



RESEARCH DIVISION

Working Paper Series

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Working Paper 2018-026D
<https://doi.org/10.20955/wp.2018.026>

December 2019

FEDERAL RESERVE BANK OF ST. LOUIS

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Assessing the Impact of Central Bank Digital Currency on Private Banks

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April 22, 2020

Abstract

I investigate how an interest-bearing central bank digital currency (CBDC) can be expected to impact a monopolistic banking sector. My framework of analysis combines the Diamond (1965) model of government debt with the Klein (1971) and Monti (1972) model of a monopoly bank. I find that the introduction of a CBDC has no detrimental effect on bank lending activity and may, in some circumstances, even serve to promote it. The intervention does, however, reduce monopoly bank profit since it induces the monopoly bank to raise its deposit rate to retain deposits that remain a relatively cheap source of funding. More attractive deposit services have the effect of increasing financial inclusion and decreasing the demand for currency. Available theory and evidence suggests that a properly-designed CBDC is not likely to threaten financial stability.

JEL Codes: E4, E5

Keywords: Digital Currency, Central Banks, Monopoly, Markups.

*I thank seminar participants at the Federal Reserve Bank of St. Louis, the Federal Reserve Bank of Kansas City and LUISS Guido Carli in Rome, as well as conference participants at the T2M Conference in Nuremberg in 2019 and the conference on the Economics of Central Bank Digital Currency in Ottawa in 2019, sponsored by the Bank of Canada and the Sveriges Riksbank. I appreciate comments received by Fabrizio Mattesini, Hervé le Bihan, Michael Kumhof, David Skeie, Maria Koumenta, Francesco Lippi and three anonymous referees. I also thank Andrew Spewak and Mahdi Ebsim for their assistance. The views expressed here are my own and should not be attributed to the Federal Reserve Bank of St. Louis or the Federal Reserve System.

1 Introduction

Central bank digital currency (CBDC) is a proposal to make central bank deposit accounts available to everyone. Proponents envisage it operating as a basic public option, offering people a number of privileges presently reserved for banks (Ricks, Crawford and Menand, 2018).¹ Opponents raise several concerns, the most significant of which cite potentially deleterious effects on the availability of bank credit and financial stability (Cecchetti and Schoenholtz, 2017). Fears over the impact that CBDC may have on the cost of bank funding, the volume of bank lending, and financial stability have also been raised by prominent policymakers; see, for example, Broadbent (2016). The goal of my paper is to assess the theoretical and empirical basis of these concerns.

The theoretical analysis below combines a version of the Diamond (1965) model of government debt with the Klein (1971) and Monti (1972) model of a monopolistic banking sector.² A model in which an outside asset is valued seems important in the present context as government bonds form a natural basis for CBDC. A model where banks have market power also seems pertinent in light of the empirical evidence in Figure 1 and the likely importance of market structure for the question at hand.³ Almost all theoretical investigations on the impact of CBDC assume a competitive banking industry, including Barrdear and Kumhof (2016), Keister and Sanches (2018), Brunnermeier and Niepelt (2019) and Williamson (2019).⁴ In these environments, the pass-through from policy rates to bank lending rates is direct, so a CBDC offering a higher deposit rate necessarily discourages investment. In non-competitive settings, policy interventions can also affect markups and monopoly profits, which mitigate (and in some cases, reverse) the effect on

¹The main benefits include relatively high deposit rates, fully-insured deposit accounts, and instant payments.

²The key assumption I make is market power. The monopoly set-up lets me invoke this assumption in the most tractable manner possible. The key results I report below are likely to hold for oligopolistic settings as well.

³See also Ketterer and Andrade (2016), Wheelock and Wilson (2020) and the discussion in Martin, McAndrews and Skeie (2016).

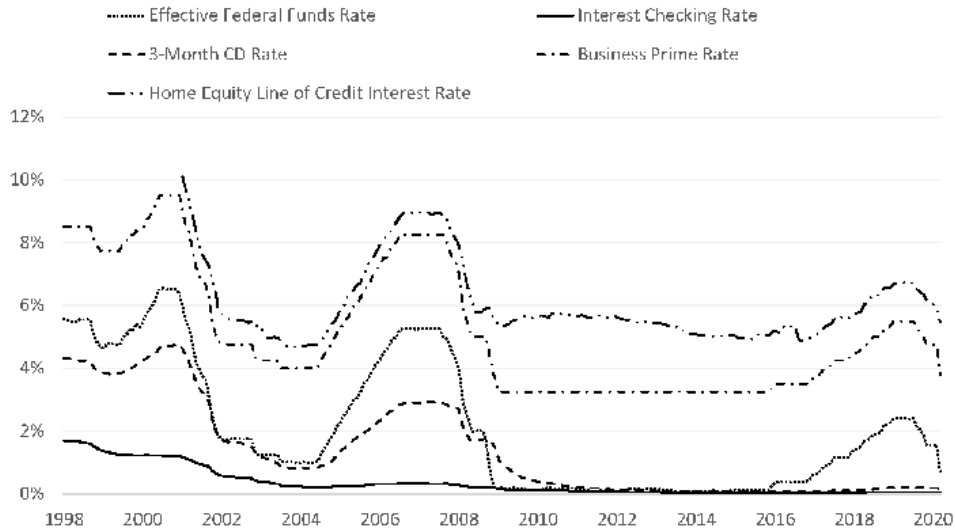
⁴An exception is Chui, Davoodalhosseini, Jiang and Zhu (2019), who model the market for deposits (but not loans) as an oligopoly.

investment.

Figure 1
Selected U.S. Interest Rates

Interest Rates

Percent, Jan 1998 - Mar 2020



Source: Federal Reserve Board and RateWatch

The bank sector in my model intermediates transactions between workers and entrepreneurs. Entrepreneurs need workers to construct capital goods that generate output in a future period. Workers, who have heterogeneous skills, offer their labor in exchange for money, some of which is saved. Money takes the form of bank deposits, both private and CBDC, as well as physical currency. Because there is no credit market, entrepreneurs must borrow money from the private bank sector (the central bank does not grant loans to nondepository institutions). These money loans are effectively collateralized by the capital projects they fund. Because of the fixed cost that is incurred in opening a bank account, only relatively rich workers have access to deposit money, while relatively poor workers are compelled to use cash. And because some of their workers will want to be paid in cash, money loans to entrepreneurs are made redeemable for cash. The bank's assets consists of interest-bearing reserves that yeild a policy-determined interest-on-reserve (IOR) rate and risk-free loans. The bank's liabilities consist solely of deposits

(there is no role for bank capital in the model).

The monopoly bank provides a vital service in this economy. In particular, it creates deposit-money in the act of lending, which permits exchanges to occur that would not otherwise take place. A competitive bank in the model would be compelled to set loan and deposit rates equal to the IOR. However, because the bank here is a monopoly, it takes advantage of its market power to mark loan rates above IOR and mark deposit rates below IOR. Absent CBDC, the monopoly deposit rate is disciplined only by the existence of cash. The presence of an interest-bearing CBDC places even greater discipline on the monopoly deposit rate, increasing the cost of deposit-funding (but not other forms of funding) for the bank.

The model delivers the following results. First, if the interest rate on CBDC is set independently of the IOR rate, then the introduction of a CBDC in no way discourages bank lending. This is because the opportunity cost of bank lending is the IOR rate and not the CBDC rate. Second, if the CBDC rate is set below the IOR rate, the monopoly bank has every incentive to match the CBDC rate for the purpose of retaining deposits. This is because the bank lends to the central bank at IOR and only pays depositors the CBDC rate. As long as this IOR-CBDC spread is positive, it pays for banks to retain deposits. Third, because the threat of CBDC induces more favorable contractual terms for depositors, it increases the supply of deposits. In the model, this occurs both through an intensive margin (existing depositors are encouraged to save more) and through an extensive margin (unbanked individuals are encouraged to pay the fixed cost of accessing the bank sector). If access to CBDC is made costless, the demand for physical currency disappears, but this in no way inhibits the efficacy of monetary policy. Fourth, if a regulatory liquidity constraint binds for the bank, the increase in deposits resulting from CBDC competition induces an *expansion* in bank lending. That is, an increase in the CBDC rate leads to an increase in the deposit rate and a *decrease* in the lending rate. This is because a higher CBDC rate induces deposit expansion, which relaxes the liquidity constraint. Fifth, there is the hardly surprising result that CBDC unambiguously reduces monopoly bank profit.

The paper is organized as follows. In Section 2, I describe the physical structure of the economy—individual preferences, demographics, and available technologies. I develop the equilibrium framework in Section 3 and use

the model to evaluate the economic implications of CBDC in Section 4. In Section 5, I discuss some of the theory and evidence that might be brought to bear on the question of CBDC and financial stability. Section 6 concludes.

2 The physical environment

The model economy is populated by two-period-lived overlapping generations (together with an *initial old* population that lives for one period only).⁵ Time is denoted by $t = 1, 2, \dots, \infty$. Let $c_t(i)$ denote consumption at date t for a person in period $i \in \{1, 2\}$ of their life. Since the initial old population only lives for one period, their welfare is monotonically increasing in $c_1(2)$. The initial young and all future generations have identical preferences represented by,

$$U(c_t(1), c_{t+1}(2)) = (1 - \beta)u(c_t(1)) + \beta c_{t+1}(2) \quad (1)$$

where $u'' < 0 < u'$ and $0 < \beta < 1$. The quasilinear structure of preferences imply that the substitution effect from an interest rate change dominates the wealth effect, and that the distribution of second-period lump-sum tax obligations does not matter for economic aggregates.

The population is constant, with four broad categories of people existing at any given date. Because people live for two periods, the population is divided into two equal measures of *young* and *old* individuals. In addition, I assume that people belong to one of two groups, each of equal size, labeled *workers* and *entrepreneurs*. Hence, at any given date t , the economy is populated by a unit mass each of: (i) young workers, (ii) old workers; (iii) young entrepreneurs; and (iv) old entrepreneurs.

Young entrepreneurs are identical—each is endowed with a investment project that takes k_t units of output at date t and transforms it into $F(k_t)$ units of output at date $t + 1$. Assume that $F'' < 0 < F'$ with $\lim_{k \rightarrow 0} F'(k) = \infty$ and $\lim_{k \rightarrow \infty} F'(k) = 0$.

Young workers are endowed with a unit of time that produces y units of output, either in the form of a consumption good or investment good. In what follows below, I refer to y as labor, since we can think of y units of output

⁵The OLG model is convenient for its tractability. The analysis could alternatively have been cast in the Lagos and Wright (2005) model with liquid government debt as developed in Andolfatto and Martin (2018).

as being produced with y units of labor. Young workers have heterogeneous levels of ability, indexed by $y \in [0, \bar{y}]$, a parameter that is distributed across the population according to an exogenous cumulative distribution function $H(a) \equiv \Pr[y \leq a]$.

In what follows, I restrict attention to stationary allocations. The total resources available for consumption and investment is given by $\int ydG(y) + F(k)$. This output is used to finance investment k and aggregate consumption. If consumption depends only on age, then the resource constraint in a stationary economy is given by,

$$2[c(1) + c(2)] + k \leq \int ydG(y) + F(k) \quad (2)$$

In the equilibrium studied below, the level of consumption will generally differ across different types of individuals. In particular, there will be inter-generational differences, differences across workers and entrepreneurs, and even differences within the class of workers owing to differences in ability.

The pattern of welfare-improving trade here is clear. In particular, young entrepreneurs want young labor in exchange for a share of the future output that can be produced through their joint effort. If the economy is dynamically inefficient, intergenerational transfers of resources from young workers to old workers may also be desirable. However, in what follows I study equilibria, not optima. Nevertheless, in any equilibrium, market forces strive to realize the gains to trade described above. Market outcomes will, however, depend on policy. This is described in the next section.

3 A model with money and banking

In what follows, I restrict attention to deterministic stationary equilibria. I therefore take the liberty of dropping time subscripts where there is little risk of confusion. In the overlapping generations model, the equilibrium inflation rate is determined by rate of growth of nominal government debt.⁶ Since my focus is not on inflation, I assume a constant stock of nominal debt. In this case, the price-level, which I denote below by p , is constant. Note, however, that while p is constant in a stationary equilibrium, it is determined endogenously in equilibrium.

⁶Net of the growth rate of the real economy, which is normalized to zero here.

3.1 Government policy

Let D denote the nominal value of outstanding government debt. I abstract from government purchases. Therefore, the only job of the fiscal authority is to service the interest expense of the debt. This interest expense depends on the composition of government debt. In the model below, debt can take potentially take the form of physical currency M^1 , bank reserves M^2 , and CBDC M^3 . Thus, as a matter of accounting, we have $M^1 + M^2 + M^3 = D$. In what follows, I restrict the currency component of the debt to be non-negative, i.e., $M^1, M^3 \geq 0$. Reserves on the other hand are permitted to be negative, though this is not critical for the main results reported below.⁷

Let R^i denote the gross nominal interest rate paid on type $i = 1, 2, 3$ debt. The interest expense of the debt is financed through tax revenue T . The government budget constraint in this case is given by,

$$T = (R^1 - 1)M^1 + (R^2 - 1)M^2 + (R^3 - 1)M^3 \quad (3)$$

Below, I assume that tax revenue is generated via a lump-sum tax on the old.

I assume that physical currency is constrained to yield a zero nominal interest rate, $R^1 = 1$.⁸ The IOR rate R^2 and the CBDC rate R^3 are policy parameters. The composition of the debt is not a direct policy choice, but can be influenced by the choice of interest rates, as will be explained below in due course.

3.2 Entrepreneurs

Entrepreneurs borrow money B at interest rate R^L to finance their consumption $c^e(1)$ and capital expenditure k . The investment generates output $F(k)$

⁷A negative reserve position here would mean that banks are net borrowers of reserves. Here, I assume reserves can be borrowed at the IOR rate. In reality, most central banks operate a channel system, so that the borrowing rate exceeds the deposit rate.

⁸In reality, evidence suggests that effective interest rate on storing large amounts of currency is clearly negative. Consider, for example, Pablo Escobar's drug empire which, at its apex, was reportedly bringing in \$420 million per week. Large quantities of cash were stashed in dilapidated warehouses or buried in pits. According to the cartel's accountant, 10% of this cash was written off owing to a variety of storage costs. Source: <https://www.businessinsider.com/pablo-escobar-and-rubber-bands-2015-9>

in the subsequent period. Revenue from sale of output is used to pay back the money loan and settle any tax obligations. After-tax profit is used to finance consumption when old $c^e(2)$. Thus, entrepreneurs face the following sequence of budget constraints,

$$c^e(1) + k = B/p \quad (4)$$

$$c^e(2) = F(k) - R^L B/p - (1/2)T/p \quad (5)$$

Substituting (4) and (5) into the objective (1) permits us to express the entrepreneurial choice problem as,

$$\max_{b,k} (1 - \beta)u(b - k) + \beta [F(k) - R^L b - \tau] \quad (6)$$

where $b \equiv B/p$ and $\tau = (1/2)T/p$. The solution (\hat{b}, \hat{k}) at an interior satisfies,

$$(1 - \beta)u'(\hat{c}^e(1)) = \beta R^L \quad (7)$$

$$F'(\hat{k}) = R^L \quad (8)$$

where $\hat{c}^e(1) = \hat{b} - \hat{k}$. Condition (7) implies that entrepreneurial consumer demand $\hat{c}^e(1)$ is decreasing in R^L . Condition (8) implies that the demand for investment \hat{k} is decreasing in R^L . Since $\hat{b} = \hat{c}^e(1) + \hat{k}$, it follows that the private demand for debt \hat{b} is also decreasing in R^L .

Lemma 1 *The entrepreneurial demand for loans $\hat{b}(R^L)$ and the demand for investment spending $\hat{k}(R^L)$ are both decreasing in the lending rate R^L .*

3.3 Workers

Because workers generate income when young and have no income when old, they constitute the natural savers in this economy. Workers can save in one of three ways: physical currency, bank deposits, and CBDC. I assume that bank deposits and CBDC have no special technological advantage over each other. Consequently, if both are to be voluntarily held, they must yield the same rate of return. If one yields more than the other, the one offering a lower return will not be held. Opening a bank account, however, requires incurring a fixed utility cost $\phi \geq 0$. There is no fixed cost associated with holding physical currency. Hence workers face a trade-off: physical currency requires no fixed cost, but deposit money generally yields interest.

Let R^D denote the maximum of the bank deposit rate and the CBDC rate. If cash is costless to store (in reality it is not), then there is a zero-lower-bound on the nominal interest rate, $R^D \geq 1$. Now, consider the consumption-saving decision of a type y worker that chooses to open a bank account,

$$W^b(y, R^D) \equiv \max_{x^b} (1 - \beta)u(y - x^b) - \phi + \beta [R^D x^b - \tau] \quad (9)$$

The solution $\hat{x}^b(y, R^D)$, at an interior, satisfies

$$(1 - \beta)u'(y - \hat{x}^b) = \beta R^D \quad (10)$$

Lemma 2 *Desired saving $\hat{x}^b(y, R^D)$ and welfare $W^b(y, R^D)$ for banked workers are both strictly increasing in worker income y and the deposit rate R^D .*

Consider next the consumption-saving decision of a type y worker that chooses to remain unbanked,

$$W^u(y) \equiv \max_{x^u} (1 - \beta)u(y - x^u) + \beta [x^u - \tau] \quad (11)$$

The solution $\hat{x}^u(y)$, at an interior, satisfies

$$(1 - \beta)u'(y - \hat{x}^u) = \beta \quad (12)$$

Lemma 3 *Desired saving $\hat{x}^u(y)$ and welfare $W^u(y)$ for unbanked workers are both strictly increasing in worker income y and invariant to the deposit rate R^D .*

From Lemmas 2 and 3, it follows that for a given $R^D \geq 1$ there exists a number $\hat{y}(R^D)$ that satisfies the equality:

$$W^b(\hat{y}, R^D) = W^u(\hat{y}) \quad (13)$$

Evidently, workers with an income level $y \geq \hat{y}(R^D)$ choose to open bank accounts while workers with income level $y < \hat{y}(R^D)$ choose to operate with cash. The critical level of income $\hat{y}(R^D)$ that defines indifference between cash and deposits is clearly decreasing in the deposit rate R^D . That is, a higher deposit rate makes it relatively more attractive for the marginal worker to switch from cash to deposits.

Lemma 4 *The reservation income level $\hat{y}(R^D)$ is decreasing in the deposit rate R^D .*

We are now in a position to examine the aggregate demand for deposits and study its properties. Before I do, however, recall that young worker income is distributed on the interval $y \in [0, \bar{y}]$ according to the exogenous cumulative distribution function $H(a) \equiv \Pr[y \leq a]$. Assume that parameters \bar{y} and $\phi > 0$ are such that $0 < \hat{y}(R^D) < \bar{y}$. The aggregate demand for deposits is constructed by adding up the desired saving of all banked workers, i.e.,

$$\hat{s}(R^D) \equiv \int_{\hat{y}(R^D)}^{\bar{y}} \hat{x}^b(y, R^D) dH(y) \quad (14)$$

By Lemma 2, $\hat{x}^b(y, R^D)$ is increasing in R^D . Thus, the supply of deposits expands with a more attractive deposit rate along an *intensive margin*—that is, banked workers find it more attractive to save. By lemma 4, $\hat{y}(R^D)$ is decreasing in R^D . This implies that the supply of deposits expands with a more attractive deposit along an *extensive margin* as well—that is, poor workers are, at the margin, induced to abandon cash in favor of deposits.

Lemma 5 *The supply of deposits $\hat{s}(R^D)$ is strictly increasing in R^D as: (i) banked workers save more (intensive margin); and (ii) unbanked workers (at the margin) open bank accounts (extensive margin).*

An implication of property Lemma 5 is that a more attractive deposit rate increases financial inclusion and reduces the demand for cash. Formally, the aggregate demand for currency is given by,

$$\hat{m}(R^D) \equiv \int_0^{\hat{y}(R^D)} \hat{x}^u(y) dH(y) \quad (15)$$

Since the real rate of return on currency is equal to zero (inflation is zero, in equilibrium), currency in circulation expands or contracts with R^D only along the extensive margin.

Note that the demand for cash and deposits comes from young workers and not young entrepreneurs. Recall that young entrepreneurs also open deposit accounts in the period they are granted a loan from the bank. However, entrepreneurs borrow only the money they need to pay workers. That

is, they spend all the money they borrow and do not carry deposits over time (since the return on their capital investments weakly dominate the return on deposits). It is the banked workers who carry deposits over time and the unbanked workers that carry cash over time.

3.4 The private monopoly bank

Consider now the behavior of the banking sector, which I model here as a consolidated bank sector along the lines of Klein (1971) and Monti (1972). Let me assume for the moment that there is no CBDC. The monopoly bank takes the central bank policy rate, the IOR rate R^I as given. For a chosen deposit rate R^D , the monopoly bank attracts nominal deposits $\hat{s}(R^D)$. These deposits are used to fund bank assets consisting of reserves and loans.⁹ For a chosen loan rate R^L , the monopoly bank attracts a loan demand equal to $\hat{b}(R^L)$. Given R^D and R^L , the monopoly bank's demand for reserves ℓ is implicitly given by its first-period balance sheet constraint,

$$\ell + \hat{b}(R^L) = \hat{s}(R^D) \quad (16)$$

Let $R^I \geq 1$ denote the IOR rate. Then this balance sheet generates an expected future real profit equal to

$$R^I \ell + R^L \hat{b}(R^L) - R^D \hat{s}(R^D) \quad (17)$$

Combining (16) with (17), the bank's problem can be expressed as,

$$\max_{R^L, R^D \geq 1} \left\{ [R^L - R^I] \hat{b}(R^L) + [R^I - R^D] \hat{s}(R^D) \right\} \quad (18)$$

The first-order necessary conditions for an optimum are given by,

$$[R^L - R^I] \hat{b}'(R^L) + \hat{b}(R^L) = 0 \quad (19)$$

$$[R^I - R^D] \hat{s}'(R^D) - \hat{s}(R^D) = 0 \quad (20)$$

where (20) holds if the lower bound on the deposit rate is not binding. Otherwise, we have $R^D = 1$.

⁹There is no role for bank equity in this economy. A meaningful role for equity finance could be introduced along the lines of Dermine (1986).

According to (19) and (20), the profit-maximizing lending and deposit rates depend on the policy rate R^I , but are otherwise set independently of each other. By Lemma 1 we know that $\hat{b}'(R^L) < 0$. Hence, the first term in condition (19) represents the marginal cost of increasing R^L ; namely, the consequent decline in loan demand multiplied by the profit margin. The second term measures the incremental interest income on the bank's loan portfolio. By Lemma 5 we know that $\hat{s}'(R^D) > 0$. Here, the first term in condition (20) represents the marginal benefit of increasing R^D ; namely, the consequent increase in the depositor base multiplied by the profit margin. The second term represents the added cost of having to pay more for deposits.

To see how R^L and R^D depend on R , it is convenient to rearrange (19) and (20), respectively, in the following way,

$$R^L = \left[\frac{\chi(R^L)}{1 - \chi(R^L)} \right] R^I \quad (21)$$

where $\chi(R^L) \equiv -R^L \hat{b}'(R^L) / \hat{b}(R^L) \in (0, 1)$, which implies $R^L > R^I$ and

$$R^D = \left[\frac{\eta(R^D)}{1 + \eta(R^D)} \right] R^I \quad (22)$$

where $\eta(R^D) \equiv R^D \hat{s}'(R^D) / \hat{s}(R^D) > 0$. Equations (21) and (22) are the standard markup (markdown) conditions for monopoly pricing. If the elasticities of loan demand to the loan rate $\chi(R^L)$ and deposit supply to the deposit rate $\eta(R^D)$ are relatively insensitive to their respective interest rates, then lending and deposit rates are roughly proportional to the policy rate R^I . The only exception to this would be for when R^I is sufficiently low to make the lower bound on deposit rates bind. Let R_0^D denote the solution to (22) and assume $R_0^D > 1$.

Let R^C denote the CBDC interest rate. The introduction of CBDC means that banked workers now have an option to hold a bank account outside the private bank sector. As I assume that the fixed cost ϕ associated with opening a bank account is the same for private and public sector accounts, depositors in this model will choose the account that offers the highest rate of return. If $R^C < R_0^D$, the existence of CBDC is irrelevant. If $R^C > R_0^D$, the existence of CBDC poses a competitive threat to the monopoly bank. Note that as long as $R_0^D < R^C < R^I$, the monopoly bank has a strict incentive to match the CBDC rate to retain its deposit base, since when $R^D = R^C$ the profit margin $[R^I - R^C]$ in (18) remains strictly positive.

Lemma 6 *If $R_0^D < R^C < R^I$, then $R^D = R^C$; that is, it is optimal for the monopoly bank to match the CBDC deposit rate.*

Note that while the monopolist bank is compelled to match the CBDC when $R_0^D < R^C < R^I$, it is evident from the monopolist's choice problem above that the higher cost of deposit funding does not affect the optimal lending rate. In particular, from condition (21), we see that the profit-maximizing lending rate is independent of the deposit rate.

Lemma 7 *If $R_0^D < R^C < R^I$, then the profit-maximizing lending rate R^L is independent of the CBDC rate R^C .*

There remains the question of how monopoly bank behavior is affected if the CBDC rate is (for whatever reason) set above the IOR rate. If $R^C > R^I$, then the bank would make a loss on deposits if it chooses to retain them. Whether the bank chooses to retain deposit funding depends on the cost and availability of alternative funding sources. In reality, one might expect higher-cost non-deposit funding to be available. In addition, private banks could borrow the funds they need from the central bank, if such a lending facility is made available. In the model here, if the monopoly bank can save and borrow at the same rate R^I , then its ability to fund private investments is independent of the availability of deposits. If $R^C > R^I$, then in this case the monopoly bank's balance sheet constraint (16) becomes $\ell + \hat{b}(R^L) = 0$ and its profits are derived solely from its lending operations. Condition (21) continues to characterize the optimal lending rate.

Lemma 8 *If $R^C > R^I$ and if the monopoly bank can borrow reserves at the IOR rate R^I , then the profit-maximizing lending rate R^L is independent of the CBDC rate R^C even as deposit funding vanishes.*

The situation is different, of course, if the central bank either refuses to lend reserves, or stands willing to lend, but at a penalty rate $R^P > R^I$ to form a "corridor" system. In a corridor system, if the CBDC rate satisfies $R^P > R^C > R^I$, the monopoly bank would continue to set its deposit rate to match the CBDC rate (as long as overall profits remain positive). That is, even though the monopoly bank would in this case make a loss on its deposit business, retaining deposits is remains cheaper than borrowing funds from

the central bank (or some other source) at an even higher penalty rate R^P . It is clear in this case that the opportunity cost of funds for the monopoly bank is R^C and not R^I . The optimal lending rate in this case continues to satisfy condition (21), but with R^C replacing R^I . The competitive pressure induced by CBDC in this case is likely to increase the bank lending rate and, hence, by Lemma 1, lead to decline in investment spending.

3.5 Equilibrium

The policy interest rates R^I and R^C are exogenous. An economy without CBDC is equivalent here to an economy with CBDC subject to condition that CBDC yield the same interest as physical currency; i.e., $R^C = 1$. For a given policy (R^I, R^C) , an equilibrium requires: (i) optimization on the part of entrepreneurs, workers, and the monopoly bank; (ii) market-clearing; and (iii) government budget balance. To be clear in what follows, I assume that $R^I > 1$ represents both a deposit and lending rate for the private bank.

The monopoly bank must choose its funding source. If $R^C < R^I$, it chooses to fund itself through deposits (Lemma 6). If $R^C > R^I$, it chooses to fund itself through borrowed reserves (Lemma 8). Either way, the equilibrium lending rate R^L faced by entrepreneurs depends only on R^I . The equilibrium deposit rate R^D depends on whether the CBDC rate R^C constrains the monopoly bank. If $R^C < R_0^D$, the constraint is slack and $R^D = R_0^D$, where R_0^D depends on R^I (see condition 22). If $R^C \geq R_0^D$, the constraint binds and $R^D = R^C$.

Given a structure of interest rates (R^L, R^D, R^I, R^C) , optimal behavior on the part of entrepreneurs implies a private demand for loans $\hat{b}(R^L)$ and planned capital expenditure $\hat{k}(R^L)$ as summarized in Lemma 1. Optimal behavior on the part of workers implies desired saving by banked workers $\hat{s}(R^D)$ and desired saving by unbanked workers $\hat{m}(R^D)$; see equations (14) and (15), respectively. Thus, the demand for outside assets is given by $\Omega(R^I, R^C) \equiv \hat{s}(R^D) + \hat{m}(R^D) - \hat{b}(R^L)$. If $\Omega(R^I, R^C) > 0$, then the equilibrium price-level must satisfy,

$$D/p = \Omega(R^I, R^C) \tag{23}$$

I discuss in Appendix A the conditions under which $\Omega(R^I, R^C) > 0$ and prove the following lemma.

Lemma 9 $\Omega(R^I, R^C)$ is increasing in R^C .

Finally, the government budget constraint (3) must be satisfied. Since the interest rate on currency is zero, we have $R^1 = 1$. Updating the notation, we have $R^2 = R^I$ and $R^3 = R^C$. The demand for reserves is given by $M^2/p = \ell$, where $\ell(R^I, R^C)$ is part of the solution to the monopoly bank problem (18) and (16). In particular,

$$\ell(R^I, R^C) = \begin{cases} \hat{s}(R^D) - \hat{b}(R^L) & \text{if } R^I \geq R^C \\ -\hat{b}(R^L) & \text{if } R^I < R^C \end{cases} \quad (24)$$

where $R^D = R_0^D$ if $R_0^D > R^C$ and $R^D = R^C$ if $R_0^D < R^C$.

Let $M^3/p = \sigma(R^I, R^C)$ denote the real demand for CBDC. Here, we have

$$\sigma(R^I, R^C) = \begin{cases} 0 & \text{if } R^I \geq R^C \\ \hat{s}(R^C) & \text{if } R^I < R^C \end{cases} \quad (25)$$

That is, the demand for deposits is always $\hat{s}(R^D)$. The only question is the determination of R^D and whether these deposits live on the balance sheet of private banks or the central bank.

The lump-sum tax $\tau = (1/2)T/p$ that appears in the budget constraints of entrepreneurs and workers is set to satisfy,

$$\tau = (R^I - 1)\ell(R^I, R^C) + (R^C - 1)\sigma(R^I, R^C) \quad (26)$$

Note that the lump-sum tax here does not feed back into saving or portfolio decisions, given the quasilinear nature of preferences and the assumption that the tax incidence occurs in old age only. The tax does, of course, affect the level of consumption in old age.¹⁰

4 The economic impact of CBDC

As mentioned earlier, an economy without CBDC in this set-up is equivalent to setting $R^C = 1$. Thus, imagine an initial steady-state equilibrium with

¹⁰The equilibrium consumption allocations can be derived by replacing the equilibrium conditions above into individual budget constraints. Given the linearity of preferences in old age, the distribution of monopoly profits does not matter for anything other than welfare. I do not report results on consumption or welfare as my analysis is focused on other matters.

$R^C = 1$ and $R_0^L > R^I > R_0^D > 1$. The analysis compares this steady-state to another in which CBDC exists. Assume that the CBDC regime entails policy interest rates such that $R^I > R^C > R_0^D$ unless otherwise specified.

The first, and perhaps most provocative result, is that introducing a CBDC that is technologically on par with bank deposits will have no effect on the ability of banks to fund profitable investments. Specifically, Lemma 7 states that CBDC will have no effect on the profit-maximizing lending rate. Combining this result with Lemma 1 implies that there is no effect on the demand for credit, so that desired investment spending remains unchanged.

This rather stark result depends on a number assumptions, including the conduct of central bank policy. First, it assumes that central bank policy is implemented through an interest rate rule R^I . This seems realistic. Second, it assumes that the CBDC rate R^C does not interact with the central bank interest rate policy rule. This seems the proper thought experiment, since R^C would otherwise only have an effect on investment through its effect on R^I . Third, for the case in which $R^C > R^I$, I assume that private banks can borrow all the reserves they need at the policy rate R^I . This assumption is unrealistic. In reality, banks would not be able to finance their entire loan portfolio with reserves or with alternative non-deposit funding at the same rate of interest. On the other hand, the assumption that $R^C > R^I$ also seems unrealistic, so perhaps this is a case we need not worry about. The result also depends on the lack of binding liquidity or capital regulations. I return to this issue below.

The second important result states that as long as the CBDC rate is set below the IOR rate, the monopoly bank has every incentive to match the CBDC rate for the purpose of retaining deposits. This is because the bank lends to the central bank at IOR and only pays depositors the CBDC rate. As long as this IOR-CBDC spread remains positive, it pays for banks to retain deposits. Of course, this has the effect of squeezing monopoly profit margins. In equilibrium, the take-up rate for CBDC is zero, so that even the threat of a CBDC or statutory minimum deposit rate accomplishes the same thing here.

In reality, one would expect at least some take-up of the CBDC option and, indeed, most people would probably hold both private and public sector accounts. The result here hinges on my assumption that private and public payment systems share the same underlying technology and that the cost

of accessing or operating this technology (as measured by ϕ) is the same. In reality, systems may differ across sectors, with each system tailored to specific constituencies or for specific purposes and each system with its own particular advantages and disadvantages.¹¹ We should also keep in mind that I have modeled competition between private and public sector as occurring solely through the deposit rate. In reality, banks offer their clients an array of services, so that increased competition may not come in the form of higher deposit rates but rather as improved customer service.

A third important result is that because CBDC induces more favorable contractual terms for depositors, it increases the demand for deposits. In the model, this occurs both through an intensive margin (existing depositors are encouraged to save more) and through an extensive margin (unbanked individuals are encouraged to pay the fixed cost of accessing the bank sector). Thus, far from diminishing the demand for bank sector deposits, the competitive pressure exerted by CBDC could actually end up expanding the bank sector's depositor base. The increased depositor base does not, however, increase bank profits since the profit margin $[R^I - R^C]$ declines by more than the increase in deposits.¹² Hence, CBDC is predicted to decrease monopoly bank profits.

The increased demand for deposits just described is associated with a decline in the demand for physical currency. There has been a long-standing question of what a cashless society might imply for central bank control of monetary policy and the determination of the price-level. Costa and De Grauwe (2001) argue that central banks would lose their traditional instruments of monetary policy. Engert, Fung and Hendry (2018), on the other hand, see no major consequences. The model developed here supports the latter conclusion. In particular, there is nothing that prevents the central bank from using standard interest rate policy in a cashless economy. When accessing deposits is costless ($\phi = 0$), cash here is driven out of circulation. But the only effect is to alter the composition of the outstanding public debt

¹¹For example, the ACH system used by U.S. banks is a net settlement system that is slow but useful for handling recurring payments like company payroll. In contrast, the Fedwire system operated by the Federal Reserve to clear interbank payments is a real-time gross-settlement system. The CBDC option discussed here would provide the retail sector with a Fedwire-like service, but would not necessarily displace an existing net settlement system.

¹²This has to be the case since the monopoly bank is maximizing profit according to condition (22) before the introduction of CBDC.

D from currency to deposits.¹³ The price-level continues to be determined by D and not by its composition; see condition (23).

Finally, the model predicts that CBDC is likely to exert a disinflationary force, at least, in the short run. Specifically, the effect of increasing R^C is to increase the demand for real outside assets; see Lemma 9. The market-clearing condition (23) then implies that for fixed level of outside assets D , the price-level must decline. The long-run rate of inflation, however, continues to be governed by the growth rate of D .

4.1 Liquidity regulations

The model above predicts that CBDC will have no effect on bank lending rates or on bank lending activity; see Lemma 7. The only regulatory constraint on my model bank is that deposit liabilities must be made redeemable on demand for government money (in the model, this takes the form of physical currency, reserves, and CBDC). However, there are no liquidity restrictions or capital requirements. How might the existence of these balance sheet constraints affect the result reported in Lemma 7?

Let us consider the effect of a liquidity regulation. A classic asset-side restriction takes the form of a minimum reserve requirement. Today, banks are subject to the Basel III liquidity-coverage-ratio (LCR) requirement that has the effect of increasing the regulatory demand for reserves (and other high-quality liquid assets). In the United States, global systemically important banks (GSIBs) are also compelled to hold reserves to facilitate orderly resolution in the event of bankruptcy. The analysis above assumes that such liquidity constraints are either absent or do not bind. But what if they do?

In the context of the model above, an LCR-like restriction can be modeled as the following constraint,

$$\ell \geq \lambda \hat{s}(R^D) \tag{27}$$

where $0 \leq \lambda \leq 1$ is a policy parameter specifying the minimum reserve-to-deposit ratio. Combining (27) with the balance sheet constraint (16) permits

¹³In the model developed here, $\phi = 0$ corresponds to a cashless economy in the sense that cash will not be held in equilibrium. This is similar to Lagos and Zhang (2018), who point out that an economy operating at the cashless limit is not the same as an economy without cash.

us to rewrite this liquidity constraint as,

$$(1 - \lambda)\hat{s}(R^D) \geq \hat{b}(R^L) \quad (28)$$

The monopoly bank problem is now to maximize profit (17) subject to the balance sheet constraint (16) and the reserve requirement (28). Let $\xi \geq 0$ denote the Lagrange multiplier for the reserve requirement (28). In what follows, I assume that $R^I > R^D = R^C$. The monopoly bank problem may therefore be stated as choosing the lending rate that maximizes,

$$[R^L - R^I]\hat{b}(R^L) + [R^I - R^C]\hat{s}(R^C) + \xi \left[(1 - \lambda)\hat{s}(R^C) - \hat{b}(R^L) \right] \quad (29)$$

The profit-maximizing lending rate satisfies,

$$[R^L - R^I]\hat{b}'(R^L) + b(R^L) = \xi\hat{b}'(R^L) \quad (30)$$

If the constraint is slack ($\xi = 0$) then (30) corresponds to (19). But if the constraint binds ($\xi > 0$), then the constrained-efficient lending rate satisfies,

$$(1 - \lambda)\hat{s}(R^C) = \hat{b}(R^L) \quad (31)$$

Since the supply of deposits $\hat{s}(R^C)$ is increasing in R^C by Lemma 5 and since the demand for loans $\hat{b}(R^L)$ is decreasing in R^L by Lemma 1, it follows from (31) that an increase in R^C here leads to a *decrease* in the lending rate R^L and a corresponding *increase* in the volume of bank lending activity.

The force of Lemma 7 is to suggest that a CBDC need not lead to a contraction in bank lending. The analysis here suggests that one might even expect banks to lower their lending rates and expand their lending activity if they are subject to binding liquidity regulations. The mechanism is as follows: (1) an increase in the CBDC rate leads banks to increase their deposit rates; which (2) increases the supply of deposits; which (3) permits liquidity constrained banks to expand their lending activity; which (4) can only be accomplished by lowering the market lending rate.¹⁴

¹⁴Assuming that capital is more expensive than deposits, a capital requirement in this model simply serves to increase the overall cost of funding. It would not affect the bank's optimal lending decision, since this would continue to be determined at the margin by the IOR rate.

5 Financial stability

While the model above does not explicitly incorporate the possibility of bank runs, it would be easy to extend the analysis in the manner of Andolfatto, Berentsen and Martin (2019). In what follows, I use the model developed above in combination with this latter paper as an heuristic device to evaluate concerns over the potential for CBDC to destabilize money markets.

Cecchetti and Schoenholtz (2017), for example, suggest that CBDC may render bank deposits a less stable form of funding, with unsophisticated depositors prone to moving their money out of private banks into CBDC at the first sign of financial distress, perhaps leading to a self-fulfilling bank crisis (Diamond and Dybvig, 1983 and Bryant, 2005). Of course, as they acknowledge, such instruments are already available in the form of cash and treasury debt. The main difference with CBDC is its apparent superiority and widespread availability as a “flight to safety” vehicle. The run-inducing incentives put in place by CBDC would, by their reckoning, require an heroic expansion of lending by the central bank in a financial crisis. Kumhof and Noone (2019) also note that while bank runs have always been possible with currency, the electronic nature of CBDC potentially makes run phenomena much easier and hence, more likely.

It is worthwhile pointing out, however, that similar concerns were expressed when the Federal Reserve Bank of the United States implemented its overnight reverse repo (ON RRP) facility in 2014; see Cecchetti and Schoenholtz (2014). Happily, the financial instability risks feared at that time have not materialized and, indeed, activity on the facility has now ceased as higher returns are available elsewhere.¹⁵ As a theoretical matter, run risk against the ON RRP facility could be mitigated by sharply lowering the ON RRP interest rate in the event of a heavy inflow of funds. As Kumhof and Noone (2019) point out, the same property can be embedded in the CBDC rate. These authors also stress that stability would require abandoning any firm commitment to redeem CBDC for reserves at a fixed exchange rate. This property is reminiscent of the recent regulatory reforms in U.S. money markets that permit the managers of prime money market funds to apply liquidity fees and impose redemption gates at their discretion in the event of heavy

¹⁵ Cochrane (2015) provides a sober assessment of the financial stability risks associated with the Fed’s ON RRP facility.

redemptions. On top of this, stability could be guaranteed by the central bank itself, standing ready to serve as a lender-of-last-resort.¹⁶

Given that the CBDC experiment has not yet been tried, there is no direct empirical evidence to answer the question of financial stability or its likely impact on banking in general. There is, however, some indirect evidence worth considering. Grodecka (2019), for example, examines how the Bank of Canada, at its establishment in 1935, slowly replaced existing private banknotes with its own issuances. Her analysis suggests that banks were impacted negatively at the time and that some banks reduced the share of loans in their portfolios. It is doubtful, however, that anyone views that intervention as crippling Canadian banks today. More importantly, Canadian banks are widely considered to be the paragon of stability. One might also draw on the experience of postal savings system. It is clear from Schuster, Jaremski and Perlman (2016), for example, that U.S. banks did not look favorably on the U.S. Postal Savings System (1911-67). Prior to federal deposit insurance, it appears that the latter facility was used as a flight-to-safety vehicle, leading to measurable outflows of deposits from commercial banks. While small deposits are presently insured, large deposits are not, suggesting that owners of large-value deposits may view CBDC as an attractive run option. On the other hand, it is possible that better designed deposit contracts—embedding the liquidity fees and redemption gates described above, for example—could serve to mitigate this risk.

Related to this issue is the question of how a fully-insured CBDC facility might impact the wholesale banking sector. Overnight repo arrangements collateralized with safe securities like U.S. treasuries are attractive relative to bank deposits for large value transactions for at least two reasons. First, government insurance on deposit accounts is typically capped at a relatively small amount (\$250,000 in the United States). Second, even where deposits are insured, the money may not be immediately available if a bank is experiencing financial difficulties. Since CBDC accounts would be free of these risks, corporate money managers may be inclined to move out of repo markets and into CBDC. To the extent that they do, the effect is likely to alleviate the demand for safe assets (Grey, 2019) and render money markets more stable.

¹⁶Andolfatto, Berentsen and Martin (2019) explain how psychologically-driven bank runs are theoretically eliminated through the use of nominal deposit contracts and a lender-of-last-resort.

6 Conclusion

The analysis above suggests the main benefit of CBDC will accrue to depositors in jurisdictions where banks (and other money services businesses) use their market power to keep deposit rates depressed (or service fees elevated) relative to what would prevail in a more competitive setting. The model predicts that CBDC is likely to increase financial inclusion and diminish the use of cash, though the quantitative magnitude of this effect is likely to depend on program parameters and the existing degree of financial development. It also suggests that CBDC need not have a negative impact on bank lending operations if the central bank follows an interest rate policy rule. Finally, I conclude on the basis of theory and evidence that a well-designed CBDC is not likely to threaten financial stability.

These conclusions are the implications that follow from a highly abstract and provisional model and so should naturally be viewed with due caution. The model abstracts from several important real world considerations, including the absence of risk and moral hazard, to name just two factors. On the other hand, it is not immediately clear how incorporating these considerations would render the conclusions above directionally incorrect. Only further research can answer this question.

7 Appendix A

For debt to be valued in this environment, we need $\Omega(R^I, R^C) \equiv \hat{s}(R^D) + \hat{m}(R^D) - \hat{b}(R^L) > 0$. Assume, for the moment, that $R^C = 1$. Note that by (14) and (15) we can express the desired saving of workers as,

$$\hat{s}(R^D) + \hat{m}(R^D) = \int_{\hat{y}(R^D)}^{\bar{y}} \hat{x}^b(y, R^D) dH(y) + \int_0^{\hat{y}(R^D)} \hat{x}^u(y) dH(y)$$

Lemma 2 implies that this expression is increasing in the deposit rate R^D . Lemma 1 implies that the demand for funds by entrepreneurs is decreasing in R^L . The markup conditions (21) and (22) suggest that lending and deposit rates are increasing in the policy rate R^I . Hence, demand for outside assets $\Omega(R^I, 1)$ is increasing in R^I , so there is a value for R^I that satisfies $\Omega(R^I, 1) > 0$. The interest expense associated with this interest rate policy can be financed with the lump-sum tax instrument; see (26). If the non-negativity constraint on old-age consumption for poor workers binds, the burden of the tax can be diverted to other agents in the economy.

Consider now the effect of increasing R^C . At some point, $R^D = R^C$, as described above. Since desired saving for workers is increasing in the deposit rate, it follows that an increase in R^C at this point serves to increase desired saving. By Lemma 7, R^C has no effect on the demand for funds by entrepreneurs. Hence, $\Omega(R^I, R^C)$ is increasing in R^C , as reported in Lemma 9.

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