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Markup Cyclicality: A Tale of Two Models

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Abstract

Many models in the business cycle literature generate counter-cyclical price markups. This paper examines if the prominent models in the literature are consistent with the empirical findings of micro-level markup behavior in Hong (2016). In particular, I test the markup behavior of the following two models: (i) an oligopolistic competition model, and (ii) a New Keynesian model with heterogeneous price stickiness. First, I explore the Atkeson and Burstein (2008) model of oligopolistic competition, in which markups are an increasing function of firm market shares. Coupled with an exogenous *uncertainty* shock as in Bloom (2009), i.e. a second-moment shock to firm productivities in recessions, this model results in a countercyclical average markup, as in the data. However, in contrast with the data, this model predicts that smaller firms reduce their markups. Second, I calibrate both Calvo and menu cost models of price stickiness to match the empirical heterogeneity in price durations across small and large firms, as in Goldberg and Hellerstein (2011). I find that both models can match the average counter-cyclicality of markups in response to monetary shocks. Furthermore, since small firms adjust prices less frequently, they exhibit greater markup counter-cyclicality, consistent with the empirical patterns. Quantitatively, however, only the menu cost model, through its selection effect, can match the extent of the empirical heterogeneity in markup cyclicality. In addition, both sticky price models imply pro-cyclical markup behavior in response to productivity shocks.

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1 Introduction

There is a long line of empirical studies regarding price markup fluctuations over business cycles. One could view the markup as the ratio of price over marginal cost, and it measures the distortion in the output market. A countercyclical markup contributes to the amplification of aggregate fluctuations. However, the determinants of markup movements are not well understood. Many models in the business cycle literature generate cyclical price markups. One approach is to assume that firm's markup follows an exogenous process and varies over time in the literature (as in Smets and Wouters 2003, 2007, and Steinsson 2003). In contrast, other models rationalize the variable markups with micro-founded models.

However, which model is the right one to consider? The aim of this paper is to choose one that is consistent with the empirical findings at the micro level. Hong (2016) finds that markups are countercyclical on average, and small firms' markups are more countercyclical than large firms'. In particular, I test the markup behavior of the following two models: (i) an oligopolistic competition model, and (ii) a New Keynesian model with heterogeneous price stickiness. First, I study the competition model in a general setting. I find that one needs varying second moment shock in firm's idiosyncratic productivities over time to generate variable markups (as in Bloom 2009). In a special case with Atkeson and Burstein (2008), firm's pricing function is increasing and convex in its own market share. And due to Jensen's inequality, the changes in dispersion of market shares generates countercyclical markup at the aggregate level. However, since convexity is stronger for large firms than small firms, large firms' markups tend to be more countercyclical, which is not consistent with the data. Second, I calibrate the New Keynesian model with heterogeneous adjustment costs. With sticky price, firms adjust price more slowly compared to changes in marginal cost. Hence, with a procyclical marginal cost, markup is countercyclical. A recent empirical study by Goldberg and Hellerstein (2011) finds that small firms adjust less frequently as large firms. I calibrate both Calvo and menu cost models of price stickiness to match the empirical heterogeneity in price durations across small and large firms as in their study. The model is subject to nominal aggregate demand shock. I find that the model could successfully generate both countercyclical markup and that small firms' markup more countercyclical than large firms'. Quantitatively, however, only the menu cost model, through its selection effect, can match the extent of the empirical heterogeneity in markup cyclicality. In addition, both sticky price models imply pro-cyclical markup behavior in response to productivity shocks, since marginal costs become countercyclical.

The rest of the paper proceeds as follows: section 2 derives theoretical results for oligopolistic competition in a general setup. Section 3 discusses the quantitative analysis of the oligopolistic competition in a specific setup, namely the Atkeson-Burstein (2008) model. Section 4 introduces and discusses the results of a New Keynesian model with new extensions. Section 5 concludes.

2 General Oligopolistic Competition Model

To think about markup cyclicality along business cycle, a natural first step is oligopolistic competition model. I start with a general imperfect competition framework as described in Burstein and Gopinath (2013) to study how nature of firm competition and underlying marginal cost process affect cyclicality of markup.

2.1 General Framework

Consider an economy consisting of n firms each indexed by $i = \{1, ..., n\}$. Each firm has a constant-returns-to-scale production technology. Firm i's optimal pricing rule is markup over marginal cost

$$p_i = \mu_i + mc_i, \tag{1}$$

where $p_i \equiv \log P_i$ is the log price of firm, $\mu_i \equiv \log \mathcal{M}_i$ is the log markup of firm, and $mc_i \equiv \log MC_i$ is the log marginal cost of firm. Markup depends on both firm's log price p_i, and log industry price index $p \equiv \log P$. In particular, log of markup takes the form of $\mu_i = \mu(p_i - p)$. Many models generate this relationship between markup and relative price. The functional form of markup $\mu(\cdot)$ and industry price index p depend on the model.

Firms compete in product market and interact with each other through industry price index p. We can see this competition framework as a special case of interaction networks as in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015). To study cyclicality of markup, I focus on small marginal cost shocks to firms so I can use first several terms of Taylor expansions around initial states. In particular, variable of main interest is first difference in log industry markup defined as

$$\Delta\mu(\mu_1, ..., \mu_n) = \sum_{i=1}^{n} S_i \Delta\mu_i, \tag{2}$$

where S_i is firm i's market share in revenue. Intuitively, log industry markup change is revenue-weighted average of individual markup change. We will see that this definition is consistent with welfare-relevant measure in the following subsection, in which I introduce a specific imperfect competition model for calibration.

First Order Approximation

First, I start with a first-order Taylor approximation of change in individual markup with respect to change in marginal costs

$$\Delta\mu_i = -\Gamma_i \left[\sum_{k=1}^{n} \frac{\partial(p_i - p)}{\partial mc_k} \Delta mc_k \right], \tag{3}$$

where $\Gamma_i \equiv -\frac{\partial\mu_i}{\partial(p_i - p)}$ is the elasticity of markup with respect to the relative price. If desired markup is decreasing in relative price, $\Gamma_i > 0$. Also, Γ_i measures strength of strategic complementarities in pricing. To see this, take a first-order approximation of equation (1):

$$\Delta p_i = -\Gamma_i(\Delta p_i - \Delta p) + \Delta mc_i,$$

which leads to

$$\Delta p_i = \frac{\Gamma_i}{1+\Gamma_i}\Delta p + \frac{1}{1+\Gamma_i}\Delta mc_i.$$

Hence, price of a firm with higher Γ_i responds more to industry price index than its own marginal cost shock, and *vice versa*. Note that two coefficients sum to one.

I use a first order approximation for change in industry price index:

$$\Delta p = \sum_{j=1}^{n} S_j \Delta p_j, \tag{4}$$

which is revenue-weighted average of individual price change. This equation holds exactly in many models, including the one I use in the following subsection. Combine equation (4) with partial differentiation of equation (1), we get the following equation for each i

$$\frac{\partial p_i}{\partial mc_k} = -\Gamma_i \left(\frac{\partial p_i}{\partial mc_k} - \sum_{j=1}^{n} S_j \frac{\partial p_j}{\partial mc_k} \right) + \mathbf{1}\{i = k\},$$

where $\mathbf{1}\{i = k\}$ is an indicator function whether $i = k$. And it is straightforward to show that:

$$\frac{\partial p_i}{\partial mc_k} = \frac{\Gamma_i}{1+\Gamma_i} \left(\sum_{j=1}^{n} \frac{S_j}{1+\Gamma_j} \right)^{-1} \left(\frac{S_k}{1+\Gamma_k} \right) + \frac{\mathbf{1}\{i = k\}}{1+\Gamma_i}. \tag{5}$$

Thus how much marginal cost shock to firm k impacts firm i depends on either if firm i has strong strategic complementarities in pricing ($\frac{\Gamma_i}{1+\Gamma_i}$), or if firm k is relatively important in the industry ($\frac{S_k}{1+\Gamma_k}$). Additionally, if the marginal cost shock hits firm i itself, it responds through its own marginal cost channel.

Putting definition of the industry markup change (2), and equation (3) & (5) leads to the following linear approximation of industry markup change as a function of underlying marginal cost change:

Theorem 1 *The first-order approximation to the industry markup change is given by*

$$\Delta\mu^{(1)} = -\left(\sum_{j=1}^{n} \frac{S_j}{1+\Gamma_j}\right)^{-1} Cov_S\left(\frac{\Gamma_i}{\Gamma_i+1}, \Delta mc_i\right).^{1} \tag{6}$$

This result shows that industry markup change is proportional to negative covariance between strategic complementarities and marginal cost shock. Hence if firms with stronger complementarities are hit with greater marginal cost shock, industry markup decreases. Also, this result implies that if all firms are hit with identical shock ($\Delta mc_i = \Delta mc$, $\forall i$), industry markup stays the same. This is easy to understand since each firm's desired markup depends on relative price difference, hence to lead to aggregate effect, we need some heterogeneities in marginal cost shocks.

However, even if marginal cost shocks are independently and identically distributed, and have mean zero and variance σ^2, we have the following corollary for the expectation of industry markup change:

Corollary 1 $E[\Delta\mu^{(1)}] = 0$.

This corollary shows that first-order expansion is not informative about interaction between the competition network and the underlying marginal cost process. Therefore, it is natural to use second-order expansion in the following.

Second Order Approximation

I start with a second-order approximation for individual markup change:

$$\Delta\mu_i = -\Gamma_i\left[\sum_{k=1}^{n} \frac{\partial(p_i - p)}{\partial mc_k}\Delta mc_k\right] + \frac{1}{2}\sum_{k=1}^{n}\sum_{r=1}^{n} \frac{\partial^2 p_i}{\partial mc_k \partial mc_r}\Delta mc_k \Delta mc_r, \tag{7}$$

where first term is the same as first-order approximation, and second term comes from the fact that

$$\frac{\partial^2 \mu_i}{\partial mc_k \partial mc_r} = \frac{\partial^2 p_i}{\partial mc_k \partial mc_r}.$$

To derive this Hessian matrix for prices, I take second partial derivative of equation (1) to get

$$\begin{aligned}\frac{\partial^2 p_i}{\partial mc_k \partial mc_r} = &-\Gamma_i\left(\frac{\partial^2 p_i}{\partial mc_k \partial mc_r} - \sum_{j=1}^{n} S_j \frac{\partial^2 p_j}{\partial mc_k \partial mc_r} - \sum_{j=1}^{n}\sum_{j'=1}^{n} \frac{\partial^2 p}{\partial p_j \partial p_{j'}}\left(\frac{\partial p_{j'}}{\partial mc_k}\right)\left(\frac{\partial p_j}{\partial mc_r}\right)\right) \\ &+ \Gamma_{ii}\left(\frac{\partial(p_i - p)}{\partial mc_k}\right)\left(\frac{\partial(p_i - p)}{\partial mc_r}\right).\end{aligned} \tag{8}$$

[1] I define $\mathrm{Cov}_S(X_i, Y_i)$ as the weighted covariance $\mathrm{Cov}_S(X_i, Y_i) \equiv \sum_i S_i X_i Y_i - \left(\sum S_i X_i\right)\left(\sum S_i Y_i\right)$, where weights sum to 1: $\sum S_i = 1$.

where $\Gamma_{ii} \equiv -\frac{\partial \Gamma_i}{\partial p_i - p}$ is superelasticity of markup, which captures convexity (or concavity) of markup. If $\Gamma_{ii} > 0$, firms with lower relative price have more strength of strategic complementarities, and *vice versa*. Furthermore, I show the following result (see the Appendix for proof):

Proposition 1 *If market share S_j is a function of relative price $S_j = S\left(\frac{P_j}{P}\right)$, then the elasticity of market share with respect to relative price $-\frac{\partial \log S_j}{\partial (p_j - p)}$ is a constant for all j. And the Hessian matrix for industry price equals:*

$$\frac{\partial^2 p}{\partial p_j \partial p'_j} = \Lambda S_j (S_{j'} - \mathbf{1}\{j = j'\}), \tag{9}$$

where Λ denotes the market share elasticity $-\frac{\partial \log S_j}{\partial (p_j - p)}$.

This proposition leads to simplification of equation (8) (see Appendix for derivation):

$$\frac{\partial^2 p_i}{\partial mc_k \partial mc_r} = \frac{\Gamma_i}{1+\Gamma_i}\left(X_i^{kr} + \sum_{j=1}^{n} S_j \frac{\partial^2 p_j}{\partial mc_k \partial mc_r}\right), \tag{10}$$

where

$$X_i^{kr} \equiv \frac{\Gamma_{ii}}{\Gamma_i}\left(\frac{\partial (p_i - p)}{\partial mc_k}\right)\left(\frac{\partial (p_i - p)}{\partial mc_r}\right) - \Lambda \sum_j S_j \left(\frac{\partial (p_j - p)}{\partial mc_k}\right)\left(\frac{\partial (p_j - p)}{\partial mc_r}\right).$$

Note that $\frac{\Gamma_{ii}}{\Gamma_i} = -\frac{\mu_i''}{\mu_i'}$ measures the convexity of markup. Combining equation (10) with equation (7) leads to the following result:

Theorem 2 *The second-order approximation to the total markup change is given by*

$$\Delta\mu^{(2)} = \Delta\mu^{(1)} + \frac{1}{2}\sum_{k=1}^{n}\sum_{r=1}^{n}\left\{\left(\sum_j \frac{S_j}{1+\Gamma_j}\right)^{-1}\left(\sum_j S_j \frac{\Gamma_j}{1+\Gamma_j} X_j^{kr}\right)\Delta mc_k \Delta mc_r\right\}, \tag{11}$$

where $\Delta\mu^{(1)}$ is first-order approximation as in Theorem 1.

To understand the intuition of this result, I take the expectation, and assume that all firms' initial states are the same to get the following (see Appendix for proof):

Corollary 2 *If all firms have the same initial states such that $S_j = \frac{1}{n}$, $\Gamma_j = \Gamma'$, $\Gamma_{jj} = \Gamma''$, then*

$$E[\Delta\mu^{(2)}] = \frac{1}{2}\sigma^2 \frac{n-1}{n}\frac{\Gamma'}{(1+\Gamma')^2}\left(\frac{\Gamma''}{\Gamma'} - \Lambda\right). \tag{12}$$

This result implies that if the convexity of markup $\frac{\Gamma''}{\Gamma'}$ is greater than the elasticity of market share Λ, change in industry markup is an increasing function of variance σ^2.

3 Quantitative Analysis: Atkeson-Burstein

In this section, I use the oligopolistic competition framework introduced by Atkeson and Burstein (2008) for quantitative simulation.

Household

The representative household has an additively separable preference over consumption and labor

$$U(C,L) = \frac{C^{1-\sigma}}{1-\sigma} - \omega \frac{L^{1+\psi}}{1+\psi}, \tag{13}$$

where $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution (IES), ω is the disutility parameter from labor, and $\frac{1}{\psi}$ is the Frisch elasticity of labor supply. Total consumption C consists of consumption from a continuum of sectors j:

$$C = \left(\int_0^1 C_j^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}, \tag{14}$$

where C_j is consumption for sector j's good, and η is the elasticity of substitution between any two different sectoral goods. Within each sector j, there are n_j firms producing differentiated goods. The household has a CES type preference over finite number of differentiated goods for each sector j:

$$C_j = \left(\sum_{i=1}^{n_j} C_{ij}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \tag{15}$$

where C_{ij} is consumption of good i in sector j, and ρ is the elasticity of substitution between any two differentiated goods within sector. It is assumed that the elasticity of substitution within sector is higher than the elasticity of substitution across sector, $\rho > \eta$.

The household chooses consumption $\{C_{ij}\}$ and labor L to maximize the utility function (13) subject to the following budget constraint

$$\int_0^1 \left(\sum_{i=1}^{n_j} P_{ij} C_{ij} \right) dj \leq WL, \tag{16}$$

where P_{ij} is the price of good i in sector j, and W is the nominal wage. The solution to the household's problem gives the demand function for C_{ij}:

$$C_{ij} = \left(\frac{P_{ij}}{P_j} \right)^{-\rho} \left(\frac{P_j}{P} \right)^{-\eta} C, \tag{17}$$

where P_j is sector j's price index defined as

$$P_j \equiv \left(\sum_{i=1}^{n_j} P_{ij}^{1-\rho} \right)^{\frac{1}{1-\rho}}, \tag{18}$$

and P is total economy price index defined as

$$P \equiv \left(\int_0^1 P_j^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{19}$$

And the consumption and labor optimality condition is the following

$$\omega\frac{L^{\psi}}{C^{-\sigma}} = \frac{W}{P}. \tag{20}$$

Firm

Firm i in sector j produces output using labor

$$Y_{ijt} = a_{ijt} l_{ijt}, \tag{21}$$

where a_{ijt} is producer-level productivity and I discuss its composition and evolution in the next sub subsection. Firms engage in Cournot competition within sector.[2] Taking wage W and demand equation (17) as given, a firm i in sector j chooses its output Y_{ijt} to maximize its profit

$$\pi_{ijt} = \max_{Y_{ijt}} \left[\left(P_{ijt} - \frac{W}{a_{ijt}}\right) Y_{ijt} - W\phi\right] \mathbf{1}\{Y_{ijt} > 0\}, \tag{22}$$

where ϕ is fixed cost of production and is denominated in units of labor. A firm can choose not to produce to avoid paying the fixed cost ϕ. Hence ϕ captures the extensive margin of the oligopolistic competition.

The solution to the firm's profit maximization problem is a markup over marginal cost

$$P_{ijt} = \frac{\varepsilon(S_{ijt})}{\varepsilon(S_{ijt}) - 1}\frac{W}{a_{ijt}}, \tag{23}$$

where firm-specific demand elasticity $\varepsilon(S_{ijt})$ is a harmonic weighted average of elasticities of substitution ρ and η

$$\varepsilon(S_{ijt}) = \left(S_{ijt}\frac{1}{\eta} + (1 - S_{ijt})\frac{1}{\rho}\right)^{-1}, \tag{24}$$

where S_{ijt} is firm's market share in sector j,

$$S_{ijt} = \frac{P_{ijt}Y_{ijt}}{\sum_{i=1}^{n_j} P_{ijt}Y_{ijt}} = \left(\frac{P_{ijt}}{P_{jt}}\right)^{1-\rho}. \tag{25}$$

Since there are finite number of firms in each sector, the firms are large enough ($S_{ijt} > 0$) to affect industry price index P_{jt}.

[2] Bertrand competition generates qualitatively the same results.

Also, firm's markup $\mathcal{M}_{ijt}$ can be expressed as

$$\frac{1}{\mathcal{M}_{ijt}} = \frac{\rho - 1}{\rho} - \left(\frac{1}{\eta} - \frac{1}{\rho}\right) S_{ijt}, \tag{26}$$

and the elasticity of markup with respect to relative price are:

$$\Gamma_i = -\frac{\partial \log \mathcal{M}_{ijt}}{\partial(\log P_{ijt} - \log P_{jt})} = (\rho - 1)\left(\frac{1}{\eta} - \frac{1}{\rho}\right) S_{ijt}\mathcal{M}_{ijt}. \tag{27}$$

Since $\rho > \eta$, markup is an increasing and convex function of market share. Respectively, the elasticity and super-elasticity of markup with respect to relative price are:

$$\Gamma_i = -\frac{\partial \log \mathcal{M}_{ijt}}{\partial(\log P_{ijt} - \log P_{jt})} = (\rho - 1)\left(\frac{1}{\eta} - \frac{1}{\rho}\right) S_{ijt}\mathcal{M}_{ijt} \tag{28}$$

$$\Gamma_{ii} = -\frac{\partial \Gamma_i}{\partial(\log P_{ijt} - \log P_{jt})} = \Gamma_i(\rho - 1 + \Gamma_i). \tag{29}$$

The market share elasticity with respect to relative price is:

$$\Lambda = -\frac{\partial \log S_{ijt}}{\partial(\log P_{ijt} - \log P_{jt})} = \rho - 1. \tag{30}$$

Hence $\frac{\Gamma_{ii}}{\Gamma_i} - \Lambda = \Gamma_i > 0$, and according to Corollary 2, change of industry markup is an increasing function of marginal cost shock variance in expectation.

Market Clearing

Denote L_t^* the optimal labor supply by the representative household, and l_{ijt}^* the labor demand of firm i in sector j. The labor market clearing condition is then

$$\int_0^1 \left(\sum_{i=1}^{n_j}(l_{ijt}^*) + \phi\right) = L_t^*. \tag{31}$$

And the good market clearing condition is

$$C_{ijt} = Y_{ijt} \quad \forall i, j, t \tag{32}$$

Aggregate Productivity and Markup

Define aggregate productivity as the following:

$$A_t \equiv \frac{Y_t}{\tilde{L}_t^*}, \tag{33}$$

where Y_t is the quantity of final output, and $\tilde{L}_t^*$ is the aggregate labor supply net of production fixed costs. From the labor market clearing condition (31), the aggregate productivity can be expressed as the quantity weighted harmonic average of individual productivity:

$$A_t = \left[\int_0^1 \left(\sum_{i=1}^{n_j} \frac{Y_{ijt}}{Y_t} \frac{1}{a_{ijt}}\right) dj\right]^{-1} \tag{34}$$

Define aggregate markup as the following:

$$\mathcal{M}_t \equiv P_t \left(\frac{W_t}{A_t}\right)^{-1}, \tag{35}$$

where P_t is the aggregate price index as defined in (19), and $\frac{W_t}{A_t}$ is the aggregate marginal cost. From equation (34), it is easy to see that the aggregate markup can be expressed as the market share weighted harmonic average of individual markup:

$$\mathcal{M}_t = \left[\int_0^1 S_{jt} \left(\sum_{i=1}^{n_j} \frac{S_{ijt}}{\mathcal{M}_{ijt}}\right) dj\right]^{-1}, \tag{36}$$

where $S_{jt} \equiv \frac{P_{jt}Y_{jt}}{P_t Y_t}$ is sector j's total revenue share of the economy.

Note that the aggregate productivity can be rewritten as

$$A_t = \left[\int_0^1 \left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t}\right)^{-\eta} a_{jt}^{\eta-1}\right]^{\frac{1}{\eta-1}}, \tag{37}$$

where $\mathcal{M}_{jt} \equiv P_{jt} \left(\frac{W_t}{a_{jt}}\right)^{-1}$ is the sectoral markup and a_{jt} is the sectoral productivity defined as

$$a_{jt} \equiv \left[\sum_{i=1}^{n_j} \left(\frac{\mathcal{M}_{ijt}}{\mathcal{M}_{jt}}\right)^{-\rho} a_{ijt}^{\rho-1}\right]^{\frac{1}{\rho-1}}. \tag{38}$$

We can compare this to the first best (FB) aggregate productivity attained by a social planner:

$$A_t^{\text{FB}} = \left(\int_0^1 a_{jt}^{\text{FB}\,\eta-1}\right)^{\frac{1}{\eta-1}}, \tag{39}$$

where the first best sectoral productivity is

$$a_{jt}^{\text{FB}} \equiv \left(\sum_{i=1}^{n_j} a_{ijt}^{\rho-1}\right)^{\frac{1}{\rho-1}}. \tag{40}$$

We see that the markup dispersion in the product market distorts the resource allocation and hence causes TFP loss in the economy. Hsieh and Klenow (2009), Restuccia and Rogerson (2008), and Edmond, Midrigan, and Xu (2015) analyze this misallocation effect in cross-section.

However, it might be a different picture if we think in terms of business cycle. Along business cycle, standard deviation of idiosyncratic productivities is countercyclical. Even though the aggregate TFP is lower than the level could be attained by FB, but the aggregate TFP might be countercyclical due to the well-known Oi-Hartman-Abel effect. I illustrate that it is indeed the case in the simulation.

Implications for Aggregate Output

In this sub subsection, I discuss how the imperfect firm competition affects the total output along the business cycles. Change in log total output can be written as

$$\Delta \log Y_t = \Delta \log A_t + \Delta \log L_t. \tag{41}$$

For the simplicity of illustration, I ignore the fixed cost for production in the analysis. From the representative household's consumption and labor optimality condition (20), I can express the labor supply as a function of the aggregate productivity and the aggregate markup

$$L_t = \left(\frac{A_t^{1-\sigma}}{\omega \mathcal{M}_t} \right)^{\frac{1}{\psi+\sigma}}. \tag{42}$$

Then change in log total output becomes

$$\Delta \log Y_t = \frac{\psi+1}{\psi+\sigma} \Delta \log A_t - \frac{1}{\psi+\sigma} \Delta \log \mathcal{M}_t. \tag{43}$$

Hence, countercyclical aggregate markup amplifies the fluctuation of output along business cycle.

And for change in log aggregate productivity, I show the following result (see the Appendix for proof)

Proposition 2 *Change in aggregate productivity can be decomposed into three parts:*

$$\Delta \log A_t = \Delta \log \tilde{A}_t - \frac{\eta}{\eta-1} \Delta \log \tilde{\mathcal{M}}_{st} - \frac{\rho}{\rho-1} \Delta \log \tilde{\mathcal{M}}_{wt}. \tag{44}$$

First, $\Delta \log \tilde{A}_t$ *is TFP loss due to misallocation*

$$\Delta \log \tilde{A}_t \equiv \int_0^1 \left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t} \right)^{-1} S_{jt} \left(\sum_{i=1}^{n_j} \left(\frac{\mathcal{M}_{ijt}}{\mathcal{M}_{jt}} \right)^{-1} S_{ijt} \Delta \log a_{ijt} \right) dj. \tag{45}$$

Second term $\Delta \log \tilde{\mathcal{M}}_{st}$ *is TFP loss due to sectoral markup cyclicality*

$$\Delta \log \tilde{\mathcal{M}}_{st} \equiv \int_0^1 \left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t} \right)^{-1} S_{jt} (\Delta \log \mathcal{M}_{jt} - \Delta \log \mathcal{M}_t) dj. \tag{46}$$

Third term $\Delta \log \tilde{\mathcal{M}}_{wt}$ *is TFP loss due to within-sector markup cyclicality*

$$\Delta \log \tilde{\mathcal{M}}_{wt} \equiv \int_0^1 \left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t} \right)^{-1} S_{jt} \left(\sum_{i=1}^{n_j} \left(\frac{\mathcal{M}_{ijt}}{\mathcal{M}_t} \right)^{-1} S_{ijt} (\Delta \log \mathcal{M}_{ijt} - \Delta \log \mathcal{M}_{jt}) \right) dj. \tag{47}$$

3.1 Calibration and Simulation

3.1.1 Household Preference Parameters

Household has a log utility in consumption ($\sigma = 1$). I set Frisch elasticity of labor supply $1/\psi = 1$, as suggested by Chang, Kim, Kwon, and Rogerson (2014). Then from equation (42), movement in labor supply is simply driven by only movement in aggregate markup: $L_t = (\omega \mathcal{M}_t)^{1/2}$. There is no effect of aggregate productivity on labor supply, since income and substitution effects cancel out perfectly due to unit intertemporal elasticity. Finally, I set disutility from labor supply parameter such that labor supply in the steady state equal to one third.

3.1.2 Elasticities of Substitution

I infer the within-sector elasticity of substitution ρ and the across-sector elasticity of substitution η by running a regression of firm's markup on firm's market share as in (48). Note that a firms's optimal pricing rule is the markup over the marginal cost, hence the markup can be expressed as:

$$\mathcal{M}_{ijt} = \frac{P_{ijt}}{W_t/a_{ijt}} = \frac{P_{ijt}Y_{ijt}}{W_t l_{ijt}},$$

where the second equality results from multiplying the denominator and the numerator by output Y_{ijt}. Hence, I can replace the dependent variable of equation (48) with the labor cost share:

$$\frac{W_t l_{ijt}}{P_{ijt}Y_{ijt}} = \gamma_0 + \gamma_1 S_{ijt}.\text{[3]} \tag{48}$$

I can infer the values of ρ and η from the ratio of the coefficient estimates γ_0/γ_1:

$$\eta = \left(\frac{1}{\rho} - \frac{\gamma_1}{\gamma_0}\left(\frac{\rho - 1}{\rho}\right)\right)^{-1}$$

The estimate of the ratio $\frac{\gamma_1}{\gamma_0}$ is -0.973. I choose $\rho = 11$ such that firms' markup equal to 1.1 under perfect competition, and hence $\eta = 1.026$.

[3] If firms have labor production elasticity β_l different from unity, equation (48) can be extended to $\frac{W_t l_{ijt}}{P_{ijt}Y_{ijt}} = \gamma_{0j} + \gamma_1 S_{ijt}$, where γ_{0j} is a dummy variable for sector j to capture heterogeneous labor production elasticities across sectors. In this case, I cannot identify $\hat{\rho}$ from $\hat{\gamma}_0$.

3.1.3 Firm Parameters

Each firm's TFP a_{ijt} consists of common TFP A_t^M and firm specific TFP a_{ijt}^F: $a_{ijt} = A_t^M \times a_{ijt}^F$. $\log A_t^M$ and $\log a_{ijt}^F$ follow AR(1) processes respectively:

$$\log A_t^M = \rho_m \log A_{t-1}^M + \nu_m \xi_t^m, \quad \xi_t^m \sim N(0,1) \tag{49}$$

$$\log a_{ijt}^F = (1-\rho_f)\ln \alpha_{ij} + \rho_f \log a_{ijt-1}^F + d_t \xi_{ijt}^f, \quad \xi_{ijt}^F \sim N(0,1). \tag{50}$$

Note that the variance of the firm-level shock d_t is itself time-varying. In the normal period, I set $d_L = 0.05$, and it spikes to $d_H = 0.15$ during the recession period.

Finally, I set the number of firms in each sector to be 30, which is close to the mean number of firms in the data.

3.2 Impulse Response

I analyze several business cycle moments with impulse response analysis. Specifically, I test with two scenarios: (i) a spike in variance of firm specific productivity d_t, and (ii) a drop in common TFP A_t^M.

3.2.1 Second Moment Shock

In this experiment, I set the variance of firm specific productivity $d_t = 0.15$ at period 0 for one period, which is three times as high as the normal period value $d_L = 0.05$. The impulse response results are in figure (1). With increased dispersion in idiosyncratic productivities and the result of *corollary 2*, the aggregate markup increases by around 2.5%. And labor supply decreases by around 1.2% accordingly. However, due to Oi-Hartman-Abel effect, the aggregate TFP actually increases in the recession. Bloom (2009) discusses this undesired effect, but since there are adjustment costs for both labor and capital usage in his model, misallocation effect dominates and aggregate TFP decreases. Finally, since increase in aggregate TFP dominates decrease in labor supply, aggregate output turns out to increase during recession.

The model also has a wrong prediction for response of small and large firms. On average, small firms have smaller markups while large firms have larger ones, the model predicts that small firm's markup is procyclical while large firm's is countercyclical (as in figure (2)). But in empirical analysis of markup cyclicality, I actually find that small firm's markup is more countercyclical than large firm's.

Moreover, with the same second moment shock, I now assume that firms have to pay operating cost to produce in the economy. Specifically, I assume that firms have to pay 4% of mean profit in the steady state. Now in the recession, the number of operating firms decrease by around 12%. Jaimovich and Floetotto (2008) emphasize this extensive margin effect on markup cyclicality. However, as seen in figure (1), we see that this effect is almost negligible. The reason is only small firms drop out of the market and they have marginal effect for large firms remaining in the market.

3.2.2 First Moment Shock

In this experiment, I set the common TFP A_t^M drops by 3% at period 0. From *Theorem 1*, It is not surprising to see that it has no effect on aggregate markup. And since movement of labor supply is only determined by markup in our parameter specification, labor supply stays constant. Hence all firms profit stay constant across the time period and hence no firm exits the market even though they have to pay operating cost.

4 Sticky Price Model

In the previous section, we have seen that the oligopolistic competition successfully generates countercyclical markup at the aggregate level, but is inconsistent with micro-level evidence. Now I examine another model that could generate countercyclical markup - sticky price model.

The reason that a standard New Keynesian model could generate countercyclical markup is the following. Under monopolistic competition and constant consumer price elasticity θ, a firm's optimal pricing strategy is a constant markup $\frac{\theta}{\theta-1}$ over marginal cost. However, with price stickiness, a procyclical marginal cost implies that in a boom, the gap between the price and the marginal cost shrinks, and hence decrease in the markup.

To match the cross-sectional markup cyclicality in the data, small firms should exhibit more price stickiness than large firms. Goldberg and Hellerstein (2011) find that it is indeed the case. They categorize firms into three equal bins by their size, and they find that the largest firms have a frequency of price adjustment 18.20%, while the smallest firms have a frequency of price adjustment 10.50%[4]. We see that large firms adjust prices almost twice as frequently as small firms. Hence, the sticky price model implies markup cyclicality that is consistent with my empirical finding qualitatively. To investigate if heterogeneity in price stickiness is large enough to generate heterogeneity

[4]Please see Table 2.

in markup cyclicality, I examine the following New Keynesian model in general equilibrium. The innovation of my model is that cost and probability of price adjustment depends on firm's size.

4.1 Household

The representative household has an additively separable preference over consumption and labor and maximizes the following

$$\max \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \omega \frac{L_t^{1+\psi}}{1+\psi} \right) \right\}, \tag{51}$$

where $\frac{1}{\sigma}$ is the inter temporal elasticity of substitution (IES), ω is the disutility parameter from labor, and $\frac{1}{\psi}$ is the Frisch elasticity of labor supply. And C_t is Dixit-Stiglitz aggregator of differentiated goods consumption over varieties i,

$$C_t = \left(\int_0^1 c_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}.$$

The budget constraint for household is

$$\int_0^1 p_{it} c_{it} di + \mathrm{E}_t[Q_{t,t+1} B_{t+1}] \leq B_t + W_t L_t + \int_0^1 \pi_{it} di.$$

A complete set of Arrow-Debreu state-contingent assets is traded, so that B_{t+1} is a random variable that delivers payoffs in period $t+1$. $Q_{t,t+1}$ is the stochastic discount factor used to price them.

The first-order conditions of the household's maximization problem is

$$\frac{W_t}{P_t} = \omega \frac{L_t^{\psi}}{C_t^{-\sigma}}$$

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}$$

Finally, I assume that the aggregate nominal value-added $S_t \equiv P_t C_t$ follows an exogenous random walk:

$$\log S_t = \log S_{t-1} + \mu_S + \eta_t, \quad \eta_t \sim N(0, \sigma_S). \tag{52}$$

We can think of this as the central bank has a targeted path of nominal value-added, and it does so by adjusting interest rate accordingly.

4.2 Firms

Each firm produces output c_{it} using a technology in labor l_{it}:

$$c_{it} = a_{it} l_{it}, \tag{53}$$

where a_{it} is firm-specific idiosyncratic productivity, which follows an AR(1) process

$$\log a_{it} = \rho_a \log a_{it-1} + \epsilon_{it}, \quad \epsilon_t \sim N(0, \sigma_a).$$

And each firm faces the following demand:

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\theta} C_t, \tag{54}$$

where p_{it} is price of good i, P_t is the aggregate price level, and C_t is the aggregate consumption.

To change its price, a firm must pay a fixed cost κ_{it} in units of labor. Structure of κ_{it} will be specified below. Hence, a firm's nominal profit equals to

$$\pi_{it} = \left(p_{it} - \frac{W_t}{a_{it}}\right)\left(\frac{p_{it}}{P_t}\right)^{-\theta} C_t - \kappa_{it} W_t I_{p_{it} \neq p_{it-1}}.$$

4.2.1 Krusell-Smith Forecast Rule

To solve the model in general equilibrium, it is necessary to keep track of distribution of firms over idiosyncratic productivities and prices, and thus determines the aggregate price level. Here, I assume that the aggregate price level itself is self predictable. In particular, I assume that each firm perceives a Krusell-Smith type law of motion for S_t/P_t

$$\log \frac{S_t}{P_t} = \gamma_0 + \gamma_1 \log \frac{S_t}{P_{t-1}}.$$

Given this conjecture, a firm's state variables are: (i) last period's individual price over the nominal value-added $\frac{p_{it-1}}{S_t}$, (ii) idiosyncratic productivity a_{it}, (iii) ratio of nominal value-added over aggregate price level $\frac{S_t}{P_t}$, and (iv) size of adjustment cost κ_{it}. And firm's problem can be written recursively in real term as

$$V\left(\frac{p_{it-1}}{S_t}, a_{it}, \frac{S_t}{P_t}, \kappa_{it}\right) = \max_{p_{it}} \left\{\frac{\pi_{it}}{P_t} + \mathrm{E}_t\left[Q_{t,t+1} V\left(\frac{p_{it}}{S_{t+1}}, a_{it+1}, \frac{S_{t+1}}{P_{t+1}}, \kappa_{it+1}\right)\right]\right\}.$$

Please see appendix for numerical solution outline.

4.3 Recursive Competitive Equilibrium

A recursive competitive equilibrium is a law of motion (γ_0, γ_1), a set of price level path $\{P_t\}$, and a set of wage path $\{W_t\}$ that are consistent with

1. Household utility maximization problem

2. Firm profit maximization problem

3. Goods market clearing
4. Arrow-Debreu market clearing
5. Evolution of nominal aggregate demand S_t and idiosyncratic productivity a_{it}

4.4 The CalvoPlusPlus Model

To match the heterogeneity in price stickiness, there are two ways to implement it. First, the cost of price adjustment (menu cost) depends on the firm size. Second, the Calvo probability of price adjustment depends on the firm size.

Nakamura and Steinsson (2010) introduces the *CalvoPlus* model, where a firm has a probability $1-\lambda$ to face an infinite menu cost, and a probability λ to face a small menu cost, but large enough that it makes some of the firms unwilling to adjust their prices still. The last assumption is different from the usual Calvo model, in which all firms adjust their prices with probability λ. In my model, both the size and probability of menu cost depend on the firm's size, hence I call this new extension *CalvoPlusPlus Model*.

4.4.1 Menu Cost Model

To adjust its price, a firm has to pay the following menu cost

$$\kappa_{it} = \kappa_0 \left(\frac{p_{it} c_{it}}{P_t} \right)^{\kappa_1}.$$

The value of the cost depends on its revenue, as in Gertler and Leahy (2008). Note that $\kappa_1 = 0$ corresponds to the case of a constant menu cost.

4.4.2 Calvo Model

A firm has a certain probability of not paying any cost to adjust its price

$$\kappa_{it} = \begin{cases} 0 & \text{w.p. } \lambda_{it} \\ \bar{\kappa} & \text{otherwise,} \end{cases}$$

where probability of zero menu cost λ_{it} depends on last period's revenue

$$\lambda_{it} = \lambda_0 \left(\frac{p_{it-1} c_{it-1}}{P_{t-1}} \right)^{\lambda_1}.$$

$\bar{\kappa}$ is set such that firms almost never pay $\bar{\kappa}$ to adjust prices.

4.4.3 Interpretation

How should we understand these heterogeneous adjustment costs? I do not see them as the literal cost of changing the menu. Instead, I see them as a general way of capturing the cost associated with adjusting the listed prices, which includes survey cost of current market condition, paying a manager to collect information, and etc. And this cost could weigh large or small relative to a firm's total revenue. Midrigan (2011), and Bhattarai and Schoenle (2014) find that multi-product firms tend to change prices more frequently than single-product firms. They construct a model where firms can pay one cost to change prices of all the underlying products, and it matches their empirical finding. Gertler and Leahy (2008) introduce a size-dependent menu cost to keep price adjustment decision of the firm homogeneous of its size. Carvalho (2006) introduces exogenous heterogeneity in price stickiness across sectors, and find that monetary shocks tend to have larger effects in the heterogeneous model, compared to an identical price stickiness model. My model is an addition to this heterogeneity in price stickiness, which depends on the firm size in particular. I leave it to the future research to study the microstructure underlying the heterogeneous adjustment costs I introduce here.

4.5 Calibration

In the model, one period equals to one month in the data. The monthly discount factor is $\beta = 0.997$. For the representative household, I assume log utility in consumption $\sigma = 1$, and infinite Frisch elasticity of labor supply $\psi = 0$ as in Hansen (1985) and Rogerson (1988). Hence, the real wage is a linear function of the aggregate consumption $W_t/P_t = \omega C_t$, this means that we do not need to keep the aggregate labor supply as a state variable.

For elasticity of substitution, I set $\theta = 5$, which is aligned with most of empirical findings. The growth rate and standard deviation of value-added S_t are taken from Nakamura and Steinsson (2010). The values I find in France data are quite close to these values. Firm's idiosyncratic productivity has persistence $\rho_a = 0.9$, and standard deviation $\sigma_a = 0.03$.

For the parameters of price adjustment cost, I set them such that the model matches top and bottom firms' price adjustment frequency. Please see Table 3 and Table 5 for parameter specifications for the Calvo model, and menu cost model, respectively.

4.6 Simulation Results

I present and discuss the simulation results of the CalvoPlusPlus model under two alternative assumptions about adjustment costs, (i) Calvo model, and (ii) menu cost model.

4.6.1 Calvo Model

The main statistics from the model is summarized in Table 4. Compared to Hellerstein and Goldberg's (2011) finding in Table 2, I find that firms increase prices more frequently in the model, and the size of price adjustment is smaller in the model, too. For example, the size of adjustment for middle is 6% in the data, while 5.38% in the model. However, most of the values are in the same magnitude as in the data. This is surprising since the only moments that I target in the calibration is price adjustment frequency of top and bottom firms.

Furthermore, I compare the markup cyclicality in the model to my empirical finding. In the simulation, I run the same regression as I run in the data: regress change in log markup $\Delta \log \mathcal{M}_{it}$ on change in aggregate output $\Delta \log Y_t$. In Figure 4, I present both markup cyclicality from the data and the model. Number 1 on the vertical axis stands for the smallest firms in terms of market share, number 2 for firms with middle market share, and number 3 for firms with largest market share. I find that the model generates the same magnitude of markup cyclicality as in the data, and it captures the heterogeneity in markup cyclicality qualitatively. Small firms adjust prices less frequently, hence more firms are unable to adjust prices while the underlying marginal cost fluctuates procyclically with the aggregate output. Therefore small firms' markup are more countercyclical relative to large firms'. However, we can see that the model does not capture the heterogeneity of markup cyclicality closely as in the data.

4.6.2 Menu Cost Model

The main statistics about the menu cost model is summarized in Table 6. The result is surprising, since the model captures all the moments astonishingly well, including size of price adjustment and etc. Furthermore, I compare the markup cyclicality in the model to empirical counterparts as I do in the Calvo model, and I find that the model captures both the magnitude and heterogeneity quite well.

The reason that the menu cost model generates more heterogeneity in markup cyclicality is the following: In a menu cost model, only a firm that has its markup substantially far away from its optimal markup $\mu^* \equiv \frac{\theta}{\theta-1}$ would adjust its price to obtain optimal profit. Upon a positive demand shock, firms that are close to the optimal mark do not adjust their price, which contributes countercyclicality to the aggregate markup. While firms that are far from the optimal markup are willing to pay the adjustment cost, and increase their price with respect to the increased nominal marginal cost, which contributes procyclicality to the aggregate markup. In contrast, in a Calvo model, the selection of which firms adjusting their prices is independent of how far they are from optimal markups; the firms chosen by a random probability λ_{it} are allowed to adjust their prices. Hence, the strong selection effect in the menu cost model generates large heterogeneity in the markup cyclicality.

4.7 Robustness

The business cycle of the benchmark model is driven by the nominal value-added shock. To check the robustness of my result, I investigate a New Keynesian model with an aggregate TFP shock in partial equilibrium. I find that markup becomes procyclical, in contrast to countercyclical markup with nominal value-added shock. The reason is that upon a positive TFP shock, the nominal marginal cost shifts downward, instead of upward upon a positive demand shock, hence with a sticky price, markup increases during a boom. The result of the model with TFP shock is not presented here, but is available upon request.

5 Conclusion

Markup cyclicality is an important magnification mechanism in the business cycle models. Previous literatures either assume an exogenous process for markup cyclicality, or use models that generate markup cyclicality without examining their validities at the micro level. In this paper, I examine two representative models, an oligopolistic competition model, and a New Keynesian model. First, I find that the oligopolistic competition model can generate the countercyclical aggregate markup, but fails to capture markup cyclicality at the firm level. Second, I introduce heterogeneous price adjustment costs into a standard New Keynesian model, and discipline the parameters to match heterogeneity in price adjustment frequencies. The resulting model successfully captures all the important moments, in the data, and in particular, the magnitude and heterogeneity in markup cyclicality in the Cobb-Douglas production function case. However, both sticky price models imply procyclical markup behavior in response to productivity shocks.

6 Appendix

6.1 Oligopolistic Competition Model

Proof of Proposition 1

If industry price index p is a continuous function of individual firm's price p_j, the symmetry of second partial derivatives holds

$$\frac{\partial^2 p}{\partial p_j \partial p_{j'}} = \frac{\partial^2 p}{\partial p_{j'} \partial p_j}.$$

Since $\frac{\partial p}{\partial p_j} = S_j$, it leads to

$$\begin{aligned} \frac{\partial S_j}{\partial p_{j'}} &= \frac{\partial S_{j'}}{\partial p_j} \\ S_j \frac{\partial \log S_j}{\partial p_j - p}(\mathbf{1}\{j' = j\} - S_{j'}) &= S_{j'} \frac{\partial \log S_{j'}}{\partial p_{j'} - p}(\mathbf{1}\{j = j'\} - S_j) \\ \Rightarrow \frac{\partial \log S_j}{\partial p_j - p} &= \frac{\partial \log S_{j'}}{\partial p_{j'} - p} \quad \forall j, j'. \end{aligned}$$

Derivation of Equation (11)

From Proposition 1, we have that $\frac{\partial^2 p}{\partial p_j \partial p_j'} = \Lambda S_j (S_{j'} - \mathbf{1}\{j = j'\})$, hence

$$\begin{aligned} &\sum_{j=1}^{n} \sum_{j'=1}^{n} \frac{\partial^2 p}{\partial p_j \partial p_{j'}} \left(\frac{\partial p_{j'}}{\partial mc_k}\right) \left(\frac{\partial p_j}{\partial mc_r}\right) \\ =& -\Lambda \sum_{j=1}^{n} S_j \left(\frac{\partial p_j}{\partial mc_k} - \sum_{j'=1}^{n} S_{j'} \frac{\partial p_{j'}}{\partial mc_k}\right) \frac{\partial p_j}{\partial mc_r} \\ =& -\Lambda \sum_{j=1}^{n} S_j \left(\frac{\partial p_j}{\partial mc_k} - \sum_{j'=1}^{n} S_{j'} \frac{\partial p_{j'}}{\partial mc_k}\right) \left(\frac{\partial p_j}{\partial mc_r} - \sum_{j''=1}^{n} S_{j''} \frac{\partial p_{j''}}{\partial mc_r} + \sum_{j''=1}^{n} S_{j''} \frac{\partial p_{j''}}{\partial mc_r}\right) \\ =& -\Lambda \sum_{j=1}^{n} S_j \left(\frac{\partial p_j}{\partial mc_k} - \sum_{j'=1}^{n} S_{j'} \frac{\partial p_{j'}}{\partial mc_k}\right) \left(\frac{\partial p_j}{\partial mc_r} - \sum_{j''=1}^{n} S_{j''} \frac{\partial p_{j''}}{\partial mc_r}\right) \\ & - \Lambda \left(\sum_{j''=1}^{n} S_{j''} \frac{\partial p_{j''}}{\partial mc_r}\right) \underbrace{\left(\sum_{j=1}^{n} \frac{\partial p_j}{\partial mc_k} - \sum_{j'=1}^{n} \frac{\partial p_{j'}}{\partial mc_k}\right)}_{=0} \\ =& -\Lambda \sum_{j=1}^{n} S_j \left(\frac{\partial p_j - p}{\partial mc_k}\right) \left(\frac{\partial p_j - p}{\partial mc_r}\right), \end{aligned}$$

and the rest follows.

Proof of Corollary 2

Since marginal cost shock Δmc_i are independently and identically distributed with mean zero, only $(\Delta mc_i)^2$ terms matter in expectation. Hence

$$\mathrm{E}[\Delta\mu^{(2)}] = \frac{1}{2}\sigma^2 \left(\sum_{j=1}^{n} \frac{S_j}{1+\Gamma_j}\right)^{-1} \left[\sum_{j=1}^{n} S_j \frac{\Gamma_j}{1+\Gamma_j} \left(\sum_{k=1}^{n} X_j^{kk}\right)\right].$$

Since I assume that all firms have the same initial states, $S_j = \frac{1}{n}$, $\Gamma_j = \Gamma'$, $\Gamma_{jj} = \Gamma''$, and $X_j^{kk} = X^{kk}$. Putting X^{kk} with equation (5) leads to

$$\begin{aligned}
&\sum_{k=1}^{n} X^{kk} \\
=&\frac{\Gamma''}{\Gamma'}\left(\frac{1}{1+\Gamma'}\right)^2 \left[\left(\sum \frac{S}{1+\Gamma'}\right)^{-2}\left(\sum\left(\frac{S}{1+\Gamma'}\right)^2\right) - 2\left(\sum \frac{S}{1+\Gamma'}\right)^{-1}\left(\frac{S}{1+\Gamma'}\right) + 1\right] \\
&- \Lambda\left(\sum S\left(\frac{1}{1+\Gamma'}\right)^2\right)\left(\sum \frac{S}{1+\Gamma'}\right)^{-2}\left(\sum\left(\frac{S}{1+\Gamma'}\right)^2\right) \\
&- \Lambda S\left(\frac{1}{1+\Gamma'}\right)^2\left[n - 2\left(\sum \frac{S}{1+\Gamma'}\right)^{-1}\left(\sum \frac{S}{1+\Gamma'}\right)\right] \\
=&\frac{\Gamma''}{\Gamma'}\left(\frac{1}{1+\Gamma'}\right)^2 \frac{n-1}{n} \\
&- \Lambda\left(\frac{1}{1+\Gamma'}\right)^2 \frac{1}{n} - \Lambda\left(\frac{1}{1+\Gamma'}\right)^2 \frac{n-2}{n} \\
=&\frac{n-1}{n}\left(\frac{1}{1+\Gamma'}\right)^2\left(\frac{\Gamma''}{\Gamma'} - \Lambda\right),
\end{aligned}$$

and the rest follows.

Proof of Proposition 2

Take full log differentiation of $\log A$, and we have

$$\begin{aligned}
d\log A &= \frac{1}{\eta-1}\left[\int_0^1 (\eta-1)\left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t}\right)^{-\eta}\left(\frac{a_{jt}}{A_t}\right)^{\eta-1}\frac{da_{jt}}{a_{jt}}dj + \int_0^1 -\eta\left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t}\right)^{-\eta-1}\left(\frac{a_{jt}}{A_t}\right)^{\eta-1} d\left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t}\right)dj\right] \\
&= \int_0^1 \left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t}\right)^{-\eta}\left(\frac{a_{jt}}{A_t}\right)^{\eta-1} d\log A_{jt} dj - \frac{\eta}{\eta-1}\int_0^1 \left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t}\right)^{-\eta}\left(\frac{a_{jt}}{A_t}\right)^{\eta-1} d\log\left(\frac{\mathcal{M}_{jt}}{\mathcal{M}_t}\right)dj.
\end{aligned}$$

For log sectoral productivity change $d\log a_{jt}$, we have

$$d\log a_{jt} = \sum_{i=1}^{n_j}\left[\left(\frac{\mathcal{M}_{ijt}}{\mathcal{M}_{jt}}\right)^{-\rho}\left(\frac{a_{ijt}}{a_{jt}}\right)^{\rho-1} d\log a_{ijt} - \frac{\rho}{\rho-1}\left(\frac{\mathcal{M}_{ijt}}{\mathcal{M}_{jt}}\right)^{-\rho}\left(\frac{a_{ijt}}{a_{jt}}\right)^{\rho-1} d\log\left(\frac{\mathcal{M}_{ijt}}{\mathcal{M}_{jt}}\right)\right]$$

Also, respectively, firm market share, and sectoral market share can be expressed as

$$
\begin{aligned}
S_{ijt} &= \left(\frac{P_{ijt}}{P_{jt}} \right)^{1-\rho} \\
&= \left(\frac{\mathcal{M}_{ijt}/a_{ijt}}{\mathcal{M}_{jt}/a_{jt}} \right)^{1-\rho},
\end{aligned}
$$

and

$$
\begin{aligned}
S_{jt} &= \left(\frac{P_{jt}}{P_{t}} \right)^{1-\eta} \\
&= \left(\frac{\mathcal{M}_{jt}/a_{jt}}{\mathcal{M}_{t}/A_{t}} \right)^{1-\eta}.
\end{aligned}
$$

Substitute these into the equations above and the result follows.

6.2 Numerical Solution for CalvoPlusPlus Model

The firm's real profit of posting price P_{it} in period t is

$$\begin{aligned}\Pi_{it}^{R}(p_{it}) &= \left(\frac{p_{it}}{P_t}\right)^{1-\theta} C_t - \frac{W_t}{P_t} L_{it} \\ &= \left(\frac{p_{it}}{S_t} - \frac{\omega}{a_{it}}\right)\left(\frac{p_{it}}{S_t}\right)^{-\theta}\left(\frac{S_t}{P_t}\right)^{2-\theta},\end{aligned}$$

where I have used the identity real wage $W_t/P_t = \omega C_t$, and $C_t = S_t/P_t$ in the second line.

Hence, I can define the state space for firm i as $\mathcal{S}_{it} = \left\{\frac{p_{it-1}}{S_t}, a_{it}, \frac{S_t}{P_t}\right\}$, and rewrite the firm's value function in real term:

$$V(\mathcal{S}_{it}) = \max\{V_N(\mathcal{S}_{it}), V_A(\mathcal{S}_{it})\},$$

where the value of not adjusting price $V_N(\mathcal{S}_{it})$ and adjusting price $V_A(\mathcal{S}_{it})$ are respectively given by

$$V_N(\mathcal{S}_{it}) = \Pi_{it}^{R}(p_{it-1}) + \beta \mathrm{E}_t\left[\frac{S_t/P_t}{S_{t+1}P_{t+1}} V(\mathcal{S}_{it+1})\right],$$

and

$$V_A(\mathcal{S}_{it}) = \max_{p_{it}} \Pi_{it}^{R}(p_{it}) - \kappa_{it}\omega\left(\frac{S_t}{P_t}\right) + \beta \mathrm{E}\left[\frac{S_t/P_t}{S_{t+1}/P_{t+1}} V(\mathcal{S}_{it+1})\right].$$

Specific form of the adjustment cost κ_{it} depends on the nature of the adjustment cost. Under a menu cost model, the adjustment cost is

$$\kappa_{it} = \kappa_0 \left(\frac{p_{it}}{S_t}\right)^{\kappa_1(1-\theta)} \left(\frac{S_t}{P_t}\right)^{\kappa_1(2-\theta)}.$$

And under a calvo model, the adjustment cost is

$$\kappa_{it} = \begin{cases} 0 & \text{w.p. } \lambda_{it} \\ \bar{\kappa} & \text{otherwise,} \end{cases}$$

where the Calvo probability can be written as

$$\lambda_{it} = \lambda_0 \left(\frac{p_{it-1}}{S_{t-1}}\right)^{\lambda_1(1-\theta)} \left(\frac{S_{t-1}}{P_{t-1}}\right)^{\lambda_1(2-\theta)}.$$

In the simulation with the calibrated parameters, λ_{it} is always below one.

Tables and Figures

Table 1: Parameter Values for Calibration

Parameter	Value	Rationale
Intertemporal Elasticity of Substitution (IES)	$1/\sigma = 1$	Unit IES
Frisch Elasticity of Labor Supply	$1/\psi = 1$	Chang, Kim, Kwon, and Rogerson (2014)
Disutility Parameter from Labor	$\omega = 7$	
Across-sector Elasticity of Substitution	$\eta = 1.026$	Labor Cost Share and Market Share
Within-sector Elasticity of Substitution	$\rho = 11$	Labor Cost Share and Market Share
Number of Firms	$n_j = 30$	Moment in the data
Fixed Cost of Production	$\phi = 4\%$	
Persistence of Firm Productivity	$\rho_f = 0.95$	
SD of Firm Productivity (Low)	$\nu_f = 0.05$	
SD of Firm Productivity (High)	$\nu_f = 0.15$	

Table 2: Summary Statistics: Goldberg and Hellerstein (2011)

Weighted Median	Top	Middle	Bottom
Frequency of Adjustment	18.20%	12.20%	10.50%
Frequency of Increases	13.60%	10.30%	8.20%
Frequency of Decreases	5.50%	1.60%	1.50%
Adjustment Size Change	5.60%	6.00%	6.00%
Upward Size Change	5.70%	5.40%	5.60%
Downward Size Change	5.60%	5.90%	6.70%

Top, Middle, and Bottom refers to terciles in terms of firms revenues. Large firms adjust prices more frequently, and adjust less than small firms.

Table 3: Parameter Values for Simulation: Calvo Model

Parameter	Value	Rationale
Monthly Discount Factor	$\beta = 0.997$	Nakamura and Steinsson (2010)
Elasticity of Substitution	$\theta = 5$	
Inverse of Intertemporal Elasticity of Substitution	$1/\sigma = 1$	
Inverse of Frisch Elasticity of Labor Supply	$\psi = 0$	
Steady State Labor Supply	$L_{ss} = 1/3$	
Nominal Aggregate Demand Growth Rate	$\mu_S = 0.0028$	Nakamura and Steinsson (2010)
Nominal Aggregate Demand Std. Deviation	$\sigma_S = 0.0065$	Nakamura and Steinsson (2010)
Idiosyncratic Productivity Persistence	$\rho_a = 0.9$	
Idiosyncratic Productivity Std.Deviation	$\sigma_a = 0.03$	moments in the data
Calvo Constant	$\lambda_0 = 3.1200$	moments in the data
Calvo Curvature	$\lambda_1 = 3.0169$	moments in the data

Table 4: Summary Statistics: Calvo Model

Weighted Median	Top	Middle	Bottom
Frequency of Adjustment	18.20%	13.90%	10.60%
Frequency of Increases	11.20%	8.80%	6.80%
Frequency of Decreases	6.90%	5.10%	3.60%
Adjustment Size Change	4.87%	5.38%	5.93%
Upward Size Change	5.12%	5.75%	6.45%
Downward Size Change	4.46%	4.75%	4.96%
$\text{Corr}(\Delta \ln \mathcal{M}_{it}, \Delta \ln Y_t)$	-0.1304	-0.1351	-0.1392
ϕ	-0.9643	-1.0155	-1.0519
Mean of Markup	1.2544	1.2591	1.2683
Std of Δ log Markup	0.0403	0.0410	0.0412

Table 5: Parameter Values for Simulation: Menu Cost Model

Parameter	Value	Rationale
Monthly Discount Factor	$\beta = 0.997$	Nakamura and Steinsson (2010)
Elasticity of Substitution	$\theta = 5$	
Inverse of Intertemporal Elasticity of Substitution	$1/\sigma = 1$	
Inverse of Frisch Elasticity of Labor Supply	$\psi = 0$	
Steady State Labor Supply	$L_{ss} = 1/3$	
Nominal Aggregate Demand Growth Rate	$\mu_S = 0.0028$	Nakamura and Steinsson (2010)
Nominal Aggregate Demand Std. Deviation	$\sigma_S = 0.0065$	Nakamura and Steinsson (2010)
Idiosyncratic Productivity Persistence	$\rho_a = 0.9$	
Idiosyncratic Productivity Std. Deviation	$\sigma_a = 0.03$	moments in the data
Menu Costs Constant	$\kappa_0 = 0.00043\%$	moments in the data
Menu Costs Curvature	$\kappa_1 = -7$	moments in the data

Table 6: Summary Statistics: Menu Cost Model

Weighted Median	Top	Middle	Bottom
Frequency of Adjustment	18.30%	14.00%	10.40%
Frequency of Increases	12.70%	10.10%	8.10%
Frequency of Decreases	5.60%	3.80%	2.20%
Adjustment Size Change	5.37%	5.83%	6.24%
Upward Size Change	5.01%	5.40%	5.77%
Downward Size Change	6.16%	6.96%	7.86%
$\text{Corr}(\Delta \ln \mathcal{M}_{it}, \Delta \ln Y_t)$	-0.1278	-0.1394	-0.1514
ϕ	-0.9407	-1.0774	-1.2071
Mean of Markup	1.2399	1.2364	1.2326
Std of Δ log Markup	0.0276	0.0289	0.0299

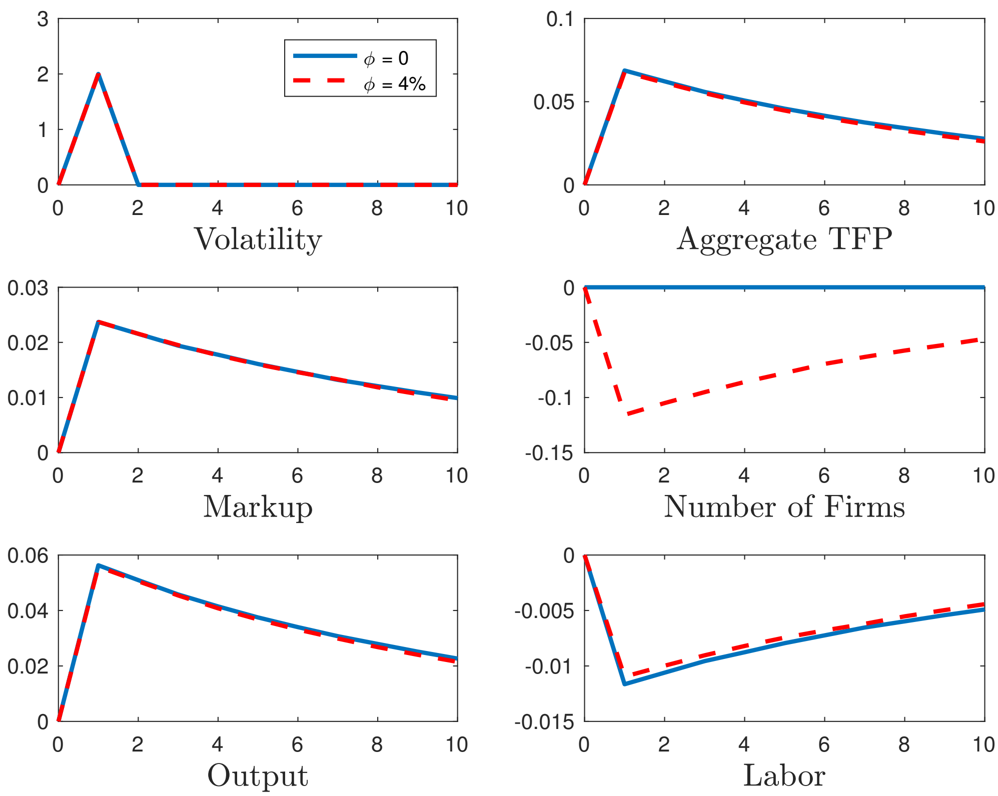


Figure 1: Second Moment Shock

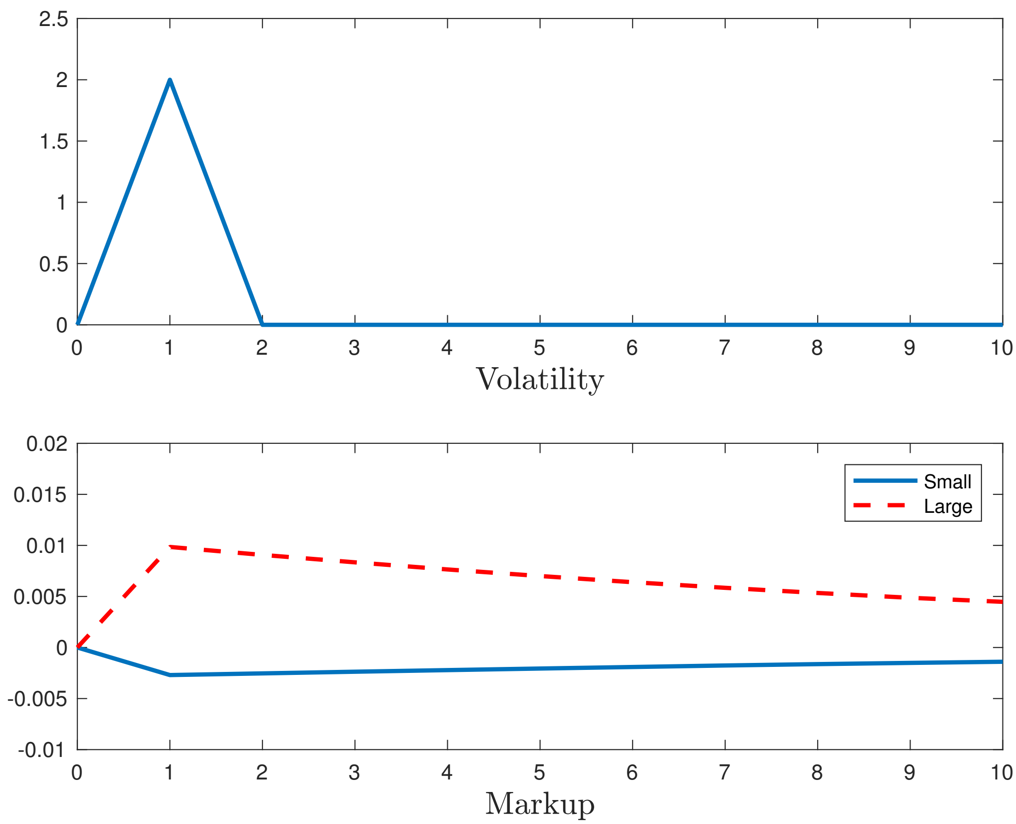


Figure 2: Response of Small Firms VS Large Firms

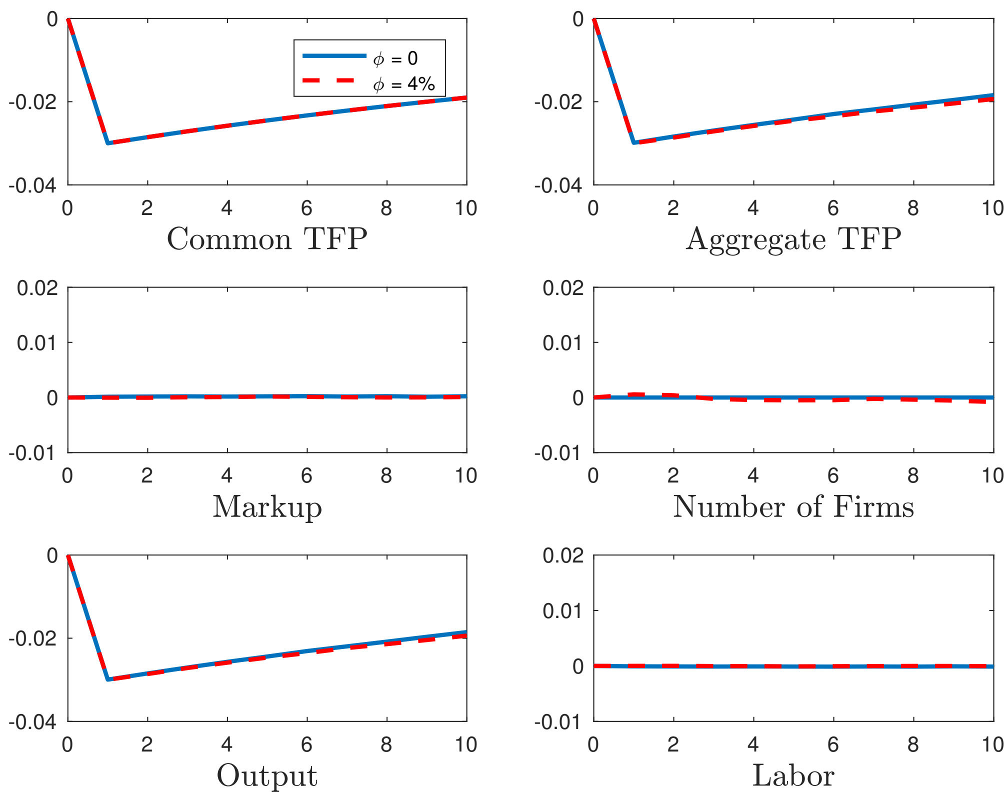
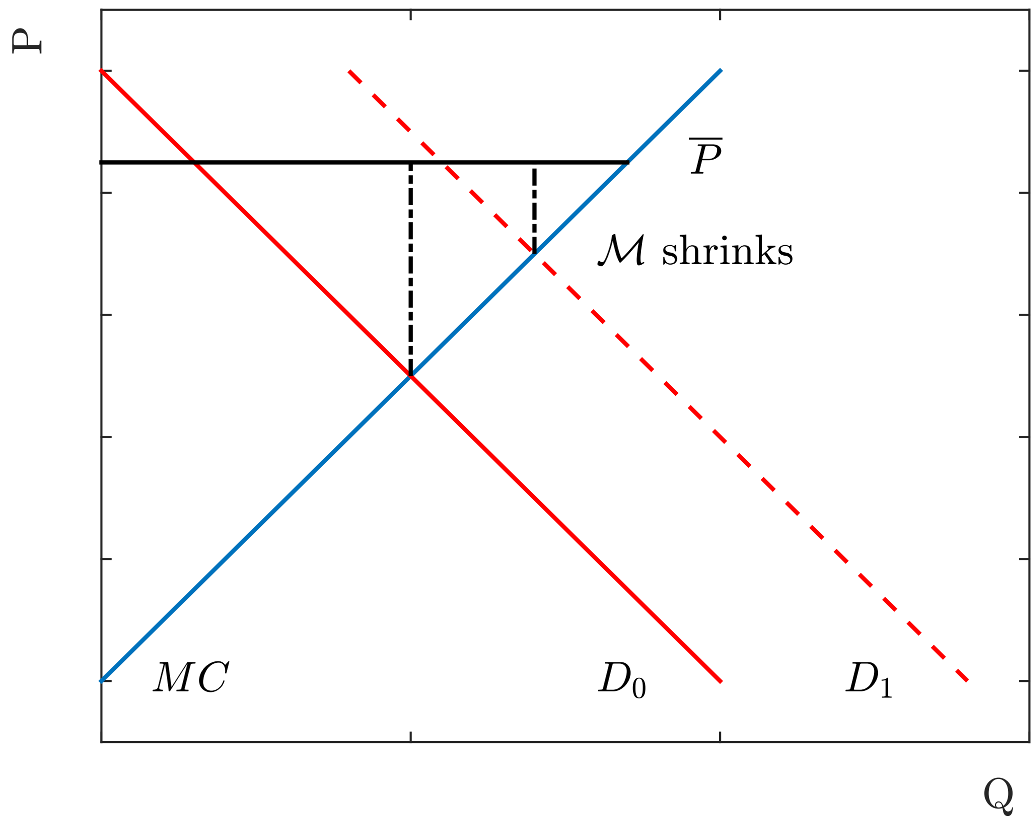


Figure 3: First Moment Shock



- Initially, firm sets price at $\overline{P}$. When demand curve shifts from D_0 to D_1, marginal cost MC increases. Due to price stickiness, price stays at $\overline{P}$, hence markup $\mathcal{M}$ shrinks.
- To match the cross-sectional markup cyclicality in the data, small firms should exhibit more price stickiness than large firms.

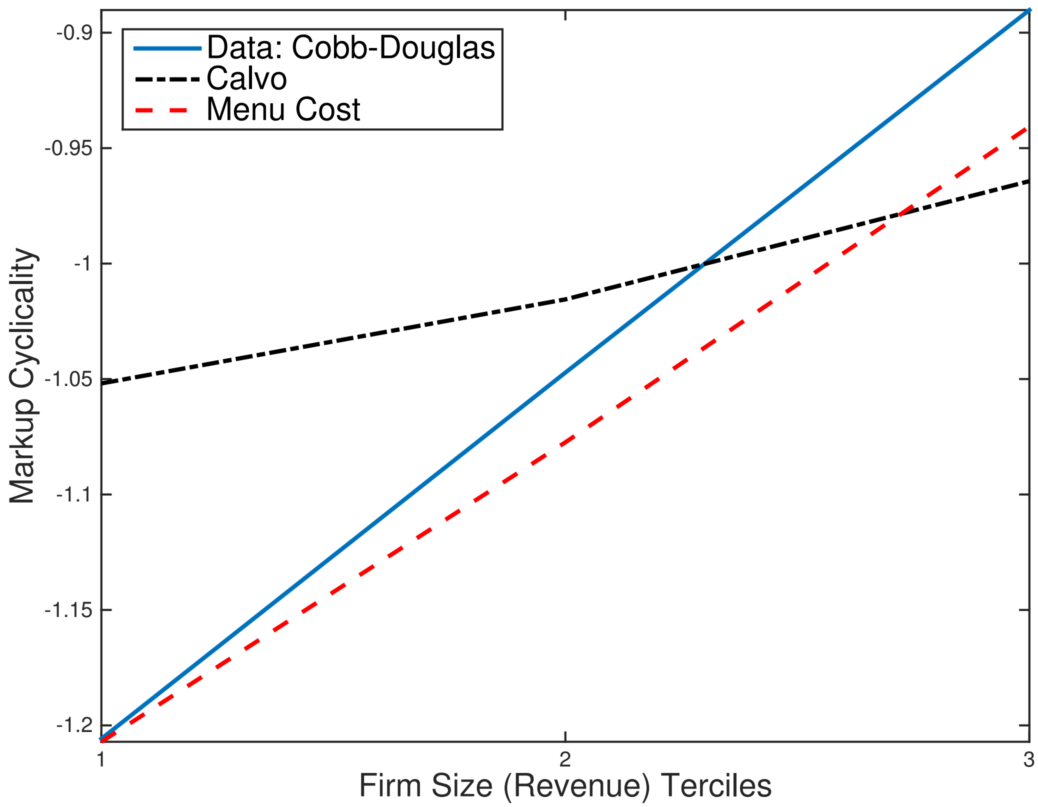


Figure 4: Comparison of markup cyclicality ϕ between data, Calvo Model, and Menu Cost model.

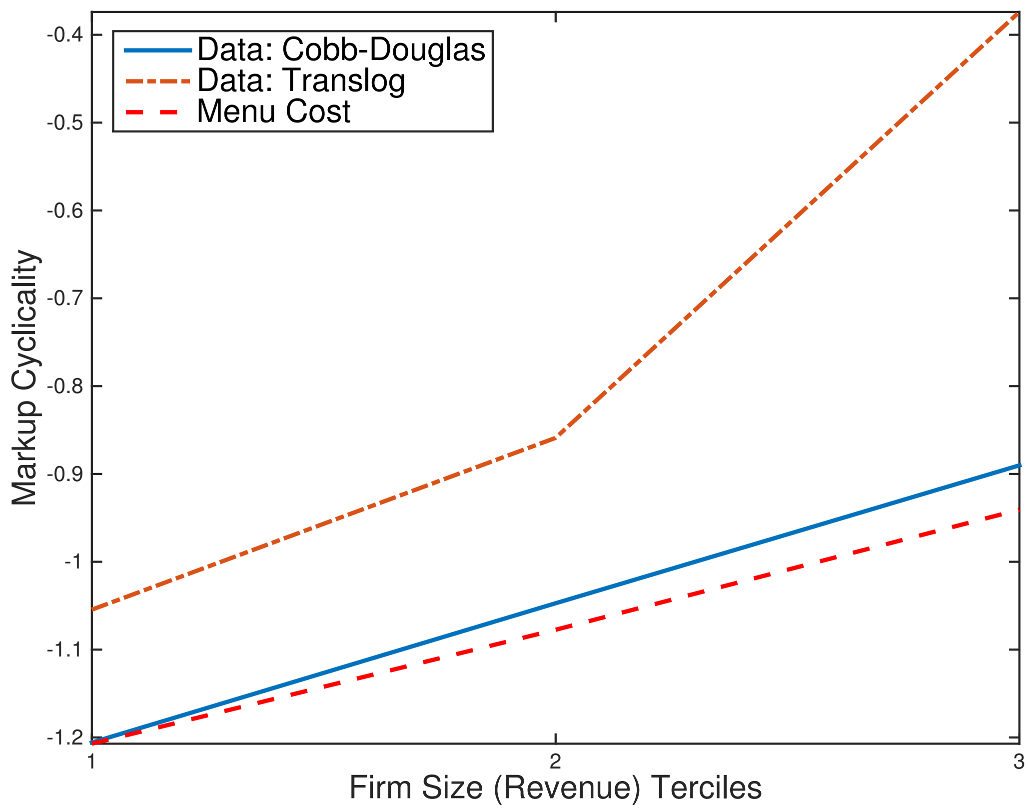


Figure 5: Comparison of markup cyclicality ϕ between data and Menu Cost model. Data includes both Cobb-Douglas and Translog cases.

References

[1] Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. Networks, shocks, and systemic risk. Tech. rep., National Bureau of Economic Research, 2015.

[2] Atkeson, A., and Burstein, A. Pricing-to-market, trade costs, and international relative prices. *The American Economic Review 98*, 5 (2008), 1998–2031.

[3] Bhattarai, S., and Schoenle, R. Multiproduct firms and price-setting: Theory and evidence from us producer prices. *Journal of Monetary Economics 66* (2014), 178–192.

[4] Bils, M., Klenow, P. J., and Malin, B. A. Resurrecting the role of the product market wedge in recessions. Tech. rep., National Bureau of Economic Research, 2014.

[5] Bloom, N. The impact of uncertainty shocks. *econometrica 77*, 3 (2009), 623–685.

[6] Burstein, A., and Gopinath, G. International prices and exchange rates. Tech. rep., National Bureau of Economic Research, 2013.

[7] Carvalho, C. Heterogeneity in price stickiness and the real effects of monetary shocks. *The B.E. Journal of Macroeconomics 6*, 3 (2006), 1–58.

[8] Chang, Y., Kim, S.-B., Kwon, K., and Rogerson, R. Individual and aggregate labor supply in a heterogeneous agent economy with intensive and extensive margins. Tech. rep., 2012.

[9] Edmond, C., Midrigan, V., and Xu, D. Y. Competition, markups, and the gains from international trade. *The American Economic Review 105*, 10 (2015), 3183–3221.

[10] Gertler, M., and Leahy, J. A phillips curve with an ss foundation.

[11] Goldberg, P., and Hellerstein, R. How rigid are producer prices? *FRB of New York Staff Report*, 407 (2011).

[12] Hansen, G. D. Indivisible labor and the business cycle. *Journal of monetary Economics 16*, 3 (1985), 309–327.

[13] Hsieh, C.-T., and Klenow, P. J. Misallocation and manufacturing tfp in china and india. *The Quarterly Journal of Economics 124*, 4 (2009), 1403–1448.

[14] Jaimovich, N., and Floetotto, M. Firm dynamics, markup variations, and the business cycle. *Journal of Monetary Economics 55*, 7 (2008), 1238–1252.

[15] Midrigan, V. Menu costs, multiproduct firms, and aggregate fluctuations. *Econometrica 79*, 4 (2011), 1139–1180.

[16] Nakamura, E., and Steinsson, J. Monetary non-neutrality in a multisector menu cost model. *The Quarterly Journal of Economics 125*, 3 (2010), 961–1013.

[17] Ravn, M., Schmitt-Grohé, S., and Uribe, M. Deep habits. *The Review of Economic Studies 73*, 1 (2006), 195–218.

[18] Restuccia, D., and Rogerson, R. Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic dynamics 11*, 4 (2008), 707–720.

[19] Rogerson, R. Indivisible labor, lotteries and equilibrium. *Journal of monetary Economics 21*, 1 (1988), 3–16.

[20] Smets, F., and Wouters, R. An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European economic association 1*, 5 (2003), 1123–1175.

[21] Smets, F., and Wouters, R. Shocks and frictions in us business cycles: A bayesian dsge approach. *The American Economic Review 97*, 3 (2007), 586–606.