Managing Macroeconomic Fluctuations with Flexible Exchange Rate Targeting

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Abstract

We show that a monetary policy rule that uses the exchange rate to stabilize the economy can outperform a Taylor rule in managing macroeconomics fluctuations and in achieving higher welfare. The differences between the rules are driven by: (i) the paths of the nominal exchange rate and the interest rate under each rule and (ii) external habits in consumption, which leads to deviations from uncovered interest parity. These differences are larger in economies, which are very open, which are more exposed to foreign shocks, or in which domestic and foreign goods are highly substitutable.

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1 Introduction

In the aftermath of the global financial crisis, unconventional monetary policies implemented by central banks in advanced economies resulted in an increase in liquidity in the financial system. This excess liquidity was channeled towards emerging economies, as investors were “searching for yield.” That search for yield exacerbated the appreciation of non-crisis countries.

In small open economies, the exchange rate is an important element of the transmission of monetary policy (Svensson (2000)). Fluctuations in exchange rates have an effect on inflation and other economic variables and, thus, challenge the ability of monetary policy to stabilize the economy. Because they are more subject to foreign shocks, central banks in emerging economies generally prefer to keep the exchange rate under tight control. In fact, many emerging economies today follow a quasi-managed floating exchange rate regime. Despite a large body of empirical research on managed floating, the theoretical literature discussing how monetary authorities deal with exchange rates has focused on corner solutions: either the currency rate is fixed by the central bank, or the government, or it is left to be determined by market forces. First, a large number of papers evaluate the costs and benefits of fixed exchange rates (including Friedman (1953) and Flood and Rose (1995)). A second approach to incorporating the exchange rate into discussions of monetary policy is to augment a closed-economy Taylor rule with the rate of currency depreciation. Under this approach the interest rate reacts not only to inflation and the output but also to movements in the exchange rate. For example, De Paoli (2009) derives an optimal monetary policy rule within a DSGE model and shows that by putting some weight on real exchange rate fluctuations, a central bank can achieve improvements in social welfare. Lombardo and Ravenna (2014) characterize optimal exchange rate policy. They emphasize the role of the composition of international trade on the optimal volatility of the exchange rate, and quantify the loss from an exchange rate peg relative to the Ramsey policy conditional on the composition of imports. Other recent papers show that optimal simple rules involve some type of adjustment of the nominal exchange rate (see, for example, Devereux, Engel, and Lombardo (2018)).

In this paper, we evaluate the properties of intermediate exchange rate regimes by considering an alternative class of policy rules where the central bank, instead of using the interest rate, adjusts the exchange rate in response to inflation and the output. The exchange rate is adjusted by the central bank in a manner similar to how it adjusts the interest rate when using it as an operating instrument. To evaluate the benefits of this rule relative to a standard interest rate rule, we build a DSGE model where the monetary authority adjusts the exchange rate in response to deviations of inflation from a target and fluctuations in the output.

Understanding the costs and benefits of an exchange rate policy rule within a fully specified model is not a trivial task. The immediate reaction is that if the model features an uncovered interest parity (UIP) condition, then interest rate and exchange rate rules might generate similar outcomes. In our model, there are two reasons why the outcomes for the two rules differ. First, the actual implementation of the exchange rate rule is important. While the central bank technically can replicate any interest rate rule by moving the exchange rate today and announcing depreciation consistent with UIP, it is not the way that our rule operates. In our model, the central bank sets the exchange rate today to react to current fluctuations of inflation and output. It then announces the depreciation rate from time $t$ to

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More recently, Hassan, Mertens, and Zhang (2016) and Itskhoki and Mukhin (2021) emphasize the benefits of a partial peg due to the risk premium effect, while Egorov and Mukhin (2020) discuss the optimality of a partial peg due to the local currency pricing and dollar pricing in international trade.

Benigno, Benigno, and Ghironi (2007) show that it is possible to implement a fixed exchange rate regime with an interest rate rule for a follower country.

The underlying assumption is that the central bank can commit to a particular exchange rate next period. This implies, for example, that the model may not feature the standard overshooting result, as the currency rate both today and at \( t + 1 \) are determined by the monetary authority. The simulations of our model suggest that this feature does generate differences between the two rules. A key factor for the exchange rate rule to be successful in reducing economic fluctuations is that the announcements of the central bank implementing the rule are credible. If the central bank is more than backed with foreign reserves and if it has built the credibility for maintaining low inflation, it will be less necessary for it to make continuous interventions and the policy will be more successful. We abstract from credibility issues in the paper and assume that the exchange rate rule can be perfectly implemented.

Furthermore, the differences between exchange rate and interest rate rules are amplified when the UIP condition fails. Indeed, Alvarez, Atkeson, and Kehoe (2007) argue forcefully that a key part of the impact of monetary policy on the economy goes through conditional variances of macroeconomic variables rather than conditional means. In terms of the UIP condition, their paper implies that the interest parity condition has a time-varying risk premium. Interest in a time-varying risk premium has been growing in recent years. Verdelhan (2010) shows how consumption models with external habit formation can generate a counter-cyclical risk premium that matches key stylized facts quite successfully.

In our model, we adopt a similar approach by allowing external habit formation. To show the importance of the counter-cyclical risk premium, we report results for a first-order approximation, which wipes out the risk premium from the UIP condition, and for a third-order approximation, which preserves time variation in the risk premium.

We start by writing down a relatively standard New-Keynesian small open economy model as in Gali and Monacelli (2005) that we extend to include external habit in consumption, as in De Paoli and Zabczyk (2013). We then analyze the performance of the model under two different policy rules: a standard Taylor rule in which the monetary authority sets interest rates and an alternative monetary rule in which the monetary authority sets the depreciation rate of the nominal exchange rate. We show that if UIP holds, these rules generate quantitatively similar responses to shocks. The Taylor rule generates higher volatility of the exchange rate and other economic variables. We then introduce deviations from UIP. The goal is to analyze the performance of the two competing rules when the one-to-one relationship between exchange rates and interest rates breaks down. In this case, the differences between the two rules, in terms of the response of the economy to shocks, are amplified. The main reason is that the implementation of the monetary rule has an effect on the volatility of the risk premium through a precautionary savings motive. The Taylor rule generates larger fluctuations in inflation and the output, as the larger volatility of exchange rates increases the risk premium. The opposite is true for the exchange rate rule, as the monetary authority adjusts its path of appreciation to smooth economic fluctuations by generating a less-volatile exchange rate. In this regard, the exchange rate rule is also different from a peg, in which the monetary authority fixes the exchange rate to a specified value.

After exposing the mechanism driving the differences between the two rules, we evaluate quantita-

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4Deviations from UIP and its consequences for asset prices have been studied by recent papers in the literature (see Engel (2016), Colacito and Croce (2013), Gabaix and Maggiori (2015), Maggiori (2017), Bacchetta and van Wincoop (2019), Valchev (2017), and Jiang, Krishnamurthy, and Lustig (2018), Iskhoki and Mukhin (2021), among others).

5An alternative route for introducing a risk premium in the UIP condition is to build in incomplete financial markets, as in Schmitt-Grohé and Uribe (2003), Turnovsky (1985), Benigno (2009), and De Paoli (2009). Deviations from UIP come from costs of adjusting holdings of foreign bonds. Limited participation models are alternative incomplete market models that generate time-varying risk premia (Alvarez and Jermann (2001), Lustig and Van Nieuwerburgh (2005), Chien and Lustig (2009), and Alvarez, Atkeson, and Kehoe (2009)). Alternatively, modeling recursive preferences and stochastic conditional variances, as in Backus, Foresi, and Telmer (1995), and Backus, Gavazzi, Telmer, and Zin (2010), could generate large and variable risk premia in models with complete markets.

6Our results are consistent with those in Chow, Lim, and McNelis (2014), who estimate a DSGE model for the Singapore economy under the two rules and find that the exchange rate rule outperforms the Taylor rule in reducing fluctuations in inflation. Different from their paper, we have a more general framework that can be applied to any small open economy. More importantly, our goal is to analyze the mechanisms behind the different performances of the two rules, especially those driven by the existence of a countercyclical risk premium that introduces deviation from UIP.
tively their performances both in terms of managing macroeconomic fluctuations and in terms of welfare. We begin by specifying a general rule in which the monetary authority reacts to fluctuations in inflation, the output and the exchange rate, with a certain degree of interest rate smoothing. We then compute for each rule the implied volatility of key economic variables. We do this for (i) a log-linearized version of our model that does not capture the existence of a time-varying risk premium and (ii) a non-linear version of our model in which we log-linearize the demand and supply conditions, while taking a third-order approximation of the equations that depend on the risk premium directly. In that way, our non-linear model isolates the role of a time-varying risk premium and, hence, deviations from the UIP condition. We find that, for a wide range of plausible parameters in the monetary rules, the exchange rate rule outperforms the interest rate rule in terms of inflation and output growth. It also outperforms the peg. This is true for a large combination of parameter values in the monetary rules. In a related paper, Schmitt-Grohé and Uribe (2016) study the optimal exchange rate policy when there is downward wage rigidity and compare it to the exchange rate peg. Different from our framework, they solve for the first-best allocation using an exchange rate policy. By letting the exchange rate react to fluctuations, they find that the monetary authority would like to implement large devaluations when there are recessions. An exchange rate peg, however, would be an inefficient policy.

Finally, we compare the performances of the two rules in terms of welfare. Note that we do not compute the Ramsey policy in which the social planner finds the optimal allocation and then obtains values for the monetary policy rule parameters that mimic such allocation. An alternative would be to compute the constrained Ramsey problem in which the planner is constrained to maximize the lifetime expected utility of the representative consumer in a competitive equilibrium. Instead, in this paper we compare simple monetary rules in which the central bank uses either the interest rate or the exchange rate as the instrument. More precisely, we evaluate lifetime utility for a wide range of parameter combinations in the two rules (in all the exercises, however, we keep the autoregressive smoothness parameter fixed). We compute welfare by taking a third-order approximation of the full model and of the utility function. In that way, we capture variations in the risk premium of the economy and we are able to evaluate how they impact the preferences of the central bank. Our welfare analysis is done using numerical approximations (see Collard and Juillard (2001), and Schmitt-Grohé and Uribe (2004)).\(^7\) We find that a policy rule in which the central bank adjusts the exchange rate to react to fluctuations in output and inflation is welfare improving with respect to a monetary rule in which the central bank uses the interest rate as its instrument. This is especially the case for very open economies and for economies in which the elasticity of substitution between domestic and foreign goods is large. A further analysis shows that the key friction driving welfare differences between the two rules is external habit formation. Indeed, removing other distortions such as monopolistic competition and the trade externality does not affect our results substantially.

Our findings suggest that (i) external habits in consumption—which drives deviations from the UIP condition—and (ii) the different implementation of the rules, generate differences in terms of business cycle fluctuations and welfare between a Taylor rule and an exchange rate rule. These two channels prevent the monetary authority from being able to mimic the properties of an exchange rate rule with a Taylor rule. Moreover, the effect of external habits in driving these differences is amplified when we allow for a higher persistence and volatility of the productivity shocks. In this case, the economy faces a larger and more volatile risk premium that gives rise to larger welfare effects as the distortion introduced by external habits gets amplified.

\(^7\)Several papers have derived analytically a welfare-based loss function of the small open economy. De Paoli (2009) uses the linear-quadratic approach developed by Sutherland (2002) and Benigno and Woodford (2006). External habits introduce an additional trade-off in the economy that leads to over-consumption, as households do not internalize the effect of their consumption on the habit level of the economy (see De Paoli and Zabczyk (2013) and Leith, Moldovan, and Rossi (2012) for a closed economy model).
The optimal choice of the monetary policy instrument was studied in the classic paper of Poole (1970). Poole (1970) shows that in a deterministic environment, the choice of the instrument is irrelevant. However, when there are stochastic terms in the model, the choice of the instrument matters because it affects the strength with which shocks pass through to variables that the policymaker cares about. Our paper is mostly related to Lubik and Schorfheide (2007), who show that the exchange rate is of significant importance in setting monetary policy in some open economies. In our paper, we investigate whether we can take this one step further by showing that in cases of very open economies, the exchange rate might serve as an instrument for implementing monetary policy. One can argue that there is a continuum or a range of policy rules that start with pure interest rate rules that ignore completely exchange rate fluctuations thus allowing for significant volatility in exchange rates ceteris paribus. Then there are rules that reduce exchange rate volatility by reacting to exchange rates with different intensity as in Lubik and Schorfheide (2007). Then there are rules introduced in this paper, where the exchange rate responds to current macroeconomic conditions and the central bank sets not only the exchange rate today but also commits to a path of future appreciation/depreciation. Finally, there are fixed exchange rates and currency unions. Our paper thus paper is a study along this continuum of rules.

The rest of the paper proceeds as follows. Section 2 lays out the details of the model. Section 3 presents the mechanism of the exchange rate rule. Section 4 performs a quantitative analysis. Section 5 provides a summary of our key findings, some ideas for future research and conclusions.

2 The Model

Our model extends Gali and Monacelli (2005) by introducing a new policy rule based on using the exchange rate to stabilize the economy and adding external habits as in Campbell and Cochrane (1999), Jermann (1998), Verdelhan (2010), and De Paoli and Sondergaard (2009). Modeling assumptions are kept at a minimum to ensure that we can study the properties of the exchange rate rule without introducing too many confounding factors.

The basic framework is a DSGE model with one country that represents a small open economy (Home) and another that represents the rest of the world (Foreign). It features complete international financial asset markets, monopolistic competition in production, and sticky prices à la Calvo. Prices are set in the producer’s currency and the law of one price holds, hence there is complete exchange rate pass-through. However, due to home bias in consumption, there are deviations from purchasing power parity and fluctuations of the real exchange rate. The economy is subject to a domestic productivity shock and a foreign output shock. We study the properties of the model under two alternative monetary policy rules: a conventional interest rate rule, in which the monetary authority uses the short-term nominal interest rate as its instrument, and an exchange rate rule, in which it uses the exchange rate as the instrument. In the remainder of the paper, we refer to these rules as IRR and ERR, respectively.

2.1 Households

In each country, there is a representative household who maximizes life-time expected utility. The utility function of the household in Home is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t - hX_t, N_t),$$  

(1)
where $N_t$ is hours of labor, $X_t$ is the level of habits defined below, and $C_t$ is a composite consumption index defined by

$$C_t = \left[ (1 - \alpha)^{\frac{1}{\delta}} (C_{H,t}) + \alpha^{\frac{1}{\delta}} (C_{F,t}) \right]^{\frac{1}{\eta}},$$

(2)

where $C_{H,t}$ denotes the consumption of domestic goods by Home consumers, $C_{F,t}$ denotes the consumption of foreign goods by Home consumers, $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\alpha \in [0, 1]$ is the degree of openness of the country. $C_{H,t}$ and $C_{F,t}$ are aggregates of intermediate products produced by Home and Foreign combined in the following way:

$$C_{H,t} = \left[ \int_0^1 C_{H,t}(i)^{\frac{1}{\delta}} \, di \right]^{\frac{1}{\eta}}; \quad C_{F,t} = \left[ \int_0^1 C_{F,t}(i)^{\frac{1}{\delta}} \, di \right]^{\frac{1}{\eta}},$$

(3)

where $\epsilon$ is the elasticity of substitution between varieties, which in turn are indexed by $i \in [0, 1]$.

As in De Paoli and Sondergaard (2009), we assume that habits are external. We allow for flexibility in assessing the importance of habits by introducing in equation (1) the parameter $h \in [0, 1]$. When $h = 0$, the model collapses to the basic version in Gali and Monacelli (2005), while $h = 1$ corresponds to the modeling assumptions in Campbell and Cochrane (1999) and Verdelhan (2010). The evolution of habits follows an AR(1) process, with accumulation of habits based on last-period’s consumption:

$$X_t = \delta X_{t-1} + (1 - \delta)C_{t-1},$$

(4)

Parameter $\delta \in [0, 1]$ captures the degree of habit persistence. Again, this parameter allows us to consider various assumptions about habits, with $\delta = 0$ corresponding to the assumptions in the earlier literature on habit-formation where habits are determined exclusively by last-period’s consumption (e.g., Campbell (2003) and Jermann (1998)).

Consumers maximize (1) subject to the following budget constraint:

$$\int_0^1 P_{H,t}(i)C_{H,t}(i) \, di + \int_0^1 P_{F,t}(i)C_{F,t}(i) \, di + E_t \{ M_{t,t+1}B_{t+1} \} \leq B_t + W_t N_t,$$

(5)

where $P_{H,t}(i)$ is the price of variety $i$ produced in Home, $P_{F,t}(i)$ is the price of variety $i$ imported from Foreign (expressed in Home currency), $M_{t,t+1}$ is the stochastic discount factor, $B_{t+1}$ is the nominal payoff in period $t + 1$ of the portfolio held at the end of period $t$, and $W_t$ is the nominal wage.

The optimal allocation of expenditures within each variety gives the demand function for each product:

$$C_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_H} \right)^{-\epsilon} C_H; \quad C_{F,t}(i) = \left( \frac{P_{F,t}(i)}{P_F} \right)^{-\epsilon} C_F,$$

(6)

where $P_{H,t} = \left[ \int_0^1 P_{H,t}(i)^{1-\epsilon} \, di \right]^{\frac{1}{\epsilon}}$ and $P_{F,t} = \left[ \int_0^1 P_{F,t}(i)^{1-\epsilon} \, di \right]^{\frac{1}{\epsilon}}$ are the price index of domestic and the price index of imported goods (expressed in units of Home currency), respectively. From expression (6), $P_{H,t}C_H = \int_0^1 P_{H,t}(i)C_{H,t}(i) \, di$ and $P_{F,t}C_F = \int_0^1 P_{F,t}(i)C_{F,t}(i) \, di$.

The optimal allocation of expenditures between domestic and imported goods is

$$C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t,$$

(7)

where $P_t = [(1 - \alpha)P_{H,t} + \alpha P_{F,t}]^{\frac{1}{\eta}}$ is the consumer price index (CPI). From the previous equations, total consumption expenditures by the domestic household is $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. Therefore,
we can rewrite the budget constraint as

\[ P_t C_t + E_t \{ M_{t,t+1} B_{t+1} \} \leq B_t + W_t N_t. \tag{8} \]

With a per-period utility function of the form

\[ U(C_t, X_t, N_t) \equiv \frac{(C_t - hX_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma}, \tag{9} \]

the first-order conditions for the household’s problem are

\[ (C_t - hX_t)^{\sigma} N_t^\gamma = \frac{W_t}{P_t}, \tag{10} \]

\[ \beta \left( \frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = M_{t,t+1}. \tag{11} \]

Taking expectations on both sides, we have the Euler equation:

\[ R_t \cdot E_t \left[ \beta \left( \frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right] = 1, \tag{12} \]

where \( R_t \equiv 1/E_t [M_{t,t+1}] \) is the gross return on a riskless one-period discount bond paying off one unit of domestic currency in \( t+1 \).

Below we elaborate on the need to use habit formation in this model, but from the Euler equation it is already clear that the marginal utility of consumption increases when consumption goes down relative to the level of habits. As we discuss below, this modeling approach generates a counter-cyclical coefficient of relative risk aversion leading to a counter-cyclical risk premium that drives a wedge between the interest rate differential and expected depreciation in the UIP condition. Both Verdelhan (2010) and De Paoli and Sondergaard (2009) make this point quite forcefully.

### 2.2 Firms

We now characterize the supply side of the economy. In each country there is a continuum of monopolistically competitive firms, \( i \in [0, 1] \), that use labor to produce a differentiated good (each firm is associated with a different variety). Labor is the only factor of production, and we assume it to be immobile across countries.

Each firm \( i \) operates the linear technology

\[ Y_t(i) = A_t N_t(i), \tag{13} \]

where domestic productivity follows an AR(1) process:

\[ \log(A_t) = \rho_A \log(A_{t-1}) + \log(U_{At}), \tag{14} \]

where \( \sigma_A \) is the standard deviation of the innovation \( \log(U_{At}) \). Firms face downward-sloping demand from domestic and foreign households and maximize expected profits by setting the price of their varieties. Prices are set as in the Calvo model. A measure \( 1 - \theta \) of randomly selected firms sets new prices every period. Since firms are identical, all resetting firms set the same new price, \( P_{H,t}(i) = P_{H,t} \), such that we
can drop the firm index $i$. It is well known that firms optimally set

$$\frac{P_{H,t}}{P_t} = \frac{\varepsilon}{\varepsilon-1} \frac{H_t}{F_t},$$

(15)

where $H_t$ and $F_t$ are auxiliary variables to express the pricing decision recursively and defined as

$$F_t \equiv \Lambda_t Y_t + \beta \theta \cdot E_t (F_{t+1} \Pi_{t+1}^\varepsilon),$$

(16)

$$H_t \equiv \Lambda_t MC_t Y_t + \beta \theta \cdot E_t (H_{t+1} \Pi_{t+1}^\varepsilon),$$

(17)

where $MC_t \equiv W_t / P_{H,t} A_t$ denotes real marginal costs (in units of the domestic goods) and $\Lambda_t \equiv (C_t - hX_t)^{-\sigma}$. In the Calvo model, domestic prices evolve following

$$P_{H,t} = (1 - \theta) \left( \frac{P_{H,t}}{P_t} \right)^{1-\varepsilon} + \theta P_{H,t-1}^{1-\varepsilon},$$

which is written in real terms as

$$P_{H,t} = \left( (1 - \theta) \left( \frac{P_{H,t}}{P_t} \right) \right)^{1-\varepsilon} + \theta \left( P_{H,t-1} \right)^{1-\varepsilon} \Pi_{t-1}^\varepsilon,$$

(18)

where $\Pi_t \equiv P_t / P_{t-1}$ is CPI inflation. We take the price of the Home consumption bundle, $P_t$, as a numeraire and express all variables in real terms.

### 2.3 The Rest of the World

Because the foreign economy (the rest of the world) is exogenous to Home (our small open economy), there is some flexibility in specifying the behavior of the foreign variables. We assume foreign output follows an AR(1) process,

$$\log (Y^*_t) = \rho \log (Y^*_t) + \log (U_{Y^*t}),$$

(19)

where $\sigma_{Y^*}$ is the standard deviation of innovations $\log (U_{Y^*t})$. With external habits in consumption in the rest of the world, the foreign household’s inter-temporal first-order condition is

$$M^*_t, t+1 = \beta \left( \frac{C^*_{t+1} - hX^*_{t+1}}{C^*_t - hX^*_t} \right)^{\varepsilon} \left( \frac{P^*_t}{P^*_{t+1}} \right),$$

(20)

where $M^*_t, t+1$ is the foreign economy’s stochastic discount factor, $C^*_t$ is the foreign consumption-bundle, and $X^*_t$, the stock of habits, accumulates as

$$X^*_t = \delta X^*_{t-1} + (1 - \delta)C^*_t,$$

(21)

and $P^*_t$ is the foreign CPI. Taking expectations of Equation (20) yields the foreign economy’s Euler equation and the foreign interest rate as $R^*_t \equiv 1/E_t \left[ M^*_t, t+1 \right]$. We normalize $P^*_t = 1$. Finally, the foreign household’s intratemporal optimization yields demand for Home goods as

$$C^*_t, H_t = \alpha \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C^*_t,$$

(22)

where $P_{H,t}$ denotes the price of Home goods in the rest of the world.

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8Full stabilization of the foreign CPI implies that the foreign interest rate is directly pinned down by the foreign economy’s Euler equation. Alternatively, we could allow the foreign consumer price index to fluctuate and introduce an interest rate monetary policy rule to determine the foreign interest rate as a function of inflation. Our results are robust to this alternative specification, since the foreign economy is fully exogenous to our small open economy.
2.4 Two Monetary Policy Rules: An IRR versus an ERR

We consider two alternative monetary policy rules. First, we analyze the model under a standard Taylor rule, in which the monetary authority sets the nominal interest rate to smooth fluctuations in CPI inflation, output, and the nominal exchange rate depreciations,

\[
\frac{R_t}{R_t} = \left( \frac{R_{t-1}}{R_t} \right)^\rho \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\Pi} \left( \frac{Y_t}{Y} \right)^{\phi_Y} \left( \frac{E_{t+1}^*}{E_t} \right)^{\phi_E} \right]^{1-\rho},
\]

where \(\rho \in (0, 1)\) is the degree of interest rate smoothing.

Second, we consider a monetary policy rule in which the central bank adjusts the path of the nominal exchange rate to stabilize CPI inflation and output. The exchange rate depreciation policy, \(\frac{E_{t+1}^*}{E_t}\), is adjusted in response to deviations of these variables from their targets,

\[
\frac{E_{t+1}^*}{E_t} = \frac{E_{t+1}}{E_t} \left\{ \left( \frac{\Pi_{t+1}}{\Pi} \right)^{-\phi_\Pi} \left( \frac{Y_{t+1}}{Y} \right)^{-\phi_Y} \right\},
\]

where \(\frac{E_{t+1}}{E_t}\) is the depreciation required to reach the long-run equilibrium exchange rate.\(^9\) We assume that there is some smoothing in the way the nominal exchange rate adjusts to its target level, that is

\[
\frac{E_{t+1}}{E_t} = \left( \frac{E_{t+1}}{E_t} \right)^\rho \frac{\left( \frac{E_{t+1}}{E_t} \right)^{(1-\rho)}}{\left( \frac{E_{t-1}}{E_t} \right)}.
\]

When inflation and output are high, the central bank announces a path of appreciation of the exchange rate that is given by \(\frac{E_{t+1}}{E_t}\) and that determines the evolution of the nominal exchange rate.\(^10\)

2.5 Market Clearing Conditions

Here, we describe the market clearing conditions in the goods market and in the financial assets markets.

**Goods Market**

Since the Home economy is small, its demand for foreign goods is small as well, and the goods market clearing condition for foreign goods can be expressed as

\[
Y_t^* = C_t^*.
\]

Demand for domestic goods is

\[
Y_t^d = C_{H,t} + C_{H,t}^*
\]

Substituting equations (7) and (22), we have

\[
Y_t = \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} \left\{ (1-\alpha)C_t + \alpha Q_t^0 C_t^* \right\},
\]

where we use the fact that the law of one price holds, \(P_{H,t} = E_t P_{H,t}^*\) (where \(E_t\) is the nominal exchange rate denoted in units of domestic currency per unit of foreign currency, as in Gali and Monacelli (2005)),

\(^9\) We follow the convention that an increase in the exchange rate implies depreciation of the domestic currency, as in Parrado (2004).

\(^10\) The ERR and its properties have been documented and estimated for the case of Singapore by Parrado (2004) and Khor, Lee, Robinson, and Supaat (2007).
and define the real exchange rate as the relative price of the foreign consumption bundle in terms of the
domestic consumption bundle, \( Q_t \equiv \frac{E_t P^*_t}{P_t} \).

**International Risk Sharing and the UIP Condition**

We assume that financial markets are complete and all contingent claims are traded internationally.
Therefore, the stochastic discount factors for Home and Foreign must be equalized in equilibrium in
terms of the domestic currency\(^{11}\):

\[
M^*_{t,t+1} = \frac{\xi_{t+1}}{\xi_t} \cdot M_{t,t+1},
\]

where \( \frac{\xi_{t+1}}{\xi_t} \) denotes the nominal depreciation of the domestic currency. Together with the Euler equations,
we obtain that the marginal rates of intertemporal substitution in real terms are equalized between Home
and Foreign and risk is perfectly shared in all states of the world.

Equation (28) also holds in expectation, implying an equilibrium relation among domestic and foreign
interest rates, nominal exchange rate depreciation, and the domestic stochastic discount factor:

\[
\mathbb{E}_t \left[ M_{t,t+1} \left( R_t - R^*_t \cdot \frac{\xi_{t+1}}{\xi_t} \right) \right] = 0.
\]

Log-linearization of Equation (29) around a perfect-foresight steady state yields the standard UIP con-
dition (small letters with a hat denote log deviations from steady state):

\[
\hat{r}^*_t = \hat{r}_t + \mathbb{E}_t[-\Delta \hat{e}_{t+1}],
\]

Alvarez, Atkeson, and Kehoe (2007) argue that assumptions leading to this simplified interest parity
condition imply dynamics that are inconsistent with the data. Under assumptions of conditional log-
normality of the stochastic discount factor, a time-varying risk premium emerges, as shown by Backus,
Gavazzoni, Telmer, and Zin (2010). Alternatively, a higher-order approximation of the Euler equation
can also generate a time-varying risk premium.\(^{12}\)

Habit-based preferences, as specified in Equations (4) and (9), give rise to counter-cyclical risk aver-
sion\(^{13}\): when consumption is close to the habit level in period \( t \), the household is more averse towards risk
in period \( t+1 \). Such counter-cyclical risk aversion is reflected in the household’s pricing kernel, \( M_{t,t+1} \),
and therefore affects the premium required by domestic households for holding foreign bonds. Foreign
bonds carry exchange rate risk from the viewpoint of the domestic investor since they pay in foreign
currency. For example, if the nominal exchange rate appreciates in states of low future consumption, a
domestic investor demands a higher return on foreign bonds to compensate for the risk exposure.

An analytical expression of the exchange rate risk premium in our model is beyond the scope of the
paper. However, to gain intuition on the components that drive the exchange rate risk premium, we
follow Backus, Gavazzoni, Telmer, and Zin (2010), who assume that the stochastic discount factors are

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\(^{11}\)The stochastic discount factor \( M_{t,t+1} \) can also be interpreted as the price of an Arrow-Debreu security paying
of one unit of domestic (foreign) currency in any given state of the world in \( t+1 \). If all securities are traded internationally,
no-arbitrage implies that returns to one unit of the domestic currency are equalized, i.e. \( 1/M_{t,t+1} = 1/\xi_{t+1}/M^*_{t,t+1}\xi_{t+1} \).

\(^{12}\)At a first-order approximation of the model, there is no risk premium; at a second-order approximation, there is a risk
premium, but it is constant.

\(^{13}\)As shown De Paoli and Sondergaard (2009) the coefficient of relative risk aversion can be defined as \( \sigma_t \equiv \sigma_t \equiv -C_{t} \frac{U_{CC}}{U_{C}} = \sigma / \eta X_t \).
jointly lognormal. Hence, Equation (29) becomes exactly\textsuperscript{14}

\[
\bar{r}_t^* = \bar{r}_t + \mathbb{E}_t \left[ -\hat{d}_{t+1} \right] + \text{Cov}_t \left[ \frac{\hat{m}_{t,t+1}^* + \hat{m}_{t,t+1}^*}{2}, -\Delta \hat{e}_{t+1} \right],
\]

(31)

where \(fxp_t\) denotes the exchange rate risk premium. Equation (31) shows that the exchange rate risk premium is a function of the conditional (on time \(t\)) covariance of the average pricing kernel with the nominal appreciation of the domestic currency.

The return on foreign bonds carries a risk premium for two reasons: First, domestic investors require a higher return on foreign bonds if the domestic currency systematically appreciates in states of low consumption and/or inflation, \(\text{Cov}_t \left[ \hat{m}_{t,t+1}^*, -\Delta \hat{e}_{t+1} \right] > 0\). This is because foreign-currency investments pay off less in states where the marginal utility of currency is high. Second, foreign investors require compensation for holding foreign bonds if the domestic currency appreciates in states where foreign consumption is low, \(\text{Cov}_t \left[ \hat{m}_{t,t+1}^*, -\Delta \hat{e}_{t+1} \right] > 0\).\textsuperscript{15}

Equation (31) illustrates that domestic financial conditions are impacted by the dynamics of the exchange rate risk premium, which provides new opportunities for monetary policy. A small open economy with fully integrated financial markets that takes world interest rates as given may be able to exploit variations in time-varying risk premia to stabilize domestic macroeconomic fluctuations. Through managing the conditional covariance structure of the economy (i.e., \(fxp_t\)), monetary policy could shield domestic interest rates from foreign interest rate fluctuations while keeping the nominal exchange rate relatively stable.

3 The Mechanism

Here, we provide an analysis of how the model works under an ERR in two ways. First, we describe for our ERR the transmission channels of monetary policy that have been studied extensively under an IRR. Second, we analyze the main differences between the two rules considered in the paper in terms of managing macroeconomic fluctuations.

3.1 ERR Transmission Channels

To illustrate the main transmission channels of the ERR, we assume that the monetary authority follows a simplified ERR that targets only CPI inflation, with some degree of interest rate smoothing. That is

\[
\Delta \hat{e}_t = \rho \Delta \hat{e}_{t-1} - (1 - \rho) \phi \Pi \hat{\pi}_t.
\]

(32)

Consider a negative domestic productivity shock. Inflation increases, as there is excess demand for Home goods in the domestic economy and the monetary authority reacts by announcing a path of appreciation of the nominal exchange rate in the following way. In period \(t\), the increase in inflation following the shock, \(d \hat{\pi}_t > 0\), translates into an appreciation of \(d \Delta \hat{e}_t = -(1 - \rho) \phi \Pi d \hat{\pi}_t\) in period \(t\), \(d \Delta \hat{e}_{t+1} = -(1 - \rho) \phi \Pi (d \hat{\pi}_t + d \hat{\pi}_{t+1})\) in period \(t + 1\), \(d \Delta \hat{e}_{t+2} = -(1 - \rho) \phi \Pi (\rho d \hat{\pi}_t + \rho d \hat{\pi}_{t+1} + d \hat{\pi}_{t+2})\) in period \(t + 2\), and so on. Therefore, changes in current inflation today appreciate the nominal exchange rate over time.

\textsuperscript{14}As mentioned in De Paoli and Zabczyk (2013), this expression also holds up to the second order without any distributional assumptions on the pricing kernels.

\textsuperscript{15}Under conditional log-normality, the foreign exchange rate risk premium can also be expressed as \(fxp_t = \frac{1}{2} \left[ \text{Var}_t [\hat{m}_{t+1}] - \text{Var}_t [\hat{m}_{t+1}^*] \right]\). In good times in the domestic economy, the variance of the domestic discount factor is low, relative to the variance of the foreign discount factor, and hence the risk premium that domestic investors demand to hold foreign bonds is lower.
We argue that this appreciation policy transmits to economic decisions through three distinct channels: (i) expenditure switching effect, (ii) intertemporal consumption-saving decisions, and (iii) endogenous exchange rate risk-premium dynamics. While the first two operate already at a first-order approximation of the model, the third is only active at third- or higher-order approximations.

The first two channels are related to the dual role of the exchange rate as a price of financial assets and of goods (Corsetti, Dedola, and Leduc (2010)). By managing the exchange rate when there are fluctuations in inflation and output, the monetary policy directly influences both the goods and the financial markets. As a result, households revise their consumption plans because of inter-temporal consumption-saving motives—as they similarly would do under conventional interest rate rules—as well as because of the intratemporal trade-off between consuming domestic goods or consuming foreign goods. We explain each channel in detail next.

**Expenditure Switching Effect** In the goods markets, the immediate nominal appreciation implemented by an ERR deteriorates the terms of trade in Home, as domestic goods become more expensive relative to foreign goods. The appreciation of the exchanges rate induces switching domestic demand for foreign goods, which is sufficient to clear the excess demand caused by the shock. As a result, prices adjust less in order to equalize demand and supply and inflation is more stable in equilibrium.

The intratemporal channel of expenditure switching effect can be illustrated by totally differentiating the demand for Home goods in the log-linerized model (see Appendix D). Noting $\hat{y}_d^e$ as the log-deviation of the demand for Home goods from the steady state,

$$d\hat{y}_d^e = -\frac{\eta\alpha(2 - \alpha)}{1 - \alpha} \left(1 + (1 - \rho)\phi_{\Pi}\right) \cdot d\hat{\pi}_t + (1 - \alpha)d\hat{c}_t + \alpha d\hat{c}_t^*.$$  \hfill (33)

The effect of the ERR on the demand for Home goods is stronger, the more domestic goods are substitutes for foreign goods ($\eta$), the more open the economy ($\alpha$), and the stronger the reaction of monetary policy to CPI inflation ($(1 - \rho)\phi_{\Pi}$).

**Intertemporal Substitution** In the financial markets, the nominal appreciation following the shock causes the expected return on foreign bond investments denominated in domestic currency, $\hat{r}_t^f + E[\Delta\hat{e}_{t+1}]$, to decrease. The ERR effectively turns foreign bonds into CPI inflation-indexed bonds by tying the nominal exchange rate to domestic monetary policy targets. If CPI inflation increases, returns on foreign bond investments decrease and vice versa. The lower return on foreign investments induces immediate capital inflows from the rest of the world, which boosts domestic asset prices and pushes down domestic interest rates, which are determined by the financial market equilibrium (Equation 30).

In contrast to the IRR, however, under the ERR households have an incentive to save more and consume less when domestic interest rates decrease. The reason is that under the ERR the domestic CPI level is stationary: forward-looking households anticipate that higher prices today imply lower prices in the future. As under an exchange rate peg, households prefer to shift — in response to current inflation — consumption into future periods, where goods are expected to become cheaper again. By appreciating the nominal currency in states of high inflation, the ERR induces capital inflows, pushing

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16 Under an IRR, on the other hand, the central bank only directly intervenes on the financial markets through setting the risk-free interest rate. Intra-temporal shifts in consumption remain solely determined by market forces.

17 Gali and Monacelli (2005) show that prices are stationary under an exchange rate peg. We verify this to be the case under our ERR as well.

18 This is different to the inter-temporal substitution channel operating under an IRR. Under an IRR, domestic CPI is not stationary, and forward-looking households do not expect CPI prices to return to their initial level in the future. Rather, inflation today implies expected inflation tomorrow, consumption only decreases in response to current inflation if the central bank increases interest rates sufficiently — i.e. more than one-for-one (the Taylor principle) — such that saving more and consuming less in the current period becomes beneficial.
down the domestic interest rate. Households thus need to save more if interest rates decrease, in order to shift consumption to the future.

To illustrate the intertemporal substitution channel under the ERR (see Appendix D), we combine the Euler equations (12) and (20) with the log-linearized UIP condition in equation (30) and the exogenous process of foreign output in equation (19) and obtain

\[
\frac{d\hat{c}_t}{C_{\text{Saving}}} = -(1-h)\sigma^{-1}(1 + (1 - \rho)\phi_{\Pi}) \cdot d\hat{\pi}_t + d\hat{m}_t. \tag{34}
\]

The consumption-saving channel is stronger the lower the degree of habit formation \((h)\), the more households are willing to smooth consumption inter-temporally \((1/\sigma)\), and the stronger the reaction of the monetary authority to CPI inflation \(((1 - \rho)\phi_{\Pi})\).

**The Exchange Rate Risk Premium and Precautionary Savings** The exchange rate risk premium provides an additional degree of freedom in the trilemma among independent domestic monetary policy, a fixed exchange rate, and free capital flows (Mundell (1963) and Fleming (1962)). For example, for given foreign interest rate and exchange rate expectations, an increase in the equilibrium exchange rate risk premium pushes down domestic interest rates.

Under the simplified ERR, the risk premium in equation (31) is a function of the conditional covariance between stochastic discount factors and CPI inflation.\(^{19}\) We follow the same reasoning as in Section 2.5.

Forward-looking households anticipate that higher CPI inflation induces a nominal currency appreciation policy. Therefore, the return on foreign bond investments is low in high-inflation states (the stochastic discount factor is low), and high in low-inflation states (the stochastic discount factor is high). If high-inflation states are associated with high consumption (e.g., after a positive foreign output shock), the stochastic discount factor is unambiguously low in high-inflation states and high in low-inflation states.\(^{20}\) Households accept a lower payoff on foreign bonds relatively to domestic bonds because of the insurance benefits of foreign bonds. The exchange rate risk premium is on average negative \((fxp_t < 0)\).\(^{21}\)

As shown by De Paoli and Zabczyk (2013), the dynamics of the risk premium under external habits are determined by fluctuations in (i) risk aversion and (ii) consumption prospects. If risk aversion is high or consumption prospects are poor, households ask for more compensation for exchange rate risk and the risk premium increases. Risk aversion is counter-cyclical and consumption prospects are pro-cyclical if habits \(\hat{h}\) and shocks \((\rho_A, \rho_Y)\) are persistent enough\(^{22}\) or future prices are expected to decrease sufficiently with increasing consumption (and vice versa)\(^{23}\). The risk premium decreases if prospects improve or only deteriorate by little with increasing consumption. Households become less precautionary (the domestic discount factor is less volatile), save less in riskless domestic bonds, and accept lower excess returns on foreign (risky) bonds. Vice versa, the risk premium increases with foreign

\(^{19}\)Plugging the ERR into the expression for the exchange rate premium shows that under ERR: \(fxp_t = Cov_T\left(\hat{m}_{t+1}, \hat{m}_{t+1}\right)/2 \cdot (1 - \rho)\phi_{\Pi\Pi}\).

\(^{20}\)On the other hand, high inflation states are associated with low consumption (e.g., after a negative domestic productivity shock), the stochastic discount factor may increase in high-inflation states. In this case, investors demand a positive risk premium \((fxp_t > 0)\) on foreign bond investments on average. We find that in our small open economy the foreign output shock dominates the conditional covariance of consumption with inflation.

\(^{21}\)This allows the small open economy, for example, to sustain higher domestic interest rates for any given foreign interest rate and currency appreciation paths (or vice versa to sustain an appreciated currency on average). This is the terms of trade externality (see for example De Paoli and Sondergaard (2009)). The small open economy has an incentive to appreciate its real exchange rate on average to increase consumption possibilities (of foreign goods) without increasing disutility from labor.

\(^{22}\)For example, if a positive shock is expected to persist, consumption prospects improve, whereas they deteriorate if the shock is rather transitory.

\(^{23}\)For example, consumption prospects may increase if prices go down sufficiently for several periods after a positive domestic productivity shock. This inflation channel is absent in both De Paoli and Zabczyk (2013) and De Paoli and Sondergaard (2009) who analyze risk premium dynamics in flexible price economies.
consumption if prospects in the rest of the world improve or only deteriorate by little. In this case, foreign households become less precautionary (the volatility of the foreign discount factor goes down) and save less in the foreign (riskless from the viewpoint of the foreign investor) bond. They accept lower excess returns on domestic bonds.

These dynamics can stabilize domestic prices: if inflation increases under the ERR, the central bank curbs demand by implementing a path of nominal currency appreciation. Through intertemporal substitution, households save more and consume less. If the risk premium is counter-cyclical – increases with lower consumption – domestic households become more precautionary and save even more and consume less. The risk premium thus reinforces the fall in consumption. Domestic prices need to go up less to clear excess demand.

3.2 Implications of the IRR and ERR for Business Cycle Dynamics

The two monetary rules have different implications for business cycle fluctuations. These differences are amplified at a third-order approximation, due to the presence of an endogenous and time-varying risk premium that breaks down the UIP condition.\(^{24}\)

To understand how the two rules imply different business cycle dynamics, we first log-linearize equations (23) and (24). From equation (23), we have

\[
\hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)[\phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t].
\]

(35)

Under a Taylor rule, the central bank increases the nominal interest rate when inflation or output are higher than their targets. Instead, if inflation or output are low, the central bank stimulates the economy by lowering the nominal interest rate.

Similarly, if we put together equations (24) and (25) and we log-linearize the expression, we have

\[
\Delta \hat{e}_t = \rho \Delta \hat{e}_{t-1} - (1 - \rho)[\phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t].
\]

(36)

Under an ERR, the central bank stimulates the economy by depreciating the currency when inflation or output is low. In this case, the nominal exchange rate is depreciated (goes up) to increase demand from the rest of the world. Notice that, unlike in the Taylor rule, when lower inflation or output lead to a lower interest rate, the gradual depreciation under the ERR leads to an increase in the interest rate, through the UIP condition.\(^{25}\) When UIP does not hold, there is an additional effect on the nominal interest rate, which is driven by a precautionary-saving motive.

To illustrate the different implications for business cycle dynamics, consider a positive domestic productivity shock. A central bank that follows an IRR decreases the nominal interest rate (see equation 35), inducing capital outflows which depreciate the domestic currency. When UIP holds, non-arbitrage determines a future appreciation of the domestic currency, which leads to an overshooting of the nominal exchange rate. Under an ERR, however, the overshooting does not happen. The reason is that, after the shock, the central bank reacts by announcing a slow depreciation of the currency (see equation (36)). Because forward-looking consumers expect that the currency will continue to depreciate, there is an excess supply of domestic bonds, which lowers the price of these bonds hence increasing the domestic interest rate. When UIP holds, the increase in the nominal interest rate equals the expected future depreciation

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\(^{24}\) At a first-order approximation of the model, there is no risk premium; at a second-order approximation, there is a risk premium but it is constant.

\(^{25}\) This is somewhat counterintuitive, as “fighting inflation” implies a lower interest rate. The communication from the Monetary Authority of Singapore is quite clear in terms of using appreciation of the currency to lower inflation. Anecdotally, one can see the decline in interest rates, e.g. the announced appreciation of the Singapore dollar in October 2007, that led to a decline in the one-month rates from 2.5% to 2.38% within a month and eventually to 1.13% before the following announcement in April 2008.
(see equation (30)). Because there is no overshooting, this increase is lower than the decrease in the domestic interest rate under the Taylor rule, and hence the ERR generates fewer fluctuations in domestic variables. Some degree of exchange rate smoothing in the rule is key to avoid overshooting under the ERR. The two rules imply differences in business cycle fluctuations due to the different responses of the exchange rate and the nominal interest rate.

External habits introduce deviations of the UIP condition at higher-order approximations through an endogenous and time-varying risk premium. This leads to additional differences in business cycle fluctuations between the IRR and the ERR.

A positive domestic productivity shock generates an excess supply of domestic goods, and firms set lower prices. The central bank stimulates demand to stabilize the economy. Under the IRR, the central bank decreases the interest rate, which leads to capital outflows and a large currency depreciation and a future appreciation (overshooting). While under the ERR, it implements a depreciation path for the currency. This also leads to capital outflows since the expected depreciation increases the return on foreign bond investments and increases domestic interest rates. Under both rules, households increase consumption and with a counter-cyclical risk premium decrease their precautionary savings. This amplifies the consumption increase under both rules. This effect is weaker under the ERR because the exchange rate is less volatile (no overshooting), and so the risk premium decreases less.

4 Quantitative Analysis

We calibrate the model and perform a quantitative analysis to compare the macroeconomic performance of the ERR and IRR. We do this for (i) a first-order approximation of the model, which does not capture the existence of a risk premium, and (ii) for a third-order approximation of the model, which does capture a time-varying risk premium. Here, as in De Paoli and Sondergaard (2009), we use the log-linear version of the demand and supply conditions, while taking a third-order approximation of the equations that directly depend on the risk premium. In that way, our non-linear model isolates the role of a time-varying risk premium. We call this the hybrid model. We use Dynare for our numerical exercises. Moments are based on simulations of 10,000 periods, and we use pruning to avoid non-stationarity of simulations at a third-order approximation.

We start by analyzing impulse responses of one-standard-deviation shocks to domestic productivity and foreign output to evaluate the business cycle properties of the different rules and shed light on the mechanism driving the differences. At a first-order approximation of the model, differences between the two rules depend on the actual implementation—that the central bank announces a gradual path of appreciation or depreciation of the currency and the direct intervention of the ERR in the goods market. At a third-order approximation, the performance of the policy rules will additionally depend on how they affect the dynamics of the risk premium (see Van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2012)).

In Section 4.3, we conduct a more formal quantitative analysis in which, first, we evaluate differences between the two rules in terms of second moments. We consider generalized monetary rules in which the central bank reacts to deviations of inflation, output, and exchange rates. We also allow for a wide range of parameter values in both rules and check whether there exists any combination of the parameters for which the two rules deliver business cycle volatility. Second, we compare the two rules in terms of welfare.  

\(^{26}\)In the appendix, we also derive the welfare-based loss function and characterize the Ramsey constrained optimal policy.
4.1 Baseline Calibration

The calibrated parameters are reported in Table A.1. We follow closely the parametrization of De Paoli and Sondergaard (2009) and Gali and Monacelli (2005). The model is calibrated at a quarterly frequency. The parameters of habit persistence are set to \( h = 0.85 \) and \( \delta = 0.99 \). The elasticity of substitution across intermediate goods, \( \varepsilon = 6 \), and between domestic and foreign goods is \( \eta = 1 \). We set the inverse of Frisch elasticity of the labor supply, \( \gamma \), equal to \( 3/1 - h \).\(^{27}\) The degree of openness, \( \alpha \), is set to 0.08 (see De Paoli and Sondergaard (2009)). The discount factor is set to \( \beta = 0.99 \), which implies a steady-state interest rate of 4% in a quarterly model. We assume the degree of price stickiness to be \( \theta = 0.67 \), which is consistent with an average period of price adjustment of three quarters, and the inverse of the intertemporal elasticity of substitution to be \( \sigma = 5 \), which is within the range found by the empirical literature of \([2, 10]\).

As in Gali and Monacelli (2005), domestic productivity is assumed to have a standard deviation of \( \sigma_A = 0.71\% \) and the foreign productivity shock is assumed to have a standard deviation of \( \sigma_{Y^*} = 0.78\% \). The shocks are assumed to be positively correlated with correlation \( \sigma_{A,Y^*} = 0.3 \).

4.2 Business Cycle Dynamics: Impulse Response Functions

For simplicity and to illustrate our mechanism, we consider simplified rules in which the monetary authority adjusts either the nominal interest rate or the nominal exchange rate to react to fluctuations in inflation only, with a certain degree of smoothing. For the Taylor rule, we set \( \phi_{\Pi} = 1.5 \), \( \phi_Y = 0 \), and \( \rho = 0.85 \), as in Lubik and Schorfheide (2007). For the ERR, there is not a clear value for \( \phi_{\Pi} \) that allows us to compare the two rules exactly. Here we consider \( \phi_{\Pi} = 1 \), a value that is consistent with estimates found in the literature for the case of Singapore (see Parrado (2004)). We assume the same degree of smoothing in both rules.

4.2.1 First-Order: Uncovered Interest Parity Holds

After a positive domestic productivity shock (Figure B.1), both output and consumption increase. Domestic inflation decreases because the economy is now more productive and there is excess demand for home goods. The central bank stimulates the economy differently under the two rules. If the central bank follows a Taylor rule, it sets a lower interest rate and the domestic currency depreciates. The initial depreciation is followed by a future appreciation, since UIP holds. There is overshooting of the nominal exchange rate. CPI inflation decreases because the initial decrease in domestic inflation dominates the depreciation of the currency. If, instead, the central bank follows the ERR, after the positive domestic productivity shock, the central bank reacts to the fall in inflation by announcing a gradual depreciation of the exchange rate. In this case, the nominal interest rate increases, since, as households expect a future depreciation of the currency, there is an excess supply of domestic bonds. Therefore, interest rates move in opposite directions under the two rules and the exchange rate is more stable under the ERR because there is no overshooting.

After a positive foreign output shock (Figure B.2), differences in business cycle dynamics caused by the two monetary rules are even more pronounced. Common to both rules, the shock has the two immediate effects that foreign interest rates fall and there is excess demand by the rest of the world for home goods. The capital inflows induced by lower foreign interest rates, however, have a different effect under each rule: while under the IRR it is the exchange rate that is determined by market forces (and

\(^{27}\)We follow De Paoli and Sondergaard (2009), who argue that under external habits, consumption would be too smooth relative to the data.
the interest rate that is set by the central bank), under the ERR it is the interest rate that is free-floating and the exchange rate is determined as a function of domestic monetary policy targets.

Under the IRR, capital inflows immediately appreciate the nominal exchange rate (i.e., there is an overshooting followed by expected depreciations through UIP) and deteriorate the small open economy’s terms of trade. This leads to large switching of aggregate demand away from Home goods and towards foreign goods. As a result there is an excess supply for domestic goods and firms decrease domestic prices and produce less. The central bank stimulates the economy by decreasing the domestic interest rate in response to deflation, which boosts consumption today and cushions the nominal currency appreciation. Under the ERR, on the other hand, capital inflows directly pass-through and push down domestic interest rates. Households save less and consume more, which amplifies the initial excess demand for Home goods by foreigners, and firms increase domestic prices. The central banks aims to curb demand and announces a path of currency appreciation in response to inflation. This causes the small open economy’s terms of trade to deteriorate, which rebalances aggregate demand towards foreign goods and decreases consumption through intertemporal substitution.

After a positive foreign output shock, the ERR, thus, stabilizes the exchange rate much more by avoiding overshooting. This shields the domestic economy’s terms of trade from deteriorating too strongly, and leads to modest inflation. This contrasts to the terms of trade deteriorating too strongly and there being a slump in demand and, consequently, a slump in output under the IRR.

Our impulse response analysis using the log-linear model shows that there exist qualitative differences in business cycle dynamics driven by the implementation of the monetary rule, especially when the small open economy experiences foreign shocks. The differences arise because with the ERR there is no overshooting of the nominal exchange rate, which helps stabilize the terms of trades without significantly increasing the volatility of monetary policy targets such as inflation and output.

These results are confirmed for our baseline calibrations by the second moments of key macroeconomic variables reported in Table (A.2, columns 1-2).

4.2.2 Third-Order: Deviations from Uncovered Interest Parity

We now show that differences in business cycle dynamics between the two rules are amplified at a third-order approximation, due to the existence of a time-varying risk premium. The time-varying risk premium reflects fluctuations in the precautionary savings motive, which may stabilize or amplify fluctuations in monetary policy targets such as inflation.

As argued before, after a positive domestic productivity shock, consumption increases under both rules, however more so under the IRR. Since consumption prospects improve upon impact – consumption is hump-shaped or decreases only slowly – domestic households become less precautionary, save less and consume more (Figure B.1 versus B.3). Hence, the presence of an endogenous risk premium amplifies fluctuations in consumption. The risk premium on foreign bonds is counter-cyclical and decreases, more so under the IRR, where the exchange rate is more volatile.

Under the IRR, where interest rates are fixed by the central bank, lower precautionary savings lead to more capital outflows, stronger immediate depreciation (overshooting) and larger expected appreciation. Thus, the lower risk premium offers an improvement in the terms of trade and rebalances aggregate demand to domestic goods. Prices would need to decrease less in order to clear a given excess supply for Home goods. However, higher consumption means also a higher marginal utility of working and higher output as well. In total, prices need to decrease more. Under the ERR, on the other hand, lower precautionary savings imply higher domestic interest rates. For a given path of current and expected deflation, households save more and consume less (the intertemporal substitution effect). Since the risk premium is very stable under the ERR, the additional price decrease is small.
After a positive foreign output shock (see Figure B.4), foreign and domestic consumption go up. Since the foreign shock is sufficiently persistent, both domestic and foreign households become less precautionary, save less and consume even more. However since foreign consumption increases more, precautionary savings decrease more in the rest of the world, leading to capital inflows under both rules. The risk premium is pro-cyclical and increases, more so under the IRR, where the exchange rate is more volatile.

Under the IRR, the additional capital inflows (due to less precautionary savings by foreign investors) lead to additional nominal appreciation and amplify the deterioration in the terms of trade. Demand rebalances away from Home towards foreign goods. Prices have to decrease more, and the central bank sets interest rates even lower to stimulate demand. The risk-premium dynamics thus destabilize the small open economy. Under the ERR, on the other hand, the higher consumption (due to less precautionary saving) increases the marginal utility of working. Output increases in response to the foreign output shock such that excess demand decreases and prices increase by less. The central bank under the ERR appreciates the currency by less, which further stabilizes the nominal exchange rate. The risk-premium dynamics thus stabilize the small open economy under the ERR.

Our qualitative results show that the two monetary policy rules generate different business cycle dynamics when UIP holds (see also Table A.2, columns 4-5). These differences are amplified by deviations from UIP. The ERR generates fewer fluctuations than the Taylor rule, especially when the economy is exposed to foreign shocks. That is, small open economies that are exposed to shocks originating in the rest of the world may benefit from rules that use the exchange rate to stabilize the economy.

### 4.3 Generalized Monetary Rules

In this section, we analyze the performance of the different rules in terms of second moments. We augment the rules from Section 4.2 along the following dimensions: (i) we allow the monetary authority to react to deviations of inflation and output (Figure B.5); (ii) we allow for the IRR to react also to the exchange rate (Figure B.6); and (iii) we compare the ERR with a rule in which the monetary authority leaves the exchange rate fixed, which we call the PEG. In all of the cases, we evaluate the rules under a wide range of plausible values for the parameters.

The particular functional forms of the monetary rules is as follows:

\[
\begin{align*}
\hat{r}_t &= \rho \hat{r}_{t-1} + (1 - \rho) [\phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t + \phi_E \Delta \hat{e}_t] \\
\Delta \hat{e}_t &= \rho \Delta \hat{e}_{t-1} - \phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t \\
\Delta \hat{e}_t &= 0
\end{align*}
\]

where \(\rho = 0.85\), \(\phi_\Pi \leq 5\), \(\phi_Y \in [0,1]\). Figure B.5 shows the volatility of key economic variables for various values of \(\phi_Y\) and \(\phi_\Pi\) under our rules. In this comparison, we assume \(\phi_E = 0\). Figure B.6 shows the volatility of key economic variables for various values of \(\phi_Y\) and \(\phi_\Pi\) under an ERR and an IRR in which we allow the central bank to react also to fluctuations in the nominal exchange rate. We consider two cases: (i) \(\phi_E = 0.2\) and (ii) \(\phi_E = 0.8\). We do the analysis for the hybrid model. In this case, the different performances between the two rules will be driven by the interaction between the implementation of the rule and the existence of a time-varying risk premium.

In Figure B.5, we find that both the ERR and the PEG outperform the IRR in terms of reducing fluctuations in all variables except for consumption growth and the domestic interest rate. These differences are larger the lower the response to inflation and the larger the response to output deviations. Under the ERR, the monetary authority is more effective at stabilizing the economy by introducing

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28Several previous studies on optimal monetary policy tend to restrict \(\phi_\Pi \leq 3\) (see Schmitt-Grohé and Uribe (2007)).
fewer fluctuations in the risk premium. We find that the PEG does better than the ERR for smoothing domestic interest rates and exchange rates; it seems to perform similarly for output and consumption growth but does worse in terms of smoothing inflation and domestic inflation. As the parameters of the rule increase in value, the differences between the PEG and the ERR are more striking. Under the PEG, the central bank’s objective is to keep the exchange rate fixed, and this sometimes comes at the expense of generating larger fluctuations in other parts of the economy. With the ERR, however, the central bank manages to smooth fluctuations in the exchange rate by allowing for a large degree of smoothing of the instrument while still being able to react to fluctuations in inflation and the output. This is especially the case when the monetary authority that follows an ERR puts a strong weight on fluctuations of inflation and output.

In Figure B.6, we consider the same rules as before, but we allow the IRR to react, in addition to deviations of inflation and output, to fluctuations in exchange rates through the parameter \( \phi_E \). We consider two values for this parameter: (i) a low and conservative value of \( \phi_E = 0.2 \) and (ii) a stronger and less-conservative value of \( \phi_E = 0.8 \). We find that the differences between the ERR and the IRR are similar when \( \phi_E = 0 \) or when \( \phi_E = 0.2 \). We start observing larger differences when the reaction to exchange rate fluctuations is larger, i.e., when \( \phi_E = 0.8 \). In that case, the IRR performs very similarly or slightly better than the ERR in terms of smoothing fluctuations in domestic inflation and CPI inflation when \( \phi_\Pi \) is large. For low values of \( \phi_\Pi \), however, the performance of the IRR in terms of smoothing economic fluctuations is much worse than that of the ERR. That is, if the central bank puts a big weight both on smoothing the volatility of the exchange rate and inflation, the IRR would perform very similarly to the ERR in terms of second moments.

### 4.4 Welfare Analysis

We compare the performance of the IRR and the ERR in terms of the implied lifetime utility of the household for a wide range of parameter combinations in the two rules (in all the exercises, we keep the smoothness parameter fixed at \( \rho = 0.85 \)). We compute welfare by taking a third-order approximation of the full model and of the utility function. In that way, we capture variations in the risk premium of the economy and we are able to evaluate how they impact the preferences of the central bank. We consider the same generalized monetary policy rules as above. The results are reported in Figure B.7. We analyze the differences between the rules in terms of welfare for (i) our baseline economy; (ii) an economy with a higher elasticity of substitution between domestic and foreign goods (i.e., \( \eta = 2 \)); and (iii) an economy with a higher degree of openness. The shaded gray area represents ranges of parameter values for \( \phi_\Pi \) and \( \phi_Y \) for which the ERR generates larger welfare – in terms of life-time utility – than the IRR.

The first row of Figure B.7 plots the results for our baseline model and for three different values of \( \phi_E \) in the IRR. When we do not allow the central bank to react to fluctuation in the exchange rate in the IRR, the ERR outperforms in terms of welfare as long as the reaction of \( \phi_\Pi \) is not too large. For values of \( \phi_\Pi \) above 3, the IRR generates larger welfare. As the reaction to exchange rate fluctuations increases (columns 2 and 3 in the first row of the figure), the IRR starts outperforming the ERR in terms of welfare for a wider set of parameters.

The second row of Figure B.7 plots the results for a version of our model that has a larger elasticity of substitution between domestic and foreign goods (i.e., \( \eta = 2 \)). In this case, the ERR outperforms the IRR in terms of welfare for every plausible combination of parameter values, when \( \phi_E = 0 \) and when \( \phi_E = 0.2 \). When the reaction to exchange rate fluctuations is stronger, that is, when \( \phi_E = 0.8 \), for large values of \( \phi_\Pi \) and \( \phi_Y \), the IRR generates larger welfare.

Finally, the third row of Figure B.7 plots the results for a version of our model with a higher degree of openness (i.e., \( \alpha = 0.4 \)). In this case, the ERR outperforms the IRR in terms of welfare for every
plausible combination of parameter values of $\phi_{\Pi}$, $\phi_{Y}$, and $\phi_{E}$. That is, as the economy becomes more open, following an exchange rate rule yields better results in terms of welfare than following a standard IRR, regardless of whether we allow the central bank to react to fluctuations in exchange rates.

In conclusion, a policy rule in which the central bank adjusts the exchange rate to react to fluctuations in output and inflation is welfare improving with respect to a monetary rule in which the central bank uses the interest rate as its instrument. This is especially the case for very open economies and for economies in which the elasticity of substitution between domestic and foreign goods is large.\(^{29}\)

### 4.4.1 The Role of Frictions

There are three main distortions in our model: (i) monopolistic competition, (ii) a terms-of-trade externality, and (iii) the presence of external habits. Distortions (i) and (ii) have been discussed extensively in the literature, and are typically removed by adding a time-invariant subsidy that depends on the mark-up and the degree of openness. De Paoli (2009) shows that even in the case of flexible prices, mark-up shocks and movements in the real exchange rate generate inefficiencies in the equilibrium so that a policy of domestic price stabilization that mimics the flexible price allocation does not implement the efficient allocation. Distortion (iii) has been less studied. Leith, Moldovan, and Rossi (2012) show in a closed economy model that external habits introduce an additional externality. When habits are internal (households care about their consumption relative to their own past consumption, rather than the consumption of other households), there is no additional externality associated with consumption habits themselves, and, given an efficient steady state, the flexible price equilibrium in the neighborhood of that steady state remains efficient. However, with external habits there is an externality associated with fluctuations in consumption and the flexible price equilibrium is not usually be efficient, thereby creating an additional trade-off for optimal policy.

To evaluate the role of each distortion in driving welfare differences between the two rules, we simulate our economy under four different specifications:

a. A specification in which all frictions are present (our baseline specification in the paper).

b. A specification that maintains external habits in consumption but removes the monopolistic competition and trade externality.

c. A specification without external habits but maintains monopolistic competition and trade externality.

d. A specification in which all distortions are removed.

We remove the external habit distortion by setting $h = 0$, and the monopolistic competition and trade externality distortions by adding a subsidy $\tau = \tau^{opt}$, where $\tau^{opt} = 1 - \frac{\sigma - 1}{\lambda} \cdot \frac{1}{\Lambda}$, with $\Lambda$ a parameter that depends on $\eta$, $\alpha$, $h$, $\delta$, and $\beta$.\(^{30}\) In all cases, we set $\phi_{\Pi} = 1.5$, $\phi_{Y} = 0.5$, and $\rho = 0.85$.

For each specification, we compute welfare $W_i$ as the life-time utility under each monetary policy rule $i \in \{ERR, IRR\}$, and differences in welfare between the two rules as $\Delta_W = (W_{ERR} - W_{IRR})/W_{IRR} \times 100$. A positive number for $\Delta_W$ indicates that the ERR outperforms the IRR in terms of welfare. Table A.3 reports the results, for both the log-linear and the hybrid cases. Recall that in the log-linear case there is not a risk premium, so differences between the rule are just coming from differences in implementation of each particular rule. The hybrid case, however, captures the role of an endogenous risk premium.

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\(^{29}\)These results are robust to variations of the rules in which (i) both rules react to deviations of domestic inflation instead of CPI inflation; and (ii) both rules react to output growth rather than deviations of output from its steady-state level. The ERR performs better than the peg for all parameter combinations.

\(^{30}\)Details on the derivation of the optimal subsidy are relegated to Appendix E in the paper.
We find that in our baseline specification where all distortions are present the ERR outperforms the IRR by 7.15%. Introducing a subsidy but keeping external habits in consumption (specification b.) delivers similar results: the ERR yields a welfare that is 7.23% higher than the IRR in this case. Instead, when we remove external habits formation, both rules deliver similar welfare results regardless of whether there are other distortions (specification c.) or not (specification d.). Hence, differences in welfare between the two rules seem to be driven by the presence of habit formation rather than by other distortions in the model.

The table also shows that welfare increases as distortions, especially habit formation, are removed. Moreover, as we increase the value of $\phi\Pi$, and hence get closer to the flexible-price equilibrium, the two rules produce similar results in terms of welfare. However, differences in welfare between the rules are higher when there is habit persistence in consumption.

Our findings can also shed light on the role of the endogenous risk premium. As we have discussed throughout the paper, external habits generate endogenous deviations from the UIP condition. In particular, when there is external habit in consumption, shocks to the economy generate counter-cyclical fluctuations of the risk premium. Having established that external habits are the main friction driving differences between the ERR and the IRR in terms of welfare also points at the importance of analyzing in more detail the role of the different rules in smoothing fluctuations of the risk premium. We recalibrate the habit process to generate larger fluctuations of the risk premium in a way that is more consistent with empirical estimates. We use the findings in De Paoli and Zabczyk (2013) who show that if the persistence of shocks and habits is sufficiently high, then one can generate cyclical properties of the risk premium that are more in accordance with the empirical estimates. We thus recalibrate our shock processes by setting the persistence parameter to 0.9977, as in De Paoli and Zabczyk (2013). In an additional simulation exercise we also increase volatility of the shock by a factor of 2. The welfare results and the moments for the risk premium dynamics are reported in Table A.4. We find that a higher persistence and/or a higher volatility of the shock processes generate a larger and more volatile risk premium. Welfare differences between the two rules are amplified in favor of the ERR as the risk premium becomes more volatile. Indeed, the risk premium is always more volatile under the IRR than under the ERR.

These results indicate that by smoothing fluctuations of the risk premium, the ERR can improve the allocation and achieve a higher welfare than the IRR. The differences between the two rules are even larger in an economy with a larger and more volatile risk premium, which reinforces our prior on the importance of external habit formation.

4.4.2 The Role of an Endogenous Risk Premium

We explore the importance of an endogenous risk premium by comparing welfare between the two rules when deviations from the uncovered interest parity (UIP) condition are engineered through UIP shocks rather than through habit persistence. Specifically, we introduce an exogenous wedge, denoted $\chi_t$, between the return on domestic Arrow-Debreu securities and foreign Arrow-Debreu securities,

$$\frac{1}{M_{t+1}} = \frac{\mathcal{E}_{t+1}}{\tilde{\mathcal{E}}_t} \cdot \chi_{t+1}$$

Using the expression for the domestic and foreign pricing kernel, the modified international risk sharing condition writes,

$$\tilde{C}_t = (Q_t \cdot \chi_t)^{\frac{1}{2}} \cdot \tilde{C}_t^*.$$
We assume an auto-regressive process for the UIP deviation as given by
\[
\log (\chi_t) = \rho \chi \log (\chi_{t-1}) + \log (U_{\chi t}), 
\]
where \( \sigma_\chi \) is the standard deviation of innovations \( \log (U_{\chi t}) \).

We follow Kollmann (2005) to calibrate this process, and set the shock persistence to \( \rho = 0.5 \) and the standard deviation \( \sigma_\chi = 3.30\% \) (as in Kollmann (2005), we also consider the case of a standard deviation of \( \sigma_\chi = 0.58\% \)). The results are reported in Table A.5.

We find that in a model with an exogenous risk premium driven by UIP shocks, the IRR outperforms the ERR in terms of welfare. In this case, the ERR does not have the ability to smooth fluctuations of the risk premium, as it is exogenous. This finding suggests that an ERR is less successful at stabilizing the economy than the IRR when deviations from the UIP condition are given exogenously. We interpret these results show that the endogenous risk premium is key in driving our welfare results.

5 Conclusion

We have shown how a theoretical model based on optimizing behavior of households and producers is able to generate powerful conclusions about the desirability of implementing monetary policy through flexible exchange rate targeting. More generally, we show that in a standard microfounded monetary model, relatively open economies can stabilize the economy better through exchange rate rules rather than the traditional rules based on interest rate adjustments.

The model reveals that there are two key sources of the reduction in macroeconomic fluctuations. First, in implementing the exchange rate rule, the central bank announces a gradual depreciation rate, which avoids the standard overshooting result. For small open economies, where a large part of the price level is determined by prices of imported goods, this policy already reduces the volatility of exchange rates and thus prices of imports. To identify the second source, we follow recent advances in international monetary economics and asset pricing, which build into standard models with counter-cyclical risk premia derived endogenously from habits in consumption. The time-varying risk premia drive a wedge between exchange rate movements and the interest rate differential thus further separating the implied dynamics of interest rate rules from the dynamics implied by adopting an exchange rate rule. The time-series properties of the risk premium differ considerably between the rules.

We have kept the model to a bare minimum in terms of its economic structure in order to identify the key factors behind the observed differences between the two rules. There are many directions in which the model can be extended to gain further insights into the desirability of exchange rate rules. First and foremost, increasing the interest rate sensitivity of key economic sectors may lead to a significant improvement in the performance of the interest rate rule. This may occur due to the importance of investment in economic fluctuations or by including a financial accelerator as in Bernanke, Gertler, and Gilchrist (1999). Second, our model does not distinguish between tradable and non-tradable goods. At the same time, secular changes in relative prices due to convergence in income per capita, for example, might present interesting problems for the exchange rate rule. Finally, in the past few years, standard monetary policy rules have been put to the test by the well-known zero lower bound on nominal interest rates. This minimum bound created a problem for economies that use the interest rate as an instrument of monetary policy, forcing them to switch to quantitative easing once interest rates reached zero. For an economy operating with an exchange rate rule, the challenge is different (Amador, Bianchi, Bocola, and Perri (2017)). When the anchor-currency country lowers rates to zero while the domestic economy overheats, the response should be future appreciation of the domestic currency. To meet the no-arbitrage condition implied by UIP, domestic rates have to go below zero. And even though in the past few years
negative rates have been observed, there is a limit to how low negative rates can go. These three extensions are not only interesting from a modeling point of view, but they are clearly relevant for the actual implementation of an exchange rate rule in small open economies.

Finally, Singapore is not the only country that has used the exchange rate to stabilize the economy through monetary policy. From 6 September 2011 to 15 January 2015, the Swiss National Bank introduced as its key monetary policy instrument the minimum exchange rate of 1.20 Swiss francs per euro. The main goal was to correct the massive overvaluation of the Swiss franc. The Czech National Bank (CNB) decided in November 2013 to start using the exchange rate as an additional instrument for easing the monetary conditions, after having lowered the interest rate to almost zero. The CNB intervened in the foreign exchange market to weaken the koruna so as to achieve the desired easing of the monetary conditions as mandated by the CNB Board. In many ways, Switzerland and the Czech Republic share similar characteristics to the Singapore economy. They are small open economies with long-term liquidity in the banking sector. It is not surprising that these economies monitor closely the exchange rate. What distinguishes them from just following the exchange rate is that they have explicitly used it as an instrument of monetary policy to achieve their monetary policy mandates. The particular implementation of the exchange rate rules in these countries differs from the one in Singapore. While both Switzerland and the Czech Republic have abandoned the use an exchange rate rule to stabilize their economies, Singapore has continued using one since 1980, and it has been quite successful at stabilizing inflation and output. Even prior to the successful introduction of the exchange rate as an instrument of monetary policy in Singapore, there were experiments with exchange-rate-based monetary policy in Latin America (the so-called Southern Cone Stabilization Plans). To fight against high inflation, Argentina, Chile and Uruguay introduced in the late 1970s preannounced schedules of depreciation for the exchange rate (tablitas). The tablitas were active crawling pegs, where the central bank preannounced the future values of the nominal exchange rate over a specified horizon. The announcements were expected to directly lower the prices of traded goods and also to lower inflationary expectations and thus future prices.

References


De Paoli, Bianca and Jens Sondergaard. 2009. “Foreign exchange rate risk in a small open economy.”


Friedman, M. 1953. “The case for flexible exchange rates.” 


Appendix

A Tables

Table A.1: Baseline Calibration of the Model’s Parameters

This table shows the calibration of the model’s parameters used in all baseline simulations if not explicitly specified differently.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>External Habits in Consumption</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$ Degree of External Consumption Habits</td>
<td>0.85</td>
<td>GM2005</td>
</tr>
<tr>
<td>$\delta$ Persistence of External Consumption Habits</td>
<td>0.99</td>
<td>DePS2009</td>
</tr>
<tr>
<td><strong>Household Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$ Time-Discount Factor</td>
<td>0.99</td>
<td>GM2005</td>
</tr>
<tr>
<td>$\sigma$ Inverse of the Elasticity of Intertemporal Substitution</td>
<td>5</td>
<td>DePS2009</td>
</tr>
<tr>
<td>$\gamma$ Inverse of the Frisch Labor Supply Elasticity</td>
<td>$3/(1-h)$</td>
<td>DePS2009</td>
</tr>
<tr>
<td>$\varepsilon$ Elasticity of Substitution between Varieties</td>
<td>6</td>
<td>GM2005</td>
</tr>
<tr>
<td>$\eta$ Trade Elasticity</td>
<td>1</td>
<td>GM2005</td>
</tr>
<tr>
<td>$\alpha$ Degree of Openness</td>
<td>0.08</td>
<td>DePS2009</td>
</tr>
<tr>
<td><strong>Production Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$ Calvo Price Stickiness</td>
<td>0.67</td>
<td>GM2005</td>
</tr>
<tr>
<td><strong>Exogenous Processes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_A$ Std. Deviation of Domestic Productivity Shock</td>
<td>0.0071</td>
<td>GM2005</td>
</tr>
<tr>
<td>$\rho_A$ Persistence of Domestic Productivity</td>
<td>0.66</td>
<td>GM2005</td>
</tr>
<tr>
<td>$\sigma_{Y^*}$ Std. Deviation of Foreign Output Shock</td>
<td>0.0078</td>
<td>GM2005</td>
</tr>
<tr>
<td>$\rho_{Y^*}$ Persistence of Foreign Productivity</td>
<td>0.86</td>
<td>GM2005</td>
</tr>
<tr>
<td>$\sigma_{A,Y^*}$ Corr. btw. Domestic Prod. and Foreign Output Shock</td>
<td>0.3</td>
<td>GM2005</td>
</tr>
</tbody>
</table>

Table A.2: Moments under Simplified Monetary Policy Rules

This table shows simulated moments of selected variables of the model under an inflation targeting interest rate rule ($\phi_n = 1.5$, IRR), exchange rate rule ($\phi_n = 1$, ERR), and a fixed exchange rate regime (PEG). Baseline calibrations are used. The economy is subject to domestic productivity and foreign output shocks. Simulations are performed in the log-linearized model (Log-Linear, columns 2-4) and in the model where all equations are approximated to the third order, with the exception of demand and supply equations (Hybrid, columns 5-7). Simulations are done in Dynare, each model is simulated for 10,000 periods, and for the hybrid model the pruning option is used.

<table>
<thead>
<tr>
<th></th>
<th>Log-Linear</th>
<th>Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IRR ERR PEG</td>
<td>IRR ERR PEG</td>
</tr>
<tr>
<td><strong>Real Growth Rates: Standard Deviations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output, $\Delta Y_t$</td>
<td>1.92% 0.49% 0.47%</td>
<td>1.64% 0.61% 0.60%</td>
</tr>
<tr>
<td>Consumption, $\Delta C_t$</td>
<td>0.42% 0.70% 0.70%</td>
<td>0.48% 0.76% 0.76%</td>
</tr>
<tr>
<td>Real Depreciation Rate, $\Delta Q_t$</td>
<td>13.88% 4.61% 4.54%</td>
<td>12.33% 2.65% 2.60%</td>
</tr>
<tr>
<td><strong>Nominal Growth Rates: Standard Deviations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic Inflation, $\Pi_{H,t}$</td>
<td>7.77% 4.37% 4.93%</td>
<td>9.26% 2.52% 2.83%</td>
</tr>
<tr>
<td>CPI Inflation, $\Pi_t$</td>
<td>8.65% 3.98% 4.54%</td>
<td>9.95% 2.29% 2.60%</td>
</tr>
<tr>
<td>Domestic Interest Rate, $R_t$</td>
<td>4.86% 8.10% 7.66%</td>
<td>5.79% 7.57% 7.33%</td>
</tr>
<tr>
<td>Foreign Interest Rate, $R^*_t$</td>
<td>7.66% 7.66% 7.66%</td>
<td>7.33% 7.33% 7.33%</td>
</tr>
<tr>
<td>Nominal Depreciation Rate, $\Delta E_t$</td>
<td>21.21% 1.04% 0.00%</td>
<td>20.44% 0.60% 0.00%</td>
</tr>
<tr>
<td>Interest Rate Differential, $R_t - R^*_t$</td>
<td>2.99% 0.89% 0.00%</td>
<td>2.03% 0.52% 0.00%</td>
</tr>
<tr>
<td><strong>FX Risk Premium: Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.00 0.00 0.00</td>
<td>-0.012 -0.001 0.00</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.00% 0.00% 0.00%</td>
<td>0.16% 0.01% 0.00%</td>
</tr>
</tbody>
</table>
This table shows welfare differences between the ERR and IRR in percent, defined as $\Delta_W = (W_{ERR} - W_{IRR}) / |W_{IRR}| \times 100$, with $\phi_H = 1.5$, $\phi_V = 0.5$, and $\rho = 0.85$. If not otherwise stated, baseline calibrations are used. The economy is subject to domestic productivity and foreign output shocks. Simulations are performed in the log-linearized model (Log-Linear, columns 3-5) and in the model where all equations are approximated to the third order, with the exception of demand and supply equations (Hybrid, columns 6-8). Simulations are done in Dynare, each model is simulated for 10,000 periods, and for the hybrid model the pruning option is used.

### Table A.3: The Role of Frictions

<table>
<thead>
<tr>
<th>Role of Distortions, $\phi_H = 1.5$</th>
<th>Log-Linear</th>
<th>Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W_{IRR}$</td>
<td>$W_{ERR}$</td>
</tr>
<tr>
<td><strong>Baseline</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $h = 0.85, \tau = 0$</td>
<td>$-22817.71$</td>
<td>$-22807.74$</td>
</tr>
<tr>
<td>b. $h = 0.85, \tau = \tau^{opt}$</td>
<td>$-20308.95$</td>
<td>$-20300.50$</td>
</tr>
<tr>
<td>c. $h = 0, \tau = 0$</td>
<td>$-50.21$</td>
<td>$-50.21$</td>
</tr>
<tr>
<td>d. $h = 0, \tau = \tau^{opt}$</td>
<td>$-51.99$</td>
<td>$-51.99$</td>
</tr>
<tr>
<td><strong>Role of Distortions, $\phi_H = 10$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $h = 0.85, \tau = 0$</td>
<td>$-22811.91$</td>
<td>$-22808.91$</td>
</tr>
<tr>
<td>b. $h = 0.85, \tau = \tau^{opt}$</td>
<td>$-20304.06$</td>
<td>$-20301.50$</td>
</tr>
<tr>
<td>c. $h = 0, \tau = 0$</td>
<td>$-50.21$</td>
<td>$-50.21$</td>
</tr>
<tr>
<td>d. $h = 0, \tau = \tau^{opt}$</td>
<td>$-51.99$</td>
<td>$-51.99$</td>
</tr>
<tr>
<td><strong>Role of Distortions, $\phi_H = 100$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $h = 0.85, \tau = 0$</td>
<td>$-22810.51$</td>
<td>$-22810.02$</td>
</tr>
<tr>
<td>b. $h = 0.85, \tau = \tau^{opt}$</td>
<td>$-20302.88$</td>
<td>$-20302.46$</td>
</tr>
<tr>
<td>c. $h = 0, \tau = 0$</td>
<td>$-50.21$</td>
<td>$-50.21$</td>
</tr>
<tr>
<td>d. $h = 0, \tau = \tau^{opt}$</td>
<td>$-51.99$</td>
<td>$-51.99$</td>
</tr>
</tbody>
</table>

### Table A.4: Risk Premium Dynamics under Alternative Specifications

This table shows welfare differences between the ERR and IRR in percent, defined as $\Delta_W = (W_{ERR} - W_{IRR}) / |W_{IRR}| \times 100$ and moments of the risk premium with $\phi_H = 1.5$, $\phi_V = 0.5$, and $\rho = 0.85$. If not otherwise stated, baseline calibrations are used. The economy is subject to domestic productivity and foreign output shocks. Simulations are performed in the model where all equations are approximated to the third order, with the exception of demand and supply equations (Hybrid). Simulations are done in Dynare, each model is simulated for 10,000 periods, and the pruning option is used.

<table>
<thead>
<tr>
<th>Welfare $\Delta_W$</th>
<th>Baseline</th>
<th>High persistence</th>
<th>High persistence and volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_W$</td>
<td>7.147</td>
<td>34.164</td>
<td>59.328</td>
</tr>
</tbody>
</table>

**FX Risk Premium: Moments**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRR</td>
<td>0.0130</td>
<td>0.1745%</td>
</tr>
<tr>
<td>ERR</td>
<td>0.0009</td>
<td>0.7717%</td>
</tr>
<tr>
<td>IRR</td>
<td>0.0135</td>
<td>0.7717%</td>
</tr>
<tr>
<td>ERR</td>
<td>0.0010</td>
<td>0.3078%</td>
</tr>
<tr>
<td>IRR</td>
<td>0.0546</td>
<td>0.4687%</td>
</tr>
<tr>
<td>ERR</td>
<td>0.0039</td>
<td>0.3078%</td>
</tr>
</tbody>
</table>
Table A.5: UIP Shocks

This table shows welfare differences between the ERR and IRR in percent, defined as \( \Delta W = \frac{(W_{ERR} - W_{IRR})}{W_{IRR}} \times 100 \) with \( \phi_\Pi = 1.5 \), \( \phi_Y = 0.5 \), and \( \rho = 0.85 \). If not otherwise stated, baseline calibrations are used. The economy is subject to domestic productivity, foreign output shocks, and UIP shocks as calibrated in Kollmann (2005). Simulations are performed in the model where all equations are approximated to the third order, with the exception of demand and supply equations (Hybrid). Simulations are done in Dynare, each model is simulated for 10,000 periods, and the pruning option is used.

<table>
<thead>
<tr>
<th>Welfare ( \Delta W )</th>
<th>Baseline ( \sigma_\chi = 0.58% )</th>
<th>( \sigma_\chi = 3.30% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRR ERR IRR ERR IRR ERR</td>
<td>IRR ERR IRR ERR IRR ERR</td>
<td></td>
</tr>
<tr>
<td>7.15</td>
<td>-37.0</td>
<td>-507.6</td>
</tr>
</tbody>
</table>

B Figures

Figure B.1: Responses to a Domestic Productivity Shock - Log-Linear Approximation

This figure shows the responses of key variables (in % deviation from steady state) to a positive domestic productivity shock of 1 percent implied by a fixed exchange rate regime (PEG), an interest rate rule (IRR), and an exchange rate rule (ERR), targeting CPI inflation only. The model is approximated to the first order.
Figure B.2: Responses to a Foreign Output Shock - Log-Linear Approximation

This figure shows the responses of key variables (in % deviation from steady state) to a foreign domestic productivity shock of 1 percent implied by a fixed exchange rate regime (PEG), an interest rate rule (IRR), and an exchange rate rule (ERR) targeting CPI inflation only. The model is approximated to the first order.
This figure shows the responses of key variables (in % deviation from steady state) to a positive domestic productivity shock of 1 percent implied by a fixed exchange rate regime (PEG), an interest rate rule (IRR), and an exchange rate rule (ERR), targeting CPI inflation only. The model is approximated to the third order.
This figure shows the responses of key variables (in % deviation from steady state) to a positive foreign output shock of 1 percent implied by a fixed exchange rate regime (PEG), an interest rate rule (IRR), and an exchange rate rule (ERR), targeting CPI inflation only. The model is approximated to the third order.
Figure B.5: Volatilities under Augmented Rules

This figure shows the standard deviations (in %) of key variables implied by a fixed exchange rate regime (PEG), an interest rate rule (IRR) and exchange rate rule (ERR), targeting CPI inflation ($\phi_{\Pi}$) and output ($\phi_Y$). The model is approximated at the third order with the exception of the supply and demand equations. Simulations are obtained in Dynare for 10,000 periods.
Figure B.6: Volatilities under Augmented Rules - IRR with Exchange Rate Target

This figure shows the standard deviations (in %) of key variables implied by an exchange rate rule (ERR) and two interest rate (IRR) rules. In addition to CPI inflation ($\phi_{\Pi}$) and output ($\phi_Y$), the interest rate rules also target nominal depreciation, with high intensity ($\phi_E = 0.8$) and with low intensity ($\phi_E = 0.2$), respectively. The model is approximated at the third order with the exception of the supply and demand equations. Simulations are obtained in Dynare for 10,000 periods.
This figure shows the space of monetary policy rule parameters, $\phi_\Pi$ and $\phi_Y$, for which the exchange rate rule (ERR) outperforms an interest rate rule (IRR), with no ($\phi_E = 0$, column 1), weak ($\phi_E = 0.2$, column 2), or strong ($\phi_E = 0.8$, column 3) nominal depreciation targeting. Rules are compared for the baseline calibration (Panel (a)) and the robustness scenarios of high trade elasticity (Panel (b)) and high openness (Panel (c)). The grey shaded area depicts the parameter space for which the ERR outperforms the IRR.
C The Model’s Equilibrium Equations

The paths of the endogenous variables,
\[ \left\{ C_t, X_t, C_{Ht}, C_{Ft}, A_t, W_t, \frac{P_{Ht}}{P_t}, \frac{P_{Ft}}{P_t}, \Pi_t, N_t, R_t, S_t, Q_t, C_t^*, e_t, Y_t, N_t, MC_t, Y_t^*, \Pi_t^*, R_t^* \right\}, \]
are determined by the following equilibrium conditions.

**Households**

\[ R_t E_t \left\{ \beta \left( \frac{C_t^{\sigma+1} - hX_t^{\sigma+1}}{C_t^{\sigma+1}} \right) - \frac{1}{\Pi_t^{\sigma+1}} \right\} = 1 \]

\[ X_t = \delta X_{t-1} + (1 - \delta)C_{t-1} \]

\[ C_{Ht} = (1 - \alpha) \left( \frac{P_{Ht}}{P_t} \right)^{-\eta} C_t \]

\[ C_{Ft} = \alpha \left( \frac{P_{Ft}}{P_t} \right)^{-\eta} C_t \]

\[ (C_t - hX_t)^\sigma N_t^\gamma = \frac{W_t}{P_t} \]

**Firms**

\[ Y_t = A_t N_t \]

\[ A_t = A_t^{\rho_g} U_{At} \]

\[ W_t = MC_t P_{Ht} A_t \]

**Price Setting**

\[ \frac{\tilde{P}_{Ht}}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{H_t}{F_t} \]

\[ F_t = \Lambda_t Y_t + \theta \beta E_t (F_{t+1}(\Pi_{t+1})^{\varepsilon-1}) \]

\[ H_t = \Lambda_t MC_t Y_t + \beta \theta E_t (H_{t+1}(\Pi_{t+1})^{\varepsilon}) \]

\[ \Lambda_t = (C_t - hX_t)^{-\sigma} \]

\[ \frac{P_{Ht}}{P_t} = \left( 1 - \theta \right) \left( \frac{\tilde{P}_{Ht}}{P_t} \right)^{1-\varepsilon} + \theta \left( \frac{P_{Ht-1}}{P_{t-1}} \right)^{1-\varepsilon} \left( \Pi_t^{\varepsilon-1} \right)^{\frac{1}{\varepsilon}} \]

**Goods Market Clearing**

\[ Y_t = C_t \left[ (1 - \alpha)S_t^\eta Q_t^{-\eta} + \alpha Q_t^{-\frac{1}{\eta}} \right] \]

**Monetary Policy Rules**

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho} \left( \frac{Y_t}{\bar{Y}} \right)^{(1-\rho)\phi_y} \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{(1-\rho)\phi_m} \]
\[
\frac{E_{t+1}}{E_t} = \left( \frac{E_{t+1}^*}{E_t^*} \right)^{(1-\rho)} \left( \frac{E_t}{E_{t-1}} \right)^\rho
\]

Terms of Trade and the Real Exchange Rate

\[
S_t = \frac{P_{Ft}}{P_{Ht}}
\]

\[
Q_t = \frac{P_{Ft}}{P_t}
\]

\[
P_{Ft} = E_t P_t^*
\]

Uncovered Interest Parity

\[
E_t \left\{ \mathcal{M}_{t,t+1} \left( R_t - R_t^* \frac{E_{t+1}}{E_t} \right) \right\} = 0
\]

Domestic Euler Equation

\[
\beta \left( \frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = \mathcal{M}_{t,t+1}
\]

Rest of the World

\[
C_t^* = Y_t^*
\]

\[
\left( \frac{Y_t^*}{Y} \right) = \left( \frac{Y_{t-1}^*}{Y} \right)^{\rho_u} U_{Y_t^*}
\]

\[
R_t^* E_t \left\{ \beta \left( \frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \right\} = 1
\]
We solve for the symmetric steady state.

\[
\begin{align*}
\tilde{C} &= (1 - h)C \\
X &= C \\
\beta R &= 1 \\
P_H &= P \\
P_F &= P \\
C_H &= (1 - \alpha)C \\
C_F &= \alpha C \\
Y &= C \\
\frac{W}{P} &= \omega = \frac{\varepsilon - 1}{\varepsilon} \\
Y &= N \\
N &= (1 - h)\tilde{\pi} \left( \frac{\varepsilon - 1}{\varepsilon} \right)^{\frac{1}{\varepsilon - 1}} \\
Q &= 1 \\
MC &= \frac{\varepsilon - 1}{\varepsilon} \\
F &= \frac{\Lambda Y}{1 - \theta \beta} \\
H &= F \cdot MC \\
\Lambda &= C^{-\sigma} (1 - h)^{-\sigma} \\
\frac{\tilde{P}_H}{P_t} &= 1
\end{align*}
\]

D Derivation of ERR Transmission Channels

This appendix derives Equations (33) and (34), illustrating the mechanism.

Expenditure Switching Effect Channel  We start by loglinearizing demand for the Home good (Equation (27)),

\[
\dot{y}_t^d = -\eta \dot{p}_{H,t} + (1 - \alpha)\tilde{c}_t + \eta \alpha \dot{q}_t + \alpha \tilde{c}_t^*, \tag{D.1}
\]

and replacing the logdeviation of the domestic price (in real terms), \( \dot{p}_{H,t} \), with

\[
\dot{p}_{H,t} = -\frac{\alpha}{1 - \hat{q}_t} \tag{D.2}
\]

from the loglinearization of the CPI index. This yields

\[
\dot{y}_t^d = \frac{\eta \alpha (2 - \alpha)}{1 - \alpha} \dot{q}_t + (1 - \alpha)\tilde{c}_t + \alpha \tilde{c}_t^*. \tag{D.3}
\]

Using the log-linear definition of the real exchange rate, \( \dot{q}_t = \Delta \hat{e}_t - \hat{n}_t + \hat{q}_{t-1} \), we can replace the log-deviation of the nominal depreciation rate with the simplified exchange rate rule (Equation (32)) and obtain

\[
\dot{y}_t^d = \frac{\eta \alpha (2 - \alpha)}{1 - \alpha} (\rho \Delta \hat{e}_{t-1} - (1 + \rho (1 - \phi_H)\hat{n}_t + \hat{q}_{t-1}) + (1 - \alpha)\tilde{c}_t + \alpha \tilde{c}_t^*. \tag{D.4}
\]

Total differentiation holding \( t - 1 \)-variables constant yields Equation (33) in the text.
**Inter-Temporal Consumption-Saving Channel**  To illustrate the intertemporal consumption-saving channel, we loglinearize the domestic Euler equation (Equation (12)),

\[
\hat{c}_t = -\sigma^{-1} (\hat{r}_t - E_t [\hat{\pi}_{t+1}]) + E_t \left[ \hat{c}_{t+1} \right],
\]

and use the uncovered interest parity condition,

\[
\hat{r}_t^* = \hat{r}_t + E_t [-\Delta \hat{c}_{t+1}],
\]

to see how foreign interest rates and currency appreciations transmit to domestic consumption choice,

\[
\hat{c}_t = -\sigma^{-1} (\hat{r}_t^* + E_t [\Delta \hat{c}_{t+1}] - E_t [\hat{\pi}_{t+1}]) + E_t \left[ \hat{c}_{t+1} \right].
\]

Together with the simplified exchange rate rule (Equation (32)), we obtain

\[
\hat{c}_t = -\sigma^{-1} (\hat{r}_t^* + \rho^2 \Delta \hat{c}_{t-1} - \rho(1-\rho)\phi_H \hat{\pi}_t - (1 + (1-\rho)\phi_H)E_t [\hat{\pi}_{t+1}]) + E_t \left[ \hat{c}_{t+1} \right].
\]

Total differentiation holding \( t-1 \)-variables constant and solving the Euler equation forward gives

\[
d\hat{c}_t = -\sigma^{-1}E_t \left[ \sum_{i=0}^{\infty} d\hat{r}_{t+i}^* - \rho(1-\rho)\phi_H \sum_{i=0}^{\infty} d\hat{\pi}_{t+i} - (1 + (1-\rho)\phi_H) \sum_{i=0}^{\infty} d\hat{\pi}_{t+1+i} \right] + \lim_{i \to \infty} E_t \left[ d\hat{c}_{t+1+i} \right].
\]

Since we consider non-explosive solutions only, \( \lim_{i \to \infty} E_t \left[ d\hat{c}_{t+1+i} \right] = 0 \). Gali and Monacelli (2005) show that under an exchange rate peg prices are stationary. We verify numerically that the ERR also implies stationary prices, which, in turn, implies that the sum of all inflation deviations from steady state is zero, \( \sum_{i=0}^{\infty} d\hat{\pi}_{t+i} = 0 \). Adding and substracting \(-1 + (1-\rho)\phi_H)d\hat{\pi}_t \) gives

\[
d\hat{c}_t = -\sigma^{-1}(1 + (1-\rho)\phi_H)d\hat{\pi}_t - \sigma^{-1} \sum_{i=0}^{\infty} E_t \left[ d\hat{r}_{t+i}^* \right].
\]

Equivalently, solving forward the foreign Euler equation,

\[
d\hat{c}_t^* = -\sigma^{-1} \sum_{i=0}^{\infty} E_t \left[ d\hat{c}_{t+i}^* \right],
\]

and using the log-linearization of habit-adjusted consumption, \( \hat{c}_t = (1-h)^{-1}(\hat{c}_t - h\hat{x}_t) \), allows us to solve for consumption,

\[
d\hat{c}_t = -(1-h)\sigma^{-1}(1 + (1-\rho)\phi_H) \cdot d\hat{\pi}_t + d\hat{c}_t^*,
\]

where \( d\hat{x}_t = \delta d\hat{x}_{t-1} + (1-\delta)\hat{c}_{t-1} = 0 = d\hat{x}_t^* \) holding \( t-1 \)-variables constant. This is equation (34) in the text.

**E Derivation of the Optimal Subsidy**

Here, we describe the steps to derive the optimal subsidy that removes the monopolistic competition and trade externality distortions. We start by deriving the expression for the time-invariant production subsidy, \( \tau \), that eliminates the model’s frictions in steady state. We do so by solving the social planner’s problem and characterizing the efficient allocation in steady state. We then derive the decentralized equi-
librium allocation in steady state. Finally, we compare the efficient with the decentralized allocation to find the subsidy \( \tau^{opt} \) that eliminates the decentralized allocation’s inefficient wedge between the marginal rate of substitution between consumption and leisure (MRS) and the marginal rate of transformation (MRT).

**Steady State Efficient Allocation.** Let \( Z_t = \frac{C_t - h_t}{C_t} \) denote surplus consumption given by the ratio of habit-adjusted consumption and consumption. The social planner directly chooses the real allocation – the paths of consumption \( C_t \), surplus consumption \( Z_t \), the real exchange rate \( Q_t \), and output \( Y_t \) – that maximizes the household’s life-time utility given (i) international risk sharing, (ii) aggregate demand for home goods, and (iii) the law of motion for habit formation. In particular, the social planner’s problem is:

\[
\max_{C_t, Z_t, Y_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ (C_t Z_t)^{1-\sigma} - \frac{\left( \frac{Y_t}{P_t} \right)^{1+\gamma}}{1+\gamma} \right]
\]

subject to:

\[
C_t Z_t = Q_t^1 C_t^* Z_t^*
\]

\[
Y_t = \left( \frac{1-\alpha Q_t^{-\gamma}}{1-\gamma} \right)^{1-\gamma} \left[ (1-\alpha)C_t + \alpha Q_t^{-\gamma} C_t^* \right]
\]

\[
C_t (1-Z_t) = h(1-\delta)C_{t-1} + \delta C_{t-1} (1-Z_{t-1})
\]

where \( PD_t = \int_0^1 \left( \frac{P_{t-1}}{P_t} \right)^{-\epsilon} \) is the domestic price dispersion, and \( \mu_{i,t} \) are Lagrange multipliers. We use the aggregate production function \( Y_t = A_t N_t P_{D_t} \) to replace hours \( N_t \) in household’s utility.

As we are interested in the steady state, we drop time subscripts and use the steady state values of exogenous variables in the first order conditions. The yields,

\[
(CZ)^{-\sigma} (1-h) + \mu_1 (1-h) - \mu_2 (1-\alpha) + \mu_3 h - \beta \mu_3 [h (1-\alpha) + \delta h] = 0
\]

\[
(CZ)^{-\sigma} + \frac{\mu_1 - \mu_3 (1-\beta \delta)}{1-\alpha} = 0
\]

\[
\frac{\mu_1}{1-\alpha} - \frac{\eta (2-\alpha)}{1-\alpha} \mu_2 \quad = \quad 0
\]

\[
\mu_2 - Y^\gamma = 0
\]

Then, replacing steady-state Lagrange multipliers yields the efficient wedge, \( \Lambda^e \), between marginal rate of substitution between consumption and leisure and the marginal rate of transformation,

\[
\frac{C}{N^\gamma} = \frac{\left[ \sigma (2-\alpha) (1-\alpha) + (1-\alpha) \right]}{(1-\alpha) (1-\alpha)} \left( \frac{\sigma \eta (2-\alpha) (1-h) + (1-\alpha)^2}{(1-\alpha) (1-h)} \right) \cdot \frac{1}{MRT}, \tag{E.1}
\]

where \( k \equiv \frac{(1-\delta)\beta}{1-\gamma} \). The efficient wedge simplifies to values known in the literature for specific parametrizations. In particular:

- without habit, \( \Lambda^e \big|_{h=0} = \frac{\sigma \eta (2-\alpha) + (1-\alpha)^2}{(1-\alpha)} \) as in De Paoli (2009),

- without habit and with \( \sigma \eta = 1 \), \( \Lambda^e \big|_{h=0, \sigma \eta = 1} = \frac{1}{1-\alpha} \) as in Gali and Monacelli (2005),

- and finally in a closed economy, \( \Lambda^e \big|_{\alpha = 0} = \frac{1}{1-hk} \) as in De Paoli and Sondergaard (2009).

**Steady State Decentralized Allocation with Production Subsidy.** A production subsidy drives a wedge between labor costs and the wage. Since we are interested in the steady state, we drop the time
subscript and firms’ marginal costs are given by

\[ MC = (1 - \tau) \frac{W}{P} \frac{1}{A}, \]  

(E.2)

where \( \tau \) is the production subsidy. In steady state, firms set prices at a constant mark-up over marginal costs given by

\[
\frac{P_H}{P} = \frac{\epsilon}{\epsilon - 1} MC = \frac{\epsilon}{\epsilon - 1} \frac{(1 - \tau) W}{P} \frac{1}{A}.
\]  

(E.4)

Together with the household’s intra-temporal optimality condition in steady state,

\[
\frac{\tilde{C}^{\gamma - \sigma}}{N^{\gamma}} = \frac{W}{P},
\]  

(E.5)

and the steady state values for real domestic prices \( \frac{P_H}{P} = 1 \), and productivity \( A = 1 \), we find that the wedge, \( \Lambda^I \), between marginal rate of substitution of consumption and leisure and marginal rate of transformation in the decentralized equilibrium is given by

\[
\frac{\tilde{C}^{\gamma - \sigma}}{N^{\gamma}} \bigg|_{MRS} = \frac{\epsilon - 1}{\epsilon} \left( \frac{1}{1 - \tau} \right), \quad \frac{1}{MRT},
\]  

(E.6)

**Optimal Production Subsidy.** To find the optimal production subsidy, we set the efficient wedge \( \Lambda^e \) equal to the inefficient wedge in decentralized equilibrium \( \Lambda^I \) and find

\[
\tau^{opt} = 1 - \frac{\epsilon - 1}{\epsilon} \cdot \frac{1}{\Lambda^e},
\]  

(E.7)

where \( \Lambda^e \) is defined above and depends on \( \sigma, \alpha, h, \delta, \) and \( \beta \).