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Natural Resources and Sovereign Risk in Emerging Economies: A Curse *and* a Blessing

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Abstract

Emerging economies that are large oil producers have sizable external debt, their country risk rises when oil prices fall, and several of them have defaulted at least once since 1979. Moreover, while oil and non-oil output reduce country risk on impact and in the long-run, oil reserves reduce it marginally on impact but *increase* it in the long-run. We propose a model of sovereign default and oil extraction consistent with these observations. The sovereign manages oil reserves strategically to make default less painful by altering the value of autarky, and hence its sustainable debt falls. All else equal, default is less likely in states in which reserves or oil prices are higher, or non-oil GDP is lower, but the equilibrium dynamics of reserves and country risk in response to oil-price shocks switch from negatively correlated on impact to positively correlated for several years.

Keywords: Country Risk, Oil Prices, Oil Reserves, Sovereign Debt.
JEL Codes: E44, F4, F34, G12, H63, L72.

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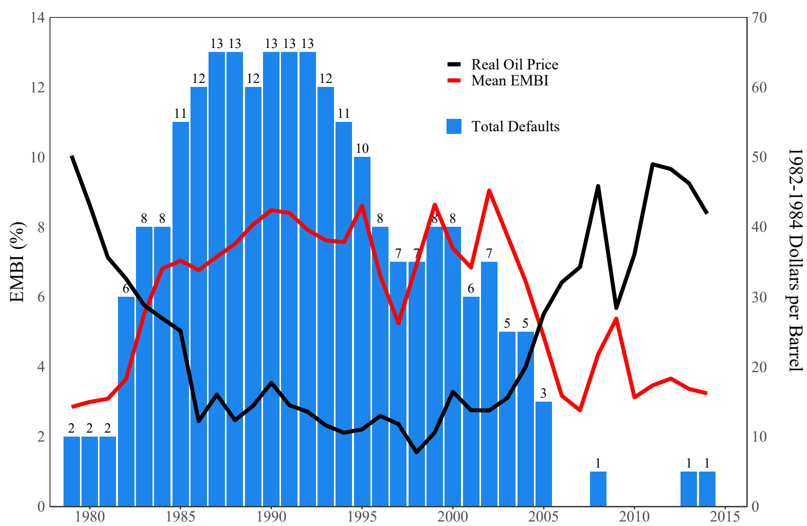
1 Introduction

Emerging economies that are large commodity producers have a turbulent history in sovereign debt markets. This is particularly the case for oil producers, for which oil-price fluctuations are an important determinant of external debt, repayment history, and country risk. Figure 1 summarizes the relationship between oil prices, default risk, and default events in the 1979-2014 period for the thirty largest oil-producing emerging economies as of 2010.[1] The bars show the number of countries that were in default or financial exclusion each year, the red curve is the mean of the Emerging Market Bond Index (EMBI) for all thirty countries (left axis), and the black curve is the real price of Brent crude oil (right axis).[2] Clearly, country risk and the number of countries in default rise sharply as the oil price falls. These co-movements are evident over the medium term (e.g, in the 1980-2005 period) and also in the short-run. For instance, in 2005-2007 the oil price was high, there were no defaults and country risk fell, but as soon as the price fell in 2008 default risk rose and one country defaulted.

Figure 1: Oil Price, EMBI and Number of Countries in Default



In this paper, we study the stylized facts connecting oil prices, production, and reserves to sovereign default and country risk and propose a model aimed at explaining them. We start with an empirical analysis that formalizes the suggestive evidence shown in Figure 1. We document in particular three facts: (1) the thirty largest oil producers have sizable external

[1]See Appendix A for the exact list of countries, data sources and transformations.

[2]The EMBI is available starting in 1998. Section 2 of the paper describes how the estimates for the period 1979-1997 were constructed.

debt (22.5% on average, weighted by the share of combined oil output), about half of them defaulted at least once, and their debt-GDP ratio and country risk are positively correlated; (2) at the cyclical frequency, country risk is positively correlated with the real price of oil, with an unconditional (weighted) correlation of 0.69; and (3) in dynamic error-correction panel regressions, higher oil production or non-oil GDP reduce country risk on impact and in the long-run, but oil reserves display a (conditional) non-monotonic effect on country risk, with a marginally positive effect on impact but a negative effect as both variables converge to their long-run trends.

We then develop a model of sovereign default in the vein of Eaton & Gersovitz (1981). In the model, the economy has an endowment of a tradable, non-storable good (non-oil GDP), and produces oil that is sold in world markets at an exogenous relative price in units of the tradable good. The sovereign makes the oil extraction and reserves decisions, and also has access to the same foreign credit market typical of Eaton-Gersovitz (EG) models where it cannot commit to repay its debt. Oil prices and non-oil GDP are subject to stochastic shocks. When the sovereign defaults, oil exports incur a piece-wise linear cost analogous to the exogenous default cost in terms of endowment income introduced by Arellano (2008). This results in a default cost that is akin to a progressive tariff on oil exports, but with an endogenous component because oil output responds to the tariff-like cost.

In the canonical EG model, the value of default is determined by the exogenous realization of endowment income net of the default cost, but in the model we propose it is also altered by the optimal plans of the sovereign. Default causes the sovereign to be excluded from the credit market but not from the world oil market, and thus oil production and exports continue even after the sovereign defaults. Hence, the value of default depends on oil reserves and thus incentivizes the sovereign who cannot commit to repay its debt to strategize over both debt and reserves. When oil prices are high, the standard incentive to extract more oil and reduce reserves competes with the incentive to build reserves to prop up consumption in the event of a default. We derive analytic results showing that, conditional on keeping other variables constant, default incentives are stronger when oil reserves or oil prices are lower. We also show that the results derived by Arellano (2008) for the effects of debt and endowment income on default incentives still hold.

We calibrate the model using the same cross-country dataset used in the empirical analysis, and solve the model numerically to derive its quantitative predictions and compare them with the stylized facts. The calibration is targeted to match the observed mean debt ratio and frequency of default highlighted in the first stylized fact we noted earlier.

The quantitative analysis relaxes some of the strong assumptions used to prove the analytic results (e.g., i.i.d. shocks, no default costs, and permanent exclusion after default) but the analytic results still hold. Quantifying the costs of default, we find that they are uniformly *lower* than in a variant of the model with constant oil extraction (i.e., treating oil output as an endowment). This illustrates the sovereign's ability to strategize over oil extraction and reserves so as to make default less costly. For the same reason, however, the sovereign's borrowing capacity weakens and the average debt it can sustain in the long-run falls.

Comparing the model's predictions with the co-movements observed in the data, we find that the model does well at approximating the second stylized fact we noted: Country risk is negatively correlated with oil prices. The model also does a good job at replicating the observed income correlations and a reasonable job at matching the rest of the oil-price correlations. In terms of the variability moments, the model does well at approximating the variability ratios of gross oil output and oil extraction, and reasonably well at approximating those of total GDP, disposable income, and the trade balance-GDP ratio, but it overestimates the variability ratios of consumption and spreads and underestimates the one for reserves. Specifically, relative to oil price variability, consumption is more variable in the model than in a variant of it where the government is committed to repay, but in both models consumption is significantly more volatile than in the data.

The model also does well at replicating the non-monotonic pattern of the conditional dynamic relationship of oil reserves and sovereign risk. We document this in two ways. First, we show that the model's cross-correlation function of spreads with future and past reserves declines with time. Second, the model's impulse response functions to a positive oil-price shock show that reserves fall and country risk rises in the first three years, but then they go through seventeen years in which both reserves and country risk fall. Thus, the joint equilibrium dynamics of reserves and country risk in response to oil-price shocks switch from negatively correlated on impact to positively correlated for several years.

Finally, we quantify the relevance of the oil sector for the long-run properties of sovereign debt and spreads and for default event dynamics by comparing the results of the proposed model with the variants with constant extraction and without default risk. Endogenizing oil extraction and allowing strategic default incentives to influence extraction and reserves decisions have large quantitative implications. In the long-run, removing the commitment to repay (i.e., in the baseline model) reduces the mean debt ratio to about 23 percent from nearly 52 percent in the model without default risk. With constant extraction (and default risk), however, the mean debt ratio rises to nearly 28 percent. Hence, strategic accumulation of oil reserves in response to default incentives reduces the sovereign's sustainable debt by 5 percentage points of GDP. Around default events, the sovereign reduces oil output and increases reserves sharply above what the risk-free and constant-extraction models predict, and these extra reserves are used to prop up consumption post-default. On the other hand, this extra margin for strategic incentives to operate implies that, pre-default, the sovereign borrow less and at a sharply higher spreads.

Our work contributes to the large sovereign default literature based on the classic work of Eaton & Gersovitz (1981), particularly the quantitative branch that followed Aguiar & Gopinath (2006) and Arellano (2008) (see the survey by Aguiar et al. (2016)). Three strands of this literature examine models related to ours in that the sovereign's decisions alter the value of default. First, Sosa-Padilla (2018) introduces banks that hold sovereign debt as an asset and make working capital loans to firms. The sovereign internalizes that the net worth of the banks alters the value of default because a default reduces working capital financing. Second, Hur & Kondo (2016), Bianchi et al. (2018), Bianchi & Sosa-Padilla (2022), and Suarez (2022) examine the optimal choice of foreign reserves and sovereign debt. In the presence of long-term debt, foreign reserves alter the cost of borrowing across states of nature and the resources available after a default. Third, Hamann (2004), Gordon & Guerron-Quintana (2018), Asonuma & Joo (2021), and Esquivel (2022) introduce capital accumulation controlled by the sovereign, so that the value of default differs with the capital stock. The effect of oil reserves on the value of default in our model differs from this literature in that they prop up consumption directly by supporting oil extraction during periods of exclusion, even with one-period debt. Also, this effect is limited only to oil reserves, an asset controlled by the government in many oil-rich emerging economies, excluding the rest of the capital stock.

We also contribute to the literature on the business cycle implications of commodity-price and terms-of-trade shocks, as in Mendoza (1995), Bornstein et al. (2017), Fernández et al. (2017), Schmitt-Grohé & Uribe (2018), Ben Zeev et al. (2017), and Di Pace et al. (2020).[3] These studies typically abstract from modeling sovereign debt explicitly and assume that

[3] See Uribe & Schmitt-Grohé (2017) for a review of this literature.

external debt is risk-free. Hence, our analysis adds to the transmission mechanism of terms-of-trade shocks effects that operate via the sovereign's default incentives. As a result, default and default risk affect fluctuations in consumption, oil reserves (i.e., investment), and net exports. Moreover, because of the lack of commitment to repay sovereign debt, the economy sustains less debt in the long run and is less able to use external borrowing to smooth consumption, relative to what RBC-like models with terms-of-trade shocks predict.

Our work is also related to empirical studies of the connection between commodity prices and country risk. Bouri et al. (2017) find significant spillovers from the conditional variance of commodity prices to that of sovereign credit default swaps. Our analysis differs in that, in addition to studying the effects of oil-price shocks on default risk, we study their effects on the dynamics of debt, oil reserves and macro aggregates. Reinhart et al. (2016) document a strong overlap between movements in capital flows, commodity prices, and sovereign defaults. In line with their results, we find that oil prices and sovereign risk are negatively correlated. Esquivel (2022) provides empirical evidence showing that country risk spreads rise after large oil discoveries, consistent with our finding that higher oil reserves increase country risk in the long-run.

Finally, our work contributes to the literature on the "curse of natural resources," initiated by the finding from Sachs & Warner (1995) showing that GDP growth is lower in developing countries with higher resource exports-GDP ratios (see the survey by van der Ploeg (2011)). Manzano & Rigobon (2001) argued that this result may be driven by external debt crises: Resource-rich developing countries borrowed heavily when prices were high in the 1970s, and then suffered debt crises when the prices fell in the 1980s. In a 1980s panel regression, the negative effect of resource abundance on long-run growth disappears once the initial debt-GDP ratio is added as a regressor. Moreover, they document that all the countries driving the negative effect of natural resource exports on growth experienced debt crises. Although our focus is not on long-run growth, we contribute to this literature by examining empirically and theoretically how debt, default, country risk, oil reserves, and macro-aggregates respond to oil-price shocks. Empirically, reserves are neutral for country risk in the short run but increase it in the long run. In the model, higher reserves weaken default incentives and reduce default risk everything else constant. But considering the full equilibrium dynamics, a positive oil-price shock initially causes reserves to fall and debt, country risk and extraction to rise, as noted by Manzano & Rigobon (2001), but this is followed by a prolonged phase in which reserves and country risk rise together. Overall, default risk makes consumption and net exports more volatile and sharply reduces sustainable debt. Hence, our findings suggest that natural resources are both a curse *and* a blessing.

The paper proceeds as follows. Section 2 presents the empirical evidence, Section 3 presents the model, Section 4 presents the calibration and quantitative analysis of the model, and Section 5 concludes.

2 Stylized Facts

This section documents key empirical regularities linking oil prices, production, and reserves with aggregate economic activity and sovereign risk. We collected data for the thirty largest oil-producing emerging economies as of 2010 covering the 1979-2014 period. The data include oil-sector statistics on production (extraction), reserves, and net exports; national accounts data on GDP, consumption, trade balance, and oil rents as a percentage of GDP; and financial data on total public debt, external public debt, net foreign assets (NFA), default episodes, and a country risk indicator.[4]

Since international databases do not report oil value added, we used oil rents as a share of GDP from the World Bank's *World Development Indicators* (WDI) to construct a measure of oil GDP. This was done by multiplying oil rents as a share of GDP times total GDP at constant local-currency prices from the same source. We then defined non-oil GDP by subtracting this oil GDP measure from total GDP. Consumption and trade balance data are also from WDI at constant local-currency prices.

Oil-sector data on reserves (in billions of barrels), production and net exports (both in thousands of barrels per day), and prices (Brent crude oil spot price, FOB, U.S. dollars per barrel) are from the US Energy Information Administration (EIA). For reserves, we used proven reserves.[5] We then constructed the real price of oil by deflating the Brent oil price with the US CPI index for all urban consumers, all items (U.S. City average, seasonally adjusted, 1982-1984=100) and gross oil output by multiplying the real price of oil times oil production.

As an indicator of country risk, we use the Institutional Investor's Index of Country

[4]Not all variables are available for all 30 countries. Appendix A provides full the details regarding data availability, sources and construction of all variables.

[5]Reserves are hard to measure due to uncertainties about the quantity and quality of oil in the ground. Available measures include ultimately recoverable resources, proven, probable and possible reserves, and oil in place.

Credit Ratings (III). The III is published biannually in the March and September issues of the *Institutional Investor* and is based on credit ratings constructed with the Institutional Investor's Country Credit Survey, which reflects information provided by senior economists and sovereign-risk analysts at leading global banks and money management and securities firms. Respondents grade each country on a scale of zero to 100, with 100 corresponding to the best rating (i.e., higher scores represent a lower probability of default), and their responses are weighted according to their institutions' global exposure. The III is intended to capture a collection of risks related to investing in a particular country, including political risk, exchange rate risk, economic risk, and sovereign risk.[6]

Total public external debt is from the World Bank's *Global Development Finance* database (GDF), and NFA from the updated and extended version of the "External Wealth of Nations" dataset, constructed by Lane & Milesi-Ferretti (2007). Default data is from Borensztein & Panizza (2009) for the 1979-2004 period and from Reinhart & Rogoff (2010) for the 2005-2014 period. A sovereign default is defined as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue, or an exchange offer of new debt that contains terms less favorable than the original issue (a restructuring).

The data we collected yields three key stylized facts:

1. Large oil-producing emerging economies have an average external public debt ratio of 22.5%, about half of them defaulted at least once, and country risk worsens as debt rises (the average correlation between the debt ratio and the III is -0.6).

Figure 2 shows the average ratio of external public debt to GDP for the countries in our sample (countries for which no data are available are shown as zeros). The lowest is around 4% for Iran and the largest is 72% for Vietnam. Across all countries, the average external debt ratio is 22.5%.[7] Figure 3 shows the number of default episodes for the same set of countries. Sixteen of the thirty countries have defaulted at least once since 1979, and several have defaulted more than once (Argentina, Ecuador, Gabon, Indonesia, Nigeria, Russia

[6]Spreads based on the Emerging Markets Bond Index (EMBI) are typically used to measure country risk. EMBI spreads, however, are only available since 1994 and for a small number of countries, which imposes limitations on the scope of the empirical analysis that we can conduct. In Appendix B we show that the III is negatively correlated with EMBI spreads in the subsample of countries we study for which both are available (i.e., both indicators tend to move together to show higher risk), with a median correlation of -0.629, and III is also positively correlated with Moody's, and Fitch credit ratings.

[7]This is a weighted average of country-specific averages using as weights shares in oil production for the group of countries in our sample, as described later in this Section.

and Venezuela).[8] External debt and country risk move together (i.e., debt and III are negatively correlated) in all but two countries for which debt data are available, with correlations ranging from -0.96 to -0.43, and the average correlation across all countries is roughly -0.6.[9]

Figure 2: External Public Debt (average 1971-2015)

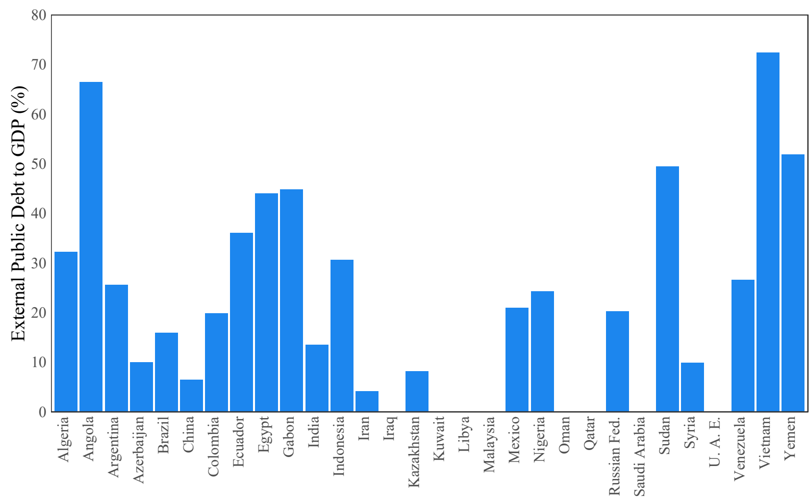
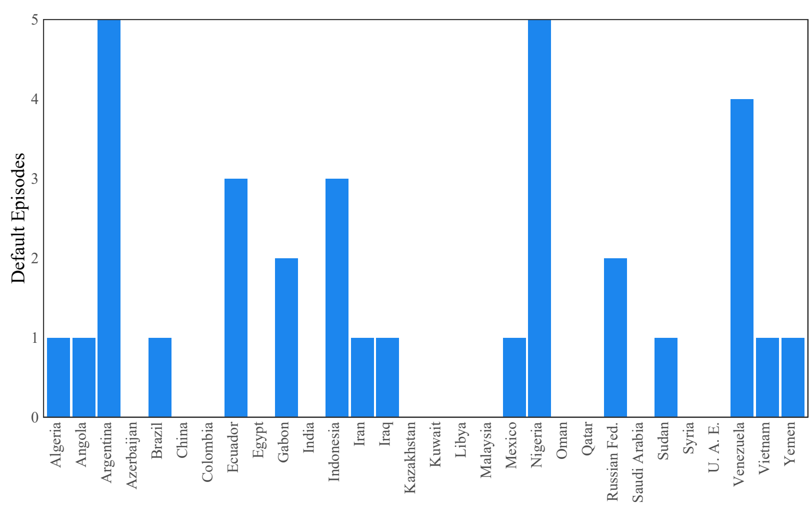


Figure 3: Number of Default Episodes per Country (1979-2014)



[8] A zero in Figure 3 denotes a country that has not defaulted, it does not indicate lack of data as in Figure 2.

[9] See Appendix J for a Table containing country-by-country statistical moments for all the dataset. The average of the country-specific correlations is again weighted using oil production shares as explained in Fact 2.

Table 1: Oil Prices and Business Cycle Moments

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Autocorr.
Oil price	0	0.182	0.111	1	0.131	0.847
Non-oil GDP	0	0.093	0.631	-0.043	-0.05	0.385
GDP	0	0.069	1	0.111	0.074	0.523
Oil production	0	0.123	0.624	0.041	0.149	0.502
Consumption	0	0.049	0.523	0.120	0.048	0.523
Gross oil output	0	0.235	0.492	0.342	0.110	0.276
Trade balance to GDP	0.073	0.090	0.106	0.190	0.133	0.630
Institutional Investor Index	47.494	11.486	0.208	0.693	0.084	0.869
Debt to GDP	0.224	0.144	-0.284	-0.612	-0.179	0.836
Reserves	76.969	20.088	0.074	0.131	1	0.833

* Note: Business cycle moments are weighted averages across the thirty countries included in the dataset. The weights were set by first computing the average of each country's share of oil production in the combined oil production of the thirty countries over the 1979-2014 period, and then normalizing the country averages so that they add to 1.

2. Real oil prices and country risk are negatively correlated over the business cycle.

Table 1 reports cross-country weighted averages of business cycle moments of the variables in our dataset, including standard deviations (or coefficients of variation), correlations with GDP, the real price of oil, and oil reserves, and first-order autocorrelations. The weighted averages are computed as follows: The panel has 30 countries indexed by i. The cross-country weighted average of a moment x is $x = \sum_{i=1}^{30} w_i x_i$, where x_i is the moment for country i and w_i is the country's weight. The weights are time-invariant and they were set by first computing the average of each country's share of oil production in the combined oil production of the 30 countries over the 1979-2014 period, and then normalizing the country averages so that they add to 1. To compute the business cycle moments, the data were logged and detrended using the Hodrick-Prescott filter with the smoothing parameter set at 100, except for the variables measured as ratios of GDP and the III, which were not detrended (the former because they were stationary and the latter because it is a bounded index between 0 and 100). For variables for which data are not available for some countries, we re-calculate the weights to exclude these countries in calculating total oil production for all countries.

Table 1 shows that the (weighted) average correlation of real oil prices with the III is 0.693. Thus, when oil prices fall over the business cycle, country risk rises (III falls). This fact also holds country by country, since the vast majority of the country-specific oil price-III correlations are positive and concentrated around the weighted average (see Appendix J).

The average correlations of oil prices with most of the rest of the variables are not very high, as Table 1 shows, but unlike the case of the oil price-III correlation, there is significant heterogeneity across countries. Appendix J shows that the correlations of real oil prices with non-oil GDP, oil production, the trade balance, and GDP vary widely. The correlation between oil prices and the trade balance is the most heterogeneous. It ranges from -0.25 for Indonesia to 0.78 for Kazakhstan. Out of the thirty countries in the sample, eight have correlations above 0.45 and nine have negative correlations. The correlation between oil prices and GDP is between -0.15 and 0.27. Seven countries have small but negative correlations, and eleven countries have correlations exceeding 0.1. The correlation between oil prices and non-oil GDP ranges between -0.24 for Algeria and 0.24 for Russia, although twenty four countries have negative correlations that are close to zero. The correlation between oil prices and oil production ranges from -0.22 for Colombia to 0.18 for Algeria. Sixteen countries have a negative correlation and fourteen a positive correlation. The mean correlation between real oil prices and gross oil output is relatively high, at 0.342, but this is mostly a valuation effect because oil production is nearly uncorrelated with the real price of oil (recall that gross oil output was defined as production multiplied by the real price of oil).

3. The conditional relationship between oil reserves and sovereign risk is non-monotonic over time. Contemporaneous changes in reserves do not have a statistically significant effect on sovereign risk, but as the series converge to their long-run trends, higher oil reserves are associated with *higher* sovereign risk.

Facts 1 and 2 focused on unconditional moments as weighted averages of country-specific moments. We now study conditional co-movements incorporating all the information in the cross-section of countries. To handle the non-stationarity of the original data and separate low-frequency co-movements, we use dynamic error-correction panel estimation.

We estimate dynamic panel regressions for three different specifications. In Model (1), we regress the III on oil production, real non-oil GDP in local currency units, oil reserves, external public debt as a share of GDP, oil discoveries, and a default dummy. In Model (2), we control for NFA and exclude the default dummy, and in Model (3) we include all control variables. In the three regressions, we control for country fixed effects (to take care of country-specific political situations, for example), as well as for time fixed effects (which include the effect of oil-prices, since the real price of oil is common to all countries). All coefficients can be interpreted as elasticities with the exception of the coefficient on oil discoveries. We ran the regressions for both balanced and unbalanced panels. Table 2 shows the results for the unbalanced panel, but the results are qualitatively the same in both panels. Table D3 in Appendix D shows the results for the balanced panel.[10]

In these regressions, the convergence coefficient measures the speed at which the III converges to its long-run trend. In each model, it has the expected sign and is statistically significant at the 1% level. Convergence in the III runs at an annual rate between 0.156% and 0.183%, which means that each year the III covers about 0.17% (depending on the model) of the distance to its trend. It should also be noted that convergence is slightly slower in Model (2), where the NFA-GDP ratio is included and default is excluded.

Table 2: Dynamic Fixed Effects Regression Results for Institutional Investor Index

	Δ Inst. Investor Index		
	Model (1)	Model (2)	Model (3)
Convergence coefficient			
Inst. Investor Index (-1)	-0.175***	-0.156***	-0.183***
	(0.019)	(0.020)	(0.020)
Short-run coefficients			
Δ Oil Production	0.052**	0.047**	0.055**
	(0.021)	(0.022)	(0.022)
Δ Non-Oil GDP	0.199***	0.231***	0.198***
	(0.058)	(0.059)	(0.057)
Δ Oil Reserves	0.006	0.014	0.010
	(0.020)	(0.020)	(0.020)
Δ Ext. pub. debt to GDP	-0.104***	-0.094*	-0.107**
	(0.038)	(0.052)	(0.051)
Δ Oil Discoveries	-0.003	-0.003	-0.003
	(0.003)	(0.004)	(0.003)
Δ NFA		-0.040	-0.046
		(0.035)	(0.034)
Long-run coefficients			
Oil Production	0.048	0.048	0.038
	(0.041)	(0.049)	(0.041)
Non-oil GDP	0.095	-0.027	0.101
	(0.106)	(0.120)	(0.100)
Oil Reserves	-0.162***	-0.141**	-0.141***
	(0.051)	(0.060)	(0.050)
Ext. pub. debt to GDP	-0.810***	-1.226***	-1.001***
	(0.140)	(0.219)	(0.178)
Default	-0.369***		-0.379***
	(0.072)		(0.068)
Oil Discoveries	0.045	0.048	0.039
	(0.028)	(0.033)	(0.027)
NFA		-0.003	-0.119
		(0.141)	(0.116)
Constant	0.245	0.767	0.219
	(0.542)	(0.546)	(0.537)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

[10]Due to data limitations, Azerbaijan, Kazakhstan, Kuwait, Iraq, Libya, Oman, Qatar, Saudi Arabia, Syria, United Arab Emirates, and Yemen are dropped from these regressions. Consequently, the estimation is performed taking into account 512, 509 and 509 observations in the regression models 1, 2, and 3 respectively.

Focusing now on the short-run coefficients, an increase of 1% in oil production decreases country risk around 5 basis points, and this effect is significant at the 5% confidence level in the three regressions. A one-percent increase in non-oil GDP reduces country risk by 20 to 23 basis points, and this result is significant at the 1% level. An increase in oil reserves decreases country risk, but the coefficient is not significant, in line with the mixed results of the country-specific correlation coefficients in Appendix J. As expected, higher external public debt increases country risk (by about 10 basis points per 100-points of debt), and this result is statistically significant. Finally, the effect of NFA on country risk (keeping external public debt constant) is not significant.

Looking at the long-run or co-integration coefficients, the long-run elasticity of the III with respect to oil production is positive, in the $0.04 - 0.05$ range, but is not statistically significant. With respect to non-oil GDP, long-run elasticities are again not statistically significant. Interestingly, all three regressions yield a statistically significant *negative* long-run elasticity of country risk with respect to oil reserves (at the 5% level for the second model and at the 1% level for the other two models). In the long-run, a rise in reserves of 1% worsens country risk by about 0.15%. Thus, the data indicate that oil-producing emerging economies are perceived as more risky in the future when they boost their reserves today. The contemporaneous effect is positive but not statistically different from zero, as indicated by the short-run coefficient for oil reserves, but the long-run elasticity is clearly negative. This suggests a non-monotonic co-movement between reserves and country risk that changes from neutral in the short-run to negative as the variables converge to their trends.

External public debt still has a negative effect on country risk in the long-run and is statistically significant at the 1% level in the three regressions. Similar to the short-run, in the long-run the *level* of net-foreign assets increases country risk, but is not statistically significant. Finally, as expected, being in default increases country risk. When a country is in default, the III drops about 37%. This last result is statistically significant at the 1% level. As for oil discoveries, an increase in oil discoveries decreases country risk but the effect is not statistically significant in all three models.

Summing up, Fact 1, showing that large oil producers display non-trivial debt ratios and default rates and positively-correlated debt and country risk, highlights important empirical regularities consistent with what the sovereign default literature has documented for emerging markets in general. Facts 2 and 3 provide key new facts that illustrate the relevance of the oil sector in the sovereign debt and risk dynamics of large oil producers. Fact 2 shows that, over the business cycle, country risk rises when oil prices fall. Fact 3 sheds light on the short- and long-run conditional dynamic relationship of country risk with oil production, reserves, and discoveries, and with key variables like non-oil GDP and NFA. The main finding is that, while higher oil production or non-oil GDP reduce country risk, the relationship between oil reserves and country-risk is non-monotonic over time. Higher reserves improve country risk in the short-run, albeit with an effect that is not statistically significant, but in the long-run they have a statistically significant effect that increases country risk.

3 A Model of Sovereign Default and Oil Extraction

The model we propose introduces oil extraction and reserves into a model in the class of the EG models of sovereign default. A benevolent social planner cannot commit to repay external debt and chooses optimally whether to default or not. The planner owns the oil industry, and thus chooses also extraction and reserves.[11] Hence, the planner has two vehicles for reallocating resources intertemporally (debt and oil reserves) and can affect the value of default by altering reserves. In addition, the planner's income, repayment capacity and default incentives are exposed to oil-price shocks.

3.1 Model structure

There are two types of goods, oil and a tradable, non-storable consumption good. The price of oil relative to the consumption good, p, is stochastic and determined in world markets. The economy has an exogenous stochastic endowment of the tradable good (non-oil GDP), y, which has an exogenous world-determined price set to 1 without loss of generality. Oil prices and non-oil GDP follow a joint first-order, stationary Markov process with known realization vectors and a transition probability matrix denoted by $\pi(p', y'|p, y)$, where primes are used to denote next-period's values.

Oil is extracted at a cost denominated in units of the consumption good. The cost of extracting x units of oil out of an existing stock of oil reserves s is determined by the cost function $e\left(x, s\right)$, so that oil GDP is $y^O \equiv px - e(x, s)$, with $e_s(\cdot) < 0$, $e_x(\cdot) > 0$ and $e_s\left(0, s\right) = 0$.

[11] This assumption is in line with the dominant role of state-owned enterprises in commodity extraction and/or exports in many emerging and developing economies.

Its functional form is as follows:

$$e(x, s) = \psi \left(\frac{x}{s}\right)^{\gamma} x. \tag{1}$$

Hence, the per-unit extraction cost ($\psi \left(\frac{x}{s}\right)^{\gamma}$) is homogeneous of degree zero in x and s.

Reserves follow the law of motion $s' = s - x + \kappa$, where κ denotes a constant amount of oil discoveries each period and s' denotes reserves carried over to the next period. Extraction cannot be negative ($x \geq 0$) and cannot exceed the sum of reserves plus discoveries ($x \leq s + \kappa$). Since oil is a form of capital with an endogenous return, it has an asset valuation that we label the "asset price of oil" defined as $q^O \equiv p - e_x(x, s) + \Delta\tilde{\psi}$, where $\Delta\tilde{\psi} \equiv [\psi^l - \psi^h]/u'(c)$ and ψ^l and ψ^h are the multipliers on the lower and upper bounds of x, respectively.[12]

The world credit market is the same as in standard EG models. The government maximizes private-sector utility, defined by a standard time-separable expected utility function with constant-relative-risk-aversion period utility $u(c) = \frac{c^{1-\mu}}{1-\mu}$ and subjective discount factor β. The sovereign sells one-period, non-state contingent discount bonds, denominated in units of the consumption good, to risk-neutral foreign investors. The outstanding bond position is denoted b and newly issued bonds are denoted b' (the sovereign is indebted when $b < 0$). The set of feasible bond positions is given by a discrete grid defined over the interval $B = [b_{min}, b_{max}]$where $b_{min} \leq b_{max} = 0$. If the sovereign defaults, it does not repay b in the current period and is excluded from the credit market, so no b' can be issued. Next period, the sovereign re-enters the credit market with probability λ.

We assume that the country continues to participate in the world oil market during the financial exclusion period. Hence, the sovereign can still export oil when it defaults. This is important because it implies that the sovereign's plans for the accumulation of oil reserves affect the value of default, since those reserves can be extracted and exported to generate income while access to credit markets remains closed.

The timing of decisions within a period is as follows: At the beginning of the period, s and b are known and the shocks p and y are realized. The sovereign then decides whether to repay or default by choosing the option that yields the highest value, as explained below.

[12] Appendix G shows that, taking as given a bond pricing function, q^O equals the expected present discounted value (discounted with the sovereign's stochastic discount factor) of the income stream composed of oil "dividends", $d^O \equiv -e_s(t) + \psi^h_{t+1}/u'(c_t)$, and the marginal revenue resulting from the effect of accumulating higher oil reserves on the price of debt.

If the sovereign defaults, it makes oil extraction and reserves decisions, since the country is excluded from the world bond market but not from the oil market, and pays extraction costs. If the sovereign repays, it sells new bonds b' to foreign investors at the price q, makes extraction and reserves decisions, and pays extraction costs. The resources generated from debt and profits from oil exports are then transferred to households and used for consumption.

The planner's payoff at the beginning of the period is:

$$V(b,s,y,p) = \max\left\{v^{nd}(b,s,y,p), v^{d}(s,y,p)\right\},$$

where $v^{nd}(b,s,y,p)$ is the value of repayment and $v^{d}(s,y,p)$ is the value of default.

The value of repayment is characterized by the following maximization problem:

$$v^{nd}(b,s,y,p) = \max_{\{c,x,b',s'\}} \left\{u(c) + \beta E\left[V\left(b',s',y',p'\right)\right]\right\} \tag{2}$$

s.t.

$$c = y - A + px - e\left(x,s\right) + b - q\left(b',s',y,p\right)b', \tag{3}$$

$$s' = s - x + \kappa, \tag{4}$$

$$0 \leq x \leq s + \kappa. \tag{5}$$

Constraint (3) is the resource constraint, (4) is the law of motion of reserves, and (5) states the feasibility constraints on extraction. In the resource constraint, $q\left(b',s',y,p\right)$ is the pricing function for the risky sovereign bond, which varies with the choices of bonds and reserves and the realizations of (p,y), and A represents autonomous (exogenous) spending allocated to investment expenditures so that the consumption-GDP ratio can be calibrated later to match the data (consumption will include private and public consumption). Note that we are assuming that extraction costs are factor payments abroad.[13]

The value of default is characterized by the following problem:

$$v^{d}(s,y,p) = \max_{\{c,x,s'\}} \left\{u(c) + \beta\left(1-\lambda\right)Ev^{d}\left(s',y',p'\right) + \beta\lambda EV\left(0,s',y',p'\right)\right\} \tag{6}$$

subject to the same law of motion of reserves and feasibility constraint as in the repayment

[13]This assumption can be relaxed by assuming that a fraction ϕ of extraction costs are domestic factor income. In which case $e(x,s)$ is replaced with $(1-\phi)e(x,s)$ in the resource constraint.

case and the following resource constraint:

$$c = y - A + h(p)x - e\left(x, s\right). \tag{7}$$

In the right-hand-side of the value of default (6), the sovereign re-enters credit markets with probability λ and a clean slate of debt ($b' = 0$), and it retains its oil reserves s'. It remains in default with probability $(1-\lambda)$ but again it retains its oil reserves s'. The resource constraint (7) includes a piece-wise default cost akin to the one proposed by Arellano (2008) for income but in terms of the price of oil: $h(p) = \hat{p}$ if $p > \hat{p}$ and $h(p) = p$ if $p \leq \hat{p}$. Intuitively, this is similar to a foreign ad-valorem tariff on the country's oil exports that rises with p above the threshold $\hat{p}$. This trade penalty is in line with the empirical observation that international trade is negatively affected by sovereign default. Alternatively, we can focus on the income default cost that $h(p)$ implies in terms of oil output or aggregate GDP. Both are affected not only by the exogenous adjustment in p but by the endogenous response of oil production (and hence of total GDP) induced by that adjustment. Hence, unlike in standard EG models, this model's default cost in terms of income includes an endogenous component. We examine this issue in more detail in Section 4.

For a given (b, s), default is optimal for the pairs $\{y, p\}$ for which $v^d(s, y, p) \geq v^{nd}(b, s, y, p)$. Hence, the default set is given by:

$$D\left(b, s\right) = \Big\{\{y, p\} : v^d(s, y, p) \geq v^{nd}(b, s, y, p)\Big\}. \tag{8}$$

The associated default decision rule is given by the function $d(b, s, y, p)$, which takes the value of 1 for $(y, p) \in D\left(b, s\right)$ and 0 otherwise (i.e. it equals 1 if the government defaults).

The probability of default one-period ahead conditional on current-period information, $P^d(b', s', y, p)$, can then be induced from the default decision rule and the Markov process of the shocks as follows:

$$P^d\left(b', s', y, p\right) = \sum_{y'}\sum_{p'} d(b', s', y', p')\pi(y', p'|y, p). \tag{9}$$

Since foreign investors are risk neutral, sovereign bond prices satisfy the standard no-arbitrage condition:

$$q\left(b', s', y, p\right) = q^*\left(1 - P^d\left(b', s', y, p\right)\right),$$

where q^* is the price of a risk-free bond such that $q^* \equiv 1/R^*$ where R^* is the world's risk-free gross real interest rate that represents the opportunity cost of funds for foreign investors.

3.2 Model properties

Appendix G proves six propositions that establish useful properties of the asset price of oil and oil profits, demonstrate that key properties of the standard EG model still hold, and characterize the effects of oil reserves and oil-price shocks. Relative to the standard EG model, obtaining analytic results is more difficult because of the endogeneity of the default payoff on the choice of oil reserves. As we explain below, this is particularly the case for deriving results related to how reserves affect default risk, what contracts are feasible under repayment when default is possible, and how default incentives respond to y and p.

The propositions rely on three conjectures: 1) Asset prices of oil are non-negative under repayment and default; 2) optimal consumption under repayment is nondecreasing in s; and 3) for (y, p) pairs in the default set (when this set is non-empty), the available contracts for new debt and choices of oil reserves under repayment yield a trade balance at least as large as the difference in oil profits across repayment and default.

Since the propositions rely on these conjectures, and some impose also parameter restrictions (i.i.d shocks, permanent exclusion after default, and no oil-price default cost or $\hat{p} = p$) and provide only sufficiency conditions, we evaluated numerically both the conjectures and the propositions using the calibration specified in the next Section. They all hold in 100 percent of the possible model evaluations that apply to each, except for Conjecture 2 which holds in 98 percent of the corresponding evaluations (see Appendix G for details). We also evaluated the non-negativity of profits included in Conjecture 1 and the trade balance conditions that are part of Propositions 5 and 6.[14] Profits are strictly positive for all optimal decision rules of s' under repayment and default. The trade balance conditions of Propositions 5 and 6 hold in 97 and 100 percent of all model evaluations, respectively. Removing these conditions, the main results of those propositions, namely that default incentives strengthen at lower y (Proposition 5) or lower p (Proposition 6) also hold in 100 percent of the model evaluations. Thus, in our calibrated model, lower p *always* strengthens default incentives and the sufficiency condition to prove it (i.e., the trade balance condition of Proposition 6) always

[14]We also checked whether the boundary conditions for x (or s') bind and found that they are never binding.

holds. Lower y *always* strengthens default incentives and the sufficiency condition to prove it (i.e., the trade balance condition of Proposition 5) holds in 97 percent of the evaluations.

Proposition 1. The repayment payoff is non-decreasing in b and default sets shrink as b rises (or grow as debt rises)

For all $b^1 \leq b^2$, $v^{nd}(b^2, s, y, p) \geq v^{nd}(b^1, s, y, p)$. Moreover, if default is optimal for b^2 (i.e., $d(b^2, s, y, p) = 1$) for some states (s, y, p) then default is optimal for b^1 for the same states (s, y, p) (i.e., $D(b^2, s) \subseteq D(b^1, s)$ and $d(b^1, s, y, p) = 1$) .

This is analogous to Proposition 1 in Arellano (2008). It implies that the country risk premium is non-decreasing in the amount of new debt issued ($q(\cdot)$ is non-decreasing in b').

Proposition 2. If asset prices of oil are positive, oil profits are increasing in s, for given s', and decreasing in s', for given s.

Given Conjecture 1, oil profits under repayment and default are increasing in $s \in [\underline{s}, \overline{s}]$, namely $M_s^{nd}(\cdot), M_s^d(\cdot) > 0$, and decreasing in $s' \in [s+\kappa-s(p/\psi)^{(1/\gamma)}, s+\kappa]$, namely $M_{s'}^{nd}(\cdot), M_{s'}^d(\cdot) < 0$.[15]

This proposition shows that, if the asset prices of oil are positive under repayment and default, the corresponding profits from oil extraction are higher if reserves carried over from the previous period are higher, for a given value of s', and lower if new reserves are higher (i.e. extraction falls) for a given value of s. We show in Appendix F that positive asset prices of oil are equilibrium outcomes in three variants of the model without default risk, one under financial autarky and two with access to world credit markets and an exogenous bond pricing function (one set equal to q^* and one with the same properties as that of the model with default). The result under financial autarky implies also that $q^{Od}(\cdot) > 0$ in the model with default and $\lambda = 0$.

Proposition 3. The default and repayment payoffs are non-decreasing in s.

For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$ and $v^d(s^2, y, p) \geq v^d(s^1, y, p)$.

This result follows from Proposition 2, and demonstrates that one of the conditions needed for the default sets to shrink in b in Proposition 1 (namely that the default and repayment payoffs are non-decreasing in b) also applies with respect to s. This is not sufficient, however, to yield the result that default sets shrink in s, as the next proposition shows.

Proposition 4. Default sets shrink as s rises (i.e. grow as reserves fall).

Assume $\hat{p} = p$ and $\lambda = 0$ for simplicity. For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, if default is optimal for

[15]The lower bound of s' follows from assuming oil profits are non-negative. The upper bound is at the point where extraction is set to zero. See Appendix G for details.

s^2 *(i.e.,* $d(b, s^2, y, p) = 1$*) for some states* (b, y, p)*, then default is optimal for* s^1 *for the same states* (b, y, p) *(i.e.,* $D(b, s^2) \subseteq D(b, s^1)$ *and* $d(b, s^1, y, p) = 1$*).*

This proposition establishes sufficiency conditions under which the result about country risk with respect to the bond position established in Proposition 1 extends to oil reserves. It relies on the three conjectures and Propositions 2 and 3 and establishes that the country risk premium is non-decreasing in the choice of s' (i.e., $q(\cdot)$ is non-decreasing in s'). This result does not follow just from analogy to Proposition 1 (and Proposition 3), because both the repayment and default payoffs vary with s, whereas in the case of b the default payoff does not vary with b. The key to this Proposition is Conjecture 3, which states that, when the default set is non-empty for a given (b, s), the available debt contracts and reserves choices associated with any (y, p) in the default set imply trade surpluses at least as large as the excess of oil profits under repayment over those under default. Intuitively, the net resources that all available debt contracts and reserves choices can generate for consumption under repayment are at most the same as those obtained with the optimal reserves chosen under default.

Proposition 5. If the trade balance is sufficiently large, default incentives strengthen as non-oil GDP falls.

Assuming i.i.d shocks, $\lambda = 0$ *and* $\hat{p} = p$*, for all* $y_1 < y_2$*, if* $y_2 \in D(b, s)$ *and* $tb(b^1, s^1, b) \geq M(s^1, s, p) - M(\tilde{s}^2, s, p)$ *(where* $b^1 \equiv b'(b, s, y_1, p)$*,* $s^1 \equiv s'(b, s, y_1, p)$ *are the optimal choices of bonds and reserves under repayment with* y_1 *and* $\tilde{s}^2 \equiv s^d(s, y_2, p)$ *is the optimal reserves choice under default with* y_2*), then* $y_1 \in D(b, s)$*.*

This proposition shows conditions under which Proposition 3 in Arellano (2008) holds in this model. It shows that the sovereign has stronger default incentives at lower levels of non-oil GDP when the optimal trade balance under repayment with low y is larger than the difference in optimal oil profits under repayment at that same low y relative to those under default at a higher y. As noted earlier, this trade balance condition holds in 97 percent of the state space and even when it fails, default incentives always strengthen as y falls (i.e., for all $y_1 < y_2$ and $y_2 \in D(b, s)$, $y_1 \in D(b, s)$).

Proposition 6. If the trade balance is sufficiently large and reserves chosen under default at high oil prices exceed those chosen under repayment at low prices, default incentives strengthen as oil prices fall.

Assuming i.i.d shocks, $\lambda = 0$ *and* $\hat{p} = p$*, for all* $p_1 < p_2$ *and* $p_2 \in D(b, s)$*, if* $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ *and* $s^1 \leq \tilde{s}^2$ *(where* b^1, s^1 *are the optimal bonds and reserves choices under* *repayment in state* (b, s, y, p_1) *and* $\tilde{s}^2$ *is the optimal reserves choice under default in state* (s, y, p_2), *then* $p_1 \in D(b, s)$.

This proposition shows sufficiency conditions under which the result in Proposition 5 with respect to non-oil GDP also applies to oil prices (namely that the sovereign has stronger default incentives when p is lower). This Proposition assumes not only a sufficiently large trade balance but also that the oil reserves chosen under default at a high p are larger than those chosen under repayment at a low p. This last property holds in all of the state space of the calibrated model. Moreover, in the calibrated model, the trade balance condition holds in all of the relevant evaluations, and even ignoring it, default incentives strengthen as oil prices fall in all of the state space (i.e., for all $p_1 < p_2$ and $p_2 \in D(b, s)$, $p_1 \in D(b, s)$).

Summing up, the above theoretical findings indicate that the model preserves the standard properties of EG models with respect to debt and that these extend to oil reserves. In particular, repayment payoffs are nondecreasing in b or s, the bond pricing function is increasing in (b, s) and default incentives are stronger at lower y or lower p. The theory also predicts that the default payoff is non-decreasing in s and that oil profits are increasing in existing reserves and decreasing in new reserves (i.e., increasing in extraction).

Next, we use these results and the findings from Appendix F to study how oil extraction and debt compare in their effects on resources disposable for consumption under default and repayment, and to examine how bond prices, debt, reserves and oil prices interact in the formulation of optimal extraction plans.

Consider first the effects of the choices for new debt b' and reserves s' on resources available for consumption. The constraints of the repayment optimization problem imply:

$$c = y - A + p(s + \kappa) - ps' - e\left(s', s\right) + b - q\left(b', s', y, p\right) b', \tag{10}$$

where we replaced x with s' as an argument of $e(.)$. Note that, since $e(.)_x > 0$ and x decreases with s', $e(.)_{s'} < 0$.

The above expression shows key similarities and differences faced by the sovereign in the choice of b' v. s' for reallocating consumption intertemporally (i.e., as assets for consumption smoothing). By borrowing more (reducing b'), resources for current consumption change according to the familiar debt Laffer curve of EG models.[16] Reducing s' is akin to borrowing

[16]When b' is low so that default risk is low or zero, additional debt always gains resources for consumption,

in that it increases resources for consumption by the amount by which $-(ps' + e(s', s))$ rises. In contrast with debt, however, there is no Laffer curve when "borrowing with reserves." Conditional on not hitting the feasibility boundaries of extraction, lower s' always increases resources available for consumption.[17] Borrowing with s' also differs from b' in that it alters resources in the default state, by increasing them by the amount $-(h(p)s' + e(s', s))$ as s' falls (i.e., at a lower rate than under repayment).[18]

Debt and reserves can also be compared in terms of how outstanding debt b and existing reserves s affect resources for current consumption. They are similar in that arriving at the repayment state with more debt (lower b) reduces resources by the amount b, while arriving with fewer reserves reduces resources by the amount ps. But they differ in that the debt repayment is non-state-contingent while the resources provided by s vary with p. It is often noted in the sovereign default literature that debt has poor hedging properties because it does not reduce the burden of repayment in "bad" states of nature (i.e., the repayment is uncorrelated with total income), but oil reserves are *worst* in this regard because the resources they provide correlate positively with oil prices (i.e., they provide fewer resources at lower p). Hence, viewing b and s as assets for hedging income fluctuations, reserves are inferior to debt. Moreover, the sovereign can default on b to reduce the debt burden ex-post.

Qualitatively, debt and oil reserves have similar effects on conditional default probabilities and default risk. With regard to debt, Proposition 1 established that, as in EG models, default sets shrink with b and as a result the conditional probability of default and default risk are non-decreasing in debt. Thus, $q(.)$ is non-decreasing in b'. On the side of oil reserves, Proposition 4 showed that default sets also shrink with s and thus the conditional probability of default and default risk are non-decreasing in reserves. Thus, $q(.)$ is non-decreasing in s'. The rationale is that, although in the case of oil reserves the default payoff is increasing in s instead of constant, the repayment payoff grows more than the default payoff as s rises. Notice that these are contemporaneous effects that refer to how country risk at date t responds to the sovereign choosing to increase debt or reduce reserves at t and that, as the propositions assumed, they consider only changes in b' or s' keeping everything else constant.

because bond prices fall little or stay at q^*, but as debt rises enough for default risk to reduce $q(\cdot)$ sufficiently, additional debt results in fewer resources for consumption.

[17] Note that $\partial c/\partial s' = -p - e_{s'}(s', s) = -(p - e_x(x, s)) < 0$ because $q^{Ond} > 0$ implies that $p - e_x(x, s) > 0$ for an interior solution of x (see Appendices F and G). Hence, borrowing with reserves always increases resources for consumption because the asset price of oil is positive.

[18] In the default state, $\partial c/\partial s' = -h(p) - e_{s'}(s', s) = -(h(p)p - e_x(x, s)) < 0$ because $q^{Od} > 0$ implies that $h(p) - e_x(x, s) > 0$ for an interior solution of x (see Appendices F and G).

Next we examine the interaction between sovereign risk and the sovereign's optimal oil extraction plans. For simplicity, so that we can conduct the analysis with familiar no-arbitrage conditions in sequential form, assume that we give to a sovereign who is committed to repay the model's equilibrium bond pricing function, $q\left(s_{t+1}, b_{t+1}, y_t, p_t\right)$, assuming that it is differentiable and satisfies other regularity properties.[19] The optimality conditions of the sovereign's problem yield the following no-arbitrage condition between the expected return on oil and the return on sovereign bonds (see Appendix F):

$$E_t\left[\tilde{R}^o_{t+1}\right] = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) - \frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \tilde{R}^o_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}. \quad (11)$$

In this expression, $R^b\left(s_{t+1}, b_{t+1}\right) \equiv \frac{1}{q(t+1)+q_b(t+1)b_{t+1}}$ is the sovereign's gross return on bonds. Since we are assuming commitment, there is no default risk, but because $q(\cdot)$ is the equilibrium pricing function of the model with default, the planner internalizes that higher debt carries a higher interest rate than R^* (since $q_b(\cdot) > 0$). Also, since debt is non-state-contingent, the Euler equation for bonds implies that at equilibrium $R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) = \frac{u'(c_t)}{\beta E[u'(c_{t+1})]}$. The term $\tilde{R}^o_{t+1} \equiv \frac{q^O_{t+1}+d^O_{t+1}}{\left[q^O_t+q_s(t+1)b_{t+1}\right]}$ is the sovereign's gross return on oil inclusive of the financial benefit of higher reserves increasing resources available for consumption by rising the price of newly-issued debt. This rate of return can be rewritten as $\tilde{R}^o_{t+1} \equiv R^o_{t+1}\left[\frac{1}{1+q_s(s_{t+1},b_{t+1})b_{t+1}/q^O_t}\right]$, where $R^o_{t+1} \equiv \frac{q^O_{t+1}+d^O_{t+1}}{q^O_t}$ is the "physical" return on oil and $\left[\frac{1}{1+q_s(s_{t+1},b_{t+1})b_{t+1}/q^O_t}\right]$ is the financial return from higher reserves increasing $q(\cdot)$.

Condition (11) implies that the sovereign's optimal extraction and reserves plans are set so that the total marginal gross return on the oil it extracts exceeds the full marginal cost of its liabilities by a premium equal to $-\frac{\text{cov}_t\left(u'(c_{t+1}),\tilde{R}^o_{t+1}\right)}{E_t[u'(c_{t+1})]}$. This is akin to a standard equity premium, with the caveat that both the return on oil and the return on bonds include financial components. The former (latter) because of the effect of lower oil reserves (higher debt) reducing the price of sovereign debt–or increasing the interest rate.

Appendix F examines the implications of condition (11) in two other scenarios: (i) per-

[19]We assume that $q(\cdot)$ is strictly concave and increasing in b_{t+1} for $b_{t+1} \in [-\overline{b}(s_{t+1}), 0]$, where $-\overline{b}(s_{t+1})$ is the threshold debt above which default is certain for a given s_{t+1} (i.e., $D(\overline{b}(s_{t+1}), s_{t+1})$ includes all (y_{t+1}, p_{t+1}) pairs, which exists because of Proposition 1), with $q(\cdot) = q^*$ for $b_{t+1} \geq 0$ and $q(\cdot) = 0$ for $b_{t+1} \leq \overline{b}(s_{t+1})$. $q(\cdot)$ is also increasing and concave in s_{t+1} for $s_{t+1} \in [\tilde{s}(b_{t+1}), s_t + \kappa]$, where $\tilde{s}(b_{t+1}) = max[s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}, \overline{s}(b_{t+1})]$ and $\overline{s}(b_{t+1})$ is the threshold oil reserves below which default is certain for a given b_{t+1} (i.e., $D(b_{t+1}, \overline{s}(b_{t+1}))$ includes all (y_{t+1}, p_{t+1}) pairs, which exists because of Proposition 4). We also assume that $\overline{b}(s_{t+1})$ is increasing in s_{t+1} and $\overline{s}(b_{t+1})$ is increasing in b_{t+1}.

manent financial autarky (which is the same as the solution of the default payoff if $\lambda = 0$) and (ii) a constant bond price set at $q = q^*$ (which renders the model akin to a small-open-economy RBC model).

Under financial autarky, the model resembles a canonical *closed-economy* RBC model, in which condition (11) reduces to $E_t[u'(c_{t+1}) R_t^O] = u'(c_t)$. Hence, the planner uses oil reserves in a manner akin to capital accumulation in the closed-economy RBC model. Markets are incomplete because there are no assets to insure away the risk of the shocks to p and y. Thus, the planner self-insures with reserves so as to facilitate consumption smoothing. There is also an implicit endogenous domestic real interest rate represented by the stochastic marginal rate of substitution in consumption. In the model with default, the planner has a similar incentive in the default state: being excluded from credit markets, it will use reserves to facilitate consumption smoothing, except that, because $\lambda > 0$ it assigns some probability to being able to re-enter the credit market.

In the case with $q = q^*$, condition (11) reduces to $E_t\left[R^o_{t+1}\right] = R^* - \frac{\text{cov}_t\left(u'(c_{t+1}), R^o_{t+1}\right)}{E_t[u'(c_{t+1})]}$ which is analogous to the one obtained in small-open-economy RBC models for the excess return on physical capital. Markets are again incomplete, but here the sovereign has access to no-state-contingent bonds for self-insurance and consumption smoothing. Oil is a risky asset and carries a risk premium, but the returns on oil and bonds and the risk premium do not include the financial terms due to the effects of debt and reserves on the price of bonds. Moreover, since the risk premium is small (as is typical in RBC models), the model is close to yielding Fisherian separation of extraction and reserves plans from savings and consumption plans. This separation holds strictly without uncertainty. As shown in Appendix F, the no-arbitrage condition without uncertainty is $R^o_{t+1} = R^*$ and yields a second-order difference equation in s that determines the extraction and reserves decision rules independently of the bonds and consumption decision rules.

In the model with default, since default is infrequent quantitatively, when debt and/or reserves (and the history of oil-price and non-oil GDP shocks) are such that the probability of default becomes positive only in the distant future, the dynamics of oil extraction and reserves will display similar features. The model will behave in a manner similar to a canonical small-open-economy RBC model. One important prediction of this model is that, when oil prices are low, and therefore expected to rise due to mean-reversion, the planner has the incentive to cut extraction and increase reserves. To see this, use the definitions of the asset price of oil and oil dividends to rewrite the no-arbitrage condition $R^o_{t+1} = R^*$ as follows (assuming an internal solution for x_t for simplicity):

$$\frac{p_{t+1} - e_x\left(x_{t+1}, s_{t+1}\right) - e_s\left(x_{t+1}, s_{t+1}\right)}{p_t - e_x\left(x_t, s_t\right)} = R^*. \tag{12}$$

Since $e(\cdot)$ is increasing in x_t and decreasing in s_t, when p_t falls relative to p_{t+1}, the planner reallocates extraction from t to $t+1$ by increasing s_{t+1}. This is a key incentive that is also a work in the model with default, but there it interacts with the planner's incentives to default and to affect the price of issuing new debt by adjusting reserves. As Propositions 4 and 6 show, the incentives to default at date t are stronger when p_t is low but, if the sovereign chooses not to default, the incentive to increase s_{t+1} in response to lower p_t *reduces* the default risk premium paid on bonds sold at t (i.e., increases the price of newly issued bonds) because default sets shrink with s.

4 Quantitative analysis

4.1 Calibration

We calibrate the model using the panel of oil-producing emerging economies described in Section 2. We describe first how we specified the Markov process of the shocks and then how we set the model's parameter values.

4.1.1 Exogenous shocks

To construct the stochastic processes of y and p, we first estimate the following standard VAR for each of the 30 countries in the panel:

$$\begin{bmatrix} p_t \\ y_t \end{bmatrix} = \begin{bmatrix} \rho_p & \rho_{yp} \\ \rho_{py} & \rho_y \end{bmatrix} \begin{bmatrix} p_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_p & \sigma_{yp} \\ \sigma_{py} & \sigma_y \end{bmatrix} \begin{bmatrix} \epsilon_{pt} \\ \epsilon_{yt} \end{bmatrix},$$

where ϵ_{pt} and ϵ_{yt} are mean-zero, i.i.d. innovations. The data for p and y are as in Section 2. Thus, p is the real price of oil, common to all countries, and y is each country's non-oil GDP. Since p turned out to be stationary but y did not, we take the logarithm of p and demean it, and for y, we log the data and extract the cyclical component using the Hodrick-Prescott filter. Hence, p and y are in percent deviations from mean and trend, respectively.

Next, we compute the weighted average of the VAR coefficients that are statistically significant across the thirty countries. As explained in Section 2, the weights are the normalized 1979-2014 average shares of each country's oil production in the total oil output of the countries in the panel. The VAR estimation produced statistically-significant coefficients mainly for the autocorrelation coefficients ρ_{pp} and ρ_{yy}, the former in all 30 countries and the latter in 21 countries. Their weighted averages are $\rho_{pp} = 0.901$ and $\rho_{yy} = 0.371$, respectively. In contrast, ρ_{py} is only significant in four countries and ρ_{yp} in two, and their weighted averages are 0.054 and 0.04, respectively. We do the same aggregation for the covariance matrix elements. That is, we compute the weighted average for all 30 countries.

Table 3 summarizes the parameterization of the shocks. Oil-price shocks are more persistent than non-oil-GDP shocks. Because of this, the standard deviation of p is nearly twice as large as that of y (18.2 v. 9.2 percent), even tough non-oil-GDP innovations have higher variance (0.007 v. 0.006). The interactions between the two shocks are weak, however, because ρ_{py}, ρ_{yp} and $\sigma_{p,y}$ are low.

Table 3: VAR Process for Non-Oil Output and Oil Prices

Parameter	Description	Value
ρ_p	oil price auto-correlation	0.90
ρ_y	non-oil output auto-correlation	0.37
ρ_{py}	oil price non-oil output correlation	0.05
ρ_{yp}	non-oil output oil price correlation	0.04
σ_p^2	variance oil price innovations	0.006
σ_y^2	variance non-oil output innovations	0.007
σ_{py}, σ_{yp}	covariance non-oil output, oil price	-0.002

The model is solved using a standard value function iteration algorithm for sovereign default models over a discrete state space. For the VAR process of p and y, we construct a discrete approximation with the coefficients shown in Table 3 using the approach proposed by Tauchen (1986) with a spanning factor of 2.15. This was chosen so as to match the standard deviations of p and y. The realization vectors have seven and five values for p and y, respectively. For b and s, we use discrete grids with 61 and 54 nodes, covering the interval $[-0.6, 0]$ for b and $[12, 15.97]$ for s.

4.1.2 Structural parameters

The model has nine structural parameters. The preference parameters β (discount factor) and μ (coefficient of relative risk aversion); the technology parameters κ (oil discoveries), γ

and ψ (elasticity and scale parameters of extraction costs); the financial parameters r^* (risk-free rate), $\hat{p}$ (oil-price default penalty), and λ (credit-market re-entry probability); and the autonomous spending coefficient A. Some of these parameters are set to standard values in the literature and others are targeted to match moments from the data. These targets are based on the cross-country weighted-averages constructed as explained in Section 2.

We set $\mu = 2$, a standard value in the literature. The risk-free rate is set to $r^* = 0.00775$, which corresponds to the average ex-post, US-CPI deflated yield on 3-month U.S. Treasury bills for the 1955-2014 period (see Bianchi et al. (2016)). Total GDP is normalized so that $y + y^o = 1$. Hence, the resource constraints can be interpreted as adding GDP shares.

In order to separate the identification of the oil technology parameters from the analysis of sovereign default, we calibrate the former to data from the countries in our panel dataset that did not default in the sample period (non-defaulters), assuming that defaulters and non-defaulters operate the same technology. This allows us to calibrate the technology parameters using the variant of the model without default risk examined in Section 3, which is like a small-open-economy RBC model but with oil reserves taking the role of the capital stock. Moreover, since the equity premium is negligible in this class of models, and hence Fisherian separation of savings and investment nearly holds, we approximate the solution by solving the model under financial autarky with a discount factor set to represent the inverse of the relevant opportunity cost of capital R^*.[20]

The autarky model is calibrated using its deterministic steady-state conditions. First, from the law of motion of reserves, if follows that in steady state $x = \kappa$. Next, as we show in Appendix G, the Euler equation for reserves (eq. (G.19)) yields this steady-state condition:

$$\psi \left(\frac{\kappa}{s}\right)^{\gamma} \left[\gamma \left(\frac{\kappa}{s}\right) + r^*(1+\gamma)\right] = r^*, \tag{13}$$

using the assumption that $1/\beta = R^*$ and normalizing the steady-state oil price to $p^{ss} = 1$. Note that $\left(\frac{s}{\kappa}\right)$ defines the years of oil reserves remaining before they are exhausted and that in steady state the share of Gross Oil Output (px) in GDP is $\frac{\kappa}{y+\kappa-\psi\left(\frac{\kappa}{s}\right)^{\gamma}\kappa}$.

The calibration strategy is to: (a) set γ to match the observed standard deviation of oil

[20]The autarky model can be solved with the same algorithm as the risk-free model by simply collapsing the grid of bonds to one element set to zero and redefining A so that $A = 1 - c$, where by construction A would include any steady-state debt service $-(\frac{r^*}{R^*})b$ present in the data. Using the data for non-defaulters, the weighted averages of the consumption- and debt-GDP ratios are 0.56 and -0.13, respectively.

extraction (13.1%) in the stochastic solution of the autarky model; (b) given γ, impose on the above Euler equation the expected years of reserves estimated from the data (70.06) to solve for ψ, and (c) impose γ, ψ and the data target of the share of Gross Oil Output in GDP (33.5%) on the definition of this share to solve for κ. These three data targets are weighted averages across the non-defaulters in our sample.

We apply an iterative procedure that starts with a guess for γ and solves for the associated values of ψ and κ as indicated in (b) and (c). Then, we solve the stochastic autarky model to compute the standard deviation of oil-extraction, the mean of $(\frac{s}{\kappa})$, and the mean of the ratio of oil rents to GDP, and iterate until these three model moments get as close as possible (up to a convergence criterion) to their data counterparts. With $\kappa = 0.3325$, $\gamma = 1.56$ and $\psi = 124.6544$, the standard deviation of x is 13.1%, the years of reserves $\frac{s}{\kappa} = 70.17$, and the Gross Oil Output-GDP ratio is 33.7%, all three very close to the data moments.

Next we calibrate the remaining parameters of the baseline sovereign default model. The annual probability of reentry is $\lambda = 0.332$, based on Richmond & Dias (2007) who found a median period of financial exclusion of three years after default in a sample of 128 sovereign defaults during the 1980-2005 period. The mean interest rate is set to $r = r^* + spread$, where the $spread$ is the weighted average of the country spreads for the period 1979-2016.[21]

The values of b and c are set to -22.45% and 58.88%, respectively, which correspond to the 1989-2016 weighted means of the GDP ratios of external debt and private plus public consumption, respectively, for all countries in the panel. To set the value of A, we use the assumption that GDP is normalized to 1 in the resource constraint evaluated at the deterministic steady-state, so that $A = 1 + (r/R)b - c$, which yields $A = 0.3936$. Similarly, we set the mean of non-oil GDP to $E[y] = 1 - y^o$ where $y^o = 0.2069$ is the weighted average of the ratio of oil rents to GDP in the data including all countries (so that $E[y] = 0.793$).

Finally, the values of β and $\hat{p}$ are jointly determined so that the stochastic baseline model matches the weighted averages of the debt-GDP ratio (22.45%) and the default rate (1.14%) in the full dataset.[22] The two parameter values are set following an iterative procedure simi-

[21] For the period 1998-2016 we use JP Morgan's EMBI+GSS spreads data. Since these data start in 1998, for the 1979-1997 period we extrapolate the spreads by first regressing the EMBI data on the III in the common sample for the 1998-2016 period, and then use this regression and observed pre-1998 III values to estimate EMBI spreads for 1979-1997 (707 bpts).

[22] All non-defaulters enter in the weighted sum that yields this estimate of the aggregate default rate with a zero default rate and their corresponding weight in total oil production. Including only defaulters, the default rate would be 2.2%.

lar to the one used for the technology parameters: Start with a guess for $(\beta, \hat{p})$, then solve the model and simulate it to compute the model's mean debt-GDP ratio and default rate, and iterate until the model and data moments differ by a convergence criterion. This procedure yields $\beta = 0.82$ and $\hat{p} = 0.64$, and with the complete parameterization the mean debt ratio is 22% (very close to the data) and the default rate is 1.19% (just slightly above the 1.14% in the data). Note, however, that since oil GDP is endogenous and κ was calibrated separately, the model yields $E[y^o] = 0.221$, slightly above 0.206 in the data, and $E[GDP] = 1.0145$, just a little above its normalized steady state.

Table 4 compares the moments used as data targets for the calibration with their counterparts produced by the model and Table 5 lists all the parameter values. There are two targeted data moments (average external debt to GDP and the default rate) which the model should match closely. The rest of the moments shown in the table are not targeted, and as such, can be contrasted with those of the data to gauge the model's ability to replicate them. The data moments are HP cyclical components as described in Section 2. The model-generated data are not detrended, since the model is stationary by construction, and thus we report coefficients of variation instead of standard deviations so that the measures of dispersion from the model and the data are both in percent of their "long-run" values. The variability of oil extraction in the model is 12%, close to its data counterpart (12.2%). The model falls short of replicating oil reserves (43 years in the model v. 62 in the data). The low discount factor (which is not uncommon in sovereign default models) incentivizes the planner to reallocate resources towards the present, but the lack of commitment limits its capacity to to do so via borrowing in bonds and thus reduces reserves in the long run.

Table 4: Data vs Model Moments

Description	Data	Model		
		Benchmark	Constant Extraction	Risk Free
Average External Debt to GDP	0.225	0.229	0.276	0.517
Default Rate	1.14%	1.19%	1.08%	0%
Standard Deviation of Oil Extraction	0.122	0.120	0.000	0.123
Average Reserves (in years)	62	43	42	42

Estimates of the proven reserves for the average oil exporting country correspond to those of the US Energy Information Administration.

Table 5: Parameter values

Parameter	Description	Value
β	discount factor	0.82
μ	risk aversion coefficient	2.00
q^*	risk-free debt price	0.99
$\hat{p}$	oil-price default cost threshold	0.64
k	discovery rate	0.33
λ	re-entry probability	0.33
γ	extraction costs curvature	1.56
ψ	extraction costs scale	124.6
A	autonomous spending	0.40

4.2 Default-repayment sets & default costs

We start the quantitative analysis by examining two aspects of the model's solution that highlight some of the analytic findings from Section 3 and the planner's strategic use of oil reserves. First, we examine the default and repayment sets and then we study default costs and the role of oil extraction in determining them.

4.2.1 Default and repayment sets

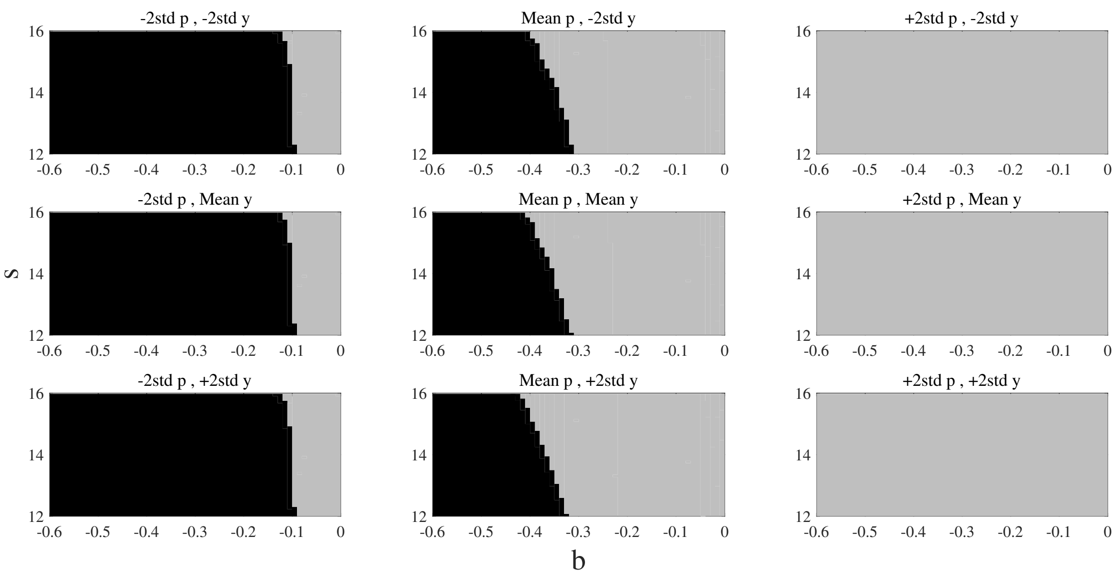
Figure 4 shows nine plots displaying the default and repayment sets across all values of (b, s) in the state space for nine (p, y) pairs that correspond to the means and +/-2 standard deviations of their means. In each plot, s (b) is shown in the vertical (horizontal) axis and the area in black (grey) is the default (repayment) set.

These plots are consistent with the results reported earlier indicating that Propositions 5 and 6 hold in the calibrated model even though the assumptions behind them do not apply, since shocks are not i.i.d, the probability of re-entry is positive and there are default costs in terms of oil prices. Proposition 6 states that default incentives strengthen as p falls, in the sense that if, for a given (b, s, y), a higher p is in the default set, a lower p must also be in the default set. This is visually evident in the plots. For any of the three levels of y shown in each row, as we move from the highest to the lowest p, namely from right to left, the repayment sets (in grey) shrink. At the highest p (and also at the second-highest, not shown), the default sets are empty for all y values and all (b, s) pairs, so the sovereign never defaults. At the lowest p, however, the repayment set spans about 1/4 of the state space of (b, s) for all y values. Thus, with p two-standard-deviations above its mean the sovereign always repays, but two-standard-deviations below it still repays if (b, s) are not "too low."

Proposition 5 is like 6 but for y: Default incentives strengthen as y falls, in the sense that if for a given (b, s, p), a higher y is in the default set, a lower y must also be in the default set. This result is harder to see in the plots. Careful examination shows that it does hold, but the default sets are nearly invariant in y. For each value of p in the columns, the default and repayment sets remain about the same as we increase y, namely from bottom to top.

The above results imply that bond prices and spreads vary with p but are nearly invariant in y (for a given (b, s)). This is an implication of the structure of default costs. Default sets respond more to a fall in p because the costs of default are linked to oil prices and they cut the oil revenues in a manner equivalent to reducing the price to a fixed value $\hat{p}$ regardless of how high the realized p is. Allowing default costs to affect y too would alter this result.

Figure 4: Default Sets



4.2.2 Default costs

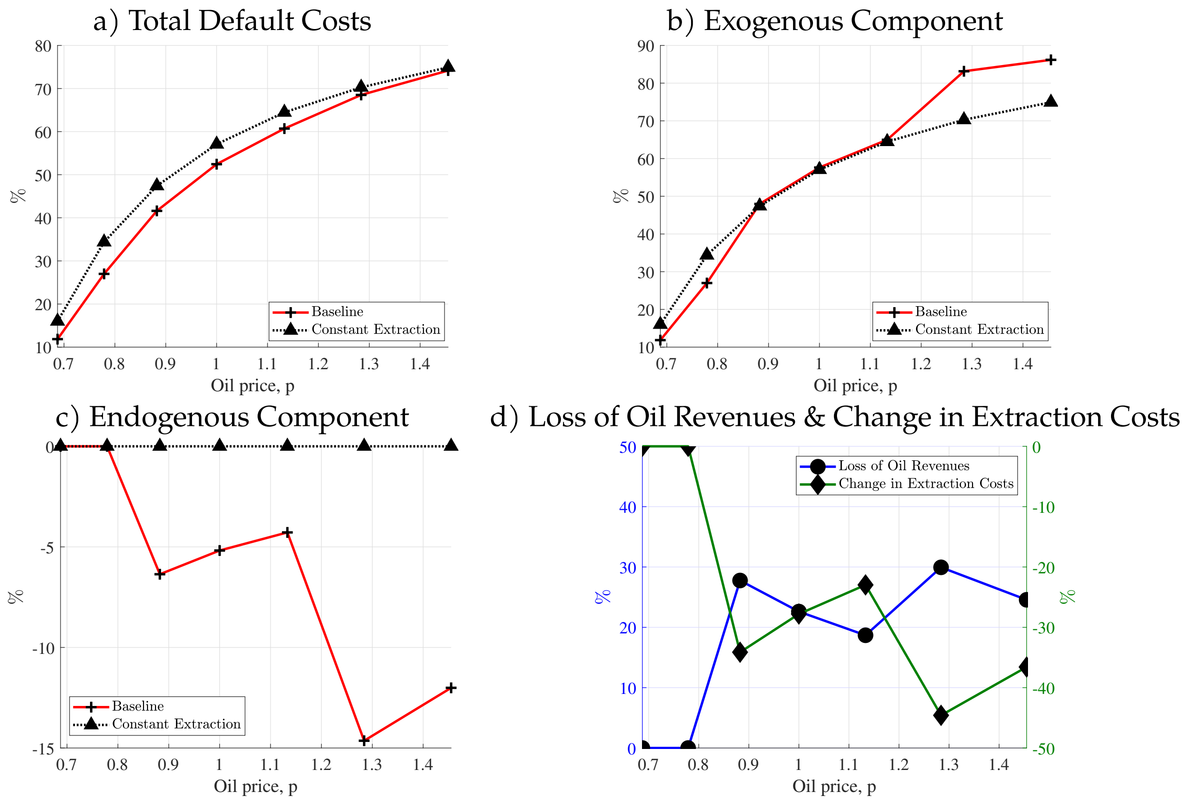
Next, we examine the magnitudes and variations of default costs across the state space. This is important because default costs in the model are determined by both the standard exogenous cost used in the literature (i.e., the piece-wise linear cost introduced by Arellano (2008), albeit applied to p in this case) and by the endogenous response of oil extraction to lower oil prices as the default cost reduces the country's effective oil price to $\hat{p}$. As noted earlier, this reduction is akin to a progressive tariff on oil exports ranging between 8% and 56%.

We measure default costs in percent of oil GDP under repayment. In the model, oil GDP under repayment and default are given by $y^{O,nd}(b,s,y,p) = px^{nd}(b,s,y,p) - e(x^{nd}(b,s,y,p),s)$ and $y^{O,d}(s,y,\hat{p}) = \hat{p}x^{d}(s,y,\hat{p}) - e(x^{d}(s,y,\hat{p}),s)$, respectively. As noted in Section 3, the lower export revenues under default, $\hat{p}x^{d}(s,y,\hat{p})$, can be viewed as a trade penalty imposed when the sovereign defaults. The total percent cost of default is $-[y^{O,d}(\cdot)/y^{O,nd}(\cdot) - 1]$. The endogeneity of this cost follows from the fact that extraction and extraction costs change because the decision rule $x^{d}(s,y,\hat{p})$ is affected by $\hat{p}$.

Plot a) of Figure 5 shows the total default costs as p varies in the baseline model (red-solid line) and in a variant of the model with constant extraction (dotted-black line). Plots b) and c) decompose the total costs in terms of exogenous and endogenous components, respectively. The exogenous component captures the loss of oil revenues due to the price penalty keeping extraction at the level under repayment, $-(\hat{p} - p)x^{nd}(\cdot)/y^{O,nd}(\cdot)$. The endogenous component captures the response in oil profits to that penalty as extraction and extraction costs change, $-\left[\hat{p}\left(x^{d}(\cdot) - x^{nd}(\cdot)\right) - \left(e(x^{d}(\cdot),s) - e(x^{nd}(\cdot),s)\right)\right]/y^{O,nd}(\cdot))$. Plot d) breaks down the two terms of the endogenous component: the loss in oil revenues if extraction falls under default and the change in extraction costs due to that fall in extraction. The sum of the two curves in Plot d) yields the curve shown in Plot c), and the sum of the curves in Plots b) and c) yields the curve shown in Plot a). In all cases, we evaluate extraction and extraction costs with (b,s,y) set at their long-run averages in the baseline model (and at the same p values) so that the state of nature is the same.

Plot a) shows two important results. First, default costs are increasing in oil prices, ranging between 11% and 73%. Hence, default is costlier in better states of nature, as Arellano (2008) showed was needed for EG models to sustain more debt and induce default in bad times. The costs are large because they are in percent of oil GDP. In percent of *total* GDP, they range between 1.6% and 24%, in line with those obtained by Areallano, ranging between 0 and 30% of GDP. Second, default costs are uniformly *lower* in the baseline model than with constant extraction , by up to 8 percentage points of GDP. Hence, the sovereign takes advantage of the possibility of strategizing over oil extraction and reserves so as to make default less costly. The gap narrows as p rises, and at the highest oil prices the difference is negligible.

Figure 5: Default Costs and Components
(in percent of Oil GDP under repayment)



Plot b) shows that the exogenous components display a similar pattern as the total costs. One key finding derived from this plot and Plots a) and c) is that, at the lowest p values, the lower default cost in the baseline model than under constant extraction is only due to the exogenous component. The smaller loss in oil revenues as p falls to $\hat{p}$ is because the sovereign in the baseline chooses lower extraction in the *repayment* state. At very low prices, p is expected to recover and hence, under repayment, the planner cuts extraction and builds reserves for future sale at better prices. This is important because, as shown later, all the defaults that occur in the model along its equilibrium path occur when p is very low.

At mid-levels of p, the exogenous components are the same in the baseline and with constant extraction. At high p, however, we observe the opposite of what happens at low p. Now the planner in the baseline model expects future prices to be lower and chooses higher extraction under repayment to take advantage of the higher current price. In contrast, however, the endogenous component is not zero as it was a low p, and as we show below, oil profits rise to nearly offset the larger exogenous cost.

Plot c) shows that, except at the lowest two values of p, the endogenous component is negative (i.e., it *reduces* default costs), and it tends to be more negative at higher p, albeit it does not change monotonically. Oil profits under default are higher than under repayment by up to 15% of oil GDP under repayment. Plot d) shows that profits are higher under default because, although extraction is lower, the loss of oil revenues due to lower extraction is more than offset by lower extraction costs (which fall with extraction). At the second-highest p, for instance, the revenue loss due to lower extraction is about 30% of oil GDP but extraction costs fall by 45%, resulting in the gain in profits of 15% shown in Plot c). These higher profits under default are about as large as the loss in oil revenues due to higher extraction under repayment (see Plot b)), so that the total default cost is about the same as with constant extraction (see Plot a)).

Jointly, the four plots illustrate the mechanism by which the sovereign uses endogenous extraction to make default less painful. This is particularly the case when oil prices are not too high. As noted above, for the lowest p values, the mechanism is driven solely by the lower exogenous cost due to lower extraction under repayment. Closer to the mean of p, however, the mechanism is driven by higher oil profits under default, which in turn are attained by reducing extraction so that, upon a default, extraction costs fall by more than the fall in revenue due to a smaller volume of oil exports. The resulting higher reserves are also important, because they prop up the value of default by contributing to sustain consumption while the economy remains in financial autarky. While defaults occur only at the lowest p along the equilibrium path, these responses at higher p levels affect the sovereign's strategic incentives and the expectations of lenders, and thus affect spreads and borrowing capacity.

4.3 Comparing model and data

We compare the model-generated data with the actual data in two key dimensions: business cycle moments and the oil reserves-sovereign risk co-movements.

4.3.1 Business Cycle Moments

Columns (1)-(2), (5)-(6), (9)-(10), and (13)-(14) of Table 6 compare variability ratios relative to oil prices, income correlations, oil-price correlations, and auto-correlations from the data with those produced by the baseline model. The Table includes additional columns showing moments produced by variants of the model with default risk but constant oil extraction (to examine the relevance of endogenizing oil production in the model with default) and with a sovereign committed to repay at the risk-free rate (to assess the implications of endogenizing default risk). These results are discussed later in this section. Below each of the data moments in cols. (1), (5), (9) and (13), which correspond to weighted averages of country-specific moments, we show in parenthesis the coefficient of variation of the corresponding moments across the thirty countries in the panel. This helps us to identify moments that have little dispersion across countries from those that differ a lot.

We show results for both disposable income (DI) and GDP because the autonomous spending A mechanically reduces the models' income variability measured as the coefficient of variation of GDP.[23] For the data, we constructed the comparable DI measure and detrended it with the HP filter, as we did with the other macro time-series.

Columns (1) and (2) show that the model does well at approximating the variability ratios of gross oil output and oil extraction, and reasonably well at approximating those of total GDP, disposable income, and the trade balance-GDP ratio, but it overestimates the variability ratios of consumption and spreads and underestimates the one for reserves.[24] For consumption, in particular, the variability ratio is 0.92 in the model v. 0.27 in the data. We discuss further this shortcoming of the model later in this Section.

In terms of income correlations, the model approximates some of them well, except it overestimates those for consumption and GDP. Importantly, since one key goal of the model is to explain the co-movements between the oil sector and country risk, the income correlations of x, px, and net exports are close to those in the data. In addition, spreads are negatively correlated with income, as in the data. The model's correlation is more negative than in the data (-0.28 v. -0.09), but in the data the correlation differs significantly across countries, with a coefficient of variation of 163%. GDP and DI are perfectly correlated by construction in the model, because $GDP = A+DI$ and A is a constant. The weakest result in these income correlations is that consumption and income are nearly perfectly correlated in the model and the model's correlation is nearly three times higher than in the data.

[23] Since $GDP = DI+A$ and A is a constant, GDP and DI have the same standard deviation, but the coefficient of variation of GDP is smaller because its mean is larger.

[24] For the spread we use the EMBI spread calculated as described in the previous subsection.

Table 6: Business Cycle moments - Data v. Model

	Variability relative to Oil Price				Correlation with DI				Correlation with Oil Price				Autocorrelation			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	Data	BSL	CE	RF	Data	BSL	CE	RF	Data	BSL	CE	RF	Data	BSL	CE	RF
Gross Oil Output	1.29	1.51	1.00	1.58	0.51	0.69	0.72	0.66	0.34	0.96	1.00	0.96	0.27	0.85	0.89	0.84
	(0.30)				(0.45)				(0.22)				(0.51)			
Total GDP	0.38	0.53	0.53	0.56	0.62	1.00	1.00	1.00	0.11	0.72	0.72	0.69	0.52	0.67	0.67	0.63
	(0.62)				(0.52)				(0.90)				(0.39)			
Disposable Income (DI)	0.56	0.86	0.86	0.92	1.00	1.00	1.00	1.00	0.12	0.72	0.72	0.69	0.42	0.67	0.67	0.63
	(0.59)				(0.00)				(0.84)				(0.41)			
Extraction	0.67	0.61	*na*	0.68	0.52	0.53	*na*	0.50	0.04	0.75	*na*	0.77	0.50	0.71	*na*	0.69
	(0.86)				(0.55)				(2.63)				(0.42)			
Consumption	0.27	0.92	0.93	0.90	0.34	0.97	0.91	0.99	0.12	0.73	0.74	0.71	0.52	0.61	0.61	0.67
	(0.65)				(0.94)				(0.80)				(0.27)			
Trade Balance/GDP	0.49	0.23	0.24	0.05	0.39	0.03	0.04	0.20	0.19	−0.15	−0.16	−0.03	0.63	0.10	0.14	−0.26
	(0.69)				(0.64)				(1.49)				(0.30)			
Reserves	1.78	0.25	*na*	0.25	0.04	−0.16	*na*	−0.19	0.13	−0.34	*na*	−0.38	0.83	0.99	*na*	0.99
	(0.68)				(3.10)				(3.09)				(0.11)			
Spread	3.00	13.63	14.34	*na*	−0.09	−0.28	−0.25	*na*	−0.46	−0.14	−0.10	*na*	0.66	0.34	0.31	*na*
	(0.54)				(1.63)				(0.75)				(0.27)			

* Actual data are for the 1979-2014 period, logged and HP-detrended, except for the TB/GDP, Debt/GDP ratios and the EMBI, which is in levels (basis points). BSL- Baseline, CE- Constant Extraction, RF- Risk Free model. Data for the three models are not detrended because the models are stationary by construction. Variability ratios for the three models are ratios of coefficients of variation divided by the standard deviation of oil prices.

Consider next oil-price correlations. The model does well at approximating Fact 2 from Section 2: Country risk (i.e., spreads) is negatively correlated with oil prices, although the model correlation is somewhat higher.[25] The model is also in line with the data in predicting that the trade balance is nearly uncorrelated with oil prices. The correlation is slightly positive (negative) in the data (model), but the correlation in the data differs sharply across countries (with a coefficient of variation of 149%). In fact, the country-specific correlations span the interval from -0.75 to 0.78, which includes the model's correlation of -0.15. The rest of the oil-price correlations in the Table have the same sign as in the data (albeit the model's correlations are markedly higher), with the exception of reserves. However, the cross-country coefficient of variation of the correlation between reserves and oil prices is very high, at 309%, with values that span the interval between -0.60 and 0.74. Hence, the model's correlation of -0.34 is within the range of correlations observed in the data.

From the model's perspective, the weakly negative correlation of the trade balance with oil prices captures also the curse-and-blessing nature of natural resources and can be explained as follows. As shown in Section 3, keeping everything else constant (particularly b, s and y), sovereign risk falls as p rises because the probability of default goes down. As a result, the sovereign has more borrowing capacity and can finance a larger trade deficit. This effect pushes for a negative correlation between oil prices and net exports. This mechanism is active in states of nature at date t in which default is possible at some realizations of p, y at $t + 1$. In contrast, when the probability of default is zero and debt is low enough and reserves high enough so that the economy is far from the default set, the model behaves as an RBC model, as explained earlier. In this case, higher p incentivizes NFA accumulation (a reduction in debt) so as to save some of the higher income of date t for future consumption. This effect pushes for a positive correlation between oil prices and net exports, although the effect is relatively weak because of the low value of β. Quantitatively, the two effects nearly offset each other to produce a trade balance-oil price correlation of only -0.15. A similar argument explains why the oil-price correlation with the spread is negative but not -1, in the sense that the effect of lower oil prices increasing country risk at date t operates also when default is possible in some states at $t + 1$.

In terms of the autocorrelations, the baseline model matches the data in that all the au-

[25]Regarding Fact 1, the model matches the observed mean debt ratio and default frequency by construction, because they were used as calibration targets.

tocorrelation coefficients are positive. The magnitudes of those for total GDP, consumption, and reserves are also similar in the model and the data, but those for the rest of the variables show sizable differences.

The main shortcoming of the model in terms of explaining the cyclical moments of the data is the high consumption volatility. Comparing the baseline (BSL) model results with the variants with constant extraction (CE) and risk-free debt (RF) sheds some light on what causes it. Notice first that, although consumption in the three models is significantly more volatile than in the data, the variability of consumption relative to income is higher in the two models where default is possible compared with the RF model. In the BSL (CE) the variability of consumption relative to DI is $0.92/0.86 = 1.07$ $(0.93/0.86 = 1.08)$, whereas the RF model yields $0.9/0.92 = 0.98$. In fact, consumption is less variable than income in the RF case. This is because without default risk, the sovereign uses both debt and oil reserves to smooth consumption without being hampered by the lack of commitment to repay debt. Still, consumption variability is high because of the presence of an ad-hoc debt limit in the RF model.[26] This hampers consumption smoothing but less so than the lack of commitment in the BSL model. Moreover, in the BSL model, the planner hits the upper bound of the bonds grid, albeit only with a frequency of 1.2 percent, and this works as an implicit savings constraint that also hampers consumption smoothing.

It is important to note that the planner uses oil reserves to smooth consumption in the BSL model without ever hitting the constraints at which there is either zero extraction in a given period or full extraction of the period's existing reserves and new discoveries. The planner adjusts s' taking into account the interaction between debt, debt prices, and reserves when aiming for the consumption path that maximizes private utility. In particular, the lack of commitment limits borrowing capacity and adds strategic incentives to the choice of reserves, but does not lead the planner to reduce s' to its lowest feasible level. Similarly, in the RF model, even when the sovereign hits the ad-hoc debt limit, it still does not choose to hit the lower bound of s'.

The CE model, in which reserves cannot be used to smooth consumption but default risk remains, is akin to a standard EG model, and its high consumption variability is in line with

[26]The RF model is solved with identical parameters and has an ad-hoc debt limit defined by the lower bound of the b grid, which is set to the largest debt in the ergodic distribution of the BSL model. Since $\beta R^* = 0.806$ is well below 1, the RF model hits this debt limit very often (88% of the time). Hence, the RF model is best viewed as a risk-free model with an exogenous borrowing constraint, whereas the BSL model has an endogenous borrowing constraint driven by the lack of commitment.

a common feature of EG models identified by Chatterjee & Eyigungor (2012): When debt is large relative to output, a change in the bond price implies large changes in consumption given that the sovereign must refinance all of the debt at the new price in one period. Because of the ladder-like shape of the equilibrium price of bonds, changes in bond prices can be large. This argument does not necessarily extend to our model, because oil reserves provide an alternative for consumption smoothing, but to the extent that extraction costs hamper consumption smoothing with reserves, the mechanism causing high consumption variability in standard EG models is still at work here.

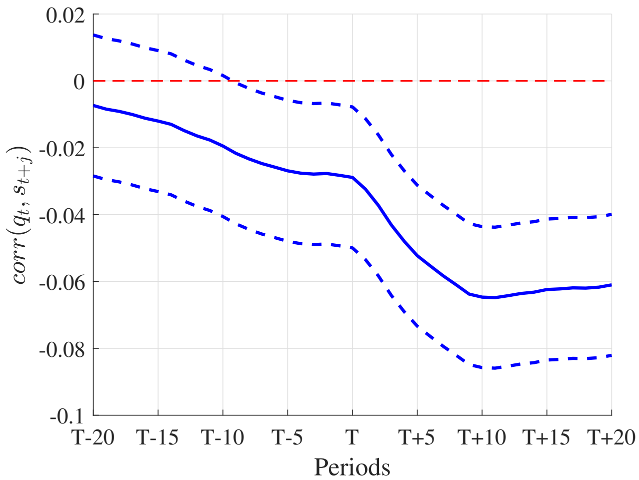
4.3.2 Oil Reserves and Sovereign Risk

Fact 3 of Section 2 showed that the relationship between oil reserves and sovereign risk is non-monotoninc over time: Contemporaneously, changes in oil reserves do not have a statistically significant effect on sovereign risk, but as the series converge to their long-run trends, higher oil reserves are actually associated with *higher* sovereign risk. Since the model is stationary, however, it does not have predictions comparable to those of the long-run (co-integration) coefficients of the dynamic panel regressions. Still, we can examine whether the model is consistent with the data in predicting a non-monotonic dynamic relationship between oil reserves and country risk as both return to their long-run averages after a shock. To this end, we use the model-simulated data to conduct two experiments.

In the first experiment, we examine the cross-correlation function of bond prices and oil reserves. Figure 6 plots the correlation between q_t and s_{t+j}, spanning the interval from $j = -20$ to $j = 20$. This plot shows that the correlation of current sovereign bond prices is more negative with future oil reserves than with current or lagged reserves. The two variables are nearly uncorrelated at the twentieth-year lag, and in fact the correlation cannot be ruled out to be zero statistically below the 10th lag. On the other hand, the correlation falls as we move into the future and converges to a statistically significant (albeit small) correlation of about -0.06 six years ahead and beyond. Thus, date-t default spreads are *positively* correlated with future oil reserves, in line with the empirical finding showing that higher future reserves increase sovereign risk in the present.

In the second experiment, we apply standard VAR techniques to derive the model's predictions about the dynamic relationship connecting oil reserves and country risk as the vari-

Figure 6: Cross-Correlation Function of Bond Prices and Oil Reserves in the Model



ables return to their stochastic steady states after a shock perturbs them. The goal is to determine if this relationship can capture a non-monotonic co-movement between oil reserves and sovereign risk qualitatively similar to that found in the data.

The VAR can be viewed as an approximation to the model's dynamical system in reduced form. It has four key equations that describe the evolution of the four state variables (p_t,y_t,b_{t+1} and s_{t+1}) and a fifth equation that describes the evolution of the bond price (q_t) as a function of the state variables. The VAR is block-recursive because p_t and y_t are independent from b_{t+1} and s_{t+1}. Hence, for oil prices and non-oil GDP, we simply impose the same VAR used in the model calibration. We then use the model-simulated data to estimate a VAR for the other three equations. Appendix K explains the details. Using the estimated VAR, we compute the IRFs of reserves, debt, and bond prices to an oil-price shock.

The five-equation system is the following:

$$
\begin{aligned}
p_t &= a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p, \\
y_t &= a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y, \\
b_{t+1} &= a_5 s_t + a_6 b_t + a_7 p_t + a_8 y_t + c_3 + \Phi_1 History_t + \psi_1 Transition_t + \epsilon_t^b, \\
s_{t+1} &= a_9 s_t + a_{10} b_t + a_{11} p_t + a_{12} y_t + c_4 + \Phi_2 History_t + \psi_2 Transition_t + \epsilon_t^s, \\
q_t &= a_{13} s_t + a_{14} b_t + a_{15} p_t + a_{16} y_t + c_5 + \Phi_3 History_t + \psi_3 Transition_t + \epsilon_t^q,
\end{aligned}
$$

where b_{t+1} and s_{t+1} refer to the optimal debt and reserves decisions at time t for time $t+1$,

(b_t, s_t, p_t, y_t) are the state variables at time t, and c_i are constant terms. The innovation terms, $\left(\epsilon_t^b, \epsilon_t^s, \epsilon_t^q\right)$, can be viewed as linearization errors since we are estimating a linear version of a non-linear model in which the only exogenous stochastic shocks are p_t and y_t.

We also include two dummy variables to control for the default and exclusion periods ($History_t$) and the transition towards a default ($Transition_t$). These are helpful to capture non-linearities associated with the run-ups to default events and with the defaults themselves. $History_t = 1$ when either the sovereign defaults or remains in exclusion from financial markets, and is zero otherwise. During these periods, q_t, b_t, and b_{t+1} take a value of zero in the model solution. $Transition_t$ controls for the fact that during normal times q_t fluctuates around the risk-free price but as a default approaches it falls rapidly. $Transition_t = 1$ when the bond price falls below a threshold and is typically associated with periods prior to a default event, and is zero otherwise. During these periods, q_t falls between 0.8 and 0.9 but the sovereign is still trading debt.

Table 7 shows the estimation results for the VAR and Figure 7 shows the impulse response functions of the variables to an oil-price shock. The Table shows that, contemporaneously and keeping the other state variables constant, a one-percent increase in s_t increases q_t by 0.1%. In contrast, the IRFs show that the dynamic response of both reserves and the price of bonds to an oil-price shock, taking into account all the feedback effects via the dynamics of the four state variables, results in reserves and bond prices moving together (both falling) for the first two periods, but then from $t = 3$ to $t = 20$, reserves fall as bond prices rise. After the 20th period, reserves and bond prices again move together, now both rising.

The oil-price innovation has positive persistence and takes about 50 periods to wash out. In response to the temporarily higher prices, the sovereign increases extraction and reduces reserves up to about $t = 20$, and after that reserves start to increase so as to return to their long-run average. The sovereign also responds by borrowing more on impact, because this helps finance the increase in extraction without too large a cut in consumption. After $t = 2$, however, debt starts to fall as bonds start reverting to their mean. The higher debt and lower reserves drive the initial drop in the price of bonds, but the drop is temporary as the mean-reversion of reserves and bonds then drive a recovery in bond prices.[27] Since debt starts shrinking sooner and faster than the reversal in reserves, we obtain the time interval in which

[27] Note that, because of the VAR structure of the shocks, the positive oil-price shock also causes a transitory drop in non-oil GDP, which adds to the sovereign's default incentives and contributes to the drop in q.

reserves are still falling while bond prices are already rising (or conversely, reserves would rise while bond prices fall in the IRF to a drop in p). Thus, while the model cannot speak to the co-movement of *trends* captured in the long-run coefficients of the error-correction panel regressions estimated with the data, it does produce a dynamic pattern of non-monotonic co-movement in country risk (or bond prices) and oil reserves at different time horizons.

Table 7: VAR for b_{t+1}, s_{t+1}, and q_t

	Debt (t+1)	Reserves (t+1)	Bond Price (t)
Reserves (t)	0.008***	0.981***	0.001***
	(0.000)	(0.000)	(0.0000)
Debt (t)	0.356***	0.019***	-0.031***
	(0.004)	(0.004)	(0.0009)
Oil Price (t)	0.387***	-0.189***	0.005***
	(0.002)	(0.002)	(0.0005)
Non-Oil GDP (t)	-0.207***	0.031***	0.025***
	(0.003)	(0.003)	(0.0008)
History	-0.040***	-0.008***	-0.991***
	(0.001)	(0.001)	(0.0003)
Transition	0.025***	-0.009***	-0.144***
	(0.001)	(0.001)	(0.0003)
Constant	-0.191***	0.422***	0.958***
	(0.006)	(0.006)	(0.0015)
Observations	8999	8999	8999
R-squared	0.959	0.998	0.999

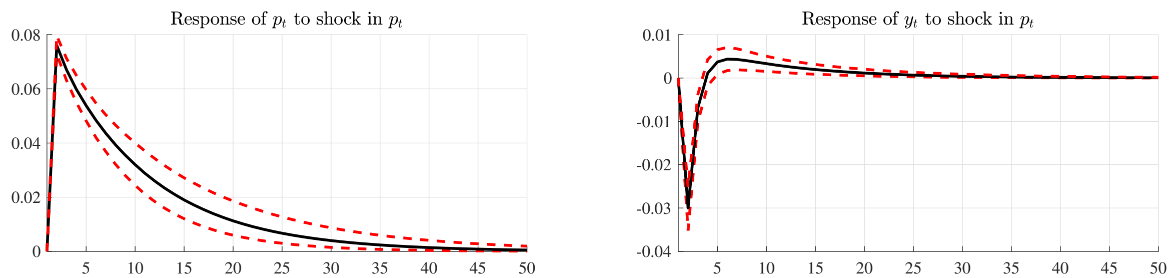
Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

4.4 The role of the oil sector

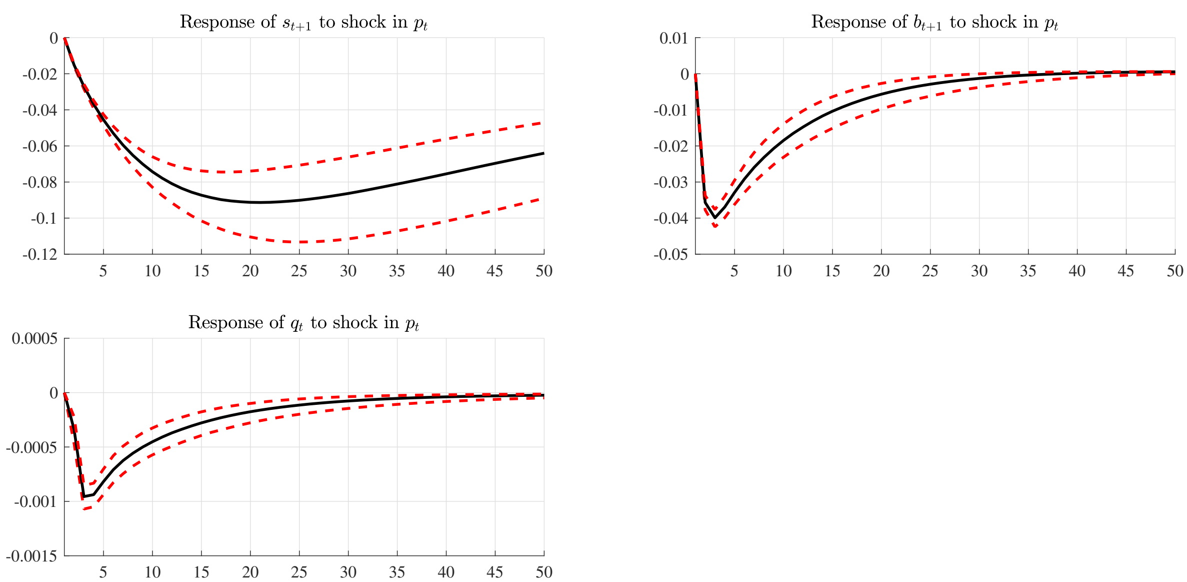
We close the quantitative analysis with an assessment of the role of the oil sector in the model by comparing the results from the BSL, CE and RF models. The BSL-CE comparison shows the importance of choosing oil extraction optimally for the sovereign's choices of debt and default. In the BSL model, the sovereign has the ability to prop up consumption when GDP is low by increasing debt, defaulting and/or reducing oil reserves, and can use oil exports to prop up consumption during periods of financial autarky. In the CE case, since extraction is constant, it cannot be used to smooth consumption or respond to default incentives. The BSL-RF comparison shows how much default risk matters for extraction and reserves decisions by studying how the choices made by a sovereign committed to repay (but facing the ad-hoc debt limit) differ. As mentioned earlier, the RF model is akin to an RBC model (albeit one where βR^* is well below 1) and thus the government can sustain more debt, whereas in the

Figure 7: Impulse Response to an oil price innovation

a) Response of exogenous variables in the baseline model



b) Response of endogenous variables in the baseline model



Note: Monte Carlo simulation for error bands (confidence level set to 95%): sample size of 1000 periods and 1000 replications.

BSL model borrowing capacity is hampered by the lack of commitment and the choice of reserves is affected by strategic incentives.

Table 4 compares the moments used as targets for the baseline calibration and Table 6 compares the rest of the business cycle moments. Comparing first the BSL and RF models, Table 4 shows that the mean debt ratio rises from 22.9% to 51.7%. Hence, full commitment to repay enhances borrowing capacity by nearly 30 percentage points of GDP. On the other hand, the debt ratio and net exports fluctuate very little, because again borrowing incentives with $\beta = 0.82$ and $R^* = 1.00775$ are very strong and make the RF model hit the ad-hoc debt limit (-0.51) 88% of the time. This should incentivize the planner to substitute debt for reserves as a vehicle for consumption smoothing and build-up precautionary reserves for self-insurance. The variability of consumption relative to that of disposable incomes does fall in the RF vis-a-vis the BSL model, as noted earlier, but mean reserves change little. This occurs because oil is not risk-free and oil reserves are a poor hedge against oil-price shocks since ps falls with p. Thus, the rate of return on oil changes as x, s' and $e(x, s')$ respond to the shocks in (p, y) . While the RF model is similar to the small-open-economy RBC model in theory, the low β that makes the ad-hoc debt limit bind frequently makes it more similar to a *closed-economy* RBC model with oil reserves taking the place of the capital stock. The ability to smooth with reserves is hampered by their endogenous return (effectively, the real interest rate of this economy is endogenous, see Section 3 and Appendix F).

Compare now the BSL and CE models. The mean debt ratio increases about 5 percentage points in the latter, which indicates that lacking the ability to use reserves to support consumption during periods of exclusion (i.e. lacking the strategic incentives on reserves) enhances the sovereign's borrowing capacity, and it also reduces slightly the frequency of defaults (about 10 basis points). Moreover, as shown earlier in this section, the ability to strategize over both reserves and debt makes default less costly in the BSL model, and this reduces the debt that the sovereign can sustain.

As noted earlier, the variability of consumption relative to that of DI is smaller in the RF model than in the other two models, although it is just below DI volatility because the debt limit binds often and the extraction costs hamper the ability to smooth consumption with oil reserves. For the same reason, debt and the trade balance fluctuate much less in the RF model than in the other two, consumption is more correlated with disposable income, and the trade balance is more procyclical and less negatively correlated with oil prices. Several of the other moments are similar between the BSL and RF models, particularly those for gross oil output and total GDP, suggesting that, even tough Fisherian separation of the consumption/borrowing decisions from the choices of reserves does not hold strictly in both models (in the former because of default risk and in the latter because of the binding debt limit), the resulting distortions on the Euler equation of oil reserves do not result in significant differences in disposable income and oil-price correlations of gross oil output and total GDP in the long-run. The same is true for the CE model, which suggests that Fisherian separation also holds approximately in terms of the long-run moments that characterize business cycles in total GDP and disposable income when default risk is introduced vis-a-vis once is removed.

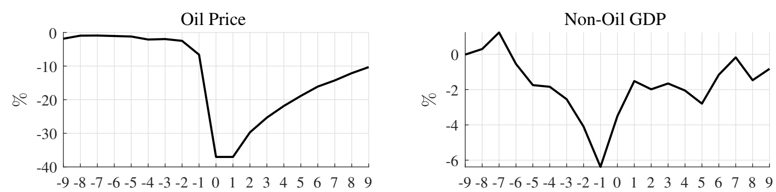
The similarity of some of the long-run cyclical moments across models does not imply that endogenizing default risk and oil extraction is irrelevant. Endogenizing extraction does lower the mean debt ratio and the average spread, and higher oil reserves increases default risk. More importantly, the three models yield very different dynamics around the default events of the BSL model, as we show next.

To compare the dynamics across models when defaults occur in the BSL model, we use again the ten-thousand period simulation of the BSL model. We identify periods in which defaults occur and construct 19-period windows centered on them, computing for each period the average of the observations across all default events. There are 107 default events, which yields a frequency of 1.1 percent, in line with the model's calibration. Then, we construct comparable time paths for the CE and RF models as follows: First, for each of the baseline model's 107 defaults, we extract the "initial" values of (b, s) in the 9th period before the default and the 19 realizations of (p, y) from the 9th period before to the 9th period after the default. Then we feed them into the decision rules of the CE and RF models to construct 107 comparable 19-period time paths for each model, and calculate the averages pertaining to each of the 19 periods across the 107 values. Finally, we subtract the RF or CE averages from the BSL model's averages and plot them in Figure 8. The red-dashed and dotted-blue lines show the results for the CE and RF models, respectively. For all variables, except those that are GDP ratios or already in percent (i.e., debt, trade balance and interest rate), we plot the difference in percent of the mean in the BSL model, so that the base of the percentages is common for the CE and RF cases. For p and y there is only one time-path, because the same sequences of shocks are used in the three models.

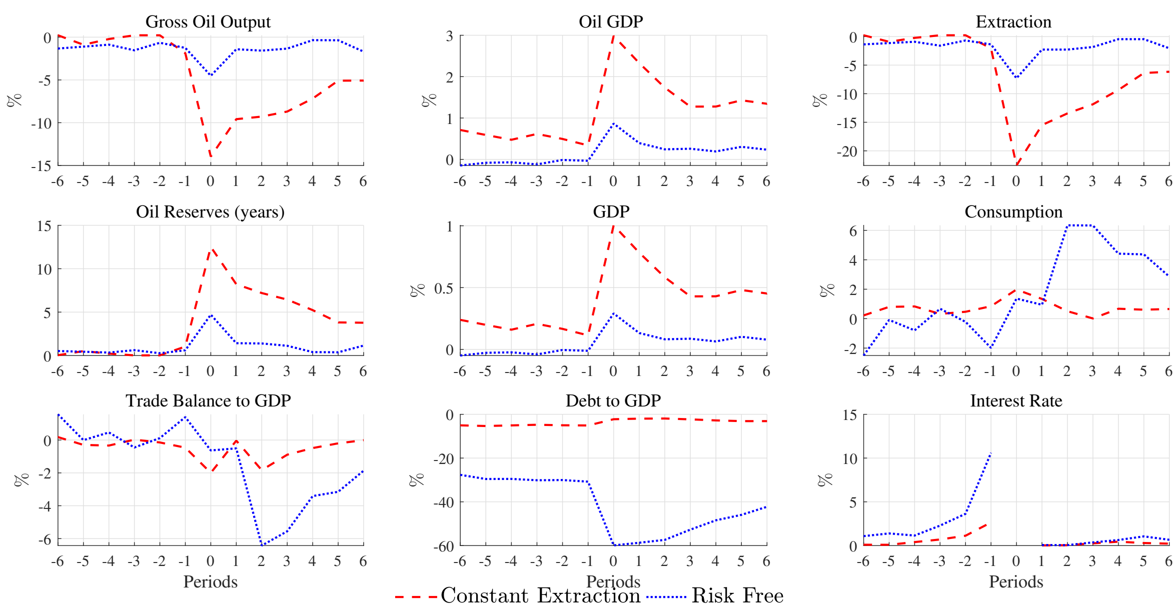
The first result to note in Figure 8 is that defaults in the baseline model occur when large, negative oil-price shocks follow a sequence of relatively neutral realizations, together with a history of relatively weak non-oil GDP realizations. When defaults occur ($t = 0$), p hits its lowest realization in the Markov process of (p, y) and y just hit a through roughly 6 percent below its long-run average a period earlier. Post-default, oil prices follow a monotonic mean-reverting path and non-oil GDP recovers, but both are still below their long-run averages. It is also worth noting that at $t = 0$, across all 107 events in which the sovereign in the BSL model always defaults, the sovereign in the CE model decides to repay in 15% of the cases. Thus, there is already a meaningful difference across the CE and BSL models in that, faced with the same initial (b, s) pair and identical sequences of (p, y) shocks, the CE sovereign does not default in some instances in which the BSL sovereign does. This result is in line with the earlier observation that defaults are less frequent in the long-run (by about 11 basis points) when the sovereign cannot strategize over both debt and reserves (see Table 4).

Figure 8: Default Event Windows: Baseline, Constant-extraction and Risk-free Models

a) Exogenous sequences of p and y that trigger defaults at date 0



b) Relative responses of endogenous variables in the BSL model



Note: All variables are reported relative to the baseline model, except for the oil price and non-oil GDP which are plotted relative to their long-run average.

The Figure also shows that the CE and BSL models display very different dynamics. At $t = 0$, when defaults occur in the BSL model, oil extraction and gross oil output (px) are 23% and 15% smaller in the BSL model than the CE model, respectively, and the BSL model builds up an extra 12 months of oil reserves. Since defaults occur when p is very low and thus expected future prices are higher, the sovereign in the BSL model cuts oil extraction and output and increases reserves. Oil GDP (i.e., oil profits) and total GDP, however, are *higher* than in the CE case, because cutting extraction reduces extraction costs by more than the decline in gross oil output. Note also that the cut in extraction and the reserves buildup are beyond what is just the response to expected higher prices, they also reflect the strategic incentives to build reserves to finance future consumption while the economy remains in financial autarky, since the BSL sovereign does default at $t = 0$.

After the default, extraction remains lower and reserves higher in the BSL than the CE model, but the differences narrow to about 8% in extraction and 4 years in reserves. Similarly, oil GDP and total GDP remain higher in the BSL case, but the differences narrow to roughly 3% and 1%, respectively. These dynamics reflect the sovereign's management of reserves to sustain consumption during the exclusion period, particularly in the early years after the default. Accordingly, the trade balance is about 2 percentage points of GDP smaller in the BSL than the CE case when default occurs and in the early periods afterwards, and consumption is about 2% higher. Moreover, prior to the default, spreads widen about 250 basis point more in the BSL model, because again strategic incentives are stronger than in the CE case.

Compare next the BSL and RF models. Since extraction is endogenous in the RF model, we observe again that at $t = 0$ extraction and gross oil output are lower in the BSL economy, and oil reserves, oil GDP and total GDP are higher, but the differences are much smaller than in the comparison with the CE model. These changes are in response to the sharp drop in p and expected higher future prices affecting the no-arbitrage condition of oil returns vis-a-vis the risk-free rate. In this case, however, there are no additional effects from strategic default incentives, and in fact the choices of (x, s') are nearly independent of the b' choice, because Fisherian separation nearly holds. Hence, the extra 15 percentage points drop in extraction and extra 7 years of reserves that the BSL sovereign builds when compared to the CE case instead of the RF case are due to those strategic incentives. After $t = 0$, again we observe the same qualitative differences in extraction, reserves, oil GDP and total GDP as in the BSL-CE comparison, but the differences are smaller. In fact, these variables are nearly the same in the BSL and RF models by $t = 6$. The gap between the CE and RF curves is again reflecting the relevance of the strategic incentives that are absent in the RF model. Six years after the default, these strategic incentives are equivalent to about an extra five percentage points in extraction and extra 3 years in reserves.

Explaining the differences in debt, spreads and consumption requires recalling first that the RF model has a mean debt-GDP ratio of 51.7% and deviates little from it, while debt has a much lower mean (22.9%) and is much more variable in the BSL model. This exercise simulates the RF economy starting from the same b as the BSL model, but that b is therefore too high in the former, and since βR^* is well below 1, the RF economy starts to move gradually back toward its larger debt levels. At $t = 0$, however, the large drop in p causes a sharp increase in debt in the RF economy, so that the debt ratio in the BSL model is 60 percentage points lower than in the RF case (in fact, the RF economy hits its ad-hoc debt limit). The trade balance, however, is still 1 percentage point lower in the BSL economy and hence consumption is about 1 percentage point higher. Post-default, the difference in debt ratios narrows and by $t = 6$ debt is about 40 percentage points larger in the RF economy, but the trade balance is still lower and consumption higher in the BSL case. Intuitively, this pattern reflects how when default occurs and early after that, the BSL sovereign attains higher consumption by defaulting than repaying. In the RF case, the sovereign borrows a lot at $t = 0$ but then reduces its debt (relative to the BSL case) afterwards, while in the BSL the sovereign defaults and then uses its accumulated reserves (and the gamble with the random chance of credit-market re-entry) to attain higher consumption for several periods. On the other hand, the anticipation of this causes spreads to rise sharply before $t = 0$ in the BSL case. At $t = -1$, the BSL sovereign pays an interest rate nearly 11 percentage points higher than R^* and spreads start to widen since $t = -6$.

Summing up, this event-analysis comparison across the BSL, CE and RF models shows that endogenizing oil extraction and allowing strategic default incentives to influence extraction and reserves decisions has large quantitative implications. The sovereign accumulates much larger reserves when it defaults and uses them to prop up consumption afterwards. On the other hand, this extra margin for strategic incentives to operate reduces even more the borrowing capacity of the sovereign and pushes up spreads significantly more.

5 Conclusions

This paper examines the implications of extraction of natural resources for the analysis of sovereign default in a framework in which the government makes both borrowing and resource-extraction decisions. The sovereign can alter the value of default by adjusting the stock of reserves of the natural resource, which are used to prop up consumption by extracting and exporting more of the natural resource while the economy is in financial autarky.

We studied the data for the thirty largest oil-producing emerging economies over the 1979-2014 period, and found that they have sizable external debt, their country risk is positively correlated with real oil prices, and several of them defaulted at least once. In addition, dynamic error-correction panel regressions showed that, while oil production and non-oil output reduce country risk on impact and in the long-run, oil reserves reduce it marginally on impact and increase it in the long-run.

In the model, the ability to increase the value of default by holding higher oil reserves gives the sovereign the incentive to strategize over both debt and oil reserves as it makes its borrowing, default and extraction decisions. We derive analytic results showing that, keeping all other variables constant, default incentives strengthen at lower oil prices or lower oil reserves. In addition, we show that when oil prices rise, the intertemporal incentive to increase extraction and reduce reserves to increase oil sales competes with the incentive to build up reserves to prop up consumption in case of a future default.

The model's quantitative predictions are examined using a calibration based on the data for the thirty largest oil producers. We found that the incetive to strategize over oil reserves is quantitatively significant. In particular, it reduces the cost of default and hence reduces the debt the sovereign can sustain in the long run relative to both a sovereign that is committed to repay and one that exports a fixed endowment of oil.

The model is consistent with the data in predicting that country risk and oil prices are negatively correlated and it does a good job at approximating observed income correlations, but it overstates the volatility of consumption. Relative to the variant of the model where the government is committed to repay, the model with default yields higher consumption variability because of the reduced ability to smooth consumption without commitment. However, in both of these models consumption is markedly more volatile than in the data. The model can also explain the non-monotonic pattern of the dynamic relationship between oil reserves and sovereign risk. In particular, impulse response functions to a positive oil-price shock show that reserves and country risk increase together on impact but then they go through a lengthy phase in which reserves fall and bond prices rise.

Two counterfactual comparisons are used to gauge the relevance of endogenous oil extraction for sovereign default by comparing the baseline model results with variants with constant extraction and without default risk. The results show that endogenizing oil extraction and allowing strategic default incentives to affect extraction and reserves decisions has large quantitative implications. The sovereign accumulates much larger reserves when it defaults and uses them to prop up consumption afterwards.

Our analysis of the dynamics connecting commodity prices, sovereign default, country risk and business cycles applies more broadly to developing countries rich in mineral and agricultural commodities other than oil. This analysis also suggests that the issuance of debt indexed to commodity prices may enhance borrowing capacity, by improving the hedging properties of external debt so that the burden of debt service weakens when commodity prices fall. Moreover, our analysis may also have important implications for evaluating the effects of climate change policies on the sovereign debt sustainability and country risk of emerging economies. In particular, the model can be used to examine how caps on fossil fuel extraction may affect sovereign debt access, country risk and macroeconomic dynamics.

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