# International Credit Markets and Global Business Cycles

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International Credit Markets
and Global Business Cycles*

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Abstract

This paper stresses a new channel through which global financial linkages contribute to the co-movement in economic activity across countries. We show in a two-country setting with borrowing constraints that international credit markets are subject to self-fulfilling variations in the world real interest rate. Those expectation-driven changes in the borrowing cost in turn act as global shocks that induce strong cross-country co-movements in both financial and real variables (such as asset prices, GDP, consumption, investment and employment). When firms around the world benefit from unexpectedly low debt repayments, they borrow and invest more, which leads to excessive supply of collateral and of loanable funds at a low interest rate, thus fueling a boom in both home and abroad. As a consequence, business cycles are synchronized internationally. Such a stylized model thus offers one way to rationalize both the existence of a world business-cycle component, documented by recent empirical studies through dynamic factor analysis, and the factor’s intimate link to global financial markets.

Keywords: World Interest Rate, International Co-Movement, Self-Fulfilling Equilibria

JEL codes: E21, E22, E32, E44, E63

1 Introduction

Empirical evidence suggests that there exists a set of common factors behind business cycles across nations. This set of global factors has affected simultaneously national outputs of many countries over the last 50 years. For example, Kose, Otrok and Whiteman (2003) document the existence of a distinct world business cycle, in a 60-country sample that spans seven regions of the world over the period from 1960 to 1990. More specifically, they show that on top of country-specific shocks, a world factor is also important and accounts for a significant fraction of country-specific business-cycle variations. In particular, this global factor accounts for about 35% of the variances of output and of consumption in the US, which turns out to be more than what the regional or the country-specific factors can explain.

Kose, Prasad and Terrones (2003) report similar results and further show that financial linkages between countries are more important for international business-cycle transmissions in economies that are more open to international capital flows (see also Imbs, 2004). More recently, Crucini, Kose and Otrok (2011) also confirm that the world-wide business cycle is as important as the nation-specific business cycles and they also find that “real interest rates are dominated by a common G-7 component, with two-thirds of the variance attributable to the common factor and almost none attributed to the nation-specific component” (page 167). In other words, in G7 countries real interest rates exhibit variations that appear to be driven by a common global factor. Therefore, not only national levels of output and consumption follow a world component, financial variables do as
In sum, the existing body of empirical evidence strongly points to the existence of a few world common factors that explain international co-movements of both output and financial variables. In addition, financial linkages between countries through international capital and debt markets, are suspected to play a key role in explaining these common factors. Of striking importance is the fact that a common component drives much of co-movements in different countries’ interest rates, at least in developed economies.

However, little is known theoretically about the nature of such common factors behind the world real output and financial variables, uncovered through dynamic factor analysis in the studies reviewed above. One obvious candidate is global technology shocks. But such an hypothesized existence of global technology shocks suffers from the lack of micro-level empirical evidence that innovations at the firm level are synchronized globally despite different institutional arrangements, such as patent laws and government efforts to protect technological secrets. It is not obvious that different countries’ technology frontiers progress and regress at the same time.

International trade is another possible source of global business cycle propagation (e.g., Miyamoto and Nguyen, 2017). But it takes time for goods and services to move across borders, shipping time often takes weeks if not months from one country to another. So the role of shocks to the speed of global shipping or trade costs in generating world business cycle is also likely to be limited.

This paper proposes that the existence of a global financial market sets the condition for the rise of financial and real global factors that cause global business cycles. We show how the existence of a global financial market makes possible for investors’ expectations to be coordinated around all corners of the planet. In particular, global sentiments originating from the world financial market can simultaneously drive both global interest rates and national outputs across the world. Intuitively, financial markets are much more tightly and timely connected across nations than goods and labor markets, yet within each country its goods and labor sectors are closely connected to its financial sector. As a result, animal spirits in the world financial market can simultaneously affect the financial and real variables in each country (especially across developed economies, which are more financially globalized).

To that end, we develop a very simple setting with two countries that are open to international financial capital flows, within which cross-national credit borrowing and lending operate under collateral constraints to alleviate defaults. We deliberately assume that production factors such as labor, land, and capital are not mobile across national borders to highlight the role of global financial linkages in generating global real business cycles.

More specifically, we assume that in each country households are international lenders and firms
are international borrowers, through a global bond market, under the standard assumption that
the former are more patient than the latter (à la Kiyotaki and Moore, 1997). Although borrowing
and lending are channeled through a world financial market, production factors in each country
cannot be traded across countries. In other words, in each country firms (households) can borrow
(lend) internationally (subject to a collateral constraint) and the world interest rate on financial
debt adjusts to reflect the tightness of the global supply and demand for loans.

We show that in this setting expectations about future world real interest rate are self-fulfilling.
As a result, sunspot/belief shocks to the world interest rate can simultaneously cause movements
in asset prices, output, consumption, employment, and investment to go in the same direction in
all countries. In other words, the optimistic/pessimistic views about the world interest rate can
generate global credit expansion/contraction, so that countries’ outputs, consumption, labor and
capital investment co-move together. Key to our results is the endogenous response of credit supply
to credit demand that renders the real interest rate counter-cyclical—a stylized fact and a long-
standing puzzle in the existing literature (see, e.g., Boldrin, Christiano and Fisher, 2001, Pintus,
Wen and Xing, 2016). In contrast, we show that country-specific shocks to productivity or leverage
do not induce co-movement across countries in output and other key aggregates, unless strong direct
(spill-over) effects are assumed. This is because an expansion in the home economy due to, say, an
increase in productivity, raises the world interest rate, which in turn hurts the foreign economy.

Related Literature: This paper contributes to the literature on open-economy extensions of
the seminal analysis in Kiyotaki and Moore (1997) - see also Iacoviello (2005), Pintus and Wen
(2013), Liu, Wang and Zha (2013), and Pintus, Wen and Xing (2016). In particular, we extend
the small-open-economy settings of, inter alia, Kocherlakota (2000), Pintus and Wen (2013), Kaas,
Pintus and Ray (2016), by considering a two-country model. The reason why self-fulfilling equilibria
arise in our two-country model is essentially similar to that detailed in the closed-economy version
with variable interest rate studied in Pintus, Wen and Xing (2016). In a nutshell, expectations about
the world interest rate can be self-fulfilling because they move credit supply and demand in the same
direction. When the interest rate is not predetermined, a lower than expected repayment triggers
a rise in demand for credit and collateral from the borrower, as well as an increase in collateral
and extended loans by the lender. Both our papers build upon and extend the seminal analysis
in Benhabib and Farmer (1994) on self-fulfilling equilibria in standard business-cycle theory by
introducing credit market frictions and global financial linkages. In addition, we use the technique
proposed by Farmer, Khramov and Nicolò (2015) to solve for the model with local indeterminacy.

Several papers emphasize that business cycles spread internationally through the channel of
international trade. For instance, in a perfect-foresight trade model, Nishimura and Yano (1993)
compare the correlation of the endogenous real business cycles in trading countries before and after
the opening up of trade. In a model with sector-specific externalities, Nishimura, Venditti and Yano (2010) investigate the interlinkages in the business cycles of two countries and its welfare implications when self-fulfilling prophecies arise in a free-trade equilibrium. In contrast, our emphasis on global financial linkages in explaining international co-movements accords with the existence of a global financial cycle documented in particular by Miranda-Agrippino and Rey (2015), in addition to the studies using dynamic factor analysis reviewed above. Differently, earlier explanations of international co-movements rely on productivity shocks that are highly correlated across countries, as in Backus and Kehoe (1992), Backus, Kehoe, and Kydland (1995). More recently, Kollman (2018) shows in a complete-market two-country model with Epstein-Zin-Weil preferences and a muted wealth effect that country-specific productivity shocks generate synchronized business cycles. In contrast, the recent evidence and our paper put international financial markets at the center stage to explain how financial spill-over effects help synchronize business cycles across countries.

Our paper also belongs to the literature on financial frictions and self-fulfilling equilibria. Liu and Wang (2014) show that the financial multiplier due to credit constraints shifts resources among firms with heterogeneous productivity, leading to endogenous aggregate TFP and increasing returns at the aggregate level. He, Wright, and Zhu (2015) study consumption loans secured by houses. They show how endogenous volatility in prices arises because housing conveys liquidity. Azariadis, Kass, and Wen (2016) focus on unsecured loans for which repayment incentives come from the value of reputation. In such a setting, they show that self-fulfilling equilibria exist because borrowers’ expectations of future credit market conditions affect their current value of reputation, which in turn determines current credit constraints. Our paper differs from the above by studying entrepreneurs’ secured borrowing activity and focusing on the equilibrium multiplicity stemming from the interaction between variable interest rate and entrepreneurs’ borrowing constraints.

The reminder of this paper is organized as follows. Section 2 sets the stage by presenting the two-country model, while Section 3 compares international business cycles driven by either country-specific or global shocks. Section 4 presents some concluding remarks and additional results are gathered in Section 5.

2 Two-Country Model

2.1 Description

The world economy is divided into two countries, home and foreign, that are each composed of households/lenders and entrepreneurs/borrowers. We assume that installed capital, land and labor are internationally immobile. To save space, we report below the problems faced by each agent
in the home country only, given that the foreign country has an identical structure. All variables with upper index "∗" are related to the foreign economy while variables without refer to the home country.

Production technology is given, in each period \( t \), by

\[
Y_t = A_t K_t^\alpha L_t^\gamma N_t^{1-\alpha-\gamma}
\] (1)

where \( K_t \) denotes capital, \( L_t \) is land, and \( N_t \) is worked hours. Total land supply is fixed and given by \( L \), with country-specific price \( Q_t \).

The entrepreneur, who takes care of production, hires labor in the domestic market with wage rate \( W_t \), and accumulates both productive capital and productive land. He participates in an international bond market where a one-period bond is exchanged, which earns the gross rate \( R_{t+1}^w \) that will be realized at date \( t+1 \). Under the assumption that the entrepreneur is more impatient than the household, he turns out to be a borrower near steady state. The entrepreneur’s bond demand is denoted by \( B_{e,t+1} \).

Finally, the entrepreneur enjoys consuming \( C_{e,t} \) in each period. The borrowing firm solves the following problem:

\[
\max_{\{C_{e,t}, K_{t+1}, L_{e,t+1}, B_{e,t+1}, N_t\}} \mathbb{E}_t \left[ \sum_{t=0}^{\infty} \beta^t \frac{C_{e,t}^{1-\sigma_b}}{1 - \sigma_b} \right]
\]

under the budget constraint:

\[
C_{e,t} + K_{t+1} - (1 - \delta)K_t + Q_t(L_{e,t+1} - L_{e,t}) + R_{t+1}^w B_{e,t} \leq B_{e,t+1} + Y_t - W_t N_t
\] (2)

as well as the collateral constraint:

\[
\mathbb{E}_t[R_{t+1}^w]B_{e,t+1} \leq \Theta_t \mathbb{E}_t[Q_{t+1}]L_{e,t+1}
\] (3)

which says that expected debt repayment cannot exceed a fraction \( \Theta_t \) of the market value of land that is expected to materialize next period.

Denoting \( \Lambda_{e,t} \) and \( \Phi_{e,t} \) the Lagrange multipliers associated to firm’s budget constraint (2) and collateral constraint (3), respectively, the following set of equations gives the firm’s first-order conditions for interior solutions:

\[
C_{e,t}^{1-\sigma_b} = \Lambda_{e,t}
\] (4)

\[
\Lambda_{e,t}Q_t = \Phi_{e,t} \mathbb{E}_t[Q_{t+1}] + \beta_\varepsilon \mathbb{E}_t \left[ \Lambda_{e,t+1} \left( \gamma \frac{Y_{t+1}}{L_{e,t+1}} + Q_{t+1} \right) \right]
\] (5)

\[
\Lambda_{e,t} = \beta_\varepsilon \mathbb{E}_t \left[ \Lambda_{e,t+1} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right) \right]
\] (6)

1To be parsimonious, we use indices \( h \) and \( e \) only for variables that differ across households and entrepreneurs in equilibrium. For example, since production comes from entrepreneurs only, \( Y_t \) is not indexed by "e". Similarly, \( N_t \) refers to hours worked in equilibrium.
Each country also has a representative household, who derives utility from consumption $C_{h,t}$ and land holding $L_{h,t}$. The household supplies labor $N_t$, which entails some loss in utility. She supplies labor to domestic market and has access to the international bond market so as to transfer consumption across time. Household’s demand for international bonds is given by $B_{h,t+1}$.

Because the household is more patient than the entrepreneur in each country (that is, $\beta_h > \beta_e$), she acts as a lender and solves:

$$\max_{\{C_{h,t}, L_{h,t+1}, B_{h,t+1}, N_t\}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta_h^t \left( C_{h,t}^{1-\sigma_l} L_{h,t}^{1-\sigma_w} \psi - \sigma_w \right) \right]$$

subject to the budget constraint:

$$C_{h,t} + Q_t(L_{h,t+1} - L_{h,t}) + B_{h,t+1} + \frac{\phi}{2} (B_{h,t+1} - B_{e,t+1})^2 \leq R_w B_{h,t} + W_t N_t$$

Note that land enters the household’s utility, and in his budget constraint (9) there appears a quadratic holding cost, the size of which is parameterized by $\phi$. This quadratic cost is expressed in terms of the net foreign asset position (or surplus), which we will denote by $S_t = B_{h,t} - B_{e,t}$. Whenever $S_t$ is positive (negative), this means that the home economy is lending to (borrowing from) the foreign economy. Of course, in that case the foreign counterpart $S_t^*$ is negative (positive). Similarly, if the deviation of $S_t$ from steady state is positive (negative), then either the home country’s lending goes up (down) or its borrowing goes down (up). The response of the foreign economy’s net foreign asset position is again the mirror image of the home economy’s.

The main reason we include a quadratic cost in terms of the net foreign asset position is to make the model stationary. Absent this cost (that is, $\phi = 0$), the model’s dynamic system has a unit root which comes from the fact that the steady-state net foreign asset position in each country is indeterminate. In such a case, the model determines only the world demand for bonds but not how it is split between countries. A unit root in our two-country setting is neither new nor surprising since there is only one world interest rate that equalizes the demand and supply of bonds at the world level, not within countries. A similar picture arises in the two-country model of, e.g., Baxter and Crucini (1995). We set $\phi = 0.0001$ throughout, so that the quadratic cost is negligible for all practical purposes.\footnote{Schmitt-Grohe and Uribe (2003) set $\phi = 0.00074$ to ensure stationarity in a small-open-economy model. Both their and our value for $\phi$ therefore implies a holding cost of net foreign assets that is negligible. The main function of the holding cost is to pin down a well-defined steady state, not to account for business cycle variances. Our qualitative and quantitative results are not sensitive to different parameter values of $\phi$. Particularly, $\phi$ is not crucial for the model’s endogenous persistence, because it is associated with an eigenvalue greater or equal to 1: when $\phi$ is zero, the...}
Therefore, the household’s first-order conditions are given, assuming interior solutions, by:

\[ C_{h,t} - \sigma L_{h,t} = \Lambda_{h,t} \]  
\[ \Lambda_{h,t} Q_t = \beta_h \mathbb{E}_t \left[ \psi L_{h,t+1} + \Lambda_{h,t+1} Q_{t+1} \right] \]  
\[ \Lambda_{h,t} \left[ 1 + \phi (B_{h,t+1} - B_{e,t+1}) \right] = \beta_h \mathbb{E}_t \left[ \Lambda_{h,t+1} R_w^{t+1} \right] \]  
\[ \Lambda_{h,t} W_t = \zeta N_t^\eta \]  

where \( \Lambda_{h,t} \) is the Lagrange multiplier associated to the budget constraint (9). Note that the first-order condition with respect to international bond demand, that is, equation (12), also holds in the foreign country. This implies, because the world interest rate is common in both country, that a unit root arises in the dynamics of the ratio of home and foreign consumptions when there is no cost of holding a nonzero foreign asset position (that is, when \( \phi = 0 \)).

Given that:

\[ S_{t+1} = B_{h,t+1} - B_{e,t+1} \]  
\[ X_t = Y_t - C_{h,t} - C_{e,t} - I_t - \frac{\phi}{2} S_{t+1}^2 \]  

The above definitions and relations apply to the foreign country symmetrically.

The market clearing condition for land is:

\[ \bar{L} = L_{e,t+1} + L_{h,t+1} \]  

In addition, the international debt market clears when

\[ B_{h,t+1} + B_{e,t+1} = B_{h,t+1} + B_{e,t+1}^*, \]  

or equivalently \( S_{t+1} + S_{t+1}^* = 0 \), while Walras’ Law implies that the world output market clears as well, that is:

\[ C_{e,t} + C_{h,t} + C_{e,t}^* + C_{h,t}^* + \frac{1}{2} \phi (B_{h,t+1} - B_{e,t+1})^2 + \frac{1}{2} \phi^* (B_{h,t+1} - B_{e,t+1}^*)^2 + I_t + I_t^* = Y_t + Y_t^* \]

where \( I_t = K_{t+1} - (1 - \delta) K_t \) denotes investment net of capital depreciation. This also implies that net exports are equal to the current account (that is, \( S_t + S_t^* \) minus the interest income from the net foreign asset position \( R_w^t - 1 \) \( S_t \)), i.e.

\[ X_t = S_{t+1} - R_w^t S_t = - (S_{t+1}^* - R_w^t S_t^*) = - X_t^*. \]  

To see this, set \( \phi = 0 \) and log-linearize equation (12) as well as its foreign analog around steady state, which gives \( c_{h,t} - c_{h,t}^* = \mathbb{E}_t [c_{h,t+1} - c_{h,t+1}^*] \), where \( c_{h,t} \) denotes the deviation of log \( C_{h,t} \) from steady state.

model has a unit eigenvalue; while a slightly positive \( \phi \) makes the corresponding eigenvalue larger than 1 to satisfy the Blanchard-Kahn conditions. Similar results with different values for \( \phi \) are not presented to save space but are available upon request.
The equilibrium allocation is described by the set of variables:

\[ \{ Y_t, C_{e,t}, C_{h,t}, L_{e,t+1}, L_{h,t+1}, K_{t+1}, B_{e,t+1}, B_{h,t+1}, S_{t+1}, X_t, N_t, \Lambda_{e,t}, \Lambda_{h,t}, \Phi_{e,t}, Q_t, W_t \}_{t=0}^{\infty} \]

for home and

\[ \{ Y^*_t, C^*_{e,t}, C^*_{h,t}, L^*_{h,t+1}, L^*_{e,t+1}, K^*_{t+1}, B^*_{e,t+1}, B^*_{h,t+1}, S^*_{t+1}, X^*_t, N^*_t, \Lambda^*_{e,t}, \Lambda^*_{h,t}, \Phi^*_{e,t}, Q^*_t, W^*_t \}_{t=0}^{\infty} \]

for foreign, while \( \{ R^w_t \}_{t=0}^{\infty} \) is the world interest rate. Those 33 variables solve home and foreign versions of equations (1)-(16) and the international debt market clearing condition (17).

### 2.2 Self-fulfilling Equilibria

The purpose of this section is to extend the proof in Pintus, Wen and Xing (2016), obtained in a closed-economy setting, to show that steady-state indeterminacy arises also in our 2-country setting. This happens when loans are such that the interest rate that applies to current debt repayment is variable, i.e. not predetermined. More specifically, a simple model with closed-form solutions is used to derive analytical results that are amenable to intuition. The setting is essentially a linear and simplified version of the model developed above: there is no capital and no labor, a linear technology transforms land in output, and the lender has linear utility. In addition, we abstract from the above quadratic cost for further simplicity.

In the simplified model, the budget constraint of the representative lender is now:

\[ C_{h,t} + Q_t (L_{h,t+1} - L_{h,t}) + B_{h,t+1} \leq R^w_t B_{h,t} \]  

while the instantaneous utility function of the lender is given by \( C_{h,t} + \psi L_{h,t} \). In other words, we assume that \( \sigma_l = \sigma_w = 0 \). In addition, a representative borrower produces goods using land and the linear technology given by \( Y_t = AL_{e,t} \), while his budget constraint is now given by:

\[ C_{e,t} + Q_t (L_{e,t+1} - L_{e,t}) + R^w_t B_{e,t} \leq B_{e,t+1} + AL_{e,t} \]

An important feature of the budget constraint is that debt repayment is not predetermined in period \( t \), as the endogenous interest rate adjusts to fundamental and possibly self-fulfilling shocks.\footnote{Under the chosen timing, the liability of the entrepreneur in period \( t \) is \( R^w_t B_{e,t} \), as opposed to \( R^w_{t-1} B_{e,t} \). That is, the interest payment generated by the outstanding debt stock \( B_{e,t} \) is subject to changes in the current market interest rate in period \( t \), when the debt repayment is due. Many financial debt contracts such as adjustable-rate mortgage contracts use variable interest rate, such as the LIBOR rate plus a markup. The proportion of variable rate debt contracts is significant in practice, e.g. Vickery (2008) shows that 70% of lines of credit in the Survey of Small Business Finance database are variable-rate.} The per-period utility function of the representative borrower is given by \( \log C_{e,t} \) (that is, \( \sigma_b = 1 \)). Finally, the borrowing faced by the borrower is still given by equation (3), with \( \Theta_t = \Theta \) now supposed to be constant over time.
The model just described turns out to have closed-form solutions. More specifically, assuming $A = 1$ and $\Theta = 1$, the first-order conditions of the lender imply that the land price is constant over time, that is, $Q = \beta_e/(1 - \beta_h)$, while the expected interest rate is constant too, that is, $\mathbb{E}_t R_{t+1}^w = \beta_h^{-1}$. In addition, the binding credit constraint gives $B_{e,t+1} = \beta_h Q L_{e,t+1}$, which once plugged into the borrower’s budget constraint gives:

$$C_{e,t} + Q(1 - \beta_h)L_{e,t+1} = X_t L_{e,t} \tag{21}$$

where $X_t \equiv 1 + Q(1 - \beta_h R_t^w)$ represents the borrower’s return on land net of interest payment. It is then easy to show that, due to logarithmic utility, the borrower’s consumption and land demand have closed-form solutions that are given by $C_{e,t} = (1 - \beta_e) X_t L_{e,t}$ and $L_{e,t+1} = X_t L_{e,t}$. On the other hand, lender’s first-order condition boils down to $\mathbb{E}_t R_{t+1}^w = \beta_h^{-1}$. It follows that self-fulfilling equilibria are simply constructed as solutions to $L_{e,t+1} = [1 + Q(1 - \beta_h R_t^w)] L_{e,t}$ and $\beta_h R_t^w = 1 + \varepsilon_t$, where the innovation $\varepsilon_t$ is any i.i.d. random variable with zero mean, given initial value $L_0 > 0$. In this simple setup, sunspot innovations $\varepsilon_t$ originate from forecasting errors on the interest rate. This means that such a simple economy with variable (non-predetermined) interest rate can be globally indeterminate so that interest rate expectations are self-fulfilling in both the home and the foreign economies.

In contrast, it is straightforward to see that steady-state determinacy holds when the interest rate is predetermined. By this we mean that the borrower’s budget and credit constraints are now:

$$C_{e,t} + Q(L_{e,t+1} - L_{e,t}) + R_{t-1}^w B_{e,t} \leq B_{e,t+1} + AL_{e,t} \tag{22}$$

$$R_{t-1}^w B_{e,t+1} \leq Q L_{e,t+1} \tag{23}$$

while the lender’s budget constraint is:

$$C_{h,t} + Q(L_{h,t+1} - L_{h,t}) + B_{h,t+1} \leq R_{t-1}^w B_{h,t} \tag{24}$$

so that the interest repayment due in period $t$ is now predetermined while the interest rate that enters the credit constraint is variable but now known in period $t$. It is then easy to show that the interest rate is constant over time, that is, $R_t^w = \beta_h^{-1}$, so that $X_t = 1$ at all dates and the economy is forever in steady state, absent fundamental shocks, hence not subject to self-fulfilling shocks.

Going back to the case of loans with a non-predetermined interest rate, the simple model described above is useful to get an intuitive sense of why self-fulfilling equilibria arise. To do so, let us derive credit demand and credit supply at the world level. Credit demand, originating from the home and foreign economies, can be derived from the binding credit constraints, that is:

$$B_{t+1}^d = \beta_h Q [L_{e,t+1} + L_{e,t+1}^*] \tag{25}$$
Figure 1: Both credit demand $B^d$ and credit supply $B^s$ shift rightward when the borrowers expect a fall in interest rate and invests more in land so that $L_{e,t+1} + L^*_e,t+1$ goes up, resulting in a self-fulfilling fall in $R^w_t$.

where $L_{e,t+1} + L^*_e,t+1$ represents total land in the hands of borrowers/producers at the world level.

On the other hand, manipulating the lender's budget constraint and taking into account that world consumption equals world output, it is straightforward to get that world credit supply is given by:

$$B^s_{t+1} = Q[L_{e,t+1} + L^*_e,t+1] - \beta X_t[L_{e,t} + L^*_e,t]$$  \hspace{1cm} (26)

and both are conveniently depicted in Figure 1. Now suppose that the borrowers around the world expect the interest rate to go down. Then the borrowers increase their consumption and land investment $L_{e,t+1} + L^*_e,t+1$ so that credit demand shifts rightward in Figure 1. In addition to being a shifter of credit demand through the collateral channel - see (25) - $L_{e,t+1} + L^*_e,t+1$ is also a shifter of credit supply through land reallocation to the borrowers - see (26). As can be seen from Figure 1, the net effect is a fall of the interest rate. This is because in view of equations (25) and (26), the credit supply curve shifts to the right by more than the credit demand curve when $L_{e,t+1} + L^*_e,t+1$ goes up: when the borrowers’ land demand goes up by $\Delta[L_{e,t+1} + L^*_e,t+1]$, the lenders’ land holdings go down by the same amount since land is in fixed supply in each country, which means that total lenders’ savings in the form of lending goes up by $Q\Delta[L_{e,t+1} + L^*_e,t+1]$. On the other hand, borrowers’ credit demand goes up by $\beta h Q\Delta[L_{e,t+1} + L^*_e,t+1]$, that is, by a little less since the loan-to-value ratio is smaller than one. The bottom line is that the interest rate goes down and the initial expectation is fulfilled. In other words, the interest rate is counter-cyclical in the indeterminate model with variable-rate loans. In contrast, this cannot happen in the economy with predetermined interest.
rate, which stays in steady state forever, absent fundamental shocks, because the interest rate is constant through time.

3 International Business Cycles

The purpose of this section is to contrast the impulse responses arising in the model developed in Section 2.1, under country-specific (fundamental) shocks and under global (expectation-driven) interest rate shocks. More specifically, we assume that country-specific shocks happen under two guises. The home country experiences a positive shock to either TFP ($A$) or the loan repayment-to-value ratio ($\Theta$, leverage for short), where both variables are assumed to follow an $AR(1)$ process with auto-correlation set to 0.95 for the sake of illustration. Notice that these two shocks to the home country have no direct (spill-over) effect on the foreign economy by construction (that is, they are not correlated across countries).

In contrast, we define a global sunspot shock as a self-fulfilling one-time reduction in the world interest rate ($R$), which by definition means that the cost of borrowing internationally goes down for both home and foreign firms. Importantly, even though the shock to the world interest rate has no exogenous persistence, the model generates endogenous persistence that is commensurate with that under persistent fundamental shocks.

A second dimension of our simulation exercises is that we compare symmetric and asymmetric countries. In the former case, both the home and the foreign economies share the exact same parameter values, that we report in Table 1. In the asymmetric case, we simply assume that the collateral constraint is tighter in the foreign economy that is otherwise similar to the home economy, and we report the impulse responses to country-specific and global shocks.

The main lesson to draw from this section is that global sunspot shocks channeled through the international credit market are much more conducive to co-movement across countries than country-specific shocks. This is because country-specific shocks that trigger an expansion in the home economy also raise the world interest rate, which adversely affects the foreign economy. In contrast, worldwide expectations of a falling interest rate generate an expansion in both regions of the world, which then co-move.

As in Pintus, Wen and Xing (2016), today’s interest rate is not uniquely pinned down and there is one-dimensional indeterminacy. Such indeterminacy comes from the fact that loans feature a variable rate, as emphasized in Section 2.2 using a simplified setting. The model is numerically solved by using the technique proposed by Farmer, Khramov and Nicolò (2015), which entails a “fundamental” forecast error $\varepsilon_t$ such that $R^n_w = E_{t-1} R^n_w + \varepsilon_t$. 

\footnote{As in Pintus, Wen and Xing (2016), today’s interest rate is not uniquely pinned down and there is one-dimensional indeterminacy. Such indeterminacy comes from the fact that loans feature a variable rate, as emphasized in Section 2.2 using a simplified setting. The model is numerically solved by using the technique proposed by Farmer, Khramov and Nicolò (2015), which entails a “fundamental” forecast error $\varepsilon_t$ such that $R^n_w = E_{t-1} R^n_w + \varepsilon_t$.}
3.1 Symmetric Countries

Our benchmark calibration for the home economy appears in Table 1. In this section, we focus on the case where the foreign economy’s parameters are also given by values reported in Table 1 and the countries are said to be symmetric.

<table>
<thead>
<tr>
<th>Table 1: Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
</tr>
<tr>
<td>0.4</td>
</tr>
</tbody>
</table>

Many parameter values in Table 1 are standard for a quarterly calibration. The share of capital is set to 0.4 while the share of land equals 0.05, which together imply that the share of labor income is 0.55. Capital depreciation is set to 2.5% while the quarterly interest rate equals 1%, given that $\beta_h = 0.99$. In addition, the borrower is only moderately more impatient than the lender, with $\beta_e = 0.98$. We focus on log utility and assume that the Frisch elasticity of labor supply is 0.1, in agreement with many micro studies. Finally, the loan repayment to value ratio $\Theta$ is set at 0.8, which means that the entrepreneur is able to borrow up to 80% of the market value of his land holdings.

Responses to Country-Specific Shocks.

We show first that country-specific shocks (either a positive TFP shock or a relaxation of the borrowing constraint) cannot generate world-wide co-movements in all key variables. In the following graphs, the blue lines represent the home economy while the red lines denote the foreign responses.

When countries are symmetric, temporary country-specific TFP shocks have very persistent effects, as can be seen in Figure 2. Additional impulse-response functions (IRFs for short) are reported in Appendix 5.2, Figure 8. But the home and foreign economies do not co-move along some key variables, such as output, investment and labor. This is because the boom in the home economy generated by rising productivity raises the world interest, which adversely affect the foreign economy in the absence of technological spill-over effects. When the home entrepreneur benefits from higher TFP, his demands for capital, land and labor go up, which in turn raises the price of land and the wage. The combination of a higher land price and of holding more land implies that the entrepreneur has more collateral, thus raising his demand for credit. In sum, because at home land is reallocated towards production, entrepreneurs accumulate more capital and households work harder, output goes up. A boom in household consumption also follows because of as strong income effect from a higher return on savings, just like what would happen in a closed economy. However, in our two-country setting, the home economy has access to a global credit market and the rise in
home demand for international bonds pushes the interest rate up next period (see Figure 2).\textsuperscript{6} In the foreign economy, the rise in tomorrow’s interest rate generates a positive income effect on the household/lender. Being richer and facing a rising return on lending, the household consumes more, works less (which pushes the wage up), and lends more. In contrast, a rising interest rate and a rising wage both hurt the entrepreneur in the foreign economy, who cuts on investment and faces a decreasing labor supply. Therefore, output falls in the foreign economy.\textsuperscript{7}

As a consequence, initially, part of the boom in the home economy is financed by the foreign economy since the home net foreign asset position goes down. This is consistent with the fact that financial resources initially flow into the home economy since it is more productive. However, pretty soon the home economy ends up with a trade surplus and a rising net foreign asset position. The reason is that while foreign households initially supply more loanable funds, this pattern is quickly reversed because both the wage income and the bond return overshoot their steady state levels. In terms of co-movement, consumptions in the home and foreign economies move in the same

\textsuperscript{6}Note that because the interest rate is subject to belief shocks, it becomes predetermined and fundamental shocks affect today’s expectations of tomorrow’s interest rate.

\textsuperscript{7}Note that because households are willing to lend more in the foreign economy, the land market adjusts and land is reallocated to the entrepreneur, just as in the home economy. As a consequence, a one-period boom in output materializes before a long-lasting recession in the foreign economy.

Figure 2: IRFs to a positive TFP shock in home economy – Symmetric countries (Home: blue lines; Foreign: red lines)
Figure 3: IRFs to a positive leverage shock in home economy – Symmetric countries (Home: blue lines; Foreign: red lines)

direction, at least during a few quarters after the positive TFP shock hits the home economy, both for households and for entrepreneurs. Similarly, land prices co-move across countries. However, national outputs, labor supplies and investment levels do not co-move.

Similarly, when countries are structurally symmetric and the home economy benefits from a temporary relaxation of the collateral constraint, key variables do not co-move, as shown in Figure 3. Specifically, while consumptions move together across countries, just as the land price does, home and foreign outputs are negatively correlated. Essentially, when the collateral constraint is relaxed in the home economy, the entrepreneur borrows more and produces more. As in the case of a positive TFP shock, the associated credit boom in the home economy raises the world interest rate (see Figure 3). This again hurts the foreign economy. In contrast with the case of the TFP shock, however, labor supplies now co-move but wages move in opposite direction.\(^8\)

\(^8\)Another difference is that the land price goes down at impact under a positive leverage shock, as shown in Figure 9 in Appendix 5.2. This is required because the credit constraint is relaxed when \(\Theta\) goes up. Specifically, both the demand curve and the supply curve for land shifts downward. From the borrower’s perspective, the land price reflects both the collateral value and the payoff value (production value and resale value), as described in equation (5). When the borrowing constraint is relaxed, the collateral value of land decreases, which shifts the demand curve for land downward. From the perspective of the lender, higher credit demand coming from the borrower pushes up the interest rate. So the lender reduces investment in land and extends loans, which shifts the supply curve of land downward. The above two effects combine to lower the land price and to reallocate land from the lender to the borrower.
Responses to Global Interest Rate Shocks

In contrast to what happens under country-specific TFP or leverage shocks, the responses of the home and the foreign economies coincide under a sunspot shock to the world interest rate, as shown in Figure 4. In other words, all variables in the home and foreign economies co-move, except of course net foreign asset positions and net exports.

This is because a self-fulfilling reduction in the interest rate means that the cost of borrowing is lowered both for home firms as well as for foreign firms. As a consequence, the demand for loans by entrepreneurs goes up all around the world. Note that the supply of loans from households accommodate demand despite a fall in consumption — and an increase in labor supply — after impact because consumption goes up later. The resulting credit boom in both economies is accompanied by a reallocation of land away from households and towards entrepreneurs, which results in a declining land price (see Figure 10 in Appendix 5.2). Because entrepreneurs borrow more, they accumulate more land and capital and they also benefit from a larger labor supply. Therefore, output goes up in both countries by the same amount since countries are symmetric and face the same reduction in the world interest rate. Last but not least, notice that although the shock to the world interest rate has no built-in persistence, its propagation mechanism generates considerable persistence endogenously. In fact, the effect of an iid interest rate forecast error is at least as persistent as the effect of a highly idiosyncratic shock.
autocorrelated shock to TFP or leverage.

In addition, while the interest rate turns out to be counter-cyclical under sunspot shocks, as in the data, it is pro-cyclical under either TFP or leverage shocks. Note that when today’s interest rate turns out to be lower than expected previously, both home and foreign firms expect a higher borrowing rate starting tomorrow, as shown in Figure 4, which is required for lenders to be willing to extend credit today.

The main result from the IRFs reported in this section, therefore, is that country-specific shocks to TFP or leverage do not generate co-movement in output, investment and many other variables across economies. In contrast, global sunspot shocks to the world interest rate do. This is further illustrated in Table 2, in which we report in the first three rows the cross-country contemporaneous correlations obtained from stochastic simulations of the model, under the three sources of disturbances. The last two rows in Table 2 also show the correlations obtained from the data, which are all positive. Table 2 suggests that global interest shocks play a key role, in our model, to explain the empirical fact that output levels co-move internationally.

Table 2: International correlations between symmetric countries (simulations for 10000 periods)

<table>
<thead>
<tr>
<th></th>
<th>corr(Y, Y^*)</th>
<th>corr(C, C^*)</th>
<th>corr(I, I^*)</th>
<th>corr(N, N^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home TFP shock (Model)</td>
<td>-0.10</td>
<td>0.25</td>
<td>-0.28</td>
<td>-0.81</td>
</tr>
<tr>
<td>Home leverage shock (Model)</td>
<td>-0.48</td>
<td>0.88</td>
<td>-0.94</td>
<td>0.48</td>
</tr>
<tr>
<td>Global interest rate shock (Model)</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>US - Eurozone (Data)</td>
<td>0.66</td>
<td>0.67</td>
<td>0.62</td>
<td>n.a.</td>
</tr>
<tr>
<td>US - Canada (Data)</td>
<td>0.78</td>
<td>0.52</td>
<td>0.77</td>
<td>0.43</td>
</tr>
</tbody>
</table>

3.2 Asymmetric Countries

In this subsection, we assume that countries have different Θ’s in steady state, thus reflecting different levels of *internal* financial development, that are however not too dissimilar (think about two developed economies). More precisely, countries are now asymmetric and while the home economy’s parameters are still given by Table 1 (with in particular $\Theta = 0.8$), the foreign economy’s $\Theta^* = 0.6$. In other words, foreign firms face a tighter borrowing constraint. Note that in the graphs reported below, the blue lines represent the home responses while the red lines denote the foreign

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9Note also that country-specific shocks generate a credit boom that is short lived in the foreign economy. As a result, cross-country correlation of debt is much weaker than under global interest rate shocks.

10We use quarterly data for output, consumption, and investment in G7 countries, from 1996Q1 to 2018Q2, as well as annual data for hours, 1950-2014. Each time series is downloaded from the FRED database, and empirical correlations in Table 2 are based on HP-filtered, log-level data.
Responses to Country-Specific Shocks

The IRFs to a positive TFP shock hitting the home economy, reported in Figure 5, do not differ much from those in the symmetric case (see Figure 2). Differently though, the recession in the foreign economy is now more brutal and more persistent. See Figure 11 in Appendix 5.3 for IRFs of additional variables. Similarly for responses to a positive leverage shock in the home economy (contrast Figure 6 with Figure 3): the recession in the foreign economy is more brutal (see also Figure 12 in Appendix 5.3). In a nutshell, country-specific shocks do not make all key variables in both economies to co-move, especially for output and investment, when countries are asymmetric. This turns out to be the case despite the fact that, at impact, a credit boom happens simultaneously at home and in the foreign economy.

Figure 5: IRFs to a positive TFP shock in home economy – Asymmetric countries (Home: blue lines; Foreign: red lines)
Responses to Global Interest Rate Shocks

As in the symmetric case, both countries still co-move when they are asymmetric, albeit not perfectly now (see Figure 7). More specifically, after the world interest rate falls, the responses of almost all variables are larger in the home economy since it is now more leveraged than the foreign country. This implies that financial amplification is more pronounced at home, compared to abroad. As a consequence, home’s output goes up by more than the foreign’s. On the other hand, the responses of land price, household’s consumption, land holdings are almost indistinguishable. An important difference with the symmetric case, though, is that the more leveraged home economy reduces its net foreign asset position by importing financial resources from abroad. As a consequence, the foreign’s capital stock goes down at impact, even though investment booms in a persistent way later on, just as in the home economy. With countries that have different levels of financial development, therefore, a reduction in the world interest rate will create credit booms that materialize simultaneously around the world, the effects of which are more pronounced in a more leveraged economy. See also Figure 13 in Appendix 5.3.

In Table 3 we report the cross-country correlations generated by the model and by the data, for the case of asymmetric countries. Note that under TFP and interest rate shocks, the correlation of investment across countries is negative due to the responses at impact but becomes positive.
after. This suggests that investment adjustment costs would improve the model. Here again, global interest rate shocks are essential to generate output co-movement across countries, as depicted in Table 3.

Table 3: International correlations between asymmetric countries (simulation for 10000 periods)

<table>
<thead>
<tr>
<th></th>
<th>corr(Y, Y*)</th>
<th>corr(C, C*)</th>
<th>corr(I, I*)</th>
<th>corr(N, N*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home TFP shock (Model)</td>
<td>-0.12</td>
<td>0.22</td>
<td>-0.43</td>
<td>-0.81</td>
</tr>
<tr>
<td>Home leverage shock (Model)</td>
<td>-0.70</td>
<td>0.90</td>
<td>-0.96</td>
<td>0.42</td>
</tr>
<tr>
<td>Global interest rate shock (Model)</td>
<td>0.99</td>
<td>0.99</td>
<td>-0.67</td>
<td>0.98</td>
</tr>
<tr>
<td>US - Eurozone (Data)</td>
<td>0.66</td>
<td>0.67</td>
<td>0.62</td>
<td>n.a.</td>
</tr>
<tr>
<td>US - Canada (Data)</td>
<td>0.78</td>
<td>0.52</td>
<td>0.77</td>
<td>0.43</td>
</tr>
</tbody>
</table>

4 Conclusion

The contribution of this paper is twofold. First we show that self-fulfilling equilibria occur in a two-country setting where in each country firms can borrow internationally from households, subject to a collateral constraint. Co-movement happens because a lowered interest rate generates credit
booms in all countries connected to the world financial market. Because more credit allows firms to expand their scale of operations, a credit boom in each country translates into booming output, investment, consumption, and hours worked. Thus global financial linkages create international synchronization and financial development means that national economies that are open to cross-border credit flows tend to co-move. In contrast, country-specific shocks to either TFP or to leverage lead to reallocation of world credit resources from one country to another, and as a consequence the home and foreign outputs do not co-move, while consumptions do because of international risk-sharing. Second, we show that the model with countries that differ in their levels of financial development can shed light on the reasons why developed economies are not perfectly synchronized.

While in this paper we modestly report qualitative results, a promising avenue for research would be to consider a seriously calibrated version of the model. We believe that the setting outlined in this paper has interesting predictions regarding why developed countries tend to co-move, while they are much less synchronized with developing countries. For example, it seems sensible to investigate the pressing issue of global imbalances, for example between the US and China, through the lens of our stylized model. One important aspect of the results reported above is that net foreign asset positions and trade surpluses can be quite persistent even if short-run variations in the world interest rate are not. This implies, theoretically, that the effects of financial globalization on business-cycle synchronization depend on the level of financial development within countries, in line with recent empirical evidence. In addition, our results suggest that estimating the model against data would reveal that global interest rate shocks are important to explain why output levels co-move internationally.
5 Appendix

5.1 Steady state

Households’ Euler equations in both home and foreign countries imply that in the steady state

\[ 1 + \phi(B_h - B_e) = \beta_h R^w = 1 + \phi(B_h^* - B_e^*), \]  

(27)

which implies

\[ S = B_h - B_e = B_h^* - B_e^* = S^* = 0 \]  

(28)

So the steady state is characterized by autarky. Then the Euler equations imply that

\[ R^w = \frac{1}{\beta_h} \]  

(29)

Define lower case variables as \( x \equiv \frac{X}{Y} \), i.e. the X-output ratio. The other equations imply the following conditions:

\[ \Phi = (\beta_h - \beta_e)\Lambda \]  

(30)

\[ Q = \frac{\xi Y}{L_e} \]  

(31)

\[ \xi = \frac{\beta\gamma}{1 - (\Theta\beta_h + (1 - \Theta)\beta)} \]  

(32)

\[ Q = \frac{\beta_h \psi c^\sigma_h}{1 - \beta h} Y^{\sigma_{\tau_i}} \]  

(33)

\[ b_e = \beta_h \Theta \xi \]  

(34)

\[ k = \frac{\alpha\beta}{1 - \beta(1 - \delta)} \]  

(35)

\[ WN = (1 - \alpha - \gamma)Y \]  

(36)

\[ c_e = \alpha + \gamma - \delta k - (R^w - 1)b_e \]  

(37)

\[ c_h = (R^w - 1)b_e + 1 - \alpha - \gamma \]  

(38)

\[ \Lambda_h = c_h^{\sigma_{\tau_i}} \]  

(39)

\[ N = \left( \frac{(1 - \alpha - \gamma)Y^{1 - \sigma_{\tau_i}}}{\zeta_{\sigma_{\tau_i}}} \right)^{\frac{1}{1 - \sigma_{\tau_i}}} \]  

(40)

\[ L_e = A^{-\frac{1}{2}} k^{-\frac{1}{2}} \left( \frac{1 - \alpha - \gamma}{\zeta} \right)^{-\frac{1 - \alpha - \gamma}{\zeta}} c_h^{\frac{1 - \alpha - \gamma}{\zeta} \sigma_{\tau_i}} \left( \frac{Y^{1 - \gamma} Y^{\sigma_{\tau_i} + \gamma}}{\zeta_{\sigma_{\tau_i}}} \right)^{\frac{1 - \alpha - \gamma}{\zeta} + \gamma + 1} \]  

(41)

We can solve the steady state \( Y \) after plugging \( L_e \) and \( c_h \) into the following nonlinear equation:

\[ \frac{\beta_h \psi c^\sigma_h}{1 - \beta_h} Y^{\sigma_{\tau_i}} = \xi \frac{Y}{L_e} \]  

(42)

After solving for \( Y \), other variables can be solved using the above explicit expressions. The steady state variables of the foreign country can be solved following the same procedures.
5.2 Symmetric Countries: Additional IRFs

In Figures 8-10 are reported IRFs for the case of symmetric countries, which extend those in Figures 2-4. Note that in all graphs reported below, the blue lines represent the home responses, the red dashed lines denote the foreign responses.

Figure 8: IRFs to a positive TFP shock in home economy – Symmetric countries

5.3 Asymmetric Countries: Additional IRFs

In Figures 11-13, we now report IRFs for the case of asymmetric countries, to complement those in Figures 5-7. Note that in all graphs reported below, the blue lines represent the home responses, the red dashed lines denote the foreign responses.
Figure 9: IRFs to a positive leverage shock in home economy – Symmetric countries

Figure 10: IRFs to a negative shock to the world interest rate – Symmetric countries
References


