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Authors	Jie (April) Cai, Nan Li, and Ana Maria Santacreu
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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

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Knowledge Diffusion, Trade, and Innovation across Countries and Sectors

Jie Cai, Nan Li & Ana Maria Santacreu*

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Abstract

We provide a unified framework for quantifying the cross-country and cross-sector interactions among trade, innovation, and knowledge diffusion. We study the effect of trade liberalization in an endogenous growth model in which comparative advantage and the stock of knowledge are determined by innovation and diffusion. We calibrate the model to match observed cross-country and cross-sector heterogeneity in production, innovation efficiency and knowledge spillovers. Our counterfactual analysis shows that a reduction in trade costs induces a re-allocation of R&D and comparative advantage across sectors. Heterogeneous knowledge diffusion amplifies the specialization effects of trade-induced R&D re-allocation, becoming an important source of growth and welfare.

Keywords: Knowledge spillovers; R&D; international trade; sectoral linkages

JEL Classification: F12, O33, O41, O47

*Cai, Shanghai University of Finance and Economics, cai.jie@mail.shufe.edu.cn; Li, International Monetary Fund, address: 700 19th Street NW, Washington DC 20431, email: nanli1@gmail.com; Santacreu, Federal Reserve Bank of St. Louis, email: am.santacreu@gmail.com. We thank Jonathan Eaton for many useful conversations. For useful comments we thank Lorenzo Caliendo, Sam Kortum, Andrei Levchenko, Dan Lu, Fernando Parro, Andres Rodriguez-Clare, Esteban Rossi-Hansberg, Alan Taylor, and Dan Trefler, as well as participants of the Federal Reserve Bank of Minneapolis, NBER Economic Growth Small Group meeting, 2019, University of Michigan, UC Davis, Yale University, 2017 and 2016 Society for Economic Dynamics, the 2017 Econometric Society Summer Meeting, IMF, Federal Reserve Bank of St. Louis, Penn State University, RIDGE, Fudan University, Peking University, and Tsinghua University. The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Banks of St. Louis or the Federal Reserve System. The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management.

1 Introduction

The world has increasingly become a highly interconnected network of countries and sectors that exchange not only goods and services but also ideas. Much is understood about how the benefits of trade may spread through production input–output linkages, thanks to the growing literature on sectoral linkages.¹ At the same time, countries and sectors are linked in another way that has not been studied nearly as much—innovation and knowledge spillovers, and how they interact with trade to affect productivity and growth. Knowledge linkages across countries and sectors are far from uniform (Acemoglu, Akcigit, and Kerr, 2016; Cai and Li, 2019). In a world with multiple sectors, trade affects the countries’ knowledge stock and composition, hence productivity. Productivity differences induced by innovation and knowledge spillovers in turn condition the patterns of trade and aggregate growth.²

This paper provides a unified framework to quantify the three-way interactions among trade, innovation, and knowledge diffusion in a multi-sector environment in which country-sectors are interconnected in both the product and knowledge spaces. We build on existing models of trade with production linkages (e.g. Costinot, Donaldson, and Komunjer, 2012; Caliendo and Parro, 2015). Our main contribution is to add dynamics related to innovation and technology diffusion across countries and sectors, thereby providing an endogenous mechanism for cross-country-sector productivity differences.

Recent papers in the literature have modeled productivity dynamics through innovation (Buera and Oberfield, 2019; Perla, Tonetti, and Waugh, 2015) or through innovation and technology diffusion (Sampson, 2019). In contrast to these papers, we emphasize the role of heterogeneous intersectoral linkages which, as we show in the paper, have important effects on income and welfare. In particular, we find that heterogeneous knowledge spillovers, when interacting with cross-country-sector differences in innovation efficiency, amplify significantly the gains from trade in a way that the previous literature has not captured. We provide empirical evidence for this heterogeneity and to study its role in determining innovation, production and trade patterns across countries and sectors in a unified framework.

¹See, for example, Arkolakis, Costinot, and Rodríguez-Clare (2012), Costinot, Donaldson, and Komunjer (2012), Caliendo and Parro (2015), and Baqaee and Farhi (2019).

²A vast empirical literature has documented the significant impact of trade on innovation (e.g., Lileeva and Treffer, 2010; Bustos, 2011; Autor et al., 2016; Aghion et al., 2018). With regard to the reverse relationship, Cameron, Proudman, and Redding (2000) and Santacreu and Zhu (2018) find that innovation and knowledge diffusion help determine trade patterns to the extent that they impact differences in productivity.

Our model consists of two “blocks”: A ‘*trade block*’ that determines the static equilibrium, taking as given country-sector productivity; and a ‘growth block’ in which productivity evolves endogenously through innovation and knowledge diffusion. The trade block builds upon the Ricardian model of trade with Bertrand competition (Bernard et al., 2003). The growth block is modeled in the spirit of Eaton and Kortum (1996, 1999), which analyze an endogenous growth model but do not allow for trade or sectoral linkages.

Firms choose their research effort to create new ideas, while building on the current stock of knowledge. Ideas diffuse across *all* sectors and countries, although the speed of diffusion is heterogeneous.³ In our model, knowledge diffusion increases the stock of knowledge in two ways. First, it increases the knowledge in the receiving country-sector. Second, that increase in turn enhances the innovation efficiency there, fostering creation of more new knowledge. All diffused ideas contribute to the stock of knowledge, but only the highest-quality ideas are adopted for production. Our model differs from recent papers on diffusion (e.g. Santacreu, 2015; Buera and Oberfield, 2019), in that we allow diffusion to take place independently from trade. Hence, we allow for other channels such as multinational production firms (Lind and Ramondo, 2018), FDI (Javorcik, 2004; Fons-Rosen et al., 2017), or migration (Bahar and Rapoport, 2018) to potentially diffuse ideas. Innovation and diffusion determine the distribution of knowledge stock across countries and sectors and, thereby, economic growth. We solve for the model’s balanced growth path (BGP) on which all countries and sectors grow at a common and constant rate. Hence, we abstract from transitional dynamics.⁴

With multiple sectors, trade liberalization induces an endogenous re-allocation of research effort across countries and sectors, increasing aggregate innovation and long-run growth. This approach contrasts with standard one-sector models, in which trade has a negligible effect on innovation and growth because the market size effect exactly offsets the competition effect

³In contrast with the literature on technology adoption, which assumes that only the best knowledge is adopted following diffusion from the economies on the technology frontier (Comin and Hobijn, 2010; Comin and Mestieri, 2018), our assumption is supported by observations that a nonnegligible number of novel inventions are initiated by economies outside the frontier. However, similar to this paper, the above studies also find that the rate at which technologies are diffused differs across countries and is higher in richer countries.

⁴Transitional dynamics in a model with exogenous innovation have been analyzed in Buera and Oberfield (2019) to explain growth miracles. Analyzing transitional dynamics in our model of endogenous innovation with many countries and sectors connected through knowledge spillovers is not a trivial task, as the dimension of the state-space increases rapidly with the number of countries and sectors. This is due to the forward-looking behavior of innovators who receive profits forever, unless a better technology arrives that surpasses the currently used technology.

(Eaton and Kortum, 2006; Atkeson and Burstein, 2010; Buera and Oberfield, 2019). In addition, comparative advantage in production is endogenous, and it results in welfare gains from trade beyond the specialization effects emphasized by static multi-sector models. In this context, heterogeneous knowledge spillovers across country-sectors could, depending on the exact pattern of diffusion, either amplify or dampen the specialization effect of trade-induced R&D re-allocation on growth and welfare. On the one hand, if diffusion forces are stronger among already innovative countries (i.e., if innovative countries diffuse ideas rapidly also among themselves) then the dispersion of comparative advantage resulting from R&D re-allocation could increase, and thus *amplify* the specialization effect. On the other hand, if less innovative country-sectors are better at absorbing knowledge, then knowledge diffusion could actually *dampen* that specialization effect by enabling faster productivity convergence and by making the stock of knowledge more similar across country-sectors. Our estimation of the diffusion pattern suggests that, in the data, the former effect dominates.

We calibrate the model to data on production, bilateral trade, R&D intensity, and patent citations for 19 OECD countries and 19 sectors (including a nontradable sector). Our main empirical contribution is the use of patent citations and R&D expenditure data to estimate key parameters related to knowledge spillovers and innovation. In particular, we calibrate the speed of cross-country and cross-sector knowledge spillovers by fitting a citation function that depends, among other factors, on the diffusion lag.⁵ There are two key advantages to this approach. First, it allows for patents in different country-sectors to vary not only in terms of their diffusion speed but also in terms of their obsolescence rates and their ability to generate spillovers. Second, it does not require us to assume that citations are mapped one-to-one onto knowledge spillovers. Because the diffusion speed parameters are estimated jointly with other parameters that also govern the citation process, this procedure helps us obtain a more accurate estimate of the parameters of interest: diffusion speed across countries and sectors.

The innovation parameters are calibrated by jointly solving the two blocks of the model. In the model’s trade block, we use an algorithm based on excess demand iterations to solve for the static trade equilibrium for a given distribution of productivity; in the growth block, a fixed-point algorithm is used to solve for the endogenous growth rate and average pro-

⁵Our method extends the approach proposed in Caballero and Jaffe (1993) to a multi-country, multi-sector environment.

ductivity. This algorithm helps us determine the exogenous cross-country and cross-sector efficiency of innovation, the elasticity of innovation with respect to research effort, and the economy’s stock of knowledge on the BGP. Our calibrated model performs well at matching untargeted data such as wages, income per capita and country-sector relative productivity. Furthermore, we can use our calibrated model to estimate the contribution of domestic and foreign sources of productivity. In more innovative countries, domestic innovation contributes more to productivity than in less innovative countries. However, the correlation between the rate of innovation of a country and the contribution of domestic innovation to productivity in that country is only 0.47, as innovative countries also benefit more from innovations developed elsewhere, which increases the importance of foreign innovation from those countries. We also find that domestic R&D intensity can explain around 27% of the variation in productivity across countries, which is consistent with other findings in the literature (see Sampson, 2019).

We conduct a counterfactual exercise to study the effect of a 25% uniform reduction in trade costs on innovation, comparative advantage, and growth along the BGP in order to quantify the mechanisms at work. Changes in trade costs have a nonnegligible effect on innovation in our model, since there is a re-allocation of R&D towards sectors in which the country has a comparative advantage. In particular, we find that after trade liberalization, research efforts are directed more into country-sectors with a comparative advantage in production, consistent with suggestive empirical evidence presented in the Appendix. Our estimated speed of diffusion implies that innovative countries are relatively faster at absorbing and diffusing knowledge from and to each other. Thus the presence of heterogeneous knowledge spillovers, coupled with research effort re-allocation, leads to greater dispersion in endogenous comparative advantage—both across countries for a given sector, and across sectors within a country. Productivity grows at a higher rate in the counterfactual BGP, increasing from 2.8% to 3.4%. These results have implications for welfare. We find that all countries experience positive welfare gains from lowering trade costs, although the size of those gains ranges from 8% in Japan to 34% in Germany.

Next, we study the role of the different model elements by considering four alternative versions of our model: (i) a static model in which productivity is taken as exogenously given, (ii) a dynamic model without input-output linkages in production, (iii) a dynamic model with homogeneous knowledge spillovers across countries and sectors, and (iv) a dynamic model

with almost negligible knowledge spillovers across countries and sectors. We recalibrate each version to match the same moments of the data.⁶ We show that the static and dynamic model without input-output linkages perform similarly in terms of welfare: Changes in trade costs have a small effect on innovation and growth, and on the dispersion of productivity. As a result, welfare gains are smaller and less disperse than in our baseline model.

More importantly, we establish that the presence of *heterogeneous* knowledge spillovers plays a key role in how trade affects innovation, growth and welfare. A model with either homogeneous or negligible knowledge spillovers cannot match the data as well as our baseline model. First, if knowledge spillovers are homogeneous across countries and sectors, then there is zero dispersion in productivity. Second, negligible knowledge spillovers, despite allowing for productivity dispersion, cannot match country-sector relative productivity as observed in the data. Furthermore, the presence of knowledge diffusion greatly enhances the growth effect of trade liberalization; the reason being that countries and sectors then have access to innovations developed elsewhere. In addition, we find that welfare gains from trade are lower and less disperse in the alternative models, which underscores the importance of considering multi-sector models with heterogeneous knowledge spillovers when seeking to quantify the effects of trade liberalization. In particular, the average gain from trade in the dynamic model doubles that in the static model, and is 30 percent larger than in a model with homogeneous or negligible spillovers.

Finally, we perform a “Brexit”-inspired exercise in which we study, through the lens of our model, the effect of protectionist trade measures. We study a situation in which the United Kingdom increases its trade barriers with respect to its main European Union trading partners by 25%, and those countries retaliate by increasing trade barriers by the same amount. BGP growth in the world decreases slightly. The United Kingdom and Ireland are most affected by “Brexit”, both in terms of innovation and welfare losses, although almost all countries lose. We also find that the reallocation of R&D and production across sectors in the United Kingdom differs substantially from the case in which we do not allow for knowledge spillovers. Sectors in the United Kingdom that are more connected through knowledge spillovers are also more affected from higher trade barriers. In the case of negligible diffusion, sectors in which the United Kingdom had an initial comparative advantage in production and innovation are more affected.

⁶In the Appendix, we also study the case in which there are no heterogeneous input-output linkages.

A few points should be mentioned regarding our calibration strategy for knowledge diffusion. Without direct measures of technology spillovers, patent citation data have been used extensively in economic research as a way to track technological diffusion across time and geographic boundaries.⁷ One patent application citing an earlier patent generally indicates that the applicant has benefited from the prior patent. Although patent citations provide valuable rare insight into knowledge spillovers, one caveat is that they are subject to certain limitations. For example, such citations do not capture technology transfer or any other types of learning (e.g., reverse-engineering, imitation, replication) that do not result in a patent. Moreover, a substantial amount of inventions are not patented but are protected through trade secrets and other informal mechanisms. That said, there is no convincing evidence that nonpatented knowledge should be expected to diffuse at a significantly different speed than patented knowledge. A second and perhaps more important point is that our estimation procedure builds on the approach proposed by Caballero and Jaffe (1993) by extending it to a multi-country, multi-sector environment. This approach is designed to incorporate—in addition to heterogeneous cross-country and cross-sector diffusion speed—how citations are affected by variations in obsolescence rates, patent quality and the extent to which such citations represent knowledge spillovers. Controlling for these additional variations within a richly structured citation process allows us to estimate diffusion speed more accurately. Third, consider the alternative regression approach in the literature that estimates the relation between domestic total factor productivity (TFP) in a certain sector and foreign R&D capital stock in another sector and then uses the estimated elasticity as a proxy for spillovers. Such estimation requires data that, for most countries, are either difficult to measure (e.g., sectoral TFP) or simply unavailable (e.g., R&D stock). Furthermore, the use of outcome-based measures may confound technology spillovers with other factors that lead to comovement between country-sectors.

Literature Review Our paper connects and extends existing theoretical literature on the relationship between trade and innovation (e.g., Grossman and Helpman, 1991; Rivera-Batiz and Romer, 1991; Atkeson and Burstein, 2010; Somale, 2017), between trade and diffusion (Perla, Tonetti, and Waugh, 2015), and between innovation and diffusion (Eaton and Kor-

⁷See, for example, Jaffe, Trajtenberg, and Henderson (1993); Peri (2005); Thompson and Fox-Kean (2005); Griffith, Lee, and Van Reenen (2011); Bloom, Schankerman, and Van Reenen (2013); Fons-Rosen et al. (2017).

tum, 1996, 1999). Yet, the three-way interactions between trade, innovation, and diffusion are rarely analyzed in a single unified framework. Notable exceptions are Eaton and Kortum (2006), Santacreu (2015), Buera and Oberfield (2019), and Sampson (2019). However, none of these studies consider a multi-sector environment with sectors being interconnected by both production and knowledge linkages. In addition, in both Buera and Oberfield (2019) and Santacreu (2015), trade is the only vehicle of diffusion, while our paper allows diffusion and trade to operate separately. Sampson (2019) develops an endogenous growth model of innovation and technology adoption with Armington trade to explain technology gaps and global income inequality. Similarly to ours, productivity gaps in his model depend on differences in innovation efficiency across countries and sectors, and the rate at which ideas diffuse within and across countries. In our paper, technology gaps depend also on intersectoral linkages in production and knowledge spillovers.

This paper also augments the literature that aims to quantify dynamic gains from trade (Perla, Tonetti, and Waugh, 2015; Akcigit, Ates, and Impullitti, 2018; Arkolakis et al., 2018; Ravikumar, Santacreu, and Sposi, 2019; Buera and Oberfield, 2019). In a Melitz-type model, Perla, Tonetti, and Waugh (2015) find that lowering trade barriers induces faster technology adoption and growth because of increased gains in the relative profit of average and marginal adopting firms. However, those authors report lower gains from trade owing to the decrease in varieties due to entry. Akcigit, Ates, and Impullitti (2018) focus on the role of strategic interaction between firms in shaping their innovation responses to policy changes (e.g., tariffs and R&D subsidies) and on dynamic gains from trade. Ravikumar, Santacreu, and Sposi (2019) analyze the role of capital accumulation on the welfare gains from trade. Arkolakis et al. (2018) studies how a reduction in the cost of multinational production affect countries' specialization patterns in production and innovation in a one-sector model. Countries with a comparative advantage in innovation specialize more in innovation, while countries with large markets concentrate more on production due to the home market effects. Although each study has a different focus, they all find that the gains from trade increase substantially compared to their model's static counterparts—a result found in our paper as well.

Our work also contributes to a burgeoning strand of research that analyzes the implications of interconnections between different sectors in closed economies (e.g., Jones, 2011; Gabaix, 2011; Acemoglu et al., 2012; Carvalho and Gabaix, 2013; Carvalho, 2014), or in open economies (Costinot, Donaldson, and Komunjer, 2012; Caliendo and Parro, 2015; Eaton

et al., 2016; Baqaee and Farhi, 2019). Most of these papers focus on factor–demand production linkages. In addition to such input–output linkages, we also consider intersectoral linkages through knowledge spillovers as in Cai and Li (2019) and Acemoglu, Akcigit, and Kerr (2016). Both papers use patent citation data across technology classes and establish that knowledge linkages across sectors are highly heterogeneous. Here, we study how cross-country cross-sector heterogeneous knowledge diffusion influences the growth effects of trade.

2 The Model

We develop a general equilibrium model of trade in intermediate goods, with sector heterogeneity and input-output linkages, in which technology evolves endogenously through innovation and knowledge diffusion. The model can be decomposed into two blocks: (i) a *trade* block which—for a given distribution of technology and trade barriers—determines the static equilibrium; and (ii) a *growth* block, which determines the dynamics of technology through innovation and knowledge spillovers.

There are M countries and J sectors. Countries are denoted by i and n and sectors are denoted by j and k . The only production factor is labor, which is assumed to be mobile across sectors within a country but immobile across countries. Ricardian trade takes place in the form of intermediate goods.

2.1 Consumers

In each country there is a representative household with lifetime utility

$$U_{nt} = \int_{t=0}^{\infty} e^{\rho t} \log(C_{nt}) dt, \quad (1)$$

where $\rho \in (0, 1)$ is the discount factor and C_{nt} is consumption of country n at time t .

The household consumes and also finances the R&D activities of entrepreneurs, and it owns all the firms. In return, the household receives labor income and the profits generated by entrepreneurs.

The household's budget constraint is given by

$$P_{nt}C_{nt} + \dot{a}_{nt} = r_{nt}a_{nt} + \Pi_{nt} + W_{nt}L_{nt}.$$

In this expression, P_{nt} is the price of the final good (to be defined later), a_{nt} is the household's holding of firms' shares, r_{nt} is the return on assets, Π_{nt} is the profit of firms that the household receives because it finances firms' R&D activities, and $W_{nt}L_{nt}$ denotes labor income.

2.2 Final Production

In each country n , a domestic final producer uses the composite output from each domestic sector j at time t (Y_{nt}^j) to produce a nontraded final output (Y_{nt}) according to the following Cobb-Douglas production function:

$$Y_{nt} = \prod_{j=1}^J (Y_{nt}^j)^{\alpha^j}; \quad (2)$$

here $\alpha^j \in (0, 1)$ is the share of sector production in total final output, and $\sum_{j=1}^J \alpha^j = 1$.

Final producers operate under perfect competition. Profits are given by

$$\Pi_{nt} = P_{nt}Y_{nt} - \sum_{j=1}^J P_{nt}^j Y_{nt}^j,$$

where P_{nt} is the price of the final product and P_{nt}^j is the price of the composite good produced in sector j from country n .

Under perfect competition, the price charged by the final producer to the consumers is equal to the marginal cost; that is

$$P_{nt} = \prod_{j=1}^J \left(\frac{P_{nt}^j}{\alpha^j} \right)^{\alpha^j}.$$

The demand by final producers for the sectoral composite good is given by

$$Y_{nt}^j = \alpha^j \frac{P_{nt}}{P_{nt}^j} Y_{nt}.$$

2.3 Intermediate Producers

In each sector j there is a continuum of intermediate producers indexed by $\omega \in [0, 1]$ that use labor, $l_{nt}^j(\omega)$ —together with a composite intermediate good from every other sector k in the country, $m_{nt}^{jk}(\omega)$ —to produce a variety ω according to the following constant-returns-to-scale technology⁸:

$$y_{nt}^j(\omega) = z_n^j(\omega) [l_{nt}^j(\omega)]^{\gamma^j} \prod_{k=1}^J [m_{nt}^{jk}(\omega)]^{\gamma^{jk}}, \quad (3)$$

where $\gamma^j + \sum_{k=1}^J \gamma^{jk} = 1$. Here γ^{jk} is the share of materials from sector k used in the production of intermediate good ω in sector j , and where γ^j is the share of value added. Firms are heterogeneous in their productivity $z_n^j(\omega)$.

The cost of producing each intermediate good ω is

$$c_{nt}^j(\omega) = \frac{c_{nt}^j}{z_n^j(\omega)},$$

where c_n^j denotes the cost of the input bundle. With constant returns to scale in production, it follows that

$$c_{nt}^j = \Upsilon^j W_{nt}^{\gamma^j} \prod_{k=1}^J (P_{nt}^k)^{\gamma^{jk}}; \quad (4)$$

here $\Upsilon^j = \prod_{k=1}^J (\gamma^{jk})^{-\gamma^{jk}} (\gamma^j)^{-\gamma^j}$ and W_{nt} is the nominal wage rate.

Intermediate producers operate under Bertrand competition. Their equilibrium prices are characterized later in the paper.

2.4 Composite Intermediate Goods (Materials)

Each sector j produces a composite good at minimum cost by combining domestic and foreign varieties from that sector. Composite producers operate under perfect competition and buy intermediate products ω from the lowest-cost supplier.

The production of a composite good in sector j and country n is captured by the Ethier

⁸In this paper we employ the following notational convention: the subscripts denote the country and the superscripts denote the sector; whenever there are two subscripts or two superscripts, the leftmost (resp. rightmost) one corresponds to the destination (resp. source).

(1982) constant elasticity of substitution (CES) function,

$$Q_{nt}^j = \left(\int e_{nt}^j(\omega)^{1-1/\sigma} d\omega \right)^{\sigma/(\sigma-1)}. \quad (5)$$

In this expression, $\sigma > 1$ is the elasticity of substitution across intermediate goods and $e_{nt}^j(\omega)$ is the demand of intermediate goods from the lowest-cost supplier in sector j .

The demand for each intermediate good ω is given by

$$e_{nt}^j(\omega) = \left(\frac{p_{nt}^j(\omega)}{P_{nt}^j} \right)^{-\sigma} Q_{nt}^j,$$

Here P_{nt}^j is

$$P_{nt}^j = \left(\int p_{nt}^j(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, \quad (6)$$

and $p_{nt}^j(\omega)$ is the lowest price of intermediate good ω across all countries n .

Composite intermediate goods are used as final goods in the final production and as materials for the production of the intermediate goods. Hence we write

$$Q_{nt}^j = Y_{nt}^j + \sum_{k=1}^J \int m_{nt}^{kj}(\omega) d\omega.$$

2.5 International Trade

Trade in goods is costly. In particular, there are iceberg transport costs so that shipping a good produced in sector j from country i to country n requires producing $d_{ni}^j > 1$ units of good in sector j of country i . We assume that the “triangular” inequality holds; thus $d_{ih}^j d_{hn}^j > d_{in}^j$. We follow Bernard, Eaton, Jensen, and Kortum (2003) and assume Bertrand competition. Under Bertrand competition, just as under perfect competition, composite producers in each sector buy from the lowest-cost supplier and the price charged by the producer is a function of the production cost of the second-lowest cost producer.

Ricardian motives for trade are assumed, as in Eaton and Kortum (2002), because productivity is allowed to vary by country-sector-firm. The productivity of producing intermediate good ω in sector j of country i is drawn from a Fréchet distribution characterized by T_{it}^j and by the shape parameter θ : $F(z_i^j) = \Pr[Z \leq z_i^j] = \exp\{-T_{it}^j z_i^{-\theta}\}$. A higher T_{it}^j

implies a higher average fundamental productivity of that country-sector, whereas a lower θ implies more dispersion of productivity across varieties. Thus T_{it}^j determines the cross-sector comparative advantage, while θ determines the intra-industry comparative advantage (see Costinot, Donaldson, and Komunjer 2012).

Given these assumptions, Bernard et al. (2003) show that the price index of goods in sector j of country n is

$$P_{nt}^j = B (\Phi_{nt}^j)^{-1/\theta}, \quad (7)$$

where $B = \left[\frac{1+\theta-\sigma+(\sigma-1)(\bar{m})^{-\theta}}{1+\theta-\sigma} \Gamma \left(\frac{2\theta+1-\sigma}{\theta} \right) \right]^{1/(1-\sigma)}$ and

$$\Phi_{nt}^j = \sum_{i=1}^M T_{it}^j (d_{ni}^j c_{it}^j)^{-\theta}. \quad (8)$$

So that prices will be well-defined, we assume that $\sigma < (1 + \theta)$.⁹

Expenditure shares Given our distributional assumptions on productivity, the probability that country i is the lowest-cost supplier of a sector j good produced for export to country n is

$$\pi_{nit}^j = \frac{T_{it}^j (c_{it}^j d_{ni}^j)^{-\theta}}{\Phi_{nt}^j}, \quad (9)$$

for c_{it}^j defined as in equation (4). Since there is a continuum of intermediate goods, it follows that π_{nit}^j is also the fraction of goods that sector j of country i sells to any sector in country n . Formally, the share that country n spends on sector- j products from country i is

$$\pi_{nit}^j = \frac{X_{nit}^j}{X_{nt}^j}. \quad (10)$$

Here $X_{nt}^j = P_{nt}^j Q_{nt}^j$ represents country n 's total expenditures on goods from sector j and X_{nit}^j denotes the value of intermediate products from sector j that country n buys from country i .

⁹Details of these derivations can be found in Bernard et al. (2003).

2.6 Total Expenditures and Balanced Trade

Total expenditures on goods from sector j in country n are given by the sum of (a) what the composite producers from each sector k and country i buy and (b) the spending by final producers. Hence, X_n^j is given by

$$X_{nt}^j = \sum_{k=1}^J \gamma^{kj} \sum_{i=1}^M X_{it}^k \pi_{int}^k + \alpha^j P_{nt} Y_{nt}. \quad (11)$$

We assume trade is balanced on a period-by-period basis, that is

$$\sum_{k=1}^J X_{nt}^k \sum_{i=1/i \neq n}^M \pi_{nit}^k = \sum_{i=1/i \neq n}^M \sum_{k=1}^J \pi_{int}^k X_{it}^k, \quad (12)$$

where left-hand-side are total imports and right-hand-side are total exports of country n .

2.7 Endogenous Growth: Innovation and Knowledge Spillovers

So far, we have described the *trade* block of the model, which determines the static equilibrium, given a distribution of technology (T_{nt}^j) and trade barriers (d_{ni}^j). Note that, our model differs from static models of trade, in that T_{nt}^j is endogenous and depends on t . Next we describe the model's *growth* block, which determines the endogenous evolution of T_{nt}^j . In this model, T_{nt}^j also represents the stock of knowledge in sector j of country n .

In each sector j and country n , there is a continuum of entrepreneurs who invest final output, R_{nt}^j , to come up with a new idea. Ideas are the “blueprints” used to produce an intermediate good more efficiently.¹⁰ Research efforts are targeted at any of the continuum of intermediate goods in that sector. In each country n and sector j , ideas are drawn at a Poisson rate given by

$$\lambda_n^j T_{nt}^j (s_{nt}^j)^{\beta_r}. \quad (13)$$

Here λ_n^j is a country- and sector-specific parameter that determines the exogenous component of the innovation efficiency; $s_{nt}^j = R_{nt}^j / \bar{Y}_t$, where \bar{Y}_t is the world output along the BGP; and $\beta_r \in (0, 1)$ is a parameter that governs the diminishing returns to R&D investment. This process ensures that there is a balanced growth path without scale effects (see Eaton and

¹⁰We follow Kortum (1997) in modeling the innovation process within each sector of a country.

Kortum 1996, 1999 and Santacreu 2015). In our specification for the Poisson arrival of new ideas, $\lambda_n^j T_{nt}^j$ determines *comparative advantage in innovation*—an advantage that depends not only on an exogenous component (λ_n^j) but also on an endogenous component (T_{nt}^j). All else being equal, countries that have accumulated more knowledge stock over time (i.e., a higher T_{nt}^j) become more efficient in their innovation.

As is standard in the literature on “quality ladders”, an idea is the realization of two random variables. One of these variables is the good ω to which the idea applies. An idea applies to only one good in the continuum, and the good ω with which an idea is associated is drawn from the uniform distribution $[0, 1]$. The second variable is the idea’s quality q , which is drawn from a Pareto distribution of qualities: $H(q) = 1 - q^{-\theta}$.

Ideas developed in country n and sector j can diffuse to other countries and sectors. Diffusion happens when the country-sector learns about ideas that have been developed elsewhere and it takes time.¹¹ We assume that an idea discovered at time t in country n and sector j diffuses to country i and sector k at time $t + \tau_{in}^{kj}$. We assume that the diffusion lag, τ_{in}^{kj} , follows an exponential distribution with parameter ε_{in}^{kj} : $Pr[\tau_{in}^{kj} \leq x] = 1 - e^{-\varepsilon_{in}^{kj}x}$. Thus, ε_{in}^{kj} is the speed of diffusion from country n sector j to country i sector k and its inverse is the mean diffusion lag.

Therefore, an idea may be the outcome of domestic research in the same sector, or may arrive from other countries or from other sectors. It could be the result of recent innovations or of a previous innovation that has since diffused. Summing over the past research conducted in all countries and sectors, the flow of ideas diffusing to country n and sector j is given by

$$\dot{T}_{nt}^j = \sum_{i=1}^M \sum_{k=1}^J \varepsilon_{ni}^{jk} \int_{-\infty}^t e^{-\varepsilon_{ni}^{jk}(t-s)} \lambda_i^k T_{is}^k (s_{is}^k)^{\beta_r} ds. \quad (14)$$

The evolution of the stock of knowledge in sector j and country n at time t depends on the past research outcome due to each other country-sector before time t and diffused at rate ε_{ni}^{jk} . We assume that every idea eventually diffuses to every other country-sector; in other

¹¹Here we adopt the Eaton and Kortum (1996, 1999) strategy in modeling the diffusion of ideas as exogenous. It is different from Buera and Oberfield (2019) in which the diffusion of ideas is tightly linked to trade, or from Sampson (2019) in which the diffusion of ideas depends on the distance to the frontier technology. Each approach has its own advantages and disadvantages. The advantage of our strategy is that we are agnostic about specific channels of diffusion but let the data discipline the heterogeneous parameters of diffusion speed. If the estimated speed of diffusion from country A to B is high, it could reflect either a close trade relationship between the two or B’s existing technology being much backward and hence B endogenously investing for faster adoption of frontier technology.

words, the values of ε_{ni}^{jk} are strictly positive.

Diffused ideas that are at least as productive as those already in use in a particular country-sector can be used to produce an intermediate good in that country-sector. We refer to this process as *adoption*. The adopted idea would be used to produce an intermediate product ω in sector j and country n with efficiency $z_n^j(\omega)$. Therefore, the efficient technology $z_n^j(\omega)$ for producing good ω in country n is the best idea for producing it yet discovered (see Eaton and Kortum 2006). Thus, although all ideas eventually diffuse and increase the stock of knowledge, only the best ideas are adopted.

The stock of ideas at time t in each sector j and country n is T_{nt}^j . Because there is a unit interval of intermediate goods, the number of ideas for producing a specific good is Poisson with parameter T_{nt}^j . This Poisson arrival implies that the quality distribution of best ideas is $F(q) = e^{-T_{nt}^j q^{-\theta}}$.¹² Therefore, the quality distribution of successful ideas inherits the distribution of productivity of the intermediate goods produced in a country. Our probabilistic distribution assumption for the quality of an idea implies that the probability of an idea being successful (i.e. being the best idea) is $1/T_{nt}^j$.

Entrepreneurs finance R&D activities by issuing equity claims to households. These claims pay nothing if the entrepreneur is not successful in introducing a new technology in the market, and it pays the stream of future profits if the innovation succeeds. Because of the probabilistic distribution of productivity, all products within a sector deliver the same expected profit and, thus entrepreneurs are indifferent to what product ω to devote their efforts. In addition, we assume that there is perfect intellectual property rights protection within a country-sector, but not across country-sectors.¹³ Therefore, innovators only get profits from those ideas that have been developed, and then diffused and adopted in their own country-sector. Diffusion to other country-sectors is modeled as a pure spillover since adopters there do not have to pay for the right to use these ideas.¹⁴

¹²Under Poisson arrival rate of new ideas, the probability of k ideas for producing a good by period t in sector j and country n is $\left(T_{nt}^j\right)^k e^{-T_{nt}^j}/k!$. If there are k ideas, the probability that the best one is below the best quality q is $[H(q)]^k$. Summing over all possible k , we have $F(q) = e^{-T_{nt}^j q^{-\theta}}$.

¹³One rationale for this assumption is that it is generally hard to verify the original source of ideas across country-sectors.

¹⁴In Appendix J, we study the other extreme case of perfect enforcement of IPR across all countries and sectors.

Formally, innovators choose the amount of R&D investment (R_{nt}^j) to maximize

$$\lambda_n^j T_{nt}^j (s_{nt}^j)^{\beta_r} V_{nt}^j - P_{nt} R_{nt}^j,$$

where V_n^j is the value of an innovation created at t and diffused and adopted later ($s \geq t$) in sector j and country n , which can be expressed as

$$V_{nt}^j = \int_t^\infty e^{-\int_t^s r_{nu} du} \left(1 - e^{-\varepsilon_{nn}^{jj}(s-t)}\right) \frac{\Pi_{ns}^j}{T_{ns}^j} ds. \quad (15)$$

Here, $1/T_{nt}^j$ governs the probability of an idea being successful, and Π_{nt}^j denotes total profits generated from selling the goods in all countries, which can be expressed as

$$\Pi_{nt}^j = \frac{\sum_{i=1}^M \pi_{int}^j X_{it}^j}{1 + \theta}.$$

The expression for V_{nt}^j introduces a competitive effect, whereby the larger the stock of knowledge in a country-sector, the lower the probability a new idea lowers the cost there. And, conditional on that idea being successful, the innovator's expected profits are determined by the likelihood that the intermediate good produced with her idea is produced at the lowest cost, which in turn is determined by π_{int}^j .

The first-order condition for optimal R&D investment is

$$s_{nt}^j = \left(\beta_r \lambda_n^j T_{nt}^j \frac{V_{nt}^j}{P_{nt} Y_{nt}} \frac{Y_{nt}}{Y_t} \right)^{\frac{1}{1-\beta_r}}. \quad (16)$$

As it will become clear later, this equation is key to analyze the effect of trade liberalization on innovation, growth, and welfare.

2.8 Resource Constraint

Final output in a given country is used for either consumption or R&D investment. The resource constraint equation is

$$Y_{nt} = C_{nt} + \sum_{j=1}^J R_{nt}^j. \quad (17)$$

3 Balanced Growth Path

We define the balanced growth path as an equilibrium in which all variables grow at a constant rate. In our model, growth along the BGP is endogenous and depends on policy parameters. Changes in trade costs have both growth and level effects. We stationarize all the endogenous variables so that they are constant on the BGP. We denote the normalized variables with a hat, and remove all time subscripts in our derivation. Here we characterize the BGP growth rate of the economy.

Cross-country and cross-sector knowledge spillovers guarantee that the stock of knowledge T_n^j grows at the constant rate g , which is common across all countries and sectors.

We can use equation (15) to express the value of an innovation on the BGP as

$$\hat{V}_n^j = \Gamma_n^j \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{(1 + \theta)}, \quad (18)$$

where $\Gamma_n^j = \left(\frac{1}{r-g/\theta+g} - \frac{1}{r-g/\theta+g+\varepsilon_{nn}^{jj}} \right)$, $\hat{V}_n^j = \frac{V_n^j T_n^j}{W_M}$ and $\hat{X}_i^j = \frac{X_i^j}{W_M}$. We impose the condition $r - g/\theta + g > 0$, with $r = g_y/\rho$. Substituting equation (18) into equation (16) allows us to write the optimal R&D intensity as follows:

$$s_n^j = \left(\beta_r \lambda_n^j \frac{1}{(1 + \theta)} \Gamma_n^j \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} \frac{\hat{Y}_n}{\hat{Y}} \right)^{\frac{1}{1-\beta_r}}. \quad (19)$$

Here $\hat{Y}_n = \frac{Y_n}{W_M}$ and $\hat{Y} = \frac{\bar{Y}}{W_M}$. Combining equation (19) with equation (14) shows that the growth rate of the stock of knowledge on the BGP is

$$g = \sum_{i=1}^M \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}}{g + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{T}_i^k}{\hat{T}_n^j} (s_i^k)^{\beta_r}, \quad (20)$$

where $\hat{T}_i^k = \frac{T_i^k}{T_M^j}$. Finally, substituting equation (19) into equation (20) yields

$$g = \sum_{i=1}^M \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}}{g + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{T}_i^k}{\hat{T}_n^j} \left(\beta_r \lambda_i^k \Gamma_i^k \frac{\sum_{n=1}^M \pi_{ni}^k \hat{X}_n^k}{(1 + \theta) \hat{Y}_i} \frac{\hat{Y}_i}{\hat{Y}} \right)^{\frac{\beta_r}{1-\beta_r}}. \quad (21)$$

The growth rate of the stock of knowledge on the BGP depends positively on the speed of diffusion, expected future profits, and the efficiency of innovation; it depends negatively

on the dispersion parameter. Following Eaton and Kortum (1999), the Frobenius theorem guarantees that there is a unique growth rate on the BGP in which all countries and sectors grow at the same rate g . The expression for the growth rate can be expressed in matrix form as

$$gT = \Delta(g)T.$$

If the matrix $\Delta(g)$ is positive definite, then there exists a unique positive BGP rate of technology $g > 0$, given research intensities and diffusion parameters. Associated with that growth rate is a vector T (defined up to a scalar multiple), with every element positive, which reflects each country-sector's relative level of knowledge along that BGP.

In Appendix B, we provide details on the derivation of the BGP, and in Appendix C, we summarize the equations of our model's equilibrium conditions after normalizing all endogenous variables.

4 The Mechanism

This section describes the mechanism through which a reduction of trade costs, d_{in}^j , affects innovation, growth, and comparative advantage. In multi-sector *static* models of trade, there is the well-known specialization effect: a decrease in d_{in}^j induces a re-allocation of production towards those sectors in which the country has a comparative advantage (Caliendo and Parro 2015). The larger the dispersion in relative productivity, the greater the comparative advantage, and hence the stronger the specialization effect.

In our multi-sector *dynamic* model with cross-country-sector spillovers, there are additional effects of trade liberalization. First, there is an R&D re-allocation effect. Second, trade-induced R&D re-allocation has an effect on the dispersion of relative productivity—hence, on comparative advantage—and on growth. Next, we formalize each of these effects and explain how knowledge spillovers play an important role on either dampening or amplifying them.

R&D resources are re-allocated according to three components: (i) the exogenous efficiency of innovation, λ_n^j , (ii) the speed of within-country-sector diffusion, ε_{nn}^{jj} , and (iii) the relative production share of sector j . The first two components are exogenous and depend on model's parameters, whereas the third component is endogenous. Through the specialization effect just described, profits increase in sectors with stronger comparative advantage

because of the well-known market size effect following the decline in trade costs. As a result, R&D resources are re-allocated to sectors that experience a greater increase in production.¹⁵ Formally, consider two sectors j and j' in country n . From equation (19), we can obtain an expression of relative expenditure on R&D between these two sectors as

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j \Gamma_n^j \sum_{i=1}^M \pi_{in}^j X_i^j}{\lambda_n^{j'} \Gamma_n^{j'} \sum_{i=1}^M \pi_{in}^{j'} X_i^{j'}}. \quad (22)$$

Everything else constant, lowering trade costs affects the economy's production patterns and shifts R&D towards sectors that experience greater increases in profits (i.e., a higher $\sum_{i=1}^M \pi_{in}^j X_i^j$). This re-allocation effect changes aggregate R&D intensity at the country level. The exact magnitude of that change is a quantitative question that we shall address in Section 5.

The following two extreme cases illustrate the R&D re-allocation effect after a change in trade costs.

Case 1 (Autarky). Suppose all countries are closed to international trade. That is, let $d_{in}^j \rightarrow \infty, \forall i, n, j$. Equation (22) can then be rewritten as

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j \Gamma_n^j X_{nn}^j}{\lambda_n^{j'} \Gamma_n^{j'} X_{nn}^{j'}}, \quad (23)$$

where X_{nn}^j denotes total domestic expenditure on the sector- j product. In comparison with another country-sector, we have

$$\left(\frac{s_n^j/s_n^{j'}}{s_{n'}^j/s_{n'}^{j'}}\right)^{1-\beta_r} = \underbrace{\frac{\lambda_n^j/\lambda_n^{j'}}{\lambda_{n'}^j/\lambda_{n'}^{j'}}}_{\text{comparative advantage in innovation}} \times \underbrace{\frac{\Gamma_n^j/\Gamma_n^{j'}}{\Gamma_{n'}^j/\Gamma_{n'}^{j'}}}_{\text{comparative advantage in within-diffusion}} \times \underbrace{\frac{X_{nn}^j/X_{nn}^{j'}}{X_{n'n'}^j/X_{n'n'}^{j'}}}_{\text{relative domestic market size}}. \quad (24)$$

This expression shows how innovation efforts are distributed across country-sectors according to: (i) the exogenous component of comparative advantage in innovation; (ii) the rate at which ideas diffuse and are potentially adopted within the country-sector, and (iii) the relative domestic market share.

¹⁵This result is similar to that obtained by Somale (2017) in a semi-endogenous growth model *without* knowledge spillovers.

Case 2 (Free Trade). In the case of free trade, $d_{in}^j = 1$. Equation (22) then becomes

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j \Gamma_n^j T_n^j(c_n^j)^{-\theta} / \sum_n T_n^j(c_n^j)^{-\theta} X^j}{\lambda_n^{j'} \Gamma_n^{j'} T_n^{j'}(c_n^{j'})^{-\theta} / \sum_n T_n^{j'}(c_n^{j'})^{-\theta} X^{j'}} \quad (25)$$

where $X^j = \sum_n X_n^j$ represents world demand for the sector- j good. According to this equation, a country's R&D resources under free trade are distributed not only as a function of sector-specific relative innovation efficiency and the within-country-sector diffusion speed, but also according to the comparative advantage in production, and the world expenditure share ($X^j/X^{j'}$).¹⁶ The latter captures the traditional market size effect of trade liberalization.

For any given sector, since the last two terms are common across countries, we can express the relative R&D allocation across countries and sectors as

$$\left(\frac{s_n^j/s_n^{j'}}{s_{n'}^j/s_{n'}^{j'}}\right)^{1-\beta_r} = \underbrace{\frac{\lambda_n^j/\lambda_n^{j'}}{\lambda_{n'}^j/\lambda_{n'}^{j'}}}_{\text{comparative advantage in innovation}} \times \underbrace{\frac{\Gamma_n^j/\Gamma_n^{j'}}{\Gamma_{n'}^j/\Gamma_{n'}^{j'}}}_{\text{comparative advantage in within-diffusion}} \times \underbrace{\frac{T_n^j(c_n^j)^{-\theta}/T_n^{j'}(c_n^{j'})^{-\theta}}{T_{n'}^j(c_{n'}^j)^{-\theta}/T_{n'}^{j'}(c_{n'}^{j'})^{-\theta}}}_{\text{comparative advantage in production}}. \quad (26)$$

The first two components of the right-hand side of equation (24) and (26) are identical and determined by country-sector specific parameters. Thus, a comparison between the extreme cases of autarky and free trade reveals that, after trade liberalization, research efforts flow more to country-sectors that have a comparative advantage in production. At the same time, a greater share of R&D investment in a sector translates, *ceteris paribus*, into a larger stock of knowledge. Thus comparative advantage in production evolves endogeneously with the distribution of innovation efforts, which in turn affect—as shown in equation (26)—the R&D allocation, generating a positive feedback loop.

The rich interactions between R&D re-allocation and heterogeneous cross-country-sector knowledge spillovers generate additional effects of trade liberalization on the dispersion of relative productivity—endogenous comparative advantage—and growth. Based on equation (20), absent cross-country /cross-sector spillovers (i.e., when $\varepsilon_{ni}^{jk} = 0$ for $nj \neq ik$), the evolution of the technology distribution ($T_n^j/T_n^{j'}$) eventually reflects the underlying specialization

¹⁶The ratio $T_n^j(c_n^j)^{-\theta} / (\frac{1}{N} \sum_n T_n^j(c_n^j)^{-\theta})$ gives the absolute advantage based on cost-adjusted technology. When we compare a sector j to another sector j' in the same country, the comparative advantage is given by $\frac{T_n^j(c_n^j)^{-\theta} / \sum_n T_n^j(c_n^j)^{-\theta}}{T_n^{j'}(c_n^{j'})^{-\theta} / \sum_n T_n^{j'}(c_n^{j'})^{-\theta}}$

in innovation $(\lambda_n^j/\lambda_n^{j'})$ and within-country-sector diffusion $(\Gamma_n^j/\Gamma_n^{j'})$. However, heterogeneous knowledge spillovers across countries and sectors, could either dampen or amplify the specialization effect induced by trade liberalization, depending on the exact pattern of diffusion.

To show this, we rearrange terms in equation (20), and write

$$gT_n^j = \sum_{i=1}^M \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}}{g + \varepsilon_{ni}^{jk}} \lambda_i^k T_i^k (s_i^k)^{\beta_r}. \quad (27)$$

If the speed of diffusion for a given source country and sector is common across all destination countries and sectors $(\varepsilon_{ni}^{jk} = \varepsilon_i^k, \forall n, j)$, then it follows from equation (27) that the stock of knowledge will be the same everywhere $(T_n^j = \bar{T}, \forall n, j)$. Yet, when the speed of diffusion is heterogeneous, the effect of R&D re-allocation on the stock of knowledge will be stronger in those country-sectors that receive knowledge more rapidly from the country-sectors that have experienced greater increases in innovation. Therefore, if less innovative country-sectors are better at absorbing knowledge, then knowledge diffusion can *dampen* the aforementioned specialization effect by enabling faster productivity convergence and by rendering the stock of knowledge to be more similar across country-sectors. However, if diffusion forces are stronger for already innovative countries—that is, innovative countries also diffuse ideas rapidly among themselves—diffusion could *amplify* the specialization effects of trade-induced R&D re-allocation, as productivity and the stock of knowledge become even more disperse. Our calibration in section 5.1.1 establishes that the latter effect receives greater support from the data.

Finally, trade liberalization in our model also has a growth effect. From equation (20), a change in R&D spending across country-sectors alters the growth rate along the BGP. More R&D investment overall increases the world growth rate. In addition, if R&D is re-allocated toward sectors that are better at doing R&D and have faster diffusion speed, the growth rate of the world also increases. The existence of knowledge spillovers reinforces this channel, because changes in R&D across sectors will then have a greater effect on BGP growth as countries and sectors can benefit from R&D in other countries and sectors.

5 Quantitative Analysis

We calibrate the model in order to quantify the effect of trade liberalization on innovation, comparative advantage and growth. To explore the role of the main channels, we simulate five versions of the model: (i) our baseline model with innovation and cross-sector and cross-country knowledge and input-output linkages; (ii) a static model in which productivity is exogenously determined; (iii) a model without input-output linkages; (iv) a model with (almost) no knowledge spillovers; and (v) a model with knowledge spillovers that are homogeneous across countries and sectors.¹⁷ We then assess how each of the channels proposed in Section 4 affect innovation, growth, and welfare gains from trade. In all cases, we recalibrate the model parameters to match the same moments of the data.

5.1 Calibration

We calibrate the model’s main parameters with data on bilateral trade flows, population, production, R&D intensity and with patent citations data. In this section, we focus on the calibration of the diffusion parameters ε_{in}^{jk} , and the parameters that govern the innovation process—the elasticity of innovation, β_r ; the efficiency of innovation, λ_i^j . The calibration of the production parameters and the trade costs, d_{in}^j , is standard and details are relegated to Appendix D, together with a description of the data used for the calibration.

5.1.1 Speed of Knowledge Diffusion

Estimating the speed of knowledge diffusion is not a trivial task, since diffusion is conceptual and difficult to measure. The diffusion literature has typically found patent citations to be a reasonable indicator of diffusion albeit with some degree of noise (e.g., Jaffe, Trajtenberg, and Fogarty 2000; Bottazzi and Peri 2003; Bloom, Schankerman, and Van Reenen 2013; Cai and Li 2019, among many others).¹⁸ When a patent is granted, its documentation must identify all citations made to previous patents upon which the current one builds. Citations

¹⁷In the Appendix, we also study the case in which there are no heterogeneous input-output linkages and the case of knowledge spillovers across countries within a sectors but not across different sectors.

¹⁸Although patent statistics have been widely used in studies of firm innovations, not all innovations are patented—especially process innovations, which are often protected in other ways (e.g., copyright, trademarks, secrecy; see Levin et al. 1987). Our measure implicitly assumes that, for any sector, patented and nonpatented knowledge both use (patented or nonpatented) knowledge from other sectors in the same manner and, in particular, with the same speed.

are therefore informative of the links between innovations. A single technology cited in numerous patents is evidently involved in many developmental efforts.

In this section we adapt the approach proposed in Caballero and Jaffe (1993) to estimate diffusion speed parameters using patent data. For consistency with our model, we extend their approach to a multi-sector, multi-country environment. We use patents as an indicator of new idea creation and view citations as indicating the use of those ideas in the creation of new ones.

An extensive literature discusses the potential issues with using patent data to proxy for ideas and spillovers.¹⁹ First, a considerable number of inventions or ideas are never patented but are protected by secrecy or other informal mechanisms. Second, sectors differ in their propensity to patent and also in their propensity to cite; hence a relatively abundant stock of patents in one sector need not imply a relatively large accumulation of ideas. Third, individual patents vary in terms of quality (the number of ideas embodied and/or the capacity to generate spillovers). Finally, not all citations represent spillovers because the decision to cite another patent sometimes rests with the patent examiner, who is supposed to be an expert in the area and able to identify relevant prior art that the applicant might have missed or concealed. The implication is that the inventor may not be aware of such earlier work and so the patent’s citations may not represent the true knowledge transmission.

A particular virtue of the Caballero and Jaffe (1993) approach is that it can deal with some of these issues by estimating sector-specific factors—such as the propensity to patent and to cite, the ability to generate spillovers, the knowledge obsolescence rate, and the discrepancy between citations and spillovers—*jointly* with the diffusion speed parameters. Given the fairly rich structure of the citation process, controlling for these additional sectoral variations helps ensure more accurate estimation of our parameter of interest: the cross-country, cross-sector speed of diffusion, $\{\varepsilon_{ni}^{jk}\}_{MJ \times MJ}$.²⁰

In particular, we first specify a citation function that describes the usefulness of an idea generated at time s in country-sector ik for the production of new knowledge in country-sector nj at time t ($t \geq s$). Let $C_{ni}^{jk}(t, s)$ be the citations from patents applied by country

¹⁹See the survey by Griliches (1990).

²⁰Eaton and Kortum (1999) use international patenting data to calibrate ε_{ni}^{jk} using GMM. Their procedure requires decomposing ε_{ni}^{jk} into $\varepsilon_{n.}^j \times \varepsilon_{.i}^k$. In contrast, our generalization of Caballero and Jaffe (1993)’s method allows us to capture heterogeneity across all nj - ik pairs in ε_{ni}^{jk} . We utilize $MJ \times MJ \times T(T-1)/2$ moments to identify $MJ \times MJ + 4MJ$ parameters. For a given value of a subset of the parameters, there are $T(T-1)/2$ moments within each nj - ik pair to identify each ε_{ni}^{jk} .

n 's sector j in year t to patents by country i 's sector k in year s , and let $P_{i,s}^k$ represent the number of patent applications by country i 's sector k in period s . Then the expected citation function can be written as follows:

$$C_{ni}^{jk}(t, s) = \phi_{n,t}^j \delta_{i,s}^k (\psi_{i,s}^k P_{i,s}^k)^{\beta_g} (\psi_{n,t}^j P_{n,t}^j)^{\beta_l} e^{-\sum_{\tau=s}^t O_{i,\tau}^k \tilde{P}_{i,\tau}^k} (1 - e^{-\varepsilon_{ni}^{jk}(t-s)}). \quad (28)$$

Here, $\phi_{n,t}^j$ is a *citing* country-sector cohort-specific parameter which governs (the inverse of) the number of actual ideas used per citation, such that $C_{ni}^{jk}(t, s)/\phi_{n,t}^j$ is the true number of ideas used by nj at time t . The sector-specific variations in propensity to cite are also captured by $\phi_{n,t}^j$, which is high when an average patent in country-sector nj at year t generates a large number of citations. The parameter, $\delta_{i,s}^k$, represents the ability to generate spillovers from patents in *cited* country-sector ik dated in period s ; it is high when an average patent in country-sector ik at year s receives a large number of citations. It is also assumed that the number of ideas embodied in each patent in country-sector nj at time t is given by $\psi_{n,t}^j$. Parameters β_g and β_l represent the elasticity of citations with respect to the number of ideas in cited and citing country-sector. A value for β_l and β_g of 1 corresponds to the multi-sector multi-country version of Caballero and Jaffe (1993).

The discount term, $e^{-\sum_{\tau=s}^t O_{i,\tau}^k \tilde{P}_{i,\tau}^k}$, captures knowledge obsolescence, where $\tilde{P}_{i,t}^k$ denotes the relative number of ideas in ik during period τ —that is, relative to the total stock of ideas in ik over the sample period, i.e. $\tilde{P}_{i,t}^k = \psi_{i,t}^k P_{i,t}^k / \sum_{\tau=1}^T \psi_{i,\tau}^k P_{i,\tau}^k$. The discount term decreases with the (normalized) size of inventions that take place between the recipient cohorts t and the source cohorts $\tau (\in [s, t])$, with a time-varying obsolescence rate $O_{i,\tau}^k$. This term captures the notion that old knowledge eventually is made obsolete by the emergence of superior new knowledge. Hence it is the accumulation of new inventions, and not simply the passage of time, that increases the rate at which the source knowledge becomes obsolete. Note that to separately identify $\psi_{i,s}^k$ from $\delta_{i,s}^k$ and $\psi_{n,t}^j$ from $\phi_{n,t}^j$, it is crucial to have the discount term, $e^{-\sum_{\tau=s}^t O_{i,\tau}^k \tilde{P}_{i,\tau}^k}$.

The last term in equation (28) represents the probability with which ideas in s have been diffused by period t . Given our model's assumption that diffusion lags follow an exponential distribution, it follows that the probability of an idea that is $t - s$ years old to diffuse is given by $1 - e^{-\varepsilon_{ni}^{jk}(t-s)}$, where ε_{ni}^{jk} is the constant diffusion speed from ik to nj during our sample period. Instantaneous diffusion is indicated by $\varepsilon_{ni}^{jk} \rightarrow \infty$, whereas $\varepsilon_{ni}^{jk} = 0$ implies

no diffusion. This is the parameter that we are especially interested in estimating. For a given citing-cited country-sector pair (nj - ik pair), the number of citations across different citation time lags ($t - s$) are the moments used to identify the constant ε_{ni}^{jk} , controlling for other factors that may also affect citation flow.

Therefore, the empirical strategy is to collect citations and patent applications data between patent cohorts for each nj - ik pair, and use these to estimate the above parameters for many t and s . It is crucial for identification that each parameter depends on either t or s or none of the two and not on (t, s) .

We obtain patent and patent citation data across countries and sectors from the U.S. Patent and Trade Office (USPTO) for the period 2001–2010. Using U.S. patent data to measure cross-country innovation performance and spillovers has been widely adopted in previous studies (e.g., Griffith, Harrison, and Van Reenen, 2006; Acharya and Subramanian, 2009; Hsu, Tian, and Xu, 2014). According to the principle of territoriality in US patent laws, anyone intending to claim exclusive rights for an invention is required to file a U.S. patent. In fact, more than half of the patents applied for in the U.S. during the 2000s were from foreign inventors. Given that the U.S. has been the world’s largest technology consumption market in the past few decades, we follow the previous studies by assuming that most important innovations from other countries have been patented in the United States.²¹ In the data set, each patent is assigned to an International Patent Classification (IPC) category. We use the probability mapping between IPC and revision 3 of the International Standard Industrial Classification (ISIC)—provided by the World Intellectual Property Organization—to assign patents into our 19 sectors. The country-sector patent counts are based on application years (as opposed to grant years) to capture the actual effective time of innovation. Patents are assigned to countries by their inventors (as opposed to assignee) to better reflect the origin of innovation activities. Overall, our sample contains 1.74 million patents and more than 10 million citations between the 19 countries and 19 sectors.

Our estimate of equation (28) uses the generalized method of moments (GMM) and is based on observations about the citation count from nj, t to ik, s —that is, on $\hat{C}_{ni}^{jk}(t, s)$ for $t \in [2001, 2010]$, $s \in [2001, t]$, $j, k \in [1, J]$, and $n, i \in [1, M]$. Let $\Theta_{nj,ik}(t, s) = \{\varepsilon_{ni}^{jk}, \phi_{n,t}^j, \psi_{n,t}^j, \delta_{n,t}^j\}$

²¹For our purposes, we need only that statements of the following type hold: If U.S. patent data shows that a patent belonging to a German inventor filing a patent in the U.S. in the electronic components sector cites a Japanese patent in the radio and television receiving equipment sector, then a similar relationship holds also for German inventors filing a patent in Europe.

be the set of parameters to be estimated, and let $\Gamma[\Theta]$ be the difference between the model-generated moments and the data moments:

$$\Gamma[\Theta_{nj,ik}(t, s)] = C_{nt,is}^{jk} - \hat{C}_{nt,is}^{jk}.$$

Our GMM estimators solve

$$\Theta^* = \arg \min_{\Theta} \sum_{n,i=1}^M \sum_{j,k=1}^J \sum_{t=2001}^{2010} \sum_{s=2001}^t \Gamma[\Theta_{nj,ik}(t, s)]^2. \quad (29)$$

The GMM estimates $\phi_{n,t}^j$, $\psi_{n,t}^j$, $\delta_{n,t}^j$, and ε_{ni}^{jk} separately through two iterating steps. First, given the initial level of $\phi_{n,t}^j$, $\psi_{n,t}^j$, and $\delta_{n,t}^j$, we find the $\hat{\varepsilon}_{ni}^{jk}$ that minimizes

$$\Theta^* = \arg \min_{\theta} \sum_{t=2001}^{2010} \sum_{s=2001}^t \Gamma[\Theta_{nj,ik}(t, s)]^2.$$

Second, we take the estimated $\hat{\varepsilon}_{ni}^{jk}$ from the first step as given and estimate the values of $\hat{\phi}_{n,t}^j$, $\hat{\psi}_{n,t}^j$, and $\hat{\delta}_{n,t}^j$ that minimize equation (29). Then we set $\hat{\phi}_{n,t}^j$, $\hat{\psi}_{n,t}^j$, $\hat{\delta}_{n,t}^j$ as initial value and iterate the preceding two steps until the two sets of estimated parameters converge.

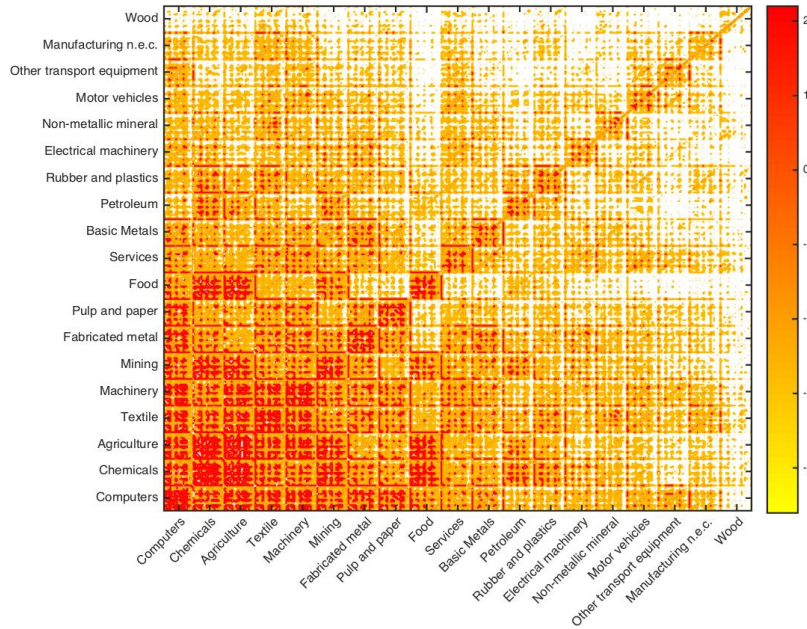
The estimated ε_{ni}^{jk} are then normalized in the manner described by Eaton and Kortum (1999). That is, we fix the within-sector adoption speed in the United States to two years (i.e., taking the midpoint of the evidence reported by Pakes and Schankerman (1984)). The adoption lag in the model is given by $1/(\bar{\varepsilon}_{US,US} + g) = 2$, which implies that $\bar{\varepsilon}_{US,US} = 0.38$ with $g = 0.12$ in our calibration. We then use this restriction to normalize all ε_{ni}^{jk} .

Several interesting findings emerge from estimating the citations function. First, there is considerable heterogeneity in diffusion speed across countries and sectors (i.e., between (nj, ik) cells), and many country-sector pairs diffuse knowledge to each other quite slowly. The mean diffusion lag (i.e., $1/\varepsilon$) is about 12 years for cross-country-sector diffusion (i.e., when $nj \neq ik$), and the within-country-sector diffusion lag averages slightly more than a year.

Figures 1 and 2 are contour maps of the estimated ε_{ni}^{jk} , where Figure 1 presents the observations ordered first by sector and then by country and Figure 2 organizes the observations first by country and then by sector. For example, the red square in the lower-left corner visualized in Figure 1 displays the estimated diffusion speed (in log) of patents in Computers sectors from country i to country j in the same sector. The lower-left sub-block in Figure 2

presents the estimated diffusion speed (in log) between U.S. sectors. The colors in the figure represent different percentiles. A darker color corresponds to a higher value of $\log \varepsilon_{ni}^{jk}$. There is clearly much heterogeneity across country–sector pairs. In general, patents in the computer, electronic, and optical equipment sector and in the chemicals and chemical products sector exhibit the highest diffusion speed; at the other extreme, patents in the wood products sector exhibit the lowest diffusion speed. Note also that sectors whose patents are cited more rapidly likewise tend to cite other sectors more rapidly.

Figure 1: Contour map of $\log(\varepsilon_{ni}^{jk})$ by sector

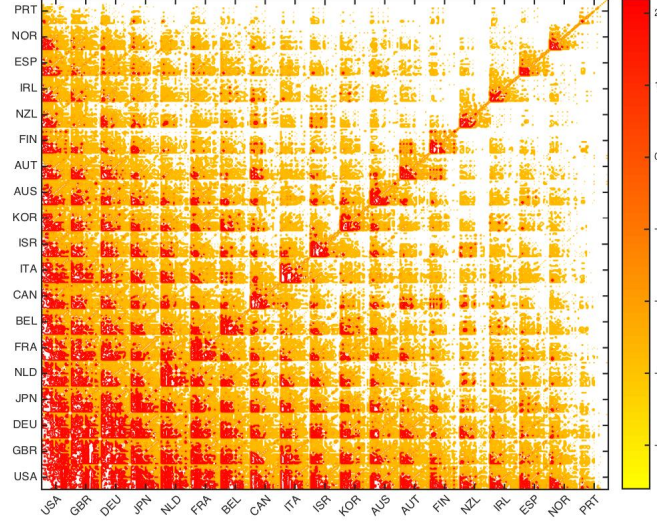


Notes: The y -axis represents citing sector-countries and the x -axis the cited sector-countries. Only sectors are labeled due to limited space. Sectors are ranked by their average speed of citation. The color in the figure represents different percentiles of $\log(\varepsilon_{ni}^{jk})$.

New knowledge created in the United States, Japan, Germany, and the United Kingdom diffuses more rapidly, on average, than does new knowledge from other countries. We find that new knowledge created in Portugal, Norway, and Spain diffuses the slowest. Countries that diffuse knowledge (are cited) rapidly also tend to acquire new knowledge from other countries (cite others) rapidly. Since the advanced innovative countries also share knowledge faster among themselves than they do with less developed or less innovative countries, as discussed in Section 4, knowledge diffusion tends to magnify the specialization effect of R&D re-allocation across country-sectors. We will quantify the role of heterogenous spillovers in

Section 6.

Figure 2: Contour map of $\log(\varepsilon_{ni}^{jk})$ by country



Notes: The y -axis represents citing country-sectors and the x -axis the cited country-sectors. Only countries are labeled in the figure due to limited space. Countries are ranked by their average speed of citation. The color in the figure represents different percentiles of $\log(\varepsilon_{ni}^{jk})$.

5.1.2 Innovation Parameters

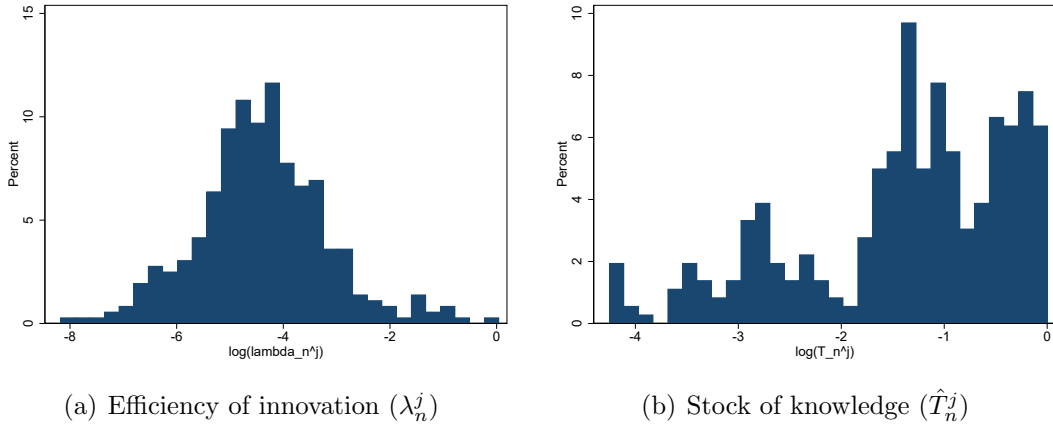
Given data for R&D intensity, together with the calibrated values for trade costs d_{in}^j (estimated by gravity regressions), for the production input-output linkages parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$ (estimated using the U.S. input-output table for 2005), and for knowledge spillovers (estimated via patent citation data), as well as calibrated values for $\{\theta, \rho\}$ (taken from the literature) and g , we use the trade and growth blocks of our model to calibrate the parameters of innovation: $\{\beta_r, \lambda_n^j\}$. With this algorithm, we also obtain a value for \hat{T}_n^j in the initial BGP associated to the growth rate g . We set the elasticity of trade $\theta = 4$ as in Waugh (2010) and the discount factor to $\rho = 0.9$, which implies an annual interest rate of 4%.²² We assume a growth of income on the BGP of $g_y = 2.8\%$. This corresponds to a growth rate for the stock of knowledge on the BGP of $g = 12\%$ (see equation B.8 in Appendix B). The

²²For simplicity, we choose a uniform trade elasticity across sectors. In our set-up this parameter also affects profits and the value of an innovation, and thus a sector-specific elasticity would significantly complicate the closed-form expressions. In a robustness exercise we compute welfare gains using heterogeneous trade elasticity across sectors from Yilmazkuday, Yi, and Giri (2018) and found that the gains are slightly larger and more disperse than with homogeneous elasticity. These results are available upon request.

algorithm used to calibrate the innovation parameters is described in detail in Appendix F. Basically, we proceed in four steps. First, we guess a value of \hat{T}_n^j . Second, we follow the standard procedure in solving for the model's static equilibrium. Third, we solve for the growth equilibrium, and fourth, we iterate to find a fixed point solution in \hat{T}_n^j .²³

We obtain a value for the elasticity of research, β_r of 0.45. This value is consistent with Acemoglu et al. (2018), who find an elasticity of successful innovation with respect to R&D of 0.5.²⁴ The efficiency of innovation (λ_n^j) and the stock of knowledge (\hat{T}_n^j) are heterogeneous across our sample of country-sectors. In particular, λ_n^j has a mean of 0.032 and a standard deviation of 0.10 (see Figure 3(a)). The stock of knowledge has a mean of roughly 0.37, with a standard deviation of 0.28 (see Figure 3(b)). Appendix E presents, by sector and by country, the dispersion in λ_n^j and \hat{T}_n^j .

Figure 3: Efficiency of innovation and stock of knowledge



Notes: This figure shows the dispersion of the exogenous component of the innovation efficiency in each country-sector (λ_n^j) and of its endogenous component, which corresponds to the stock of knowledge of each country-sector (\hat{T}_n^j). Logarithm values are used for both variables.

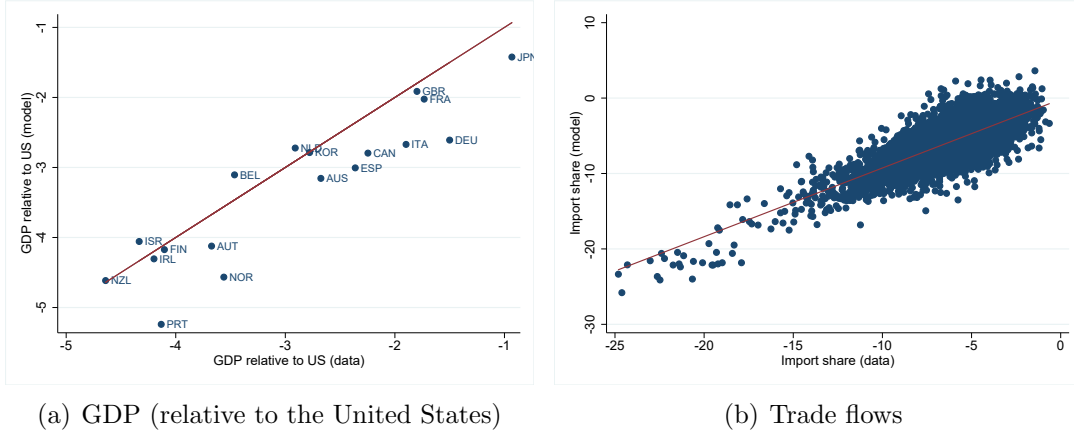
²³Allen, Arkolakis, and Li (2015) provide sufficient conditions for existence and uniqueness of the trade block of the model, given T_n^j . The Frobenius theorem and the growth block of the model provide sufficient conditions for uniqueness of g and T_n^j given the static trade equilibrium. Kucheryavyy, Lyn, and Rodríguez-Clare (2016) study the existence and uniqueness of equilibrium in a multi-sector model with external economies and without knowledge spillovers. In our quantitative analysis, we check that systematically varying the initial guess of T_n^j in our given calibration leads us to the same fixed point solution for \hat{T}_n^j .

²⁴It is also consistent with an external economies of scale parameter of 0.125 ($= \beta_r/\theta$), which is in the ballpark of what Kucheryavyy, Lyn, and Rodríguez-Clare (2016) found.

5.2 Validation

Our calibration strategy delivers wages, relative income, sector productivity, and trade flows that are broadly consistent with those observed in the data. The correlation between gross domestic product (GDP) in our model and in the data is about 0.98. The model performs well at matching the dispersion of GDP per capita across countries, which is roughly 0.3 in both cases (see Figure 4(a)). The correlation between wages in the model and in the data is around 0.65. Finally, the correlation between trade flows in the model and in the data is roughly 0.8 (see Figure 4(b)).²⁵

Figure 4: GDP and trade flows in the model and in the data



Notes: This figure provides validation of our calibration on GDP (relative to the United States) and trade shares, which are computed as $\log(\pi_{in}^j / \pi_{nn}^j)$.

Next, following Levchenko and Zhang (2016) we compare the productivity estimates obtained by our algorithm, $(T_n^j)^{1/\theta}$, with data on labor productivity computed as real value added per worker (VAL_n^j) by running the following regression:

$$\log VAL_n^j = \beta \log (T_n^j)^{1/\theta} + \delta_n + \delta_j + \varepsilon_n^j. \quad (30)$$

Country and sector fixed effects (δ_n and δ_j , respectively) are included so the average productivity levels in individual countries and sectors are netted out. Table 1 shows that there is a positive and statistically significant relationship between labor productivity in the data and productivity in our model.

²⁵We do not match exactly trade flows from the data because our trade costs are obtained using gravity regressions at the sector level, from which it follows that predicted trade flows are measured with some error.

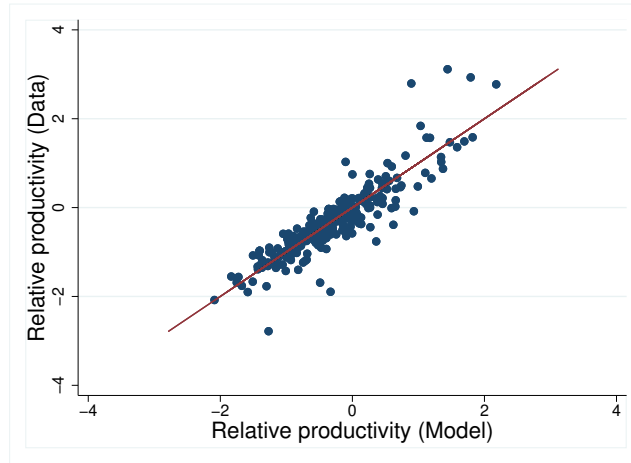
Table 1: Comparison of our calibrated productivity and labor productivity in the data

	(1)	(2)	(3)
$\log(T_n^j)^{1/\theta}$	0.705*** (0.174)	1.248*** (0.234)	1.105*** (0.153)
N	281	281	281
R-squared	0.056	0.548	0.819
Country FE	No	No	Yes
Sector FE	No	Yes	Yes

Notes: This table reports the results of comparing our productivity with empirical labor productivity without fixed effects, with sector fixed effects and with both country and sector fixed effects. Robust standard errors are reported in parentheses; *** indicates significance at 0.1%.

Figure 5 then compares the relative productivity from the data with the predicted value from regression (30). It shows that the model does a good job in fitting the labor productivity dispersion across countries and this result is not driven by outliers.

Figure 5: Relative productivity in the model and in the data



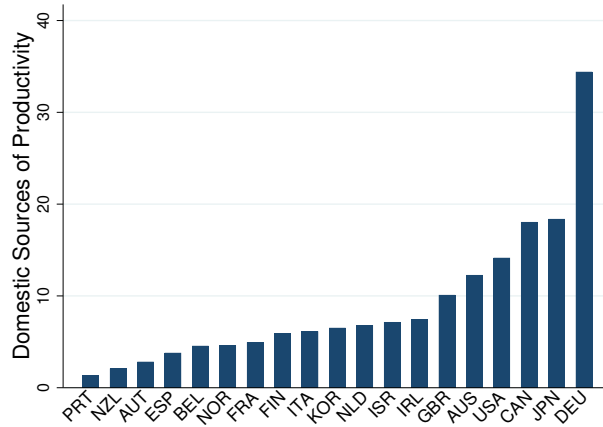
Notes: This figure provides validation of our calibration on productivity (relative to the United States).

5.3 Decomposing the Sources of Growth

For each country in our sample, we can use equation (27) to decompose quantitatively the contribution of domestic and foreign R&D on that country's productivity. There is large

variation across countries in the contribution of domestic R&D to productivity. In Portugal, less than 2% of productivity is explained by domestic innovation, while in Germany, domestic innovation accounts for more than one-third of total productivity followed by almost 20% in Japan and Canada. In the United States, despite being the most innovative country in our sample, domestic innovation accounts for 14% of total productivity (see Figure 6). The reason is that the United States benefits from foreign R&D through knowledge spillovers, especially so from other innovative countries. In fact, the correlation between R&D intensity and the contribution of domestic sources of productivity is only 0.47, as heterogeneous knowledge spillovers play a big role in determining the contribution of foreign sources of innovation. Moreover, the United States specializes more in highly connected sectors such as Computers and Chemicals sectors (see Figure 1) which allows it to benefit from cross-sector knowledge spillovers as well.

Figure 6: Domestic Sources of Productivity



Notes: This figure shows the percentage contribution of domestic sources to productivity computed based on equation (31).

We find that 27% in the variation of productivity across country-sectors is explained by domestic innovation intensity, and 73 % is explained by knowledge spillovers. Since innovation efficiency and R&D intensity are highly correlated in our baseline calibration, these results are consistent with the findings in Sampson (2019), who finds that variation in innovation efficiency explains around 30% of income per capita differences.

Equation (27) can also be used to decompose the main sources of knowledge around the world. We denote with \hat{T}_{ni}^{jk} the contribution of each country-sector ik to the stock of

knowledge of each country-sector nj . Using Equation (20) this can be written as

$$\hat{T}_{ni}^{jk} = \frac{\varepsilon_{ni}^{jk}/g}{\varepsilon_{ni}^{jk} + g} \lambda_i^k \hat{T}_i^k (s_i^k)^{\beta_r}.$$

Summing across each country-sector nj yields the contribution of each country-sector ik to the total stock of knowledge of the world:

$$\sum_{n=1}^M \sum_{j=1}^J \hat{T}_{ni}^{jk} = \underbrace{\lambda_i^k \hat{T}_i^k (s_i^k)^{\beta_r}}_{\text{Innovation intensity}} \underbrace{\sum_{n=1}^M \sum_{j=1}^J \frac{\varepsilon_{ni}^{jk}/g}{\varepsilon_{ni}^{jk} + g}}_{\text{Knowledge spillovers}}. \quad (31)$$

The first component of equation (31) captures the contribution of *innovation* in country-sector ik to the world's stock of knowledge; the second component captures the contribution of knowledge diffusion originating in country-sector ik to the world's stock of knowledge. The contributions to that stock of knowledge from country-sectors are increasing in the speed at which their knowledge diffuses.

Aggregating across all k sectors, we find that the United States, Germany and Japan are the main contributors to world knowledge, both because they are the most innovative countries and because they are better at spreading knowledge around the world. In contrast, Portugal, Ireland, Norway and New Zealand are the countries that contribute the least. Aggregating across all i countries, we find that the Computer, electronic and optimal equipment, Chemicals and chemical products, and Machinery and equipment sectors are the main contributors to the world stock of knowledge, while Wood and wood product, Manufacturing n.e.c. and recycling, and Coke and refined products are the sectors that contribute the least to the world stock of knowledge.

Furthermore the correlation between the contribution of country-sector ik to the world knowledge stock and the innovation intensity component is around 0.97, indicating that more innovative country-sectors contribute more to world productivity. The correlation between the contribution of country-sector ik and the strength of knowledge spillovers is around 0.80, indicating that spillover effects play an important role in understanding a country-sector's contribution to world productivity. Interestingly, the innovation intensity and knowledge spillovers components are positively correlated, with a correlation of around 0.65. That is, in our sample of analysis, those country-sectors that are more innovative are also better at

spreading knowledge around the world.

5.4 Counterfactual Analysis

We analyze the effect of trade liberalization on innovation, long-run growth, and comparative advantage. In particular, we simulate a uniform and permanent reduction of 25% in trade costs (in terms of $d_{in}^j - 1$) across all sectors j and country pairs i, n . All other parameters are kept fixed at their calibrated values. We start by briefly describing the algorithm we developed to compute the counterfactual BGP. Next, we report the main results from our multi-country and multi-sector endogenous growth model, which features heterogeneous interlinkages between production and knowledge flows. Finally, we analyze the role of the main channels in our model: (i) endogenous productivity, (ii) input-output linkages in production, and (iii) heterogeneous knowledge spillovers.

5.4.1 The Algorithm

In our calibration procedure, we took the economy's growth rate, g , as given, and obtained a value of \hat{T}_n^j associated to that growth rate. Yet, in our endogenous growth model, g and hence \hat{T}_n^j change across counterfactuals when there are changes in trade costs. Changes in g and \hat{T}_n^j are induced by changes in the innovation intensity, s_n^j , and also by knowledge diffusion, as explained in Section 4. Our algorithm to solve for the counterfactual equilibrium exploits the properties of the Frobenius theorem. Using our calibrated values for $\{\theta, \rho, \gamma^j, \gamma^{jk}, \alpha^j, \beta_r, \lambda_n^j, \hat{T}_n^j, \varepsilon_{ni}^{jk}\}$ and the new value of trade costs d_{in}^j , we can compute the new static trade equilibrium. We thus obtain a new optimal R&D intensity s_n^j and use the Frobenius theorem to derive the new growth rate (g) and associated stock of knowledge (\hat{T}_n^j). This process is repeated by iterating over equation (20) until $g(t-1) = g(t)$. The associated eigenvector delivers the new \hat{T}_n^j .

5.4.2 Innovation, Growth, and Comparative Advantage

We now quantify the effects of trade liberalization—on innovation, growth, and comparative advantage—that arise via the mechanisms described in Section 4.

In Section 4 we derived an analytical expression for the R&D allocation across countries and sectors in two extreme cases—autarky and free trade. Here, we examine whether

our counterfactual experiment is consistent with the intuition underlying the mechanism described in that section. Following trade liberalization, R&D tends to be re-allocated toward a country's sectors that have a comparative advantage in production—thereby, strengthening the specialization effects of trade liberalization.

Motivated by equation (26), we examine the following regression using both the baseline and counterfactual outcomes:

$$\log \left(\frac{s_n^j}{\sum_j s_n^j} \right) = \beta_0 + \beta_1 \log(\text{ICA}_n^j) + \beta_2 \log(\text{DCA}_n^j) + \beta_3 \log(\text{PCA}_n^j) + f_n + f_j + \mu_n^j. \quad (32)$$

In this equation, ICA_n^j is the exogenous component of the comparative advantage in innovation (i.e. based on λ_n^j), DCA_n^j denote the comparative advantage in within-country-sector diffusion speed (based on Γ_n^j) and PCA_n^j is the comparative advantage in production (based on $T_n^j(c_n^j)^{-\theta}$). All of these terms are measured as in Hanson, Lind, and Muendler (2015) by applying the double normalization—we first obtain a measure of the country's absolute advantage in a sector via normalizing λ_n^j , Γ_n^j or $T_n^j(c_n^j)^{-\theta}$ by the global mean for that sector; then, we normalize the absolute advantage by its countrywide mean.²⁶ The variables f_n and f_j represent country and sector fixed effects, respectively. The sector fixed effects absorb the role of world demand and world aggregate technology, as shown in equation (26).

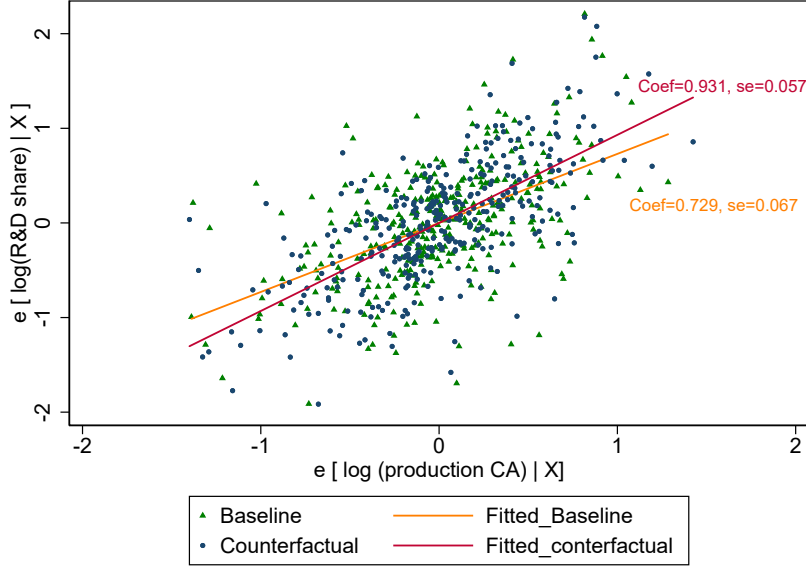
Figure 7 plots the post-estimation partial relationship between R&D share and comparative advantage in production, while controlling for other explanatory variables. As expected, sectors with a higher comparative advantage in production receive a greater share of R&D investment—in the baseline case and also when trade costs are reduced. In the counterfactual scenario, though, the relationship is stronger as manifested by the steeper fitted line and an increase, from 0.729 to 0.931, in the coefficient of production comparative advantage (β_3). This result implies that comparative advantage in production now plays an even larger role in directing research efforts.²⁷

Next, we evaluate the effect of trade liberalization on comparative advantage. Comparative advantage in production is endogenous in our model because T_n^j evolves endogenously in response to changes in trade costs and in the subsequent movements of innovation allocation.

²⁶More precisely, $\text{PCA} = \frac{T_n^j(c_n^j)^{-\theta} / \frac{1}{M} \sum_n T_n^j(c_n^j)^{-\theta}}{\frac{1}{J} [\sum_j T_n^j(c_n^j)^{-\theta} / \frac{1}{M} \sum_n T_n^j(c_n^j)^{-\theta}]}$, $\text{DCA} = \frac{\Gamma_n^j(c_n^j)^{-\theta} / \frac{1}{M} \sum_n \Gamma_n^j(c_n^j)^{-\theta}}{\frac{1}{J} [\sum_j \Gamma_n^j(c_n^j)^{-\theta} / \frac{1}{M} \sum_n \Gamma_n^j(c_n^j)^{-\theta}]}$ and $\text{ICA} = \frac{\lambda_n^j / \frac{1}{M} \sum_n \lambda_n^j}{\frac{1}{J} [\sum_j \lambda_n^j / \frac{1}{M} \sum_n \lambda_n^j]}$.

²⁷These results are consistent with suggestive empirical evidence provided in Appendix H.

Figure 7: Re-allocation of R&D and production comparative advantage



Notes: This figure shows the partial residual plots of regression (32). The coefficients correspond to β_3 for the baseline and counterfactual cases, respectively. Standard errors associated with the estimated coefficients are also included.

Hence, we now examine how the dispersion in T_n^j changes following liberalization. First, if we lump all countries and sectors together, then the standard deviation of $\log(T_n^j)$ increases from 1.05 to 1.10. Second, when decomposing this increase in dispersion into country and sector dimensions, we find that—in most countries—the comparative advantage in T_n^j across sectors becomes more disperse following a trade liberalization as most countries see an increase in the standard deviation of their comparative advantage in production across sectors (dispersion increases from 0.79 to 0.81). For a given sector, countries similarly exhibit more dispersion in their T_n^j after the reduction in trade costs (from 0.62 to 0.66).

The re-allocation of R&D across sectors has both growth and welfare effects. We focus now on growth effects and analyze welfare effects in Section 5.4.3. As a result of the re-allocation effect of R&D, growth jumps to a higher value in the new BGP. Growth of the stock of knowledge, g increases from 12% to 13.9%—an increase that is nonlinear with the extent of the trade liberalization, as shown by Figure 11. After a 25% (resp. 50%) trade liberalization growth increases to 13.9% (resp. 18.5%).

5.4.3 Welfare Gains from Trade

We compare the baseline and the counterfactual BGP in terms of welfare gains from trade following trade liberalization. In our model, welfare is defined in equivalent units of consumption. Equation (1) can be used to obtain lifetime utility in the initial (baseline) BGP as

$$\bar{U}_i^* = \int_{t=0}^{\infty} e^{-\rho t} \log \left(\hat{C}_i^* e^{g^{*t}} \right) dt,$$

and in the counterfactual BGP as

$$\bar{U}_i^{**} = \int_{t=0}^{\infty} e^{-\rho t} \log \left(\hat{C}_i^{**} e^{g^{**t}} \right) dt;$$

here the superscript with * and ** mark the baseline and counterfactual BGP, respectively.

Welfare gains are defined as the amount of consumption a consumer is willing to forgo in the counterfactual BGP to remain at the same level as in the initial BGP. We use λ_i to represent this amount, which is obtained as

$$\bar{U}_i^*(\lambda_i) = \bar{U}_i^{**}.$$

This equation yields

$$\rho \log \left(\hat{C}_i^* \lambda_i \right) + g^* = \rho \log \left(\hat{C}_i^{**} \right) + g^{**}. \quad (33)$$

Welfare gains depend on changes in normalized consumption between the BGPs and the change in growth rates. Because we are analyzing only changes across BGPs, dynamic gains do not include the transition. These gains are “dynamic” because they reflect the gains that account for changes in the stock of knowledge across counterfactuals. Dynamic gains are therefore computed by letting \hat{T}_i^j vary across counterfactuals.

Welfare gains from trade are large and disperse, with a mean of 23% and a standard deviation of 7%. They range from 8% in Japan to 34% in Germany (see Figure 17 in Appendix G for the distribution of welfare gains across countries).

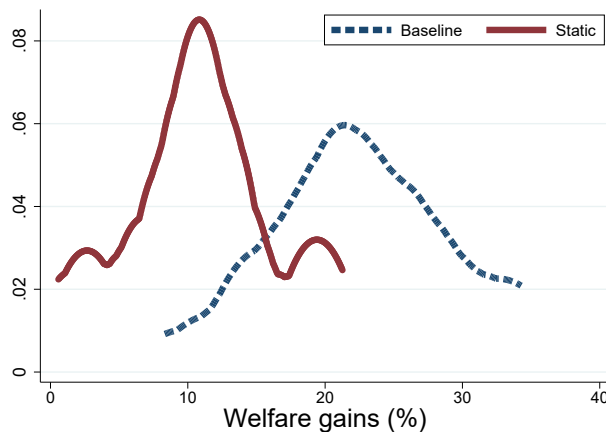
6 The Role of the Main Channels in the Model

Our mechanism hinges on three key ingredients: (i) endogenous productivity, (ii) multiple sectors and (ii) heterogeneous knowledge spillovers across countries and sectors. In this section, we explore the importance of each ingredient in driving our main results.

6.1 Endogenous Productivity

We recalibrate a version of our baseline model in which we do not allow for innovation or diffusion; hence productivity is taken as given. That is, we simulate our model while holding \hat{T}_i^j constant across counterfactuals. This specification corresponds to Caliendo and Parro (2015). Figure 8 compares welfare gains from trade in our baseline model with those static gains in which the stock of technology is kept unchanged across counterfactuals. It shows that welfare gains from a 25 % uniform trade liberalization are smaller and less disperse than those in our baseline model. Indeed, these are the gains which, as predicted by standard static models of trade, are driven by increased specialization and comparative advantage. The difference between the two gains is a measure of the dynamic gains from trade. The cross-country distribution of static gains is shifted to the left and have a lower standard deviation, which implies that dynamic gains are sizable and more disperse (see Table 3).

Figure 8: Welfare gains from trade (% change): The role of endogenous productivity

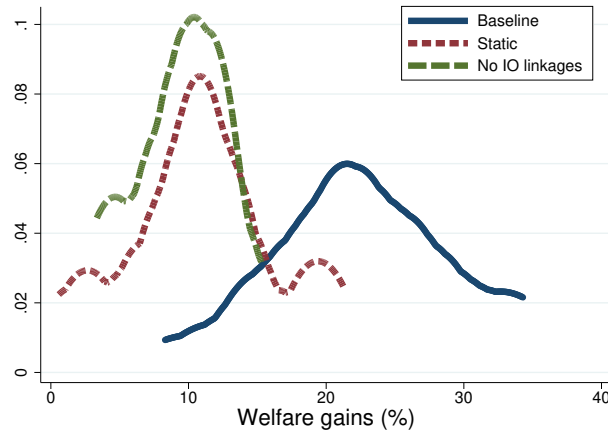


Notes: This figure plots the distribution of welfare gains across countries and compares gains from trade in our baseline dynamic model (solid line) with those of a static model in which \hat{T}_n^j is kept unchanged across counterfactuals (dashed line).

6.2 Input-Output Linkages

To explore the importance of input-output linkages in driving our results, we shut down intersectoral input-output linkages by assuming that a producer in a sector j can only source the composite good from the same sector. That is, we set $\gamma^{jk} = 1 - \gamma^j$ for $k = j$ and $\gamma^{jk} = 0$ otherwise. We then recalibrate the model to match the same moments of the data as those we matched in our baseline calibration. We find that the elasticity of innovation in this case is $\beta_r = 0.57$. With the same counterfactual trade liberalization of a 25 percent uniform reduction in trade barriers, we find that trade liberalization has a small effect on R&D intensity. Hence, the trade-induce R&D reallocation effect in this case is smaller than in our baseline exercise. As a result, the dispersion of productivity remains almost unchanged and the effect of trade on growth is much smaller than in our baseline exercise. These results translate into lower and less disperse welfare gains from trade. Indeed, Figure 9 shows that the welfare gains from trade in this case behave similarly to the static case where there are no productivity dynamics.

Figure 9: Welfare gains from trade (% change): The role of input-output linkages



Notes: This figure plots the distribution of welfare gains across countries and compares gains from trade in our baseline dynamic model (solid line) with those of a model in which \hat{T}_n^j is kept constant across counterfactuals (dashed line) and a model without intersectoral production linkages (long-dashed line).

6.3 Heterogeneous Cross-country-sector Knowledge Spillovers

To examine the effect of knowledge diffusion on growth, we recalibrate our baseline model in two ways. First, we consider the case of homogeneous diffusion across all country-sector

pairs, in which we set $\varepsilon_{ni}^{jk} = \varepsilon, \forall i, n, j, k$. Here ε is the average speed of diffusion estimated in the data. Second, we consider the case of no intersectoral diffusion by setting the diffusion parameters ε_{ni}^{jk} to a very small value of 0.0001, for all $i \neq n$ and $k \neq j$ while keeping ε_{nn}^{jj} the same as estimated in the data.²⁸ We recalibrate the parameters, β_r, λ_n^j and \hat{T}_n^j , while keeping the same input-output linkage parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$, R&D intensity (s_n^j) and growth rate (g) values than in the baseline model.

Calibration results We obtain $\beta_r = 0.37$ in the case of homogeneous diffusion and $\beta_r = 0.10$ in the case of no diffusion. When diffusion is homogeneous, all countries and sectors reach the same level of productivity on the BGP as shown by equation (27). Hence, welfare effects driven by standard specialization effects are no longer present. If diffusion across country-sectors (i.e., the off-diagonal of the diffusion matrix) were equal to zero, country-sectors would not necessarily grow at a common rate and we could not apply the Frobenius theorem. Their growth rate would be determined by their R&D intensity. With almost zero spillovers across country-sectors, the growth rate will be the same despite very different research intensities in the data. For that reason, our calibration finds a very low value of β_r , putting less weight to the research intensity in determining growth rates in order to be consistent with a common growth rate.

We find that, in contrast to our baseline model with heterogeneous knowledge spillovers, the two alternative models considered in this section cannot match the relative productivity as observed in the data. In the case of homogeneous knowledge spillovers, there is no dispersion in production either across countries nor across sectors along the BGP, as all countries and sectors benefit from world innovation equally. In the case of negligible knowledge spillovers the correlation between productivity in the model and in the data is very small and statistically non-significant, as shown in Table 2 which reports the results from estimating equation (30) using the calibrated productivity from the model with negligible diffusion.

²⁸The Frobenius theorem is valid only if there is at least some diffusion across all country-sector pairs. Setting ε_{ni}^{jk} to a very low number enables us to use the Frobenius theorem's properties while allowing for very slow (or virtually no) diffusion.

Table 2: Comparison of the calibrated productivity with negligible spillovers to measured labor productivity

	(1)	(2)	(3)
$\log (T_n^j)^{1/\theta}$	-0.0912 (0.441)	0.0776 (0.387)	0.0996 (0.250)
N	281	281	281
R-squared	0.000	0.458	0.798
Country FE	No	No	Yes
Sector FE	No	Yes	Yes

Notes: This table reports the results of comparing the estimated productivity from a model with negligible spillovers with empirical labor productivity based on the OECD STAN database. Robust standard errors are reported in parentheses.

Trade-driven R&D reallocation, growth and welfare In Section 4, we showed that heterogeneous knowledge spillovers—as measured in the data—increase the dispersion of productivity and comparative advantage across country-sectors, because innovative country-sectors are also better at absorbing knowledge from each other. For the same reason, we also argued that heterogeneous knowledge spillovers amplify the trade-driven R&D reallocation effect. We show that it is indeed the case.

First, we find that when cross-country-sector spillovers are homogeneous or negligible comparative advantage in production is less disperse across country-sectors than in our baseline model—as demonstrated by a smaller range in the x-axis of the two panels of Figure 10 compared to that in Figure 7. Second, the elasticity of research intensity with respect to comparative advantage in production is lower: It drops from 0.73 in our baseline model with heterogeneous spillovers to 0.51 in the case of negligible cross-country-sector spillovers, and to 0.63 in the case of homogeneous diffusion. Third, trade liberalization induces a weaker resource reallocation effect, as manifested by a smaller shift in the slope of the fitted lines of Figure 10. Although innovation resources are still allocated more towards sectors with comparative advantage—the elasticity changes from 0.505 to 0.582 in the case of negligible diffusion and from 0.627 to 0.772 in the case of homogeneous diffusion—this shift is much less noticeable than in our baseline model with heterogeneous knowledge spillovers (see Figure 7).

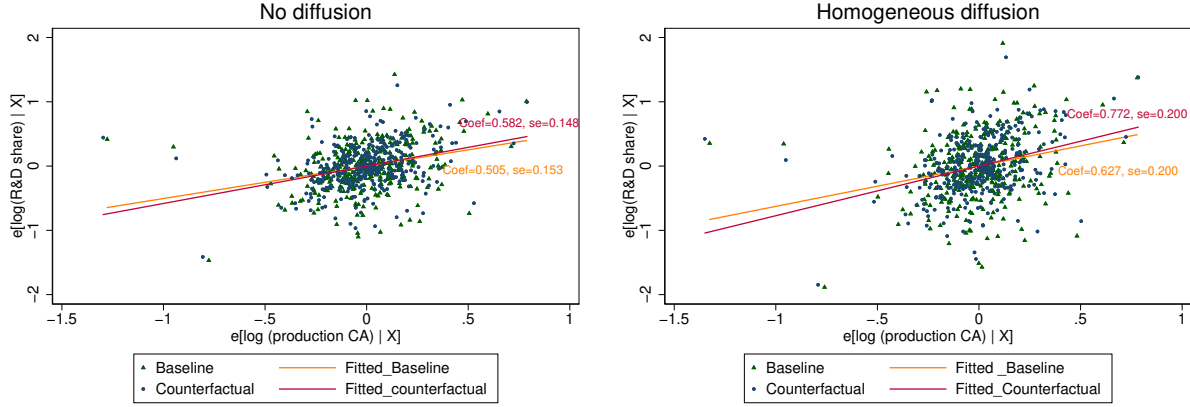
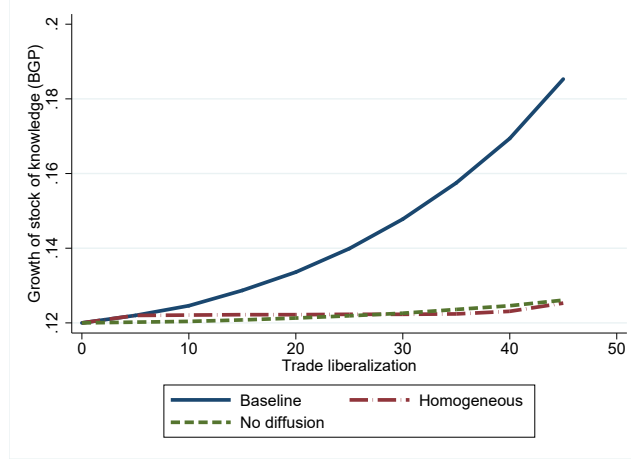


Figure 10: Re-allocation of R&D and production comparative advantage

Notes: This figure shows the partial residual plots of regression (32) for the case of negligible knowledge spillovers (left panel) and homogeneous knowledge spillovers (right panel). Coefficients correspond to β_3 for baseline (orange) and counterfactual (red) for each case. Robust standard errors are included.

Consistent with a weaker specialization effect in the cases of negligible diffusion and homogeneous diffusion, trade liberalization has a lower impact on growth rates than in our baseline model. A 25% reduction in trade costs increases the growth rate to 0.122 (resp. 0.1202) when there is homogeneous diffusion (resp. negligible diffusion). Furthermore, the rate of increase of BGP growth with the size of trade liberalization is slower than in our baseline model with heterogeneous knowledge spillovers (see Figure 11).

Figure 11: Effect of trade liberalization of BGP growth



Notes: This figure shows the effect on the BGP growth rate of the stock of knowledge depending on the magnitude of the trade liberalization. The x -axis represents the decline in $d_{in}^j - 1$. The solid, dashed and dotted correspond to: (i) baseline, (ii) (almost) no cross-country and cross-sector knowledge spillovers, and (iii) homogeneous diffusion across countries and sectors.

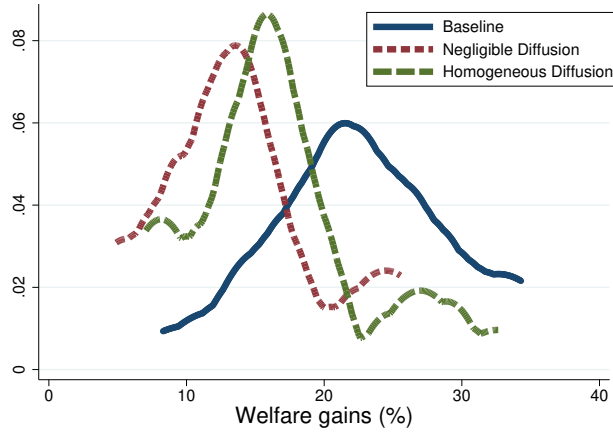
Finally, we compare welfare gains from trade in our baseline model to those in the alternative models. Compared to our baseline, the cases of homogeneous or negligible diffusion also deliver lower and less disperse gains from trade (see Table 3). The reason why the gains from trade with homogeneous diffusion are less disperse is because, in that case, BGP relative productivity is the same for all countries and sectors. Hence, the standard specialization gains from static models of trade are no longer present. The dispersion of gains both in the negligible spillover case and in the case of homogeneous diffusion is mainly driven by dispersion in the efficiency of innovation. Hence, modeling heterogeneous cross-country-sector knowledge spillovers has important consequences for the magnitude and distribution of welfare gains from trade across countries.

Table 3: Welfare gains (%) from trade

Model	Mean	Std. Dev.	Min	Max
Baseline	22.63	6.90	8.33	34.27
Static	10.92	5.87	0.58	21.26
Homogeneous diffusion	16.60	6.80	7.05	32.64
Negligible diffusion	16.53	6.08	4.89	25.54

Note: This table reports welfare gains under four versions of our model: (i) baseline model; (ii) static model; (iii) homogeneous diffusion; and (iv) (almost) negligible cross-country cross-sector knowledge spillovers.

Figure 12: Welfare gains from trade (% change): The role of heterogeneous knowledge spillovers



Notes: This figure plots the distribution of welfare gains across countries and compares gains from trade in our baseline model (solid line) with those of a model with negligible cross-country-sector diffusion (dashed line) and homogeneous spillovers (long-dashed line).

7 The Economic Consequences of Protectionism: Brexit

In this section, we evaluate the economic consequences of protectionist measures in a subgroup of countries. In particular, we consider a “Brexit”-inspired exercise in which the United Kingdom’s trade barriers with respect to its European Union (EU) trading partner increase by 25%, which triggers a simultaneous tit-for-tat retaliation by EU countries.²⁹ Note

²⁹This is considered the worst case scenario according to studies done by the International Monetary Fund. For details, see <https://www.imf.org/en/Publications/CR/Issues/2018/11/14/United-Kingdom-2018-Article-IV-Consultation-Press-Release-Staff-Report-Staff-Statement-and-46353>.

that since we do not model tariffs explicitly, tariff revenues are not included in our model; trade costs are driven by iceberg transport costs. Therefore, we perform our “Brexit” exercise by changing these iceberg trade costs, which include both tariffs and non-tariff barriers. We then study the role of diffusion in driving the results.

Using our baseline model, we find that the “Brexit” counterfactual lowers the world BGP growth rate from 3% to 2.97%. All countries, except for Norway, experience welfare losses. The United Kingdom experiences the highest loss (-10%), followed by Ireland (-5%) and the Netherlands (-2%).³⁰ The United States loses by -0.5%. Norway experiences an increase in welfare of 0.32%. Norway is not part of the EU, but it belongs to the EU’s Single Market. The increase in welfare may reflect diversion of production away from the United Kingdom as European countries start demanding more products from Norway.

Our “Brexit” exercise has important consequences for R&D investment and output, not only in the United Kingdom, but also in the rest of the world. The United Kingdom and Ireland experience the largest decrease in R&D investment (-8.5% and -2%, respectively) and output (-9% and -5%, respectively). Interestingly, other EU countries experience increases in R&D, as there is a trade diversion away from the United Kingdom and towards these countries.

Changes in R&D and output at the country level are associated with changes in R&D investment and production across sectors within a country. In the United Kingdom, all sectors experience a reduction of innovation, but R&D intensity reallocates from Chemicals and chemical products, Motor vehicles, and Computer, electronic and optical equipment towards Other transport equipment, Coke, refined petroleum products and Food products. Production, instead, reallocates from Rubber and plastic products and Machinery and Equipment towards Mining and Chemicals and Chemical Products.

Next, we explore the role of knowledge spillovers in driving these results. To do that, we consider the same within-country-sector diffusion parameters as in the baseline model, but shut down across-country-sector knowledge spillovers. We find that the welfare losses from Brexit are lower than in our baseline model. Hence, diffusion tends to propagate negative shocks more strongly across countries. Furthermore, without diffusion, Australia, Israel, and New Zealand do not experience welfare losses, as these countries tend to be more isolated in

³⁰In a recent study, Dhingra et al. (2017) find welfare losses for the United Kingdom from leaving the EU to be between 6.3% and 9.4%.

terms of knowledge spillovers.

We also find that, when there is negligible cross-country-sector knowledge spillovers, the effects on output and R&D spending are less negative. In the United Kingdom, not all sectors lose from increases in trade barriers when there is negligible diffusion. Coke and refined petroleum and Wood sectors increase their R&D investment, as these sectors are more isolated in the knowledge space; Rubber and plastic products, Chemicals and chemical products, and Basic metals experience the largest losses, as these sectors are highly connected through knowledge spillovers. Production in the United Kingdom reallocates from Rubber and plastic products and Machinery and equipment towards Chemical and chemical products and mining.

8 Concluding Remarks

We develop a quantitative framework to study the effect of interlinkages among trade, knowledge flows and production on innovation, comparative advantage, and growth. Changes in trade costs have a quantitatively important effect on innovation. Following trade liberalization, R&D investment is re-allocated towards sectors in which the country has a comparative advantage in production. Knowledge diffusion amplifies this effect, as comparative advantage is re-allocated towards sectors with greater knowledge flows. Furthermore, knowledge spillovers allow sectors in a country to benefit from a larger pool of ideas, increasing growth.

We evaluate welfare gains from trade, and distinguish between static gains, which are driven by increased specialization, and dynamic gains from trade, which are driven by innovation and knowledge diffusion. Our model generates quantitatively larger and more dispersed gains than models without our channels (i.e. endogenous comparative advantage, heterogeneous knowledge spillovers, heterogeneous input linkages).

We can use our model to perform additional counterfactual exercises. For instance, we can analyze the effect of sector-specific tariffs on the structure of production of a country or a group of countries through the effect on innovation. We leave these issues for future research.

ONLINE APPENDIX

A Model Equations

For each $i = 1 \dots M$ and $n = 1 \dots M$, the endogenous variables are

$$\{\pi_{int}^j, T_{it}^j, c_{it}^j, W_{it}, P_{nt}^j, X_{nit}^j, X_{nt}^j, P_{nt}, Y_{nt}, \Phi_{nt}^j, C_{nt}, s_{nt}^j, V_{nt}^j, \Pi_{nt}^j\}$$

The corresponding equations are as follows.

(1) Probability of Imports

$$\pi_{nit}^j = T_{it}^j \frac{(c_{it}^j d_{ni}^j)^{-\theta}}{\Phi_{nt}^j}, \quad (\text{A.1})$$

(2) Import shares

$$X_{nit}^j = \pi_{nit}^j X_{nt}^j. \quad (\text{A.2})$$

(3) Cost of production

$$c_{nt}^j = \gamma^j W_{nt}^{\gamma^j} \prod_{k=1}^J (P_{nt}^k)^{\gamma^{jk}}. \quad (\text{A.3})$$

(4) Intermediate good prices in each sector

$$P_{nt}^j = A^j (\Phi_{nt}^j)^{-1/\theta}. \quad (\text{A.4})$$

(5) Cost distribution

$$\Phi_{nt}^j = \sum_{i=1}^M T_{it}^j (d_{ni}^j c_{it}^j)^{-\theta}. \quad (\text{A.5})$$

(6) Price index

$$P_{nt} = \prod_{j=1}^J \left(\frac{P_{nt}^j}{\alpha^j} \right)^{\alpha^j}. \quad (\text{A.6})$$

(7) Labor market clearing condition

$$W_{nt}L_{nt} = \sum_{j=1}^J \gamma^j \sum_{i=1}^M \pi_{int}^j X_{it}^j. \quad (\text{A.7})$$

(8) Sector production

$$X_{nt}^j = \sum_{k=1}^J \gamma^{kj} \sum_{i=1}^M X_{it}^k \pi_{int}^k + \alpha^j P_{nt} Y_{nt}. \quad (\text{A.8})$$

(9) Income

$$P_{nt}C_{nt} = W_{nt}L_{nt} + \frac{\sum_{j=1}^J \sum_{i=1}^M \pi_{int}^j X_{it}^j}{1 + \theta}. \quad (\text{A.9})$$

(10) Resource constraint

$$Y_{nt} = C_{nt} + \sum_{j=1}^J R_{nt}^j. \quad (\text{A.10})$$

(11) Innovation

$$\dot{T}_{nt}^j = \sum_{i=1}^M \sum_{k=1}^J \varepsilon_{ni}^{jk} \int_{-\infty}^t e^{-\varepsilon_{ni}^{jk}(t-s)} \alpha_s^k T_s^k (s_{is}^k)^{\beta^k} ds. \quad (\text{A.11})$$

(12) R&D expenditures

$$\beta^j \lambda_{nt}^j V_{nt}^j (s_{nt}^j)^{\beta^j - 1} = P_{nt} \bar{Y}_t. \quad (\text{A.12})$$

(13) Value of an innovation

$$V_{nt}^j = \int_t^{\infty} e^{-\int_t^s r_{nu} du} \left(1 - e^{-\varepsilon_{nn}^{jj}(t-s)} \right) \frac{\Pi_{ns}^j}{T_{ns}^j} ds \quad (\text{A.13})$$

(14) Profits

$$\Pi_{nt}^j = \frac{1}{(1 + \theta)} \sum_{i=1}^M X_{it}^j \pi_{int}^j. \quad (\text{A.14})$$

(15) Trade balance

$$\sum_{k \neq j}^J X_{nt}^k \sum_{i \neq n}^M \pi_{nit}^k = \sum_{i \neq n}^M \sum_{k \neq j}^J \pi_{int}^k X_{it}^k. \quad (\text{A.15})$$

B Derivations on the BGP

Here, we derive an expression for the growth rate of the economy along the BGP. We remove time subscripts when we refer to variables on the BGP.

First, note that through technology diffusion, the level of knowledge-related productivity, T_n^j , grows at the same rate for every country n and sector j . Therefore, we can pick country M and sector J 's technology level to normalize every T_n^j . Normalized variables are denoted with a hat. In particular, $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$.

From equation (A.7), X_i^j is normalized as $\hat{X}_i^j = \frac{X_i^j}{W_M}$ for all j . Hence, expenditures grow at a constant rate for all sectors, since π_{in}^j is constant in the BGP (see equations (A.1) and (A.5)). From equations (A.7) and (A.9), $P_n Y_n$ grow at the rate of W_M . Note that $g_{w_n} = g_w$ for all n .

From equation (A.10), by which consumption, C_n and final output, Y_n grow at the same constant rate on the BGP together with the fact that output in each country grows at the same rate, hence $\frac{Y_n}{Y}$ is constant in the BGP implies that the fraction of world output that is invested into R&D, s_n^j is constant on the BGP. This result, together with equation (16), implies that $\frac{V_n^j T_n^j}{W_M}$ is constant along the BGP. We then have from equation (15):

$$\hat{V}_n^j = \Gamma_n^j \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{(1 + \theta)},$$

in which $\hat{V}_n^j = \frac{V_n^j T_n^j}{P_n Y_n}$, $\hat{X}_i^j = \frac{X_i^j}{W_M}$, and $\hat{Y}_n = \frac{P_n Y_n}{W_M}$, with W_M being the nominal wage in the numeraire country M . We impose $r - g/\theta + g > 0$. From equation (10), π_{in}^j is constant along the BGP.

To derive an expression for the BGP growth rate of the real output per capita, Y_n , we start from the fact that $\frac{W_n}{P_n Y_n}$ is constant in steady-state. Hence,

$$g_{Y_n} = g_w - g_{P_n}.$$

Using equation (A.6),

$$g_{P_n} = \sum_{j=1}^J \alpha^j g_{p_n^j}.$$

We then derive the expression for $g_{p_n^j}$ from equations (A.3), (A.4) and (A.5). First, we rewrite equation (A.3) as

$$\frac{c_n^j}{W_n} = \prod_{k=1}^J \left(\frac{p_n^k}{W_n} \right)^{\gamma_n^{jk}}.$$

In growth rates, it becomes

$$g_{\tilde{c}_n^j} = \sum_{k=1}^J \gamma_n^{jk} g_{\tilde{p}_n^k}, \quad (\text{B.1})$$

where $\tilde{c}_n^j = \frac{c_n^j}{W_n}$ and $\tilde{p}_n^k = \frac{p_n^k}{W_n}$. From equation (A.5),

$$g_{\Phi_n^j} = g - \theta g_{c_n^j} = g - \theta g_{c_i^j}.$$

Hence, $g_{c_n^j} = g_{c^j}$ for all n . Normalizing by wages,

$$g_{\tilde{\Phi}_n^j} = g - \theta g_{\tilde{c}_n^j}, \quad (\text{B.2})$$

where $\tilde{\Phi}_n^j = \frac{\Phi_n^j}{W_n^{-\theta}}$

Combining equation (A.4) and (B.2) implies that

$$g_{\tilde{p}_n^k} = -\frac{1}{\theta} g + g_{\tilde{c}^k}. \quad (\text{B.3})$$

Substitution into (B.1) and using $\sum_{k=1}^J \gamma^{jk} = 1 - \gamma^j$, we get

$$g_{\tilde{c}^j} = -\frac{(1 - \gamma^j)}{\theta} g + \sum_{k=1}^J \gamma^{jk} g_{\tilde{c}^k}. \quad (\text{B.4})$$

We can express the previous expression in matrix form so that

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{1}{\theta} g \begin{bmatrix} 1 - \gamma^1 \\ 1 - \gamma^2 \\ \vdots \\ 1 - \gamma^J \end{bmatrix} + \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} \end{bmatrix} \begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} \quad (\text{B.5})$$

From here

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{g}{\theta}(I - A)^{-1} \begin{bmatrix} 1 - \gamma^1 \\ 1 - \gamma^2 \\ \vdots \\ 1 - \gamma^J \end{bmatrix} \quad (\text{B.6})$$

where

$$A = \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} \\ \vdots & \vdots & \vdots & \ddots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} \end{bmatrix}$$

Therefore, the cost of production c_n^j can be normalized as

$$\hat{c}_n^j = \frac{c_n^j}{W_M(T_M^J)^{-\frac{1}{\theta}\Lambda_j}}, \quad (\text{B.7})$$

where Λ_j is the j th entry of the vector $\Lambda = (I - A)^{-1} \begin{bmatrix} 1 - \gamma^1 \\ 1 - \gamma^2 \\ \vdots \\ 1 - \gamma^J \end{bmatrix}$.

With this, we can obtain an expression for the growth rate of real output as

$$g_{Y_n} = g_w - \sum_{j=1}^J \alpha^j g_{p_n^j}.$$

From equation (B.3), we have

$$g_{Y_n} = g_w - \sum_{j=1}^J \alpha^j \left(\frac{-1}{\theta} g + g_{c^j} \right).$$

Based on equation (B.7), the above equation becomes

$$g_{Y_n} = g_w - \sum_{j=1}^J \alpha^j \left(\frac{-1}{\theta} g + g_w - \Lambda_j g \right).$$

Therefore,

$$g_{Y_n} = \frac{1}{\theta} \left(1 + \sum_{j=1}^J \alpha_j \Lambda_j \right) g = g_y, \forall n. \quad (\text{B.8})$$

Note that in a one-sector economy in which $\gamma^{jk} = 0, \forall n, k$ and $\gamma^j = 1, \forall j$, the growth rate is

$$g_y = -\frac{1}{\theta} g.$$

as in Eaton and Kortum (1996, 1999). With multiple sectors, however, the growth rate of the economy is amplified by the input-output linkages.

C Model Equations (Normalized) along the BGP

In what follows, we report the equations of the model after normalizing the endogenous variables so that they are constant in the BGP. We follow the results obtained in Appendix B.

(1) Probability of imports

$$\pi_{ni}^j = \hat{T}_i^j \frac{(\hat{c}_i^j d_{ni}^j)^{-\theta}}{\hat{\Phi}_n^j}, \quad (\text{C.1})$$

where $\hat{T}_n^j = \frac{T_n^j}{T_M^j}$ and $\hat{\Phi}_n^j = \frac{1}{T_M^j} \frac{\Phi_n^j}{(W_M)^{-\theta} (T_M^j)^{\Lambda_j}}$ with Λ^j defined in Appendix B.

(2) Import shares

$$\hat{X}_{ni}^j = \pi_{ni}^j \hat{X}_n^j. \quad (\text{C.2})$$

(3) Cost of production

$$\hat{c}_n^j = \gamma^j \hat{W}_n^{\gamma^j} \prod_{k=1}^J (\hat{P}_n^k)^{\gamma^{jk}}. \quad (\text{C.3})$$

(4) Intermediate good prices in each sector

$$\hat{P}_n^j = B \left(\hat{\Phi}_n^j \right)^{-1/\theta}. \quad (\text{C.4})$$

(5) Cost distribution

$$\hat{\Phi}_n^j = \sum_{i=1}^M \hat{T}_i^j (d_{ni}^j \hat{c}_i^j)^{-\theta}. \quad (\text{C.5})$$

(6) Price index

$$\hat{P}_n = \prod_{j=1}^J \left(\frac{\hat{P}_n^j}{\alpha^j} \right)^{\alpha^j}. \quad (\text{C.6})$$

(7) Labor market clearing condition

$$\hat{W}_n L_n = \sum_{j=1}^J \gamma^j \sum_{i=1}^M \pi_{in}^j \hat{X}_i^j. \quad (\text{C.7})$$

(8) Sector production

$$\hat{X}_n^j = \sum_{k=1}^J \gamma^{kj} \sum_{i=1}^M \pi_{in}^k \hat{X}_i^k + \alpha^j \hat{Y}_n. \quad (\text{C.8})$$

where $\hat{Y}_n = \frac{P_n Y_n}{W_M}$.

(9) Income

$$\hat{C}_n = \hat{W}_n L_n + \frac{\sum_{j=1}^J \sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{1 + \theta}. \quad (\text{C.9})$$

(10) Resource constraint

$$\hat{Y}_n = \hat{C}_n + \sum_{k=1}^J s_n^k \hat{Y}. \quad (\text{C.10})$$

with

$$\hat{Y} = \sum_{m=1}^M \hat{Y}_m$$

(11) Innovation

$$g = \sum_{i=1}^M \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}}{g + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{T}_i^k}{\hat{T}_n^j} \left(\frac{1}{r - g_y + g} \beta_r \lambda_i^k \frac{1}{(1 + \theta)} \frac{\sum_{n=1}^M \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1 - \beta_r}}.$$

(12) R&D expenditures

$$\beta_r \lambda_n^j \hat{V}_n^j (s_n^j)^{\beta_r - 1} = \hat{Y}. \quad (\text{C.11})$$

(13) Value of an innovation

$$\hat{V}_n^j = \Gamma_n^j \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{(1 + \theta)}, \quad (\text{C.12})$$

(14) Profits

$$\hat{\Pi}_n^j = \frac{1}{(1 + \theta)} \sum_{i=1}^M \hat{X}_i^j \pi_{in}^j. \quad (\text{C.13})$$

(15) Trade balance

$$\sum_{k=1, k \neq j}^J \hat{X}_{nt}^k \sum_{i=1, i \neq n}^M \pi_{nit}^k = \sum_{i=1, i \neq n}^M \sum_{k=1, k \neq j}^J \pi_{int}^k \hat{X}_{it}^k. \quad (\text{C.14})$$

D Data Description and Calculation

This appendix describes the data sources and the construction of various variables for the paper. Nineteen countries are included in our analysis based on data availability (developed OECD countries: Australia, Austria, Belgium, Canada, Finland, France, Germany, Israel, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Poland, Portugal, Spain, the United Kingdom, and the United States). The model is calibrated for 2005. Eighteen tradable sectors and one aggregate nontradable sector are under consideration and reported in Table 4.

Bilateral trade flows at the sectoral level Bilateral trade data at the sectoral level (expenditure by country n of sector j goods imported from country i , X_{ni}^j) are obtained from the OECD STAN Bilateral Trade Dataset. Values are reported in thousands of U.S. dollars at current prices. Sectors are recorded at the ISIC (rev. 3) 2-3 digit level and are aggregated into the 19 sectors as listed in Table 4. We use the importer reported exports in each sector as the bilateral trade flows because it is generally considered to be more accurate than the exporter reported exports.

Value added and gross production Domestic sales in sector j , X_{nn}^j , are estimated based on the *domestic* input-output table provided by the OECD STAN database, which contains data at the ISIC 2- and 3-digit level that can be easily mapped into our 19 sectors. OECD provides separate IO tables for domestic output and imports. We sum up the values for a given row before the column “Direct purchases abroad by residents (imports)” to obtain X_{nn}^j . We compare this way of estimating the domestic expenditure on domestic product with an alternative calculation based on $X_{nn}^j = Y_n^j - \sum_{i \neq n}^M X_{in}^j$, where both gross production of country n in sector j , Y_n^j , and the total exports from n to i in sector j , $\sum_{i \neq n}^M X_{in}^j$, are from the OECD STAN Database for Structural Analysis. The first method proves to be superior, as the second generates a number of negative observations for some country-sectors.

Trade barriers and gravity equation variables Data for variables related to trade costs used in gravity equations (such as geographic distance and common border dummies) at the country-pair level are obtained from the comprehensive geography database compiled by CEPII. The WTO’s RTA database provides information on regional trade agreements. The currency union indicator is obtained from Rose (2004) and was updated to reflect Euro-area membership.

Factor shares and final consumption shares In our analysis, we used the U.S. factor shares in 2005 for all countries. Data on the share of materials from sector k used in the production in sector j , γ^{jk} , as well as the labor share of production in sector j , γ^j , come from the Input-Output Database maintained by OECD STAN. The I-O table gives the value of the intermediate input in row k required to produce one dollar of final output in column j . We then divide this value by the value of gross output of sector j to obtain γ^{jk} . Similarly, the labor share is calculated as the ratio of value added to gross output, as capital input does not exist in the model. In addition, the final consumption expenditure shares of each sector, α_n^j , also come from the I-O matrix.

R&D data R&D expenditures at the country-sector level are obtained from the OECD database of Business Enterprise R&D expenditure by industry (ISIC Rev 3). Since R&D data for several sectors in some countries are missing, we obtain estimates of these missing observations using the following approach. First, we run a regression using existing country-

sector specific R&D and patent data from USPTO for 2005:

$$\log(R_n^j) = \beta_0 + \beta_1 \log(PS_n^j) + \mu_n + \gamma_j + \varepsilon_n^j, \quad (\text{D.1})$$

where R_n^j is the R&D dollar expenditure of country i in sector j and PS_n^j is the patent stock of country i in sector j . μ_i and γ_j are country and sector fixed effects. This relation is built on the observations that (i) in the steady state, R&D expenditure should be a constant ratio of R&D stock and (ii) innovation input (R&D stock) is significantly positively related to innovation output (patent stock). In fact, the coefficient β_1 is large and significant at 99% and the R^2 is close to 0.90.

We obtain the aggregate R&D expenditure as a percentage of GDP, $R\&D/GDP_n^{WB}$, for each country from the World Bank World Development Indicator database. The country-sector specific R&D can then be estimated as $s_n^j = \hat{r}_n^j \times R\&D/GDP_n^{WB}$. For the countries with missing sectors, we estimate the fitted value using the same procedure. To maintain consistency across countries, we correct the OECD data-generated total R&D with the World Bank total R&D.

$$s_n^j = R\&D/GDP_n^{WB} \times \frac{R_n^{j,OECD}}{\sum_j R_n^{j,OECD}}$$

This estimated s_n^j is the R&D intensity used in our quantitative analysis.

Estimation of d_{ni}^j : Gravity Equation at the Sector Level To estimate the trade costs for tradable sectors, $j \leq J - 1$, we estimate the model-consistent gravity equations for each sector in 2005 using bilateral trade flow data. We start from the trade shares in equation (10):

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \frac{T_i^j (c_i^j d_{ni}^j)^{-\theta}}{\Phi_n^j}. \quad (\text{D.2})$$

Dividing the trade shares by their domestic counterpart as in Eaton and Kortum (2002) and assuming $d_{nn}^j = 1$, we have

$$\frac{\pi_{ni}^j}{\pi_{nn}^j} = \frac{X_{ni}^j}{X_{nn}^j} = \frac{T_i^j (c_i^j d_{ni}^j)^{-\theta}}{T_n^j (c_n^j)^{-\theta}}. \quad (\text{D.3})$$

Taking logs of both sides, yield

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j (c_i^j)^{-\theta}\right) - \log\left(T_n^j (c_n^j)^{-\theta}\right) - \theta \log(d_{ni}^j), \quad (\text{D.4})$$

where the log of trade costs can be written as

$$\log(d_{ni}^j) = D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + ex_i^j + \nu_{ni}^j. \quad (D.5)$$

Following Eaton and Kortum (2002), we use $D_{ni,k}^j$ to denote the contribution to trade costs of the distance between countries n and i falling into the k^{th} interval (in miles), defined as [0,350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, maximum). The other control variables include common border effect, B_{ni} , common currency effect, CU_{ni} , and regional trade agreement RTA_{ni} , between country n and country i . We include an exporter fixed effect, ex_i^j , which has been shown to better fit the patterns in both country incomes and observed price levels (see Waugh 2010). The error term is ν_{ni}^j .

Substituting (D.5) back into (D.4) results in the following gravity equation at the sector level:

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j (c_i^j)^{-\theta}\right) - \theta ex_i^j - \log\left(T_n^j (c_n^j)^{-\theta}\right) - \theta(D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + \nu_{ni}^j). \quad (D.6)$$

Define $\hat{F}_i^j = \log\left(T_i^j (c_i^j)^{-\theta}\right) - \theta ex_i^j$ and $F_n^j = \log\left(T_n^j (c_n^j)^{-\theta}\right)$. We then estimate the following equation using fixed effects and observables related to symmetric trade barriers, taking θ as known:

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \hat{F}_i^j - F_n^j - \theta(D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + \nu_{ni}^j). \quad (D.7)$$

If we use the coefficient estimates of equation (D.7), then we can derive $\log(d_{ni}^j)$ based on equation (D.5). The exporter fixed effect in trade cost, ex_i^j , can now be estimated using importer and exporter fixed-effects estimates from the gravity equation (D.7): $ex_i^j = (F_i^j - \hat{F}_i^j)/\theta$. We assume a value of $\theta = 4$, as in Waugh (2010). There is variation in the estimated trade costs that we obtain. We find that the average trade cost is around 6.2 and the standard deviation is 19.³¹

³¹Note that the estimated trade cost using gravity regressions depends on the elasticity of trade, θ , that we assume. With a larger elasticity of trade, e.g. $\theta = 8.28$ as in Levchenko and Zhang (2016), the resulting trade cost would be much lower.

Table 4: List of Industries

Sector	ISIC	Industry Description
1	C01T05	Agriculture, hunting, forestry and fishing
2	C10T14	Mining and quarrying
3	C15T16	Food products, beverages and tobacco
4	C17T19	Textiles, textile products, leather and footwear
5	C20	Wood and products of wood and cork
6	C21T22	Pulp, paper, paper products, printing and publishing
7	C23	Coke, refined petroleum products and nuclear fuel
8	C24	Chemicals and chemical products
9	C25	Rubber and plastics products
10	C26	Other non-metallic mineral products
11	C27	Basic metals
12	C28	Fabricated metal products, except machinery and equipment
13	C29	Machinery and equipment, nec
14	C30T33X	Computer, electronic and optical equipment
15	C31	Electrical machinery and apparatus, n.e.c.
16	C34	Motor vehicles, trailers and semi-trailers
17	C35	Other transport equipment
18	C36T37	Manufacturing n.e.c. and recycling
19	C40T95	Nontradables

E Figures

Figure 13: Efficiency of innovation by country

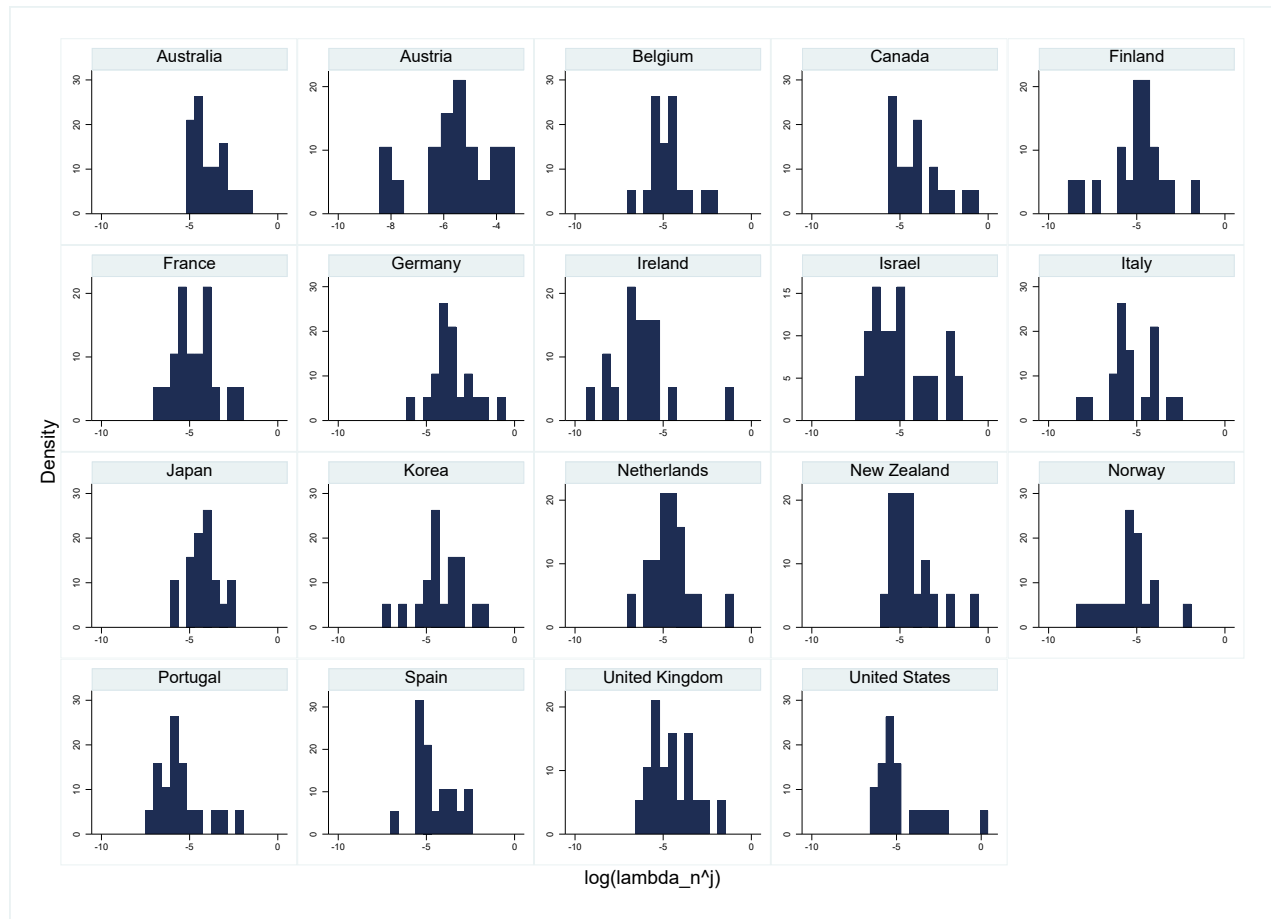


Figure 14: Efficiency of innovation by sector

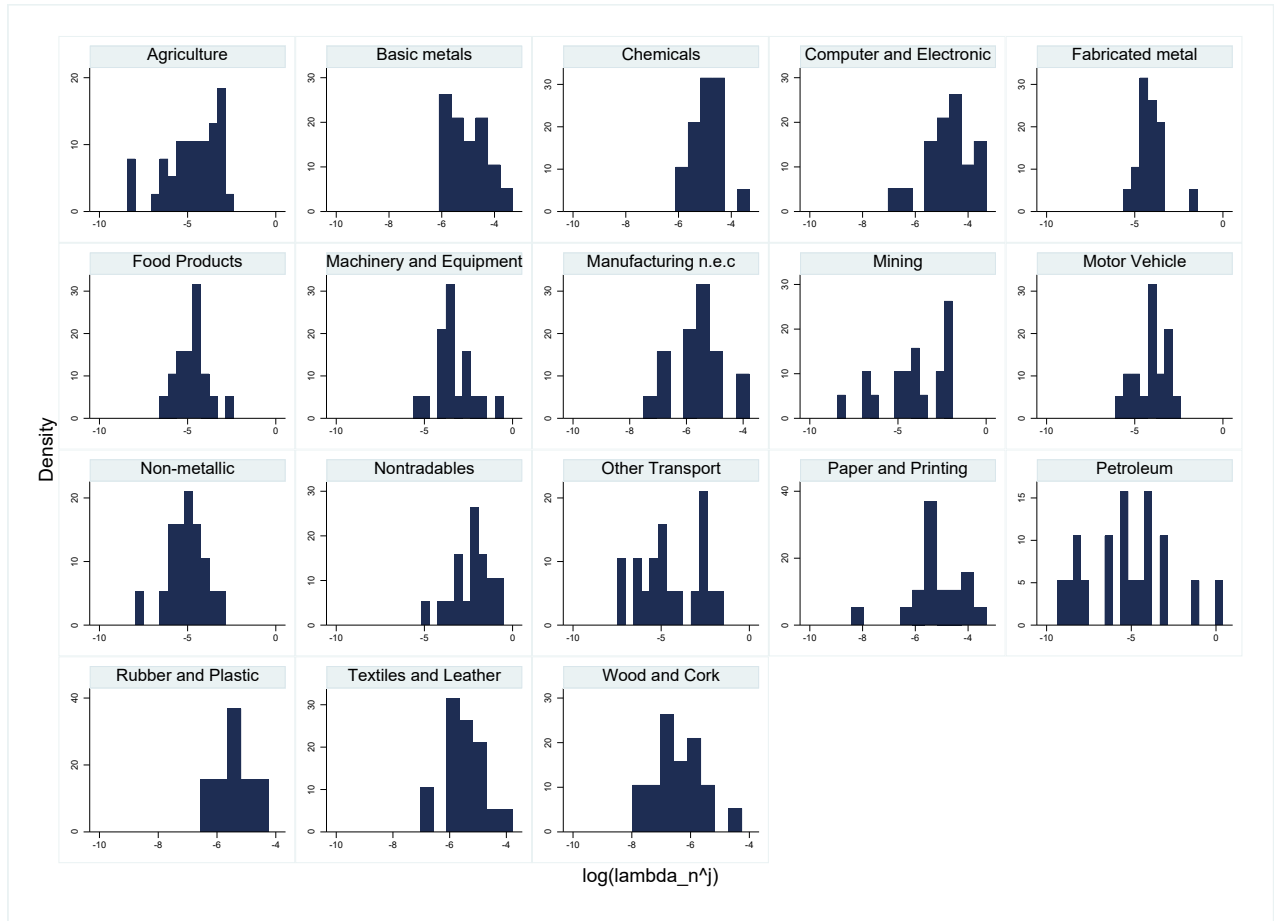


Figure 15: Stock of knowledge by country

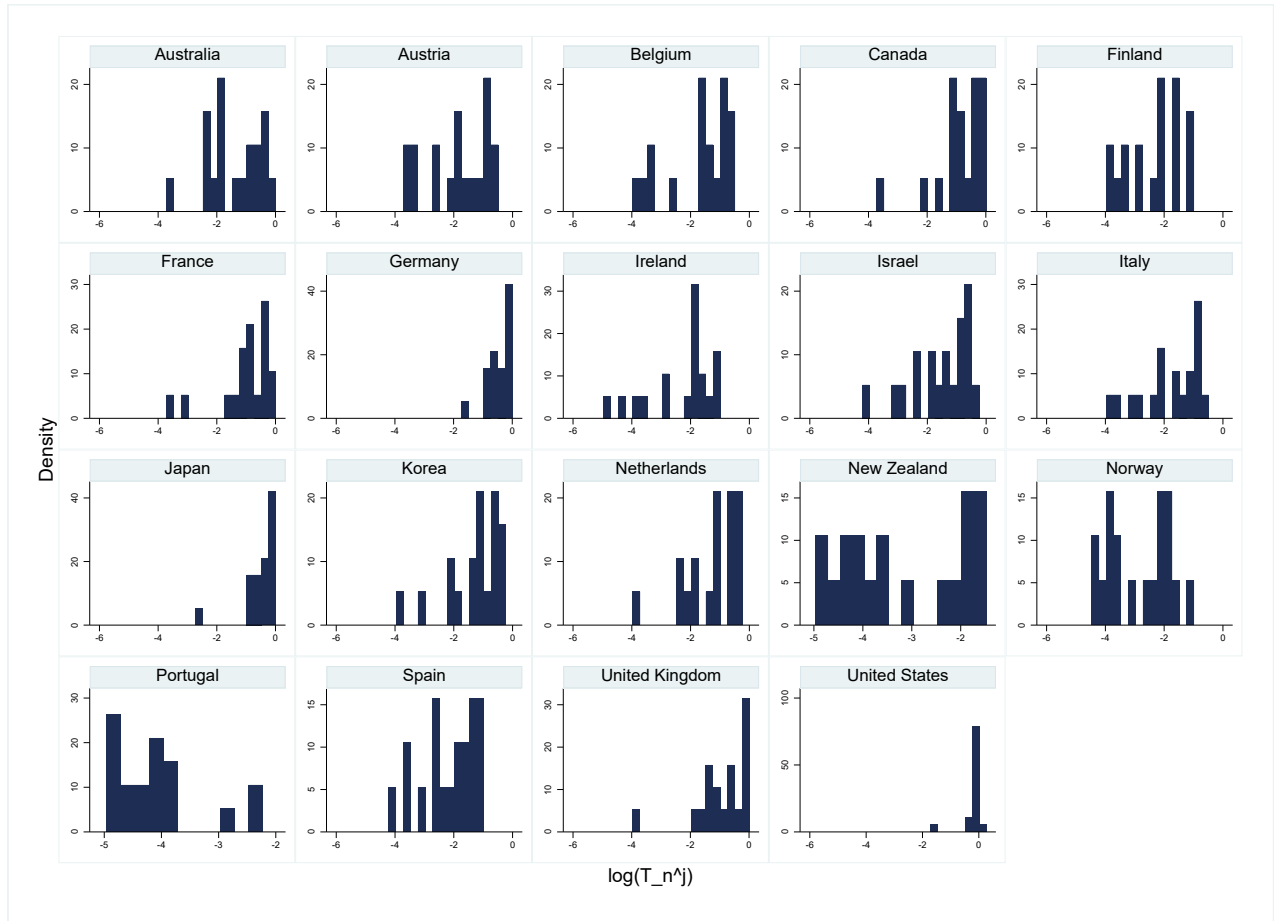
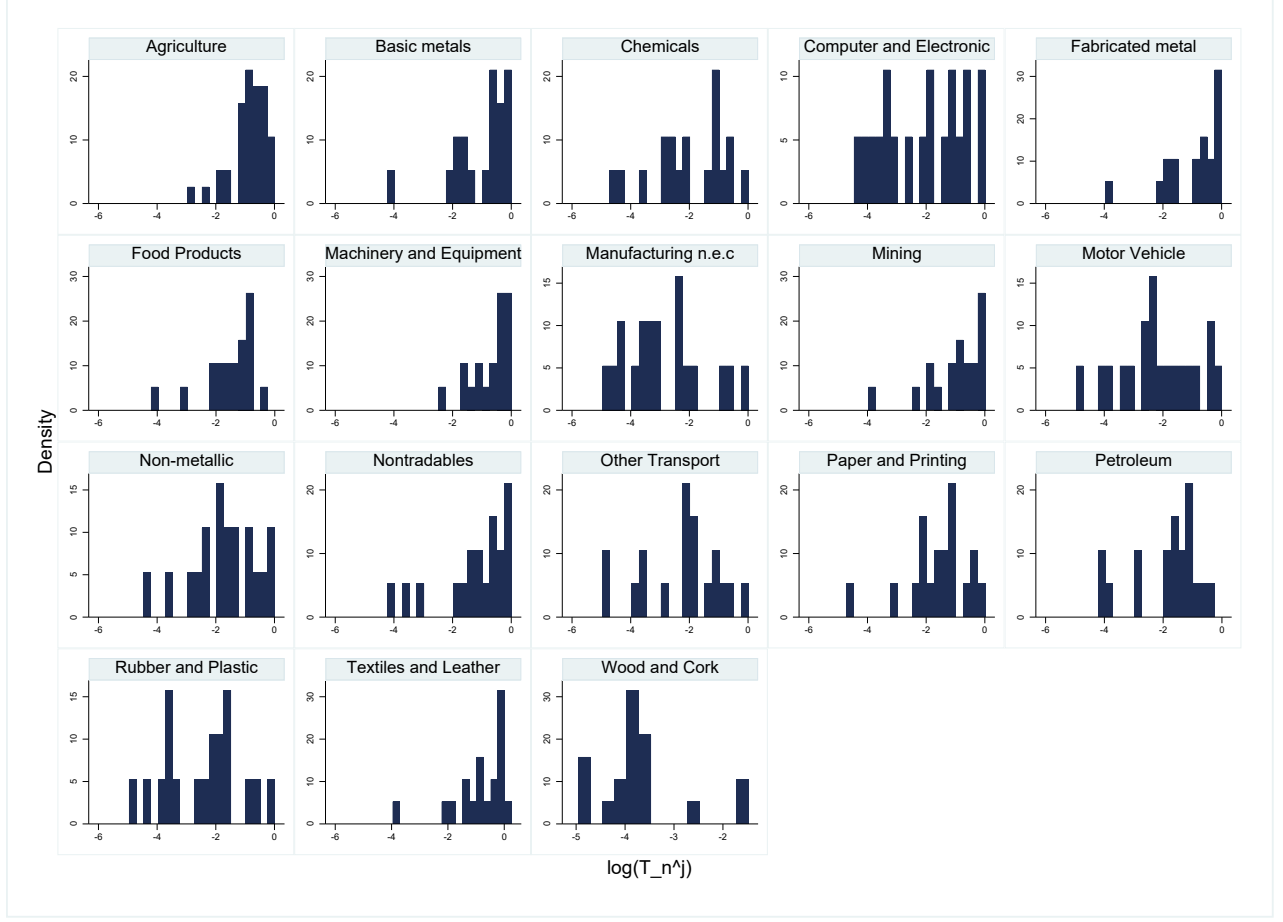


Figure 16: Stock of knowledge by sector



F Algorithm for calibration

Here, we describe in detail the steps we follow to calibrate $\{\beta_r, \lambda_n^j, \hat{T}_n^j\}$ in our model:

1. We guess a value for \hat{T}_n^j
2. We use the guess from step 1, the calibrated values for $\{\gamma^j, \gamma^{jk}, \alpha^j, d_{in}^j, \theta\}$ — together with data on R&D intensity and the equations that define the trade block of the model to obtain wages, prices, expenditures, trade shares, and output.³²
3. We use the results from step 2 and the growth block of the model to calibrate the innovation parameters as follows:

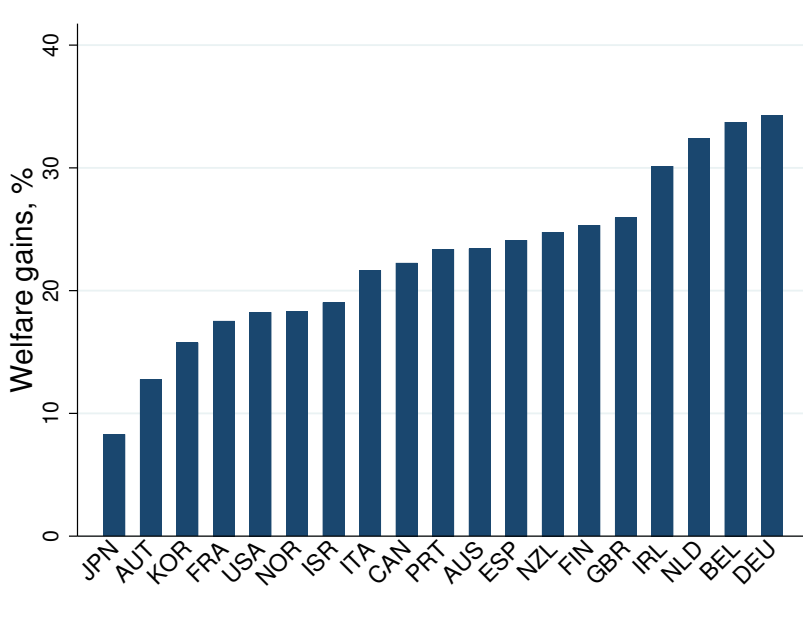
³²See equations (C.1), (C.2), (C.3), (C.4), (C.5), (C.7), (C.8), (C.9), and (C.10) in Appendix C.

- (a) We start by setting a value for the growth rate of the economy in the BGP, g_y . This value corresponds to a growth rate for the stock of knowledge along the BGP of $g = \theta \left(1 + \sum_{j=1}^J \alpha_j \Lambda_j\right)^{-1} g_y$.
 - (b) With the results from the trade block of the model computed in step 2 and the value for g , we can use equation (18) to obtain \hat{V}_n^j .
 - (c) Given the calibrated values for ε_{in}^{jk} , data on R&D intensity, s_n^j , and the value for g , we iterate over equations (18), (19), and (20) and use the Frobenius theorem to obtain $\{\beta_r, \hat{T}_n^j\}$. The Frobenius theorem guarantees the existence of a unique balanced growth path in which all countries and sectors grow at the same rate g .
 - (d) We update β_r so that the growth rate of the stock of knowledge is g and then obtain \hat{T}_n^j from the eigenvector associated with $\Delta(g)$.
4. Taking the values for β_r and \hat{T}_n^j , we go back to steps 2 and 3, and continue iterating until we find a fixed point solution for \hat{T}_n^j .
 5. Finally, we compute λ_n^j so that R&D intensity matches the data through equation (19).

G Welfare Gains, Output and Home Trade Share

We use equation (33) to compute the welfare gains from trade. We find that these welfare gains are heterogeneous across countries; they range from 8% to 34%, with a cross-country average of 23% and a standard deviation of 7% (see Figure 17).

Figure 17: Welfare gains (%)



Notes: This figure shows welfare gains from trade in our multi-sector model with innovation and knowledge spillovers.

Following trade liberalization, output is higher in every country; on average, output increases by 20%. The increase in output is heterogeneous across countries, with Japan experiencing the least increase (8%) and Germany the greatest increase (40%). The home-trade share of each country-sector nj share decreases on average by 15% (see Figure 18), and sector productivity—calculated as $\frac{T_n^j}{(\pi_{nn}^j)^\theta}$ —exhibits an average increase of 5%. Aggregating across countries, the average country experiences a productivity increase of 12.5%. This increase is driven by both a decrease in the home trade share and an increase in the stock of knowledge.

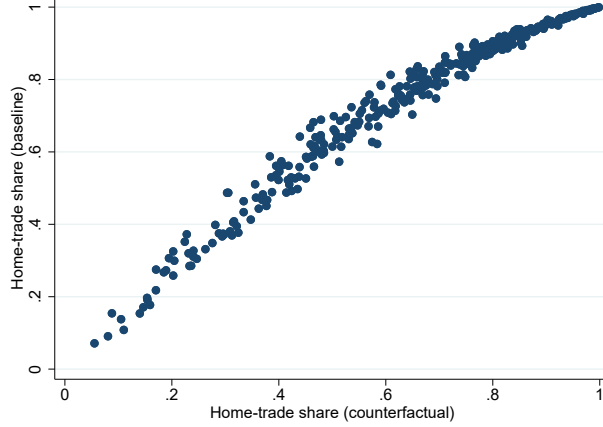


Figure 18: Home-trade share (baseline vs counterfactual)

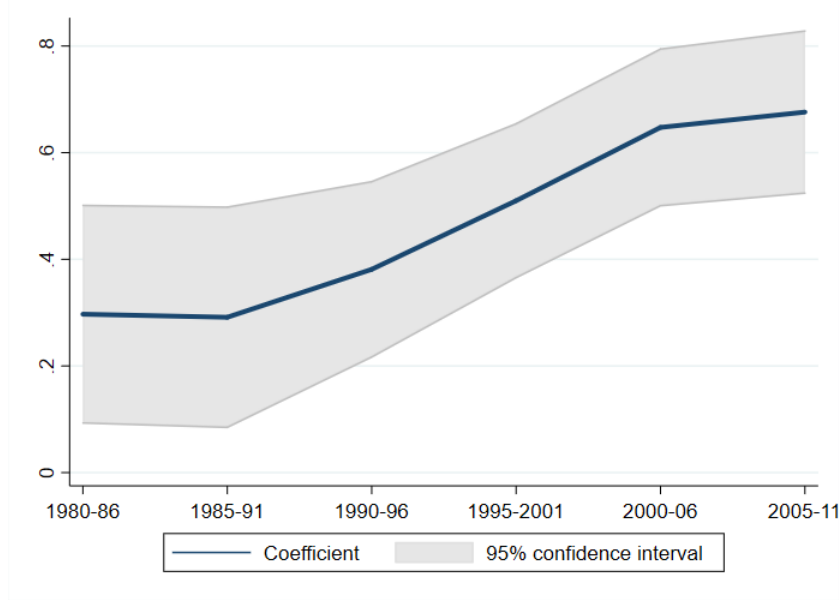
Notes: This figure shows the home-trade share in the baseline and in the counterfactual BGP. Our counterfactual exercise considers a drop of 25% in trade costs.

H Suggestive empirical evidence on the mechanism

We provide suggestive empirical evidence for our mechanism on the trade-driven reallocation of R&D according to comparative advantage. Using data for R&D intensity at the sector level for our sample of 19 countries and 19 sectors and the period 1970 to 2011, we perform a non-overlapping 5-year regression of our measure of R&D share, $s_{nt}^j / \sum_j s_{nt}^j$, on the revealed comparative advantage, RCA_{nt}^j , including industry-year and country-year fixed effects (f_{jt} and f_{nt}) as in equation 32. We then analyze the evolution of β_3 over time. The idea is that the worldwide decline in trade costs that we observe over the past decades, especially since the mid-80s (Santacreu and Zhu (2018) documents this fact), would imply a stronger correlation between the R&D share and the production comparative advantage.

Figure 19 plots the rolling coefficient, β_3 , together with a 95% confidence interval. The figure indeed lends support for our model's prediction that R&D efforts are distributed according to production comparative advantage and this relationship becomes stronger as trade costs decline up to the global financial crisis. While trade liberalization has stalled or even reversed following the crisis, this relationship has become weaker, consistently with the model's predictions.

Figure 19: Re-allocation of R&D and comparative advantage in production (Data)



Notes: This figure shows the coefficient of a 5-year rolling window regression of the log of R&D share on the log of revealed comparative advantage, together with 95% confidence intervals. The coefficients correspond to β_3 in equation (32).

I The role of heterogeneous input-output linkages

Here, we evaluate the role of heterogeneous input-output linkages by setting $\gamma^j = \gamma, \forall j$ and $\gamma^{jk} = 1 - \gamma/J, \forall j, k$. We then perform the same 25% uniform and permanent trade liberalization and analyze the effects on the reallocation of R&D and welfare.

Figure 20 shows that with homogeneous input-output linkages the reallocation of R&D after trade liberalization is less driven by comparative advantage in production than in our model with heterogeneous spillovers across countries and sectors. The coefficient increases from 0.72 to 0.77, whereas in our baseline model it increased from 0.72 to 0.93.

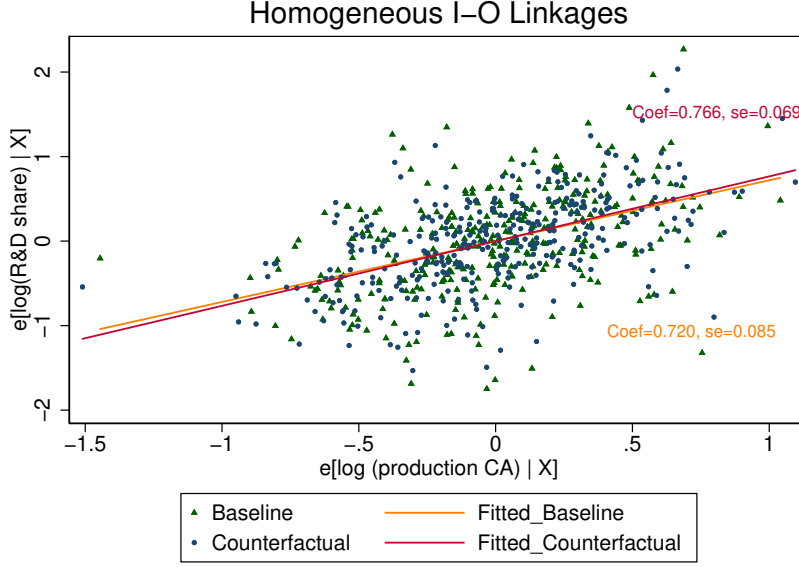
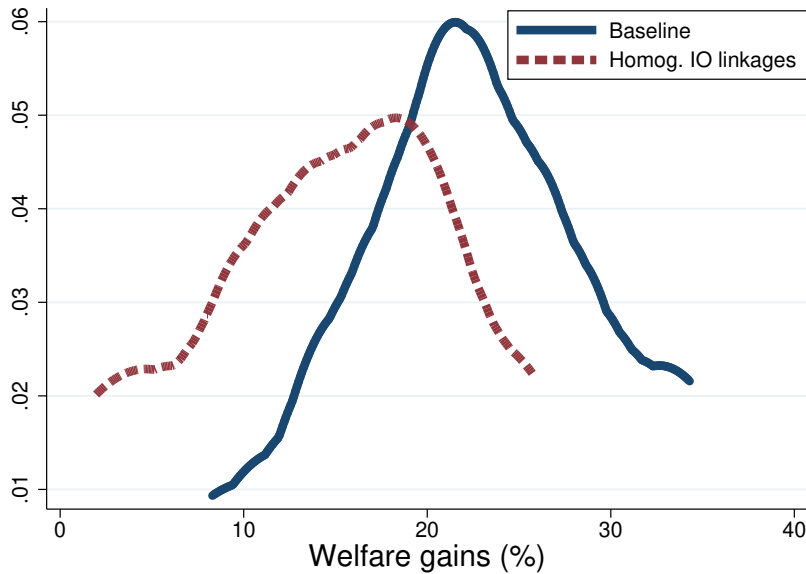


Figure 20: Re-allocation of R&D and production comparative advantage

Notes: This figure shows the partial residual plots of regression (32) for the case of homogeneous input-output linkages. The coefficients correspond to β_3 for the baseline and counterfactual cases, respectively. Standard errors associated with the estimated coefficients are also included.

Next, we compare welfare gains from trade in our baseline model to those in a model with homogeneous input-output linkages. With respect to our baseline, the case of homogeneous input-output linkages delivers lower and slightly disperse gains from trade. Figure 21 shows the distribution of welfare gains across the different models.

Figure 21: Welfare gains from trade (% change): The role of heterogeneous input-output linkages



Notes: This figure plots the distribution of welfare gains across countries and compares gains from trade in our baseline model (solid line) with those of a model with homogeneous input-output linkages (dashed line).

J Perfect Enforcement of Intellectual Property Rights

In our baseline model, intermediate producers in each country-sector can adopt diffused ideas from other country-sectors to produce an intermediate good. Innovators get the profits from their ideas that have been adopted in their own country-sector. However, innovators cannot get profits from their adopted ideas in another country-sector. In this sense, there is perfect enforcement of intellectual property rights (IPR) within a country-sector, but cross-country and cross-sector knowledge diffusion is modeled as a pure spillover effect. Here, we study the case of perfect enforcement of intellectual property rights (IPR) not only within the country-sector but also across different countries and sectors. In this case, innovators can also obtain profits from those ideas that have diffused and then been adopted by an intermediate producer in another country-sector. These profits represent royalty payments to the innovator.³³

³³A more realistic scenario would lie in between the two extreme cases of pure knowledge spillovers and perfect enforcement of IPR. Eaton and Kortum (1999) derive the case of imperfect IPR by explicitly introducing the innovator's decision to patent in a model without international trade.

We begin by describing the equations that change when there is perfect enforcement of IPR with respect to our baseline model.³⁴ The main equation that changes in the trade block of is that of trade balance (i.e., equation 12). Because of cross-country royalty payments, trade is no longer balanced trade on a period-by-period basis. Equation (12) is now be expressed as

$$EX_{nt} = IM_{nt} - RP_{nt} \quad (\text{J.1})$$

with $RP_{nt} = \sum_{j=1}^J RP_{nt}^j$ and

$$RP_{nt}^j = \sum_{i \neq n} \sum_{k \neq j} \left(\omega_{int}^{kj} \Pi_{it}^k - \omega_{nit}^{jk} \Pi_{nt}^j \right) \quad (\text{J.2})$$

with ω_{nit}^{jk} the fraction of profits that country-sector ik pays out in royalties to country-sector nj .

In the growth block, the main equation that changes is the value of an innovation (equation 15). When there is perfect enforcement of IPR, the value of an innovation takes into account that the innovator of each country-sector nj obtains profits from those diffused and adopted ideas in each other country-sector ik . Hence, allowing for diffusion of ideas from nj to ik , the value of an innovation in nj can be expressed as

$$V_{nt}^j = \sum_{i=1}^M \sum_{k=1}^J \int_t^\infty e^{-\int_t^s r_{nu} du} \left(1 - e^{-\varepsilon_{in}^{kj}(t-s)} \right) \frac{\Pi_{is}^k}{T_{is}^k} ds \quad (\text{J.3})$$

Next, we derive the BGP of this version of the model and stationarize all the variables so that they are constant along the BGP. As we did in previous sections, we represent these variables with a hat. We focus on the main variables that change in the new version of the model.

We start by deriving an expression for $\hat{\omega}_{in}^{kj}$, which represents the fraction of ideas available in country i sector k that proceed from country n sector j , i.e. $\frac{\hat{T}_{in}^{kj}}{\hat{T}_i^k}$

$$\hat{\omega}_{in}^{kj} = \frac{\varepsilon_{in}^{kj}/g}{g + \varepsilon_{in}^{kj}} \lambda_n \frac{\hat{T}_n^j}{\hat{T}_i^k} (s_n^j)^{\beta_r} \quad (\text{J.4})$$

Note that, from equation (20), $\sum_{n=1}^M \sum_{j=1}^J \hat{\omega}_{in}^{kj} = 1$.

³⁴Chaney (2008) also derives a model where there are royalty payments across countries.

Equation (J.3) can be expressed as

$$\hat{V}_n^j = \sum_{i=1}^M \sum_{k=1}^J \left(\frac{1}{r - g/\theta + g} - \frac{1}{r - g/\theta + g + \varepsilon_{in}^{kj}} \right) \frac{\hat{\Pi}_i^k}{\hat{T}_i^k} \quad (\text{J.5})$$

Then, the optimal investment in R&D is

$$s_n^j = \left(\beta_r \lambda_n^j \hat{T}_n^j \frac{1}{1 + \theta} \frac{1}{\hat{Y}} \sum_{i=1}^M \sum_{k=1}^J \Gamma_{in}^{kj} \frac{\hat{\Pi}_i^k}{\hat{T}_i^k} \right)^{\frac{1}{1-\beta_r}} \quad (\text{J.6})$$

with $\Gamma_{in}^{kj} = \left(\frac{1}{r-g/\theta+g} - \frac{1}{r-g/\theta+g+\varepsilon_{in}^{kj}} \right)$.

It is important to note that the innovators of an idea in country-sector nj do not compete directly with the intermediate producers that use that idea in another country-sector ik since, as there is a continuum of goods in each country-sector, we assume that the idea is used to produce a different variety in each country-sector.

We now analyze the effect of trade liberalization on the re-allocation of R&D across countries and sectors in this version of the model. We follow the procedure of Section 4 and take the ratio of equation (J.7) with respect to an arbitrary sector j' and an arbitrary country n' as follows.

$$\left(\frac{s_n^j / s_n^{j'}}{s_{n'}^j / s_{n'}^{j'}} \right)^{1-\beta_r} = \frac{\lambda_n^j / \lambda_n^{j'} \hat{T}_n^j / \hat{T}_n^{j'}}{\lambda_{n'}^j / \lambda_{n'}^{j'} \hat{T}_{n'}^j / \hat{T}_{n'}^{j'}} \frac{\Phi_n^j / \Phi_n^{j'}}{\Phi_{n'}^j / \Phi_{n'}^{j'}} \quad (\text{J.7})$$

with $\Phi_n^j = \sum_{i=1}^M \sum_{k=1}^J \Gamma_{in}^{kj} \frac{\hat{\Pi}_i^k}{\hat{T}_i^k}$.

When there is perfect enforcement of IPR, the re-allocation of R&D across countries and sectors depends on: (i) the *exogenous* comparative advantage in innovation, as determined by differences in λ_n^j ; (ii) the *endogenous* comparative advantage in innovation, as determined by differences in \hat{T}_n^j ; and (iii) differences weighted average of expected profits of each country-sector ik , weighted by the strength of knowledge spillovers from nj to ik (i.e., ε_{in}^{kj}), as determined by differences in Φ_n^j .

Everything else constant, R&D is re-allocated to those country-sectors that diffuse more knowledge to country-sectors that are either more likely to be the lowest cost suppliers (i.e., larger π_{ni}^k), or more likely to adopt ideas (i.e., lower $1/\hat{T}_i^k$). Therefore, after trade liberalization, R&D will flow to those country-sectors that send more knowledge to country-

sectors experiencing larger increases in their comparative advantage.³⁵

Given the re-allocation of R&D across countries and sectors, the growth and specialization effects through changes in the dispersion of relative productivity work exactly as in our baseline model.

³⁵Note that, if $\varepsilon_{in}^{kj} = \varepsilon_n^j$, $\forall i, k$, R&D re-allocation will only be determined by (i) since, in that case, $\hat{T}_n^j = \hat{T}$, $\forall n, j$. This contrasts to our baseline model in which (iii) depends only on the profits of the innovator.

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