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Optimal Fiscal Policy in Overlapping Generations Models*

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Abstract

In this paper, we explore the proposition that the optimal capital income tax is zero using an overlapping generations model. We prove that for a large class of preferences, the optimal capital income tax along the transition path and in steady state is non-zero. For a version of the model calibrated to the US economy, we find that the model could justify the observed rates of capital income taxation for an empirically reasonable intertemporal utility function and a robust demographic structure.

Keywords: Optimal taxation, uniform commodity taxation.
J.E.L. classification codes: E62, H21.

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1 Introduction

This paper explores the proposition that the optimal capital income tax is zero. The standard view is that capital returns should not be taxed at all. This view is built on a well-established theory of optimal fiscal policy. In standard neoclassical growth models with infinitely-lived consumers, Judd (1985) and Chamley (1986) show that the optimal policy predicts zero capital taxes in the long run.[1]

This paper explores the proposition that the optimal capital income tax is zero using overlapping generations economies. The main contribution of the paper is to show that in a standard overlapping generations model it is very difficult to obtain zero optimal capital taxes either in steady state or along the transition path. We provide sufficient conditions in preferences for zero capital taxation in these models, and show that these conditions are more restrictive than the standard uniform commodity tax result needed in infinitely-lived models with perfect competition. For a general class of preferences, the optimal policy implies a non-zero capital income tax violating the standard uniform commodity tax result that specifies under which circumstances taxing all goods at the same rate is optimal.

To provide some intuition it is useful to relate the present findings with the economic intuition presented in Judd (1999) for an infinitely-lived consumer economy. From general equilibrium theory we know that the static Arrow-Debreu model can be applied to a dynamic context, so does the principle from the commodity tax literature. As a result, a positive capital income tax is equivalent to a commodity tax on the time t good that grows exponentially in t. In infinitely-lived consumer economies, if preferences are separable and exhibit some degree of substitutability, this policy entails an ever-growing distortion between the marginal rate of substitution and the marginal rate of transformation. Given that individuals have a preference to smooth consumption, they prefer a constant consumption tax to an ever-increasing consumption tax. This policy can be implemented by removing the tax rate on capital income and replacing it with a tax on labor income. In contrast, if individuals live a finite number of periods the distortions associated with this policy are not that important because for a given generation today's consumption and period T consumption are not perfectly substitutable. Hence the effect of capital distortions is much smaller and not necessarily bigger than distortions caused by other taxes. The key to the general result is the existence of consumers of different ages making the same type of decisions (consumption/savings and labor supply) at a given point in time. Given that consumption and hours worked are not constant over the life-cycle even in the steady state, consumption should be taxed when it is relatively higher. The government can imperfectly affect consumption by

[1]Several papers have extended this result to more general class of economies that include endogenous growth (Jones, Manuelli and Rossi (1997)), aggregate shocks (Chari, Christiano and Kehoe (1994)), and open economies (Razin and Sandka (1995)) and find similar results, capital returns should not be taxed at all.

setting a non-zero capital tax. In particular, if we interpret consumption at different ages as different goods that can be taxed at a different rate, in general, we find that the optimal policy implies an increasing consumption tax over the life-cycle. Restricting the tax policy to **age-independent** taxes imposes additional constraints to the government problem that leads to this result. However, relaxing this assumption and allowing **age-dependent** taxes can restore the zero capital income tax result in overlapping generation economies.

These theoretical findings can reconcile the quantitative work by Escolano (1992) that uses a large scale quantitative overlapping generations model and finds positive optimal capital income taxes with a large theoretical literature that uses two-period overlapping generations and finds zero optimal capital income taxes, for example Pestieau (1974), Atkinson and Sandmo (1980), Atkinson and Stiglitz (1980), and Chari and Kehoe (1999). The discrepancy comes from the fact that Escolano (1992) computes the **age-independent** optimal tax policy whereas the other papers use **age-dependent** taxes.

The findings in this paper are similar to parallel work by Erosa and Gervais (2002) who study the same problem in an environment where the government uses **age-dependent** taxes. They find that if the government can condition taxes on age, the zero capital income tax results of the infinite-lived consumer model can be extended to life-cycle economies. In similar vein to Escolano (1992), they perform a quantitative exercise and show that the optimal steady state capital income tax is non-zero when the government can only use **age-independent** taxes. Relative to Erosa and Gervais (2002), this paper provides a set of sufficient theoretical conditions that imply non-zero capital income taxes with **age-independent** taxes by exploiting properties of the uniform commodity tax result.

To confirm the connection between commodity taxation and capital income taxation we simulate the model for an empirically reasonable intertemporal utility function and a robust demographic structure. Given some plausible choices of parameter values, when the government cannot condition taxes on age the optimal policy can be consistent with the observed tax rates.[2] Nevertheless, the optimal capital tax predicted by the model **can** change when the government can condition taxes on age. For preferences that satisfy the uniform commodity tax result the optimal capital tax across ages is zero. This zero capital income tax also implies an equivalent commodity tax of zero as in the infinite-lived consumer model. For preferences that violate the uniform commodity tax result, the optimal capital income tax changes over the life-cycle. For young households borrowing the optimal capital income tax is negative, increasing the cost of borrowing. For middle age and older households, the optimal capital income tax is positive, reducing the return from savings. When taxes cannot be conditioned on age, this nonlinear schedule across ages is imperfectly replicated by setting

[2] Mendoza, Tesar and Razin (1994) document that most OECD economies have effective capital income taxes that are different from zero. In particular, for the US the average capital income tax over the period 1965-95 is around 35%, and in the U.K. and Germany it is around 37% and 23.5% respectively.

a non-zero capital tax.

The contributions of this line of work using overlapping generations models to evaluate optimal tax rates has been followed and extended by other papers. For example, Conesa and Garriga (2008) design an optimal transition from a pay-as-you-go social security system to a fully-funded system. Conesa, Kitao and Krueger (2009) quantitatively demonstrate the optimality of positive capital taxation as a way to mimic the optimal age-dependent labor taxation in a model with incomplete markets, uninsurable income risk, and parametric tax functions. In a similar fashion, Gervais (2012) explores the connection between the progressivity of income taxation and age-dependent capital income taxes. Other work has explored the role of age-dependent taxation in a dynamic Mirrleesian framework, see Weinzierl (2011).

The paper is organized as follows. Section 2 describes the behavior of the market economy, and section 3 defines the government problem and derives the sufficient conditions for the zero capital tax result. Section 4 further characterizes the optimal policy and illustrates the basic results using numerical simulations. Finally, section 5 concludes.

2 The economy

The economy is an overlapping generations model with production and two goods, a consumption-capital good and labor. Agents live $I \geq 2$ periods and each cohort is populated by identical households. Without loss of generality, the population is assumed to be stationary and its total size is constant.[3]

There is a representative firm that produces aggregate output Y_t using a constant returns to scale production function $F(K_t, L_t)$, using aggregate capital K_t and aggregate labor L_t as primary inputs. Labor is measured in efficiency units. The production function $F : R_+^2 \to R_+$ is strictly concave, monotone, continuously differentiable. Capital depreciates each period at a constant rate $\delta \in (0, 1)$ and there is no exogenous technological change. These assumptions imply that in competitive factor markets firms will make zero profits, hence it is unnecessary to specify firms' ownership. Then, each period prices are determined by

$$r_t = F_{Kt} - \delta, \tag{1}$$

$$w_t = F_{Lt}, \tag{2}$$

where r_t denotes the interest rate net of depreciation and w_t is the wage rate per efficiency unit of labor. Let C_t and L_t denote aggregate consumption and labor respectively

$$C_t = \sum\nolimits_{i=1}^{I} c_t^i \qquad \forall t, \tag{3}$$

$$L_t = \sum\nolimits_{i=1}^{I} \epsilon^i l_t^i \qquad \forall t, \tag{4}$$

[3]This is not an important assumption for the basic results and simplifies notation.

where c_t^i denotes consumption of an individual of age i at time t, ϵ^i denote her efficiency units, and l_t^i is hours worked.

The government in this economy finances an exogenous sequence of expenditure $\{G\}_{t=0}^{\infty}$ using proportional capital taxes θ_t, consumption taxes η_t, labor taxes τ_t and debt D_t. The government intertemporal budget constraint is

$$G_t + R_t D_t \leq \eta_t C_t + \tau_t w_t L_t + \theta_t r_t K_t + D_{t+1} \qquad \forall t, \tag{5}$$

where R_t denotes the return on government debt. Let $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^{\infty}$ be a tax policy consisting of an infinite sequence of proportional taxes and government debt, where D_0 is given at $t = 0$. Solving the government budget constraint forward gives the intertemporal constraint

$$D_t = \sum\nolimits_{j=1}^{\infty} (T_{t+j} - G_{t+j}) / \prod_{j}^{\infty} R_{t+j},$$

for $t \geq 0$ and $T_t = \eta_t C_t + \tau_t w_t L_t + \theta_t r_t K_t$. We have ruled out Ponzi schemes by imposing the transversality condition. The period resource constraint is

$$C_t + K_{t+1} - (1-\delta) K_t + G_t \leq F(K_t, L_t) \qquad \forall t. \tag{6}$$

Each generation has an endowment of one unit of time at each period and a life cycle profile of efficiency units of labor $\epsilon = (\epsilon^1, ..., \epsilon^I)$. The endowment of an individual of age i can be transformed into ϵ^i units of input in the production function. Households in this economy have standard preferences defined over a stream of consumption and labor/leisure and are represented by a time separable utility function

$$\sum\nolimits_{i=1}^{I} \beta^{i-1} U(c_{t+i}^i, l_{t+i}^i) \qquad \forall t, \tag{7}$$

where $\beta > 0$ is the subjective discount factor, c_{t+i}^i and l_{t+i}^i represent the consumption and the time devoted to work by an individual of age i at time $t + i$. The utility function $U : R_+^2 \to R_+$ is C^2, strictly concave, increasing in consumption $U_c(c, l) > 0$ and decreasing in labor $U_l(c, l) < 0$. At each period, taking prices and taxes as given, individuals choose consumption, labor supply, and asset holdings. The consumer at each period maximizes (7) subject to

$$(1+\eta_t) c_t^i + a_{t+1}^{i+1} \leq (1-\tau_t) w_t \epsilon^i l_t^i + (1 + r_t(1-\theta_t)) a_t^i \qquad 1 \leq i \leq I \quad \forall t, \tag{8}$$

$$a_t^1 = 0, \quad 0 \leq l_t^i \leq 1, \quad c_t^i, a_t^{I+1} \geq 0 \qquad \forall t.$$

Each generation is born with no assets, and can accumulate wealth a_{t+1}^{i+1} by buying one-period government debt and lending to firms. Markets are complete, so different generations can intertemporaly trade assets to smooth consumption over the life-cycle.[4]

At the initial period, $t = 0$, the stock of capital and debt is distributed among the initial, s, generations (individuals of age 2 to I). Let $\overline{a}_0^s = (k_0^s + d_0^s)$ be the endowment of capital and debt distributed among the initial generations, where $K_0 = \sum_s k_0^s$ and $D_0 = \sum_s d_0^s$. The period-0 budget constraint is

$$(1+\eta_0)c_0^s + a_1^{s+1} \leq (1-\tau_0)w_0\epsilon^s l_0^s + (1+r_0(1-\theta_0))\overline{a}_0^s \qquad 2 \leq s \leq I. \tag{9}$$

Market clearing conditions in the capital markets imply

$$K_{t+1} = \sum\nolimits_{i=1}^{I} a_{t+1}^i - D_{t+1} \qquad \forall t. \tag{10}$$

Next we proceed by defining the notion of competitive equilibrium.

Definition 1 (Competitive Equilibrium): *Given a tax policy π and a sequence of government expenditure $\{G_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy is a sequence of individual allocations $\{\{c_t^i, l_t^i, a_{t+1}^i\}_{i=1}^{I}\}_{t=0}^{\infty}$, production plans $\{K_t, L_t\}_{t=0}^{\infty}$, government debt $\{D_{t+1}\}_{t=0}^{\infty}$, and relative prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, such that:*

1. Consumers born at time $t \geq 1$ maximize (7) *subject to* (8)*. Similarly, consumers born at $t \leq 0$ maximize utility subject to* (9).

2. In the production sector (1) *and* (2) *are satisfied for all t.*

3. Factor markets (4) *and* (10) *clear.*

4. The government budget constraint (5) *is satisfied.*

5. Feasibility (6) *is satisfied for all t.*

3 Government problem

With the behavior of the market economy described in the previous section, we turn to the problem faced by the government. The objective of the government is to choose a tax policy π^* to maximize the welfare of all (present and future) generations, and the policy has to be consistent with the private sector equilibrium. As it has been pointed out by Kydland and Prescott (1977), the optimal policy might be time-inconsistent because the government can have incentives to deviate from the announced policy. In this paper we abstract from these issues and it is assumed that the government can commit to future policies.

[4]This assumption implies that agents are not credit constrained. Aiyagari (1995) shows that in an economy with uninsurable income risk the optimal capital tax is positive. The basic intuition works as follows. Because of the incomplete insurance markets, there is a precautionary motive for accumulating capital. In addition, the possibility of being borrowing constrained in the future leads to some additional savings. These facts increase the capital stock. Then, a positive capital tax is needed to reduce capital accumulation and equalize the interest rate of the economy to the rate of time preference.

In a representative consumer model, the social welfare function for the benevolent government has to be the consumer's utility function. In an economy with an infinite number of generations, the government needs to assign weight to the different consumers, and these weights are somewhat arbitrary. Let $\omega_t \in R_+$ be the weight of a generation born at time t. In order to have a well-defined problem it is necessary to assume that the sequence of weights $\{\omega_t\}_{t=-(I-1)}^{\infty}$ is summable, $\sum_{t=-(I-1)}^{\infty} \omega_t < \infty$. Formally, the government objective function is

$$W(\{c_t^i, l_t^i\}) = \sum\nolimits_{t=0}^{\infty} \sum\nolimits_{i=1}^{I} \omega_{t+1-i} \left(\beta^{i-1} U(c_t^i, l_t^i)\right). \tag{11}$$

To find an asymptotic steady state for the government problem it is necessary to impose some structure on the sequence of weights[5], such as $\lim_{t\to\infty} \frac{\omega_t}{\omega_{t+1}} = \frac{1}{\lambda} > 1$ where $\lambda \in (0, 1)$ is the relative weight between of present and future generations. Specifying the social welfare function to be of this form imposes some restrictions, because it rules out steady state "golden-rule" equilibria, as in Samuelson (1958).

The government problem of choosing the optimal policy is solved using the so-called *primal approach,* developed in Atkinson and Stiglitz (1980). One way to think of it is having the government choosing directly from the set of implementable allocations given a tax policy π. Then from the allocations it is possible to back out policies and prices from the market economy. The set of implementable allocations is characterized by the period resource constraint and an implementability constraint for each generation. The implementability constraint represents the households' present value budget constraint after substituting the consumer's and firm's first-order conditions to eliminate prices and taxes. The next proposition describes how to characterize the set of implementable allocations for a given tax policy $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^{\infty}$.

Proposition 1 (Set of Implementable Allocations): *Given a tax policy π a competitive equilibrium allocation $x = \{\{c_t^i, l_t^i\}_{i=1}^{I}, K_{t+1}\}_{t=0}^{\infty}$ satisfies the following set of conditions:*

i) period resource constraint:

$$\sum\nolimits_{i=1}^{I} c_t^i + K_{t+1} - (1-\delta)K_t + G_t = F(K_t, \sum\nolimits_{i=1}^{I} \epsilon^i l_t^i) \qquad \forall t, \tag{12}$$

ii) implementability constraints for all newborn generations:

$$\sum\nolimits_{i=1}^{I} \beta^{i-1} \left(c_{t+i-1}^i U_{c_{t+i-1}^i} + l_{t+i-1}^i U_{l_{t+i-1}^i}\right) = 0 \qquad t \geq 0, \tag{13}$$

iii) implementability constraints for the initial old generations at $t = 0$:

$$\sum\nolimits_{i=s}^{I} \beta^{i-s} \left(c_{i-s}^i U_{c_{i-s}^i} + l_{i-s}^i U_{l_{i-s}^i}\right) = U_{c_0^s} a_0^s \quad s = 2, ..., I, \tag{14}$$

[5]Using a two period overlapping generations model, Atkinson and Sandmo (1980) derive the steady state optimal capital tax for different government discount factors. It can be easily shown that all are particular cases of this formulation.

iv) marginal rates of substitution between consumption and labor, and consumption today and tomorrow are equal across consumers:

$$\frac{U_{c_t^1}}{U_{l_t^1}}\epsilon^1 = \ldots = \frac{U_{c_t^I}}{U_{l_t^I}}\epsilon^I \qquad \forall t, \tag{15}$$

$$\frac{U_{c_t^1}}{U_{c_{t+1}^2}} = \ldots = \frac{U_{c_t^{I-1}}}{U_{c_{t+1}^I}} \qquad \forall t. \tag{16}$$

Furthermore, given allocations that satisfy (12), (13), (14), (15), *and* (16), *we can construct a tax policy* $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^{\infty}$ *and relative prices* $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, *that together with the allocation* x, *constitute a competitive equilibrium.*

Proof. See Appendix ■

In a representative consumer economy, the set of implementable allocations is uniquely determined by the period resource constraint and the implementability constraint. In an economy with heterogeneous consumers, these two conditions do not necessarily guarantee that the marginal rates of substitution across consumers are equal at a given period t. Unless specified, the government might find it optimal to tax different consumers with different tax rates. In this particular problem, the government might choose to condition taxes on age.[6] Consider the following case where the government can use age-dependent taxes, $\pi^i = \{\{\eta_t^i, \tau_t^i, \theta_t^i\}_{i=1}^{I}, D_t\}_{t=0}^{\infty}$. For each generation, the implementability constraints associated with the new tax system coincides with the one where the government cannot condition taxes on age. Hence, if taxes cannot be conditioned on age, the set of implementable allocations has to include additional constraints to ensure that the marginal rates of substitution are equal across generations.

Inspection of the first-order conditions of the consumer problem (displayed in the appendix) shows that if an allocation belongs to the set of implementable allocations, then it can be decentralized under a variety of tax schemes.

Corollary 1: *Given a sequence of* $\{G\}_{t=0}^{\infty}$ *and an initial distribution of wealth* $\{\overline{a}_0^s\}_{i=2}^{I}$, *if* $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^{\infty}$ *is the tax policy given by proposition 1 associated with an allocation* x, *then there exists another tax policy* $\pi' = \{\eta_t', \tau_t', \theta_t', D_t'\}_{t=0}^{\infty}$ *that supports the same allocation.*

Proof. See Appendix ■

The primal approach implements optimal wedges between the marginal rates of substitution and marginal rates of transformation, but it does not prescribe any particular type of instruments. As a result, the optimal policy can be supported as a competitive equilibrium under a variety of tax schemes. Such a system could include those with only consumption

[6]In the case of intratemporal heterogeneity, the government might find it optimal to condition taxes on age and type.

and labor income taxes, or more complicated tax systems. In this particular paper we are interested in capital income taxation, so we set consumption taxes, $\eta_t = 0$, in all periods.

In an infinite-lived consumer economy, the government has incentives to tax heavily the initial stock of capital at $t = 0$ and achieve a Pareto efficient allocation. To avoid an effective lump-sum tax, it is generally assumed that the government takes as the initial capital tax θ_0. In an overlapping generations economy, the individuals that face a front-loading tax policy are different than the ones that benefit from the reduction of distortionary taxes in the future. As a result the government faces a trade-off between efficiency and intergenerational redistribution. Hence, a tax in the initial distribution of wealth $\{\overline{a}_0^s\}_{s=0}^I$ is equivalent to a lump-sum tax only on the initial generations alive a $t = 0$. Through-out the paper we assume that the initial capital tax $\overline{\theta}_0$ is given.

The government problem is to maximize the social welfare function over the set of implementable allocations. Formally,

$$\max_{\{\{c_t^i, l_t^i\}_{i=1}^I, K_{t+1}\}_{t=0}^\infty} \sum_{t=0}^{\infty} \sum_{i=1}^{I} \omega_{t+1-i} \left[\beta^{i-1} U(c_t^i, l_t^i)\right], \tag{17}$$

$$s.t. \quad \sum_{i=1}^{I} c_t^i + K_{t+1} - (1-\delta)K_t + \overline{G}_t = F(K_t, \sum_{i=1}^{I} \epsilon^i l_t^i) \qquad \forall t, \tag{18}$$

$$\sum_{i=1}^{I} \beta^{i-1} \left(c_{t+i-1}^i U_{c_{t+i-1}^i} + l_{t+i-1}^i U_{l_{t+i-1}^i} \right) = 0 \qquad t \geq 0, \tag{19}$$

$$\sum_{i=s}^{I} \beta^{i-s} \left(c_{i-s}^i U_{c_{i-s}^i} + l_{i-s}^i U_{l_{i-s}^i} \right) = U_{c_0^s} a_0^s \quad s = 2, ..., I, \tag{20}$$

$$\frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}} = ... = \frac{U_{c_t^{I-1}}}{U_{c_{t+1}^I}} \qquad \forall t, \quad i = 1, ..., I, \tag{21}$$

$$\frac{U_{c_t^1}}{U_{l_t^1}} \epsilon^1 = ... = \frac{U_{c_t^I}}{U_{l_t^I}} \epsilon^I \qquad \forall t, \quad i = 1, ..., I. \tag{22}$$

where the initial distribution of wealth $a_{s,0}$, $K_0 = \overline{K} > 0$ and $\{G_t\}_{t=0}^\infty$ are given and $c_t^i \geq 0$, $l_t^i \in [0, 1]$.

The allocation x that solves the government problem is constrained efficient, in the sense that there exists no other constrained efficient allocation x' belonging to the set of implementable allocations that dominates the optimal.

To solve the government problem we consider a relaxed version of the problem with the constraints (21) and (22) dropped. If the solution of the relaxed version satisfies the constraints dropped, then it solves the original problem. Then, we look for a sufficient condition on preferences that implies zero capital taxes and also satisfies the additional constraints (21) and (22). To derive the first-order conditions, it is useful to redefine the Lagrangian by introducing the implementability constraint on it. Let μ_{t-i} be the Lagrange multiplier of the implementability constraint[7] for the agent born in period $t-i$. Then let's define

$$W(c_t^i, l_t^i, \mu_{t-i}) = U(c_t^i, l_t^i) + \mu_{t-i}(c_t^i U_{c_t^i} + l_t^i U_{l_t^i}). \tag{23}$$

the additional term measures the effect of distortionary taxes on the utility function. In particular, it captures the effect of the distortion on the marginal rate of substitution. If the implementability constraint binds, the first-order conditions of the consumer problem are distorted unless the term in the parenthesis is equal to zero. With the new notation the government problem is given by

$$\max_{\{\{c_t^i, l_t^i\}_{i=1}^I, K_{t+1}\}_{t=0}^\infty} \sum\nolimits_{t=0}^{\infty} \sum\nolimits_{i=1}^{I} \omega_{t+1-i} \left[\beta^{i-1} W(c_t^i, l_t^i, \mu_{t-i})\right] - \sum\nolimits_{s=2}^{I} \mu_{1-s} U_{c_0^s} a_0^s, \tag{24}$$

$$s.t. \quad \sum\nolimits_{i=1}^{I} c_t^i + K_{t+1} - (1-\delta)K_t + G_t = F(K_t, \sum\nolimits_{i=1}^{I} \epsilon^i l_t^i) \qquad \forall t. \tag{25}$$

Let ψ_t be the Lagrange multiplier of the resource constraint, then, the first-order necessary conditions for an interior solution at $t > 0$ are

$$\begin{array}{lrl} [c_t^i] & \omega_{t+1-i}\beta^{i-1} W_{c_t^i} - \psi_t & = 0, \\ [c_t^{i+1}] & \omega_{t-i}\beta^{i} W_{c_t^{i+1}} - \psi_t & = 0, \\ [l_t^i] & \omega_{t+1-i}\beta^{i-1} W_{l_t^i} + \psi_t F_{Lt}\epsilon^i & = 0, \\ [K_{t+1}] & -\psi_t + \psi_{t+1}(1-\delta+F_{Kt+1}) & = 0, \end{array}$$

together with the period resource constraint (25) and the transversality condition for the optimal capital path

$$\lim_{t\to\infty} \psi_t K_{t+1} = 0. \tag{26}$$

Throughout the paper we assume that the solution of the Ramsey allocation problem exists and that the time paths of the solutions converge to a steady state. Neither of these assumptions is innocuous. The sufficient conditions for an optimum involve third derivatives of the utility function. Therefore, the solutions might not represent a maximum, or the system might not have a solution because there does not exist a feasible policy that satisfies the intertemporal government budget constraint. However, assuming that the solution to the government problem exists and is interior, it will satisfy the above first-order conditions. Hence, the optimal taxation analysis will apply to these cases only. Rearranging terms, we have

$$\omega_{t+1-i} W_{c_t^i} = \omega_{t+2-i} W_{c_{t+1}^i} (1-\delta+F_{Kt+1}) \qquad \forall i, t, \tag{27}$$

$$\frac{W_{l_t^i}}{W_{c_t^i}} = -F_{Lt}\epsilon^i \qquad \forall i, t, \tag{28}$$

[7] If the government has access to lump-sum taxes, the implementability constraint will not be binding, $\eta_{t-i} = 0$, and it will not be optimal to use distortionary taxes and the economy would achieve a full efficient allocation.

$$W_{c_t^i} = \frac{\omega_{t-i}}{\omega_{t+1-i}} \beta W_{c_t^{i+1}}, \qquad \forall i, t. \tag{29}$$

Notice that Equation (27) is slightly different from the consumer Euler equation. The government equates the derivative of the objective function of a newborn generation at different times. Equation (28) is the intratemporal condition between consumption and labor, that determines the amount of effective hours worked by each generation at a given period t. Finally equation (29) is the static redistributive condition, and implies that the government will assign consumption among two different generations according to the ratio of their relative weights. This condition does not appear in the equilibrium conditions for the private sector, but it is very useful to derive the optimal capital taxes. Updating (29) one period and substituting in (27) we obtain

$$W_{c_t^i} = \beta W_{c_{t+1}^{i+1}} (1 - \delta + F_{Kt+1}) \qquad \forall i, t, \tag{30}$$

This new expression resembles a life-cycle Euler equation, but instead of having the marginal utility with respect to consumption it involves the derivative of $W(\cdot)$ with respect to consumption. For the initial s generations at $t = 0$, the first-order necessary conditions incorporate the initial distribution of asset holdings. Formally,

$$\frac{W_{l_0^s} - \mu_{1-s} \left[U_{c_0^s l_0^s} \left((1 + F_{K_0}(1 - \overline{\theta}_0) a_0^s \right) + U_{c_0^s} F_{K l_0^s} (1 - \overline{\theta}_0) a_0^s \right]}{W_{c_0^s} - \mu_{1-s} U_{c_0^s c_0^s} \left((1 + F_{K_0}(1 - \overline{\theta}_0) a_0^s \right)} = -F_{L_0} \epsilon^s, \tag{31}$$

$$\frac{W_{c_0^s} - \mu_{1-s} U_{c_0^s c_0^s} \left((1 + F_{K_0}(1 - \overline{\theta}_0) a_0^s \right)}{W_{c_0^{s+1}} - \mu_{2-s} U_{c_0^{s+1} c_0^{s+1}} \left((1 + F_{K_0}(1 - \overline{\theta}_0) a_0^{s+1} \right)} = \beta \frac{\omega_{s+1}}{\omega_s}. \tag{32}$$

To derive the optimal tax policy π^* we have to substitute the allocations obtained from the government problem into the private sector equilibrium conditions. The resulting expressions for the optimal capital and labor tax rate are

$$\theta_{t+1}^* = \frac{1}{\beta r_{t+1}} \left[\frac{W_{c_t^i}}{W_{c_{t+1}^{i+1}}} - \frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}} \right] \qquad \forall t. \tag{33}$$

$$\tau_t^* = 1 - \frac{U_{c_t^i}}{U_{l_t^i}} \frac{W_{l_t^i}}{W_{c_t^i}} \qquad \forall t. \tag{34}$$

If we drop the age subscripts from the first-order conditions of the government problem, the associated expressions for the optimal tax policy are equivalent to the ones obtained in an infinite-lived consumer economy. Judd (1985, 1999) and Chamley (1986) prove two important results for this class of economies. First, for a general class of utility functions capital taxes should be zero in the long run (consumption is constant, therefore $U_{c_t} = U_{c_{t+1}}$). Second, for a particular class of functions, that satisfy $W_{c_t}/W_{c_{t+1}} = U_{c_t}/U_{c_{t+1}}$, the optimal capital income taxes are zero after a finite number of periods. The conditions for the zero capital tax result in the transition path are generally viewed as an application of the uniform commodity taxation principle (see Atkinson and Stiglitz (1980)), that specifies conditions under which taxing all goods at the same rate is optimal.

In overlapping generations economies if the government cannot use age-dependent taxes the previous results are not generally true. Notice that the marginal utility is not constant over the life-cycle, even in steady state. Hence, we cannot expect zero taxes on capital returns unless two conditions are satisfied. First, for a given utility function the ratio $W_{c_t^i}/W_{c_{t+1}^{i+1}}$ must be equal to the solution of the private sector equilibrium $U_{c_t^i}/U_{c_{t+1}^{i+1}}$. Second, given that we are solving a relaxed version of the government problem, we have to ensure that constraints (21) and (22) are satisfied as well.[8] The next proposition provides a sufficient condition on the consumer utility function that guarantees the zero capital income tax result. Unfortunately, most of the preferences do not satisfy the condition.

Proposition 2: *When the government cannot condition taxes on age, that is taxes are age-independent, and preferences satisfy*

$$\frac{c_t^i U_{cc_t^i} + l_t^i U_{lc_t^i}}{U_{c_t^i}} = \frac{l_t^i U_{ll_t^i} + c_t^i U_{cl_t^i}}{U_{l_t^i}} \qquad \forall t > 1 \tag{35}$$

then, the optimal capital and labor income tax are zero for $t \geq 2$, *providing that the present value government budget constraint is satisfied.*

Proof. We need to show two results under this condition. First, that the optimal capital income tax is zero from $t \geq 2$. Second, that if preferences satisfy this property, then the solution of the less constrained government problem is also a solution to the more constrained problem. We proceed by rewriting condition (35) as follows

$$c_t^i U_{cc_t^i} + l_t^i U_{lc_t^i} = A U_{c_t^i}, \tag{36}$$

$$l_t^i U_{ll_t^i} + c_t^i U_{cl_t^i} = A U_{l_t^i}. \tag{37}$$

where A is a constant. Now let's consider the first-order conditions of the government problem with respect to c_t^i

$$(1+\eta_{t-i})U_{c_t^i} + \mu_{t-i}\left[c_t^i U_{cc_t^i} + l_t^i U_{lc_t^i}\right] = \psi_t,$$

where μ_{t-i} and ψ_t denote the Lagrange multipliers of the implementability constraint of a generation born at period $t-i$ and the period t resource constraint respectively. Substituting Equation (36) in the first order conditions, we can rewrite the expression as

$$U_{c_t^i}(1 + \mu_{t-i}(1+A)) = \psi_t, \tag{38}$$

[8]Clearly, if the first condition is satisfied, the set of constraint (21) is also satisfied but it does not imply that constraints (22) are satisfied too. Hence, if the government cannot condition labor income taxes on age, the restrictions on the set of instruments have an important impact on the sufficient conditions for the zero capital income tax result.

since this equations holds for time t and $t+1$, then we combine the equation with the first-order condition with respect to capital and obtain

$$\frac{U_{c_t^i}}{\beta U_{c_{t+1}^{i+1}}} = 1 + F_{Kt+1} - \delta.$$

Clearly all consumers face the same prices, hence constraint (21) is satisfied if the utility function satisfies (36).[9] Now we want to show that condition (36) together with condition (37) are sufficient to ensure that the solution of the less constrained problem is a solution of the more constrained problem. Using the same argument we have

$$U_{l_t^i}(1 + \mu_{t-i}(1 + A)) = -\psi_t F_{L_t} \epsilon^i. \tag{39}$$

Since Equations (38) and (39) hold for all generations at a given period t, then the marginal rates of substitution between consumption and labor are equal across generations. So, constraint (22) is also satisfied.

At $t=1$, capital and labor taxes are different from zero because the implementability constraints of the initial generations include the initial distribution of capital stock that prevents capital taxes from being zero. At the initial period $t=0$ the capital taxes are given, $\overline{\theta}_0$. Under this policy the government only collects taxes at $t=0,1$, but it is able to affect the cumulative discount rate, with the initial taxes on capital. Given that we have not imposed any bound on the capital tax rate $\theta \leq 1$, the optimal tax during these periods is effectively taxing the wealth of the initial generations alive at $t=1$. ■

Imposing bounds on the optimal tax rate modifies the result by having some periods with capital income taxation. Nevertheless, the important result is that in general most preferences defined over consumption and leisure as

$$U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} + v(1-l) \tag{40}$$

or

$$U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} v(1-l) \tag{41}$$

do not satisfy the sufficient condition. In these two cases the optimal policy implies non-zero capital income tax. However, in the next section we show that preferences that do not satisfy the sufficient conditions can be consistent with zero capital income taxation for some plausible parameter values.

It is important to remark that l denotes hours worked and $(1-l)$ is leisure. If we redefine the utility function to depend on leisure l and $(1-l)$ hours worked, then the associated

[9]This condition is sufficient to ensure zero capital taxes in an infinitely-lived consumers model, but in these types of economies it does not guarantee that the additional constraints on the marginal rates of substitution between consumption and leisure are satisfied.

implementability constraint has to be modified to include $(1-l)U_l$ instead of lU_l. This modification does not change the previous results.

Inspection of the sufficient condition gives some insight into the requirements for the non-zero capital tax. In a sense this condition requires a constant elasticity of the marginal utility. This condition cannot be satisfied by preferences that are non-homothetic with respect to labor. Consider utility functions $U(c,l)$ where c and l are homothetic. We then have

Proposition 3: *If the utility function is homothetic with respect to consumption and hours worked, then the steady state capital tax is zero.*

Proof. This class of preferences satisfies

$$cU_{cc} + lU_{lc} = AU_c, \tag{42}$$

$$lU_{ll} + c_t^i U_{cl} = BU_l. \tag{43}$$

where A and B are different constants. Next, we prove that under these assumptions the optimal labor tax satisfies the additional constraint (21) in steady state. In this case, the first-order conditions of the government problem are

$$\frac{(1+\mu)U_{c^i} + \mu\left[c^i U_{cc^i} + l^i U_{lc^i}\right]}{(1+\mu)U_{l^i} + \mu\left[l^i U_{ll^i} + c^i U_{cl^i}\right]} = \frac{1}{F_L \epsilon^i}, \tag{44}$$

where μ, the Lagrange multiplier of the implementability constraint, is constant.[10] All newborn generations face the same prices and taxes over the life-cycle. Substituting Equation (42) and (43) into the first-order conditions, we can rewrite Equation (44) as

$$\frac{U_{c^i}}{U_{l^i}} = \left[\frac{(1+\mu(1+B))}{(1+\mu(1+A))}\right]\frac{1}{w\epsilon^i}, \tag{45}$$

where $1-\tau = (1+\mu(1+A))/(1+\mu(1+B))$ is the optimal labor tax. Since this expression holds across generations, the additional constraint (22) is satisfied. Outside the steady state, the Lagrange multipliers of the implementability constraints are not constant across generations. ■

An example of utility function that satisfies this property is

$$U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\phi}}{1+\phi} \tag{46}$$

where $A = -\sigma$ and $B = \phi$, so for a positive Lagrange multiplier, the optimal labor tax is positive.

In general, the conditions under which capital income taxes are zero are viewed as an application of the uniform commodity tax result. In overlapping generation economies, if

[10] The Lagrange multipliers have been previously normalized by the government discount rate. If the economy converges to a steady state this normalization requires $\psi_t \lambda^t$.

labor is non-homothetic and the government cannot use age-dependent taxes, we cannot expect capital taxes to be zero. From the government perspective labor supply of different generations is viewed as a different commodity, so it has an incentive to tax it when it is relatively more inelastic. When taxes cannot be conditioned on age, the government uses a non-zero capital tax as an indirect instrument that can be used to tax leisure. In general, there is no reason to expect optimal uniform labor income taxes. Even when leisure is homothetic, it is optimal to tax it indirectly through non-uniform taxation. In an overlapping generations model, the distortions associated with this policy are not that important because for a given generation today's consumption and period T consumption are not perfectly substitutable. Hence the effect of capital distortions is much smaller and not necessarily bigger than distortions caused by other taxes like labor income taxes.

We believe that these results improve the existing literature in several ways. First, it considers a general model where individuals live I periods, and analyzes the optimal policy on the transition path. Second, the results show that policy analysis using two period OLG economies or conditioning taxes on age can generate misleading results. When the government cannot condition taxes on age, the additional constraints that this restriction imposes in the set of tax instruments plays an important role in the determination of the optimal policy. In the numerical simulations it will be clear that these additional constraints lead to different optimal policies. However, if generations live only two periods or taxes can be conditioned on age, preferences that satisfy the uniform commodity tax result imply zero optimal capital income taxes both along the transition path and in steady state. We summarize this finding in the next proposition,

Proposition 4: *The uniform commodity tax condition* (36) *is a sufficient condition to ensure zero capital taxes in steady state and also along the transition path from period* $t \geq 2$ *if:*

1) The government has access to age-dependent taxes, or

2) generations live two periods and the old does not supply labor.

Proof. If the government can use age-specific taxes, then the optimal taxation problem ignores constraints (21) and (22). In this case, utility functions of the form (40) and (41) imply zero capital taxes from period two onwards together with age-dependent labor taxes. ■

Equivalently, two period OLG economies where the old generation does not supply labor explicitly assumes away constraints (21) and (22). Usually, the young generation supplies labor in the market while the old generation supplies capital.[11]

[11] One might think that one way to get around the non-zero capital tax result is to consider inelastic labor supply. With inelastic labor supply the government can ignore the constraints (22). Next we show in a simple two period model that this intuition is not correct. The implementability constraint associated with the problem would be $c_{1t}U_{c1t} + \beta c_{2t+1}U_{c2t+1} = U_{c1t}w_t$. From the firm problem we know that $w_t =$

Next, we want to explore, as Judd (1999) suggested, if the model can justify the observed rates of capital income for an empirically reasonable intertemporal utility function and a robust demographic structure.

4 Quantitative Results

4.1 Parameterization

This section describes the choice of the functional forms for the numerical simulations and the parameterization process. The functional forms are chosen to have comparable results with Chari and Kehoe (1999). The utility function is

$$U(c,l) = \frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma},$$

where $\gamma \in (0,1)$ is the consumption share in the utility function and σ denotes the inverse of the intertemporal elasticity of substitution. In the benchmark economy σ is set equal to 1 as in Chari and Kehoe (1999) that is a logarithmic utility function. Clearly, this utility function does not satisfy the sufficient conditions for zero capital income taxes. The technology is a constant returns to scale Cobb-Douglas production function, $F(K,N) = K^{\alpha}N^{1-\alpha}$.

A period in the model is one year, and we assume that agents live 59 periods. Hence, the model can be interpreted as one in which individuals are born economically at age 20 and live up to a maximum of 79 years. The empirical evidence shows that hours worked are not constant over the life-cycle. Ghez and Becker (1975) and Juster and Stafford (1991) find that households allocate one third of their discretionary time in market activities. Setting $\gamma = 0.4$ in the model implies that individuals work an average of 33% of their time endowment over the life-cycle. In the benchmark economy, the intertemporal elasticity of substitution for consumption is set equal to one, $\sigma = 1$, (i.e., this is equivalent to using logarithmic preferences). The discount factor β is chosen together with the intertemporal elasticity of substitution to match the observed capital/output ratio of the economy. Setting $\beta = 0.99$ matches the observed average capital/output ratio of 2.4 for the market economy. The labor earnings age profile $\{\epsilon_j\}$ is derived using PSID data. In the model, labor services are homogeneous, so there is a single wage per efficiency unit of labor. Hence, $\{\epsilon_j\}$ is chosen to match the age profile of average wages in the cross-section of US data.

In the technology, the estimate of $\alpha = 0.33$ comes from the computation of capital's share in national income, which includes durables as a part of the capital stock. The aggregate

$f(k_t) - k_t f'(k_t)$. Given that the government problem can only depend on allocations, the IC should be written as $c_{1t}U_{c1t} + \beta c_{2t+1}U_{c2t+1} = U_{c1t}(f(k_t) - k_t f'(k_t))$. Then, even if preferences satisfy the uniform commodity taxation condition the implied capital income taxes are different from zero, unless the effective lump-sum labor tax is sufficient to finance government expenditure. In this case, obviously the optimal capital income tax is zero.

depreciation rate, δ, depends on the aggregate investment/output ratio. Given that we do not explicitly include growth this number must be larger to match investment. A value of $\delta = 0.08$ matches the observed average investment-output ratio of 16.1%. The capital/output ratio together with the depreciation rate imply a gross interest rate of 5.6%. The government consumption is set to 19%, which corresponds to the average of the last decade. In determining taxation on factor earnings, we follow the methodology of Mendoza, Tesar and Razin (1994). They develop consistent measures of the effective tax rate on factors' income for OECD countries. The competitive economy is calibrated using the last decade average tax rates, which imply a capital income tax of 35% and a labor income tax of 24%. Given G/Y and the tax policy π, the ratio D/Y is endogenously determined by the model. The resulting debt/output ratio in the market economy is 24%. This figure is roughly consistent with the average observed in the last few decades, which is 23%.

4.2 Findings

In this section we start by computing the initial steady state equilibrium for the market economy. Then, we assume that the government implements the optimal policy and the economy converges to a new steady state.[12] We then quantify the optimal capital and labor income taxes for two different tax regimes: age-independent and age-dependent taxes. To compute the optimal policy we need to specify a value for the relative weight that the government places between present and future generations. This parameter has no counterpart in the market economy and the quantitative results can drastically change for different values. We then choose to compare the results for a broad range of values for this parameter.

Table 1 presents a summary of the steady state results obtained for the case where the government cannot condition taxes on age. The first row displays the policy used to calculate the market equilibrium in the benchmark economy. There are several important quantitative features of the optimal policy. The parameterized model can justify the observed tax rates for capital and labor income for some plausible values of the government discount rate. In particular for values of λ that are consistent with the observed capital/output ratio, the implied optimal policy predicted by the model is very similar to the benchmark taxation displayed in the first row. Changes in λ have important redistributive effects because the government changes its relative weight between the young and old generations. If the government increases λ it lowers the gross interest rate of the economy in steady state (given by $\frac{1}{\lambda} = 1 - \delta + \alpha Y/K$) and increases the capital/output ratio. The optimal tax on capital returns is inversely related with the government discount factor. For some parameter value, $\lambda = 0.97$, the optimal capital income tax is roughly zero, and for higher values the government

[12]A feature of this model is that if the economy converges to the steady state, then it has the modified golden rule property and it is independent of the initial conditions (see Escolano (1992)).

subsidizes investment.[13] The optimal level of debt is also inversely related with λ. For some values of λ the government owns part of the capital stock and lends to firms.

Another parameter of importance for the optimal capital-income tax is the intertemporal elasticity of substitution. Table 2 summarizes the results for different values of σ when the government discount rate is set to $\lambda = 0.947$ matching the average capital/output ratio of 2.4 for the market economy. We observe that changes in the intertemporal elasticity of substitution have important effects on the optimal capital income tax. As we increase σ, households have a higher preference for consumption smoothing, and in order to reduce the distortions the government responds by lowering the optimal capital income tax. We find that the labor income tax does not respond much in comparison with the capital income tax. This result comes from the fact that the consumption/leisure decision does not depend directly on σ. We thus conclude that if the government cannot condition taxes on age the optimal policy can justify the observed tax rate for some plausible choice of parameter values. Nevertheless, for some values of λ or σ, the optimal policy implies zero capital income taxes.

Next, we analyze the optimal policy when the government can condition taxes on age (age-dependent taxation). Formally, that implies dropping constraints (21) and (22) from the government problem. Figure I shows the optimal tax policy (capital and labor income taxes) with the benchmark elasticity of substitution for consumption ($\sigma = 1$) and the same average capital/output ratio of 2.4 ($\lambda = 0.947$). This choice of parameter for the government weight is convenient because the optimal policy mainly redistributes and the efficiency gains are minimal. Thus, allowing comparisons with the findings of Escolano (1992). For the benchmark case, the age-specific capital tax is constant across households and equal to zero. This result is consistent with Proposition 3 because the utility function satisfies the uniform commodity tax result.[14] Since consumption and leisure move together over the life-cycle, the government has an incentive to tax labor when it is more inelastic. In the benchmark case, that clearly occurs at the early stage of the life-cycle when households are accumulating wealth for retirement. With a flat profile of efficiency units, we would observe a decreasing labor income tax over the life-cycle. The hump in the distribution of labor taxes occurs at the ages where the efficiency units of labor over the life-cycle exhibit a hump. For this class of preferences, changes in the government discount rate do not affect the optimal tax on capital returns. Then, it is important to remark that the zero capital income tax result is obtained independently of the relative weight that the government places on present and future generations. Changes in λ only affect the distribution of labor income taxes mainly due to the effect on the relative prices. The level of debt is adjusted to satisfy the desired

[13]Escolano (1992) also found that for some values of the government discount factor the optimal tax on capital returns is zero.

[14]In this case the additional conditions that restrict the marginal rates of substitution between consumption today and tomorrow are not binding. In order to satisfy the sufficient conditions for zero capital taxation we must allow labor taxes to differ across agents.

capital/output ratio. For higher values of the intertemporal elasticity of substitution the optimal capital tax across generations is different from zero. Figure 2 displays the optimal age-specific taxes for different values of σ. With non-separable preferences the optimal capital tax across ages depends on borrowing/savings behavior. For young households that borrow against their future income, the government charges a negative tax increasing the cost. For savers, the government taxes a positive tax on capital. For these particular functional forms, an increase in σ does not substantially affect the distribution of capital taxes across generations, but it lowers the labor-specific taxes for all ages.

5 Conclusions

This paper explores the proposition that the optimal capital income tax is zero. In contrast with previous studies, we consider an overlapping generations version of the neoclassical growth model to analyze the optimal fiscal policy along the transition path to a long-run steady state. In this context, we provide sufficient conditions for the zero capital income tax result and we show that it is very difficult to obtain zero optimal capital taxes if the government cannot condition taxes on age. When the government cannot condition taxes on age, the additional constraints that this restriction imposes in the set of tax instruments plays an important role in the determination of the optimal policy. However, we find that the uniform commodity tax result is a sufficient condition to ensure zero optimal capital taxes if either the government can condition taxes on age, or generations live two periods and the old does not supply labor.

For a version of the model calibrated to the US economy we find that the model could justify the observed rates of capital income taxation for some plausible choice of parameters and functional forms. These results answer Judd's (1999) suggestion that further work is needed to see the robustness of the optimality of zero capital income taxes in overlapping generations models with realistic demographic specifications and empirically reasonable intertemporal utility functions. The general result shows that intergenerational heterogeneity can alter the basic results and generate a non-zero capital income tax either in the transition path or in the long-run.

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7 Appendix

Proof of Proposition 1: We first proceed by showing that the allocations in a competitive equilibrium must satisfy (12), (13), (14), (15), and (16). Condition (12) is straightforward from substituting the labor market clearing condition (4) into (6).

The implementability constraint for each generation is constructed by substituting the consumer first-order conditions

$$\frac{U_{l_t^i}}{U_{c_t^i}} = \frac{(1-\tau_t)w_t\epsilon^i}{(1+\eta_t)} \qquad \forall t, i, \tag{47}$$

$$\frac{U_{c_t^i}}{(1+\eta_t)} = \frac{\beta U_{c_{t+1}^{i+1}}}{(1+\eta_{t+1})}(1+r_{t+1}(1-\theta_{t+1})) \qquad \forall t, i. \tag{48}$$

into the intertemporal budget constraint

$$\sum\nolimits_{i=0}^{I-1} p_{t+i}(1+\eta_{t+i})c_{t+i}^i \leq \sum\nolimits_{i=0}^{I-1} p_{t+i}(1-\tau_{t+i})w_{t+i}\epsilon^i l_{t+i}^i, \tag{49}$$

where p_{t+i} denotes the Arrow-Debreu price for the consumption good at period $t+i$. We then use the definition of p_{t+i} to substitute for the interest rate in the intertemporal condition. For the initial generations in the economy at time $t=0$ the distribution of asset holdings appears on the right hand side of the the budget constraint. That explains the additional term (14) on the implementability constraint. If the government is restricted to use the same proportional taxes for all generations the set of implementable allocations needs to include constraints (15), and (16).

Now we prove the second part of Proposition 1. Now we prove that given an allocation x that satisfies the previous conditions, it is possible to construct a sequence of prices and a policy π that together with the allocation constitute a competitive equilibrium. From the aggregate capital stock, K_t, and the aggregate labor supply, L_t, we construct the relative prices using the firm's first-order conditions (1) and (2). To derive the government policy $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^{\infty}$, we substitute the allocations $\{\{c_t^i, l_t^i\}_{i=1}^{I}\}_{t=0}^{\infty}$ and the equilibrium prices $\{r_t, w_t,\}_{t=0}^{\infty}$ into households' first-order conditions (47) and (48). To obtain, the intertemporal budget constraint for the households we have to substitute $U_{c_t^i}$ and $U_{l_t^i}$ into (13), (14). All these conditions determine a system of equations from where we obtain π. The sequence of government debt $\{D_t\}_{t=0}^{\infty}$ is adjusted to satisfy the market equilibrium capital/output ratio

$$D_{t+1} = \sum\nolimits_{i=1}^{I} a_{t+1}^i - K_{t+1}. \tag{50}$$

Finally, combing feasibility and the households budget constraint for a given period the government budget constraint also has to be satisfied. For a given period t, add up all the generations budget constraint:

$$\sum_{i=1}^{I}(1+\eta_t)c_t^i - \sum_{i=1}^{I}(1-\tau_{t+i})w_t\epsilon^i l_t^i \leq \sum_{i=1}^{I}(1+r_t(1-\theta_t))a_t^i - \sum_{i=1}^{I} a_{t+1}^{i+1},$$

aggregating variables and using the market clearing condition in the capital market we obtain

$$\eta_t C_t + \tau_t w_t L_t + \theta_t r_t K_t + R_t B_t + B_{t+1} \leq F_{L_t} L_t + F_{K_t} K_t - C_t - K_{t+1} + (1-\delta)K_t,$$

Combining the previous expression with the resource constraint we obtain the period government budget constraint. ■

Proof of Corollary 1: We want to show how different tax policies can be consistent with the same allocation x. Clearly, the relative prices $\{r_t, w_t\}$ and the resource constraint depend uniquely on the allocation x. The two policies π and π' satisfy

$$\frac{(1+\eta_t)}{(1-\tau_t)} = \frac{(1+\eta_t')}{(1-\tau_t')} \qquad \forall t, i, \tag{51}$$

and,

$$\frac{(1+r_{t+1}(1-\theta_t))(1+\eta_t)}{(1+\eta_{t+1})} = \frac{(1+r_{t+1}(1-\theta_t'))(1+\eta_t')}{(1+\eta_{t+1}')} \qquad \forall t, i, \tag{52}$$

because the relative prices haven't change. Substituting (51) and (52) in the households' budget constraint:

$$(1+\eta_t)c_t^i - \frac{(1-\tau_t')(1+\eta_t)}{(1+\eta_t')}w_t\epsilon^i l_t^i = (1+r_t(1-\theta_t'))\frac{(1+\eta_{t-1}')(1+\eta_t)}{(1+\eta_t')(1+\eta_{t-1})}a_t^i - a_{t+1}^{i+1}, \tag{53}$$

and multiplying in both sides by $(1+\eta_t')/(1+\eta_t)$:

$$(1+\eta_t)c_t^i - (1-\tau_t')w_t\epsilon^i l_t^i = (1+r_t(1-\theta_t'))\widetilde{a}_t^i - \widetilde{a}_{t+1}^{i+1} \qquad \forall t, i,$$

where $\widetilde{a}_t^i$ and $\widetilde{a}_{t+1}^{i+1}$ are the equivalent distribution of asset holdings:

$$\widetilde{a}_t^i = \frac{(1+\eta_{t-1}')}{(1+\eta_{t-1})}\widetilde{a}_t^i \qquad \forall t, i. \tag{54}$$

■

Table 1: Optimal Fiscal Policy (Case $\sigma = 1$)

Market Economy (Benchmark)

Taxes		Net Interest		
Capital	**Labor**	**Rate**	**Debt/GDP**	**Capital/GDP**
35.0%	24.0%	3.6%	24.0%	2.4

Optimal Fiscal Policy for Different Government Weights (λ)

	Taxes		Net Interest		
λ	**Capital**	**Labor**	**Rate**	**Debt/GDP**	**Capital/GDP**
0.93	58.1%	17.9%	3.1%	18.1%	2.2
0.94	49.2%	19.8%	3.2%	12.8%	2.3
0.947*	40.8%	21.4%	3.3%	8.7%	2.4
0.95	36.6%	22.1%	3.3%	6.8%	2.5
0.96	17.7%	24.7%	3.4%	-0.3%	2.7
0.97	0.3%	19.3%	3.1%	-30.1%	3.0
0.98	-6.5%	10.4%	2.2%	-91.5%	3.3

Source: Author calculations

(*) Discount rate that ensures the same K/Y ratio as the benchmark economy

Table 2: Optimal Fiscal Policy
Sensitivity analysis (Case λ =0.947)

	Taxes		**Net Interest**	
σ	**Capital**	**Labor**	**Rate**	**Debt/GDP**
1.0	40.8%	21.8%	3.3%	8.7%
1.5	23.0%	27.0%	4.3%	9.5%
2.0	8.9%	27.9%	5.1%	2.6%
2.5	3.4%	22.2%	5.4%	-9.7%
3.0	-1.2%	17.7%	5.6%	-18.8%
4.0	-6.9%	10.2%	6.0%	-32.2%

Source: Author calculations

Figure I: Capital and labor age-dependent taxes (Case $\sigma = 1$)

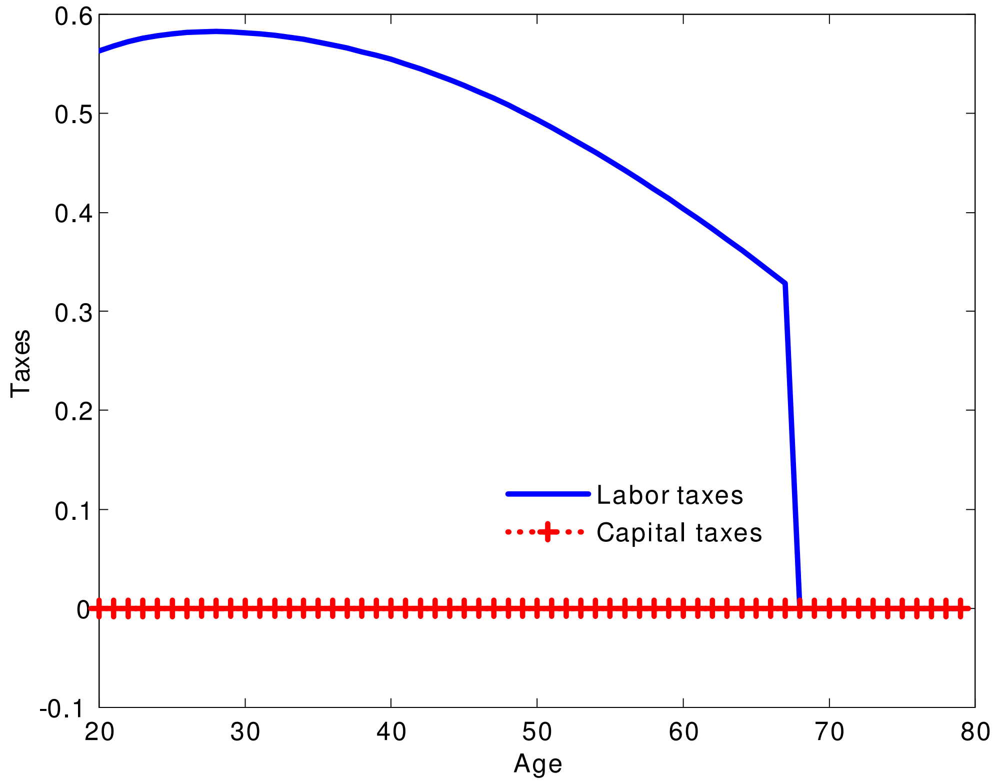
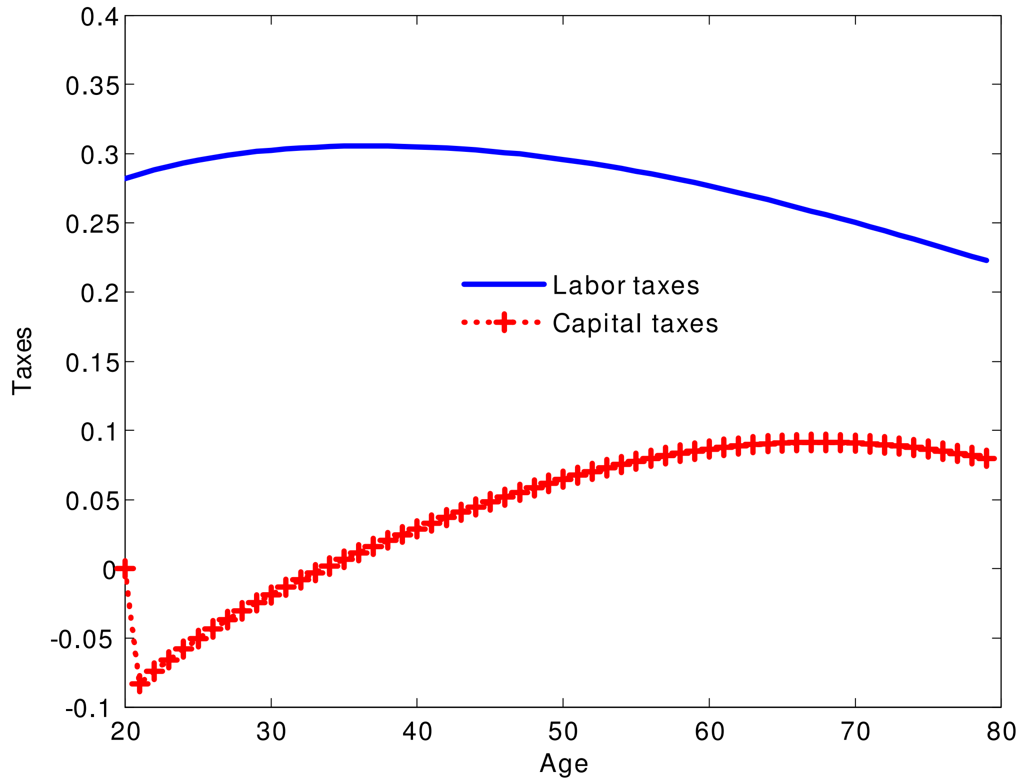


Figure II: Sensitivity analysis

Case $\sigma = 2$



Case $\sigma = 3$