock and Watson (2012) argued that “heightened uncertainty” was a key factor that triggered the 2007-2009 recession, but the sharp rise in oil prices and financial market disruptions were also important.

3 Data and Inference

3.1 Measuring Uncertainty

Because uncertainty is unobserved, a key challenge is devising a proxy. An early attempt to measure uncertainty developed by Bloom (2009) used actual and implied stock market volatility. More recent attempts to measure uncertainty use a more formal econometric framework. Jurado, Ludvigson, and Ng (2015) use multiple series, exploiting the common variation in forecast errors as a measure of uncertainty. Rossi and Sekhposyan (2015) use the Survey of Professional Forecasters (SPF), to measure upside and downside uncertainty. Campbell (2007) also used a forecasting approach that relied on the SPF. Baker, Bloom, and Davis (2016) constructed an Index of Economic Policy Uncertainty (EPU) by using the frequency of newspaper articles containing several key
search terms.\footnote{In theoretical models, uncertainty is typically characterized as Knightian, where risks are unknown. These proxies, however, may include both known and unknown risks, making them not pure measures of Knightian uncertainty.}

Our intention is not to enter the debate about the optimal measure of uncertainty. Instead, we use off-the-shelf uncertainty series taken from other sources. Our benchmark measure of uncertainty is the Baker, Bloom, and Davis (2016) EPU, which is publicly available for the sample 1985-2018 from the Federal Reserve Bank of St. Louis FRED website.\footnote{For robustness, we also consider the Chicago Board of Exchange VIX measure of implied stock market volatility as a measure of uncertainty in Section 6. This approach is consistent with Bloom (2009) and Caggiano, et. al. (2017), among others. The data are available on the Federal Reserve Bank of St. Louis FRED database and the CBOE website. We use the average of the daily VIX over each quarter and again construct $\tilde{Z}_t$ with $m = 4.$} We convert monthly series to quarterly by taking the average value over the months in each quarter. Using the EPU, we set $m = 4$ to construct the $\tilde{Z}_t$ series by considering the maximum value of uncertainty over the previous 4 quarters. Figure 1 shows the quarterly EPU (dashed line, right axis) from 1985-2018, plotted with the mean value over the full sample, and the max uncertainty series (solid line, left axis). The shaded vertical bars represent NBER-dated recessions.

It can be clearly seen that uncertainty is high around recessions. The period of heightened uncertainty associated with the global financial crisis and Great Recession persists well after the NBER defined the end of the recession in 2009:Q2.

We find 18 different events lasting a total of 29 quarters during which $\tilde{Z}_t > 0.$ Of these, six events occur after 2008:Q4, once the federal funds rate hit the zero lower bound. Figure 1 also shows the max uncertainty series with historical events associated with some of the substantial spikes. We note five major uncertainty events. In 1998:Q3, Russia defaulted on its externally-held debt. The spike in uncertainty during 2001:Q3 is associated with the terrorist attacks on September 11, the collapse in technology stock prices, and corporate governance scandals. The next two uncertainty spikes—2008:Q1 (Bank of America’s purchased Countrywide Financial and JPMorgan Chase purchased
Bear Stearns) and 2008:Q4 (the aftermath of Lehman Brothers bankruptcy in September 2008, Fannie Mae and Freddie Mac placed into government receivership, Bank of America purchase Merrill Lynch, the bailout of AIG, the failure of Washington Mutual Bank, and Citigroup purchase Wachovia Securities)—were associated with key events during the 2007-2009 Great Recession and Financial Crisis.\(^7\) The spike in 2011:Q3 was associated with Europe’s banking and sovereign debt crisis and Standard and Poor’s announcement on August 5 that it downgraded U.S. sovereign debt from AAA to AA+.

### 3.2 Macroeconomic Data

In addition to the EPU index, we include five macroeconomic variables in our baseline VAR. Transforming the data to ensure stationarity, we use the first differences of quarterly log real gross domestic product (GDP), log Personal Consumption Expenditures chain price index (INF), and log total nonfarm employment (EMP). Additionally, we include the effective federal funds rate and the 10-year Treasury note yield in levels. All macroeconomic data are obtained from the Federal Reserve Bank of St. Louis FRED database.

In a subsequent section considering through which channels uncertainty propagates, we include first differences of log real personal consumption expenditures (CONS) and log real total investment (INV). We then consider disaggregate consumption and investment series. In particular, we include first differences of log real personal durable consumption expenditures (DUR), log real personal nondurable (NON) consumption expenditures, and log real personal service consumption expenditures (SERV). As disaggregate investment series, we include first differences of log real private inventories (VEN), log real residential fixed investment (RES), and log real nonresidential business

\(^7\)See the St. Louis Fed’s Financial Crisis Timeline for more details on critical events during this time frame: https://www.stlouisfed.org/financial-crisis/full-timeline.
fixed investment (BFI).\textsuperscript{8}

Since the EPU index is available starting in 1985, we estimate the VAR using data from 1985:Q1-2018:Q2. The interest rates are taken as the average value over each quarter. We considered lag orders from one to four for the VAR and found the BIC to favor one lag. Therefore, we report the results of the estimation of a VAR(1) in the six variables (macroeconomic plus the EPU).

3.3 Estimation and Inference

One benefit of the setup (3) is that the homoskedastic model can be estimated simply using conventional methods. While the model allows us to estimate the VAR using OLS, we will utilize Bayesian methods. We impose a normal-inverse Wishart prior on the coefficients of the reduced-form model and assume that the parameters have mean zero and are uncorrelated. Let $\Psi$ represent the full vector of parameters, let $\Psi_{-\psi}$ represent the full vector of parameters less the parameter $\psi$, and let $Y$ collect the data. The sampler has two blocks: (1) the reduced-form VAR parameters, $B(L)$ and (2) the reduced-form constant variance-covariance matrix, $\Omega$. Given the prior, the sampler is a standard normal–inverse-Wishart conjugate draws.

One drawback of the model is that impulse responses will depend on the history of the uncertainty variable and both the size and direction of the uncertainty shock and, therefore, cannot be constructed in the usual way. Instead, we can construct generalized impulse response functions (GIRFs), developed by Koop, Pesaran and Potter (1996). The GIRFs are constructed using Monte Carlo methods from random draws from the history of uncertainty and are described in the Appendix.\textsuperscript{9} Since GDP, PCE inflation,

\textsuperscript{8}Total fixed investment is the sum of nonresidential and residential fixed investment. Total gross private domestic investment is thus measured as total fixed investment plus the change in private inventories. Here, we include the log of real private inventories in first differences to express the magnitude in comparable terms to the other variables in the VAR, as percentage changes.

\textsuperscript{9}In our model, the history of the $X_t$ variable does not affect the response to an uncertainty shock.
and EMP all enter the VAR in first differences of logs, we express the cumulative impulse responses of these variables to see the log-level response. In order to compare across different constant-volatility model specifications, we consider a shock with magnitude equal to one standard deviation of the $Z$ data series.\textsuperscript{10}

After discarding the first $2000$ draws, we use $8000$ draws from the sampler, thinning at each $10^{\text{th}}$ draw, to construct generalized impulse responses.

\section{Measuring the Effect of Uncertainty Shocks}

Initially, we consider three permutations of the nonlinear VAR outlined in equation (3): (i) uncertainty has only linear effects—i.e., where $\hat{b}^{xz}(L) = 0$; (ii) uncertainty has only nonlinear effects—i.e., where $b^{xz}(L) = 0$; and (iii) uncertainty can have both linear and nonlinear effects—i.e., where we leave $b^{xz}(L)$ and $\hat{b}^{xz}(L)$ unrestricted. In this last model, uncertainty shocks have linear effects in periods of low uncertainty. When uncertainty rises above the threshold, the second term on the right-hand-side of (3) produces additional nonlinear effects. In our initial experiments, we use the baseline macroeconomic dataset, $X_t = [GDP_t, INF_t, EMP_t, FFR_t, 10Y_t]^\prime$.

Our first exercise is to consider the fit of each of the three alternative specifications. For each of the three models listed above, we compute the BIC at each iteration of the Gibbs sampler to obtain the mean BIC.\textsuperscript{11} The model with the lowest average BIC is permutation (ii), the max uncertainty VAR where $b^{xz}(L) = 0$, which we adopt as our benchmark model. This initial result suggests that nonlinearities are important in quantifying the effects of uncertainty shocks. Moreover, once we account for these threshold nonlinearities, the linear term $b^{xz}(L)$ contributes less to increasing in-sample

\textsuperscript{10}For the EPU, this results in a shock equal to 28.99 index points. For the VIX, the shock is equal to 7.81 index points.

\textsuperscript{11}Kass and Raftery (1995) argue that the BIC closely approximates the computation of Bayes factors.
fit than the corresponding increase in estimation error associated with the additional parameters. Thus, real activity, inflation, and interest rates are only affected when a shock raises uncertainty above a local maximum and firms and households to begin to pay attention.

4.1 Impulse Responses

As we alluded to above, the impulse responses one obtains from the linear VAR are invariant with respect to the events leading up to the time of the shock: Whether uncertainty has been high or low in recent history does not affect the future propagation of a shock at time $t$. On the other hand, the responses for our nonlinear max uncertainty VAR will depend on the history (at least through the window $m$) of uncertainty up to the time of the shock. One approach to computing the responses would be to average over all possible histories. Instead, we compare the responses under two alternative histories leading up to the shock at time $t$: (1) $\hat{Z}_{t-1} = \ldots = \hat{Z}_{t-p} = 0$, and (2) $\hat{Z}_{t-1} > 0$ and $\hat{Z}_{t-2} = \ldots = \hat{Z}_{t-p} = 0$. The first scenario represents times when the economy has not experienced a spike in uncertainty in recent history. The second scenario represents the case for which uncertainty has just reached a high level in the previous period.

Figure 2 shows the impulse responses of macro variables and interest rates to a one-standard-deviation shock to the EPU index for the max uncertainty VAR. The solid line and light-shaded band, respectively, represent the posterior median and 68-percent posterior coverage of the responses when uncertainty has not recently crossed the threshold (Scenario 1). The dashed line and dark-shaded band show the responses

\footnote{In our baseline model, an extreme case is obtained when the level of uncertainty does not cross the local max threshold for the duration of the response period; in this case, uncertainty shocks will have no effect on real variables.}

\footnote{Our scenarios do not encompass all possible histories. Also, note that the first scenario includes histories in which the shock raises uncertainty above the threshold as well as histories in which the threshold is not crossed for the duration of the response horizon.}
when uncertainty has recently crossed the threshold, $\tilde{Z}_{t-1} > 0$ (Scenario 2).

As expected, an increase in uncertainty produces recessionary conditions in both scenarios, leading to declining output, prices, and employment. We also find reductions in the federal funds rate and the interest rate on the 10-year Treasury Bill. This may reflect the systematic response of monetary policy intended to mitigate the contractionary effects of the uncertainty shock.\(^{14}\) When uncertainty has recently been high, the contractionary effects are stronger, leading to larger reductions in economic activity and inflation.

### 4.2 Comparison with the Linear VAR

We next compare the responses from a linear VAR with those estimated from the max uncertainty VAR. Figure 3 plots the posterior median responses of the linear model (open squares), the max uncertainty VAR under Scenario 1 (solid lines), and the max uncertainty VAR under Scenario 2 (dashed lines). For the first six quarters, the responses from the linear VAR are less contractionary but more persistent than those from either scenario in the max uncertainty VAR. When we introduce the nonlinear transmission of uncertainty shocks, all variables contract more quickly following the shock but also recover more quickly once uncertainty either stabilizes or declines (i.e., once $\tilde{Z}_{t+k} = 0$ for some future $k$).

As depicted in the top left panel of Figure 3, the shock to $Z_t$ is persistent and has long-term effects regardless of whether nonlinear effects are included or not. In the linear model, the persistence of the uncertainty shock produces economic effects across

\(^{14}\)Using the VIX, Caggiano, Castelnuova, and Pellegrino (2017) find statistically stronger negative real effects of uncertainty when monetary policy is at the zero lower bound (defined as the subsample from 2008:Q4-2015:Q4). A closer look at the EPU index reveals that the observations from 2008:Q3-2013:Q4 are above the sample average, coinciding with much of the ZLB period. Our max uncertainty variable takes on values greater than zero in 10 quarters after the end of 2008. Thus, our results are comparable to those of Caggiano, Castelnuova, and Pellegrino (2017) in that many of the high-uncertainty episodes occur once the economy faces the zero lower bound.
the entire response horizon. The contraction in output continues after uncertainty has stabilized because the longer-run level of uncertainty is higher than it was pre-shock. On the other hand, in the max uncertainty VAR, once $Z_t$ stabilizes around its new, higher level, households and firms become accustomed to the new environment building up a form of uncertainty tolerance. After a short period, output begins to recover.

### 4.3 Comparing to the Threshold VAR

Next, we compare our max uncertainty VAR—where the state change occurs when uncertainty is locally high—to a constant threshold VAR—where a state change occurs when uncertainty is above sample mean of the EPU index. As seen in Figure 1, the index is above the mean during all three recessions that occur in the sample (1990:Q3-1991:Q1, 2001:Q2-2001:Q4, and 2008:Q1-2009:Q2). The EPU also suggests uncertainty was above average from the beginning of the sample through the end of 1986. After the Great Recession ends in 2009, the EPU stays above average through the end of 2013.

We construct GIRFs from the threshold VAR for two scenarios: (A) uncertainty was above the threshold in the previous period and (B) uncertainty was below the threshold in the previous period. We then compare these two threshold VAR scenarios to the two max uncertainty VAR scenarios described in the previous section. Figure 4 shows the posterior median responses to equal-sized uncertainty shocks for the four scenarios: open circles and dots, respectively, for the threshold VAR scenarios A and B and solid line and dashed lin, respectively, for the max uncertainty VAR scenarios 1 and 2.

When compared to the sharp declines in the macro variables exhibited from the max uncertainty VAR, GIRFs from the threshold VAR are relatively more shallow but

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15Comparing only the median responses, we see larger reductions in all real activity variables following an increase in uncertainty when initially below the threshold. This is likely due to the fact that a one-standard-deviation shock to $Z_t$ when uncertainty is initially low represents a much larger spike than when uncertainty was already high to begin with. Further increases in uncertainty when the economy already is facing high uncertainty produce similar contractionary effects.
remain more persistent. Similar to the comparison of our model with the linear VAR, the real variables recover more slowly in the threshold VAR than in the max uncertainty VAR. Thus, it seems that both accounting for the nonlinear transmission of heightened uncertainty and time-variation of threshold are important for tracing out the effects of uncertainty shocks.

5 Identifying Propagation Channels

Previous studies have proposed theories about the channels through which uncertainty could affect real economic variables. We have highlighted a few of the papers which argue that uncertainty could act through firm investment [Bernanke (1983); Dixit and Pindyck (1994)], household purchases [Basu and Bundick (2017)], or both [Gilchrist, et al. (2014); Pastor and Veronesi (2012)]. Moreover, a few recent papers have argued for the importance of monetary policy in affecting the transmission of uncertainty shocks [e.g., Caggiano, Castelnuovo and Nodari (2017); Colombo and Paccagnini (2017)]. These papers examine the interaction between monetary policy, focusing on the systematic response of monetary policy to changes in uncertainty.

In this section, our objective is to disentangle some of the channels through which uncertainty can have effects on real variables. In particular, we consider through which of these two sectors—investment or consumption—uncertainty propagates to the real economy. We then examine the effect of suppressing the systematic component of monetary policy to determine policy’s role in mitigating the effects of uncertainty shocks. While similar to that conducted in Caggiano, Castelnuovo and Nodari (2017), our experiment differs from theirs along a number of important dimensions. First, regime changes in their model are driven by real variables; thus, the experiment is disconnected from the variable that drives the regime change. Second, their threshold is
time-invariant. In the previous section, we showed that the constant threshold assumption leads to an increase in the persistence of the effects of uncertainty shocks, which could lead to important differences in the monetary policy response.

To do this, we consider a few variants of the baseline VAR. First, we augment the baseline VAR with real aggregate investment to determine the extent to which uncertainty shocks propagate through investment behavior. We then further disaggregate investment into business fixed investment, residential investment, and inventories. Next, we augment the baseline VAR with real consumption expenditures. We then further disaggregate consumption into durable, non-durable, and service consumption.

5.1 Investment and Consumptions Channels

The first panel of Figure 5 shows that aggregate investment experiences a significant downturn following a shock increasing uncertainty. Because investment is the most volatile component of aggregate output, it is not surprising to see it contract sharply in Scenario 2, when the economy faces relatively uncertain times.

Next, we examine how uncertainty propagates through disaggregated measures of investment including business fixed investment, residential investment, and inventories. We augment the baseline VAR with the first differences of log real spending on these three categories, ordered directly after GDP. The second and third panels of Figure 5 show the cumulative responses of the three subcategories of investment to uncertainty shocks for the two scenarios. For both histories of uncertainty, residential investment falls more on impact and declines more sharply than the other two investment series, but it also rebounds more quickly. Business fixed investment continues to fall for a

\[\text{For brevity, we do not include figures showing the impulse responses for all variables implied by this model. These results are available from the authors upon request.}\]

\[\text{All real expenditure variables enter the VAR in the first difference of logs. To remain consistent with earlier results, we report the cumulative impulse responses so that we can interpret the effects of uncertainty shocks on log-levels of these variables.}\]
longer duration after the shock and exhibits a more persistent contraction. Under both starting scenarios, residential investment reaches its minimum 3 quarters after the shock while business fixed investment continues to fall through 6 quarters.\textsuperscript{18} Following the uncertainty shock, the initial decline in inventories is small—although larger in Scenario 2—but persistent. These results imply that businesses adapt to the uncertain environment rather quickly, adjusting their decision-making process in accordance with volatile economic conditions. Inventories adjust downward even as consumers reduce spending, thus suggesting that businesses might be cutting back both on investment as well as production.

The first panel of Figure 6 shows that consumption, like the other real variables, declines significantly after the shock to uncertainty. The effects are persistent under both scenarios, with a more severe contraction if the recent history of uncertainty has been high. The magnitude of the consumption response is about one-fifth that of the investment response.

Next, we estimate the baseline VAR including the first differences of log real consumption spending on durables, non-durables, and services, ordered after GDP. The second and third panels of Figure 6 plot the cumulative responses of the three subcategories of consumption to uncertainty shocks for the two scenarios. Regardless of whether uncertainty has recently been high or low, all three categories of consumption decline on impact in response to the shock. Also, regardless of the uncertainty conditions at the time of the shock, durables consumption falls by a larger magnitude and exhibits a more persistent contraction than the other consumption categories. This result supports the view that households exercise precautionary reductions in spending, in particular related to durables spending which would be comparable to the real-option

\textsuperscript{18}Kim and Kung (2017) find that firms using less redeployable assets—an important feature of investment irreversibility—reduce capital investment more after increases in uncertainty. This behavior explains the severe contraction of business fixed investment.
effects of irreversible investment spending.

5.2 The Interaction Between Uncertainty and Monetary Policy

One of the prevailing themes in the current literature is that monetary policy can be used as a tool to mitigate the effects of uncertainty. In our baseline results, the Fed accommodates the uncertainty shock by lowering the fed funds rate. Thus, the responses in the preceding sections rely on the behavior of the Fed remaining consistent. As an alternative, one might be interested in evaluating the effect of uncertainty shocks in isolation, where the Fed is not responding to the shock. This both provides a benchmark response to uncertainty shocks and demonstrates the extent to which the systematic monetary response can lessen the effect of the uncertainty shocks.

The experiment that isolates the effect of the uncertainty shock and suppresses the response of monetary policy is outlined in Bernanke, Gertler, and Watson (1997) and consists of constraining the value of the federal funds rate to remain at its pre-shock level. For example, to determine the response of GDP to uncertainty shocks when the interest-rate channel is shut down, we compute the counterfactual response of GDP while removing the future expected path of the federal funds rate from the simulation.

Figure 7 plots the posterior median of the cumulative GIRFs of GDP from the benchmark with the counterfactual analysis shutting down the interest rate channel in either (i) the baseline VAR, (ii) the baseline VAR augmented with investment, or (iii) the baseline VAR augmented with consumption. We observe that the contraction in GDP is larger in all cases when monetary policy does not systematically respond to the negative effects of the uncertainty shock.

Table 1 shows the ratio of the 12-quarter cumulative response of GDP under the restricted counterfactual to the unrestricted benchmark for each of the three VARs
discussed above. For each VAR, we compute the GIRFs using the two starting scenarios we define in Section 4.1. Values greater than 1.00 suggest that GDP declines more in the counterfactual than in the benchmark. Not surprisingly, in every case, GDP declines more when we restrict monetary policy’s ability to react to the uncertainty shock.\footnote{Caggiano, Castelnuovo, and Nodari (2017) conduct a similar exercise but compare counterfactual scenarios in which systematic monetary policy does not react in either recessionary or expansionary economic conditions. They find that monetary policy is influential for avoiding a recession if the uncertainty shock occurs in a strong economy. Alternatively, monetary policy attenuation has little effect if the uncertainty shock occurs when the economy is already facing a recession.} In particular, monetary policy reduces the effect of uncertainty shocks between 31 and 57 percent, depending on the model specification and the initial conditions at the time of the shock. The channel through which monetary policy has the largest effects is the case where we explicitly model the uncertainty channel through consumption.

6 Robustness

6.1 Alternative Uncertainty Series

For robustness, we consider two alternative measures of uncertainty: the CBOE VIX measure of implied stock market volatility and the macroeconomic uncertainty series constructed by Jurado, Ludvigson, and Ng (2015), henceforth referred to as "JLN". In Figure 8, we plot the EPU index (left axis - solid line) with the VIX (right axis - dashed line). The two series behave similarly throughout the sample and exhibit a correlation of 0.43. The indices differ considerably in magnitude where the EPU has a standard deviation equal to 28.99 index points while that of the VIX is only equal to 7.81 points. Constructing $\hat{Z}_{t}^{\text{VIX}}$ analogously, the VIX produces 31 quarters during which $\hat{Z}_{t}^{\text{VIX}} > 0$. Figure 9 plots a comparison of the $\hat{Z}_{t}$ from the EPU index (left axis - solid line) with that of the VIX (right axis - dashed line). While the VIX produces two additional observation of a non-zero $\hat{Z}_{t}^{\text{VIX}}$, most of the non-zero values are of much
smaller magnitude than those associated with the EPU.

Alternatively, Figure 10 plots the EPU index (left axis - solid line) with the JLN index (right axis - dashed line). With a correlation of only 0.34, we find more variation in the behavior of these two series where the JLN index rises only slightly in the recessions of the early 1990’s and early 2000’s but spikes dramatically during the financial crisis. Like the VIX, the JLN series also takes on values of a much smaller magnitude than the EPU with a standard deviation of only 0.08. Figure 11 plots $\hat{Z}_{t}^{JLN}$ constructed with the JLN series, highlighting 35 quarters in which $\hat{Z}_{t}^{JLN} > 0$. Only 15 of these quarters overlap with dates for which the EPU $\hat{Z}_{t} > 0$. $\hat{Z}_{t}^{JLN}$ spikes prior to, rather than during, the recession in the early 1990’s. We also find large, non-zero values of $\hat{Z}_{t}^{JLN}$ in 1996:Q1 and 2005:Q3, while our $\hat{Z}_{t}$ based on the EPU stays at zero during both of these episodes.

We construct generalized impulse responses of all variables in the VAR to an increase in uncertainty under the same two environments considered previously: (1) when $\hat{Z}_{t-1} = \ldots = \hat{Z}_{t-p} = 0$, and (2) when $\hat{Z}_{t-1} > 0$ and $\hat{Z}_{t-2} = \ldots = \hat{Z}_{t-p} = 0$. To be concise, we do not include figures showing the impulse responses when the VIX or the JLN series is substituted as the measure of uncertainty.²⁰ The results are qualitatively similar: an increase in uncertainty when the economy has recently hit a local max produces a larger contraction in economy activity and inflation and a larger reduction in both the federal funds rate and the interest rate on 10-year Treasury Bills. Furthermore, when comparing the max uncertainty VAR to linear or threshold VARs, all variables contract more steeply and recover more quickly in the former model.

²⁰These are available upon request.
6.2 Allowing Linear Responses to Uncertainty

The benchmark model that includes the restriction that $b^{xz}(L) = 0$ shuts down the linear effect of uncertainty on the macroeconomy, damping out the effect of small uncertainty shocks (as they are less likely to cross the threshold and activate the second term in (3)) and zeroing out the effect of decreases in uncertainty. While our specification tests suggest our benchmark model is preferred, we estimated the model leaving $b^{xz}(L)$ unrestricted for comparison. Figure 12 compares the responses of GDP from the restricted model with the unrestricted model for scenarios 1 and 2, respectively.\footnote{The responses of the other variables in the VAR are available from the authors upon request.} These figures show qualitatively comparable contractionary effects for uncertainty shocks in both models, although the differences between the two scenarios are less pronounced. The contraction in output is more persistent in the unrestricted model where we estimate the full set of coefficients in the linear portion of the VAR. This captures the behavior evident from the linear model in which output continues to contract over the longer-term as the shock to uncertainty is so persistent. Thus, we are able allow for a similar dynamic in which the heightened uncertainty that persists long after the initial spike to max uncertainty might continue to suppress economic activity over longer time horizons.

6.3 Alternative Lag Length in the VAR

As discussed in Section 3.2, we considered lag orders from one to four for the VAR and found the BIC to favor one lag. Therefore, all of the results presented thus far include one lag of all variables in the VAR. Given that we have quarterly data, we also looked more closely at the results when replicating all of our exercises for a VAR(4) instead. All of the qualitative conclusions are consistent across lag lengths: economic activity and prices contract more severely following a shock when uncertainty has recently been high.
The relationships between the results suggested by our max uncertainty, linear, and threshold models are also comparable when increasing the lag length in the VAR. We also generate similar conclusions regarding the role of the investment and consumption channels in the broader macroeconomy when facing heightened uncertainty.

7 Conclusion

We construct a model with nonlinearities and a deterministic time-varying threshold. In our model, uncertainty must rise above recent historical highs to trigger the nonlinearity. The model has the advantage of being relatively easy to estimate, in part because of the deterministic threshold. In addition, unlike models with time-varying unobserved thresholds, the deterministic threshold is easy for a policymaker to interpret as the economy’s proximity to the nonlinearity is known.

Our results are consistent with existing literature in finding that increases in uncertainty lead to economic downturns. Furthermore, we find empirically relevant differences between the macroeconomic responses to uncertainty shocks under conditions of high and low uncertainty. Compared to linear models and a number of other nonlinear alternatives, we find that the effects of uncertainty shocks are deep and sharp but not as persistent. This is perhaps due households and firms ignoring fluctuations in uncertainty during tranquil economic times that leads to considerable variation in the sensitivity to shocks that create volatility.
References


A Computing the GIRFs

Stacking the elements of the VAR, let’s define \( y_{t+k} = [X'_{t+k}, Z'_{t+k}]' \). We can think of an impulse response as the difference between the expectation of the variable conditional on the shock and the expectation of the variable conditional on no shock:

\[
IRF_k(\delta) = E_t [y_{t+k} | \Omega_t, v_t = \delta] - E_t [y_{t+k} | \Omega_t, v_t = 0],
\]

where \( IRF_k(\delta) \) is the impulse response at horizon \( k \) after a shock of magnitude \( \delta \) at time \( t \), \( v_t \) is the structural shock, and \( \Omega_t \) is the information (history) at time \( t \).

To construct the impulse response, we first compute the path of the variables for no shock to uncertainty at time \( t \). That is, we compute:

\[
\begin{bmatrix} Z_t \\ X_t \end{bmatrix} = \begin{bmatrix} b^{zz}(L) & b^{zx}(L) \\ b^{xz}(L) & b^{xx}(L) \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ X_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{b}^{zz}(L) \end{bmatrix} \hat{Z}_{t-1},
\]

at time \( t \). For the duration of the response, we simulate innovations out to horizon \( H \) by drawing random values for \( \varepsilon_{t+k} \) from the \( N(0, \Omega) \) distribution:

\[
\begin{bmatrix} Z_{t+k} \\ X_{t+k} \end{bmatrix} = \begin{bmatrix} b^{zz}(L) & b^{zx}(L) \\ b^{xz}(L) & b^{xx}(L) \end{bmatrix} \begin{bmatrix} Z_{t+k-1} \\ X_{t+k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{b}^{zz}(L) \end{bmatrix} \hat{Z}_{t+k-1} + \begin{bmatrix} \varepsilon^z_{t+k} \\ \varepsilon^x_{t+k} \end{bmatrix}
\]

for \( k = 1, \ldots, K \). Obviously, the propagation of the shock will be different if uncertainty is sufficiently high that \( \hat{Z}_{t+k-1} \) is nonzero. Thus, we construct the response under two alternative scenarios: (1) when \( \hat{Z}_{t-1} = \ldots = \hat{Z}_{t-p} = 0 \), and (2) when \( \hat{Z}_{t-1} > 0 \) and \( \hat{Z}_{t-2} = \ldots = \hat{Z}_{t-p} = 0 \). The second scenario represents the case for which uncertainty has just reached a high level in the previous period.

To compute \( E_t [y_{t+k} | \Omega_t, v_t = \delta] \) in general, we have
\[
\begin{bmatrix}
Z_t \\
X_t
\end{bmatrix} =
\begin{bmatrix}
b^{xz}(L) & b^{zx}(L) \\
b^{xz}(L) & b^{zx}(L)
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} +
\begin{bmatrix}
0 \\
\hat{\theta}^{xz}(L)
\end{bmatrix}
\begin{bmatrix}
\omega_{11} & 0 \\
\omega_{21} & \omega_{22}
\end{bmatrix}
\begin{bmatrix}
\delta
\end{bmatrix},
\]

where

\[
\text{chol} (\Omega) =
\begin{bmatrix}
\omega_{11} & 0 \\
\omega_{21} & \omega_{22}
\end{bmatrix}.
\]

In the first case, where when \( \hat{Z}_{t-1} = ... = \hat{Z}_{t-\max(p,m)} = 0 \), we have

\[
\begin{bmatrix}
Z_t \\
X_t
\end{bmatrix} =
\begin{bmatrix}
b^{xz}(L) & b^{zx}(L) \\
b^{xz}(L) & b^{zx}(L)
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} +
\begin{bmatrix}
0 \\
\hat{\theta}^{xz}(L)
\end{bmatrix}
\begin{bmatrix}
\omega_{11} & 0 \\
\omega_{21} & \omega_{22}
\end{bmatrix}
\begin{bmatrix}
\delta
\end{bmatrix}.
\]

Should the shock to uncertainty of magnitude \( \delta \) lead to \( \hat{Z}_{t-k-1} > 0 \) for any \( k = 1, ..., K \), this would turn on the channel through which uncertainty affects the macroeconomic variables via \( \hat{\theta}^{xz}(L) \hat{Z}_{t+k-1} \).

In the second scenario, where \( \hat{Z}_{t-1} > 0 \), we compute the GIRF with

\[
\begin{bmatrix}
Z_t \\
X_t
\end{bmatrix} =
\begin{bmatrix}
b^{xz}(L) & b^{zx}(L) \\
b^{xz}(L) & b^{zx}(L)
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
X_{t-1}
\end{bmatrix} +
\begin{bmatrix}
0 \\
\hat{\theta}^{xz}(L)
\end{bmatrix}
\begin{bmatrix}
\omega_{11} & 0 \\
\omega_{21} & \omega_{22}
\end{bmatrix}
\begin{bmatrix}
\delta
\end{bmatrix}.
\]

For as long as \( \hat{Z}_{t+k-1} > 0 \), the \( \hat{\theta}^{xz}(L) \hat{Z}_{t+k-1} \) term perpetuates the uncertainty shock through the response.
Table 1: **Shutdown Experiments.** Ratio of the cumulative response of GDP after 12 quarters under the counterfactual scenario, with the interest-rate channel shutdown, versus the benchmark, unrestricted response. Values greater than 1.00 suggest a larger contraction of the log level of GDP when the interest rate is unable to respond to the uncertainty shock.
Figure 1: Max Uncertainty Series (solid line - left axis) with the Baker, Bloom, and Davis (2016) Economic Policy Uncertainty Index (dashed line - right axis) and mean EPU over the sample. The Max Uncertainty series is labeled with significant historical events associated with large spikes in uncertainty.

Figure 2: Impulse responses from the benchmark VAR (ii) with only non-linear effects of uncertainty. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: $\hat{Z}_{t-1} = \ldots = \hat{Z}_{t-p} = 0$. Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: $\hat{Z}_{t-1} > 0$ and $\hat{Z}_{t-2} = \ldots = \hat{Z}_{t-p} = 0$. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.
Figure 3: Comparison of impulse responses from the benchmark VAR (ii) with only non-linear effects of uncertainty versus model (i), the linear VAR. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: $\tilde{Z}_{t-1} = \ldots = \tilde{Z}_{t-p} = 0$. Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: $\tilde{Z}_{t-1} > 0$ and $\tilde{Z}_{t-2} = \ldots = \tilde{Z}_{t-p} = 0$. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.
Figure 4: Comparison of impulse responses from the benchmark VAR (ii) with only non-linear effects of uncertainty versus the fixed-threshold VAR. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: \( \ddot{Z}_{t-1} = ... = \ddot{Z}_{t-p} = 0 \). Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: \( \ddot{Z}_{t-1} > 0 \) and \( \ddot{Z}_{t-2} = ... = \ddot{Z}_{t-p} = 0 \). We set the fixed threshold at the mean value of the \( Z \) series and compute generalized impulse responses when the economy was above or below this threshold in the period before the shock. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.
Figure 5: First panel: Generalized impulse responses of aggregate investment from the benchmark VAR (ii). Second and third panels: Impulse responses of the subcategories of investment from the benchmark VAR (ii), augmented with investment, with only non-linear effects of uncertainty. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: $\tilde{Z}_{t-1} = ... = \tilde{Z}_{t-p} = 0$. Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: $\tilde{Z}_{t-1} > 0$ and $\tilde{Z}_{t-2} = ... = \tilde{Z}_{t-p} = 0$. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.
Figure 6: First panel: Generalized impulse responses of aggregate consumption from the benchmark VAR (ii). Second and third panels: Impulse responses of the subcategories of consumption from the benchmark VAR (ii), augmented with consumption, with only non-linear effects of uncertainty. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: $\tilde{Z}_{t-1} = ... = \tilde{Z}_{t-p} = 0$. Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: $\tilde{Z}_{t-1} > 0$ and $\tilde{Z}_{t-2} = ... = \tilde{Z}_{t-p} = 0$. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.
Figure 7: Impulse responses from the benchmark VAR (ii) with only non-linear effects of uncertainty. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: \( \hat{Z}_{t-1} = \ldots = \hat{Z}_{t-p} = 0 \). Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: \( \hat{Z}_{t-1} > 0 \) and \( \hat{Z}_{t-2} = \ldots = \hat{Z}_{t-p} = 0 \). We shut down the response of the federal funds rate and compare the responses of GDP in the restricted and unrestricted cases. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.

Figure 8: Quarterly Data on the Baker, Bloom, and Davis (2016) Economic Policy Uncertainty Index (solid line - left axis) and the CBOE VIX measure of implied stock market volatility (dashed line - right axis)
Figure 9: Max Uncertainty Series with the Baker, Bloom, and Davis (2016) Economic Policy Uncertainty Index (solid line - left axis) and the VIX (dashed line - right axis)

Figure 10: Quarterly Data on the Baker, Bloom, and Davis (2016) Economic Policy Uncertainty Index (solid line - left axis) and the Jurado, Ludvigson, Ng (2015) measure of uncertainty (dashed line - right axis)
Figure 11: Max Uncertainty Series with the Baker, Bloom, and Davis (2016) Economic Policy Uncertainty Index (solid line - left axis) and the Jurado, Ludvigson, Ng (2015) measure of uncertainty (dashed line - right axis).

Figure 12: Comparison of impulse responses of GDP from the benchmark VAR (ii) with only non-linear effects of uncertainty versus model (iii), the full VAR with both linear and non-linear effects of uncertainty. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: $\tilde{Z}_{t-1} = \ldots = \tilde{Z}_{t-p} = 0$. Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: $\tilde{Z}_{t-1} > 0$ and $\tilde{Z}_{t-2} = \ldots = \tilde{Z}_{t-p} = 0$. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.