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Long-Term Finance and Investment with Frictional Asset Markets

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Abstract

This paper develops a theory of investment and maturity choices and studies its implications for the macroeconomy. The novel ingredient is an explicit secondary market with trading frictions, which leads to a liquidity spread that increases with maturity and generates an upward sloping yield curve. As a result, trading frictions induce firms to borrow and invest at shorter horizons than in a frictionless benchmark. Economies with more severe frictions exhibit a steeper yield curve, which further affects maturity and investment choices of firms. The model is calibrated to the US, and counterfactual exercises suggest that reductions in trading frictions—a new channel of financial development—can promote economic development. A policy intervention with government-backed financial intermediaries in the secondary market can improve liquidity and reduce the cost of long-term finance, which promotes investment in longer-term projects and generates substantial welfare gains.

JEL Classifications: E44, G30, O16.

Keywords: Debt maturity, Liquidity, Over-the-counter market, Secondary markets.

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Rome was not built in a day.

Li Proverbe au Vilain, 1190

1 Introduction

Firms in emerging economies tend to borrow and invest at shorter maturities compared to those in advanced countries, which may have adverse effects on the aggregate economy. The lower prevalence of long-term financing and investment is thought of by many as a contributor to poor aggregate performance. As a result, there is a policy debate on how to stimulate long-term finance.[1] This paper contributes to this literature and policy debate by developing a theoretical framework of maturity choices and their aggregate effects. It builds on the following key feature of capital markets: Both in advanced and emerging economies, corporate debt markets exhibit trading frictions. The main result is that more severe frictions make long-term finance relatively more expensive, inducing firms to borrow and invest at shorter horizons with detrimental effects on productivity and the aggregate economy.

The central mechanism of this paper is that a long-maturity asset will trade in the secondary market more times than a short-maturity one. Hence, the lack of liquidity in secondary markets—a severe trading friction—affects long-maturity assets more than short ones, generating two important results for the yield curve. First, the liquidity spread increases with maturity. Second, economies with less-liquid secondary markets have a steeper yield curve, and firms invest at shorter horizons and in lower productivity projects. A calibrated model matches key features of the yield curve in the US. Counterfactual exercises disciplined with financial data from Argentina predict about one-half of the differences in maturity between these countries, which leads to large aggregate effects. Finally, a policy intervention with subsidized financial intermediaries can improve liquidity, stimulate long-term finance, and induce investment at longer horizons, generating substantial welfare gains.

The modeling framework combines a fairly standard production economy with an over-the-counter (OTC) secondary market for debt as in Duffie et al. (2005). To finance investment, firms borrow in the primary market and choose the maturity structure of their liabilities. When deciding the gestation period of their projects, firms balance the trade-off between projects of higher productivity and more expensive financing due to an upward sloping yield curve.

Liquidity in secondary markets shapes interest rates in two ways. First, the liquidity spread is increasing in maturity. Liquidity-need shocks hit debt holders, which cause them to become

[1] For empirical evidence see Demirgüç-Kunt and Maksimovic (1998) and Levine (2005), among others. Some policy concerns are expressed in World Economic Forum (2011); European Comission (2013); OECD (2013); Group of Thirty (2013), and World Bank (2015).

potential sellers. However, trading frictions prevent them from immediately selling the asset as they need to search for a counterpart and bargain over the terms of trade. Alternatively, the maturity of the asset also provides liquidity to debt holders. Hence, gains from trade in the secondary market increase with the maturity of the asset, which delivers an upward-sloping yield curve. Second, liquidity is more important for long-term assets in the sense that a reduction of the trading friction not only reduces the liquidity spread for all maturities but also lowers the slope of the yield curve. Therefore, improvements in liquidity generate a flattening of the yield curve.

Decentralized asset markets affect investment costs, which propagate to the real economy. Long-maturity projects have high returns but need financing for a prolonged horizon. Hence, if the yield curve is upward sloping, short-term projects are relatively more attractive than longer ones. As a result, variations in trading frictions change the steepness of the yield curve and distort financial and investment choices, which affect the aggregate economy.

Free entry to the secondary market determines the equilibrium liquidity. To evaluate variations in trading frictions—i.e., financial development—we consider changes in the matching efficiency of the secondary market. In more efficient financial markets, more buyers are willing to enter and liquidity increases in equilibrium. Therefore, financial development generates more liquid markets, in which liquidity spreads diminish for all maturities but in particular for long-term debt, and induces firms to invest in more profitable and high-productivity, longer-term projects. This result suggests that the empirical evidence that firms in emerging economies borrow at shorter maturities can be the result of a substitution effect between maturity and liquidity of secondary markets.

Next, we propose a new empirical strategy to identify the slope of liquidity spreads with respect to maturity, which relies on comparing credit spreads of a given firm issuing two bonds on the same day at different maturities. We uncover the slope of liquidity spreads under the additional assumption that default spreads are constant in maturity. We provide several exercises to validate the assumption, such as considering only safe but illiquid assets, and using credit default swaps to measure the default component. This estimation only needs data on credit spreads so it can be implemented in several countries. We apply the empirical strategy to the US and Argentina, and find that liquidity spreads increase more with maturity in Argentina than in the US. These estimates are useful to calibrate the model and perform counterfactual experiments.

We calibrate the model to the US, targeting the slope of the yield curve among other standard moments. Next, we validate the estimation with additional measures of liquidity. In particular, the model does a reasonable job on also matching the level of the liquidity spread, which is not a target of the calibration. Counterfactual experiments show that variations in trading frictions generate sizable effects on maturity choices and the aggregate economy. For example, if disciplining the search frictions with the estimates for Argentina, the experiment explains about one-half of the maturity and output differences between Argentina and US. Although the model is stylized and tractable, the quantitative results suggest that the theory captures important features of corporate debt markets.

The presence of frictional asset markets suggests that a policy intervention that increases the liquidity of financial markets and improves credit conditions for the corporate sector can generate benefits for the real economy. Based on existing policies like Government-Sponsored Enterprises (i.e., Fannie Mae and Freddie Mac) or large-scale asset purchases (i.e., quantitative easing), this paper evaluates one intervention that subsidizes financial intermediaries in the secondary market, named *Government-Sponsored Intermediaries* (GSIs). The government has four instruments: the size of the intervention, the prices at which the government-sponsored dealers buy and sell from private investors, and a distortionary tax rate to finance the costs of GSIs. Under the optimal policy, government intermediaries buy at higher prices than those in private meetings to provide more gains from trade to private sellers. On the other hand, government agents sell securities at a lower price than those in private meetings to stimulate the entry of potential private buyers. The optimal policy increases the liquidity, flattens the yield curve, and stimulates the use of long-term finance. Quantitatively, this policy generates an increase in maturity of five months for an economy such us the US and eight months for an economy with a less-developed financial system such us Argentina.

The main results are robust both quantitatively and qualitatively to several extensions. First, in the benchmark model, firms can borrow only at the beginning of the project, so the maturity of the project matches the maturity of financing by assumption. An extension of the model allows entrepreneurs to rollover short-term contracts to finance long-term projects with a fixed cost of issuance. Quantitatively, the effect of a change in liquidity on the choice of projects is similar both with and without rollover opportunities regardless of the value of the issuance cost. Second, in the benchmark model, there is a single secondary market for assets of different maturities. This paper also considers a specification in which buyers direct themselves to markets segmented by maturity. The main takeaway is that even though the market tightness (defined as the ratio of sellers-to-buyers) for short-term debt increases, the tightness for long-term assets remains similar to the benchmark model with a single market. As a result, the yield curve is similar to the curve for the benchmark economy.

Related literature This paper is related to the literature on OTC markets following the seminal work of Duffie et al. (2005). Some papers applied the theory to corporate bonds markets (e.g., Chen et al., 2012; He and Milbradt, 2014; Chen et al., 2017), while others consider the interaction between primary and secondary markets (Bruche and Segura, 2017; Arseneau et al., 2017; Bethune et al., 2017a). In particular, the financial structure is a hybrid between He and Milbradt (2014) and Bruche and Segura (2017). On the one hand, He and Milbradt (2014) study the interaction between liquidity and default and abstract from the effects on maturity. They apply the model to the US corporate debt market in a framework with exogenous elements such as output, maturity, and liquidity (in the sense that meeting intensities are not an equilibrium outcome). In contrast, this paper studies the interaction between liquidity and maturity to understand cross-country differences, and, for this application and the policy analysis, it is key to have output, maturity, and liquidity as equilibrium outcomes. On the other hand, Bruche and Segura (2017) apply the theory to US commercial paper in a framework with endogenous maturity and liquidity. However, they assume exogenous profits, maturity is simplified to Poisson arrivals rather than at a deterministic date, and do not provide quantitative evaluation or policy analysis, which are important contributions of this paper.

Many papers studied the term structure of interest rates through the lens of the consumption-based capital asset pricing model (see Gürkaynak and Wright, 2012, for a recent review). Those papers extend the expectation hypothesis framework, which posits that long-term interest rates are expectations of future average short-term rates. This paper is closer to a classic idea present in several discussions such as empirical papers or textbooks (e.g., Mishkin, 2015) that attribute the shape of the yield curve to liquidity considerations. Geromichalos et al. (2016) propose a monetary-search model, with assets of two maturities, to rationalize the yield curve. The contribution of this paper is to present a model with a continuum of maturities and study both theoretically and quantitatively the implications for maturity choices of the corporate sector, the effects on the real economy, and the role of policy interventions. The version of the model with segmented markets provides a rationale for the model of Vayanos and Vila (2009) in which investors have preferences for specific maturities, so interest rates are influenced by demand and supply shocks local at that maturity.

Finally, this paper also contributes to the literature on maturity choice by proposing a novel channel based on trading frictions in the secondary market, which generates an upward sloping yield curve. In the canonical models of Diamond (1991) and Leland and Toft (1996), frictions between lenders and borrowers shape maturity choices, while in this paper the friction is within lenders in financial markets. More broadly, this paper provides a different perspective on how financial development can influence aggregate outcomes, the subject of a large body of work (e.g., Greenwood et al., 2010; Buera et al., 2011; Moll, 2014; Midrigan and Xu, 2014; Cole et al., 2016, among others). All these papers focus on contracting frictions between lenders and borrowers and interpret financial development as a reduction of that friction. Instead, this paper considers trading frictions within lenders in financial markets, in which financial development is an increase in the liquidity of the market and focuses on the choice of maturity, which is absent in previous analyses. Choudhary and Limodio (2018) explore a natural experiment in Pakistan and show that liquidity can affect long-term finance and aggregate outcomes.

The rest of the paper is organized as follows. Section 2 presents the model and Section 3 characterizes the equilibrium. Section 4 describes the empirical strategy and results, while Section 5 quantifies the model and performs counterfactual experiments. Next, Section 6 performs policy analysis and Section 7 extends the model in several dimensions. Finally, Section 8 concludes. Proofs and additional results are gathered in the Appendices.

2 Theory

Time is continuous, starts at $t = 0$, and goes on forever. The economy is populated by agents in the production and the financial sector. Firms in the production sector choose investment projects from a menu of opportunities such that the return increases with the duration of investment. To finance the projects, firms borrow from the financial sector. Corporate bonds trade in an over-the-counter (OTC) secondary market by members of the financial sector.

2.1 Production Sector

Every period a measure μ^0 of identical entrepreneurs choose a new project from a menu of potential investments differentiated by the life-cycle of returns. We consider a particular micro-foundation based on a simple production model in which firms invest in productivity. However, the qualitative results hold for a large class of models such that there are back-loaded projects that require long-term financing.

Life-cycle Two empirical facts motivate the assumptions about the life-cycle of projects. First, small firms grow faster than large firms (e.g., Akcigit and Kerr, 2017). Second, small firms are more financially constrained, particularly for research and development (Midrigan and Xu, 2014; Itenberg, 2015). To capture these facts in a stylized model, we make a stark assumption and divide the life-cycle of firms into two stages: (i) *investment* for $t \leq \tau$ and (ii) *production* for $t > \tau$. A newborn firm chooses τ, the age at which it starts production. Therefore, a new firm has a menu of potential projects summarized by $\tau \geq 0$. With Poisson arrival rate δ, the firm is hit by an exit shock and the value of the project goes to zero.[2]

[2] To reduce notation, assume that this process has the same intensity for firms in the investment and production stages. However, it is simple to consider two different processes.

Investment stage A young firm invests in research and development (R&D) to improve its productivity. Let $z(t)$ be the productivity of the firm such that $z(0) = 0$ and $\dot{z}(t) = \zeta$ for $t \leq \tau$. At maturity τ productivity is $z(\tau) = \zeta\tau$. The flow cost of investment is κ per unit of time doing R&D. We label z as productivity, but it can be broadly interpreted as any factor of production that takes time to build, such as the quality of the product, capital, or demand accumulation, among many others.

Production stage At age τ the firm stops doing R&D and starts the production phase with a production technology that is linear in productivity $y = z$.[3] The net present value of a firm that spent τ periods doing R&D is $F(\tau) = z(\tau)\int_0^\infty e^{-(\rho+\delta)t}dt$, where ρ is the discount factor. Let $Z = \frac{\zeta}{\rho+\delta}$ so $F(\tau) = Z\tau$. All firms are identical, choose the same maturity τ, and have the same productivity z. Hence, in a steady state equilibrium there is a measure $\mu^F(\tau) = \frac{\mu^0 e^{-\delta\tau}}{\delta}$ of identical firms that produce aggregate output equal to $Y(\tau) = \mu^F(\tau)z(\tau)$.

Both qualitative and quantitative results do not depend on this specific production model. Qualitative results also hold under a general return function $F(\tau)$ such that it is increasing and concave in τ. Appendix B shows that a similar return function arise in models of the quality ladder or a model of time-to-build capital. Quantitative results depend on the parametric forms and the estimation. Section 7.3 considers an alternative production function that combines productivity z and labor l to produce output with technology $y = z^{1-\sigma}l^{\sigma}$. We repeat the main quantitative analysis under this specification and show that results are relatively similar regardless of the value of the labor share σ.

Borrowing Firms do not have internal funds and need to borrow. They can borrow only at the beginning of the project; i.e., they match the maturity of the project and the debt. This assumption helps us to obtain a sharper characterization of the equilibrium and captures the idea that it might be costly or risky to borrow short-term to finance long-term projects. In fact, empirical evidence shows that firms tend to match the maturity of assets and liabilities.[4] Section 7.1 relaxes this assumption and allows firms to issue short-term debt to finance long-term projects. In that extension we assume a fixed cost of issuance and find similar results as in the benchmark model.

Corporate bonds have no coupon, default arrival rate δ, and face value one. The firm deposits the proceeds from the issuance in a bank account with risk-free rate ρ and withdraws

[3] Assume that a young firm does not produce. However, results are similar if a young firm produces and use internal funds for investment while κ are the external funds needed for investment.

[4] See Stohs and Mauer (1996); Guedes and Opler (1996); Frederick C. Scherr (2001); and Dew-Becker (2012) for the US and Schiantarelli and Sembenelli (1997); Schiantarelli and Srivastava (1997); and Schiantarelli and Jaramillo (2002) for international evidence. Myers (1977) provides a theory for maturity matching.

κ per unit of time for investment. Hence, a firm needs initial funds equal to $I(\tau) = \kappa\frac{1-e^{-\rho\tau}}{\rho}$ to invest for τ periods. The firm takes the price of a bond with maturity τ and liquidity λ, $P(\tau, \lambda)$, as given and chooses maturity τ and issue debt B to maximize its value[5]

$$\max_{\tau,B} e^{-(\rho+\delta)\tau} \left(F(\tau) - B\right) \qquad \text{s.t. } BP(\tau, \lambda) = I(\tau). \tag{1}$$

2.2 Primary Market

In the primary market, firms from the production sector issue corporate bonds and lenders from the financial sector buy those securities. Assume that there are no frictions in this market and that there is a large mass of potential lenders willing to buy the assets. Free entry into the primary market implies that the price of the bond is equal to the value of holding the asset for a lender

$$P(\tau, \lambda) = D^H(\tau; \lambda), \tag{2}$$

where $D^H(\tau, \lambda)$ is the value of holding a bond with time-to-maturity τ when the liquidity of the secondary market is λ.

2.3 Secondary Market

The financial sector trades corporate bonds in an OTC market as in Duffie et al. (2005). An agent of the financial sector can have either zero or one asset.[6] An agent without the asset can pay a search cost c, enter into the secondary market, and search for a counterpart. There is a large measure of potential entrants, which implies a free-entry condition to the market.

An agent can buy the asset in either primary or secondary markets, but it always starts as a *high valuation* agent. However, he faces an idiosyncratic liquidity risk of becoming *low valuation*. With Poisson intensity η a high valuation agent becomes low valuation and has to pay a holding cost h per unit of time.[7] This idiosyncratic risk generates differences in valuations, causing motives for trade in the secondary market. Note that asset holders are heterogeneous in two dimensions. First, they can be either high or low valuation. Second, they hold assets with time-to-maturity $y \in [0, \tau]$. Let $\mu^H(y)$ and $\mu^L(y)$ denote the measure of high- and low-valuation

[5]Section 2.3 defines liquidity λ as the intensity at which a seller finds a buyer, an equilibrium variable. It is a sufficient statistic to capture trading frictions in the secondary market.

[6]This portfolio restriction is common in the literature because it simplifies the tractability of the model. See Lagos and Rocheteau (2009) for a model with unrestricted asset holdings.

[7]The modeling assumptions about high- and low-valuation agents is standard in the literature (e.g., Duffie et al., 2005; He and Milbradt, 2014). A low investor may have (i) high discounting, (ii) high financing costs, (iii) hedging reasons, (iv) tax disadvantage, or (v) lower personal use of the asset.

agents holding an asset of time-to-maturity y, respectively.

All the low-valuation agents are the sellers in the secondary market. There is random matching in this market, so assets of different time-to-maturity trade in the same market and the total mass of sellers is $\mu^S = \int_0^\tau \mu^L(y)dy$. Section 7.4 extends the model to consider markets segmented by maturity and shows that the results are similar to the model with a single secondary market. On the other hand, a measure μ^B of buyers are agents without an asset searching in the secondary market. Assume a constant-returns-to-scale matching function between buyers and sellers $M\left(\mu^S, \mu^B\right) = A\left(\mu^S\right)^\alpha\left(\mu^B\right)^{1-\alpha}$, and define the market tightness as the ratio of sellers-to-buyers, $\theta = \frac{\mu^S}{\mu^B}$.

A seller finds a counterpart at rate $\lambda = A\theta^{\alpha-1}$, and a buyer finds a counterpart at rate $\beta = A\theta^\alpha$. Upon a match, with probability $\frac{\mu^L(y)}{\mu^S}$ a buyer meets with a seller of an asset with time-to-maturity y. It is useful to define the liquidity of the secondary market as λ because it is the key object through which the secondary market feeds back into the primary market and affects the borrower's problem.[8]

Let $P^S(y;\lambda)$ be the price in the secondary market for an asset of time-to-maturity y and assume a Nash Bargaining protocol. Let γ be the bargaining power of the seller so that

$$P^S(y) = \arg\max_{P^S(y)} \left(P^S(y;\lambda) - D^L(y;\lambda)\right)^\gamma \left(D^H(y;\lambda) - P^S(y;\lambda)\right)^{(1-\gamma)}, \tag{3}$$

where $D^H(y;\lambda)$ is the value of holding an asset for a high-valuation agent—the buyer—and $D^L(y;\lambda)$ is the value of holding an asset for a low-valuation agent—the seller.

The value of search in the secondary market for a buyer, $D^S(\lambda)$, is

$$\rho D^S(\lambda) = -c + \beta\int_0^\tau \frac{\mu^L(y)}{\mu^S}\left(D^H(y;\lambda) - D^S(\lambda) - P^S(y;\lambda)\right)dy. \tag{4}$$

The discounted value of search is equal to the search cost and the expected gains from trade. With intensity $\beta\frac{\mu^L(y)}{\mu^S}$ a buyer matches with a seller of a bond with time-to-maturity y, and gains from trade are $D^H(y;\lambda) - D^S(\lambda) - P^S(y;\lambda)$. The buyer becomes a high-valuation agent, with value $D^H(y;\lambda)$, and pays the price $P^S(y;\lambda)$. Free entry in the secondary market implies that, in equilibrium, $D^S(\lambda) = 0$.

2.4 Equilibrium

Definition 1 states the steady-state equilibrium.

[8] Note that $\beta = A\left(\frac{\lambda}{A}\right)^{\frac{\alpha}{\alpha-1}}$. It is equivalent to define functions depending on the market tightness θ, but it is easier to derive the intuition of the results thinking in the space of the selling intensity λ.

Definition 1. *A steady-state equilibrium is a selling intensity λ, debt maturity τ, prices in the primary market $P(y;\lambda)$, prices in the secondary market $P^S(y;\lambda)$ and measures $\mu^H(y)$, and $\mu^L(y)$ such that:*

1. *Firms in the corporate sector solve* (1)*;*
2. *Free entry into the primary market solves* (2)*;*
3. *$P^S(y,\lambda)$ solves the Nash Bargaining problem* (3)*; and*
4. *$D^S(\lambda) = 0$ solves free entry into the secondary market* (4).

3 Equilibrium Characterization

This section characterizes the solution of the model. First, we solve for the distribution of agents in the financial sector. Then, the main results show how the liquidity of the secondary market affects prices in the primary market, interest rates, and maturity choices. Next, a fixed point between the maturity choice τ and liquidity λ characterizes the equilibrium of the model. Finally, counterfactual exercises examine the effects of financial development.

3.1 Lenders

Buyers from primary and secondary markets start as high valuation. Over time, some agents receive liquidity shocks while others trade in the secondary market. The laws of motion for the measure of high- and low-valuation agents are

$$-\dot{\mu}^H(y) = -(\eta+\delta)\,\mu^H(y) + \beta\frac{\mu^L(y)}{\mu^S}\mu^B, \tag{5}$$

$$-\dot{\mu}^L(y) = \eta\mu^H(y) - (\delta+\lambda)\,\mu^L(y), \tag{6}$$

with boundary conditions $\mu^H(\tau) = \mu^0$ and $\mu^L(\tau) = 0$. Equations (5) and (6) show that as we move closer to maturity (lower y), a fraction η of high-valuation agent becomes low-valuation agents, and a fraction δ of both types of agents is holding an asset that is hit by a default shock. Moreover, a measure $\beta\frac{\mu^L(y)}{\mu^S}\mu^B$ of buyers find a counterpart in the secondary market and becomes high-valuation agents. Finally, a measure λ of low-valuation agents is able to sell in the secondary market. Lemma 1 characterizes the steady-state distribution of financiers.

Lemma 1. *The distribution of financiers is given by*

$$\mu^H(y) = \frac{\mu^0 \eta}{\eta + \lambda} \left(e^{\delta(y-\tau)} \frac{\lambda}{\eta} + e^{(\eta+\lambda+\delta)(y-\tau)} \right), \tag{7}$$

$$\mu^L(y) = \frac{\mu^0 \eta}{\eta + \lambda} \left(e^{\delta(y-\tau)} - e^{(\eta+\lambda+\delta)(y-\tau)} \right). \tag{8}$$

When the secondary market is well-functioning—the selling intensity, λ, is relatively high—the mass of low-valuation agents $\mu^L(y)$ is small. When λ diminishes, the secondary market is more illiquid and the mass of low valuation agents increases.[9] Note that $\mu^L(y)$ enters into the free-entry condition for the secondary market (4), so we use (8) to solve for the equilibrium liquidity.

Private valuations The values for high- and low-valuation agents holding an asset are

$$\rho D^H(y;\lambda) = -\frac{\partial D^H(y;\lambda)}{\partial y} + \eta \left(D^L(y;\lambda) - D^H(y;\lambda) \right) + \delta \left(0 - D^H(y;\lambda) \right), \tag{9}$$

$$\rho D^L(y;\lambda) = -h - \frac{\partial D^L(y;\lambda)}{\partial y} + \lambda \left(P^S(y;\lambda) - D^L(y;\lambda) \right) + \delta \left(0 - D^L(y;\lambda) \right). \tag{10}$$

At maturity both types of investors receive the face value of the asset, implying boundary conditions $D^H(0;\lambda) = D^L(0;\lambda) = 1$. Equation (9) defines the value of high-valuation agents. The left-hand side is the required return from holding the bond. The first term on the right-hand side represents the change in value due to it being closer to maturity. The second term captures the liquidity shocks that transform the investor into a low-valuation agent, which occurs at intensity η. The third term captures the risk of default of the bond. Equation (10) captures the value of low-valuation agents and follows a similar intuition as the previous equation. A low-valuation investor incurs a holding cost h, and with intensity λ the investor meets a counterpart and sells his bond at price $P^S(y;\lambda)$.

Price in secondary market The price in the secondary market, $P^S(y;\lambda)$, is the solution of the Nash Bargaining problem between the seller and the buyer in (3)

$$P^S(y;\lambda) = D^L(y;\lambda) + \gamma \left(D^H(y;\lambda) - D^L(y;\lambda) \right). \tag{11}$$

The gains from trade are $D^H(y;\lambda) - D^L(y;\lambda)$, and the seller gets a fraction γ of them.

[9] It is straightforward to show that the measure of low-valuation agents is decreasing in the meeting intensity.

3.2 Effects of Liquidity on the Primary Market

The central result of this paper is the characterization of how the liquidity of the secondary market feeds back into prices in the primary market. Proposition 1 solves Equations (9) and (10) using the equilibrium price in the secondary market (11) and characterizes the price in the primary market.

Proposition 1. *The price in the primary market is*

$$P(\tau, \lambda) = e^{-(\rho+\delta)\tau} - \mathcal{L}(\tau, \lambda), \tag{12}$$

where the illiquidity cost $\mathcal{L}(\tau, \lambda)$ *is*

$$\mathcal{L}(\tau, \lambda) = h \quad \frac{\eta}{\eta + \lambda\gamma} \quad \int_0^\tau e^{-(\rho+\delta)y} \left(1 - e^{-(\eta+\lambda\gamma)y}\right) dy. \tag{13}$$

The illiquidity cost satisfies the following properties:

1. $\mathcal{L}(\tau, \lambda)$ *is nonnegative.*
2. *Sensitivity with respect to liquidity shocks* η*:*
   (a) *If there are no liquidity shocks,* $\eta = 0$*, then* $\mathcal{L}(\tau, \lambda) = 0$*;*
   (b) *If* $\eta \to \infty$ *(i.e., the holder always has to pay the cost* h*), then*
   $$\lim_{\eta \to \infty} \mathcal{L}(\tau, \lambda) = h \frac{1 - e^{-(\rho+\delta)\tau}}{\rho + \delta}.$$
3. *Sensitivity with respect to maturity* τ*:*
   (a) $\mathcal{L}(\tau, \lambda)$ *is increasing in* τ*;*
   (b) $\mathcal{L}(\tau, \lambda)$ *has a finite limit,* $\lim_{\tau\to\infty} \mathcal{L}(\tau, \lambda) = h \frac{\eta}{(\rho+\delta)(\rho+\delta+\eta+\gamma\lambda)}$.
4. *Sensitivity with respect to liquidity* λ*:*
   (a) $\mathcal{L}(\tau, \lambda)$ *is decreasing in* λ*;*
   (b) *If there are no secondary markets,* $\lambda = 0$*, the liquidity term only represents the expected holding costs; i.e.,*
   $$\mathcal{L}(\tau, 0) = h \int_0^\tau e^{-(\rho+\delta)y} \left(1 - e^{-\eta y}\right) dy;$$
   (c) *If secondary markets are totally liquid (i.e.,* $\lambda \to \infty$*), then* $\mathcal{L}(\tau, \lambda) = 0$.

5. *Liquidity is more important for long-term assets:* $\frac{\partial^2 \mathcal{L}(\tau,\lambda)}{\partial\tau\partial\lambda} \leq 0$.

Proposition 1 shows that we can decompose the price in the primary market $P(\tau, \lambda)$ in two terms. The first component represents the *frictionless* solution: The value of a promise to pay one unit in τ periods when the discount rate is ρ and the default intensity is δ. Note that absent the second term, the expectation hypothesis holds: Long-term interest rates are equivalent to the average of short-term rates. The second term, $\mathcal{L}(\tau, \lambda)$, represents the *illiquidity cost*. When this term is different from zero, the expectation hypothesis does not hold and borrowing at longer horizons becomes more expensive than the average of short-term rates.

The illiquidity cost captures the expected discounted time that the holder of the asset is low valuation and has to pay the holding cost. If there are no secondary markets, the illiquidity cost is equivalent to the holding cost h times the expected discounted length of time between the stopping time in which the agent receives the idiosyncratic shock—which occurs at intensity η—and maturity. On the other hand, if there are no frictions in the secondary market, upon a shock the agent can sell the asset instantaneously and recover the fundamental value, which implies that the illiquidity cost would be equal to zero. Hence, both how easy is to sell in the secondary market, captured by λ, and how much of the gains from trade a seller can retain, measured by γ, shape the illiquidity cost.

To derive some intuition about the illiquidity cost, let $s^H(y)$ and $s^L(y)$ be the adjusted probabilities that a security of age y is held by agents with high and low valuations, respectively.[10] This is an adjusted probability because the transition takes into account the bargaining power

$$\dot{s}^H(y) = -\eta s^H(y) + \lambda\gamma s^L(y),$$
$$\dot{s}^L(y) = \eta s^H(y) - \lambda\gamma s^L(y),$$

with initial condition $s^H(0) = 1$ and $s^L(0) = 0$. Then, the illiquidity cost is

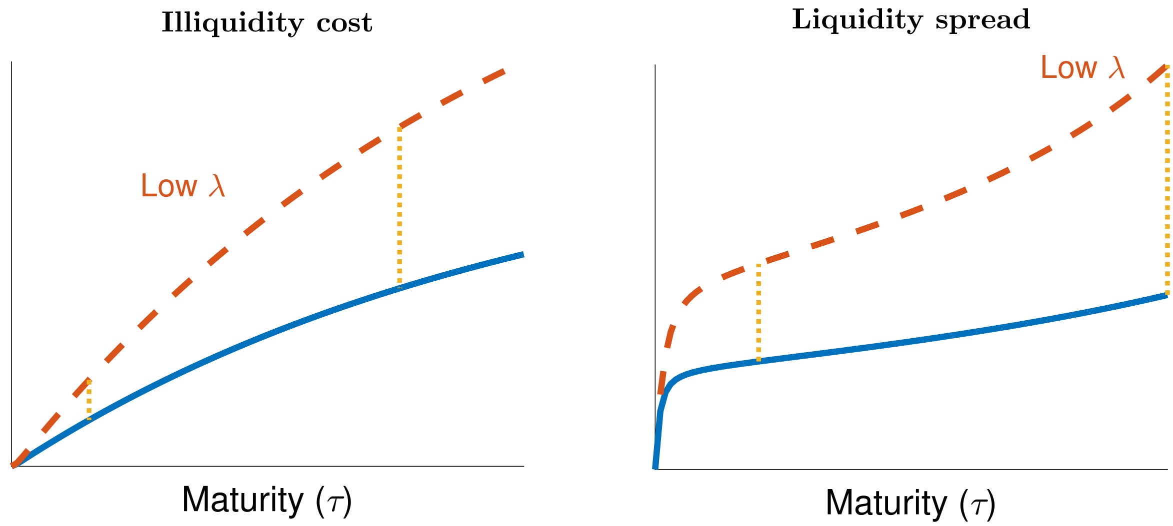
$$\mathcal{L}(\tau, \lambda) = h \int_0^\tau e^{-(\rho+\delta)y} s^L(y) dy.$$

Therefore, the illiquidity cost is proportional to the expected discounted length of time that the holder of the asset is low valuation.

Proposition 1 establishes several results about the illiquidity cost. The first two properties show that $\mathcal{L}$ is nonnegative, that if there are no idiosyncratic shocks $\mathcal{L}$ is equal to zero, and that if the agent always has to pay the holding cost then the illiquidity cost is equal to the net present

[10] Note that y is the age of the asset, not the time to maturity. Also, note that it is absent of default, as we include the default rate δ in the discount factor.

Figure 1: **Illiquidity cost and spread**



Note: Illiquidity cost and liquidity spread are increasing in maturity and decreasing in liquidity. Parameter values are discussed in Section 5.

value of paying h. More interestingly, the left panel of Figure 1 summarizes how maturity and liquidity affect the illiquidity cost. First, the illiquidity cost is increasing in maturity. Longer securities spend more time in the market, which increases the expected length of time that they are held by low-valuation agents. Second, the illiquidity cost is decreasing in liquidity λ. If the liquidity of the secondary market increases, the holders of that security spend less time paying the holding cost, which implies a lower illiquidity cost.

The central result is that the trading friction is more important for long-term assets; i.e., the cross-partial derivative of the illiquidity cost with respect to maturity and liquidity is negative: $\frac{\partial^2 \mathcal{L}(\tau,\lambda)}{\partial\tau\partial\lambda} \leq 0$. An investor that wants to exit a financial position can either sell in the secondary market or wait until maturity. Hence, the role of the secondary market is more important for an agent holding a longer-term asset. As a consequence, a reduction of the trading friction benefits more long- than short-term securities. This result highlights the importance of decentralized asset trading to study long-term finance.

Inspection of equation (13) reveals that the product of λ times γ captures the feedback of secondary market liquidity to prices in primary markets, a standard result in the literature. The first term, λ, is the selling intensity in the secondary market—an equilibrium object. If λ increases, it becomes easier to sell in the secondary market. The second term, γ, is the bargaining power of the seller in the secondary market—a parameter. When γ increases, sellers keep a larger fraction of the gains from trade. Therefore, λ captures the feedback of the friction in the secondary market to prices in the primary market and investment choices.

Yield curve For the empirical analysis in Section 4 it is useful to transform the price in the primary market into an interest rate schedule. Define $r(\tau, \lambda)$ as the compound interest rate that solves $P(\tau, \lambda) = e^{-r(\tau,\lambda)\tau}$, which is the value of the asset conditional on it being held to maturity without default or trading. The interest rate for a bond of maturity τ is

$$r(\tau, \lambda) = \rho + \delta + \frac{1}{\tau} \log \left(\frac{1}{1 - e^{(\rho+\delta)\tau} \mathcal{L}(\tau, \lambda)} \right). \tag{14}$$

The first term, ρ, is the risk-free rate, while the remaining two terms capture the credit spread. We can decompose the spread into a default and a liquidity component, following He and Milbradt (2014). Consider a marginal investor with no idiosyncratic liquidity risk. Such investor requires an interest rate equal to $\rho + \delta$. Therefore, the credit spread due to default is δ. Finally, define the credit spread due to liquidity by subtracting the default component. Hence, the third term corresponds to the liquidity spread

$$cs^{liq}(\tau, \lambda) = \frac{1}{\tau} \log \left(\frac{1}{1 - e^{(\rho+\delta)\tau} \mathcal{L}(\tau, \lambda)} \right), \tag{15}$$

and $r(\tau, \lambda) = \rho + cs^{def} + cs^{liq}(\tau, \lambda)$. Note that in this model the term premium—the difference between long- and short-term rates—is equivalent to the credit spread due to liquidity. Hence, variations in interest rates across maturities are only explained by differences in the liquidity component of the security. All assets are traded in the same secondary market; however, the liquidity spread varies with maturity as the importance of the secondary market is different across assets. Section 4 exploits this observation to identify the liquidity component in the data. Lemma 2 describes the properties of the liquidity spread.

Lemma 2. *The liquidity spread $cs^{liq}(\tau, \lambda)$ is:*

1. *Increasing in maturity $\frac{\partial cs^{liq}(\tau,\lambda)}{\partial \tau} \geq 0$;*
2. *Decreasing in liquidity $\frac{\partial cs^{liq}(\tau,\lambda)}{\partial \lambda} \leq 0$;*
3. *Increasing in the default intensity $\frac{\partial cs^{liq}(\tau,\lambda)}{\partial \delta} \geq 0$.*

Lemma 2 establishes three important properties about the liquidity spread. First, it is increasing in maturity and decreasing in liquidity. The right panel of Figure 1 shows the liquidity spread as a function of maturity for two levels of liquidity. Note that an increase in λ has a larger effect on the long-end of the yield curve. These results are also present in the illiquidity cost $\mathcal{L}(\tau, \lambda)$, and the yield curve preserves the properties.

Liquidity spreads are increasing in maturity because the measure of low-valuation agents is also increasing in maturity. The underlying intuition comes from Lemma 1. At issuance there is no low-valuation agents. However, as time evolves, a fraction of the high-valuation agents become low valuation, but not all of them are able to sell in the secondary market because of the search friction. As a result, the measure of low-valuation agents increases with maturity. This result implies that the prevalence of search frictions is more important for long-term rates as shown in Lemma 2.

Lemma 2 also shows that there is a feedback-loop between default and liquidity; the liquidity spread is increasing in the default intensity δ. Note that δ has the same role as the discount factor ρ. An increase in the discount factor decreases the value of illiquidity at maturity, which increases the liquidity spread. Section 7.2 extends the analysis and studies how changes in default risk interact with maturity choices.

The liquidity spread increases with maturity, while the default spread is constant. This result holds exactly in this model, but it is likely to hold in a large class of models for the following reasons. On the one hand, an aggregate default shock affects the value of the asset for all agents and is independent of the time-to-maturity. In particular, in this model the value is equal to zero. On the other hand, an idiosyncratic liquidity shock does not affect the value of the asset for other potential buyers or the value that the investor recovers at maturity. However, the closer the maturity of the asset is, the lower the cost associated with holding the security is due to the frictions in the secondary market. Hence, in more sophisticated models this result might not hold exactly, but these forces indicate that the liquidity component will still affect the term structure, while the default component can generate either upward- or downward-sloping yield curves.[11]

Bid-ask spreads Define the proportional bid-ask spread as the gains from trade normalized by the mid-price

$$BA\left(y;\lambda\right)=\frac{D^{H}\left(y;\lambda\right)-D^{L}\left(y;\lambda\right)}{1/2\left(D^{H}\left(y;\lambda\right)+D^{L}\left(y;\lambda\right)\right)}.$$

Lemma 3 shows that the proportional bid-ask spread is increasing in maturity. An asset of longer maturity has larger gains from trade, and as a result the bid-ask spread increases with maturity. Importantly, all the predictions in Lemmas 2 and 3 are consistent with the empirical evidence (see, for example, Edwards et al., 2007; Bao et al., 2011).

Lemma 3. *The proportional bid-ask spread is increasing in maturity.*

[11] Indeed, in traditional models of corporate default (e.g., Merton, 1974; Duffie and Singleton, 1999) the change in the term premium with respect to a change in maturity can be either positive or negative.

3.3 Entry to the Secondary Market

The free-entry condition to the secondary market characterizes liquidity λ as a function of the maturity at issuance of the bonds, τ. Replace the equilibrium price in the secondary market (11) in the free-entry condition (4) so that

$$c = \beta(1-\gamma)\int_0^\tau \frac{\mu^L(y)}{\mu^S}\left(D^H(y;\lambda) - D^L(y;\lambda)\right) dy. \tag{16}$$

Proposition 2 analyzes the free-entry condition (16). It describes how the selling intensity changes with the maturity at issuance, $\lambda(\tau)$.

Proposition 2. *$\lambda(\tau)$ is increasing in τ and $\lambda(\tau) : \mathbb{R}_+ \mapsto [0, \overline{\lambda}]$.*

When assets are of zero maturity there are no gains from trade, which implies no entry into secondary markets and a selling intensity equal to zero, $\lambda(0) = 0$. Gains from trade are increasing in τ, which implies that there are more incentives to enter into the secondary market as τ increases. Hence, $\lambda(\tau)$ is increasing. When τ goes to infinity the gains from trade are bounded, which implies that λ converges to a finite number. Section 3.5 solves the equilibrium between lenders and borrowers in which $\lambda(\tau)$ represents the *lenders' curve*.

3.4 Optimal Maturity

The solution of the firm's problem (1) is characterized by the following trade-off

$$\frac{\partial F(\tau)}{\partial \tau} = (\rho+\delta) F(\tau) + \frac{\partial I(\tau)}{\partial \tau} e^{r(\tau,\lambda)\tau} + I(\tau) e^{r(\tau,\lambda)\tau} cs^{liq}(\tau,\lambda)\left(1 + \epsilon_{cs^{liq},\tau}(\tau,\lambda)\right), \tag{17}$$

where $\epsilon_{cs^{liq},\tau}(\tau,\lambda)$ is the elasticity of the liquidity spread with respect to maturity

$$\epsilon_{cs^{liq},\tau}(\tau,\lambda) = \frac{\partial cs^{liq}(\tau,\lambda)}{\partial \tau}\frac{\tau}{cs^{liq}(\tau,\lambda)}.$$

Consider a marginal increase in τ. The left-hand side of Equation (17) represents the benefits of operating a firm with higher productivity, and the right-hand side captures three associated costs. First, a project in which returns are more back-loaded requires more time to become profitable. This implies a higher time-discount on future profits. Second, a larger firm requires more investment. Note that even without financial frictions (i.e., constant interest rates, $r(\tau) = \rho + \delta$) we have an interior solution for τ.[12] Intuitively, there is an interior solution because as the firm chooses a larger τ, it is both more costly and it takes more time to complete.

[12] Note that this is also true even if there is no default, $\delta = 0$.

The third term of Equation (17) captures the effect of the financial cost. First, as maturity increases, the firm has to pay the liquidity spread for a longer period. Second, Lemma 2 shows that the liquidity spread increases with maturity, which is captured by the elasticity $\epsilon_{cs^{liq},\tau}$. These two forces induce the firm to choose a shorter maturity than in the frictionless economy. Let $\tau(\lambda)$ be the optimal maturity when the selling intensity is equal to λ.

Proposition 3. *The optimal maturity is increasing in the liquidity of the secondary market and* $\tau(\lambda) : [0, \overline{\lambda}] \mapsto [\underline{\tau}, \overline{\tau}]$ *with* $0 \leq \underline{\tau} \leq \overline{\tau} < \infty$.

The optimal maturity increases with the liquidity of the secondary market. By Lemma 2, the liquidity spread is lower when secondary markets are more liquid. This implies that it is cheaper to borrow, particularly at longer horizons. Hence, when λ increases firms choose projects of longer maturity.

3.5 Equilibrium

The equilibrium is a fixed point between maturity and liquidity. On the one hand, firms take the liquidity of the secondary market as given and choose maturity—Equation (17)—which delivers a curve $\tau(\lambda)$. On the other hand, agents in the financial sector take the maturity of assets as given, and the free-entry condition to the secondary market—Equation (16)—delivers $\lambda(\tau)$. An equilibrium is (τ^*, λ^*) such that $\tau(\lambda(\tau^*)) = \tau^*$. Proposition 4 states that an equilibrium exists. The proof follows directly from Propositions 2 and 3. Figure 2 shows the characterization of the equilibrium in the space of liquidity and maturity. The solid red and blue curves show the lenders' and borrowers' choices, respectively. The intersection of these curves characterizes the equilibrium of the economy.

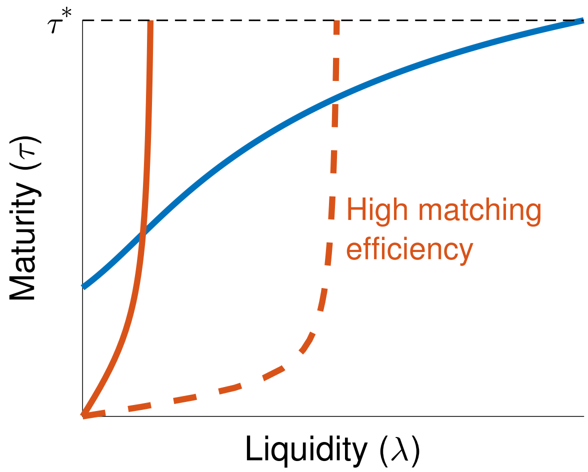
Proposition 4. *A steady-state search and matching equilibrium always exists.*

3.6 Financial Development

We interpret financial development as improvements in the efficiency of the secondary market. In the model it corresponds to improvements in the efficiency of the matching efficiency A.[13] On the one hand, the optimal maturity, characterized by the curve $\tau(\lambda)$, depends on the equilibrium λ but not directly on A, which implies that the curve $\tau(\lambda)$ is independent of A. On the other hand, the curve $\lambda(\tau)$ depends directly on the matching efficiency. For a given maturity and market-tightness, a higher efficiency increase the selling intensity λ. Moreover, it also induces more potential buyers to enter the market, which reduces the market tightness

[13] Reductions in the search cost c generate similar results.

Figure 2: **Equilibrium characterization and financial development**



Note: Equilibrium characterization and financial development. The parameter values are discussed in Section 5.

and further increases λ. As a result, the curve moves to the right. The red-dashed curve in Figure 2 shows the new locus for the equilibrium condition under financial development: Both liquidity and maturity increase. This exercise shows that financial development—an increase in the efficiency of secondary markets—causes an increase of debt maturity. In Section 5 we evaluate this mechanism quantitatively and show that the liquidity of the secondary market can generate quantitatively large movements in maturity choices.

What is *financial development*? First, there is a literal interpretation of it being related to the technology to execute trades. In developed markets, there are clearing houses such as *Euroclear* or *Clearstream*, while in emerging economies the time to execute a trade is delayed by technological constraints. For example, in Argentina investors liquidate securities in one place but make payments in a different bank.[14] Second, a broader interpretation is to think about the participants in the market. In developed financial markets, there are large and active mutual funds, which are agents that trade more frequently than the rest of the market's participants. Because these funds are either small or nonexistent in emerging economies, this could imply lower liquidity. Finally, Bethune et al. (2017b) shows that private information about the valuation of the security creates informational rents and can reduce trading. Hence, their model predicts that markets will be more illiquid when there are larger informational rents, which might be the case for developing countries, for example, due to weaker credit bureaus.

[14]Recently in Argentina, BYMA—the local exchange market—is trying to unify these operations to increase the liquidity of the market.

4 Empirical Analysis: Slope of Liquidity Spreads

Previous research has shown that liquidity has stronger effects for long-term assets than shorter ones. For example, Longstaff et al. (2005) study 68 firms during 2001-2002 and control for the default component with Credit Default Swaps (CDS). They find that there is a strong positive relation between the non-default component of credit spreads and the time-to-maturity of the bond. Similarly, Edwards et al. (2007) and Bao et al. (2011) use trading data from secondary markets to estimate several measures of liquidity and find that bonds closer to maturity are more liquid. Gilchrist and Zakrajšek (2012) find that credit spreads are increasing in maturity after controlling for distance to default and bond-specific characteristics (amount outstanding, coupon rate, callable, industry fixed effects, and credit rating fixed effects). We contribute to the existing empirical literature by presenting a new empirical strategy and evidence on the slope of liquidity spreads with respect to maturity. Importantly, the strategy only uses credit spread data, which allow us to estimate outside of the US. In particular, we use data from Argentina and find that those liquidity spreads are steeper than in the US.

Data sources We consider corporate debt issuances in the US on the Mergent Fixed Income Securitiexs Database (FISD). We keep corporate bonds of domestic borrowers in local currency (i.e., US dollars) and with a fixed interest rate. We follow Gilchrist and Zakrajšek (2012) to define credit spreads that are not subject to the "duration mismatch" by constructing a synthetic risk-free security that mimics exactly the cash flows of the corresponding corporate debt instrument.[15]

Table 1 describes the data in the final sample.[16] Our sample period is January 2000 to December 2017. There are 994 issuers and 35,513 bond issuances; 23,182 bonds are rated and the median rating from Moodys is A2. On average, a firm that is issuing bonds in a given month makes 6.69 different issuances; however, there is a large variation across firms. The average maturity is 6.95 years with an average credit spread of 60 basis points. Again, note the large dispersion in maturity and credit spreads across issuances.

[15] We use the US Treasury yield curve estimated by Gürkaynak et al. (2007). Empirical results are similar to an alternative definition of spreads. For example, we find similar results when we define credit spreads as the difference in coupon rates between corporate and sovereign bonds of similar maturity.

[16] Our final sample considers the set of firms that in a given period issues two or more bonds of different maturities. In the benchmark specification we define the period as a day and perform robustness exercises for definitions at the week and month level. As the length of the period increases (from day to week to month), there are more issuances within each group allowing us to also include firm-period fixed effects. The reason for this sample selection is important for the identification and discussed later in detail. To ensure that our results are not driven by a small number of extreme observations, we trimmed the data at the top and bottom 1 percent.

Table 1: **Summary Statistics of Corporate Bond Characteristics**

Bond characteristic	Mean	Median	SD
# of Bond Issuances per Firm/Month	6.69	3.00	7.46
Maturity at Issue (years)	6.95	5.00	6.45
Coupon Rate (pct.)	3.28	3.70	2.70
Nominal Effective Yield (pct.)	3.34	3.74	4.10
Nominal Effective Treasury Yield (pct.)	2.73	2.50	1.60
Credit Spread (bps.)	60	59	369

Note: Number of issuers = 994; number of bonds = 35,513, of which 23,182 bonds are rated.

Empirical specification Let $s_{i,t,m}$ be the credit spread for a bond issued by issuer i, on period t, and at maturity m (note that this spread contains both default and liquidity components). Consider the set of firms that in a given period issues two or more bonds of different maturities. The main empirical specification is

$$s_{i,t,m_j} - s_{i,t,m_1} = \beta(m_j - m_1) + \gamma \mathbf{X}_{i,t} + \epsilon_{i,t,m_1,m_j}. \tag{18}$$

The coefficient β measures the slope of credit spread with respect to maturity. If $\beta > 0$, long-term spreads are larger than short-term ones. The controls $\mathbf{X}_{i,t}$ include time, industry, and/or credit-rating fixed effects.

We find a significant slope of credit spreads to maturity. The first three columns of the top panel of Table 2 show different specifications with time and/or industry fixed-effects.[17] When we include time and industry fixed effects (third column), the slope is about 5 basis points per year. We also exploit the variation within issuer-period assets. We define a "group" as all issuances of the same issuer in a given month and add a fixed-effect at the group level.[18] The fourth column shows that maturity differences are significant and quantitatively similar to the previous estimates.

4.1 Slope of Liquidity Spreads

The previous estimate cannot distinguish between default and liquidity components. To uncover the slope of liquidity spreads we propose and validate a novel identification scheme. We assume that default spreads are constant in maturity, so the difference between the short- and the long-term issuance identifies the slope of liquidity spreads. Theoretically, section 2 shows that

[17] We define time fixed-effects at the month level, but results are robust to alternative length periods.

[18] Results are robust for different definitions of period (month instead of week) and if we restrict the sample so that issuers have more than three, four, or five issuances per issuer-period.

if default arrives at a constant Poisson rate, then the credit spread due to default is constant in maturity. However, other theories of default (e.g., Merton, 1974; Duffie and Singleton, 1999) propose that default spreads can be either increasing or decreasing in maturity. More importantly, we perform several empirical exercises to validate the identification assumption. First, we use information on credit rating. Following Krishnamurthy and Vissing-Jorgensen (2012), we estimate Equation (18) only for the set of assets that are safe but illiquid and recover similar estimated coefficients. Second, we use sovereign and corporate CDS to show that default spreads have a smaller estimated slope than the estimated liquidity component. Third, we focus on the set of assets in which we can match corporate bonds with their corresponding CDS to measure the non-default component and show that it has a similar slope than the benchmark specification.

The underlying intuition for the validity of the identification is that liquidity spreads arise from idiosyncratic shocks to bond holders and, as shown by the model, the time-to-maturity is a key summary statistic for the costs of these shocks. If the asset is close to maturity the agent can exit the liquidity shock through repayment at maturity, and this liquidity cost is relatively low. However, if it is a long asset the liquidity cost is larger. On the other hand, when default occurs the time-to-maturity of the bond is not relevant for the recovery value, and therefore should not affect credit spreads. In fact, after default all bonds of different time-to-maturity enter together in the renegotiation process and usually holders of different bonds receive similar haircuts.[19]

Safe but illiquid bonds We now include information on credit ratings. First, we repeat the main estimation on the sample of rated bonds because not all bonds are rated and the smaller and more illiquid bonds are probably not in the sample. The result is in the first column of the second panel of Table 2. We find an estimated coefficient of about 4 basis points per year. The second column adds credit-rating fixed effects and finds a significant and quantitatively similar coefficient for maturity differences.

Next we do the estimation only on the sample of issuances rated above A. These assets represents the High Quality Market (HQM) of corporate bonds and are safe but illiquid assets, so we can abstract from default considerations for this asset class. Appendix E shows that expected credit losses and default rates are very small for these securities on average and were small during the 2008 financial crisis. For example, the expected credit losses of a security rated A in 2008 were only 0.37%. Another concern is that long-term assets are more likely to be

[19] Nevertheless, maturity may affect default spreads if default intensities depend on maturities. However, it is very hard to get direct empirical estimates of default intensities (see Campbell et al., 2008), and there is no empirical evidence for the term structure of default intensities. Hence, we assume that it is constant on maturity.

downgraded and then default. However, we also show in the appendix that five-years cumulative transitions probabilities for these securities are quite small. Hence, we can abstract from default considerations for this group of securities and interpret the estimates as the liquidity component. The third column shows that the coefficient on maturity is significant and quantitatively similar to the previous estimates. These results validate the identification assumption and suggest that the measured coefficients can be attributed to liquidity. The fourth column considers bonds only rated Aaa and also finds a significant and quantitatively similar coefficient, reinforcing the finding.

Corporate CDS The previous estimates rely either on the assumption that default spreads are constant with maturity or in the sample of safe but illiquid assets. On the empirical side, it is hard to get direct estimates of default intensities (e.g., Campbell et al., 2008). One common strategy is to use CDS to measure the credit spread due to default. However, it is worth noticing that CDS also trade in OTC markets, so CDS's prices also contain liquidity spreads and are not a pure measure of default. Nevertheless, for robustness we can look at corporate CDS and estimate how they change with maturity.

We use corporate CDS for US entities from Markit for the years 2000-2017.[20] Let $cds_{i,t,m}$ be the implied yield for the CDS for issuer i, at month t, and maturity m. We take as the short maturity the one-year CDS and compute the slope for CDS with maturities equal to two, three, four, five, seven, ten, fifteen, twenty, and thirty years. The empirical specification replicates the estimates on total credit spreads (Equation 18)

$$cds_{i,t,m_j} - cds_{i,t,m_1} = \beta(m_j - m_1) + \gamma \mathbf{X}_{i,t} + \epsilon_{i,t,m_1,m_j}. \tag{19}$$

As before, the coefficient β measures the slope of the default spread, and the controls $\mathbf{X}_{i,t}$ include time, industry, and/or firm-month fixed effects.

The third panel of Table 2 shows that for the five different specifications the estimated coefficients are significant. Quantitatively, for each additional year the spreads on CDS increase between 2 and 2.5 basis points. If we compare them with the slope of corporate yields (first panel), the slope of CDS represents between one-third and one-half of the total slope. Moreover, as corporate CDS also trade in OTC markets, this slope also contains liquidity spreads and are not a pure measure of default. In the next section we estimate the slope on sovereign CDS, which trade in more liquid markets, and find that the slope on sovereign CDS is smaller than that on corporate CDS.

[20] We use the end-of-month yields.

Table 2: **Empirical evidence: Liquidity Spreads Increasing in Maturity**

	(1)	(2)	(3)	(4)	(5)
			A. Credit Spreads Differential		
Maturity difference	7.876***	6.247***	4.877***	5.836***	
	(0.234)	(0.359)	(0.299)	(0.325)	
Observations	23,614	23,614	23,614	19,320	
R-squared	0.046	0.104	0.173	0.858	
FE	No	Time	Time, Industry	Firm-Time	
			B. Safe but Illiquid Bonds		
Maturity difference	3.545***	3.684***	4.046***	3.620**	
	(0.261)	(0.245)	(0.336)	(1.081)	
Observations	15,471	15,471	11,956	867	
R-squared	0.103	0.135	0.126	0.212	
FE	Time	Time, Rating	Time	Time	
Sample	All	All	Aaa-A	Aaa	
			C. Credit Default Swaps		
Maturity difference	2.494***	2.412***	2.359***	2.073***	2.219***
	(0.046)	(0.230)	(0.224)	(0.201)	(0.042)
Spread short				-0.318***	
				(0.013)	
Observations	1,119,540	1,119,540	1,119,540	1,119,540	1,119,540
R-squared	0.003	0.023	0.027	0.624	0.860
FE	No	Time	Time, Industry	Time, Industry	Firm-Time
			D. Direct Measure of Liquidity		
Maturity difference	13.035**	9.215**	5.247***		
	(3.272)	(2.672)	(0.885)		
Observations	2,479	2,479	2,479		
R-squared	0.180	0.361	0.903		
FE	Time	Time, Industry	Firm-Time		

Note: The first panel shows the estimates of the benchmark specification. The second panel estimates the model on the set of *safe but illiquid bonds*. The third panel shows that the slope on CDS is smaller than in the benchmark specification. The last panel uses CDS to compute a direct measure of liquidity. See text for details. Standard errors in parentheses, *, **, *** denote statistical significance at the 10, 5, and 1 percent level, respectively.

Non-default component Finally, we match corporate bond issuers with the CDS data, so we get a direct measure of the liquidity component (i.e., we match the corporate bond with the CDS for the same issuer at the same maturity). In the data we don't have CDS with the exact same maturity, so we do a linear interpolation between the yields of the closest two maturities, but results are robust to alternative interpolation schemes. Define liquidity as

$liq_{i,t,m} = s_{i,t,m} - cds_{i,t,m}$ and estimate

$$liq_{i,t,m_j} - liq_{i,t,m_1} = \beta(m_j - m_1) + \gamma \mathbf{X}_{i,t} + \epsilon_{i,t,m_1,m_j}. \tag{20}$$

The last panel of Table 2 shows that the estimated coefficients on the maturity difference are significant for alternative specifications. If we focus on the third column, which includes a firm-month fixed effect, the coefficient is about 5 basis points per year. Note that this coefficient is similar to those on the first panel in which we assume that the default component is constant in maturity. We conclude that a significant fraction of the slope of credit spreads can be attributed to liquidity considerations.

4.2 Estimates for Argentina and Quantitative Targets

In this section we exploit the identification assumption to extend the empirical estimates on liquidity to less-liquid markets and validate the results using sovereign CDS. In particular, we consider issuances in Argentina. We look at all active corporate bonds issued in 2017 in the domestic market (*Mercado Abierto Electronico, MAE*).[21]

We find that credit spreads are steeper in Argentina than in the US. We estimate Equation 18 for Argentina and the US in 2017. The first and third columns on the first panel of Table 3 report that the slope of credit spreads is about 6 basis points per year in the US, while it increases to 50 basis points per year in Argentina. Next, we use sovereign CDS data and argue that a large fraction of these slopes can be attributed to the liquidity component of credit spreads.

Sovereign CDS For robustness we look at sovereign CDS and estimate how they change with maturity. There are two reasons to consider sovereign instead of corporate CDS. First, we only have data on sovereign CDS for Argentina. Another advantage of using sovereign CDS is that they are more liquid than corporate CDS so the bias due to liquidity should be smaller. We use data from Markit for the year 2017 and find that the estimated slope is smaller for sovereign CDS than for credit spreads. The second column of Table 3 shows that one additional year of maturity increases the CDS yield by less than 1 basis point. If we compare with the slope of corporate yields (first column), the slope of CDS represents about one-sixth of the total slope. This result also holds in Argentina. The fourth column of Table 3 shows that the estimated coefficient for sovereign CDS is about one-fifth of the total slope for corporates, a similar ratio

[21]We keep issuances in local currency, with 100% amortization and with the interest rate expressed as a spread on the *Badlar* rate (the reference short-term rate in Argentina). These are floating interest rates bonds with a fixed spread, so the credit spread is just the spread on the Badlar rate because non-arbitrage implies that agents can swap the variable Badlar rate for a fixed rate.

Table 3: **US and Argentina**

A. Spread differentials for corporate and sovereign CDS

	US		Argentina	
	Corporate	Sovereign CDS	Corporate	Sovereign CDS
Maturity difference	6.462***	0.895***	50.04***	9.529***
	(0.947)	(0.0357)	(7.377)	(0.104)
Observations	2,102	99	35	99
R-squared	0.019	0.728	0.930	0.577
FE	Time	Time	Time	Time

B. Maturities

	US		Argentina	
	Median	Mean	Median	Mean
Maturity	4.00	5.37	2.25	2.35
Short maturity	1.50	2.33	1.50	1.52
Long maturity	5.00	6.38	3.00	3.17
Maturity spread	2.58	4.03	1.5	1.64

Note: Data for 2017. Standard errors in parentheses, *, **, *** denote statistical significance at the 10, 5, and 1 percent level, respectively.

to that for the US. We conclude that credit spreads are steeper in Argentina than in the US and that a large fraction of this slope can be attributed to liquidity considerations.

Quantitative targets For the quantitative evaluation it is useful to summarize the empirical results with the effect at the average maturity (second panel of Table 3). We attribute to the liquidity spread the total slope on corporate bonds net of the slope on sovereign CDS. In the US the average maturity is 5.37 years, and when maturity increases from 2.33 to 6.38 years, the estimated coefficients imply an increase on total credit spreads of 26.04 basis points and 3.60 for sovereign CDS. Hence, we attribute an increase of 22.43 basis points to the liquidity component. On the other hand, in Argentina the average maturity is 2.35 years, and when maturity increases from 1.52 to 3.17 years, credit spreads increase by 82.07 basis points and CDS by 15.62, so we attribute 66.44 to the liquidity component. Note that these moments capture the slope of the credit spread with respect to maturity but not the level. In the quantitative evaluation we use moments for the US to calibrate the model, and validation exercises show that the level of the liquidity spread predicted by the model is consistent with additional estimates for the US. The estimates for Argentina discipline counterfactual experiments.

One caveat about the mapping of the model to the data is that in the benchmark model there is only one bond issuance, while to recover the cost of liquidity in the data we consider the spread between two bond issuances. However, we argue that the empirical estimates recover the cost of finance at different maturities, which is the relevant measure for the model irrespective of the number of issuances on the same day. There are several extensions of the model that can capture the multiple issuances observed in the data but it is not key for the main point of the paper. For example, a realistic assumption is that the life cycle of the project is more complicated than the proposed in the model and that projects generate different cash flows at different points in time. In this case, it would be optimal to issue bonds of different maturities at period zero to match the cash-flow payments of the project.

5 Quantitative Analysis

This section presents a quantitative evaluation of the theory presented in Section 2. We use the estimates from Section 4 for the US to discipline the calibration of the model and exploit additional measurements as validation exercises. Next, the estimation for Argentina provides guidance for counterfactual experiments. Overall, credit frictions explain about 50% of maturity differences between the US and Argentina and have detrimental effects for productivity and the real economy.

5.1 Calibration for the US

We match moments from the US corporate debt market. Some parameters can be calibrated "externally," while others must be calibrated "internally" from the solution of the model. We proceed in five steps to calibrate each of the parameters of the model. Table 4 summarizes the parameter values and the target moments.

Externally calibrated One unit of time is equivalent to one year. The discount factor is set to $\rho = 0.02$ as it is standard in the literature (e.g., He and Milbradt, 2014), and the default rate is $\delta = 0.03$ to match the default rate of speculative-grade firms (Moody's, 2015). Normalize the measure of new firms by $\mu^0 = 1$. For the secondary market assume a constant-return-to-scale Cobb-Douglas matching function with elasticity $\alpha = 0.5$. Further, assume that the bargaining power of sellers is $\gamma = 0.5$.

Matching technology First, target an expected time to sell of two weeks (He and Milbradt, 2014) so that $\frac{1}{\lambda} = \frac{2}{52}$. Recall that $\lambda = A\theta^{\alpha-1}$ and normalize $\theta = 1$ as we do not have reliable information on the market tightness. Hence, the matching efficiency has to be equal to $A = 26$ to match the expected time to sell.

Table 4: **Parameters and moments**

Parameter		Value	Target/source	Model	Data
Financial sector					
Matching efficiency	A	26.00	Expected time to sell	2.00	2.00
Intensity of liquidity shocks	η	0.58	Turnover rate	0.57	0.57
Holding cost	h	0.29	Slope liquidity	22.44	22.43
Search cost	c	0.28	Free entry		
Production sector					
$F(\tau) = Z\tau$	Z	1.91	Maturity	5.37	5.37
$I(\tau) = \kappa\frac{1-e^{-\rho\tau}}{\rho}$	κ	1.00	Normalization		
Matching					
Share of sellers	α	0.50	Normalization		
Bargaining power of sellers	γ	0.50	Normalization		
Others					
Discount factor	ρ	0.02	He Milbradt (2014)		
Default rate	δ	0.03	Moodys (2015)		

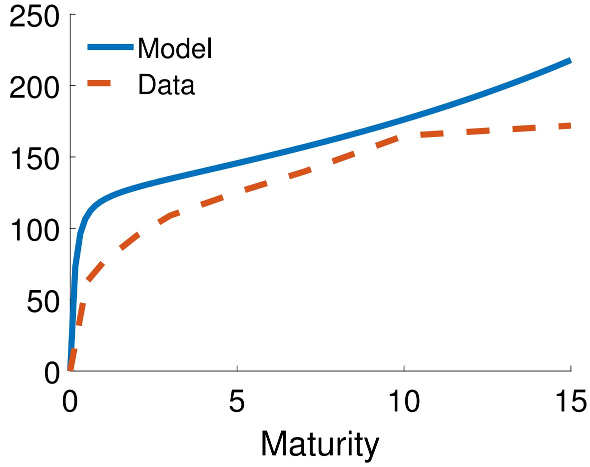
Turnover Second, target an annual turnover rate of 57% (He and Milbradt, 2014; Chen et al., 2017). Turnover is approximately equal to $(\eta^{-1} + \lambda^{-1})^{-1}$, so we can directly calibrate $\eta = 0.58$ to match this moment.

Liquidity Third, we target the slope of the liquidity spread from Table 3. The median maturity for the short and long issuances are 2.33 and 6.38 years, respectively. We target the increase in the liquidity spread between these two maturities to be 22.43 basis points; i.e., $cs^{liq}(6.38, \lambda) - cs^{liq}(2.33, \lambda) = 22.43$. Recall that this target controls for the slope attributed to liquidity using the implied slope on CDS. Note that the only parameter missing to measure this moment is the holding cost h, which is set equal to 0.29 to match the target.

Maturity Fourth, target a maturity choice of 5.37 years from Table 3. By inspection of equation (17) note that we can normalize $\kappa = 1$ and internally calibrate ζ to match the target on maturity. Recall that $F(\tau) = Z\tau$ and $Z = \frac{\zeta}{\rho+\delta}$. This moment requires $Z = 1.91$.

Free-entry Finally, the free-entry condition delivers the value of the search cost c such that in equilibrium $\theta = 1$ and the free-entry condition holds. In particular, the entry cost has to be equal to 0.28.

Figure 3: **Liquidity spread: Data versus Model**



Note: Liquidity spread in the data is computed as the spread between treasury and corporate yield curves. See text for details.

5.2 Validation and Empirical Evidence

The predictions of the model are consistent with several dimensions of the data not directly targeted in the calibration.

Yield curve To measure the term-structure of liquidity spreads we compare assets that have similar risk of default but that trade in different markets. In particular, we focus on the positive spread between the yields on Treasuries and safe but illiquid corporate bonds, colloquially known as the *convenience yield* (e.g., Krishnamurthy and Vissing-Jorgensen, 2012). In Section 4.1 we show that this spread increases with maturity. For the liquid asset we consider the zero-coupon yield curve for Treasuries. For the illiquid asset we use the zero coupon yield curve for the high-quality market of corporate debt. These securities only include bonds with rating above A, such that we can abstract from default risk, but such bonds are subject to secondary market illiquidity (default credit losses are minimal, see Appendix E). The red dotted line in Figure 3 shows that the spread between Treasuries and corporate bonds is positive and increases with maturity.

The calibration target is the slope of the liquidity spread at a specific point. Figure 3 shows that the model also predicts the level of the liquidity spread yield curve at different maturities, which was not a target of the calibration. For example, the model-implied liquidity spread at a maturity of 10 years is 176 basis points in the model and 164 in the data. Hence, the calibrated model is consistent with both the level and the slope of the liquidity spread yield curve.

Liquidity and debt maturity There is empirical evidence supporting the main mechanism of the paper: When financial markets are more liquid, firms issue bonds of longer maturities. First, Saretto and Tookes (2013) compare issuances of firms with and without CDS and argue that securities of companies with CDS trade in more-liquid financial markets. They find that firms with CDS increase maturity of their issuances between 0.68 and 1.79 years relative to those without CDS. To map these estimates to the model, assume that bonds without CDS trade in a frictional market as the benchmark model and that those with CDS trade in a centralized secondary market.[22] Table 5 shows that in the model, maturity increases by 1.7 years, which is close to the non-target estimate of Saretto and Tookes (2013) and further validates the estimation.

Second, Cortina Lorente et al. (2016) study firms in emerging economies issuing bonds in both domestic and international markets. The paper finds that maturity increases by 1.6 years for foreign issuances relative to domestic ones. This evidence also corroborates the mechanism of this paper, as foreign markets are more liquid than domestic ones for emerging market firms. To map this estimate to the model we assume that the domestic market has the same liquidity as the benchmark calibration, while the international market is a centralized one. Table 5 shows that the model predicts an increase of maturity similar to the data counterpart.

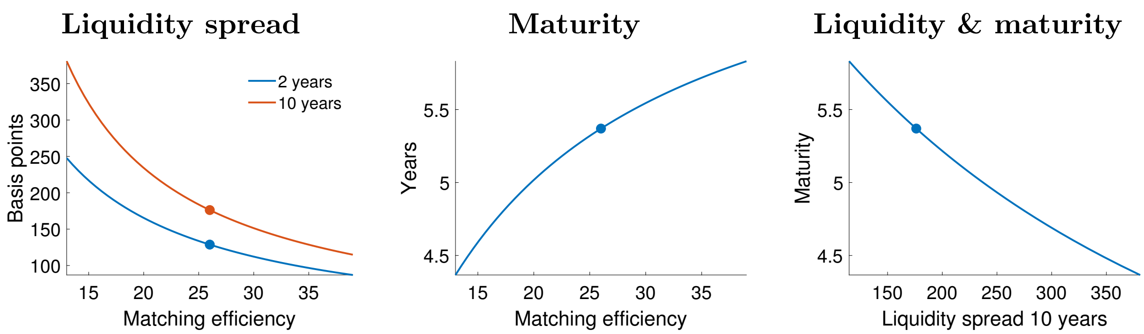
Table 5: **Validation: Debt Maturity For Different Markets**

	Data	**Model**
Maturity difference: CDS	0.68-1.79	1.70
Maturity difference: International issuances	1.6	1.70

Term premia, debt issuances, and investment The second mechanism of the paper argues that when the term premia increases, firms choose to invest in shorter-term projects. Dew-Becker (2012) uses data from the US and concludes that the duration of investment decreases when the term premium increases. Yamarthy (2016) also finds that profitability and investment rates are higher when firms shift their long-term debt ratio to longer maturities. Foley-Fisher et al. (2016) use cross-sectional variation to show that companies with more dependence on long-term debt benefit more when the yield curve flattens. Garicano and Steinwender (2016) use firm-level data from Spanish firms and find that long-term investments fell more than shorter ones after the 2008 financial crisis, which can be interpreted through the lens of the model as a financial markets freeze. Therefore, the empirical evidence supports that variations in term premia can affect investment and profitability of the firm as predicted by the model.

[22] In the data those bonds without CDS probably trade in a more illiquid market than the benchmark, while those with CDS trade in a frictional market. However, to the best of my knowledge, we do not have data to estimate the exact liquidity of these two different groups.

Figure 4: **Experiments: Matching efficiency**



Note: Effects of different matching efficiencies on liquidity spread and maturity choices.

5.3 Experiments: Lower Matching Efficiency

We can use the calibrated model to evaluate the importance of trading frictions for maturity choices, investment, and the aggregate economy. We consider variations in the matching efficiency A while we keep the rest of the parameters at the calibrated values. Note that variations in the search cost c yield similar results.

The equilibrium liquidity λ diminishes under a lower matching efficiency A because of two effects. First, for a given market tightness, the selling intensity reduces. Second, the incentives to enter into the secondary market also reduce, and the market tightness increases (i.e., more sellers per buyer). This second effect further diminishes the equilibrium liquidity. Figure 4 shows how changes in the matching efficiency affect the liquidity spreads and the maturity choices. The first panel considers the liquidity spread at maturities of 2 and 10 years. Note that the change in the liquidity spread due to more-severe trading frictions is more pronounced for the long-term asset as predicted by Proposition 1. For example, when the matching efficiency reduces from $A = 26$ to $A = 20$, the liquidity spread for a two-year bond increases from 128 to 165 basis points, while the spread for the 10-year bond increases from 176 to 234 basis points.

Firms choose to borrow and invest at shorter horizons when trading frictions are more severe because long-term finance becomes too expensive. The second panel of Figure 4 shows that firms choose to reduce their maturity from 5.36 to 5.01 years when the matching efficiency reduces from $A = 26$ to $A = 20$. The third panel shows the relationship between the liquidity spread at 10 years and the choice of maturity. Of course, the model is non-linear. However, as an approximation the figure reveals that firms choose to reduce the maturity by about 6 months when the liquidity spread at 10 years increases by 100 basis points. Hence, variations in trading frictions affect the level and slope of the yield curve, which has quantitatively large influences on the duration of investment projects.

Counterfactual for Argentina The estimates from Table 3 help us discipline a possible value for the matching efficiency in Argentina. The median maturities for the short and long issuances are 1.52 and 3.17 years, respectively. Hence, we target the increase in the liquidity spread between these two points to be 67 basis points; i.e., $cs^{liq}(3.17, \lambda) - cs^{liq}(1.52, \lambda) = 67$.[23] To match this target, the matching efficiency reduces from 26 to 8.03. Moreover, the entry decision also adjusts and the market tightness (defined as the ratio of sellers-to-buyers) increases from 1.0 to 1.18. As a result, the liquidity spread increases and the yield curve becomes steeper.

Table 6 shows that firms in an economy like the US but with the financial system of Argentina would borrow at a maturity of 3.6 years. In the data, the average maturity for Argentina is 2.4 years. Therefore, trading frictions can explain about one-half of the maturity differences in the data.

Real economy When long-term finance becomes more expensive, firms tilt their maturity choices toward the short-end. On the real side of the economy, this implies that entrepreneurs invest in shorter-term projects, which have lower productivity and, therefore, lower aggregate output. Table 6 shows that aggregate output falls by 30% under the Argentinean financial system. In the data this difference is about 60%. Hence, trading frictions can account for about one-half of the differences in output between US and Argentina due to changes in the duration of investment and aggregate productivity.

The effects on the real economy depend on assumptions about the production technology. Section 7.3 considers an alternative production model that combines labor with productivity and shows that for different parameter values the change in the liquidity spread generates a large effect on aggregate output (between 17% and 25%).

Table 6: **Counterfactual: US and Argentina**

	US		Argentina	
	Data	Model	Data	Model
Matching efficiency		26.00		8.03
Market tightness		1.00		1.18
Liquidity (bps)				
Increase 6.37 - 2.33 years	22	22	163	134
Increase 3.17 - 1.52 years	9	10	67	67
Maturity (years)	5.4	5.4	2.4	3.6
Output	1.0	1.0	0.4	0.7

[23] Recall that the slope is computed as the total slope on credit spreads net of the slope on sovereign CDS.

Alternative measures of liquidity The experiment for Argentina relies on the empirical estimates. Appendix D considers an alternative empirical strategy that does not rely on the estimates of Section 4. We calibrate the model for the US and discipline the liquidity frictions by the difference between Treasuries and the high-quality market of corporate bonds. Next, the net interest margin (a common measure of intermediation in the literature) disciplines the non-default component of credit spreads across countries. Overall, counterfactual experiments support the same results: Trading frictions are an important driver of maturity choices across countries.

6 Policy Intervention

While it is outside the scope of this paper to fully characterize the optimal policy, we explore the effects of a government intervention designed to increase the liquidity of financial markets, namely *Government-sponsored intermediaries* (GSIs). The intervention consists of government agents acting as intermediaries in the secondary market that buy and sell bonds at different prices than in private bilateral meetings. The government charges a distortionary profit tax on the corporate sector to finance the policy. We interpret this exercise as a lower bound for the effects of government interventions. In particular, we assume that the government cannot avoid the search frictions or holding costs so that government agents face the same constraints as private agents. However, public agents can participate in secondary markets and take different actions than they do in the private sector.

This intervention has some similarities with financial policies in the US and in emerging economies. First, Government Sponsored Enterprises (GSEs), such as Freddie Mac and Fannie Mae, are institutions intended to improve credit flows to households (see Fieldhouse et al., 2017, for a recent review). The proposed policy is similar to GSEs but targets the efficiency of credit to the corporate sector instead of households. Second, during the 2008 financial crisis, the central bank performed large-scale asset purchases (known as QE1, QE2, and QE3). Gertler et al. (2013) argue that this policy was effective because of limits to arbitrage in private intermediation. In a similar way, GSIs will intermediate assets in financial markets.

There are examples of policies implemented or proposed in emerging economies that also share several features of GSIs. First, "priority-sector lending" in India requires banks to lend at least 40% of their net credit to the "priority sector" and also establishes specific targets for different sub-sectors (which include agriculture and small-scale industry, among others).[24] An

[24] For details see Banerjee and Duflo (2014) and Reserve Bank of India master circular of July 2015, "Priority Sector Lending Targets and Classification" available at https://rbi.org.in/scripts/bs_viewmascirculardetails.aspx?id=9857. Importantly, most of the banks in India are in the public sector; hence, they could be interpreted as the government agents in the model.

interesting feature of this policy is that banks have to trade to meet all the specific targets, and therefore the policy increases the liquidity of the secondary market of assets in the priority sector (a similar motive for trade as in the Fed Funds Market in the US; see Afonso and Lagos, 2015). One difference between "priority-sector lending" and GSIs is that the policy in India is for short-term debt (usually below one year), while this paper argues that liquidity is relatively more important for long-term assets. Hence, we propose similar policies but for the long-end of the market.

Another example comes from Brazil, where a private capital markets association (Anbima) has launched a project to facilitate long-term financing. Some of the measures aim to increase secondary market liquidity by the creation of a "Liquidity Improvement Fund," in which private agents manage public resources to act as market makers, similar to the proposed GSIs.[25]

6.1 Government-Sponsored Intermediaries

The government agency intermediates assets in the secondary market to improve the liquidity of financial markets. Government agents are subject to the same idiosyncratic risk of holding costs as private agents, so they act as both buyers and sellers in the secondary market.[26] One possible interpretation for these idiosyncratic shocks for public agents can be balance sheet requirements, such as the priority sector lending in India discussed before. However, the government can choose different prices than those charged in private meetings. If they buy (sell) at a high (low) price they will run a deficit, which is financed by distortionary taxes on the corporate sector.

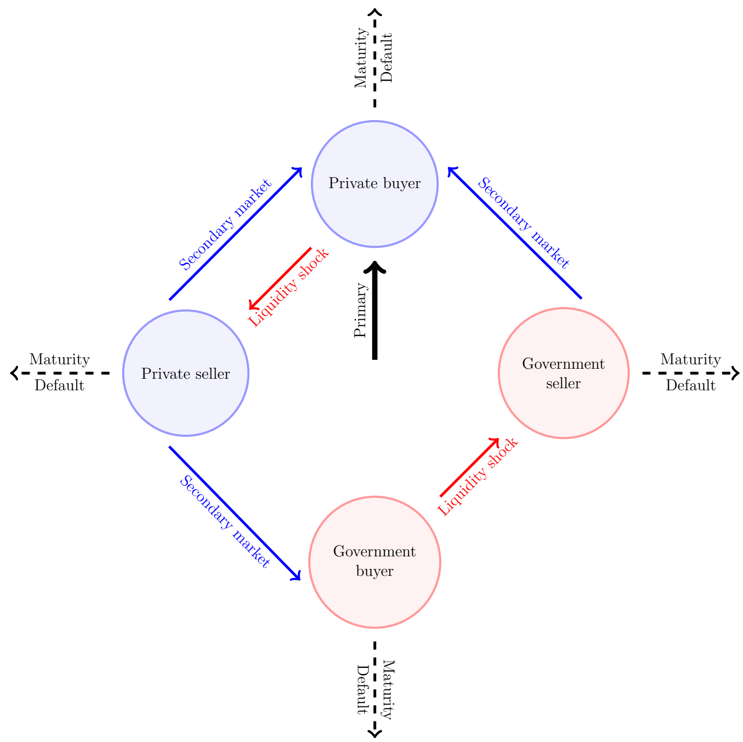
Figure 5 shows a schematic representation of the model with GSIs. Note that private sellers can now sell both to private and government agents. Moreover, private buyers can now buy in the secondary market from either private or government sellers. In this section, we describe the key features of the model with GSIs while Appendix C.1 contains additional details.

The government has four instruments: the size of GSIs, prices for buying and selling for their trading agents, and the corporate tax rate. The objective is to maximize aggregate steady-state welfare subject to running a balanced budget and equilibrium conditions. Recall that both primary and secondary financial markets are competitive—i.e., participants make zero profits in expectation. However, the production sector—i.e., the borrowers—have positive profits in equilibrium. Hence, we define the objective of the government as maximizing the value of the corporate sector.

[25] See Park (2012) for further details.

[26] The formulation of government agents is similar to Aiyagari et al. (1996).

Figure 5: **Schematic representation of GSIs**



Matching There is random matching between sellers and buyers. In the benchmark policy we assume that both government and private agents have the same efficiency to find a counterpart. However, for robustness exercises of Section 6.4, we consider a general formulation in which government and private agents may differ in the efficiency to find a counterpart. Let $e^{i,j}$ be the efficiency for $i = p, g$ (private and government agents, respectively) of $j = b, s$ (buy and sell, respectively). In the benchmark model we assume that $e^{i,j} = 1$ for all i, j.

The total mass of sellers, μ^s, is composed of private and government agents. Private sellers are $\mu^{p,s} = \int_0^\tau e^{p,s}\mu^{p,l}(y)dy$, where $\mu^{p,l}(y)$ is the measure of low-valuation private agents holding an asset and willing to sell. Similarly, government sellers are $\mu^{g,s} = \int_0^\tau e^{g,s}\mu^{l,g}(y)dy$, where $\mu^{l,g}(y)$ is the measure of low-valuation government agents holding an asset and willing to sell. The total measure of buyers includes private buyers $\mu^{p,b}$, determined by a free-entry condition, and government buyers $\mu^{g,b}$, which is a policy instrument chosen by the government. Hence, $\mu^b = e^{p,b}\mu^{p,b} + e^{g,b}\mu^{g,b}$.

The market tightness is $\theta = \frac{\mu^s}{\mu^b}$ which affects the buying and selling intensities $\beta = A\theta^\alpha$, and $\lambda = A\theta^{\alpha-1}$, respectively. Let λ^{s-b} be the intensity at which a seller of type $s = p, g$ meets a buyer of type $b = p, g$. Similarly, let $\beta^{s-b}(y)$ be the intensity at which a buyer of type $b = p, g$ meets a seller of type $s = p, g$ with an asset of time-to-maturity y. The matching technology implies that

$$\lambda^{p-p} = \lambda e^{p,s} e^{p,b} \frac{\mu^{p,b}}{\mu^b} \qquad \lambda^{p-g} = \lambda e^{p,s} e^{g,b} \frac{\mu^{g,b}}{\mu^b} \qquad \lambda^{g-p} = \lambda e^{g,s} e^{p,b} \frac{\mu^{p,b}}{\mu^b}$$

$$\beta^{p-p}(y) = \beta e^{p,s} e^{p,b} \frac{\mu^{l,p}(y)}{\mu^s} \quad \beta^{g-p}(y) = \beta e^{g,s} e^{p,b} \frac{\mu^{l,g}(y)}{\mu^s} \quad \beta^{p-g}(y) = \beta e^{p,s} e^{g,b} \frac{\mu^{l,p}(y)}{\mu^s}$$

Finally, we have to specify what happens after a meeting between a government buyer and a government seller. The idea is to interpret the government as a large player and private agents as atomistic. However, for tractability, we assume that all investors can hold either zero or one asset. To bypass this restriction, we assume that a government seller cannot trade with a government buyer; i.e., $\lambda^{g-g} = 0$. Note that this is a conservative assumption as the cost of the policy is smaller if intra-government trades can occur. In fact, Section 6.4 solves the model with this type of trade and shows that there are larger effects.

Prices in secondary markets There are three types of meetings in secondary markets. Let $P^{S,s-b}(y)$ be the price when a seller of an asset with time-to-maturity y of type $s = p, g$ meets a buyer of type $b = p, g$. In a meeting between private agents, the price is determined by Nash Bargaining in which the seller has bargaining power γ so

$$P^{S,p-p}(y) = D^L(y) + \gamma(D^H(y) - D^L(y)).$$

The prices that involve either a government buyer or seller are determined by the government. In the quantitative solution we restrict prices to be in the following parametric family

$$P^{S,g-p}(y) = D^L(y) + \gamma^{g,s}(D^H(y) - D^L(y)), \tag{21}$$

$$P^{S,p-g}(y) = D^L(y) + \gamma^{g,b}\left(D^H(y) - D^L(y)\right), \tag{22}$$

and let the government choose $\gamma^{g,s}$ and $\gamma^{g,b}$ in $[0, 1]$. Note that prices are similar to those in private meetings but that the government can choose a different bargaining power. As we will show latter, it is optimal to set $\gamma^{g,s} = 0$ and $\gamma^{g,b} = 1$. This implies that the government gives all the bargaining power to the private sector—i.e., the government buys at a *high* price and sells at a *low* price.

Of course, this is an important restriction on government prices, but it follows from the objective of finding a lower bound on the effects of the policy. For example, one can use the model with segmented markets presented in Section 7.4 and allow the government to set different prices according to maturity. However, we will show that even without this flexibility, the effects of GSIs are quantitatively significant and the extension of targeting prices according to maturity is likely to improve the results from the lower bound identified in this exercise. Importantly, note that this alternative specification would work through the same channel as the mechanism described in the benchmark policy.

Private valuations The value of holding an asset for a high-valuation private agent is equivalent to the benchmark model, Equation (9). However, the value of a low-valuation private agent is different as now the agent can sell the asset to both private and government buyers. Under the government prices specified in (21), the price that the government offers is equivalent to that offered in private meetings but in which the seller has a different bargaining power. Hence, the value for a low-valuation agent is equal to the benchmark model, Equation (10), with an augmented selling intensity: $\lambda = \lambda^{p-p}\gamma + \lambda^{p-g}\gamma^{g,b}$.

Let λ^{GSI} and λ^{EQ} be the equilibrium liquidity in the economy with and without GSIs, respectively. If $\lambda^{GSI} > \lambda^{EQ}$, Lemma 2 implies that the liquidity spread will be lower in an economy with GSIs. However, borrowers have to pay a distortionary tax to finance the intervention. Hence, ex-ante, we don't know if the policy increases welfare for borrowers.

Private buyers can meet with private and government sellers. The free entry condition is

$$c = \int_0^\tau \beta^{p-p}(y)\left(D^H(y) - P^{S,p-p}(y)\right)dy + \int_0^\tau \beta^{g-p}(y)\left(D^H(y) - P^{S,g-p}(y)\right)dy.$$

Cost of GSIs The government runs a balanced budget. The constraint is

$$\begin{aligned} &\mu^f(\tau)x^c f(\tau) + \left[\mu^{g,h}(0) + \mu^{g,l}(0)\right] + \lambda^{g-p}\int_0^\tau \mu^{g,l}(y)P^{S,g-p}(y)y \\ =&\mu^{g,b}c + \mu^{g,b}\int_0^\tau \beta^{p-g}(y)\,P^{S,p-g}(y)dy + h\int_0^\tau \mu^{g,l}(y)dy. \end{aligned} \tag{23}$$

The left-hand side of Equation (23) represents the government's income. First, the government charges a proportional corporate tax x^c to producing firms μ^f, where flow profits are $f(\tau) = z(\tau)$. Second, some of the securities held by government agents mature. Third, some low-valuation government agents sell the securities to the private sector.

The right-hand side of Equation (23) captures the expenditures. A measure $\mu^{g,b}$ of agents are searching in secondary markets, and some of them buy a bond. Moreover, some government agents are low-valuation and have to pay the holding cost h.

Optimal policy The objective of the government is to maximize steady-state profits of the production sector subject to the equilibrium conditions and the budget constraint (23)[27]

$$\max_{x^c, \mu^{g,b}, \gamma^{g-b}, \gamma^{g-s}} \mu^f(\tau) e^{-(\rho+\delta)\tau} \left((1-x^c) F(\tau) - I(\tau) e^{r(\tau)\tau} \right) \quad \text{s.t. (23) and equilibrium conditions.}$$

GSIs cause both a direct and an equilibrium effect. On the one hand, a larger intervention needs higher taxes, which lower welfare. On the other hand, if the policy increases the equilibrium liquidity, credit spreads for long-term borrowing decline, which benefits borrowers. Therefore, the optimal policy solves the trade-off between these two effects.

6.2 Optimal GSIs in the US

First, consider the optimal policy under the calibration for the US. The bargaining power when the government acts as a buyer, $\gamma^{g,b}$, directly affects the value of low- and high-valuation private agents. The optimal policy sets $\gamma^{g,b} = 1$ so private sellers get more gains from trade when trading with the government. This generates a direct effect on increasing the value of private agents in the financial sector and reduces financial costs for the production sector.

The bargaining power when the government acts as a seller, $\gamma^{g,s}$, has a direct effect on the incentives of private agents to search in the secondary market. The optimal value is $\gamma^{g,s} = 0$. Given the results in this section, the next exercises set $\gamma^{g,b} = 1$ and $\gamma^{g,s} = 0$ and let the government choose $\mu^{g,b}$ and the tax rate.[28]

Finally, the measure of government agents searching in the secondary markets is optimally chosen to maximize the welfare gains. If $\mu^{g,b} = 0$, the economy is equivalent to no intervention, while as $\mu^{g,b}$ increases, the tax rate also increases to balance the budget. Under the optimal policy, there is an increase in liquidity, which generates a drop on the five-years spread from 146 to 101 basis points. As a result, the optimal maturity increases from 5.37 to 5.76, and the welfare gains are about 4.74% (first and second row of Table 7).

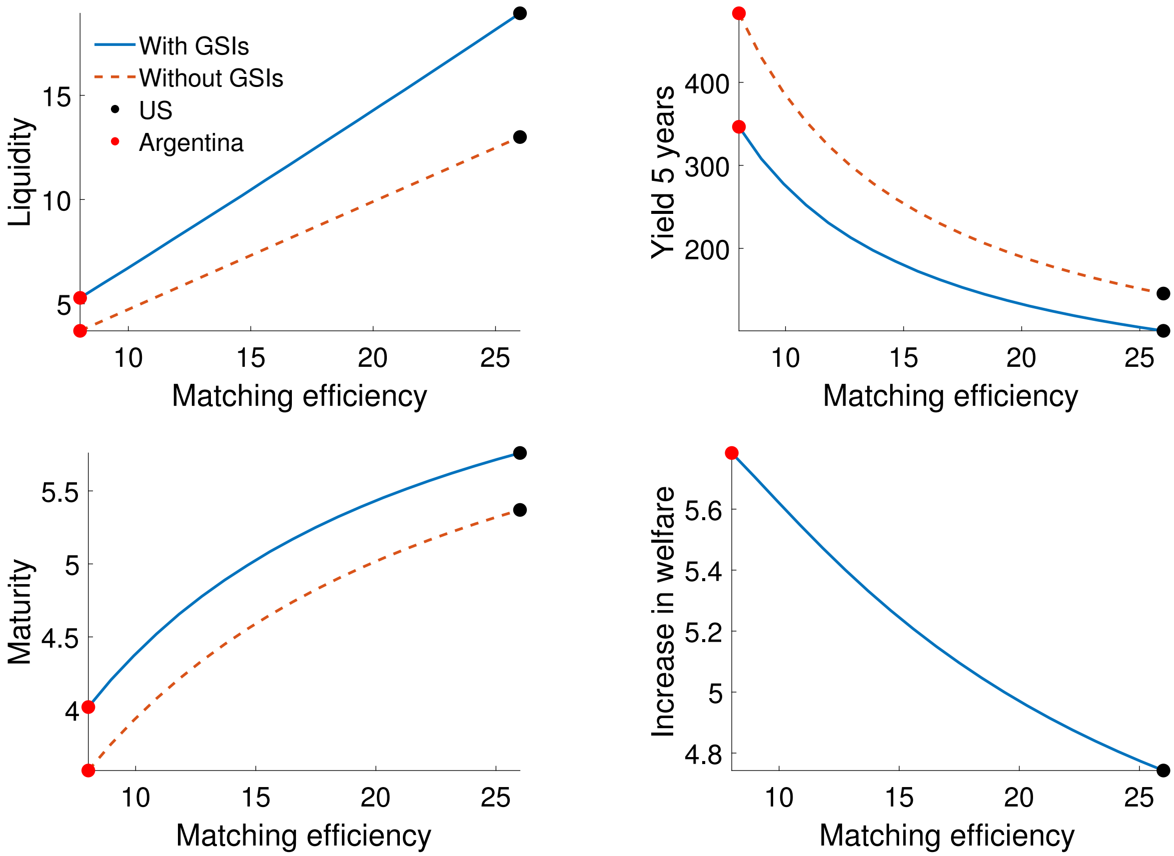
6.3 GSIs for Different Matching Efficiencies

To evaluate GSIs for economies with higher trading frictions, consider variations in the matching efficiency. We set the lower and higher value of A to match the calibration for Argentina and the US, respectively.

[27] We consider steady-state welfare because the transitions involve manipulating the boundary conditions of the distributions. However, note that this is a conservative assumption because during a transition old generations holding a security issued before the intervention are better off because asset prices increase.

[28] In fact, in all exercises we verified that if the government can choose bargaining power then they choose these values. However, this restriction simplifies the description of the results without adding additional intuition.

Figure 6: **GSIs: Effects for different trading frictions**



Note: Results with and without GSIs for different trading frictions.

The intervention has non-linear effects across countries. Figure 6 compares economies with and without GSIs for different levels of trading frictions. The top-left panel shows that the policy is more efficient at improving liquidity of financial markets when search frictions are relatively low (i.e., higher matching efficiency). However, the right-top panel shows that the flattening of the yield curve due to the policy is more effective when there are larger frictions (i.e., lower matching efficiency). In an advanced financial market, the marginal effect of an increase in liquidity is smaller than in a less-developed financial system. Hence, even though the improvement in liquidity is lower in emerging markets the consequences might be larger.

The bottom-left panel shows that GSIs increase the equilibrium maturity of corporate debt by about 4.7 months in the US. Note that this effect is larger in less-developed financial markets. For example, in a system similar to Argentina, GSIs increase the maturity of corporate debt by 5.2 months. Finally, the bottom-right panel shows that the increase in welfare due to GSIs is 4.74% for the US and 5.78% for Argentina.

6.4 Robustness: Alternative Policies

The intervention considered so far should be thought of as a lower bound on the effects of GSIs. Table 7 explores alternative assumptions that can improve the effects of government interventions for two levels of financial frictions. The first panel considers a matching efficiency at the level calibrated for the US, while the second panel considers an economy with trading frictions similar to Argentina.

First, consider government agents that are more efficient at searching for counterparts. The third and fourth rows of each panel of Table 7 show the result of increasing the search efficiency of government agents by 10% and 50%, respectively (i.e., $e^{g,b} = e^{g,s} = 1.1$ and $e^{g,b} = e^{g,s} = 1.5$, respectively). Overall, the results show that as the efficiency of the government increases, the intervention becomes more effective in increasing the liquidity of the economy; the yield curve flattens even more; and firms issue at longer maturities.

Finally, recall that the benchmark policy assumes that $\lambda^{g-g} = 0$; i.e., a government seller cannot trade with a government buyer. For a given size $\mu^{g,b}$, the cost of GSIs decreases if the government can reallocate securities among its trading agents. The last row of Table 7 shows that if government agents can trade among themselves, GSIs are more efficient and the effects on credit spreads, maturity, and welfare improve.

There are legitimate reasons to imagine that government agents might have more flexibility than private agents. Hence, the results of the benchmark policy should be considered as a lower bound on the implications for GSIs. For example, Table 7 shows that the gains from government intervention can be larger if government agents are more efficient at finding counterparts or can trade among themselves.

7 Extensions

This section extends the model in four dimensions and shows that results do not hinge on many of the assumptions of the benchmark theory. The first extension studies how borrowers choose and finance investment projects when they can rollover short-term debt to finance long-term projects. Second, we evaluate how changes in the default probability interact with maturity choices. Third, the effects on the real economy are quantitatively similar to alternative production functions. Finally, the yield curve has the same shape regardless of whether buyers can direct themselves to markets segmented by maturity instead of having one market with random matching across maturities.

Table 7: **GSI: Alternative policies**

	Liquidity	Spread 5 years	Maturity	Welfare gains	Output
Low trading frictions (US)					
No GSIs	13.00	146	5.37		
Benchmark policy	18.94	101	5.76	4.74	6.02
Gov. 10% more efficient	19.22	99	5.78	5.16	6.31
Gov. 50% more efficient	20.08	95	5.84	6.46	7.19
Gov. transactions	21.48	89	5.91	7.98	8.36
High trading frictions (Argentina)					
No GSIs	3.70	483	3.58		
Benchmark policy	5.28	346	4.02	5.78	10.67
Gov. 10% more efficient	5.39	340	4.05	6.49	11.39
Gov. 50% more efficient	5.72	321	4.15	9.04	13.74
Gov. transactions	6.12	301	4.28	13.68	17.01

7.1 Rollover

This extension shows that changes in the liquidity of the secondary market have similar effects on borrower choices even if borrowers can rollover short-term debt to finance long-term projects. Consider a firm that chooses the maturity of the project τ and issues zero coupon bonds to finance investment costs. A bond of maturity y has a fixed cost of issuance Φ and an interest rate $r(y)$.[29] Let J be the total number of issuances up to age τ. It is easy to show that a fix cost of issuance implies a finite number of issuances.

The firm chooses the maturity of the project τ, the number of issuances J, the amount borrowed B_j, and the maturity structure of their liabilities y_j to solve

$$\max_{\tau, J, y_j, B_j} e^{-(\rho+\delta)\tau}\left(F(\tau) - B_j\right) \tag{24}$$

$$\text{s.t.} \quad B_j P(y_j, \lambda) = B_{j-1} + \Phi + I(y_j) \quad \text{for } j = 1, \ldots, J$$

$$B_0 = 0 \qquad \text{and} \qquad \sum_{j=1}^{J} y_j = \tau.$$

Each issuance borrows to rollover existing debt B_{j-1}, cover the issuance cost Φ, and invest for

[29] In the data, issuance costs include management fees, selling concessions, registration fees, underwriter fees, underwriter spread (the difference between the offering price and the guaranteed price to the issuer), underpricing (the difference between the market price and the offering price), and printing, legal and auditing costs. For the Eurobond market, Melnik and Nissim (2003) find that the total issuance cost is 37 basis points. Lee et al. (1996) find similar costs and reports evidence of economies of scale, reflecting that a significant fraction is a fixed cost.

the next y_j periods, $I(y_j)$.[30] Iterate on B_j to cast the firm's problem (24) as

$$\max_{\tau} e^{-(\rho+\delta)\tau} F(\tau) - \text{FIN}^{\text{COST}}(\tau)$$

in which the financial cost is

$$\text{FIN}^{\text{COST}}(\tau) = e^{-(\rho+\delta)\tau} \min_{J,\{y_j\}_{j=1}^{J}} \sum_{i=1}^{J} (\Phi + I(y_i))\, e^{\sum_{s=i}^{J} r_s y_s}$$
$$\text{s.t.} \quad \sum_{j=1}^{J} y_j = \tau.$$

Financing cost Consider a project of a given maturity τ. The financial cost $\text{FIN}^{\text{COST}}(\tau)$ chooses the number of issuances J and the maturity structure y_j to minimize the net present value of issuance costs Φ and investment needs $I(y_j)$. Both the issuance costs and the liquidity spread affect financial decisions.

We evaluate the financial choices for different issuance costs Φ, given a project τ. The top panel of Figure 7 shows the number of issuances, the total financial cost, and the dispersion of maturities for different issuance costs Φ.[31] Naturally, as the issuance cost increases, the number of issuances decreases and the total financial cost increases. Note that if Φ is sufficiently large, the firm optimally chooses to issue only one time, matching the maturity of the project and the liabilities. In the benchmark model we focus on this particular case with $J = 1$ and derive a sharper analytical characterization of the effects of secondary-markets liquidity on the project's choice.

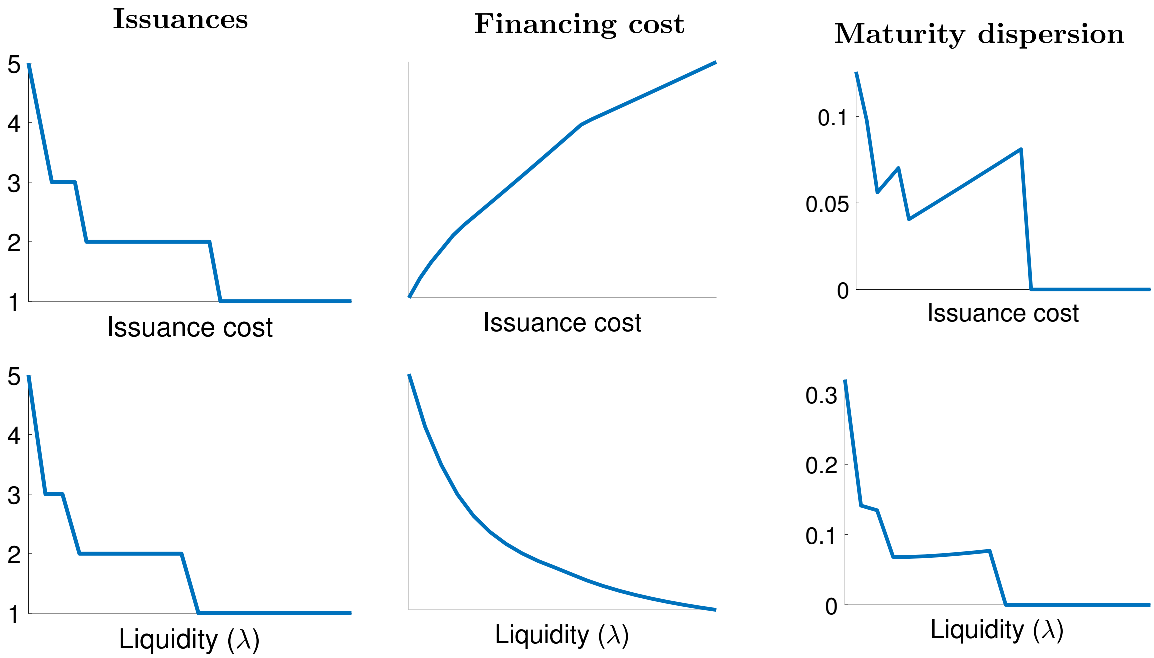
The bottom panel of Figure 7 evaluates how the liquidity of the secondary market affects financial choices. In markets with lower trading frictions—higher λ—long-term finance is more attractive (Proposition 1), which induces firms to rollover less often and borrow at longer maturities, reducing the total financial cost. Hence, for a given project τ, the financial cost diminishes when λ increases. The next exercise shows that this effect induces firms to invest in projects of longer horizons.

Maturity structure The optimal maturity structure solves the trade-off between equalizing credit spreads across different issuances and decreasing future fixed issuance costs. In equilibrium, the maturity structure is decreasing—i.e., $y_1 \geq y_2 \geq \cdots \geq y_J$. On the one hand, Figure

[30] A firm with positive cash holdings will always wait until it runs out of money to issue new debt. Hence, without loss of generality, we only have to consider the choices of the firm when outstanding debt matures which coincides with the moment in which the firm runs out of money.

[31] Maturity dispersion is defined as the coefficient of variation of individual maturities y_j.

Figure 7: **Financial cost with rollover**



Note: Financial cost for a given maturity τ and different issuance cost Φ and liquidity λ. Parameter values are discussed in Section 5. Maturity dispersion is defined as the coefficient of variation.

7 shows that, conditional on the number of issuances, when the issuance cost increases firms choose to increase the maturity dispersion.[32] Intuitively, as Φ increases, firms want to postpone the fixed-cost payments of later issuances, and, as a result, they extend the maturity of the first issuances and decrease later ones. On the other hand, the bottom panel shows that for financial markets with larger trading frictions (lower λ), the dispersion of maturities diminishes to generate similar liquidity spreads across issuances.

Investment choice Proposition 3 and the quantitative exercises in Section 5 show that the liquidity of the secondary market is important for investment decisions when firms match the maturity of assets and liabilities. Moving from an OTC secondary market with liquidity as in the US economy ($\lambda = 26$), to a shut-down of the market ($\lambda = 0$) reduces the project's maturity by 3.8 years (from 5.4 to 1.6, first panel of Table 8, rows two and three).

If the firm can rollover short-term debt, the effects of trading frictions on the project's choice can be substantially weaker because as λ decreases, the firm can rollover more often, as suggested by Figure 7. Note that this adjustment should be more important when issuance costs are low. The second panel of Table 8 considers the case of a low-issuance cost such that

[32]Note that the *jumps* on the dispersion coincide when the firm decides to change the number of issuances. Hence, the dispersion is decreasing conditional on the number of issuances.

under an OTC secondary market, the firm chooses a project of 8.1 years of duration and issues bonds 18 times (i.e., bonds have a maturity of 4.8 months on average). However, when there is a shut-down of the secondary market, the firm chooses to rollover more often (23 times) and shortens the duration of investment to 4.7 years (bonds have a maturity of 2.4 months on average). Hence, the reduction in the duration of the project due to changes in trading frictions is 3.4 years, similar to the effect when the firm cannot rollover (3.8 years). The third and fourth panels of Table 8 consider the case of higher issuance costs and find that the effect of trading frictions on maturity choices is similar, around 3 years. These exercises suggest that the results in Section 5 about how trading frictions affect investment decisions do not depend on the assumption of matching the maturity of the project and the bond.

An increase in the issuance cost generates two effects. First, conditional on the duration of the project, firms issue less frequently and increase debt maturity. Second, the firm reduces the duration of the project. Quantitatively, most of the adjustment is made by the number of issuances, while the choice of projects is relatively unaffected. This result also confirms that abstracting from rollover decisions in the main analysis of the paper is rather inconsequential for the real side of the economy.

One potential concern about these exercises is that the liquidity of the secondary market and rollover costs might be correlated. In the model, liquidity costs are endogenous while rollover costs are exogenous and fixed, so they do not respond to changes in the liquidity of secondary markets, and a potential Lucas critique may apply. However, we expect that when the secondary market becomes more liquid, both issuance and rollover costs should diminish. Hence, it is conservative to assume fixed issuance costs. As in Table 8, when liquidity of secondary markets improves, the firm adopts a project of longer duration. Similarly, when issuance costs diminish, the firm chooses longer-term projects. If the two effects are present (an increase in liquidity and a reduction of issuance costs), the firm will extend the maturity of the project even more than with fixed issuance costs.

7.2 Default

Default affects credit spreads and investment decisions. Lemma 2 shows that the liquidity spread increases with the default intensity. Quantitatively, long-term rates react more than short-term rates to changes in default. Figure 8 shows how the default rate affects the liquidity spread at maturities of 1 and 10 years, relative to the benchmark of $\delta = 0.03$. The liquidity spread for long maturities reacts more to changes in the default rate. Hence, when δ increases, the yield curve shifts upwards because of both default and liquidity. As a result, the firm chooses shorter-term projects (see rows four to six in Table 8).

Table 8: **Solution under alternative specifications**

Default	Secondary market	Issuance cost	Issuances	Maturity Project	Maturity Bond	Interest rate	Credit spread Default	Credit spread Liquidity
				No rollover				
Yes	Centralized		1	7.1	7.1	5.0%	3.0%	0.0%
Yes	OTC		1	5.4	5.4	6.5%	3.0%	1.5%
Yes	Shut down		1	1.6	1.6	16.8%	3.0%	11.8%
No	Centralized		1	23.9	23.9	2.0%	0.0%	0.0%
No	OTC		1	11.4	11.4	3.5%	0.0%	1.5%
No	Shut down		1	1.8	1.8	14.8%	0.0%	12.8%
				Rollover with low issuance cost				
Yes	Centralized	0.01	11	9.3	0.8	5.0%	3.0%	0.0%
Yes	OTC	0.01	18	8.1	0.4	6.0%	3.0%	1.0%
Yes	Shut down	0.01	23	4.7	0.2	6.7%	3.0%	1.7%
No	Centralized	0.01	1	23.9	23.9	2.0%	0.0%	0.0%
No	OTC	0.01	36	16.9	0.5	3.0%	0.0%	1.0%
No	Shut down	0.01	23	4.9	0.2	3.7%	0.0%	1.7%
				Rollover with medium issuance cost				
Yes	Centralized	0.10	3	8.6	2.9	5.0%	3.0%	0.0%
Yes	OTC	0.10	4	7.3	1.8	6.3%	3.0%	1.3%
Yes	Shut down	0.10	8	4.6	0.6	9.5%	3.0%	4.5%
No	Centralized	0.10	1	23.9	23.9	2.0%	0.0%	0.0%
No	OTC	0.10	5	15.4	3.1	3.3%	0.0%	1.3%
No	Shut down	0.10	8	4.9	0.6	6.7%	0.0%	4.7%
				Rollover with high issuance cost				
Yes	Centralized	0.46	1	7.1	7.1	5.0%	3.0%	0.0%
Yes	OTC	0.46	1	5.3	5.3	6.5%	3.0%	1.5%
Yes	Shut down	0.46	2	2.3	1.2	13.5%	3.0%	8.5%
No	Centralized	0.46	1	23.9	23.9	2.0%	0.0%	0.0%
No	OTC	0.46	2	13.6	6.8	3.4%	0.0%	1.4%
No	Shut down	0.46	2	2.6	1.3	11.5%	0.0%	9.5%

Note: The column "Secondary market" considers the cases of a centralized market (no trading frictions, $\lambda = \infty$), "OTC" (benchmark frictions, $\lambda = 26$), or "shut down" (there is no trading in secondary markets, $\lambda = 0$). We choose the value of the issuance cost so that under the "OTC" exercise the number of issuances varies from 1 to 18.

Moreover, when there is no default risk and secondary markets are centralized, firms have no incentive to issue short-term debt regardless of the issuance cost. However, when default risk is positive, even if secondary markets are centralized, firms choose to rollover debt when the issuance cost is not too high. Hence, both default risk and trading frictions shape rollover choices.

7.3 Alternative Production Functions

One might worry that the quantitative effects on productivity and output depend on assumptions and the calibration of the production function. This extension considers an alternative production model and shows that the effects of liquidity on the real economy are quantitatively similar.

Consider a production function that combines labor l and productivity z to produce output with technology $y = z^{1-\sigma} l^{\sigma}$. Let z be the productivity developed in the investment stage as in the benchmark model. To assess the importance of the curvature of the production function σ, we calibrate and do a counterfactual analysis related to the liquidity of the financial market for different values of the labor share, σ.

Static profits are

$$\pi(z) = \max_{l} z^{1-\sigma} l^{\sigma} - wl,$$

and labor demand is $l = z\left(\frac{\sigma}{w}\right)^{\frac{1}{1-\sigma}}$. Note that static profits are linear in z as in the benchmark model

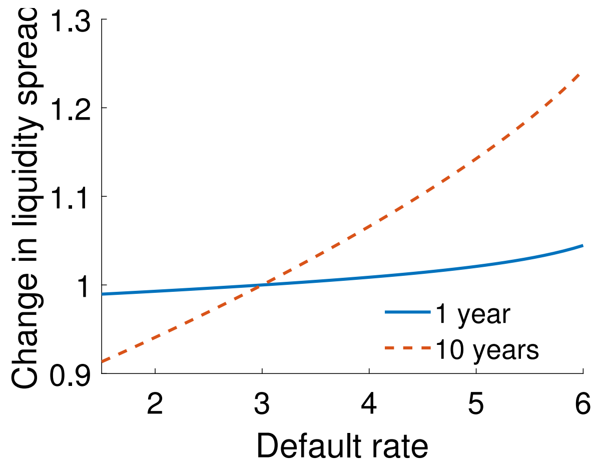
$$\pi(z) = z\left(\frac{\sigma}{w}\right)^{\frac{\sigma}{1-\sigma}}(1-\sigma).$$

Assume an exogenous labor supply normalized to one. In a steady state there is a measure $\mu^F = \frac{e^{-\delta\tau}}{\delta}$ of identical firms with productivity z. Labor market clearing implies $\mu^F l = 1$ so $w = \sigma\left(\mu^F z\right)^{1-\sigma}$. Finally, aggregate output is

$$Y = \mu^F z^{1-\sigma} l^{\sigma} = \left(\mu^F z\right)^{1-\sigma}.$$

Calibration For each $\sigma \in \{0.2, 0.5, 0.8\}$ we calibrate the model to match the same moments as in the benchmark case. In particular, recall from the calibration on Section 5 that the parameters regarding the financial sector are independent of the maturity choice. Hence, we only have to change ζ for each value of σ in order to target the maturity choice.

Figure 8: **Liquidity-default interactions**



Note: The figure shows how the liquidity spread at 1 and 10 years changes with δ with respect to the benchmark of 0.03.

Counterfactual For each σ we reduce the matching efficiency A to the level in Argentina as in the benchmark quantitative counterfactual exercise. Table 9 shows that as σ increases (i.e., an increase in the labor share), the change in the liquidity spread has a lower effect on maturity and aggregate output. Quantitatively, moving from a labor share of 0.2 to 0.8, the effect on maturity declines by about five months and the effect on aggregate output declines by 8 percentage points. Table 6 shows that in the benchmark model, the same change in credit spreads generates a decrease in maturity and aggregate output of 1.8 years and 30%, respectively. Hence, we conclude that changes in the liquidity spread are important for maturity choices and the real economy, independently of the curvature of the production function.

Table 9: **Alternative production functions**

Labor share	0.2	0.5	0.8
Δ Maturity (years)	-1.52	-1.25	-1.08
Δ Output (%)	-25	-20	-17

Note: Model is re-calibrated for each value of σ to match the target moments.

7.4 Segmented Markets

In the benchmark model, assets of different maturities are traded in a single secondary market. A potential concern could be that assets with short maturities, with small gains from trade, preclude the entry of buyers into the secondary market. This section considers secondary markets segmented by the time-to-maturity of assets. The main takeaway is that even though the market tightness for short-term bonds increases, the tightness for long-term assets remains similar to the case with only one market. Hence, the secondary market in the benchmark model is effectively a market for long-term assets. The intuition for this result is that, in equilibrium, the single market is not dominated by short-term assets, so there are always sufficient gains from trade.

Intuitively, in long-term markets, there are more gains from trade and therefore more entry of buyers. However, because there is Nash Bargaining over the gains from trade and buyers keep only a fraction $(1-\gamma)$ of the gains, the increase in the entry of buyers into long-term markets is not enough to compensate for the increase in the importance of the secondary markets for longer securities. As a result, the yield curve increases with maturity even with segmented markets. We describe the key features of the model in the main text and relegate to Appendix C.2 the full characterization of this extension.

Let τ be the initial maturity and consider the case in which secondary markets are segmented in N markets. Let $0 = \tau_1 < \cdots < \tau_{N+1} = \tau$, so each market $j = 1, \ldots, N$ trades assets with time-to-maturity $t \in [\tau_j, \tau_{j+1}]$.

Matching and distribution of agents Let $\mu^j(y) = \left[\mu^{H,j}(y), \mu^{L,j}(y)\right]$ be the measure of high- and low-valuation agents holding an asset with time-to-maturity t in market j. We start with market N and solve for the distribution of agents backwards. The boundary condition is $\mu^N(\tau) = [\mu^0, 0]$. Next, we iterate toward markets of shorter maturities with boundary conditions $\mu^j(\tau_{j+1}) = \mu^{j+1}(\tau_{j+1})$ for $j = 1, \ldots, N-1$. Lemma 4 characterizes the distribution of agents in each market.

Lemma 4. *The measure of agents for markets $j = 1, \ldots, N$ is given by the following backward recursion*

$$\begin{bmatrix} \mu^{H,N+1}(\tau) \\ \mu^{L,N+1}(\tau) \end{bmatrix} = \begin{bmatrix} \mu^0 \\ 0 \end{bmatrix}$$

and

$$\begin{aligned} \mu^{H,j}(y) =& \frac{\eta}{\eta+\lambda^j}\left[\frac{\lambda^j}{\eta} e^{\delta(y-\tau_{j+1})}\left(\mu^{H,j+1}(\tau_{j+1}) + \mu^{L,j+1}(\tau_{j+1})\right)\right. \\ & \left. - e^{\left(\eta+\lambda^j+\delta\right)(y-\tau_{j+1})}\left(-\mu^{H,j+1}(\tau_{j+1}) + \lambda^j \mu^{L,j+1}(\tau_{j+1})\right)\right] \\ \mu^{L,j}(t) =& \frac{\eta}{\eta+\lambda^j}\left[e^{\delta(y-\tau_{j+1})}\left(\mu^{H,j+1}(\tau_{j+1}) + \mu^{L,j+1}(\tau_{j+1})\right)\right. \\ & \left. + e^{\left(\eta+\lambda^j+\delta\right)(y-\tau_{j+1})}\left(-\mu^{H,j+1}(\tau_{j+1}) + \lambda^j \mu^{L,j+1}(\tau_{j+1})\right)\right]. \end{aligned}$$

where λ^j is the selling intensity in market $j = 1, \ldots, N$.

Valuations Let $D^j(y) = \left[D^{H,j}(y), D^{L,j}(y)\right]$ be the values for high- and low-valuation agents of holding an asset with time-to-maturity y in market $j = 1, \ldots, N$. To solve for the value of holding the asset, start with the first market, in which the boundary condition is that at maturity the value is equal to one, and then iterate forward, toward longer-term markets. The boundary condition for market $j = 1$ is $D^1(\tau_1) = [1, 1]$. Value matching for market $j = 2, \ldots, N$ implies $D^j(\tau_j) = D^{j-1}(\tau_j)$, and the Hamilton-Jacobi-Bellman equations are the same as in the benchmark model, Equations (9) and (10).

Free entry Free entry in each market implies that

$$c = (1-\gamma) \int_{\tau_j}^{\tau_{j+1}} \beta^j(y) \left(D^{H,j}(y) - D^{L,j}(y)\right) dy,$$

where β^j is the intensity at which a buyer finds a seller in market $j = 1, \ldots, N$. Appendix C.2 provides analytical solutions for the value functions and the free-entry condition.

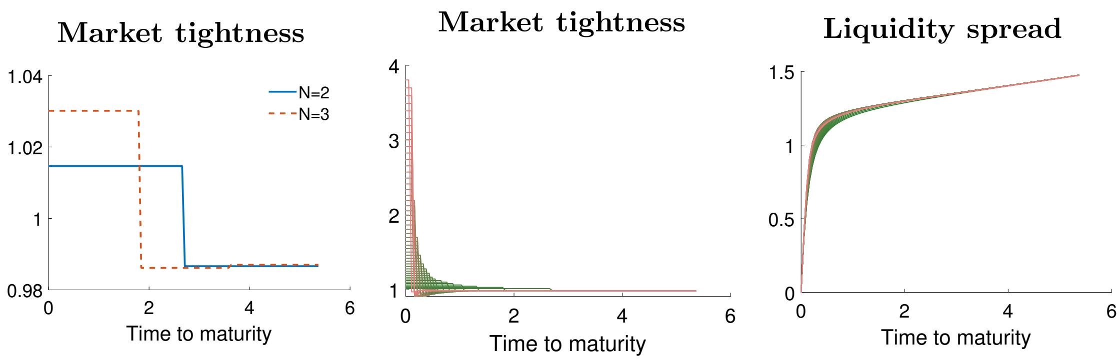
Results The first panel of Figure 9 shows the market tightness relative to the case of only one market when $N = 2$ and $N = 3$. With segmentation, markets for short-term assets are tighter (more sellers to buyers), as there are fewer gains from trade. However, for long-term bonds we find a tightness similar to the case of no segmentation. The second panel repeats the exercise under different degrees of segmentation ($N = 1, \ldots, 50$). Note that even with 50 different markets, the tightness for markets with maturity above four years is almost identical to the case of no segmentation.

The third panel of Figure 9 shows the effects on the liquidity spread for different models with $N = 1$ to $N = 50$. As the market tightness after four years is identical in all these models, the implied liquidity spread is also the same. For short-term assets (maturities up to 4 years), there are some differences in the market tightness. However, they generate small variations in the yield curve. Therefore, we conclude that the secondary market in the benchmark model with $N = 1$ is effectively a market for long-term assets.

8 Conclusion

This paper studies the linkages between the maturity of corporate debt, the liquidity of financial markets, and the real economy. Long-term finance is particularly more expensive in economies with severe trading frictions that induce firms to invest at shorter horizons. A calibration of the model suggests that even though it is a stylized and tractable model, the theory reconciles data on maturities, credit spreads, and the real economy. Finally, an intervention like GSIs can

Figure 9: **Segmented markets**



Note: The first (second) panel shows the market tightness relative to no segmentation for $N = 2, 3$ $(N = 2, \ldots, 50)$*. The third panel shows the liquidity spread for* $N = 1, \ldots, 50$*.*

improve the liquidity of financial markets, reduce long-term financial cost, and induce firms to borrow and invest at longer horizons. Several extensions suggest that the results of the paper do not hinge on particular modeling assumptions.

We assume that firms have access to corporate bonds since these assets are already well studied in the literature about trading frictions (e.g., He and Milbradt, 2014). However, similar frictions also affect other sources of finance such as bank loans, venture capital, or private equity funds.[33] The secondary markets for these assets are probably more frictional than they are for corporate debt, which indicates that aggregate effects can potentially be larger.

The framework and results developed in this paper transcend the particular application to corporate bonds and can be used to study other markets for long-term finance, such as households borrowing for real estate (mortgages) or education (student debt). Interestingly, Hicks (1969) argues that the products manufactured during the first decades of the Industrial Revolution had been invented much earlier. The critical innovation that ignited growth in England in the 18th century was capital market liquidity so that savers could easily sell their assets if needed, while at the same time the capital was committed for longer periods for investment (see Bencivenga et al., 1995).

[33] There exists a secondary market for bank loans in the US (see Altman et al., 2010; Drucker and Puri, 2008). However, the maturity of bank loans tends to be shorter than the maturity of corporate bonds (Cortina Lorente et al., 2016), and financial systems become market-based during the process of economic development (Demirgüç-Kunt et al., 2013). Moreover, banks need to raise capital to extend long-term loans. Hence, the frictions analyzed in this paper also affect banks borrowing rates, which are likely to affect loans rates.

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A Appendix: Proofs

This section provides the proofs of the main results of the paper.

A.1 Lenders

A.1.1 Distribution of financiers

Proof of Lemma 1. Let $\mu(y) = \left[\mu^H(y), \mu^L(y)\right]$. Matching implies $\mu^B \beta \frac{\mu^L(y)}{\mu^S} = \lambda \mu^L(y)$. Then, (5)-(6) imply that $\dot{\mu}(y) = A\mu(y)$ with

$$A = \begin{bmatrix} \delta + \eta & -\lambda \\ -\eta & \delta + \lambda \end{bmatrix}$$

The boundary condition is $\mu(\tau) = [\mu^0, 0]$. Note that A has two real and distinct eigenvalues. Let R be the vector with the eigenvalues and V be the matrix with eigenvectors of A. Define $B = (V)^{-1}\mu(\tau)$ so

$$V = \begin{bmatrix} -1 & \frac{\lambda}{\eta} \\ 1 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \eta + \lambda + \delta \\ \delta \end{bmatrix} \qquad B = \frac{\eta\mu^0}{\eta + \lambda}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

It is standard to show that

$$\mu^H(y) = \sum_{i=1}^{2} e^{R_i(y-\tau)} B_i V(1,i) \qquad \mu^L(y) = \sum_{i=1}^{2} e^{R_i(y-\tau)} B_i V(2,i)$$

Finally, a few lines of algebra deliver

$$\mu^H(y) = \frac{\mu^0\eta}{\eta + \lambda}\left(e^{\delta(y-\tau)}\frac{\lambda}{\eta} + e^{(\eta+\lambda+\delta)(y-\tau)}\right)$$
$$\mu^L(y) = \frac{\mu^0\eta}{\eta + \lambda}\left(e^{\delta(y-\tau)} - e^{(\eta+\lambda+\delta)(y-\tau)}\right)$$

□

A.1.2 Value functions

Proof of Proposition 1. Replace the price of the asset in the secondary market $P^S(y;\lambda)$ in (9)-(10) so

$$\begin{aligned}(\rho+\delta) D^H(y;\lambda) =& -\frac{\partial D^H(y;\lambda)}{\partial y} + \eta\left(D^L(y;\lambda) - D^H(y;\lambda)\right)\\ (\rho+\delta) D^L(y;\lambda) =& -h - \frac{\partial D^L(y;\lambda)}{\partial y} + \lambda\gamma\left(D^H(y;\lambda) - D^L(y;\lambda)\right)\end{aligned}$$

Let $H(y;\lambda) = D^H(y;\lambda) - D^L(y;\lambda)$, then

$$(\rho+\delta+\eta+\lambda\gamma) H(y;\lambda) = h - \frac{\partial H(y;\lambda)}{\partial y}$$

with $H(0;\lambda)=0$. It is straight forward to see that $H(y;\lambda) = h\frac{1-e^{-c_1 y}}{c_1}$ where $c_1 = \rho+\delta+\eta+\lambda\gamma$.

Next, solve for $D^H(y;\lambda)$ as

$$(\rho+\delta) D^H(y;\lambda) = -\frac{\partial D^H(y;\lambda)}{\partial y} - \eta h\frac{1-e^{-c_1 y}}{c_1}$$

with boundary $D^H(0;\lambda)=1$. The solution is $D^H(y;\lambda) = A + Be^{-(\rho+\delta)y} + Ce^{-c_1 t}$ with constants

$$A = -\frac{1}{\rho+\delta}\frac{\eta h}{c_1} \qquad C = -\frac{1}{\eta+\lambda\gamma}\frac{\eta h}{c_1} \qquad B = 1 + \frac{\eta h}{(\eta+\lambda\gamma)(\rho+\delta)}$$

Finally, a few lines of algebra deliver

$$\begin{aligned}D^H(y;\lambda) =& e^{-(\rho+\delta)y} - \mathcal{L}(y,\lambda)\\ \mathcal{L}(y,\lambda) =& \frac{\eta h}{\eta+\lambda\gamma}\left(\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta} - \frac{1-e^{-(\rho+\delta+\eta+\lambda\gamma)y}}{\rho+\delta+\eta+\lambda\gamma}\right)\\ \mathcal{L}(\tau,\lambda) =& h \quad \frac{\eta}{\eta+\lambda\gamma} \quad \int_0^\tau e^{-(\rho+\delta)y}\left(1-e^{-(\eta+\lambda\gamma)y}\right)dy\end{aligned}$$

The value of a low-valuation agent is $D^L(y;\lambda) = D^H(y;\lambda) - H(y;\lambda)$, that is

$$D^L(y,\lambda) = e^{-(\rho+\delta)y} - h\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta} + \frac{\lambda\gamma}{\eta}\mathcal{L}(y,\lambda).$$

Properties of the illiquidity cost:

1. **Positive:** $\mathcal{L}(\tau,\lambda)$ is positive as $\rho+\delta+\eta+\lambda\gamma \geq \rho+\delta$.

2. **Sensitivity with respect to maturity τ:**

(a) $\mathcal{L}(\tau,\lambda)$ is increasing in τ:

$$\frac{\partial\mathcal{L}(\tau,\lambda)}{\partial\tau} = h \quad \frac{\eta}{\eta+\lambda\gamma} \quad e^{-(\rho+\delta)\tau}\left(1-e^{-(\eta+\lambda\gamma)\tau}\right) \geq 0$$

(b) The limit of $\mathcal{L}(\tau,\lambda)$ when τ goes to infinity is

$$\begin{aligned}\lim_{\tau\to\infty}\mathcal{L}(\tau,\lambda) &= h\frac{\eta}{\eta+\lambda\gamma}\left(\frac{1}{\rho+\delta}-\frac{1}{\rho+\delta+\eta+\lambda\gamma}\right)\\ &= h\frac{\eta}{(\rho+\delta)(\rho+\delta+\eta+\lambda\gamma)}\end{aligned}$$

3. **Sensitivity with respect to liquidity shocks η:**

(a) If there are no liquidity shocks ($\eta=0$), then $\mathcal{L}(\tau,\lambda)=0$.

(b) If $\eta\to\infty$ (i.e., always has to pay the cost h), then

$$\lim_{\eta\to\infty}\mathcal{L}(\tau,\lambda) = h\frac{1-e^{-(\rho+\delta)\tau}}{\rho+\delta}$$

4. **Sensitivity with respect to liquidity of the secondary market λ:**

(a) $\mathcal{L}(\tau,\lambda)$ is decreasing in λ. Note that the illiquidity cost is

$$\begin{aligned}\mathcal{L}(\tau,\lambda) = \eta h&\left(\frac{1}{(\rho+\delta)(\rho+\delta+\eta+\lambda\gamma)}-\frac{e^{-(\rho+\delta)\tau}}{(\eta+\lambda\gamma)(\rho+\delta)}\right)\\ &+\eta h\frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta+\lambda\gamma)(\rho+\delta+\eta+\lambda\gamma)}\end{aligned}$$

so

$$\begin{aligned}\frac{\partial\mathcal{L}(\tau,\lambda)}{\partial\lambda} &= \eta h\left(-\frac{1}{(\rho+\delta)(\rho+\delta+\eta+\lambda\gamma)^2}+\frac{e^{-(\rho+\delta)\tau}}{(\rho+\delta)(\eta+\lambda\gamma)^2}\right)\\ &-\eta h\left(\frac{\tau e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta+\lambda\gamma)(\rho+\delta+\eta+\lambda\gamma)}+\frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta+\lambda\gamma)^2(\rho+\delta+\eta+\lambda\gamma)}\right)\\ &-\eta h\frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta+\lambda\gamma)(\rho+\lambda^D+\eta+\lambda\gamma)^2}\end{aligned}$$

Let $a=\eta+\lambda\gamma$ and $b=\rho+\delta$ so

$$\frac{\partial\mathcal{L}(\tau,\lambda)}{\partial\lambda} = \eta h\left(-\frac{1}{b(a+b)^2}+\frac{e^{-b\tau}}{ba^2}\right)-\eta h\frac{e^{-(a+b)\tau}}{a(a+b)}\left(\tau+\frac{2a+b}{a(a+b)}\right)$$

We want to show that

$$\frac{e^{-b\tau}}{ba^2} \leq \frac{1}{b\left(a+b\right)^2} + \frac{e^{-(a+b)\tau}}{a\left(a+b\right)}\left(\tau + \frac{2a+b}{a\left(a+b\right)}\right) \tag{25}$$

Define $L(\tau)$ and $R(\tau)$ as the left- and right-hand-sides of (25), respectively. Note that $R\left(0\right) = L\left(0\right) = \frac{1}{ba^2}$. Hence, it is sufficient to show that the slope of $L\left(\tau\right)$ is lower than the slope of $R\left(\tau\right)$ for all τ. Note that

$$\frac{\partial L\left(\tau\right)}{\partial \tau} = -\frac{e^{-b\tau}}{a^2} \qquad \frac{\partial R\left(\tau\right)}{\partial \tau} = -\frac{e^{-(a+b)\tau}}{a}\left(\tau + \frac{1}{a}\right)$$

Hence, the slope of L is lower than the slope of R because $a\tau \geq \log\left(a\tau + 1\right)$.

(b) If there are no secondary markets; i.e., $\lambda = 0$, then the illiquidity cost represents the expected holding costs; i.e.,

$$\mathcal{L}\left(\tau, 0\right) = h\int_0^\tau e^{-\left(\rho+\lambda^D\right)y}\left(1 - e^{-\eta y}\right)dy$$

(c) If secondary markets are totally liquid (i.e., $\lambda \to \infty$), then $\mathcal{L}\left(\tau, \lambda\right) = 0$.

5. **Liquidity is more important for long-term assets:** Recall that

$$\frac{\partial \mathcal{L}\left(\tau,\lambda\right)}{\partial \tau} = h\frac{\eta}{\eta + \lambda\gamma}e^{-(\rho+\delta)\tau}\left(1 - e^{-(\eta+\lambda\gamma)\tau}\right)$$
$$\frac{\partial \mathcal{L}\left(\tau,\lambda\right)}{\partial \tau} = \eta h e^{-(\rho+\delta)\tau}\int_0^\tau e^{-(\eta+\lambda\gamma)y}dy$$

therefore

$$\frac{\partial \mathcal{L}\left(\tau,\lambda\right)}{\partial \tau \lambda} = -\eta h e^{-(\rho+\delta)\tau}\int_0^\tau y e^{-(\eta+\lambda\gamma)y}dy \leq 0$$

□

A.1.3 Liquidity spread

Proof of Lemma 2. We show the following:

1. **The liquidity spread $cs^{liq}(\tau,\lambda)$ is increasing in maturity τ:**

$$\frac{\partial cs^{liq}\left(t,\lambda\right)}{\partial t} = \frac{1}{t^2}\log\left(1 - e^{(\rho+\delta)t}\mathcal{L}\left(t,\lambda\right)\right) + \frac{e^{(\rho+\delta)t}}{t}\frac{\left(\rho+\delta\right)\mathcal{L}\left(t,\lambda\right) + \frac{\partial \mathcal{L}(t,\lambda)}{\partial t}}{1 - e^{(\rho+\delta)t}\mathcal{L}\left(t,\lambda\right)}$$

Recall that $\log(x) \geq \frac{x-1}{x}$. Hence

$$\log\left(1 - e^{(\rho+\delta)t}\mathcal{L}(t,\lambda)\right) \geq \frac{-e^{(\rho+\delta)t}\mathcal{L}(t,\lambda)}{1 - e^{(\rho+\delta)t}\mathcal{L}(t,\lambda)}$$

which implies that

$$\frac{\partial cs^{liq}(t,\lambda)}{\partial t} \geq \frac{1}{t^2}\frac{e^{(\rho+\delta)t}\mathcal{L}(t,\lambda)}{1 - e^{(\rho+\delta)t}\mathcal{L}(t,\lambda)}\left(t(\rho+\delta) + \frac{\partial \mathcal{L}(t,\lambda)}{\partial t}\frac{t}{\mathcal{L}(t,\lambda)} - 1\right)$$

Let $\varepsilon_{\mathcal{L},t} = \frac{\partial \mathcal{L}(t,\lambda)}{\partial t}\frac{t}{\mathcal{L}(t,\lambda)}$, and note that

$$\varepsilon_{\mathcal{L},t} = t\left[e^{-(\rho+\delta)\tau} - e^{-(\rho+\delta+\eta+\lambda\gamma)t}\right]\left[\frac{1-e^{-(\rho+\delta)t}}{\rho+\delta} - \frac{1-e^{-(\rho+\delta+\eta+\lambda\gamma)t}}{\rho+\delta+\eta+\lambda\gamma}\right]^{-1}$$

A sufficient condition is $t(\rho+\delta) + \varepsilon_{\mathcal{L},t} - 1 \geq 0$. Let $a = \rho+\delta$ and $b = \eta+\lambda\gamma$, and define

$$E(t,a,b) = t\left(a + \left[e^{-at} - e^{-(a+b)t}\right]\left[\frac{1-e^{-at}}{a} - \frac{1-e^{-(a+b)t}}{a+b}\right]^{-1}\right) - 1$$

It is easy to show numerically that $E(t,a,b) \geq 0$ for all $t,a,b \geq 0$. Hence, the liquidity spread is increasing in maturity. Finally, it is straight forward to see that the liquidity spread is decreasing in liquidity λ.

2. **The liquidity spread is increasing in the default intensity** δ**:** Note that

$$\begin{aligned} e^{(\rho+\delta)\tau}\mathcal{L}(\tau,\lambda) &= \frac{\eta}{\eta+\lambda\gamma}\int_0^\tau e^{(\rho+\delta)(\tau-t)}\left(1-e^{-(\eta+\lambda\gamma)t}\right)dt \\ \frac{\partial\left(e^{(\rho+\delta)\tau}\mathcal{L}(\tau,\lambda)\right)}{\partial\delta} &= \frac{\eta}{\eta+\lambda\gamma}\int_0^\tau (\tau-t)\,e^{(\rho+\delta)(\tau-t)}\left(1-e^{-(\eta+\lambda\gamma)t}\right)dt > 0 \end{aligned}$$

□

Proof of Lemma 3. The mid-price is

$$\frac{1}{2}\left(D^H(y;\lambda) + D^L(y;\lambda)\right) = e^{-(\rho+\delta)y} - \frac{1}{2}\left(h\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta} + \left(\frac{\eta-\lambda\gamma}{\eta}\right)\mathcal{L}(y,\lambda)\right)$$

where

$$\left(\frac{\eta-\lambda\gamma}{\eta}\right)\mathcal{L}(y,\lambda) = h\frac{\eta-\lambda\gamma}{\lambda^H+\lambda\gamma}\left(\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta} - \frac{1-e^{-\left(\rho+\delta+\lambda^H+\lambda\gamma\right)y}}{\rho+\delta+\eta+\lambda\gamma}\right)$$

The mid-price is

$$e^{-(\rho+\delta)y}-\frac{h}{\eta+\lambda\gamma}\left(\eta\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}-\frac{(\eta-\lambda\gamma)}{2}\frac{1-e^{-(\rho+\delta+\eta+\lambda\gamma)y}}{\rho+\delta+\eta+\lambda\gamma}\right)$$

Define the gains from trade as

$$GT\left(y\right)=h\frac{1-e^{-\left(\rho+\delta+\lambda^{H}+\lambda\gamma\right)y}}{\rho+\delta+\eta+\lambda\gamma}$$

so

$$BA\left(y\right)=GT\left(y\right)\left[e^{-\left(\rho+\lambda^{D}\right)y}-\frac{1}{\eta+\lambda\gamma}\left(h\eta\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}-\frac{(\eta-\lambda\gamma)}{2}GT\left(y\right)\right)\right]^{-1}$$

$$BA\left(y\right)=\left[\frac{e^{-(\rho+\delta)y}}{GT\left(y\right)}-h\frac{\eta}{\eta+\lambda\gamma}\frac{\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}}{GT\left(y\right)}+\frac{1}{2}\frac{\eta-\lambda\gamma}{\eta+\lambda\gamma}\right]^{-1}$$

Note that $\frac{e^{-(\rho+\delta)y}}{GT(y)}$ is decreasing in y because of $e^{-(\rho+\delta)y}$ is decreasing and $GT\left(y\right)$ is increasing in y. Note that $\frac{\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}}{GT(y)}$ is increasing in y because of the discount in GT is larger than in the numerator. Hence, with the negative sign it is decreasing. Therefore, all the square bracket is decreasing in y, and as it is to the power of -1, the $BA\left(y\right)$ is increasing in y. $\square$

A.1.4 Free entry

Proof of Proposition 2. Gains from trade are

$$D^{H}\left(y;\lambda\right)-D^{L}\left(y;\lambda\right)=h\frac{1-e^{-c_{1}y}}{c_{1}}\qquad\qquad c_{1}=\rho+\delta+\eta+\lambda\gamma$$

The buyer gets $(1-\gamma)$ of the gains from trade. Hence, the free entry condition reads

$$c=(1-\gamma)\int_{0}^{\tau}\beta\frac{\mu^{L}(y)}{\mu^{S}}h\frac{1-e^{-c_{1}y}}{c_{1}}dy$$

And $\theta=\frac{\mu^{S}}{\mu^{B}}$. Also, recall that $\mu^{S}=\int_{0}^{\tau}\mu^{L}\left(y\right)dy$ Hence, the free entry condition is

$$c=\frac{(1-\gamma)\,h}{c_{1}}A\theta^{\alpha}\int_{0}^{\tau}\frac{\mu^{L}\left(y\right)}{\mu^{S}}\left(1-e^{-c_{1}t}\right)dy$$

$$c=\frac{(1-\gamma)\,h}{c_{1}}A\theta^{\alpha}\left(1-\frac{\int_{0}^{\tau}e^{-c_{1}y}\mu^{L}\left(y\right)dy}{\int_{0}^{\tau}\mu^{L}\left(y\right)dy}\right)$$

Define $c_2 = \eta + \delta + \lambda$ and note that

$$\int_0^\tau e^{-c_1 t} \mu^L (y)\, dy = \mu^0 \frac{\eta}{\eta + \lambda} \left(\frac{e^{-c_1 \tau} - e^{-\delta \tau}}{\delta - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1} \right)$$

As a result, the ratio of integrals in the free-entry condition reads

$$\left(\frac{e^{-c_1 \tau} - e^{-\lambda^D \tau}}{\delta - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1} \right) \left(\frac{1 - e^{-\tau \delta}}{\delta} - \frac{1 - e^{-(\eta + \delta + \lambda)\tau}}{\eta + \delta + \lambda} \right)^{-1} \tag{26}$$

And the free-entry condition boils down to

$$c = \frac{(1 - \gamma) h}{c_1} A \theta^\alpha \left(1 - \left(\frac{e^{-c_1 \tau} - e^{-\delta \tau}}{\delta - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1} \right) \left(\frac{1 - e^{-\tau \delta}}{\delta} - \frac{1 - e^{-c_2 \tau}}{c_2} \right)^{-1} \right)$$

First, note that it is easy to show that Equation (26) is increasing in τ. Next, consider $\tau = 0$. Note that the ratio of integrals in the free-entry condition is equal to 1 as

$$\begin{aligned}
&\lim_{\tau \to 0} \left(\frac{e^{-c_1 \tau} - e^{-\delta \tau}}{\lambda^D - c_1} - \frac{e^{-c_1 \tau} - e^{-c_2 \tau}}{c_2 - c_1} \right) \left(\frac{1 - e^{-\delta \tau}}{\delta} - \frac{1 - e^{-c_2 \tau}}{c_2} \right)^{-1} \\
&= \lim_{\tau \to 0} \left(\frac{-c_1 e^{-c_1 \tau} + \delta e^{-\delta \tau}}{\delta - c_1} - \frac{-c_1 e^{-c_1 \tau} + c_2 e^{-c_2 \tau}}{c_2 - c_1} \right) \left(\frac{\delta e^{-\tau \delta}}{\delta} - \frac{c_2 e^{-c_2 \tau}}{c_2} \right)^{-1} \\
&= \lim_{\tau \to 0} \left(\frac{(c_1)^2 e^{-c_1 \tau} - (\delta)^2 e^{-\delta \tau}}{\delta - c_1} - \frac{(c_1)^2 e^{-c_1 \tau} - (c_2)^2 e^{-c_2 \tau}}{c_2 - c_1} \right) \left(\frac{-(\delta)^2 e^{-\tau \delta}}{\delta} - \frac{-(c_2)^2 e^{-c_2 \tau}}{c_2} \right)^{-1} \\
&= \left(\frac{(c_1)^2 - (\delta)^2}{\delta - c_1} - \frac{(c_1)^2 - (c_2)^2}{c_2 - c_1} \right) \left(\frac{-(\delta)^2}{\lambda^D} - \frac{-(c_2)^2}{c_2} \right)^{-1} \\
&= \left(\frac{(c_1 + \delta)\left(c_1 - \lambda^D\right)}{\delta - c_1} - \frac{(c_1 + c_2)(c_1 - c_2)}{c_2 - c_1} \right) (c_2 - \delta)^{-1} \\
&= \left(-(c_1 + \delta) + (c_1 + c_2) \right) (c_2 - \delta)^{-1} = \left(c_2 - \lambda^D \right) (c_2 - \delta)^{-1} = 1
\end{aligned}$$

where we applied L'Hopital's rule in the second and third line. As a result, the free-entry condition is satisfied if and only if $\lim_{\tau \to 0} \theta = \infty$. Hence, $\lim_{\tau \to 0} \lambda = 0$. That is, $\lambda(0) = 0$.

Next, consider the case of $\tau \to \infty$. The ratio of integrals in the free-entry condition is equal to zero. Hence $c = \frac{h}{c_1} (1 - \gamma) A \theta^\alpha$. Recall that $c_1 = \rho + \delta + \eta + \lambda \gamma$ and $\lambda = A \theta^{\alpha - 1}$. Hence,

$$\rho + \delta + \eta + \gamma A \theta^{\alpha - 1} = \frac{h(1 - \gamma)}{c} A \theta^\alpha$$

As $\alpha \in (0, 1)$ the left-hand side is decreasing in θ and the right-hand side is increasing in θ. As a result, there exists a unique $\theta \in \mathbb{R}_+$. That is, $\lim_{\tau\to\infty} \lambda(\tau) = \overline{\lambda} \in \mathbb{R}_+$. $\square$

A.2 Borrowers

Proof of Proposition 3. Let $J(\tau,\lambda)$ be the value of the firm with maturity τ and liquidity λ and let $Z = \frac{\zeta}{\rho+\delta}$. The first-order condition is

$$\begin{aligned} J_\tau(\tau,\lambda) =& e^{-(\rho+\delta)\tau} Z \left(1-(\rho+\delta)\tau\right) \\ &- e^{cs^{liq}(\lambda,\tau)\tau}\left[\frac{\partial I(\tau)}{\partial \tau} + (\Phi + I(\tau))\, cs^{liq}(\lambda,\tau)\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right)\right] \end{aligned}$$

Note that τ is increasing in λ if the derivative of the first-order condition with respect to λ is positive

$$\begin{aligned} J_{\tau\lambda}(\tau,\lambda) =& -e^{cs^{liq}(\lambda,\tau)\tau}\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}\tau\left[\frac{\partial I(\tau)}{\partial \tau} + (\Phi + I(\tau))\, cs^{liq}(\lambda,\tau)\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right)\right] \\ &- e^{cs^{liq}(\lambda,\tau)\tau}\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}(\Phi + I(\tau))\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right) \\ &- e^{cs^{liq}(\lambda,\tau)\tau}(\Phi + I(\tau))\, cs^{liq}(\lambda,\tau)\frac{\partial \varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda} \end{aligned}$$

Recall that $\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda} \leq 0$. so the first and second terms are positive. However, the last term involves $\frac{\partial \varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda}$ for which we do not know the sign. We can write $J_{\tau\lambda}(\tau,\lambda)$ as

$$\begin{aligned} J_{\tau\lambda}(\tau,\lambda) =& -e^{cs^{liq}(\lambda,\tau)\tau}\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}\frac{\partial I(\tau)}{\partial \tau}\tau - e^{cs^{liq}(\lambda,\tau)\tau}(\Phi + I(\tau)) \\ &\left[\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right)\left(\tau cs^{liq}(\lambda,\tau)+1\right) + cs^{liq}(\lambda,\tau)\frac{\partial \varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda}\right] \end{aligned}$$

The first term is positive. A sufficient condition for $J_{\tau\lambda}(\tau,\lambda) \geq 0$ is that the second term is also positive. This implies

$$\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right)\left(\tau cs^{liq}(\lambda,\tau)+1\right) \leq -cs^{liq}(\lambda,\tau)\frac{\partial \varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda}$$

This expression depends only on $cs^{liq}(\lambda,\tau)$. By Lemma 2 we can approximate the liquidity spread as a linear function increasing in τ and decreasing in λ. Let $cs^{liq}(\lambda,\tau) = c_\tau\tau + c_\lambda\lambda$ with $c_\tau \geq 0$ and $c_\lambda \leq 0$. Then $\varepsilon_{cs^{liq},\tau}(\lambda,\tau) = \frac{c_\tau\tau}{c_\tau\tau+c_\lambda\lambda}$ and $\frac{\partial \varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda} = -\frac{c_\tau\tau c_\lambda}{(c_\tau\tau+c_\lambda\lambda)^2}$. The sufficient condition reads

$$\left(c_\tau\tau + cs^{liq}(\lambda,\tau)\right) cs^{liq}(\lambda,\tau)\,\tau + cs^{liq}(\lambda,\tau) \geq 0$$

which is satisfied. Therefore, $J_{\tau\lambda}(\tau,\lambda) \geq 0$ and $\frac{\partial\tau(\lambda)}{\partial\lambda} \geq 0$. Finally it is straightforward to see that $\underline{\tau} = \tau(0) \leq \lim_{\lambda\to\infty}\tau(\lambda) = \tau^* < \infty$. $\square$

A.3 Existence of equilibrium

Proof of Proposition 4. First, Proposition 2 defines a schedule for the lenders $\tau^L(\lambda)$. Note that $\tau^L(0) = 0$ and there exists $\overline{\lambda}$ such that $\tau^L(\overline{\lambda}) = \infty$.

Second, Proposition 3 define $\tau^B(\lambda)$ and notice that $\tau^B(0) = \underline{\tau} > 0$ and $\tau^B(\lambda) \geq 0$ for all λ.

Finally, define $F(\lambda) = \tau^L(\lambda) - \tau^B(\lambda)$ and note that $F(0) = -\underline{\tau} < 0$ and $F(\overline{\lambda}) = \infty$. Hence, as F is continuous, Bolzano's theorem implies that there exits $\lambda^\star$ such that $F(\lambda^\star) = 0$, which defines the equilibrium. $\square$

Online Appendix

This material is for a separate, on-line appendix and not intended to be printed with the paper.

B Alternative Demands for Long-Term Finance

In this Appendix, we provide alternative microfoundations for the demand of long-term finance.

B.1 Quality-Ladder Model

We follow the standard quality-ladder model with a continuum of intermediate goods producers with monopolistic power and final goods producers in perfect competition (e.g., Akcigit and Kerr, 2017). The final good is produced with labor and a continuum of intermediate goods $j \in [0, 1]$ with the production technology

$$Y = \frac{L^{\beta}}{1-\beta} \int_0^1 q_j^{\beta} k_j^{1-\beta} dj$$

where L is the labor input, k_j is the quantity of intermediate goods j, and q_j is its quality. Without loss of generality, normalize the price of the final good Y to one in every period. The final good is produced competitively with input prices taken as given. The inverse demand of intermediate inputs reads $p_j = L^{\beta} q_j^{\beta} k_j^{-\beta}$ and the labor demand is $L = \frac{\beta Y}{w}$.

Each intermediate good j is produced with a linear technology $k_j = \overline{q} l_j$ where $\overline{q} = \int_0^1 q_j dj$ is the average quality, and l_j is the labor input. Intermediate producers take the inverse demand and maximize static profits

$$\pi(q) = \max_{k_j \geq 0} p_j k_j - \frac{w}{\overline{q}} k_j \qquad \text{s.t. } p_j = L^{\beta} q_j^{\beta} k_j^{-\beta}$$

There is an exogenous labor supply normalized to one. Labor market clearing implies $L + \int_0^1 l_j = 1$. It is easy to show that static profits are linear in quality and aggregate output is linear in the average quality of the economy

$$\pi(q) = \underline{\pi} q_j \qquad Y = \underline{Y}\, \overline{q} \tag{27}$$

where the constants $\underline{\pi}$ and $\underline{Y}$ are

$$\underline{\pi} = \beta L \left(\frac{\overline{q}}{w} \right)^{\frac{1-\beta}{\beta}} (1-\beta)^{\frac{1-\beta}{\beta}} \qquad L = \frac{\beta}{(1-\beta)^2 + \beta} \qquad \underline{Y} = \frac{\beta^{\beta} (1-\beta)^{(1-2\beta)}}{(1-\beta)^2 + \beta}.$$

Life cycle of intermediate goods producers There are two important empirical facts that motivate assumptions about the back-loaded profile of investment projects. First, the *Arrow's replacement effect* establishes that small and young firms are more innovative than large and old firms (e.g., Arrow, 1962; Itenberg, 2015; Akcigit and Kerr, 2017). Second, small firms are more financially constrained, in particular for research and development (Midrigan and Xu, 2014; Itenberg, 2015). Based on these facts, we assume that a newborn firm chooses a project maturity τ such that it is *young* for $t \leq \tau$ and *mature* otherwise.

A young firm invests in research and development to improve the quality of the product. The evolution of quality is given by $\dot{q}(t) = \delta_1 + \lambda^Q \delta_2$. The first component, δ_1, captures a deterministic growth on quality. We assume δ_1 is large enough so at maturity the firm wants to repay the debt.[34] Second, as it is standard in the literature, we assume that quality makes jumps of size δ_2 that arrive at Poisson rate λ^Q. Doing research is costly. The firm pays κ per unit of time doing research. Moreover, the firm is financially constrained when young. It borrows to finance investment, which generates a demand for long-term finance.

At age τ, the firm becomes mature, stops R&D and starts production with quality $q(\tau, N)$, where N is a counting process with the number of jumps on quality before τ. Static profits are linear in quality $\pi(q) = \underline{\pi} q$. Hence, the net present value of a mature firm with quality q is given by $F(q) = \frac{\pi(q)}{\rho+\delta}$ where δ is the Poisson arrival rate of an exogenous exit shock.

Hence, this model deliver a similar structure to the benchmark model, where we can interpret the investments as improvements in the quality of the product rather than in firm productivity.

B.2 Time-to-build Capital

Rioja and Valev (2004) find that finance boosts growth in rich countries primarily by speeding up productivity growth, while finance encourages growth in poorer countries primarily by accelerating capital accumulation. In this section, we propose an alternative microfoundation for back-loaded projects based on time-to-build capital.

At every moment a new cohort of identical firms μ^0 enters the economy and chooses a project to implement. There is a continuum of projects differentiated by the time-to-build $\tau \geq 0$. For $t \in [0, \tau)$, the firm is *young* and is investing. For $t \geq \tau$, the firm is *mature* and is producing.

A young firm starts with no capital, $K(0) = 0$, and investment is subject to a time-to-build constraint as in Majd and Pindyck (1987) such that $dK = idt$ and $i \leq \kappa$. The investment rate i per unit of time cannot exceed κ. Given the linearity, it is optimal to build at maximum capacity. Hence, a firm with a project of duration τ concludes its investment stage with capital equal to $K = \tau\kappa$. This firm is subject to an exogenous exit shock that arrives at Poisson intensity δ, in which case the project is destroyed and the firm defaults.

A mature firm uses the capital to produce, and profits are given by $\pi = zK^\sigma$, where z is the productivity and σ are the returns to scale. This firm is subject to an exogenous exit shock

[34] If $\delta_1 = 0$ some firms will prefer to default at maturity. This might be an interesting setup to study corporate default.

that arrives at Poisson intensity δ. The net present value for a mature firm with project τ is

$$F(\tau) = \frac{zK^{\sigma}}{\rho + \delta}. \tag{28}$$

Note that the return on the project F increases with the time spent on investment. Hence, time-to-build is an alternative formulation for back-loaded projects.

C Extensions

This appendix describes the solution of the model with GSIs and segmented markets.

C.1 Government-Sponsored Intermediaries

This appendix describes how to solve the distribution of financiers with GSIs. The total assets with time-to-maturity t are $\mu(t) = \mu^0 e^{-\delta t}$. These assets are held by four types of agents: $\mu(t) = \mu^{p,h}(t) + \mu^{p,l}(t) + \mu^{g,h}(t) + \mu^{g,l}(t)$. The laws of motions for the private sector are

$$\begin{aligned}
-\dot{\mu}^{p,h}(t) &= -(\eta + \delta)\mu^{p,h}(t) + \left(\beta^{p-p}(t) + \beta^{g-p}(t)\right)\mu^{p,b} \\
-\dot{\mu}^{p,l}(t) &= \eta\mu^{p,h}(t) - \left(\delta + \lambda^{p-p} + \lambda^{p-g}\right)\mu^{p,l}(t)
\end{aligned}$$

with boundary conditions $\mu^{p,h}(\tau) = \mu^0$ and $\mu^{p,l}(\tau) = 0$. The law of motions for government agents are

$$\begin{aligned}
-\dot{\mu}^{g,h}(t) &= -(\eta + \delta)\mu^{g,h}(t) + \left(\beta^{p-g}(t) + \beta^{g-g}(t)\right)\mu^{g,b} \\
-\dot{\mu}^{g,l}(t) &= \eta\mu^{g,h}(t) - \left(\delta + \lambda^{g-p} + \lambda^{g-g}\right)\mu^{g,l}(t)
\end{aligned}$$

with boundary conditions $\mu^{g,h}(\tau) = \mu^{g,l}(\tau) = 0$. Matching implies

$$\begin{aligned}
\mu^{p,b}\beta^{p-p}(t) &= \mu^{p,l}(t)\lambda^{p-p} \\
\mu^{p,b}\beta^{g-p}(t) &= \mu^{g,l}(t)\lambda^{g-p} \\
\mu^{g,b}\beta^{p-g}(t) &= \mu^{p,l}(t)\lambda^{p-g} \\
\mu^{g,b}\beta^{g,b-g}(t) &= \mu^{g,l}(t)\lambda^{g-g}
\end{aligned}$$

Define $\mu(t) = \left[\mu^{p,h}(t), \mu^{p,l}(t), \mu^{g,h}(t), \mu^{g,l}(t)\right]$. The boundary condition is $\mu(\tau) = [\mu^0, 0, 0, 0]$

and the system is $\dot{\mu}(t) = A\mu(t)$ where

$$A = \begin{bmatrix} \eta+\delta & -\lambda^{p-p} & 0 & -\lambda^{g-p} \\ -\eta & \delta+\lambda^{p-p}+\lambda^{p-g} & 0 & 0 \\ 0 & -\lambda^{p-g} & \eta+\delta & -\lambda^{g-g} \\ 0 & 0 & -\eta & \delta+\lambda^{g-p}+\lambda^{g-g} \end{bmatrix}$$

The solution of this system is standard. The only caveat is that we should pay attention to the real and complex eigenvalues of the matrix A.

C.2 Segmented Markets

This appendix has the results for the model with segmented markets.

Distributions of financiers

Proof of Lemma 4. Total assets are $\mu^j(t) = e^{(t-\tau)\delta}\mu^0$. The evolution for high- and low-valuation agents in market j are

$$\begin{aligned} -\dot{\mu}^{H,j}(t) &= -(\eta+\delta)\,\mu^{H,j}(t) + \mu^{L,j}\beta^j(t) \\ -\dot{\mu}^{L,j}(t) &= \eta\mu^{H,j}(t) - \left(\delta+\lambda^j\right)\mu^{L,j}(\tau_j) \end{aligned}$$

Matching implies that $\mu^{B,j}\beta^j(t) = \mu^{L,j}(t)\,\lambda^j$. Hence, the system is

$$\dot{\mu}^j(t) = \begin{bmatrix} \eta+\delta & -\lambda^j \\ -\eta & \delta+\beta^j \end{bmatrix} \begin{bmatrix} \mu^{H,j}(t) \\ \mu^{L,j}(t) \end{bmatrix}$$

with eigenvalues δ and $\eta+\lambda^j+\delta$. Define V^j to be the matrix with the eigenvectors and R^j the diagonal matrix with the eigenvalues and $B^j = \left(V^j\right)^{-1}\mu^{j+1}(\tau_{j+1})$. Then

$$\begin{aligned} \mu^{H,j}(t) &= \sum_{i=1}^{2} e^{R^j(i)(t-\tau_{j+1})} B^j(i)\, V^j(1,i) \\ \mu^{L,j}(t) &= \sum_{i=1}^{2} e^{R^j(i)(t-\tau_{j+1})} B^j(i)\, V^j(2,i) \end{aligned}$$

For $j=1,\ldots,N-1$ we have that

$$\begin{aligned}
\mu^{H,j}(t) =& \frac{\eta}{\eta+\lambda^{j}}\left[\frac{\lambda^{j}}{\eta}e^{\delta(t-\tau_{j+1})}\left(\mu^{H,j+1}(\tau_{j+1})+\mu^{L,j+1}(\tau_{j+1})\right)\right] \\
&-\frac{\eta}{\eta+\lambda^{j}}\left[-e^{\left(\eta+\lambda^{j}+\delta\right)(t-\tau_{j+1})}\left(-\mu^{H,j+1}(\tau_{j+1})+\lambda^{j}\mu^{L,j+1}(\tau_{j+1})\right)\right] \\
\mu^{L,j}(t) =& \frac{\eta}{\eta+\lambda^{j}}\left[e^{\delta(t-\tau_{j+1})}\left(\mu^{H,j+1}(\tau_{j+1})+\mu^{L,j+1}(\tau_{j+1})\right)\right] \\
&+\frac{\eta}{\eta+\lambda^{j}}\left[+e^{\left(\eta+\lambda^{j}+\delta\right)(t-\tau_{j+1})}\left(-\mu^{H,j+1}(\tau_{j+1})+\lambda^{j}\mu^{L,j+1}(\tau_{j+1})\right)\right]
\end{aligned}$$

□

Value functions Let $Z^{j}(t)=D^{H,j}(t)-D^{L,j}(t)$ and $c_{j}=\rho+\delta+\eta+\gamma\lambda^{j}$, then $c_{j}Z^{j}(t)=h-\dot{Z}^{j}(t)$. The solution is $Z^{j}(t)=A^{Z,j}e^{-c_{j}t}+B^{Z,j}$ with $B^{Z,j}=\frac{h}{c_{j}}$ and the boundary condition pins down $A^{Z,j}$.

For $j=1$ the boundary condition is $D^{H,1}(\tau_{1})=D^{L,1}(\tau_{1})=1$ so $A^{Z,1}=-\frac{h}{c_{1}}$

For $j=2,\ldots,N$ we have that $Z^{j}(\tau_{j})=Z^{j-1}(\tau_{j})$ which implies $A^{Z,j}=e^{c_{j}\tau_{j}}\left(A^{Z,j-1}e^{-c_{j-1}\tau_{j}}+\frac{h}{c_{j-1}}-\frac{h}{c_{j}}\right)$

Next, we can solve for the value of high- and low-valuation agents using Z^{j} and the initial conditions. For high-valuation agents

$$(\rho+\delta)D^{H,j}(t)=-\dot{D}^{H,j}(t)-\eta\left(A^{Z,j}e^{-c_{j}t}+\frac{h}{c_{j}}\right)$$

The solution is $D^{H,j}(t)=A^{H,j}+B^{H,j}e^{-\left(\rho+\lambda^{D}\right)t}+C^{H,j}e^{-c_{j}t}$, with $A^{H,j}=-\frac{\eta h}{(\rho+\delta)c_{j}}$, $C^{H,j}=\frac{\eta A^{Z,j}}{\eta+\gamma\lambda^{j}}$, and the boundary condition pin down $B^{H,j}$.

For $j=1$, we have that $D^{H,1}(\tau_{1})=1$ and $\tau_{1}=0$, so $B^{U,1}=1-A^{U,j}-C^{U,j}$. For $j=2,\ldots,N$, we have that $D^{H,j}(\tau_{j})=D^{H,j-1}(\tau_{j})$ so

$$B^{H,j}=e^{(\rho+\delta)\tau_{j}}\left(A^{H,j-1}-A^{H,j}+B^{H,j-1}e^{-(\rho+\delta)\tau_{j}}+C^{H,j-1}e^{-c_{j-1}\tau_{j}}-C^{H,j}e^{-c_{j}\tau_{j}}\right)$$

which defines a recursion in $B^{H,j}$.

Free entry The free-entry condition in each market is

$$c_{j}=(1-\gamma)\int_{\tau_{j}}^{\tau_{j+1}}\beta^{j}(t)\left(D^{H,j}(t)-D^{L,j}(t)\right)dt$$

where $\beta^{j}(t)=A\left(\theta^{j}\right)^{\alpha}\frac{\mu^{C,j}(t)}{\mu^{C,j}}$ and both the measures and value functions are the sum of exponential functions. Hence, it is easy to solve for the integrals on the free-entry condition in each market.

D Cross-Country Evidence and Quantitative Results

This appendix summarizes the empirical evidence on corporate debt maturity across countries. Next, the cross-country evidence disciplines additional quantitative exercises.

D.1 Empirical Evidence Across Countries: Maturity

There is a vast empirical literature showing that firms in developing countries tend to borrow at shorter maturities than those in advanced economies. First, Demirgüç-Kunt and Maksimovic (1998) use firm-level balance sheet data—i.e., the data include different securities, not only corporate bonds—and find that the ratio of long-term debt (defined as maturity greater than one year to total liabilities) is typically lower in developing countries than in advanced ones, even after controlling for firm characteristics.[35] Second, the World Bank (2015) report on long-term finance uses bank-level balance sheet data and finds similar results. Third, Cortina Lorente et al. (2016) arrive at the same conclusion looking at corporate bond issuances in domestic markets. As that data set is the closest to the theory, we use it for the quantitative exercise.

Cortina Lorente et al. (2016) use Thomson Reuters Security Data Corporation (SDC) Platinum database to compile an extensive dataset of corporate bond issuances in domestic markets for 1991-2014 across 80 countries (41 developed and 39 developing economies).[36] The left panel of Figure 10 shows the empirical distributions of corporate debt maturities for developed and developing countries. In developing countries, the median maturity is about seven years, while it is about 11 years in advanced ones. The right figure shows a positive relationship between corporate debt maturity and output per capita across regions. For example, in the US the average maturity is 12.2 years, while in Latin America the average maturity is about 7.6 and GDP per capita is about 30% relative to the US.

D.2 Calibration for the US

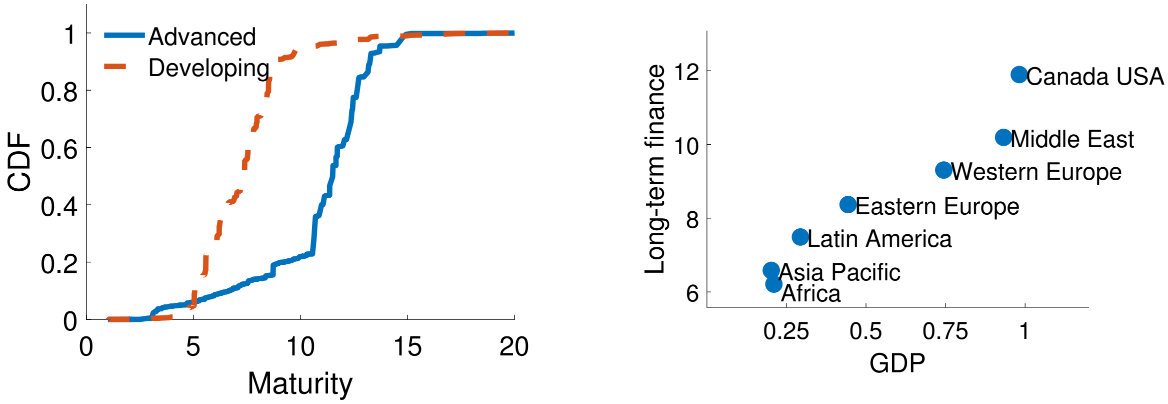
We match moments from the US corporate debt market. Except the liquidity spread and maturity, the rest of the calibration is the same as in Section 5.1. To discipline the component of the credit spread driven by liquidity in the data, we use the spread between Treasuries and high-quality corporate bonds from Section 5.2.[37] The calibration targets this spread only at the equilibrium maturity, i.e., one point of this yield curve, but as a validation the model replicates the data at different maturities.

[35] See also Caprio and Demirgüç-Kunt (1998); Demirgüç-Kunt and Maksimovic (1999); Booth et al. (2001); Fan et al. (2012).

[36] I thank the authors for sharing their data. For details, see Appendix E.

[37] Recall that high quality corporate bonds are rated above A so we can abstract from the default component.

Figure 10: **Maturity of corporate bonds markets across countries**



Note: Maturity is computed as the weighted average maturity (in years) of corporate bonds at issuance in domestic markets. Source: Cortina Lorente et al. (2016). See Appendix E for details.

For the maturity data, we use the estimates from (Cortina Lorente et al., 2016) so we can compare across a larger set of countries. Hence, for the US, we target a debt maturity at issuance of 12.2 years. Table 10 shows the calibrated parameters and moments.

Table 10: **Parameters and moments**

Parameter		**Value**	**Target/source**	**Model**	**Data**
Financial sector					
Matching efficiency	A	26.00	Expected time to sell	2.00	2.00
Intensity of liquidity shocks	η	0.58	Turnover rate	0.57	0.57
Holding cost	h	0.20	Liquidity spread	125	125
Search cost	c	0.19	Free entry		
Production sector					
$F(\tau) = Z\tau$	Z	5.38	Maturity	12.20	12.20
$I(\tau) = \kappa\frac{1-e^{-\rho\tau}}{\rho}$	κ	1.00	Normalization		
Matching					
Share of sellers	α	0.50	Normalization		
Bargaining power of sellers	γ	0.50	Normalization		
Others					
Discount factor	ρ	0.02	He and Milbradt (2014)		
Default rate	δ	0.03	Moody's (2015)		

D.3 Counterfactual Trading Frictions

For sovereign debt markets, Broner et al. (2013) show that it is more expensive to borrow long-term in emerging countries than in advanced ones. However, there is very little evidence on the yield curve for corporate bonds across countries. A moment commonly used in the literature to compare intermediation costs across countries is the bank's net interest margin, which measures the difference between interest income and payments to lenders using bank balance sheet data from Bankscope (the data is available at The World Bank Global Financial Development Database). For example, Greenwood et al. (2013) attribute these spreads to the intermediation costs related to acquiring information about borrowers. In this paper, we explore a different reason for these spreads, namely the illiquidity cost. Select the same countries as in the maturity data from Cortina Lorente et al. (2016) and split them between advanced and developing countries. According to these measures, credit spreads are 295 basis points higher and maturity is 3.6 years shorter in emerging countries than in advanced ones. The net interest margin is not a perfect measure of the illiquidity cost. However, it is in line with the estimate for the US and Argentina in Section 4.

Table 11 evaluates the counterfactual economy in which the matching efficiency reduces to match a change in the liquidity spread of 295 basis points as in the data. As a result, firms borrow at 3.45 years shorter, which is close to the data counterpart of 3.60 years.

The third column of Table 11 considers an alternative model in which the interest rate is exogenous and independent of maturity ($r = cs^{def} + cs^{liq}$ for all τ). Start with a constant liquidity spread of 125 basis points and increase it by 295 basis points as in the benchmark model. As a result, firms borrow at a shorter maturity but the effect is lower than in the model with endogenous liquidity. In the benchmark model, a change in the matching efficiency not only increases the liquidity spread but also changes the slope of the yield curve, which induces firms to borrow at shorter maturities. Hence, providing a theory of liquidity spreads is not only important to understand the frictions and study policy implications, but also to perform quantitative evaluations.

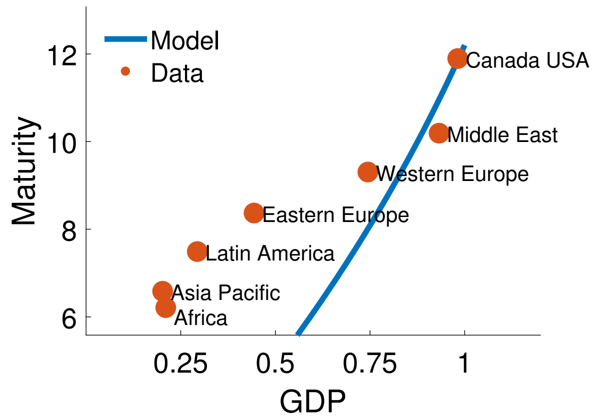
Table 11: **Credit spreads across countries**

	Data	**Model**	
		Endogenous	Exogenous
Δ Liquidity spread (bps)	295	295	295
Δ Maturity (years)	-3.60	-3.45	-3.09
Δ Output (%)	-60	-20	-17

Note: Credit spread data is from the World Bank Financial Structure and Economic Development Database. Maturity data is from Cortina Lorente et al. (2016).

Real economy When the search cost increases, long-term finance becomes relatively more expensive and firms tilt their maturity choices toward the short end. On the real side of the

Figure 11: **Maturity and the real economy**



Note: The blue line shows the equilibrium allocations of maturity and GDP under different frictions in the secondary market and the data is the same as in Figure 10.

economy, this implies that entrepreneurs invest in shorter-term projects, which have lower productivity and, therefore, lower aggregate output. Hence, changes in the real side of the model are driven by changes in the endogenous productivity that is an outcome of the maturity of investment. The third row of Table 11 shows that aggregate output reduces by 20% when the liquidity spread increases by 295 basis points and firms shorten the maturity by 3.45 years. Trading frictions can account for about one-third of the differences in maturities between advanced and developing countries due to changes in the duration of investment and aggregate productivity.

Finally, use the cross-country data on maturity and aggregate output to evaluate the performance of the model. For each country, change the matching efficiency to match the data on maturity and then evaluate the performance on aggregate output. Figure 11 shows that the model predicts a relationship between maturity and output across countries quite similar to the data counterpart. Note that the model has high explanatory power for developed countries. However, for emerging economies, trading frictions have lower explanation power. Even though we present a stylized and tractable model, the quantitative results suggest that the model captures important features of corporate debt markets and their effect on productivity and output across countries.

E Data

This appendix describes the data sources.

Credit spreads in Argentina Consider all the active corporate bonds in August 2017 in the domestic market (MAE) and keep issuances in local currency, with 100% amortization, and interest rate as a spread on the Badlar rate (which is the reference short-term rate in Argentina). These are floating interest rates bonds with a fixed spread, so the credit spread is just the spread on the Badlar rate because non-arbitrage implies that agents can swap the variable Badlar rate for a fixed rate.

High-quality corporate bond yield curve The corporate yield curve corresponds to the high-quality market (bonds rated above A), and it is available at https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/. Define the corporate yield curve as the monthly average for the year 2017. Tables 12 and 13 show the default rates, default credit losses, and the transition probabilities of credit ratings for high-quality issuers.

Table 12: **Default Credit Losses**

Rating	**Default credit losses** Average 1982-2014	**Default credit losses** Maximum 2008	**Default rates** Average 1920-2014	**Default rates** Maximum 2008
Aaa	0.00%	0.00%	0.000%	0.00%
Aa	0.03%	0.48%	0.061%	0.724%
A	0.03%	0.37%	0.096%	0.547%

Source: Moody's 2015.

Table 13: **Five-year Transitions (cumulative)**

	Aaa-A	**Baa-B**	**Caa-C**	**Default**
Aaa-A	88.70%	10.62%	0.15%	0.52%

Corporate debt maturity across countries The data for maturities in Figure 10 was shared by Cortina Lorente et al. (2016). The source is Thomson Reuters Security Data Corporation Platinum database and details about the data can be found in Cortina Lorente et al. (2016). In particular, the dataset comprises 80 economies, 41 of them are developed and 39 developing. For GDP we use GDP per capita relative to US for 2014, downloaded from the World Bank database (GDP per capita PPP, constant 2011 international $, series NY.GDP.PCAP.PP.KD).