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Majority Voting in a Model of Means Testing*

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Abstract

We study a model of endogenous means testing where households differ in their income and where the in-kind transfer received by each household declines with income. Majority voting determines the two dimensions of public policy: the size of the welfare program and the means-testing rate. We establish the existence of a sequential majority-voting equilibrium and show that the means-testing rate increases with the size of the program but the fraction and the identity of the households receiving the transfers are independent of the program size. Furthermore, the set of subsidy recipients does not depend on households' preferences, but depends only on income heterogeneity.

JEL codes: D70, D72, H20
Keywords: Sequential majority voting, Means testing, Political support, Targeting

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1 Introduction

Many public expenditures in developed economies are in the form of in-kind transfers. Currie and Gahvari (2008) document that public expenditures on health, housing, child care, and education account for almost 11 percent of GDP in Australia, 13 percent in the United States and Germany, 14 percent in the United Kingdom, and 17 percent in Denmark (Table 1, p. 335). Often such transfers are means-tested. Examples in the United States include Medicaid, food stamps, housing vouchers, education vouchers, Pell grants, and child care subsidy programs such as the Child Care Development Fund (CCDF). CCDF provides assistance based on an income-dependent sliding scale; food stamps provided by the Supplemental Nutrition Assistance Program decline by 30 cents for each additional dollar of net monthly income. Epple, Romano, and Urquiola (2017) provide evidence that most of the educational voucher programs in the United States are income-targeted.

In this paper, we develop a model of means testing where the size of the program and the degree of means testing are determined through majority voting. In our model, there is a continuum of households. Each household optimizes over a consumption good and a subsidy good such as health care, food, or education. Households differ in their (endowed) income, which is the only source of heterogeneity. Government provides subsidies that decline linearly with income. These subsidies are in-kind transfers—i.e., they can be used to buy only the subsidy good. The subsidies are financed by a flat tax on income. Households can supplement the subsidy by purchasing the good in a competitive market.

The sole purpose of public policy in our model is to redistribute the endowments via in-kind transfers.[1] The public policy is two-dimensional. One dimension is the budget for the welfare

[1] See Glomm, Ravikumar, and Schiopu (2011) for a survey of the political economy of an in-kind transfer, educational expenditures, and Bellani and Ursprung (2016) for a survey of the political economy of redistributing endowments.

program—the tax rate. The other dimension is the rate at which the subsidy declines with income—the means-testing rate. We use *sequential* majority voting to determine political outcomes. Sequential voting helps us avoid the well-known existence problems associated with multidimensional majority voting; see Plott (1967) and Ordeshook (1986).[2] We follow the sequence in Alesina, Baqir, and Easterly (1999) and De Donder, Le Breton, and Peluso (2012): Households first vote on the tax rate and then vote on the means-testing rate. When households vote on the tax rate, they anticipate the *majority decision rule* for means testing.[3]

First, we show that households' preferences over the policy are single-peaked at each stage. This allows us to invoke the median voter theorem of Black (1958) and prove the existence of a majority-voting equilibrium in a standard manner.[4] Second, at the stage of voting on the means-testing rate, the decisive voter is the household with the median income. Conditional on the tax rate, the means-testing rate does not depend on preferences over the two goods. Third, even though a universal subsidy (i.e., means-testing rate equals zero) is feasible, it is not chosen by the majority. However, the majority-preferred set of subsidy recipients includes more than 50 percent of the households. None of these results rely on any unusual properties of preferences or income distribution. Fourth, at the stage of voting on the tax rate, the median-income household is decisive when the elasticity of substitution between the subsidy good and other goods is greater than 1, while the decisive household is below the median when the elasticity is less than 1. While similar results exist in one-dimensional voting problems (see Epple and Romano, 1996; Gouveia, 1997; Glomm and Ravikumar, 1998;

[2] Epple, Romano, and Sarpca (2018) use a representative democracy model to solve a multidimensional public choice problem with two-dimensional heterogeneity.

[3] In Alesina et al. (1999) and De Donder et al. (2012), the budget is determined first and the distribution of the budget is determined next. Alesina et al. (1999) note that the order of voting where tax rates are voted on first "resembles common budget procedures in which the size of the budget is decided before its composition." Alesina and Perotti (1999) motivate this voting sequence by procedures in the United States Congress since the Budget Act of 1974.

[4] In our model, each voter is of measure zero so non-existence issues related to sequential voting by strategic voters illustrated in Ordeshook (1986), Chapter 6, do not arise.

Bearse, Cardak, Glomm, and Ravikumar, 2013), our two-dimensional setup is complicated by the fact that the indirect utility over taxes contains an endogenous majority decision rule for means testing that is a *function* of the tax rate. Finally, our sequential-voting equilibrium is also a Shepsle (1979) equilibrium: Our equilibrium means-testing rate and tax rate are such that each one is a majority-voting equilibrium given the other.

Our model implies a separation theorem: The majority-preferred size of the welfare program in the first stage does not affect who gets the means-tested subsidies in the second stage. That is, the fraction and the identities of households that receive the subsidy are independent of the size of the program. (i) The separation occurs despite the fact that households take into account the second-stage outcome when they are voting in the first stage. (ii) Our result is robust to alternative means-testing specifications where the subsidies decline nonlinearly with income. (iii) Combined with the result that the means-testing rate does not depend on households' preferences, our separation result implies that cross-economy differences in the set of subsidy recipients must be due to differences in income heterogeneity.

Our separation result does *not* imply independence across the two stages of voting. First, given an arbitrary set of subsidy recipients, the majority-preferred size of the program would be different from the one that would prevail if the subsidy recipients were chosen via majority voting. Thus, exogenous targeting affects the endogenous size of the program, similar to Gelbach and Pritchett (2002) and De Donder and Hindriks (1998). Second, the size of the program affects the subsidy amount received by each household even though the entire subsidy is distributed among the same households. Thus, unlike Cremer, De Donder, and Gahvari (2004), the first-stage vote influences the second-stage outcome on subsidies.

On the broader issue of targeting and political support for redistribution, the relationship between the two (endogenous) variables in our model has two noteworthy features. First, at least half the population receives the subsidy. We never reach a critical level of targeting such that the political support collapses.[5] Second, as the set of subsidy recipients shrinks, the size of the pie to be redistributed increases.

The structure of our paper is as follows. In Section 2, we develop our model of means testing. In Section 3 we characterize the sequential majority-voting equilibrium. We establish our separation theorem in Section 4. In Section 5, we use functional forms for preferences and income distribution and provide testable implications. Concluding remarks are in Section 6. Proofs are relegated to the Appendix.

2 Model

The economy is populated by a continuum of households. We normalize the size of the population to 1. Households differ only by endowed income, y, which is distributed according to the c.d.f. F (p.d.f. f). The p.d.f. is assumed to be continuously differentiable. The support of F is $\mathbb{R}_+$, and mean income, Y, exceeds median income, y_m. We label households by their income and refer to a household with income y as "household y."

Households derive utility from a numeraire consumption good c, or "consumption" for short, and another good d. We will refer to good d as the subsidy good. The common utility function is $u(c,d)$, which is strictly increasing in both arguments, strictly quasiconcave, and twice continuously differentiable. We also impose the following boundary condition:

Assumption 1 *For $c_1 > 0$, $d_1 > 0$, $c_2 \geq 0$, and $d_2 \geq 0$,*

$$u(c_1, d_1) > \max\{u(c_2, 0), u(0, d_2)\}.$$

[5] De Donder and Peluso (2018) show that, with exogenous transfers, targeting transfers to less than 50 percent of the population can be sustained by majority voting if the voters view the transfers as stochastic.

The markets for c and d are assumed to be perfectly competitive with a large number of producers facing identical technologies exhibiting constant marginal and zero fixed costs. We measure units of d such that its price is one unit of consumption. Both consumption and the subsidy good are assumed to be normal goods.

The government collects a tax on income at the rate $\tau \in [0, 1]$. Total tax revenue is τY and is used to finance the subsidy good d. The provision of the subsidy good to the households is means tested in the sense that the in-kind transfer received by the household depends negatively on income. Formally, the transfer to household y is given by

$$s(y; \alpha, \beta) = \max\{\alpha - \beta y, 0\}, \quad \alpha \geq 0, \ \ \beta \geq 0. \tag{1}$$

Under the specification (1), a higher β implies more severe means testing, while $\beta = 0$ implies a universal subsidy. For strictly positive α and β, there is an income threshold $(\frac{\alpha}{\beta})$ above which the subsidy is zero. (We use "subsidy" and "transfer" interchangeably to refer to the in-kind public provision of the good d.)

We assume that the government runs a balanced budget; i.e.,

$$\int_0^\infty s(y; \alpha, \beta) f(y)\, dy = \tau Y.$$

Since the subsidy is 0 for households $y \geq \frac{\alpha}{\beta}$, we can write the balanced budget as

$$\alpha F\left(\frac{\alpha}{\beta}\right) - \beta \int_0^{\frac{\alpha}{\beta}} y f(y)\, dy = \tau Y. \tag{2}$$

We refer to (2) as the Government Budget Constraint (GBC). Let $\widetilde{\alpha}(\tau, \beta)$ be the value of α satisfying (2), given (τ, β).

The collective choice problem is to determine α, β, and τ via sequential majority voting subject to the GBC. Before solving the collective choice problem we solve each household's optimization problem taking the collective choice variables as given.

2.1 Household Optimization

Each household treats α, β, and τ as given and chooses the pair (c, d) so as to maximize utility $u(c, d)$ subject to the budget constraint

$$c + d \leq (1 - \tau)\, y + s\,(y; \alpha, \beta)\,, \quad c \leq (1 - \tau)\, y. \tag{3}$$

Denote the optimal choices of household y by $\widehat{c}\,(y; \alpha, \beta, \tau)$ and $\widehat{d}\,(y; \alpha, \beta, \tau)$ and the indirect utility of household y by $V\,(y; \alpha, \beta, \tau) \equiv u\left(\widehat{c}\,(y; \alpha, \beta, \tau)\,, \widehat{d}\,(y; \alpha, \beta, \tau)\right)$. Note that there are no secondary markets where the subsidy good can be sold after the transfer. The presence of such markets will essentially transform the in-kind transfers to cash transfers and consumption will not be bounded by the after-tax income in equation (3).

Remark 1 *When $\alpha = 0$ or $\beta = \infty$, no household obtains a subsidy and all expenditures are privately financed. When $\tau = 0$, the GBC requires $\alpha = 0$; no household receives a subsidy. When $\tau = 1$, equations (2) and (3) imply that all households get zero consumption; Assumption 1 rules this out as a potential equilibrium.*

Household y supplements its subsidy if and only if

$$R\,(y) \equiv \frac{u_1\,((1 - \tau)\, y, s\,(y; \alpha, \beta))}{u_2\,((1 - \tau)\, y, s\,(y; \alpha, \beta))} < 1,$$

where the subscripts denote partial derivatives of u. In other words, the income has to exceed a critical level for the in-kind transfer to be supplemented. We further restrict preferences to ensure that there exist low income households who do not supplement their subsidy as well as rich households who do supplement their subsidy.

Assumption 2 *For all $\alpha > 0,\ \beta \in (0, \infty)$, and $\tau \in (0, 1)$,*

$$\text{(i)} \quad \lim_{y \searrow 0} R(y) > 1, \quad \text{and,}$$

$$\text{(ii)} \quad \lim_{y \nearrow \infty} R(y) < 1.$$

The voting problem involves two variables, τ and β. Once these are determined, the value of α is pinned down by GBC (2). We determine the pair (τ, β) through majority voting in two stages. In the first stage, individuals vote on the tax rate τ anticipating how the means-testing rate β will be chosen in the second stage and how β might depend on τ. In the second stage, β is voted on taking τ from the first stage as given.

Definition 1 *A* ***sequential majority-voting equilibrium*** *is an allocation (c, d) across households and a public policy (α, β, τ) satisfying:* ***(i)*** *Each household's choice of (c, d) is individually rational given public policy (α, β, τ);* ***(ii)*** *given τ, β is a majority winner in the second stage;* ***(iii)*** *anticipating how τ affects the majority winner β, τ is a majority winner in the first stage; and* ***(iv)*** *the government runs a balanced budget; i.e., $\alpha = \widetilde{\alpha}(\tau, \beta)$.*

We define majority voting in the usual sense of binary comparisons between all pairs of policies. We treat voters as sincere in that they will vote for the policy that yields a higher utility in any binary comparison between policies. We say that a policy is a *majority winner* at each stage if and only if no other policy satisfying the GBC at that stage is strictly preferred by a strict majority of the population.

3 Majority-voting equilibrium

As noted earlier, the households in our model have to collectively choose two policy variables—τ and β—sequentially. In this section, we establish the existence of a majority-voting equilibrium and characterize the equilibrium.

3.1 Second Stage: Voting Over Means-Testing Rate

At this stage, the households have already voted on a tax rate τ and the problem is to choose a means-testing rate β. To this end, we have to determine the majority-preferred β for each τ. Household y chooses β to maximize its indirect utility subject to GBC (2). Formally, given $\tau \in (0,1)$, the optimal β is $\arg\max \ V\left(y; \widetilde{\alpha}\left(\tau,\beta\right), \beta, \tau\right)$.

Remark 2 *When $\tau = 0$, there are no subsidies and the choice of β is irrelevant. The case of $\tau = 1$ as an equilibrium is ruled out by Assumption 1.*

The following lemma establishes properties of the GBC that are helpful in solving the decisive voter's optimization problem.

Lemma 1 *(Concavity of GBC) At any given tax rate τ, $\widetilde{\alpha}\left(\tau,\beta\right)$ is monotonically increasing and strictly concave in β, with a maximum gradient of $\frac{\partial\widetilde{\alpha}(\tau,\beta)}{\partial\beta} = Y$ at $\beta = 0$.*

Given τ, the size of the pie to be redistributed is fixed. The GBC is illustrated in the (α, β) space in Figure 1. Since our means-testing formula bestows larger subsidies to poorer households, no household above mean income Y will benefit from this formula and all such households will prefer $\beta = 0$. Thus, political support for a positive means-testing rate must come from households with incomes less than Y.

To understand the trade-offs faced by each voter receiving a subsidy, consider an arbitrary household y whose income is less than Y. Suppose that household y is constrained by the subsidy; i.e., its consumption is $(1-\tau)y$ and its expenditure on the subsidy good is $\alpha - \beta y$. Then, on the margin, any increase in β has to be offset by an increase in α to keep this household indifferent. More precisely, every unit increase in β would require an increase in α by y units. So, in the (α, β) space, the slope of the indifference curve for household y is y.

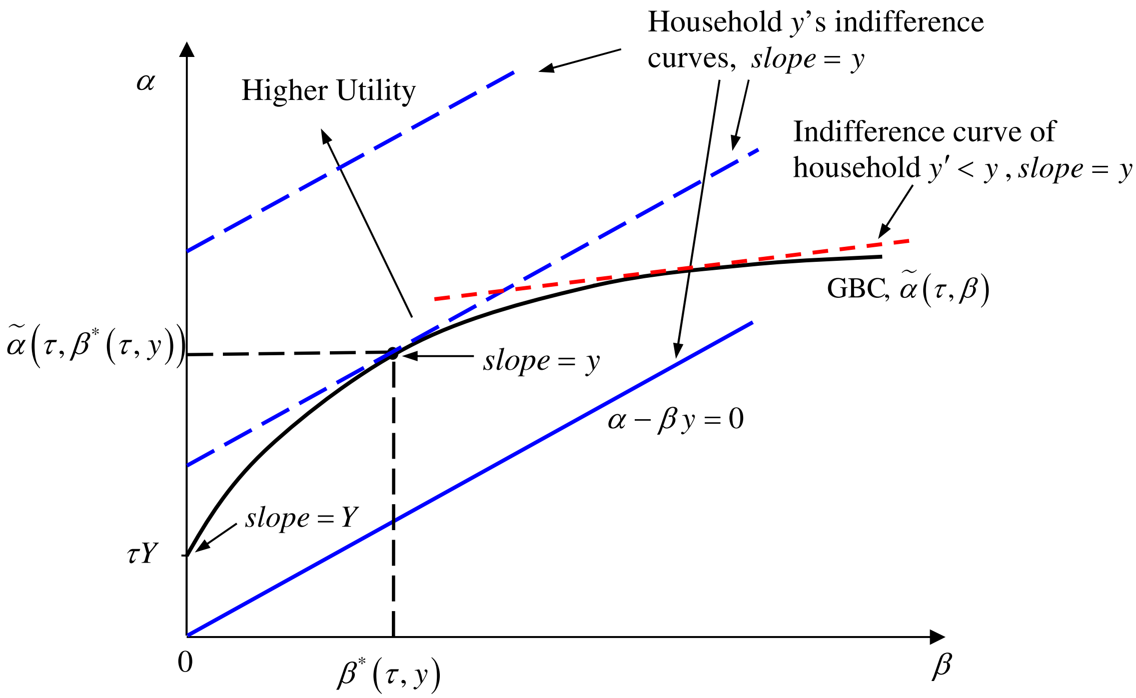

Figure 1: Government Budget Constraint and indifference curves of households

Now suppose that household y is not constrained by the subsidy amount, in which case the household will optimally allocate its resources $(1-\tau)y + \alpha - \beta y$ between consumption and the subsidy good. (Recall that the optimal choices are denoted by $\widehat{c}$ and $\widehat{d}$.) On the margin, a unit increase in β decreases this household's utility by $y \times \left\{ u_1 \left(\widehat{c}, \widehat{d}\right) + u_2 \left(\widehat{c}, \widehat{d}\right) \right\}$, whereas a unit increase in α increases this household's utility by $\left\{ u_1 \left(\widehat{c}, \widehat{d}\right) + u_2 \left(\widehat{c}, \widehat{d}\right) \right\}$. To keep the household indifferent, α has to increase by y units for every unit increase in β. Again, in the (α, β) space, the slope of the indifference curve for household y is y. Thus, we have

Proposition 1 *(Majority-preferred means testing) The household with the median income y_m is the decisive voter on the means-testing rate β. Given $\tau \in (0,1)$, $\widehat{\beta}(\tau)$ is the majority* *winner, where $\widehat{\beta}(\tau)$ solves*

$$y_m = \frac{\partial \widetilde{\alpha}(\tau, \beta)}{\partial \beta}.$$

The preferred (α, β) for household y is where its indifference curve is tangent to the GBC, denoted by $\beta^*(\tau, y)$ and $\widetilde{\alpha}(\tau, \beta^*(\tau, y))$ in Figure 1. The tangency point $\Rightarrow \alpha - \beta y > 0$.

Proceeding along the same lines, a poorer household $y' < y$ would have a flatter indifference curve in the (α, β) space and, hence, would prefer a higher (α, β) pair. Thus, the preferred means-testing rate is a decreasing function of income—richer households prefer a lower β, with households $y \geq Y$ preferring $\beta = 0$. This monotonicity helps us establish that there exists a majority-voting equilibrium in the second stage and that household y_m is the decisive voter.

Remark 3 *The second stage voting outcome does not depend on preferences. This is because, in the second stage, households want to maximize the transfer they receive, whether they are constrained on the subsidy-good expenditures or not. In the (α, β) space, the slope of each household's iso-transfer curve is its income. The parameters of the means-testing function are completely pinned down by the income distribution and the tax rate. The equilibrium tax rate, however, depends on preferences, as shown in Section 3.2.*

To determine the majority-preferred tax rate, we have to characterize the function $\widehat{\beta}(\tau)$ in Proposition 1; i.e., how does the majority-preferred β change as the tax rate changes? To this end, the following properties of the GBC are helpful.

Lemma 2 *(Properties of GBC) Fix $\tau \in (0,1)$ and let (α, β) be a point on the GBC. Denote this GBC as GBC_1. Consider another GBC_j with tax rate $\tau_j = j\tau \in (0,1)$. Then, (i) the pair $(j\alpha, j\beta)$ is on GBC_j, and (ii) the slope of GBC_j at $(j\alpha, j\beta)$ is the same as the slope of GBC_1 at (α, β).*

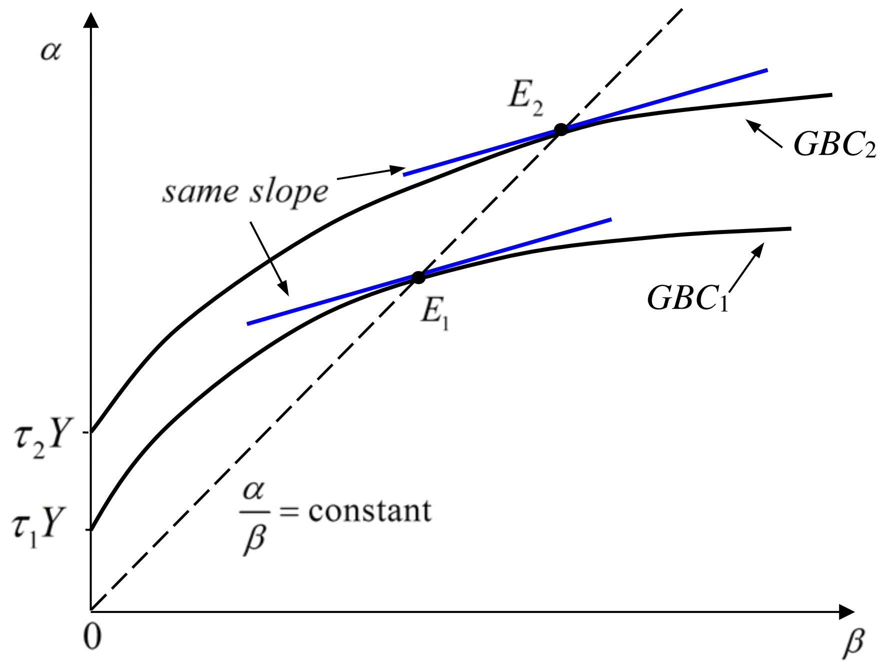

Figure 2: Homogeneity of GBC in (α, β)

Lemma 2 characterizes the set of feasible (α, β) pairs for each τ. For every change in τ, proportionate changes in α and β are in the feasible set, according to part (i). Part (ii) implies if (α, β) was the most-preferred pair for household y on GBC_1, then $(j\alpha, j\beta)$ is its most-preferred pair on GBC_j. See Figure 2. Thus, the functional relation between the tax rate and the *majority-preferred* β can be characterized as follows.

Proposition 2 *(Decision rule for the majority-preferred β and α) For all $\tau \in (0, 1)$,*

$$\widehat{\beta}(\tau) = k_\beta \cdot \tau, \quad \textit{for some constant } k_\beta > 0.$$

Furthermore, the unique $\widehat{\alpha}(\tau) \equiv \widetilde{\alpha}(\tau, k_\beta \tau)$ that satisfies the GBC can be written as

$$\widehat{\alpha}(\tau) = k_\alpha \cdot \tau, \quad \textit{for some constant } k_\alpha > 0.$$

Note that in the second stage we have to characterize the collective choice of β for every feasible tax rate, not just the equilibrium tax rate. This characterization helps us determine the equilibrium tax rate in the first stage.

As noted in Remark 3, the constants k_α and k_β do not depend on preferences. Furthermore, the GBC implies

$$k_\alpha \tau F\left(\frac{k_\alpha}{k_\beta}\right) - k_\beta \tau \int_0^{\frac{k_\alpha}{k_\beta}} y f(y)\, dy = \tau Y.$$

That is,

$$k_\alpha F\left(\frac{k_\alpha}{k_\beta}\right) - k_\beta \int_0^{\frac{k_\alpha}{k_\beta}} y f(y)\, dy = Y. \tag{4}$$

3.2 First Stage: Voting Over the Tax Rate

To determine the majority-preferred τ in the first stage, each household takes as given the majority-preferred *functions* $\widehat{\beta}(\tau)$ and $\widehat{\alpha}(\tau)$ and chooses its most-preferred τ. With a slight abuse of notation, household y chooses τ to

$$\max V(y;\tau) \equiv V(y; k_\alpha \tau, k_\beta \tau, \tau). \tag{5}$$

To help establish the existence of a majority-voting equilibrium, we will demonstrate that the households' preferences over τ are single-peaked.

First, it is easy to see that households with income $y \geq \frac{k_\alpha}{k_\beta}$ would prefer a tax rate of zero since they do not receive any subsidy at all. Second, the set of households who prefer a tax rate of zero, in fact, includes some whose income is less than $\frac{k_\alpha}{k_\beta}$. Consider all households who receive non-negative subsidies. The critical household for which the subsidy exactly offsets taxes satisfies: $\alpha - \beta y = \tau y$ or $k_\alpha \tau - k_\beta \tau y = \tau y$, or $y = \frac{k_\alpha}{1+k_\beta}$. For $y > \frac{k_\alpha}{1+k_\beta}$, the household receives less in subsidy than what it pays in taxes, so (a) the most-preferred tax rate of such households is zero, and (b) the household's utility is monotonically declining

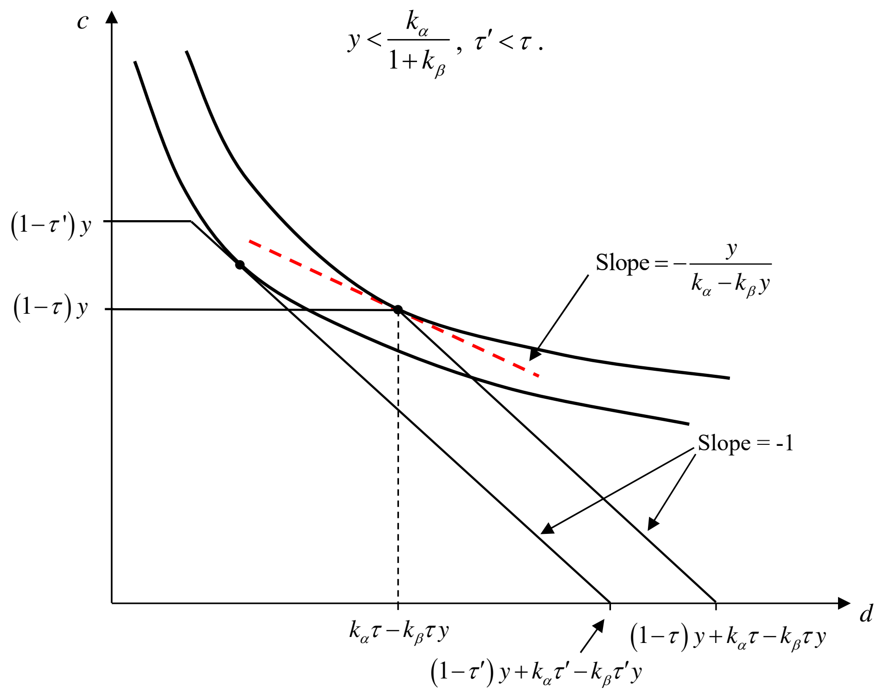

Figure 3: Trade-offs for the decisive voter on tax rates

in τ since the gap between taxes and subsidies—$\tau y - (k_\alpha \tau - k_\beta \tau y)$—is increasing in τ; i.e., the resources available to household y are decreasing in τ. Third, households with $y < \frac{k_\alpha}{1+k_\beta}$ would choose an interior bundle in the budget set for low tax rates, as illustrated in Figure 3. This is because the constraint $c \leq (1-\tau)y$ in (3) does not bind. An increase in the tax rate from low values would, on the margin, result in more transfers relative to the tax payments—$y < k_\alpha - k_\beta y$—and, hence, the households' utility would be increasing in the tax rate. Further increases in the tax rate eventually make the constraint binding, as in Figure 3, and household y faces a trade-off between more overall resources—$(1-\tau)y + (k_\alpha \tau - k_\beta \tau y)$—and tighter constraint on consumption—$c = (1-\tau)y$. Utility would eventually start declining in the tax rate. Thus, all households above and below $\frac{k_\alpha}{1+k_\beta}$ have single-peaked preferences, yielding the following proposition.

Proposition 3 *(Existence: Majority-preferred tax rate) Given* $\widehat{\beta}(\tau)$ *and* $\widehat{\alpha}(\tau)$ *from Proposi-*

tion 2, all households' preferences over τ are single-peaked and, hence, there exists a majority-voting equilibrium tax rate.

Since the households with incomes above $\frac{k_\alpha}{1+k_\beta}$ prefer a zero tax rate, for the equilibrium tax rate to be positive the decisive voter must come from the group $y < \frac{k_\alpha}{1+k_\beta}$ and the group must include a majority of households i.e., $F\left(\frac{k_\alpha}{1+k_\beta}\right)$ must be greater than 0.5. The latter is easy to verify. Recall that household y_m is decisive in the second stage, so the transfers to household y_m must exceed its tax payments: $k_\alpha\tau - k_\beta\tau y_m > \tau y_m$, for all $\tau > 0$. Thus, $k_\alpha > (1+k_\beta)y_m$, or $F\left(\frac{k_\alpha}{1+k_\beta}\right) > F(y_m) = 0.5$.

Denote the decisive voter's income by y_d. The proposition below states the necessary conditions for the majority-voting equilibrium tax rate to be positive.

Proposition 4 *(Characterization: Majority-preferred tax rate) Suppose that the majority-preferred tax rate is positive. Then, (i) the decisive voter's income $y_d \in \left[0, \frac{k_\alpha}{1+k_\beta}\right]$; (ii) the most-preferred tax rate $\widehat{\tau}(y_d)$ of the decisive voter is such that he is constrained—i.e., his consumption and subsidy good expenditures are given by*

$$\widehat{c} = (1 - \widehat{\tau}(y_d))\, y_d, \quad \widehat{d} = k_\alpha\widehat{\tau}(y_d) - k_\beta\widehat{\tau}(y_d)y_d; \text{ and}$$

(iii) the most-preferred tax rate $\widehat{\tau}(y_d)$ is the unique solution to

$$\frac{u_2\left(\left(1-\tau\right) y_d, k_\alpha\tau - k_\beta\tau y_d\right)}{u_1\left(\left(1-\tau\right) y_d, k_\alpha\tau - k_\beta\tau y_d\right)} = \frac{y_d}{k_\alpha - k_\beta y_d}. \tag{6}$$

The decisive voter, if unconstrained by the subsidy amount, can make himself better off with a higher tax rate. The marginal gain in resources from an increase in tax rate is $k_\alpha - k_\beta y - y > 0$, which translates into higher c and d. He can increase the tax rate until the constraint binds, and his marginal rate of substitution of consumption for the subsidy good is less than 1, reaching the equality in part (iii) of Proposition 4 (see Figure 3).

What remains to be determined is: Who is the decisive voter? To this end, we focus on how the marginal utility of taxes varies with income, $\frac{\partial^2 V(y;\tau)}{\partial y \partial \tau}$. Since $V(y;\tau)$ is endogenous, we consider a specific utility function to investigate this question. We use a CES functional form for preferences and provide assumptions on the primitives.

Proposition 5 *(Decisive household for the tax rate) Let*

$$u(c,d) = \frac{1}{1-\sigma}\left(c^{1-\sigma} + \delta d^{1-\sigma}\right), \quad \sigma > 0, \;\; \sigma \neq 1, \;\; \delta > 0. \tag{7}$$

(i) If $\sigma < 1$, the decisive voter is the median income household y_m.

(ii) If $\sigma > 1$, the decisive household y_d is implicitly determined by

$$1 - F\left(\frac{k_\alpha}{1+k_\beta}\right) + F(y_d) = 0.5. \tag{8}$$

(iii) If $u(c,d) = ln(c) + \delta ln(d)$, all households $y < \frac{k_\alpha}{1+k_\beta}$ vote for the same positive tax rate.

In each case, the majority-preferred tax rate is a solution to equation (6) where y_d is the income of the decisive voter in that case.

Note that the identity of the decisive household for cases (i) and (ii) depends only on ranges of σ: $\sigma < 1$ or $\sigma > 1$, not on the specific value of σ in these ranges.

For the utility specification (7), the elasticity of substitution between the subsidy good and other consumption goods is $\frac{1}{\sigma}$. In case (i), $\frac{1}{\sigma} > 1$ and the substitutability is high. Consider the extreme case of perfect substitutability. Households in this case will choose τ to maximize resources—$(1-\tau)y + (k_\alpha \tau - k_\beta \tau y)$. Recall that for a given tax rate the tax bill rises and the subsidy falls with income, so the most-preferred tax rate decreases with income and is zero for households with $y > \frac{k_\alpha}{1+k_\beta}$. Thus, the decisive household is y_m, as $F\left(\frac{k_\alpha}{1+k_\beta}\right) > 0.5$. In case (ii), the substitutability is low. Consider the extreme case with Leontief preferences and $\delta = 1$, so the households would want to equalize c and d. Proposition 4 implies $c = (1 - \tau)\, y$ and $d = (k_\alpha - k_\beta y)\, \tau$. Consider household $y' > y$ but less than $\frac{k_\alpha}{1+k_\beta}$. If $c = d$ for household y at tax rate τ, then it must be the case that household y' will have its consumption exceed the subsidy good expenditures at the tax rate τ. To equalize c and d, household y' would prefer a higher tax rate. Thus, the most-preferred tax rate increases with income up to $y = \frac{k_\alpha}{1+k_\beta}$, beyond which zero tax is preferred. As a consequence, the majority-voting equilibrium is supported by the extremes of the distribution of voters, similar to Casamatta, Cremer, and Pestieau (2000), with the decisive voter characterized by equation (8). Case (iii) is the Cobb-Douglas specification and implies constant expenditure proportions for each good. Hence, all households with $y < \frac{k_\alpha}{1+k_\beta}$ have the same preferred tax rate and form the decisive voting coalition, as $F\left(\frac{k_\alpha}{1+k_\beta}\right) > 0.5$.

Knowing the identity of the decisive voter in the first stage from Proposition 5, the equilibrium tax rate (or the size of the welfare program) is pinned down by Proposition 4.

Shepsle equilibrium Our sequential-voting equilibrium is also a structure-induced equilibrium (Shepsle, 1979). To demonstrate this, we examine the (collective) best responses of the two policy dimensions.

Proposition 6 *(i) Given the sequential-voting equilibrium τ, the sequential-voting equilibrium pair $(\alpha,\, \beta)$ in Proposition 2 is the majority-preferred best response.*

(ii) Given the sequential-voting equilibrium numbers k_α and k_β, the sequential-voting equilibrium τ in Propositions 4 and 5 is the majority-preferred best response.

4 Recipients of Means-Tested Subsidies

Proposition 2 implies that the subsidies, not surprisingly, are more generous as the size of the program increases. However, even though the subsidies are more generous, they are distributed only among the households with $y < \frac{k_\alpha}{k_\beta}$. This leads us to our main separation result: The extensive margin or the fraction of households and the identity of the households that receive the subsidy are invariant to the size of the welfare program.

Theorem 1 *The size of the welfare program does not affect the fraction or the identity of the subsidy-receiving households.*

The theorem draws on Proposition 1 and Lemma 2. First, from Proposition 1, the median-income household y_m is decisive on β at all tax rates. At any given tax rate, household y_m will choose a pair (α, β) where the slope of the GBC $\frac{\partial \tilde{\alpha}(\tau,\beta)}{\partial \beta} = y_m$. Second, based on Lemma 2, a change in the tax rate implies that proportionate changes in α and β will be optimal choices for household y_m. Thus, changing τ will not change the equilibrium value of $\frac{\alpha}{\beta}$.

Figure 2 helps illustrate the median-income household's optimal choices for two different tax rates. It is easy to see that the threshold income above which the subsidy is zero, defined by the slope of the line $\frac{\alpha}{\beta} = \frac{k_\alpha}{k_\beta}$, does not change with the tax rate.

Furthermore, since $\frac{k_\alpha}{1+k_\beta} > y_m$, the threshold $\frac{k_\alpha}{k_\beta}$ must also be greater than y_m. That is, even though the median-income household is decisive, the set of subsidy recipients includes households with incomes above the median. (Figure 2 also implies that $\frac{k_\alpha}{k_\beta} > y_m$.)

In sum, the first-stage voting on the size of the welfare program is irrelevant in order to determine the fraction and the identity of households who receive the subsidy. No matter what the size of the program is, it is always distributed among the *same* households, those below the income level $\frac{k_\alpha}{k_\beta}$. Furthermore, the fraction of the households receiving the subsidy exceeds 50 percent. The first-stage voting, however, matters for the subsidy received by households below the income level $\frac{k_\alpha}{k_\beta}$.

4.1 Robustness

One concern with Theorem 1 might be the linearity in income of the means-testing specification. Below, we consider nonlinear means-testing specifications.

Consider a means-testing specification that is nonlinear in y:

$$s_1(y, \theta; \alpha, \beta) = \max\left\{\alpha - \beta y^\theta, 0\right\}, \quad \theta > 0, \ \ \theta \neq 1.$$

For this specification, the indifference curves are linear as in Section 3 and the slope of the indifference curve for household y would be y^θ. It is easy to see that Proposition 1 and Lemma 2 hold for this specification. If the tax rate changes from τ to $j\tau$, then the majority-preferred means-testing pair changes from (α, β) to $(j\alpha, j\beta)$. This is because $(j\alpha, j\beta)$ is feasible on the GBC with tax rate $j\tau$, and the slope at $(j\alpha, j\beta)$ for household y_m is y_m^θ—same as the slope at (α, β) on the GBC with tax rate τ. Thus, Theorem 1 holds: $\frac{\alpha}{\beta}$ is invariant to τ.

The proof of Lemma 2 relies on the subsidy function's homogeneity of degree 1 in (α, β), not on the linearity in y. To emphasize the homogeneity, consider a subsidy function:

$$s(y; K\alpha, K\beta) = Ks(y; \alpha, \beta), \ \text{ for } K > 0.$$

With this specification, the GBC in Section 2 implies that for proportionate changes in the tax rate, the corresponding proportionate changes in (α, β) are feasible and the slope of GBC is the same at all multiples of (α, β). Hence, the equilibrium (α, β) satisfies Theorem 1.

We should note that Lemma 2 fails to hold for specifications such as $s(y; \alpha, \beta) = \alpha - y^\beta$. In this case, the GBC is given by $\alpha F\left(\alpha^{\frac{1}{\beta}}\right)-\int_0^{\alpha^{\frac{1}{\beta}}} y^{\beta} f(y)\,dy=\tau Y$. Suppose we scale up the tax rate to $j\tau \in (0,1)$ and consider the pair $(j\alpha, j\beta)$. Then, $j\alpha F\left((j\alpha)^{\frac{1}{j\beta}}\right)-\int_0^{(j\alpha)^{\frac{1}{j\beta}}} y^{j\beta} f(y)\,dy \neq j\tau Y$, implying that $(j\alpha, j\beta)$ is not on the GBC associated with the tax rate $j\tau$. So, part (i) of the lemma fails to hold. And, the asymmetric manner in which α and β enter $s(\cdot)$ implies that part (ii) of the lemma also fails to hold. Thus, the homogeneity of the means-testing function in (α, β) is critical to the result, not the linearity in y.

4.2 On the Irrelevance of the First-Stage Voting

Since Theorem 1 implies the irrelevance of the tax rate for the set of households receiving subsidies, one might infer that the majority-preferred tax rate would not be affected if the subsidy-receiving set was exogenous. That is, if we were to fix the ratio $\frac{\alpha}{\beta}$ to some arbitrary value, the equilibrium τ would not be affected by the value of the ratio. This inference would be incorrect. The irrelevance result holds only in one direction: The tax rate does not affect the *majority-preferred* ratio $\frac{\alpha}{\beta}$, but the value of $\frac{\alpha}{\beta}$ does affect the equilibrium tax rate.

To see this, consider the following example. Let $\frac{\alpha}{\beta}=k$ (exogenous) with $F(k)>0.5$, and let the preferences be $c^{\frac{1}{2}}+d^{\frac{1}{2}}$. We can then write the GBC (2) as

$$\begin{aligned} \beta k F(k)-\beta \int_0^k y f(y) dy &= \tau Y, \text{ or} \\ \beta\left[k F(k)-\int_0^k y f(y) dy\right] &= \tau Y. \end{aligned}$$

Thus, $\beta=\kappa(k)\tau$, is a linear function of τ, where $\kappa(k)$ is a function of the exogenous k and depends only on the parameters of the income distribution. And, $\alpha=\beta k$ is also linear in τ. It is easy to show that the decisive household that determines the equilibrium tax rate is constrained and that the decisive household is y_m. The equilibrium tax rate solves

$$(1-\tau)^{-\frac{1}{2}} y_m^{\frac{1}{2}}=\tau^{-\frac{1}{2}}\left[k\kappa(k)-\kappa(k) y_m\right]^{\frac{1}{2}}.$$

Clearly, as we vary k, the majority-preferred tax rate changes.

If we fix k equal to $\frac{k_\alpha}{k_\beta}$, the endogenous threshold from Proposition 2, then

$$\kappa\left(\frac{k_\alpha}{k_\beta}\right)=\frac{k_\beta Y}{\left[k_\alpha F\left(\frac{k_\alpha}{k_\beta}\right)-k_\beta\int_0^{\frac{k_\alpha}{k_\beta}} yf\left(y\right)dy\right]}.$$

It is easy to see from equation (4) that the right hand side of the above equation equals k_β, so $\beta=k_\beta\tau$ and $\alpha=k_\alpha\tau$. Thus, the majority-preferred tax rate is the same as the one from the first stage of the two-stage voting problem in Section 3.2.

5 *Example*

In this section, we assume specific functional forms for income distribution and preferences, and solve for the set of subsidy recipients, the means-testing rate, and the size of the program.

Let the income distribution be Pareto:

$$F\left(y\right)=1-\left(\frac{y_{\min}}{y}\right)^{\lambda},\quad y_{\min}>0,\ \ \lambda\geq 2. \tag{9}$$

Mean and median incomes are $Y=\frac{\lambda y_{\min}}{\lambda-1}$ and $y_m=y_{\min}\sqrt[\lambda]{2}$, respectively.

The subsidy function is given by equation (1). Using the GBC (2) and exploiting the result from Proposition 1 that the slope of the GBC in (α,β) space equals y_m at the majority-preferred pair of α and β, we get:

$$\left(\frac{\alpha}{\beta}\right)^{-\lambda}\left[\lambda y_{\min}^{\lambda}\left(\frac{\alpha}{\beta}\right)+\left(1-\lambda\right)y_{\min}^{\lambda+1}\sqrt[\lambda]{2}\right]=y_{\min}\left(\lambda+\left(1-\lambda\right)\sqrt[\lambda]{2}\right).$$

Denote the solution $\frac{\alpha}{\beta}=k^*$.

From Proposition 2 we know α and β are linear in τ. Since $\alpha = k^*\beta$ and $\beta = k_\beta \tau$, we can use the GBC to solve for the unknown k_β as a function of k^*:

$$k_\beta = \left(\frac{\frac{y_{\min}\lambda}{1-\lambda}}{(k^*)^{1-\lambda}\left[\frac{y_{\min}^{\lambda}}{1-\lambda}\right] - k^* - \frac{y_{\min}\lambda}{1-\lambda}} \right).$$

Since $\alpha = k_\alpha \tau$, it must be the case that

$$k_\alpha = k^* k_\beta.$$

To determine the first-stage vote over τ, let each household's preferences over consumption and the subsidy good be described by a CES utility function (7) as in Proposition 5. The indirect utility of household y is

$$V(\tau) = \frac{1}{1-\sigma}\left(((1-\tau)\,y)^{1-\sigma} + \delta\,(k_\alpha\tau - k_\beta\tau y)^{1-\sigma}\right).$$

The equilibrium tax rate is determined by the decisive household y_d as

$$\tau = \frac{y_d^{\frac{\sigma-1}{\sigma}}}{\delta^{-\frac{1}{\sigma}}\,(k_\alpha - k_\beta y_d)^{\frac{\sigma-1}{\sigma}} + y_d^{\frac{\sigma-1}{\sigma}}}.$$

From Proposition 5, the decisive voter y_d can be identified as follows. When $\sigma \in (0,1)$, $y_d = y_m$. For $\sigma = 1$ (log utility), all households vote for the same tax rate $\frac{\delta}{1+\delta}$. When $\sigma > 1$, y_d solves equation (8), which yields

$$y_d = \frac{y_{\min}}{\left(0.5 + \left(\frac{y_{\min}\left(1+k_\beta\right)}{k_\alpha}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}.$$

Targeting and Political Support for Redistribution We use the example above to provide testable implications for targeting and political support for redistribution. (Recall that both are endogenous in our model.) As noted earlier, variations in targeting (i.e., the set

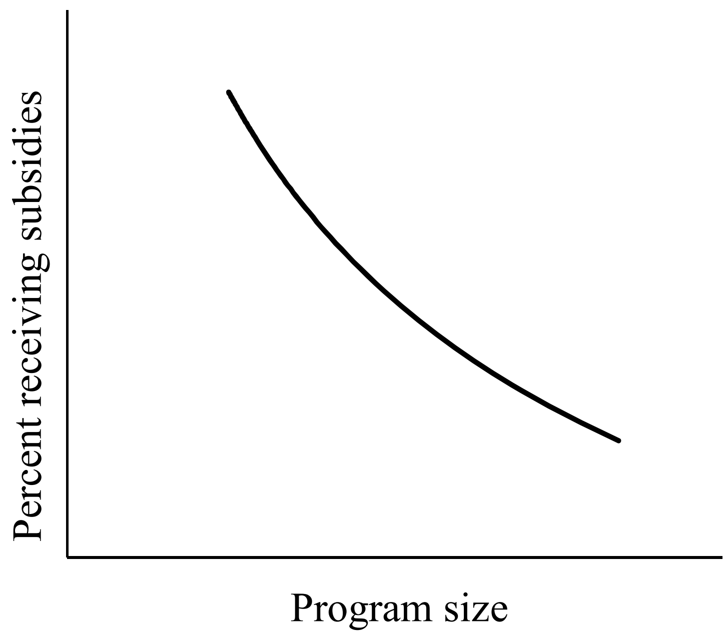

Figure 4: Targeting and Political support for redistribution

of subsidy recipients) in our model can arise only from variations in the income heterogeneity. Suppose we index economies by λ. As λ decreases, both income inequality and mean income increase.

Across these economies, Figure 4 illustrates that the subsidy-program size (τY) and the fraction receiving the subsidies ($\frac{\alpha}{\beta}$) are negatively correlated. That is, targeting fewer people for subsidies is associated with a larger program size. If the targeting were *exogenous*, then the correlation would be positive, as in De Donder and Peluso (2018).

6 Concluding Remarks

We study publicly funded linear means-tested subsidies when funding decisions are made through majority voting. The voting problem has two policy variables—the tax rate and the means-testing rate. We assume sequential voting where households first vote on the tax rate while anticipating how it will affect the means-testing rate and then vote on the means-testing rate. We show that a majority-voting equilibrium exists and characterize the majority-preferred tax rate and means-testing rate. We establish that the fraction and the identity of the households receiving the subsidy is independent of the tax rate used to fund the subsidy. This result is robust to nonlinear means-tested subsidies.

The separation result—the first-stage voting on the tax rate is irrelevant for determining the set of subsidy recipients—holds only in one direction. The majority-preferred tax rate would be different if the subsidy-receiving households were exogenously specified.

In our model, household income is exogenous, so the tax rate does not distort household behavior on earnings. The initial endowments are redistributed in the form of in-kind transfers. Thus, the distortion is only on the expenditures on the subsidy good versus other consumption goods. Incorporating distortionary taxes would complicate the second-stage voting in our model. The distribution of earnings and, hence, the government budget constraint would change with the tax rate in the first stage. It would be interesting to examine whether the separation result holds in this case.

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Appendix Proofs

Proof of Lemma 1. Holding τ fixed and applying the implicit function theorem to (2), the slope and curvature of the GBC are given by

$$\frac{\partial\widetilde{\alpha}\left(\tau,\beta\right)}{\partial\beta}=\frac{\int_0^{\widetilde{\alpha}/\beta}yf\left(y\right)dy}{F\left(\widetilde{\alpha}/\beta\right)}>0,\quad \frac{\partial^2\widetilde{\alpha}\left(\tau,\beta\right)}{\partial\beta^2}=-\frac{\tau^2Y^2f\left(\widetilde{\alpha}/\beta\right)}{\beta^3F\left(\widetilde{\alpha}/\beta\right)^3}<0$$

so that the GBC is increasing and strictly concave. Furthermore, taking the limit as β decreases to zero, $\lim_{\beta\searrow 0}\frac{\partial\widetilde{\alpha}(\tau,\beta)}{\partial\beta}=Y$, which is the maximum slope of the GBC given strict concavity. ■

Proof of Proposition 1. We prove the statement in three steps. First, we show that the indifference curves for household y are linear in the (β,α) plane. Second, we show that the most-preferred β is a decreasing function of household income. Finally, we show that the majority-preferred β is chosen by the household with median income, y_m. See Figure 1.

Indifference curves of household y: Recall that the indirect utility of the household is

$$V(y,\widetilde{\alpha},\beta,\tau)\equiv u\left(\widehat{c}\left(y,\widetilde{\alpha},\beta,\tau\right),\widehat{d}\left(y,\widetilde{\alpha},\beta,\tau\right)\right).$$

For those points in the (β,α) plane with $\frac{\alpha}{\beta}\geq y$, the slope of household y's indifference curve is

$$\left.\frac{\partial\widetilde{\alpha}}{\partial\beta}\right|_{V(y,\widetilde{\alpha},\beta,\tau)=constant}=-\frac{\partial V\left(y,\widetilde{\alpha},\beta,\tau\right)/\partial\beta}{\partial V\left(y,\widetilde{\alpha},\beta,\tau\right)/\partial\widetilde{\alpha}}=y>0.$$

Thus, the indifference curves for household y are of the form $\alpha-\beta y=$ constant.

most-preferred β on the GBC for each household: Since Lemma 1 shows $\lim_{\beta\searrow 0}\frac{\partial\widetilde{\alpha}(\tau,\beta)}{\partial\beta}=Y$, indirect utility $V\left(y,\widetilde{\alpha},\beta,\tau\right)$ is maximized at $\beta=0$ for all households $y\geq Y$. For $y<Y$, the indirect utility $V\left(y,\widetilde{\alpha},\beta,\tau\right)$ is maximized at a unique

$$\left\{\beta>0:y=\frac{\partial\widetilde{\alpha}\left(\tau,\beta\right)}{\partial\beta}\right\}.$$

Recall that the most-preferred β of household y is denoted by $\beta^*\left(\tau,y\right)$. It is easy to see from Figure 1 that for $y<Y$, $\beta^*\left(\tau,y\right)$ is decreasing in y. For $y\geq Y$, $\beta^*\left(\tau,y\right)=0$.

To be internally consistent, we have to verify whether households with $\beta^*\left(\tau,y\right)>0$ do indeed receive positive subsidies i.e., does $\beta^*\left(\tau,y\right)$ satisfy the inequality $\widetilde{\alpha}\left(\tau,\beta^*\left(\tau,y\right)\right)-\beta^*\left(\tau,y\right)y>0$ for all $y<Y$? It is easy to see from Figure 1 that every household $y<Y$ will choose a β such that it gets a positive subsidy. This is because $\alpha-\beta y=0$ is a lower indifference curve for household y

than the indifference curve that is tangent to the GBC.

Majority-preferred β: Recall that $\widehat{\beta}(\tau) \equiv \beta^*(\tau, y_m)$ is the most-preferred β for household y_m and that our income distribution has $y_m < Y$. Consider any candidate $\beta_c < \widehat{\beta}(\tau)$ on the GBC. All households with $y \leq y_m$ strictly prefer $\widehat{\beta}(\tau)$ to β_c since $\beta^*(\tau, y)$ is decreasing in y. Consequently, $\beta_c < \widehat{\beta}(\tau)$ cannot garner a majority. Next, consider a candidate $\beta_c > \widehat{\beta}(\tau)$ on the GBC. All households with $y \geq y_m$ prefer $\widehat{\beta}(\tau)$ to β_c. Hence, $\beta_c > \widehat{\beta}(\tau)$ cannot get a majority vote. Thus, $\widehat{\beta}(\tau)$ is the majority-preferred β on the GBC. ■

Proof of Lemma 2. (i) The left hand side of GBC (2) is homogeneous of degree 1 in (α, β) and the right hand side of GBC is homogeneous of degree 1 in τ. Hence, proportionate increases in α, β and τ will satisfy the GBC.

(ii) The proof of Lemma 1 provides

$$\frac{d\widetilde{\alpha}}{d\beta} = \frac{\int_0^{\frac{\widetilde{\alpha}}{\beta}} y dF(y)}{F\left(\frac{\widetilde{\alpha}}{\beta}\right)}.$$

Clearly, proportionate increases in α and β have no effect on the slope of the GBC. ■

Proof of Proposition 2. For $\tau \in (0,1)$, let $\widehat{\alpha}$ and $\widehat{\beta}$ be the most-preferred pair of household y_m i.e., the majority-preferred pair on the GBC satisfies

$$\left.\frac{d\widetilde{\alpha}}{d\beta}\right|_{(\widehat{\alpha},\widehat{\beta})} = y_m.$$

Consider an arbitrary j such that $j\tau \in (0,1)$. For the tax rate $j\tau$, Lemma 2 establishes that the pair $\left(j\widehat{\alpha}, j\widehat{\beta}\right)$ satisfies the GBC associated with $j\tau$ and that

$$\left.\frac{d\widetilde{\alpha}}{d\beta}\right|_{(j\widehat{\alpha},j\widehat{\beta})} = y_m.$$

Hence, the most-preferred pair on the new GBC is $\left(j\widehat{\alpha}, j\widehat{\beta}\right)$. We thus have $\frac{\widehat{\alpha}}{\widehat{\beta}}$ is constant, implying $\widehat{\alpha}$ is proportional to $\widehat{\beta}$. Using this proportionality, GBC (2) can be rewritten to show that $\widehat{\beta}$ is proportional to τ. Finally, the proportionality between $\widehat{\alpha}$ and $\widehat{\beta}$ implies the final result, that $\widehat{\alpha}$ is proportional to τ. ■

Proof of Proposition 3. For households $y \geq \frac{k_\alpha}{1+k_\beta}$, the utility is monotonically declining in τ and their most-preferred tax rate is zero. We will show that the utility of households $y < \frac{k_\alpha}{1+k_\beta}$ is also single-peaked. Existence of the majority-voting equilibrium then follows from Black (1958).

To establish single-peakedness for $y < \frac{k_\alpha}{1+k_\beta}$, define two functions: $\underline{V}$ where household y is constrained by the subsidy for all τ and $\overline{V}$ where the household is unconstrained for all τ.

$$\begin{aligned}\overline{V}(y;\tau) &\equiv u\left(\overline{c}\left((1-\tau)y+k_\alpha\tau-k_\beta\tau y\right),\overline{d}\left((1-\tau)y+k_\alpha\tau-k_\beta\tau y\right)\right)\\ \underline{V}(y;\tau) &\equiv u\left((1-\tau)y,k_\alpha\tau-k_\beta\tau y\right)\end{aligned}$$

where the functions $\overline{c}$ and $\overline{d}$ describe interior solutions given resources $(1-\tau)y+k_\alpha\tau-k_\beta\tau y$ and no additional constraints. It is easy to see that $\underline{V}\leq\overline{V}$ since the resource constraint is the same, but $\underline{V}$ has an additional constraint on subsidy good expenditure. Define $\overline{\tau}(y)$ such that

$$\underline{V}(y;\overline{\tau})=\overline{V}(y;\overline{\tau})$$

i.e., at $\overline{\tau}$ household y's interior choice of subsidy good expenditure is exactly the same as the subsidy amount or the subsidy constraint is just barely binding. It is easy to see that there is a unique $\overline{\tau}(y)$ (set $(1-\tau)y=\overline{c}\left((1-\tau)y+k_\alpha\tau-k_\beta\tau y\right)$ and solve for τ). Clearly, for a tax rate higher than $\overline{\tau}$, household y would be constrained. We can then write the indirect utility of household y as

$$V(y;\tau)=\begin{cases}\overline{V}(y;\tau) & \text{if} \quad \tau<\overline{\tau}(y)\\ \underline{V}(y;\tau) & \text{if} \quad \tau\geq\overline{\tau}(y)\end{cases}.$$

For household $y<\frac{k_\alpha}{1+k_\beta}$, $\overline{V}$ is increasing in τ since $k_\alpha\tau-k_\beta\tau y>\tau y$. For this household, it is also easy to see that $\underline{V}$ is strictly concave in τ. Thus, the indirect utility for household y, $V(y;\tau)$, is (i) the same as $\overline{V}(y;\tau)$ for $\tau<\overline{\tau}(y)$ and, hence, increasing and (ii) the same as $\underline{V}(y;\tau)$ for $\tau\geq\overline{\tau}(y)$ and, hence, strictly concave. At $\overline{\tau}(y)$, by construction, $\underline{V}=\overline{V}$, so there is no discontinuity in $V(y;\tau)$ at $\overline{\tau}(y)$.

Now, $\underline{V}$ is single-peaked at $\widehat{\tau}(y)$ where $\widehat{\tau}(y)$ is the unique solution to

$$yu_1\left((1-\tau)y,k_\alpha\tau-k_\beta\tau y\right)=(k_\alpha-k_\beta y)\,u_2\left((1-\tau)y,k_\alpha\tau-k_\beta\tau y\right).$$

Furthermore, $\widehat{\tau}(y)>\overline{\tau}(y)$. This is because (i) at $\tau=\overline{\tau}(y)$,

$$u_1\left((1-\tau)y,k_\alpha\tau-k_\beta\tau y\right)=u_2\left((1-\tau)y,k_\alpha\tau-k_\beta\tau y\right)$$

since household y's optimal choice (based on $\overline{V}$) of consumption is exactly the after-tax income and subsidy good expenditure is exactly the subsidy amount and, hence, (ii)

$$\left.\frac{\partial\underline{V}}{\partial\tau}\right|_{\tau=\overline{\tau}(y)}=u_1\left((1-\overline{\tau}(y))y,k_\alpha\overline{\tau}(y)-k_\beta\overline{\tau}(y)y\right)\{k_\alpha-(1+k_\beta)y\}>0$$

for all $y<\frac{k_\alpha}{1+k_\beta}$. Thus, $V(y;\tau)$ is single-peaked for all households. ■

Proof of Proposition 4. (i) If the decisive voter's income is greater than $\frac{k_\alpha}{1+k_\beta}$, then his most-preferred tax rate is zero.

(ii) From the proof of Proposition 3, for all households $y < \frac{k_\alpha}{1+k_\beta}$, indirect utility is single peaked at $\widehat{\tau}(y) > \overline{\tau}(y)$. Thus, for these households the most-preferred tax rate involves a constrained allocation. Given that the decisive household must come from $y < \frac{k_\alpha}{1+k_\beta}$ to support a positive majority-preferred tax rate, the decisive household must be constrained at the majority-preferred tax rate with its optimal allocation given by the equations in part (ii) of Proposition 4.

(iii) At the constrained allocation $c = (1-\tau)y_d$ and $d = k_\alpha\tau - k_\beta\tau y_d$. A marginal increase in τ implies a loss of y_d in consumption and a gain of $k_\alpha - k_\beta y_d$ in the subsidy amount. The decisive voter will set the most-preferred tax rate such that

$$y_d u_1\left((1-\tau)\,y_d, k_\alpha\tau - k_\beta\tau y_d\right) = \left(k_\alpha - k_\beta y_d\right) u_2\left((1-\tau)\,y_d, k_\alpha\tau - k_\beta\tau y_d\right),$$

so the utility loss on the margin is equal to the utility gain. ■

Proof of Proposition 5. Solving the voting problem specified in equation (5), we have the following first order condition for household y

$$\frac{\partial V(y;\tau)}{\partial\tau} = -yu_1\left((1-\tau)\,y, k_\alpha\tau - k_\beta\tau y\right) + \left(k_\alpha - k_\beta y\right) u_2\left((1-\tau)\,y, k_\alpha\tau - k_\beta\tau y\right) = 0. \tag{10}$$

Applying the implicit function theorem to equation (10), we have the following relationship between income and most preferred tax rate:

$$\frac{\partial\tau}{\partial y} = -\frac{\frac{\partial^2 V(y;\tau)}{\partial\tau\partial y}}{\frac{\partial^2 V(y;\tau)}{\partial\tau^2}}$$

and since $\frac{\partial^2 V(y;\tau)}{\partial\tau^2} < 0$, the sign of $\frac{\partial\tau}{\partial y}$ is the same as $\frac{\partial^2 V(y;\tau)}{\partial\tau\partial y}$.

To prove part (i) of Proposition 5 we rely on the fact that $\frac{\partial^2 V(y;\tau)}{\partial y\partial\tau} < 0$ for $\sigma < 1$. This implies that $\frac{\partial\tau}{\partial y} < 0$ and households $y \in \left[0, \frac{k_\alpha}{1+k_\beta}\right]$ will vote for a lower tax rate as y increases. All households $y > \frac{k_\alpha}{1+k_\beta}$ will vote for $\tau = 0$. As a consequence, if $\sigma < 1$, the decisive voter is household y_m. Substituting $y = y_m$ into equation (10) and rearranging, it is easy to verify the equilibrium tax rate is given by equation (6).

To prove part (ii) of Proposition 5, recall that $\sigma > 1$ implies $\frac{\partial^2 V(y;\tau)}{\partial y\partial\tau} > 0$. All households $y \in \left[0, \frac{k_\alpha}{1+k_\beta}\right]$ will vote for a higher tax rate as y increases. The lowest-income households will form a voting coalition with the households $y > \frac{k_\alpha}{1+k_\beta}$ who vote for $\tau = 0$. As the preferred tax rate is rising in income and as $1 - F\left(\frac{k_\alpha}{1+k_\beta}\right) < 0.5$, the decisive voter y_d will be the lowest-income household that delivers a majority:

$$1 - F\left(\frac{k_\alpha}{1+k_\beta}\right) + F\left(y_d\right) = 0.5.$$

Substituting $y = y_d$ into equation (10) and rearranging, we get equation (6).

In part (iii) of Proposition 5, $\sigma = 1$, so $\frac{\partial^2 V(y;\tau)}{\partial y \partial \tau} = 0$. As a consequence $\frac{\partial \tau}{\partial y} = 0$ and equation (10) for all households $y \in \left[0, \frac{k_\alpha}{1+k_\beta}\right]$ will be solved by the same most-preferred tax rate. Since these households have mass $F\left(\frac{k_\alpha}{1+k_\beta}\right) > 0.5$, they will form a decisive voting coalition where the equilibrium tax rate solves equation (10) (or, equivalently, equation (6)) for all $y \in \left[0, \frac{k_\alpha}{1+k_\beta}\right]$. ■

Proof of Proposition 6. (i) As noted in Remark 3, the second stage outcome is independent of preferences, and k_α and k_β in Proposition 2 are constants, independent of the tax rate. Thus, if we substitute the sequential-voting equilibrium τ from Section 3.2 into the equations in Proposition 2, we will recover the sequential-voting equilibrium α and β.

(ii) Given the sequential-voting equilibrium constants k_α and k_β, the threshold income $\frac{\alpha}{\beta} = \frac{k_\alpha}{k_\beta}$, for every $\tau \in (0,1)$. In the sequential-voting equilibrium, α is proportional to β. GBC (2) implies that

$$\frac{k_\alpha}{k_\beta}\beta F\left(\frac{k_\alpha}{k_\beta}\right) - \beta \int_0^{\frac{k_\alpha}{k_\beta}} y f\left(y\right) dy = \tau Y.$$

So,

$$\beta\left[k_\alpha F\left(\frac{k_\alpha}{k_\beta}\right) - k_\beta \int_0^{\frac{k_\alpha}{k_\beta}} y f\left(y\right) dy\right] = k_\beta \tau Y.$$

Equation (4) then implies that β is proportional to τ in order to satisfy the GBC associated with the tax rate τ, with the constant of proportionality being k_β. Similarly, $\alpha = k_\alpha \tau$. Hence, at the tax rate τ, the means-tested subsidy for household $y \leq \frac{k_\alpha}{k_\beta}$ is $k_\alpha \tau - k_\beta \tau y$. It is then easy to see that τ in Propositions 4 and 5 is the majority-preferred best response. ■

Proof of Theorem 1. First, recall from (1) the income threshold for subsidy recipients is $\frac{\alpha}{\beta}$, above which the subsidy is zero. Second, Proposition 2 implies $\frac{\alpha}{\beta} = \frac{k_\alpha}{k_\beta}$, which is independent of τ. Thus, changes in the size of the subsidy program, dictated by τ, do not affect income threshold, leaving the fraction and identity of subsidy receiving households unchanged. ■