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Flight to What? Dissecting Liquidity Shortages in the Financial Crisis*

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Abstract

We endogenize the *liquidity* and the *quality* of private assets in a tractable incomplete-market model with heterogeneous agents. The model decomposes the convenience yield of government bonds into a "liquidity premium" (flight to liquidity) and a "safety premium" (flight to quality) over the business cycle. When calibrated to match the U.S. aggregate output fluctuations and bond premiums, the model reveals that a sharp reduction in the *quality*, instead of the *liquidity*, of private assets was the culprit of the recent financial crisis, consistent with the perception that it was the subprime-mortgage problem that triggered the Great Recession. Since the provision of public liquidity endogenously affects the provision of private liquidity, our model indicates that excessive injection of public liquidity during a financial crisis can be welfare reducing under either conventional or unconventional policies. In particular, too much intervention for too long can depress capital investment.

Key Words: Liquidity Shortage, Resaleability Constraint, Information Asymmetry, Flight to Liquidity, Flight to Quality, Financial Crisis, Unconventional Policy.

JEL Codes: E44, E58, G01, G10

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1 Introduction

An emerging consensus in the macro-finance literature is that a shortage of private liquidity is the culprit of financial crises and business cycles because it hinders firms' ability to borrow and finance fixed investment, thus depressing aggregate employment and output. Classic works in this area include Kiyotaki and Moore (1997) and Holmström and Tirole (1998).

In particular, Kiyotaki and Moore (1997) show that a decrease in firms' borrowing limit can hinder firms' borrowing capacity, consequently triggering boom-bust cycles in aggregate output. On the other hand, Holmström and Tirole (1998) show that a sudden shortage in private liquidity can interrupt firm investment and generate an economy-wide recession because such shocks cannot be insured or diversified by firms through the private asset market or financial intermediaries. Their model thus provides a rationale for government intervention through the provision of public liquidity during financial crises.

Following these seminal works, a growing literature has emerged to address the issue of private liquidity shortages during financial crises by explicitly embedding financial-market frictions into otherwise standard DSGE models. For example, Kiyotaki and Moore (2012) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2016) (Del Negro et al. hereafter) model financial crises as the consequence of a sudden decline in the resaleability of private assets, which hinders firms' ability to finance fixed investment, causing an economy-wide meltdown through general equilibrium effects. Such shocks to firms' ability to resell (or liquidate) private assets also trigger flight to public liquidity, as in the model of Holmström and Tirole (1998), leading to sharp rises in the liquidity premium of government bonds. Due to the zero lower bound constraint on the nominal interest rate, these authors argue that unconventional policies that allow the government to directly purchase private assets can significantly mitigate the financial crisis by boosting firm investment.

Kurlat (2013) and Bigio (2015), on the other hand, argue that financial crises can also be triggered by other types of financial shocks, such as intensified asymmetric information problems in the private asset market.[1] For example, a sudden increase of low-quality assets in the private asset market can also hinder firms' ability to finance investment projects, and trigger economy-wide recession or even the collapse of the private asset market. In fact, the recent global financial crisis started in the U.S. subprime-mortgage market in 2006. It was this crisis that led to the global financial crisis in 2008 and the subsequent worldwide Great Recession.[2]

However, different types of financial shocks may have dramatically different implications for asset price movements and bond yields as well as government policies. For example, Figure 1 shows that the "liquidity premium" and the "safety (quality) premium" across financial assets can behave very

[1] Kurlat (2013) and Bigio (2015) build their models on Eisfeldt (2004), an early work on the aggregate implications of adverse selection. Also see Malherbe (2014) for a three-period model on self-fulfilling liquidity dry-ups due to adverse selection in financial markets.

[2] See more details in Section 4 about the subprime-mortgage crisis. See Section 5 for a more detailed literature review.

differently over the business cycle: They comove strongly in some periods, especially the 2008 financial crisis, but they move against each other in some other episodes, such as the mid-1970s, the mid-1980s and early 2000s.[3] Such imperfect correlations suggest that the "flight to liquidity" and the "flight to quality" are distinct (albeit related) animals, so their cyclical movements may reveal important information about the types of underlying financial shocks and the associated business-cycle propagation mechanisms.[4]

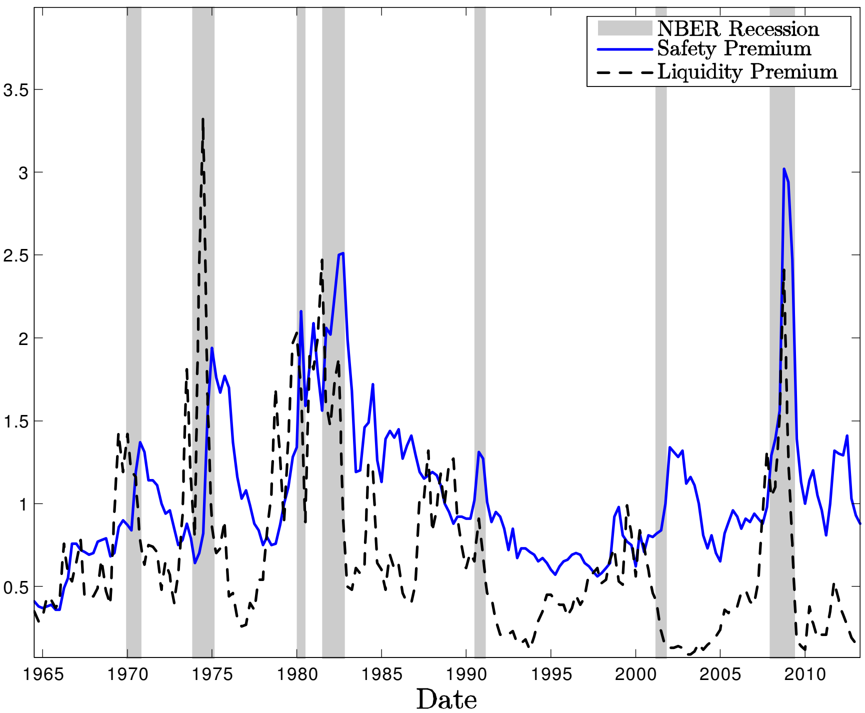

Figure 1. Cyclical Properties of the "Safety Premium" (solid line) and "Liquidity Premium" (dashed line).

Hence, several important questions arise naturally based on the existing macro-finance literature: (i) What financial shocks are quantitatively more important as a cause of the financial crisis and business cycle: the shortage of *liquidity* or the shortage of *quality (safety)* or both?[5] (ii) Can we distinguish flight to liquidity and flight to safety during the business cycle, or is it possible and sensible to decompose, *theoretically* and *empirically*, government bond yields into a liquidity premium

[3] See Appendix A for detailed data description.

[4] There is a large finance literature dealing with the importance of differentiating flight to quality and flight to liquidity, although they are highly correlated with each other. Influential empirical works in this regard include the work of Krishnamurthy and Vissing-Jorgensen (2012), who decompose the convenience yield of government bonds into liquidity and quality (also called safety) premiums. Also see Beber, Brandt and Kavajecz (2009), who empirically address the subtle difference between flight to quality and flight to liquidity. Also see He, Krishnamurthy and Milbradt (2016) and Gorton (2016) for theoretical and historical analysis of safe assets (also called reserve assets) and the shortage of safe assets. In particular, as Gorton (2016) emphasizes, a "safe asset" is an asset that can be used to transact without fear of adverse selection.

[5] In this paper we use *quality* and *safety* interchangeably.

and a quality premium? (iii) Should government respond differently to the different types of premium changes caused by different financial shocks? (iv) How does the supply of public liquidity through either conventional or unconventional policies affect the market resaleability (liquidity) and quality (safety) of private assets? (v) Do there exist limits of government intervention during financial crises?

These questions are intriguing because, as we will show shortly, when both the liquidity and the quality of private assets are endogenized and thus mutually affecting each other, the liquidity-shortage shock considered by Del Negro et al. (2016) implies not only that prices of private assets are countercyclical, but also that the liquidity premium and the safety premium of government bond yields move against each other. Namely, the liquidity premium is countercyclical while the quality premium is procyclical. In contrast, the financial shock considered by Kurlat (2013) and Bigio (2015) implies that the quality premium and the liquidity premium comove strongly with each other; namely, they are both countercyclical while asset prices are procyclical over the business cycle.

This paper provides a unified framework to address these intriguing questions. In particular, we build a tractable heterogeneous-agent model with multiple financial frictions to facilitate the decomposition of government bond premium into a *liquidity* component and a *quality* component, both of which are endogenously determined in the model and are time-varying over the business cycle. We show how the liquidity premium and the quality premium respond differently to different types of financial shocks. We use this knowledge to study the relative importance of different liquidity shocks in explaining the business cycle and especially the recent financial crisis. We also investigate how the provision of public liquidity (either through an injection of government bonds or direct purchase of troubled assets) during recessions affects the incentive structure of the private asset market under different aggregate shocks.

Our main findings are summarized as follows: By decomposing the bond premium into a "liquidity premium" and a "safety premium" over the business cycle, our calibrated model suggests that a sharp reduction in the *quality*, instead of *liquidity*, of private assets was the culprit of the recent financial crisis, which is consistent with the perception that subprime-mortgage problems triggered the Great Recession. In other words, our analysis shows that the flight to safety dominates the flight to liquidity during the recent financial crisis but not necessarily in other episodes of the business cycle. We also show that since provision of public liquidity endogenously affects (crowds out) the provision of private liquidity, proper policy intervention is desirable but excessive injection of public liquidity can be welfare reducing under either conventional or unconventional policies. In particular, too much government intervention can paralyze or even collapse the private asset market, exacerbating the financial crisis.[6]

In the remainder of the paper, Section 2 sets up the core part of the model — heterogeneous firms with multiple financial frictions — and shows the main theoretical properties of the model

[6] This finding is consistent with the empirical evidence provided by Krishnamurthy and Vissing-Jorgensen (2015), who show that safe and liquid government debt crowds out private liquidity. Also see Greenwood, Hanson and Stein (2015) for similar findings.

in this partial-equilibrium setting, especially the market endogeneity of the *liquidity* and *quality* of private assets. Toward the end of that section, we introduce government bonds and show how to decompose the convenience yield into a liquidity premium and a quality premium in our model. Section 3 closes the model by introducing a representative household and studies the general-equilibrium properties of the model. Section 4 calibrates the model and examines its quantitative predictions under different aggregate shocks. That section also quantitatively investigates the different contributions to the business cycle from different types of financial shocks. Section 5 studies the endogenous responses of the liquidity and quality of private assets as well as social welfare from the provision of public liquidity (from both conventional and unconventional policies) under different shocks, and reveals the trade-off between public liquidity and private liquidity. At the end of that section we provide a more detailed literature review. Section 6 concludes the paper with remarks for future research. Appendices A-C contain data descriptions, proofs of propositions, and the model's dynamic system and its log linearization.

2 The Model

Our model is built on Kiyotaki and Moore (2012), Del Negro et al. (2016), and Kurlat (2013) and Bigio (2015), among others.[7]

2.1 Production and Cash Flows

There is a unit measure of heterogeneous firms. In each period $t \geq 0$, firms use capital k_t and labor n_t to produce output y_t through a constant-returns-to-scale production technology:

$$y_t = A_t k_t^{\alpha} n_t^{1-\alpha},$$

where A denotes aggregate total factor productivity (TFP) shocks. To simplify analysis, define W_t as the market wage and

$$R_t^K k_t \equiv \max_{n_t \geq 0} \left\{ A_t k_t^{\alpha} n_t^{1-\alpha} - W_t n_t \right\}$$

as firms' internal cash flow (capital's value added). Choosing labor input optimally and then substituting out optimal labor demand in the cash flow gives the marginal product of capital:

$$R_t^K = \alpha A^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}}. \tag{1}$$

[7] Also see Wang and Wen (2012).

2.2 Adverse Selection and Private Liquidity

The only private asset in this economy is capital. Capital can be used both as a production factor and as a financial asset (or liquidity). Namely, firms can purchase or sell used capital in the financial market.

Analogous to Kiyotaki and Moore (2012) and Wang and Wen (2012), there are idiosyncratic shocks to firms' investment returns (or investment efficiency). That is, 1 unit of investment can be transformed into ε_t units of new capital, where ε_t is an *iid* shock and has the cumulative distribution $F(\varepsilon)$ with support $\mathcal{E} = [\varepsilon_{\min}, \varepsilon_{\max}]$ and mean $\mathbb{E}(\varepsilon) = 1$. Firms are borrowing constrained and cannot issue debt, so they can finance investment projects through internal cash flows ($R_t^K k_t$) and funds raised in the financial market by liquidating (selling) used capital. The efficiency shock ε_t implies that newly installed capital may be more valuable than vintage (used) capital. Hence, firms may opt either to sell old capital to raise funds for new investment if Tobin's q is sufficiently high, or to purchase used capital in the anticipation that future demand for liquidity (cash funds) may be high if a better investment opportunity arrives.

As in Kurlat (2013) and Bigio (2015), there are two types of used capital in the economy: "good" and "bad." The fraction of bad capital in the economy is $\pi_t \in [0, 1]$, which is public knowledge but stochastic. Specifically, in each period t after production, π_t fraction of the used capital held by all firms becomes lemons (bad capital) that will evaporate completely at the end of the period. Only the remaining $1 - \pi_t$ fraction of capital (high-quality assets) can be carried over to the next period, subject to a normal depreciation rate $1 - \gamma \in [0, 1]$. This means capital's marginal product is the same regardless of its quality during the production process, as capital turns into lemons only after use. Then the average survival rate of capital is $\gamma(1 - \pi_t) \equiv 1 - \delta_t$. Of course, when $\pi_t = 0$, then $1 - \delta_t = \gamma$, so we are back to the standard neoclassical model. Also, if $\gamma = 1$, then $\delta_t = \pi_t$. For now we assume $\gamma = 1$ in the theoretical analysis and will calibrate it to the data in the quantitative analysis.

Although firms are subject to the same aggregate shock π_t to the quality of used capital stock after production, the true quality of each specific unit of capital is a firm's private information. Namely, a firm can tell the quality of its own capital in hand but knows nothing about other firms' capital quality until after capital is purchased, except that on average π_t fraction of other firms' capital is of low quality. The asset market for used capital opens after production and closes after investment decisions are made. For simplicity, firms are allowed to trade only once in the asset market in each period.

Firms opt to trade (purchase and sell) used capital for several reasons: (1) to raise additional funds to finance new investment, (2) to get rid of lemon assets before they evaporate, or (3) to accumulate liquidity (used capital) as a store of value to buffer future investment shocks. Hence, the resaleability of used capital means that capital can serve not only as a factor of production but also as a store of value (liquidity) to insure against idiosyncratic investment risks under borrowing constraints.

The timeline of events within a period is graphed in Figure 2. Specifically, production takes place in the beginning of each period (after all aggregate shocks are realized). After production, the asset market for used capital opens for trade, the quality of each specific unit of capital (asset) is revealed to capital owners, and the investment-efficiency shock is realized. Depending on the realization of the idiosyncratic shock ε_t, firms may opt to trade used capital to enhance their investment positions before making investment decisions. At this point, since each firm already knows the quality of its own capital, they opt to sell (fire-sell) all lemon assets (bad capital) but may or may not want to sell their high-quality assets (good capital) because of adverse selection. In particular, since the market price of good capital is the same as that of bad capital, only those firms receiving a sufficiently favorable shock to investment efficiency may opt to liquidate their good assets, so as to raise additional funds to finance new investment projects. Firms receiving less-favorable shocks either undertake investment without liquidating good capital in hand or opt to purchase used capital from the asset market (despite the lemon problem under asymmetry information) without undertaking any investment (by waiting for better future investment opportunities). After the asset market closes and all investment decisions are made, all lemon assets in the economy evaporate and firms pay wages and dividends to the household (owners of the firms) by the end of the period.

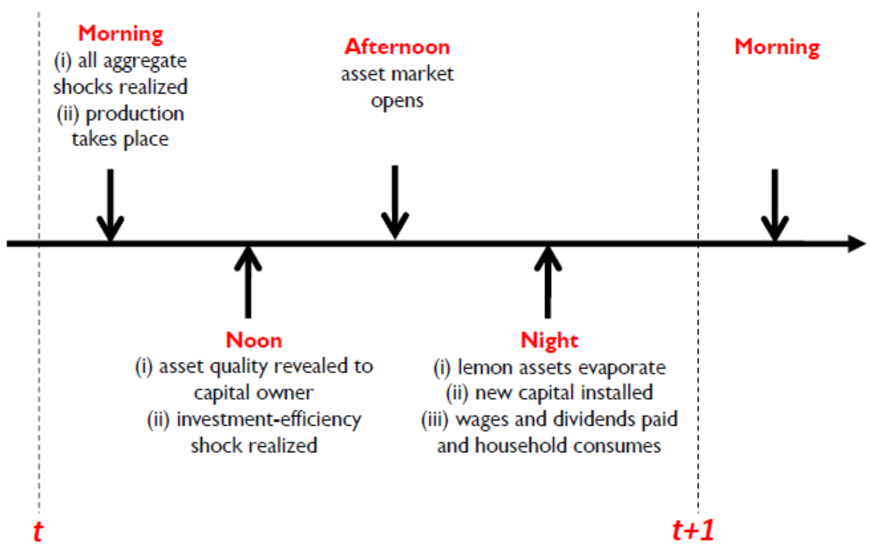

Figure 2. Timeline of Events

2.3 Asset Trading Strategy and Investment Decisions

For each firm in the beginning of period t, its initial stock of capital carried over from last period is k_t, which does not depreciate before the asset market opens if $\gamma = 1$. Denote k_t^a as used capital purchased or acquired (a) from the asset market after production and k_t^s as used capital sold (s) to the asset market after production. Of course, firms cannot distinguish the quality of other firms' used capital when acquiring used capital in the asset market, so the effective amount of used capital a firm acquires is $\theta_t k_t^a \leq k_t^a$, where $\theta_t \in [0,1]$ is the equilibrium fraction of good capital in the market, which is taken as given by firms. Since firms may opt to sell both bad and good capital, we have $k_t^s = k_t^{s,g} + k_t^{s,b}$, where the first item denotes the amount of good capital and the second item the amount of bad capital sold to the market. Since lemon assets (bad capital) evaporate at the end of the period, a firm's capital stock evolves according to the following law of motion:

$$k_{t+1} = (1 - \pi_t) k_t + \theta_t k_t^a - k_t^{s,g} + \varepsilon_t i_t, \tag{2}$$

where i_t denotes new investment and ε_t the shock to investment efficiency. This equation says that by the end of the day only high-quality capital is carried over to the next period.

Denoting P_t^K as the market price of used capital (asset price in the model), the firm's dividend d_t is given by

$$d_t = R_t^K k_t - i_t + P_t^K \left(k_t^{s,g} + k_t^{s,b} - k_t^a \right), \tag{3}$$

where low-quality capital sold to the market ($k_t^{s,b}$) has the same market price as high-quality capital ($k_t^{s,g}$). Due to adverse selection, the market price of used capital will be lower than the shadow value of capital (to be shown below).

Denote $V_t(k_t, \varepsilon_t)$ as the value of a firm in period t and $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$ as the stochastic discounting factor. Then the problem of the firm is to maximize the present value of dividends by solving

$$V_t(k_t, \varepsilon_t) = \max_{\left\{d_t, i_t, k_t^b, k_t^{s,g}, k_t^{s,b}\right\}} \left\{ d_t + \mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} V_{t+1}(k_{t+1}, \varepsilon_{t+1}) \right\},$$

subject to (2), (3), and the following resaleability constraints and non-negativity constraints:

$$k_t^{s,g} \leq \bar{\omega}_t (1 - \pi_t) k_t, \tag{4}$$

$$k_t^{s,b} \leq \bar{\omega}_t \pi_t k_t, \tag{5}$$

$$\left\{ i_t, d_t, k_t^a, k_t^{s,g}, k_t^{s,b} \right\} \geq 0, \tag{6}$$

where $\bar{\omega}_t \in [0,1]$ is a stochastic parameter restricting the resaleability of used capital (a la Kiyotaki and Moore, 2012). It applies to both good and bad assets symmetrically. The constraint $i_t \geq 0$ indicates irreversibility of investment, and $d_t \geq 0$ indicates that firms cannot borrow directly from the household.

Firms' decision rules for asset trading and new investment are presented in Appendix B. The following three propositions summarize the most important properties of the model. In this model, firms' decision rules for fixed investment and asset trading are characterized by cutoff strategies, which effectively classify all firms into two groups (so firms are ex ante identical but ex post heterogeneous): non-investing (inactive) firms and investing (active) firms. Obviously, firms will undertake new investment if their idiosyncratic shocks exceed a threshold (cutoff), $\varepsilon_t \geq \varepsilon_t^*$. Inactive firms do not undertake new investment because their idiosyncratic shocks are below the cutoff, $\varepsilon_t < \varepsilon_t^*$. Obviously, all firms opt to fire-sell their bad capital up to the resaleability constraint, $k_t^{s,b} = \bar{\omega}_t \pi_t k_t$.

However, there exists a second cutoff $\varepsilon_t^{**} > \varepsilon_t^*$ such that active firms will liquidate their good capital if and only if $\varepsilon_t \geq \varepsilon_t^{**}$. The reason the second cutoff ε^{**} lies above the first cutoff ε^* is that information friction (asymmetry) about asset quality generates a wedge between the shadow value Q_t of good capital (for the capital owner) and the market value (price) P_t^K of used capital. Due to adverse selection, the market price of used capital is lower than the shadow value of capital, $P_t^K < Q_t$. Thus, liquidating good capital leads to capital losses. This wedge reduces a firm's incentive to liquidate its high-quality capital unless its investment-efficiency shock is sufficiently high or above ε_t^{**}. So for firms willing to sell their good capital, we have $k_t^{s,g} = \bar{\omega}_t (1 - \pi_t) k_t$.

Proposition 1 *The two cutoff values $\{\varepsilon_t^*, \varepsilon_t^{**}\}$ depend only on aggregate states and not on the history of individual firms, and they fully characterize the distribution of firms. Furthermore, they are related to each other by*

$$\varepsilon_t^* = \theta_t \varepsilon_t^{**}, \tag{7}$$

where $\theta_t \in [0,1]$ is the equilibrium fraction of high-quality assets in the asset market.

Proof. See Appendix B. ■

Intuitively, we would have $\varepsilon_t^{**} = \varepsilon_t^*$ either in the case without private information or in the case with $\pi_t = 0$. Namely, the higher cutoff converges to the lower cutoff either in the case of symmetric information (for any given value of $\pi_t > 0$) or in the case of nonexistent lemons ($\pi_t = 0$).

Proposition 2 *The liquidity (resaleability) and the quality of private assets are endogenously determined in general equilibrium. In particular, given the aggregate (average) fraction of high-quality capital $(1 - \pi_t)$ in the economy, the true asset quality θ_t (i.e., the fraction of high-quality assets in the asset market) is given by*

$$\theta_t = \frac{(1 - \pi_t)\left(1 - F\left(\varepsilon_t^{**}\right)\right)}{\pi_t + (1 - \pi_t)\left(1 - F\left(\varepsilon_t^{**}\right)\right)} < 1 - \pi_t, \tag{8}$$

and the endogenously determined resaleability ω_t^* of private assets is given by

$$\omega_t^* = \left[\pi_t + (1 - \pi_t)\frac{1 - F\left(\varepsilon_t^{**}\right)}{1 - F\left(\varepsilon_t^*\right)}\right]\bar{\omega}_t < \bar{\omega}_t. \tag{9}$$

Proof. See Appendix B. ■

Comment 1. *Endogenous Quality* θ_t. Note that the average asset quality in the asset market is less than the average fraction of good assets in the economy, $\theta_t < 1 - \pi_t$, due to adverse selection. In other words, the effective portion of lemon assets in the market is larger than that in the overall economy, $1 - \theta_t > \pi_t$. This is the case because only a $[1 - F(\varepsilon^{**})]$ fraction of firms opt to sell good capital in the asset market, while all firms opt to fire-sell bad capital. Hence, the fraction of high-quality capital in the asset market is $\frac{(1-\pi_t)(1-F(\varepsilon_t^{**}))}{\pi_t+(1-\pi_t)(1-F(\varepsilon_t^{**}))} < 1 - \pi_t$. Consequently, adverse selection under asymmetric information intensifies the lemon problem. Clearly, $\theta_t = 1$ either in the absence of asymmetric information (i.e., asset quality is public information) or the absence of lemons (i.e., $\pi_t = 0$). In the former case, lemon assets have no value in the market, so the denominator becomes identical to the numerator in θ_t. In these two cases we have $\varepsilon_t^* = \varepsilon_t^{**}$. Namely, in the absence of adverse selection, firms always opt to sell high-quality capital to finance new investment whenever $\varepsilon_t \geq \varepsilon_t^*$.

Comment 2. *Endogenous Liquidity (Resaleability)* ω_t^*. Since firms can resell a maximum $\bar{\omega}_t$ portion of their used capital (either good or bad) when so desired, and since firms do not always resell good capital due to asset-price distortions under adverse selection, the marginal propensity to resell used capital (an endogenous measure of resaleability in our model) is given by ω_t^* in equation (9), where the first term π_t and the second term $(1 - \pi_t) \frac{1-F(\varepsilon_t^{**})}{1-F(\varepsilon_t^*)}$ in the brackets denote, respectively, the fraction of lemon assets and the fraction of high-quality assets in the market. So $\omega_t^* k_t$ is the actual portion of total assets a firm is willing to sell to the market. This endogenous measure of resaleability is lower than the exogenously specified resaleability $\bar{\omega}_t$ because adverse selection reduces firms' propensity to resale their good capital even when liquidity is needed. In other words, firms want to dump all lemons into the market with the upper limit $\pi_t \bar{\omega}_t$. On the other hand, since $\frac{1-F(\varepsilon_t^{**})}{1-F(\varepsilon_t^*)} = \Pr(\varepsilon > \varepsilon_t^{**} | \varepsilon > \varepsilon_t^*)$ is less than 1 and it measures the conditional probability of reselling high-quality assets given that the firm is undertaking investment, $(1 - \pi_t) \frac{1-F(\varepsilon_t^{**})}{1-F(\varepsilon_t^*)} \bar{\omega}_t$ is the actual fraction of high-quality assets supplied to the market. Clearly, $\frac{1-F(\varepsilon_t^{**})}{1-F(\varepsilon_t^*)} = 1$ when $\varepsilon_t^* = \varepsilon_t^{**}$ (or when $\theta_t = 1$); in this case, we have $\omega_t^* = \bar{\omega}_t$. Therefore, the marginal propensity (ability) to resell used capital is the true measure of resaleability in the model, which is lower than and bounded above by $\bar{\omega}_t$ — the *fundamental* resaleability constraint. Most importantly, the endogenous resaleability ω_t^* is state dependent and thus responsive to aggregate shocks. In particular, ω_t^* is a monotonically decreasing function of $\pi_t \in [0, \pi_{\max}]$, where $\pi_{\max} < 1$ is the critical value of π_t such that the asset market collapses for $\pi_t \geq \pi_{\max}$, because too many lemons can crash the market by driving out all good assets.

This implication is in sharp contrast to the model of Kiyotaki and Moore (2012) and will be discussed in detail in a later section.

Proposition 3 *The shadow value of capital Q_t is related to the lower cutoff ε_t^* by*

$$Q_t = \frac{1}{\varepsilon_t^*},$$

and the market price of used capital is related to the higher cutoff by

$$P_t^K = \frac{1}{\varepsilon_t^{**}} = \theta_t Q_t < Q_t.$$

Proof. See Appendix B. ■

Intuitively, the marginal cost of investment is 1 (in terms of output or consumption goods) and the shadow value of 1 unit of new capital is Q (units of output). Since 1 unit of investment can yield ε units of new capital (under the efficiency shock), which are worth εQ units of output, then a firm will undertake investment if and only if $\varepsilon Q \geq 1$, or $\varepsilon \geq \frac{1}{Q} \equiv \varepsilon^*$. This is Tobin's q theory — individual firm's q is simply εQ, where $Q_t = \frac{1}{\varepsilon_t^*}$.

Since 1 unit of used (high-quality) capital is as good as 1 unit of new capital if kept in hand, its shadow value is Q. On the other hand, 1 unit of used capital is worth only P^K real dollars (units of output) in the asset market; so if a firm opts to resell used (high-quality) capital in the asset market to finance new investment, then it must be the case that $\varepsilon Q P^K \geq Q$, or $\varepsilon \geq \frac{1}{P^k} \equiv \varepsilon^{**}$. Since $\varepsilon^{**} = \varepsilon^*/\theta$, we have $P^K = \theta Q < Q$; namely, the market price of used capital is lower than the shadow value of good capital under adverse selection.

Clearly, in the absence of adverse selection, either because of the lack of asymmetric information (i.e., asset quality is public information) or because there are no lemons in the economy (i.e., $\pi = 0$), we have $P^K = Q$ and $\varepsilon^* = \varepsilon^{**}$.

2.4 Bond Premium and Its Decomposition

This subsection introduces government bonds as an alternative store of value for firms. We assume (consistent with the empirical work of Krishnamurthy and Vissing-Jorgensen, 2012) that government bonds are perfectly safe and liquid (i.e., without any information frictions and resaleability constraints), so that they command a premium over capital (equity) returns.[8] Since private asset (capital) returns are affected by both resaleability constraints and information risk, our model provides a natural framework to decompose the bond premium into a liquidity component and a safety (quality) component, for which we can analyze their business-cycle properties.

[8] As shown by Gomme, Ravikumar and Rupert (2011), the return to capital is identical to the return to equity in neoclassical models.

Given the productivity of capital, firms will hold both public liquidity and private liquidity under the no-arbitrage condition. Obviously, firms will demand more public liquidity in the case of a negative shock to ω_t simply because private assets become harder to sell (less liquid), so bond premium rises as a result of "flight to liquidity", leading to a higher "liquidity premium."

Firms also demand more government bonds in the case of a positive shock to π_t when private assets become riskier, so the bond premium rises as a consequence of the flight to quality, leading to a higher quality premium.

However, because in general equilibrium the resaleability (ω_t^*) and quality (θ_t) of private assets are both endogenous, the liquidity premium and the quality premium behave differently and contribute differently to fluctuations in the bond premium under different aggregate shocks.

Denoting P_t^B as the market price of government bonds and b_t as a firm's end-of-period holding of government bonds, the problem of the firm is modified slightly to

$$V_t\left(k_t, b_t, \varepsilon_t\right) = \max_{\left\{d_t, i_t, k_t^b, k_t^{s,g}, k_t^{s,b}\right\}} \left\{d_t + \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(k_{t+1}, b_{t+1}, \varepsilon_{t+1}\right)\right\}, \tag{10}$$

subject to

$$d_t = R_t^K k_t - i_t + P_t^K \left(k_t^{s,g} + k_t^{s,b} - k_t^a\right) - P_t^B b_{t+1} + b_t, \tag{11}$$

$$b_{t+1} \geq 0, \tag{12}$$

and the same constraints in equations (2)-(6).

Proposition 4 ***(Decomposition of Yields)*** *The convenience yield (CY) (spread between the rate of return to private assets and the rate of return to government bonds) can be decomposed into a liquidity premium (r_t^{ω}) and a safety premium (r_t^{π}):*

$$CY_t = r_t^{\omega} + r_t^{\pi},$$

where $\lim_{\omega \to 1} r_t^{\omega} = 0$ and $\lim_{\pi \to 0} r_t^{\pi} = 0$. That is, the liquidity premium vanishes when $\omega = 1$ and the quality premium vanishes when $\pi = 0$.

Proof. See Appendix B. ■

3 General Equilibrium Analysis

We show how to exploit the differences in the liquidity premium and quality premium to shed light on the recent financial crisis in the U.S. To facilitate the analysis, we close the model by adding a household sector below.

3.1 Household

The problem of a representative household is given by

$$\max_{\{C_t, N_t, s^i_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \chi \frac{N_t^{1+\xi}}{1+\xi} \right),$$

subject to

$$C_t + \int_{i\in[0,1]} s^i_{t+1} \left(V^i_t - d^i_t\right) di = \int_{i\in[0,1]} s^i_t V^i_t di + W_t N_t - T_t, \tag{13}$$

where β is the time discount factor, C_t is consumption, N_t is the labor supply, and V^i_t and s^i_t are, respectively, the equity price of firm $i \in [0,1]$ and the associated share holdings. The household receives labor income $W_t N_t$ and pays lump-sum tax T_t to the government.

Denoting Λ_t as the Lagrangian multiplier of the household budget constraint, the first-order conditions (FOCs) with respect to consumption (C_t), the labor supply (N_t), and the holdings of firm shares $\left(s^i_{t+1}\right)_{i\in[0,1]}$ are given, respectively, by

$$\Lambda_t = \frac{1}{C_t}, \tag{14}$$

$$W_t \Lambda_t = \chi N_t^{\xi}, \tag{15}$$

$$V^i_t = d^i_t + \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V^i_{t+1}, \tag{16}$$

where the last equation pertains to the firm's objective function studied earlier.

3.2 Steady-State Properties

The system of dynamic equations in general equilibrium is provided in Appendix C. Here we show that there exist two steady states in the model (for certain parameter values): (i) a high steady state with a positive rate of return to bonds, $R^B_H \equiv 1/P^B > 1$ and (ii) a low steady state with a negative return to government bonds, $R^B_L < 1$. The dashed blue lines in the left panel in Figure 3 shows a backward-bending curve of bond yields, which intersects with the vertical bar (bond supply) at two different values of bond yields R^B.[9]

Multiple steady states arise in this model because of a participation externality in the asset market: The equilibrium asset price is higher when more high-quality assets are traded and lower when fewer

[9] The panels in Figure 3 show the yield curve of government bonds, a relationship between bond returns and aggregate bond supply. The horizontal axes denote the level of total bond supply $\bar{B}$ and the vertical axes denote the equilibrium yield (rate of return) to bonds. The vertical bars indicate a particular level of the bond supply. The solid red lines represent bond yields in the case of perfect information ($\widetilde{R}^B$) and are upward sloping. The dashed blue lines represent bond yields under imperfect information.

high-quality assets are traded. Which equilibrium prevails depends on firms' beliefs about other firms' actions. Intuitively, the second (low) steady state arises if firms believe that the lemon problem is so severe such that they are willing to hold bonds even if they yield a negative rate of return, as in the case of inventories (Wen, 2011), because in this case the safety and liquidity of bonds make them such a desirable store of value when used capital is too illiquid due to severe adverse selection. This case is close but not identical to the case where the private asset market is shut down completely.

Proposition 5 *The steady state of the model can be solved recursively in a loop. In particular, given the private-asset price P^K, we can solve all aggregate variables in the following list sequentially (one by one): $\{\varepsilon^{**}, \theta, Q, \varepsilon^{*}, P^B, R^B, R^K, \frac{B}{K}, \frac{I}{K}, \frac{Y}{K}, \frac{C}{K}, N, K, Y, I, C, B\}$. Then we can use the bond-market clearing condition, $B\left(P^K\right) = \bar{B}$, to pin down the capital price P^K.*

Proof. See Appendix B. ■

The bond-market clearing condition solves for private asset price P_t^K in the model, suggesting that the provision of public liquidity interacts with the incentive structure of the provision of private assets. This important property is missing in the aforementioned existing literature.

The reason is that bond yields affects the demand and supply of private assets, hence the *liquidity* and *quality* of assets traded in the market. This endogenous relationship between bond yields and private asset returns holds until the limit when total bond supply becomes zero, $\bar{B} = 0$; but the relationship breaks down if the total supply of bonds is too large, which will collapse the private asset market.

In particular, a higher bond supply raises bond yield R^B, inducing firms to hold more public liquidity and less private liquidity in their portfolio. When the supply of government bonds is high enough, say $\bar{B} \geq \bar{B}_{\max}$, the asset market with lemons collapses because demand for used capital as a store of value vanishes. In this case, firms hold capital only as a factor of production and hold bonds only as a store of value, which causes a discontinuous jump in the rate of return to bonds from R^B to $\bar{R}^B > R^B$, where $\bar{R}^B$ denotes the rate of return to bonds without information frictions (see the red line in the top of the left panel in Figure 3), as in the model of Del Negro et al. (2016).

This discontinuity happens because the rate of return to bonds without information frictions ($\bar{R}^B$) lies strictly above the rate of return to bonds with private information for any given level of bond supply $\bar{B}$. Hence, when the private asset market for used capital collapses, it is as if the information problem disappears because public liquidity is now the only store of value and does not suffer from the information problem nor the resaleability problem. In this case, our model is reduced to that of Del Negro et al. (2016).

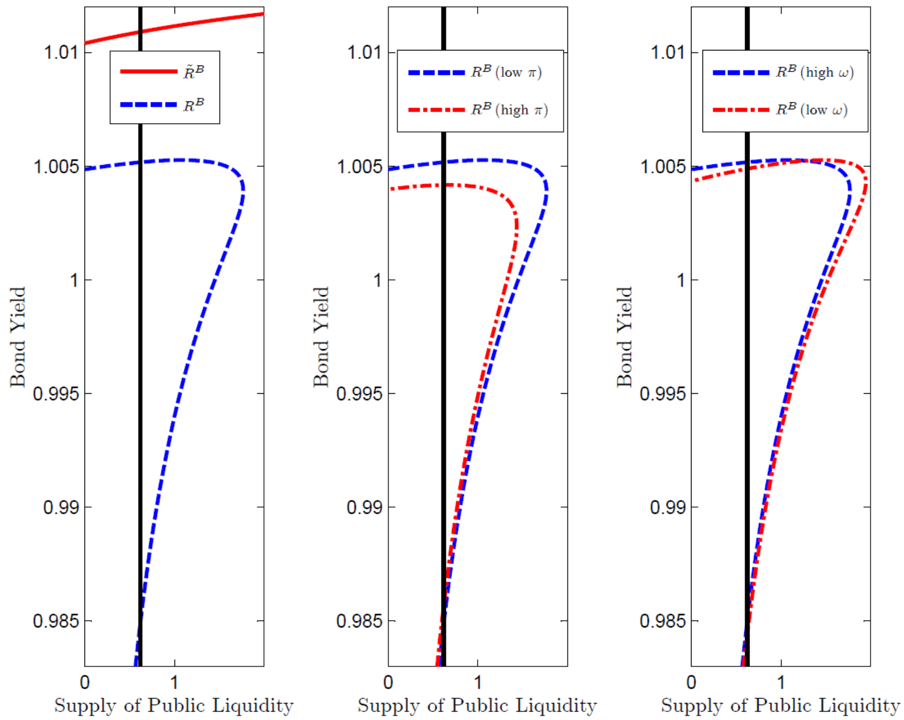

Figure 3. Bond Yields ($\tilde{R}^B$: without frictions; R^B: with frictions).

In other words, as we shrink the bond supply (by shifting the vertical bar) gradually from right to left along the horizontal axes in Figure 3 (left panel), arbitrage between private liquidity and public liquidity causes the equilibrium bond return to drop from the top solid line down to the dashed line at a critical level $\bar{B}_{\max}$ and then bifurcates along two paths afterwards (along the backward-bending dashed blue lines). That is, for any level of bond supply $\bar{B} \in [0, B_{\max})$, there exist two equilibrium values of bond return $\{R_L^B, R_H^B\}$, such that $R_L^B < R_H^B < \bar{R}^B$, where the high-yield equilibrium R_H^B indicates a higher fraction of high-quality capital in the asset market and the low-yield equilibrium R_L^B indicates a lower fraction of high-quality capital in the asset market. These equilibrium bond returns lie strictly below the rate of return to bonds under perfect information ($\bar{R}^B$).

At the lower equilibrium (R_L^B), trading volume, aggregate investment, consumption and output are extremely low; the marginal product of capital is extremely high; and the rate of return to bonds is negative (or $P^B > 1$). When the lemon problem is so severe, firms are willing to hold government bonds even if they pay a negative rate of return, exactly like the case of inventories in Wen's (2011) model, which has a negative rate of return because of the high liquidation value offered by inventories (or government bonds) when alternative assets are so illiquid in meeting firms' liquidity demand.

The middle panel in Figure 3 shows the effect of the flight to *quality* after an increase in the parameter value of π. The backward-bending curve shifts down and inward, so the original high-yield equilibrium R_H^B drops vertically from the dashed blue line to the dashed red line. This drop in bond yields is due primarily to the flight to quality. However, if this shock is big enough, the backward-bending curve may shift down too much to the origin and consequently no longer intersect with the vertical bar, suggesting a collapse of the asset market when the problem of adverse selection intensifies beyond a critical level.

In contrast, the right panel in Figure 3 shows the effect of the flight to *liquidity* after a decrease in the resaleability parameter value $\bar{\omega}$, which makes private assets less liquid and resalable. The backward-bending curve shifts down and outward (instead of inward), and the high-yield equilibrium R_H^B drops only slightly. The effect becomes ambiguous if the vertical bar is sufficiently far away from the origin. Most importantly, the asset market appears more robust (in terms of the likelihood of collapse) to the shock that reduces the liquidity (resaleability) of private assets (ω) than to the shock that reduces the quality of private assets (π).

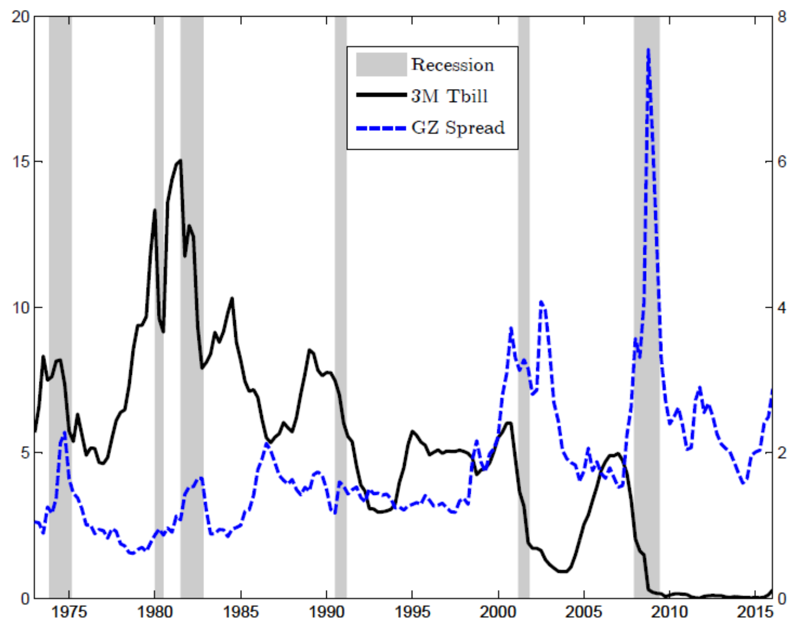

Figure 4. U.S. 3-Month Treasury Bill Yields (3M T-bill: solid black line, GZ spread: dashed blue line).

Figure 4 shows that the rate of return to U.S. Treasuries has declined dramatically since the beginning of the recent financial crisis. In the mean time, because of high demand for government bonds, the premium (as measured by the Gilchrist-Zakrajšek-credit spreads) increased dramatically, especially during the NBER-dated recession period.[10] The intriguing question is this: Was such a sharp drop in bond yields or sharp rise in the bond premium the result of a flight to *liquidity*, or a

[10] See Appendix A for detailed data description.

flight to *safety*, or both? We believe that our theoretical model can help answer such a question and shed light on the mechanisms behind the recent financial crisis.

The rest of this paper considers business-cycle dynamics only around the high steady state with positive bond returns, which is saddle stable and subject to only fundamental shocks. We leave the business-cycle analysis around the second steady state and the associated sunspot-driven fluctuations to a separate project.

4 Business-Cycle Analysis

4.1 Calibration

The time period is one quarter. We set $\alpha = 0.36$ to match the labor share of 64%, $\xi = 0.5$ to yield a Frisch elasticity of 2, and $\delta = 0.025$ to match the 10% annual depreciation rate of capital. In addition, assume that the investment efficiency ε follows the Pareto distribution, $F(\varepsilon) = 1 - (\varepsilon/\varepsilon_{\min})^{-\sigma}$ with $\varepsilon \geq \varepsilon_{\min}$ and $\sigma > 1$.

Table 1. Calibration

Parameter	Value	Definition
β	0.985	discount factor
α	0.36	capital share
ξ	0.5	inverse Frisch elasticity
χ	6.42	coefficient of labor disutility
$\bar{\omega}$	0.82	resaleability coefficient
π	0.30%	portion of lemon assets
γ	0.978	physical depreciation rate
σ	7.14	coefficient of Pareto distribution
$\varepsilon_{\min}$	0.86	lower bound of ε shock
$\bar{B}$	0.64	total government debt

Since the four parameters $(\pi, \bar{\omega}, \sigma, \bar{B})$ are specific to our model, we use the following moment conditions to jointly pin them down: (i) Following Del Negro et al. (2016) and Cui (2016), we set the debt-to-GDP ratio $\bar{B}/Y = 60\%$ in the steady state. (ii) As standard in the literature, we set the investment-to-GDP ratio $I/Y = 0.2$. (iii) Regarding the frequency of each firm's investment, $\zeta = 1 - F(\varepsilon^*)$, Del Negro et al. (2016) set $\zeta = 0.09\%$, Kiyotaki and Moore (2012) set $\zeta = 5\%$, Doms and Dunne (1998) set $\zeta = 6\%$, and Cooper et al (1999) set $\zeta = 10\%$, among others. We take the average of these values and set $\zeta = 5.5\%$. (iv) Following Del Negro et al. (2016), we set the equilibrium value of resaleability to $\omega^* = 31\%$.

These calibrations imply $\pi = 0.3\%$, $\bar{\omega} = 82\%$, $\sigma = 7.14$, and $\bar{B} = 0.64$. They in turn imply $\gamma = \frac{1-\delta}{1-\pi} = 0.978$, which is close to the value of 0.96 proposed by Kurlat (2013). In addition, as specified in Section 2, we normalize the investment efficiency shock at $\mathbb{E}(\varepsilon) = 1$. Since $\mathbb{E}(\varepsilon) = \frac{\sigma}{\sigma-1}\varepsilon_{\min}$, we then have $\varepsilon_{\min} = 1 - \frac{1}{\sigma} = 0.86$. Moreover, as calculated by Del Negro et al. (2016), the annualized real return of government bonds is around 2.2%. Thus we set $R^B = 1.0055$ for one quarter, which implies the time discount factor $\beta = 1/\left[R^B\left(1 + \frac{\zeta}{\sigma-1}\right)\right] = 0.985$. Finally, we set $\chi = 6.42$ so that the fraction of hours worked $N = 0.25$, as is standard in the literature. Table 1 summarizes the calibrations.

The aggregate amount of used capital traded in the asset market is

$$S = \left[\pi + \left(1 - F\left(\varepsilon^{**}\right)\right)(1-\delta)\right]\omega K, \tag{17}$$

and the aggregate new investment is

$$I = \left[B + r^K K + \pi\omega q^K K\right]\left(1 - F\left(\varepsilon^{*}\right)\right) + \omega(1-\delta)P^K K\left[1 - F\left(\varepsilon^{**}\right)\right]. \tag{18}$$

Therefore, the ratio of investment in used capital to total investment is $\frac{S}{S+I}$. The market for used capital in the U.S. is non-trivial. Using Compustat dataset, Eisfeldt and Rampini (2005) show that the ratio $\frac{S}{S+I} = 25\%$ by 2002. With the same dataset but updated to a later time period, Cui (2014) finds that $\frac{S}{S+I} = 40\%$ by 2012. Under the parameterization in Table 1, our model predicts that $\frac{S}{S+I}$ is around 30%, right in the interval of $[25\%, 40\%]$.

4.2 Impulse Responses

We assume three aggregate shocks in the model: (i) a TFP shock (A_t), which follows the law of motion in log-linear form: $\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_t^A$; (ii) an information shock (π_t), which follows $\hat{\pi}_t = \rho_\pi \hat{\pi}_{t-1} + \varepsilon_t^\pi$; (iii) and a resaleability shock (ω_t), which follows $\hat{\omega}_t = \rho_\omega \hat{\omega}_{t-1} + \varepsilon_t^\omega$.

Figure 5 shows the impulse responses of output (Y_t), asset prices (P_t^K), the bond yield (R_t^B) and bond premiums (r_t^ω and r_t^π) in the model to a negative TFP shock in the top-row panel, a negative information shock to $1 - \pi_t$ (a rise in the fraction of lemon assets in the economy) in the middle-row panel, and a negative liquidity shock to $\bar{\omega}_t$ (a reduction in the resaleability of private assets) in the bottom-row panel. So from the left to right in each row panel, the 1st window is output, the 2nd window is asset prices, the 3rd window is bond yields, and the 4th window plots the liquidity premium r_t^π (solid line) and the safety premium r_t^ω (dashed line).

Clearly, all three shocks are able to generate a deep recession in output (1st window in each row panel) and a sharp decline in bond yields (3rd window in each row panel). However, the price of private asset declines under TFP shock and information shock (2nd window in the top and bottom panels), but rises under resaleability shock (2nd window in the middle panel), making the resaleability shock inconsistent with the U.S. data along this dimension.

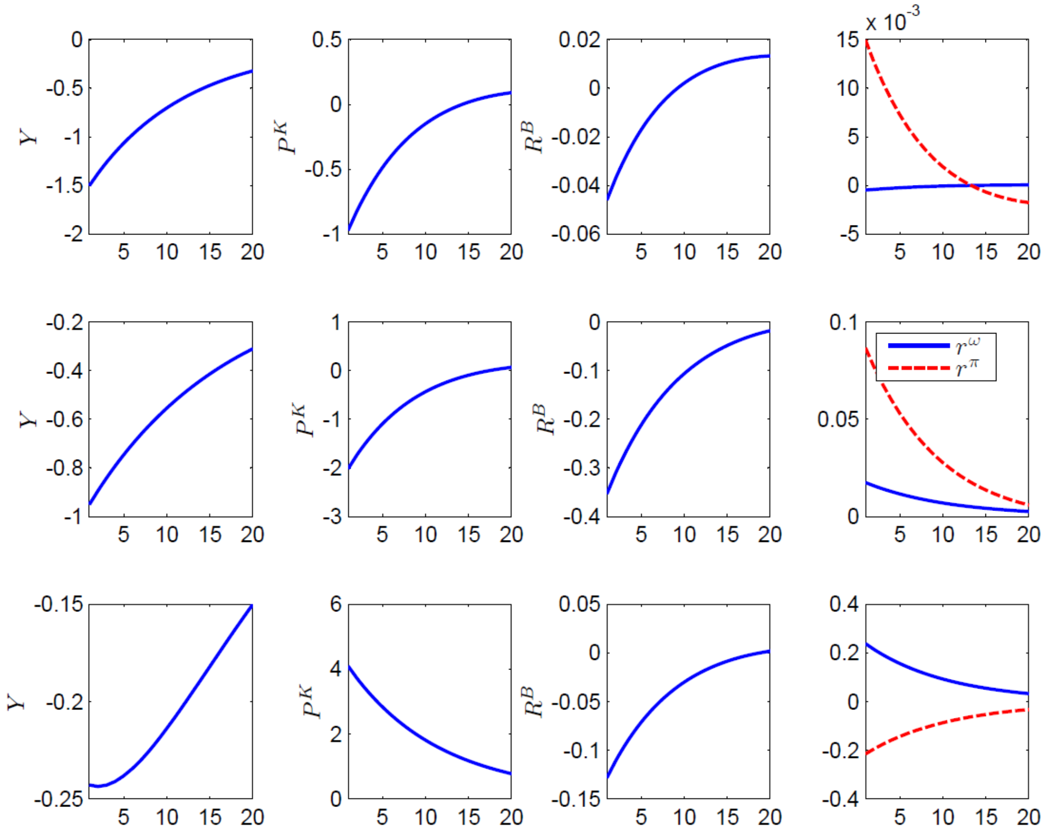

Figure 5. Impulse Responses to TFP Shock $A_t \downarrow$ (top panel), Information Shock $\pi_t \uparrow$ (middle panel), and Resaleability Shock $\omega_t \downarrow$ (bottom panel).

The liquidity premium remains nearly constant under a TFP shock, and rises under both an information shock and a resaleability shock (solid lines in the 4th column). The quality premium rises under a TFP shock and an information shock, but declines under a resaleability shock (dashed lines in the 4th column). In particular, the liquidity premium and the safety premium move against each other under a resaleability shock.

Putting it together, in the bottom-row panel, the rising equity (private asset) prices and falling bond yields suggest that the resaleability shock causes a shortage in the *supply* of private liquidity, not in its demand. A supply-side shortage in private liquidity also causes flight to liquidity under the assets-substitution effect, hence we see a sharp rise in the liquidity premium (solid line). However, there is also a sharp decrease in the quality premium. The procyclical movement in the quality premium is interesting — it can happen only if there is an increase in the average quality of private assets in the asset market so that firms are willing to hold more private assets than government bonds. The reason is that when a tightened supply-side constraint on the resaleability of private assets causes a rise (instead of a fall) in asset prices, the high cutoff ε_t^{**} $\left(= 1/P_t^K\right)$ is lowered so that selling off high-quality assets is encouraged. That is, when the intensive margin of resaleability is tightened, the economy adjusts through the extensive margin by encouraging more firms with relatively low investment efficiency to liquidate high-quality assets to finance investment, which in turn raises the average quality of private assets in the market, making equity a safer asset to hold than without the shock, despite the fact that capital is now less liquid. Hence, the quality premium drops sharply while the liquidity premium rises sharply.

On the other hand, under the information shock $\pi_t \uparrow$, there are more lemon assets in the economy, causing a flight to quality and a rise in the safety premium. In the meantime, since the *demand* for private assets is reduced, the endogenously determined resaleability ω_t^* declines, causing the liquidity premium also to rise as if there is an exogenous shock to the resaleability of private assets.

Hence, once the resaleability (ω_t^*) and the quality (θ_t) of private assets are endogenized in our model, these critical differences in the responses of the quality premium and the liquidity premium under the resaleability shock $\bar{\omega}_t$ and the information shock π_t as well as the TFP shock can provide a litmus test for macro-finance theories. In what follows, we will utilize the recent financial crisis as a natural experiment, during which both the quality premium and the liquidity premium increased sharply (see Figure 1), to quantify the contributions of different financial shocks.

4.3 Contributions of Different Shocks

Following Azariadis, Kaas and Wen (2015), we conduct the following exercise. As shown in Appendix C, the log-linearized dynamic system for aggregate variables in our model implies the following relationship between endogenous variables and the exogenous shocks (the state space):

$$\begin{pmatrix} \hat{K}_{t+1} \\ \hat{Y}_t \\ \hat{r}_t^{\pi} \\ \hat{r}_t^{\omega} \end{pmatrix} = \underbrace{H}_{4\times 4} \begin{pmatrix} \hat{K}_t \\ \hat{A}_t \\ \hat{\pi}_t \\ \hat{\omega}_t \end{pmatrix}, \tag{19}$$

where the aggregate capital stock is also in the state space. We can invert this equation to obtain

$$\begin{pmatrix} \hat{K}_t \\ \hat{A}_t \\ \hat{\pi}_t \\ \hat{\omega}_t \end{pmatrix} = H^{-1} \begin{pmatrix} \hat{K}_{t+1} \\ \hat{Y}_t \\ \hat{r}_t^{\pi} \\ \hat{r}_t^{\omega} \end{pmatrix}, \tag{20}$$

which suggests that given time-series information about the aggregate variables on the right-hand side of the equation, we can back-solve the three unobservable shocks $\{A_t, \pi_t, \bar{\omega}_t\}$ based on the calibrated model. Then we can examine each shock's business-cycle properties and their respective contributions to the recent financial crisis.

The back-solved three shocks, TFP ($\hat{A}_t$), information ($\hat{\pi}_t$), and resaleability ($\hat{\omega}_t$), are graphed in Figure 6. In general, $\hat{A}_t$ and $\hat{\omega}_t$ are strongly procyclical, while $\hat{\pi}_t$ is countercyclical, which are intuitive and in line with the literature. Note that comparing the 1973 oil crisis and the 2008 financial crisis, the middle panel and the bottom panel suggest that resaleability ($\hat{\omega}_t$) was a serious issue during the oil crisis, while adverse selection ($\hat{\pi}_t$) was much worse in the recent financial crisis.

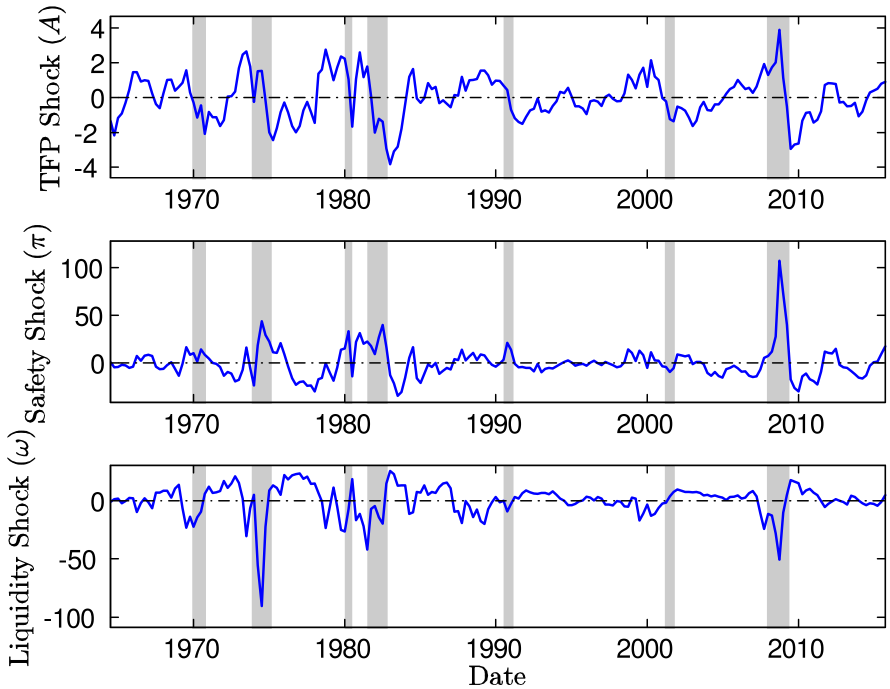

Figure 6. Back-Solved Shocks for $\{A_t, \pi_t, \omega_t\}$.

With the three imputed shocks, we can analyze their respective contributions to the business cycle, especially to output and bond yields during the recent financial crisis. Figure 7 shows how HP-filtered aggregate output in the data (solid line in each panel) and the model-implied output (dashed lines) behave under only TFP shocks (top panel), only information shocks (middle panel), and only resaleability shocks (bottom panel).

Overall, resaleability shocks ($\hat{\omega}_t$) are not as important as the other two shocks in driving output fluctuations — the model-implied aggregate output barely moves under such shocks. Especially for the recent Great Recession period, both TFP shocks ($\hat{A}_t$) and information shocks ($\hat{\pi}_t$) are most important. In particular, the dashed line in the top panel shows that if there were only aggregate TFP shocks driving output, then the predicted output would have continued to the rise from the 2008-2009 period until the middle of the Great Recession. But the actual output (solid line) started to decline before 2008. The reason can be found in the middle panel — private asset quality had already started to deteriorate significantly before 2008, so the model-predicted output under asset-quality shocks ($\hat{\pi}_t$) started to decline in 2007. Thus, it was the combination of the TFP shocks and the asset-quality shocks that generated the actual path of output contraction in the U.S. economy during the recent financial crisis. In the mean time, despite a sharp drop in the resaleability ($\bar{\omega}_t$) of private assets (the bottom panel in Figure 6), this drop did not have much impact on aggregate output (the bottom panel in Figure 7). Similar stories hold for the two recessions in the mid-1970s and mid-1980s during the oil crisis.[11]

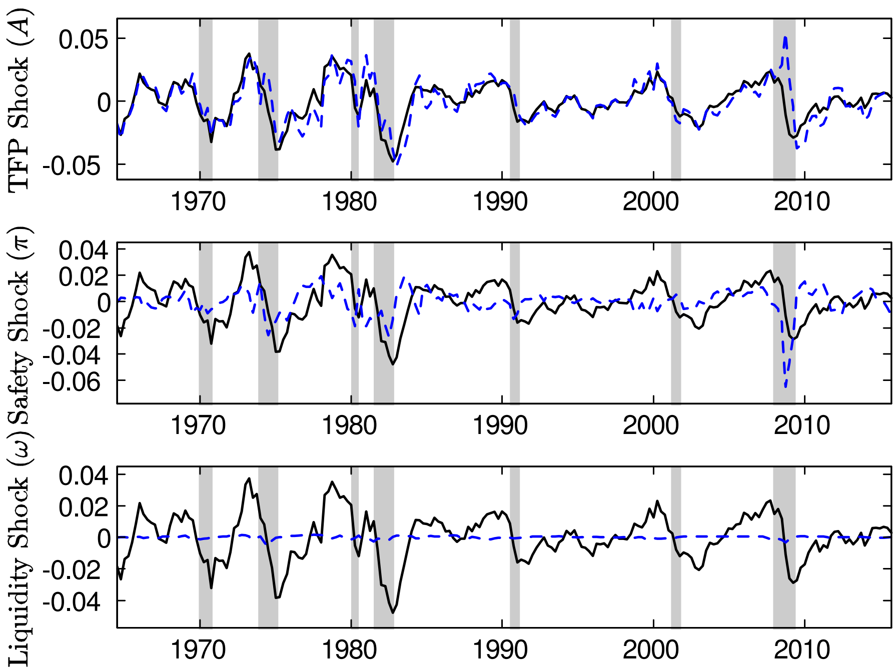

Figure 7. Counterfactual Analysis for Output (data in solid line and model in dashed line)

Figure 8 shows the business-cycle movements in U.S. government bond yields (solid lines) and the model-implied counterparts under each shock (dashed lines). Specifically, the top panel and the bottom panel show that TFP shocks and resaleability shocks played little role in explaining the sharp drop in bond yields during the recent financial crisis, which was almost entirely driven by a sharp decline of the quality of private assets (the middle panel). Since we did not include bond yields in our VARs (equation 19) to back-solve the three shocks, the sum of the three shocks together do not necessarily explain 100% of the bond yields. Thus, Figure 8 provides an independent test to our story — that asset *safety* (adverse selection) instead of *liquidity* (resaleability) per se was the culprit of the recent financial crisis.[12]

This result is consistent with the history and a large literature's claim that the subprime-mortgage crisis started in 2007 triggered the global financial crisis and the Great Recession. Prior to the crisis, home loans in the subprime-mortgage market were often packaged together, and converted into

[11] Figure C.1 in the Appendix C shows that back-solved resaleability shocks can explain most of the business-cycle movements in the measured liquidity premium in the U.S. data (r_t^ω) shown in Figure 1, but such shocks cannot generate enough fluctuations in output and bond yields in the model.

[12] The over-shooting of the model-implied bond yields during the 2008 financial crisis under information shocks could be due to the zero lower bound constraint on bond yields in the U.S. data, which is not imposed in our model.

financial products called mortgage-backed securities. These securities were sold to investors around the world. Many investors assumed these securities were trustworthy and asked few questions about their actual value. But in 2008, more and more investors realized that this was simply not true; a world-wide financial crisis then took place.

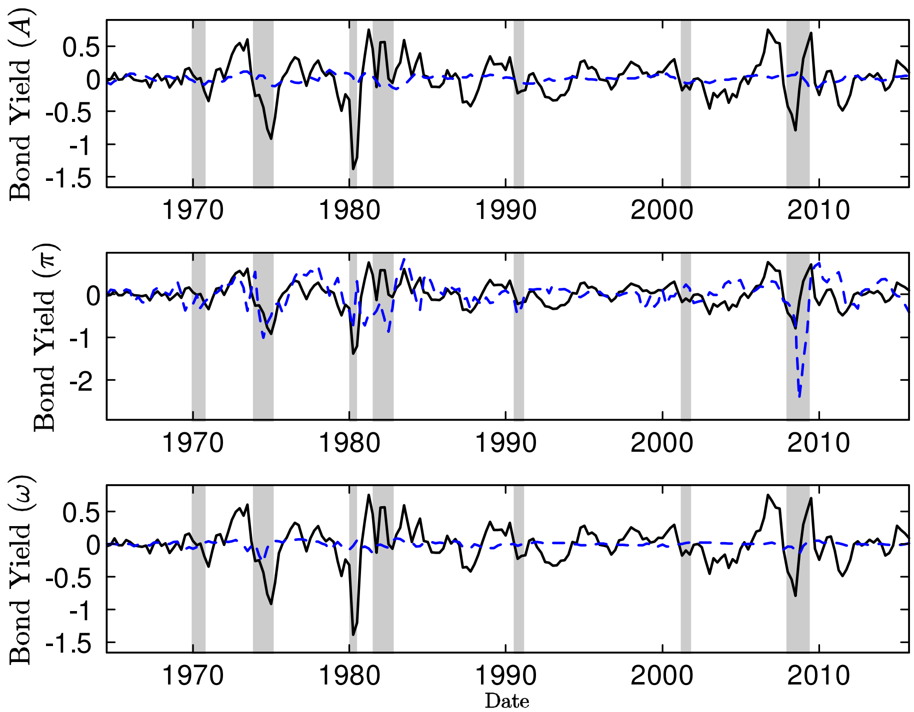

Figure 8. Counterfactual Analysis of Bond Yields (data in solid line and model in dashed line)

Empirical support of the significant relevance of adverse selection in financial markets, especially for the recent financial crisis, includes the work of Keys, Mukherjee, Seru, and Vig (2010), who show that the quality of loans with a lower probability of being securitized is higher than for those with a higher probability. Besides, Gorton and Metrick (2010, 2012) demonstrate the haircut rate of repos rose significantly following the start of the crisis, which also reflected a negative shock to adverse selection in the financial markets. Literature on information asymmetry in financial markets prior to the recent financial crisis includes Downing, Jaffee and Wallace (2009), Demiroglu and James (2012), Krainer and Laderman (2014), and Piskorski, Seru and Witkin (2015).

5 Optimal Provision of Public Liquidity

The impulse-response analysis in the previous section shows that economic recessions can be caused by a shortage of private liquidity such that the demand for public liquidity (e.g., government bonds) increases during recessions, where the shortage can come either from the supply side (such as lower asset resaleability) or from the demand side (such as lower asset quality). Hence, a countercyclical provision of public liquidity by the government may be productive, as suggested by the seminal work of Holmström and Tirole (1998), among many others.

However, our analysis in this section shows that this is not necessarily the case — namely, provision of public liquidity can be welfare reducing, depending on the parameter values and the extent of the government intervention.

We consider two types of government provision of public liquidity during an economic crisis: (i) the issuing of more government bonds and (ii) the direct purchase of private assets. We call the first policy conventional and the second policy unconventional and consider them in turn. We will show that under reasonable parameter values, countercyclical provision of public liquidity through the conventional policy can be welfare reducing unless private assets are extremely illiquid and risky, in contrast to the findings and arguments of Holmström and Tirole (1998). We also show that countercyclical provision of public liquidity through the unconventional policy is unambiguously welfare improving, but only up to a limit — beyond which the purchase of private assets by the government is also welfare reducing, in contrast to the results of Del Negro et al. (2016).

5.1 Conventional Policy

We assume that the government bond supply follows a state-contingent policy in the following simple form:

$$\overline{B}_{t+1} = \overline{B}_t \cdot (L_t/L)^{-\phi},$$

where $\overline{B}_{t+1}$ denotes bond supply in period t as a reaction function of the level of private liquidity in the economy (relative to its steady state value) and $L_t \equiv P_t^K S_t$ denotes the total market value of private liquidity, where S_t is the aggregate volume of trade of private assets in the market, as defined in equation (17). The log-linearized form of the conventional policy around the steady state is $\hat{B}_{t+1} = \hat{B}_t - \phi \hat{L}_t$, where ϕ measures the responsiveness (elasticity) of the bond supply to changes in private liquidity. The government budget constraint is given by

$$T_t + P_t^B \bar{B}_{t+1} = \bar{B}_t. \tag{21}$$

We compute the welfare gains of such a state-contingent policy as a function of ϕ. Let $U \equiv \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \chi \frac{N_t^{1+\xi}}{1+\xi} \right)$ denote the expected lifetime household utility when the policy is inactive ($\phi = 0$), and $U^* = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(\ln(1+\Delta) C_t - \chi \frac{N_t^{1+\xi}}{1+\xi} \right)$ the expected lifetime household utility when the policy is active ($\phi \neq 0$). Then the welfare gains of the countercyclical policy under a particular aggregate shock process (as specified in the previous impulse response section) is given by

$$\Delta(\phi) = [\exp((1-\beta)(U^* - U)) - 1] \times 100\%.$$

Figure 9 plots the welfare gains, where the top-row panel pertains to parameter calibrations in Table 1 and the bottom-row panel to alternative parameter calibrations (to be specified below). In each row panel, the left window shows welfare gains under a 1% negative TFP shock ($A_t \downarrow$), the middle window under a 1% information shock ($\pi_t \uparrow$), and the right window under a 1% resaleability shock ($\bar{\omega}_t \downarrow$). Notice that the welfare gain $\Delta(\phi)$ is estimated over the entire impulse response period and the steady state after an aggregate shock.

Interestingly, regardless of the shocks (top-row panel), the conventional policy is counter-productive. In particular, welfare is a decreasing function of ϕ for $\phi \geq 0$. This is the case because public liquidity crowds out private liquidity by reducing firms' incentives for fixed investment. This crowding-out effect reduces welfare because capital is also a production factor.

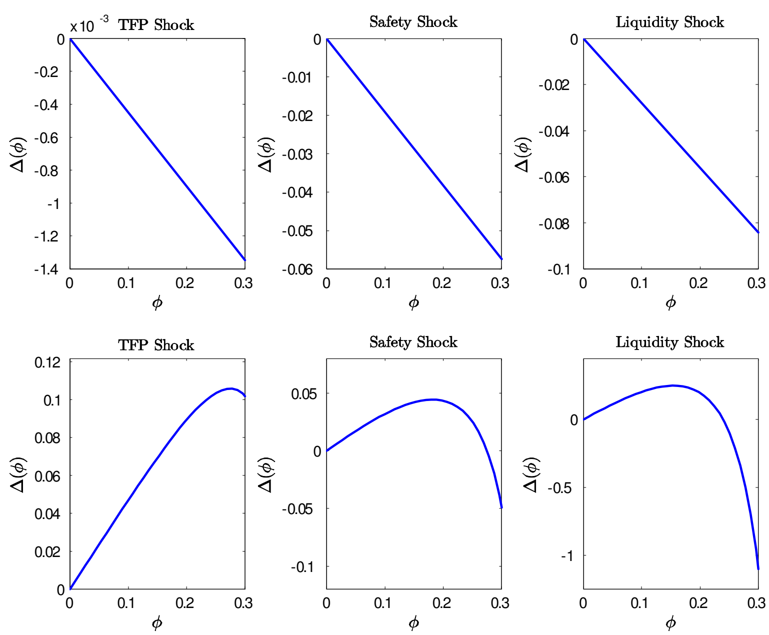

Figure 9. Welfare Gain Under Conventional Policy (parameters in upper and lower panels comes respectively from Table 1 and footnote 13).

However, if we recalibrate the steady-state values of $\{\bar{\omega}, \pi\}$ such that both the resaleability limit and the information quality of assets are extremely low,[13] then the provision of public liquidity through the conventional policy can be welfare improving up to a limit, as suggested in the lower panel of

[13] Under the calibration in Table 1, we have $\bar{\omega} = 0.82$ and $\pi = 0.3\%$, which imply trading volume $S = 0.27$ and output level $Y = 1.08$. However, under the extremely low value of $\bar{\omega} = 0.2$ and a high lemon share $\pi = 0.5\%$, we have trading volume $S = 0.14$ (50% lower than before) and output level $Y = 0.995$.

Figure 9. In particular, regardless of the shocks, the welfare-gain function is strictly positive for $\phi \in [0, \phi_{\max}]$, where $\phi_{\max}$ is around 0.28 for TFP shocks and around 0.2 for the other two shocks. But the welfare-gain function starts to decline monotonically and eventually becomes negative for $\phi \geq \phi_{\max}$, suggesting that the countercyclical bond supply from the government is beneficial to the economy if not too aggressive, provided that private liquidity is extremely low in the steady state.

The intuition behind these results in the lower panel of Figure 9 is straight forward: If the private asset market does not function well, then more public liquidity surely improves firms' positions for investment financing and thus the aggregate welfare. However, excessive provision of public liquidity could paralyze the private asset market, thus leading to welfare losses. Keep in mind that the government does not use the borrowed resources productively.

Figure C.2 in Appendix C lends further support to the intuitions provided above. It shows that welfare in the steady state is a decreasing function of the steady-state bond-to-GDP ratio under the calibration in Table 1 but an increasing and hump-shaped function of the steady-state bond-to-GDP ratio under the alternative calibrations.

The same results hold true under other fundamental shocks. For example, the top-middle window shows that a countercyclical policy reduces welfare for any value of $\phi \geq 0$ under a shock to asset quality ($\pi_t \uparrow$) unless the asset market does not function very well due to severe resaleability constraints and information problems (bottom-middle window). Similarly, under a shock to resaleability ($\omega_t \downarrow$) of private liquidity, provision of public liquidity is a bad idea (top-right window) unless the asset market is extremely poorly functioning to start with (bottom-right window).

5.2 Government Purchase of Troubled Assets

This subsection studies the effects of unconventional policies. During the recent financial crisis, one of the most notable policy responses was the direct purchase of toxic assets by the U.S. government, as through the Trouble Asset Purchase Program (TARP). Here we use our model to evaluate the dynamic general equilibrium effects of such unconventional policies.

We make the following simplifying assumptions. The government can accumulate private assets on its balance sheet. The government starts with $K_0^G \geq 0$ units of high-quality assets (owned by the government) and can use tax income to purchase private assets from the asset market during a financial crisis. Private assets are unproductive in the hands of the government and cannot be rented out to firms (i.e., they remain idle on the government's balance sheet until sold back to the market). In responding to a financial crisis, the government purchases troubled assets from the market and sells (only) high-quality assets back to the market under the same resaleability constraint ω. Private assets held by the government do not suffer from the π_t shock except at the time of purchase.

Denoting I_t^G as the amount of assets newly purchased by the government in period t, the government's budget constraint is then given by

$$T_t + P_t^B \bar{B}_{t+1} + \bar{\omega}_t P_t^K K_t^G = \bar{B}_t + P_t^K I_t^G,$$

where the left-hand side is income flows, which include revenue from lump-sum taxes T_t, income from newly issued bonds $P_t^B \bar{B}_{t+1}$, and asset sales $\omega_t P_t^K K_t^G$, and the right-hand side is total expenditures, which include assets carried over from the last period $\bar{B}_t$, and new purchase of troubled assets $P_t^K I_t^G$.

Because only θ_t fraction of the newly purchased assets are of high quality and do not evaporate, the law of motion of government-held assets is given by

$$K_{t+1}^G = (1 - \bar{\omega}_t) K_t^G + \theta_t I_t^G, \tag{22}$$

where the first term on the right-hand side is the unsold assets and the second term is the newly accumulated assets subject to the quality adjustment θ_t.

Clearly, if $\theta_t I_t^G = \bar{\omega}_t K_t^G$, i.e., if asset sales and (effective) asset purchases are equal, then the government balance sheet remains constant: $K_{t+1}^G = K_t^G = K^G$; if $\theta_t I_t^G > \bar{\omega}_t K_t^G$, then the balance sheet expands over time; and if $\theta_t I_t^G < \bar{\omega}_t K_t^G$, then the balance sheet shrinks. The steady-state balance sheet is reached when (effective) government purchases equal government sales. In the asset market, total demand from the government is I_t^G, and the total supply from the government is $\bar{\omega}_t K_t^G$. Since the government's demand for troubled assets I_t^G exceeds its supply $\bar{\omega}_t K_t^G$ of good assets in the asset market, this unconventional policy serves to purify (cleanse) the asset market overtime.

Similar to Del Negro et al. (2016), we set the state-dependent government purchase program as

$$I_t^G = \left[\left(\frac{L_t}{L} \right)^{-\phi} - 1 \right] \cdot S_t, \tag{23}$$

where $L_t = S_t P_t^K$ denotes the value of trading volume in the market, so the steady-state asset purchase by the government is zero, i.e., $I^G = 0$, which also implies that the government balance sheet is reduced to zero in the long run, i.e., $K^G = 0$.

In each period, given the stock of assets owned by firms and the government, $\left\{K_t^F, K_t^G\right\}$, the aggregate amount of capital stock is $K_t = K_t^F + K_t^G$. However, since assets owned by the government remain idle, total output is given by $Y_t = A_t \left(K_t^F\right)^\alpha N_t^{1-\alpha}$. Meanwhile, the proportion of good assets traded in the market (or asset quality) is now given by

$$\theta_t = \frac{(1 - \pi_t)(1 - F(\varepsilon_t^{**})) K_t^F + \bar{\omega}_t K_t^G}{[\pi_t + (1 - \pi_t)(1 - F(\varepsilon_t^{**}))] K_t^F + \bar{\omega}_t K_t^G}, \tag{24}$$

which is an increasing function of K_t^G (holding everything else fixed) with the limiting properties $\lim\limits_{K_t^G \to 0} \theta_t = \frac{(1-\delta)(1-F(\varepsilon_t^{**}))}{\pi+(1-\delta)(1-F(\varepsilon_t^{**}))}$ and $\lim\limits_{K_t^G \to \infty} \theta_t = 1$.

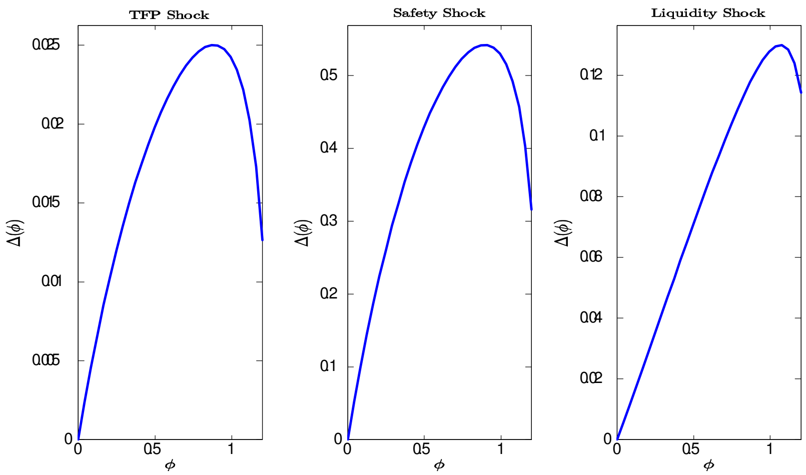

Figure 10. Welfare Gain Under Unconventional Policy

Consequently, a trade off arises: The unconventional government policy has both a *positive* effect on the economy — through its cleansing effect on asset quality — and also a *negative* effect on the economy — as high-quality assets are unproductive in the hands of the government.

The welfare-gain function $\Delta(\phi)$ under the unconventional government policy is graphed in Figure 10. Unlike the case of conventional policy, the figure shows that unconventional policy is quite effective in improving welfare for $\phi \in [0, \phi_{\max}]$ under the parameter calibrations in Table 1, suggesting that direct purchase of troubled assets is preferred to issuing more bonds regardless of the aggregate shocks triggering the financial crisis (including TFP shocks).

However, an excessive response to the financial crisis (with $\phi \geq \phi_{\max}$) can also be welfare reducing. This is so because private assets are unproductive in the hands of the government, thus accumulating too many private assets on the government balance sheet for too long is not necessarily wise.

5.3 Literature Review

The empirical motivation of our paper is based on the influential works of Krishnamurthy and Vissing-Jorgensen (2012) and Gilchrist and Zakrajšek (2012). These papers discuss the causes and the consequences of a credit spread. In particular, Krishnamurthy and Vissing-Jorgensen (2012) not only empirically investigate the effects of government debt on asset prices, but also decompose the spread into a safety premium and a liquidity premium, which motivates us to construct a theory for such decompositions over business cycles and to connect with the phenomenon of the flight to quality and flight to liquidity. Also see Beber, Brandt and Kavajecz (2008) and Baele, Bekaert, Inghelbrecht and Wei (2013) for the empirical evidence of flight to quality and flight to liquidity. Gilchrist and Zakrajšek (2012) examine the predictive effects of credit spreads on economic activities over the business cycle by constructing a new credit-spread index, called the "GZ credit spread" shown in our Figure 4.

As emphasized by Krishnamurthy and Vissing-Jorgensen (2012), the convenience yield of government bonds is due to the fact that government bonds used for public liquidity are not only safe but also liquid. We capture this dual property of government bonds in our theoretical model. We also consider welfare consequences of the provision of public liquidity in this context.

In this regard, our work is mainly motivated by the seminal works of Woodford (1990) and Holmström and Tirole (1998), who are among the first to address the provision of public liquidity. Both papers show that public liquidity generates a crowding-in effect. Our paper shows that public liquidity may generate adverse effects by crowding out private liquidity in the presence of information frictions in the financial market.

Kiyotaki and Moore (2012) and Del Negro et al. (2016) offer a macro framework to characterize liquidity shortages as represented by asset-resaleability constraints, which can address the issue of flight to liquidity. On the other hand, Eisfeldt (2004), Kurlat (2013) and Bigio (2015), among others, introduce adverse selection into a macro framework to study the issue of flight to quality. However, this literature studies the adverse-selection problem and the resaleability problem in isolation, both market liquidity and market safety of private assets do not interact with each other nor respond to policies over the business cycle.

We contribute to this literature by developing a tractable dynamic general equilibrium model, which endogenizes both the market liquidity and the market quality of private assets, thus enabling us to structurally and quantitatively account for the joint behaviors of the liquidity premium and safety premium in government bonds and the associated phenomena of the flight to quality and flight to liquidity that have long intrigued and puzzled financial economists. Moreover, our work predicts that safe and liquid government bonds can crowd out private liquidity, which is consistent with the recent empirical findings by Krishnamurthy and Vissing-Jorgensen (2015) and Greenwood, Hanson and Stein (2015).

Our work is also related to the burgeoning literature on the shortage of safe assets after the outbreak of the global financial crisis. This literature includes Benigno and Nisticò (2017), Caballero and Farhi (2013, 2017), Caballero, Farhi and Gourinchas (2016), Gorton and Ordonez (2013), Barro, Fernández-Villaverde, Levintal, and Molleru (2014), Andolfatto and Williamson (2015), Caballero, Farhi, and Gourinchas (2016), and He, Krishnamurthy, and Milbradt (2016a, 2016b). See Gorton (2016) and Golec and Perotti (2017) for surveys of this literature.

Cui and Radde (2015), and Dong, Wang and Wen (2016) adopt a search-theoretic approach to modeling frictions in the resale of assets. This literature endogenizes asset resaleability through search frictions, but does not consider the endogenous interactions of the liquidity premium and safety premium over the business cycle or the effect of public liquidity on private liquidity. A notable exception is Cui (2016), who uses the framework of Cui and Radde (2015) to analyze the monetary and fiscal interactions for asset markets with search and matching frictions. Cui (2016) endogenizes resaleability constraints and then obtains an endogenous liquidity premium. Since there is no information asymmetry in his model, a safety premium is thus absent therein. Complementary to Cui's (2016) work, we show that information frictions can serve as a potential micro foundation for the search and matching frictions in financial markets by discouraging asset trading and thus endogenizing the resaleability of assets. In this regard, our paper is the first attempt at providing a structural and unified framework to quantitatively address the decomposition and interactions between the liquidity premium and the safety premium.

Our paper is also related to the large literature on adverse selection under private information, in addition to the macro papers by Eisfeldt (2004), Kurlat (2013) and Bigio (2015) aforementioned. See Williamson and Wright (1994) for an early application of information asymmetry to the first-generation models of monetary search. Guerrieri, Shimer and Wright (2010) propose a model of competitive search under adverse selection, and Guerrieri and Shimer (2014) propose a model of dynamic flight to quality with adverse selection. Additionally, the recent progress in the monetary/liquidity search literature on information frictions include Lester, Postlewaite and Wright (2012), Rocheteau (2011), and Li, Rocheteau, and Weill (2012).

Finally, our paper discusses both conventional and unconventional liquidity policies and their welfare consequences. In addition to the aforementioned literature in the Introduction for unconventional government policies, Gertler and Karadi (2013) and Wen (2013, 2014) study the macro effects of large-scale asset purchases (LSAPs). In addition, House and Masatlioglu (2015) show that government-asset purchase programs tend to generate favorable impacts on private investment.

6 Conclusion

In this paper we endogenize the *liquidity* and the *quality* of private assets in an incomplete financial-market model with heterogeneous agents. The analytical tractability of our model facilitates the decomposition of the convenience yields into a liquidity premium and a safety premium over the business cycle. When calibrated to match the U.S. aggregate output fluctuations and asset premiums, the model reveals that a sharp reduction in the *quality*, instead of the *liquidity*, of private assets was the culprit of the recent financial crisis, consistent with the perception that it was the subprime mortgage problem that triggered the recent financial crisis and the Great Recession. Since the provision of public liquidity endogenously affects the provision of private liquidity in our model, we are able to show that excessive injection of public liquidity during a financial crisis can be welfare reducing under either conventional or unconventional policies. In particular, too much intervention for too long can paralyze the private asset market.

Our model also features multiple steady states. In this paper, we have focused on the local dynamics around the steady state with positive bond returns. In future research we intend to study the low liquidity state with negative bond yields, which may have interesting implications not only for endogenous persistence and propagation of the financial crisis, but also for the negative nominal-interest-rate phenomenon in Europe after the global financial crisis (see Dong and Wen, 2017).

Another interesting extension of our model is to connect it with the New Keynesian framework with nominal rigidity and a zero lower bound on the nominal interest rate. Such an extension can allow us to study the interaction between fiscal and monetary policies in the presence of adverse selection in financial markets.

Our model can also be extended to study both market liquidity and funding liquidity. In this paper, we focus on the interplay among adverse selection, the resaleability of private assets, and the provision of public liquidity, but we have not considered funding liquidity or other forms of firm debts (as in Wang and Wen, 2012). In addition, although our paper admits endogenous asset quality and liquidity in a production economy, an intriguing extension is to endogenize the creation of lemon assets, as in Li, Rocheteau and Weill (2012), Neuhann (2016) and Caramp (2016).

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Appendix

A Data Description

Safety/Liquidity Premium: We follow Krishnamurthy and Vissing-Jorgensen (2012) to construct safety and liquidity premiums in Figure 1. On the one hand, the safety premium refers to the spread of pairs of assets with similar liquidity but different safety (higher- and lower-grade corporate bonds and commercial paper). In particular, we use the spread between AAA and BAA bonds to measure the safety premium. Those two kinds of assets share similar turnover rates (liquidity) and the main difference lies in the safety (adverse selection or default risk). On the other hand, the liquidity premium refers to the spreads of pairs of assets with similar safety but different liquidity. More specifically, we use the spread between FDIC-insured CDs and Treasury bills to measure liquidity premium. Both of those are backed by the government, and therefore both are free of safety risk. The main difference therefore comes from liquidity. Alternatively, we can also use the spread between AAA bonds and Treasury bills since AAA bonds are almost free of safety risk and the correlation between AAA bonds and FDIC-insured CDs is around 0.85 and their means are close to each other. The main results are robust. All the aforementioned raw data are downloaded from Federal Reserve Economic Data (FRED) at `https://fred.stlouisfed.org/`.

GZ (Gilchrist and Zakrajšek, 2012) Credit Spread and 3-Month Treasury Bill Rate: For the credit spread series in Figure 4, we consider quarterly averages of the monthly series, updated until the first quarter of 2016. We downloaded their data from `http://people.bu.edu/sgilchri/Data/data.htm`. We obtained the quarterly data for the 3-month T-bill rate from FRED.

B Proof of Propositions

Proof of Proposition 1 to Proposition 3: First, we guess (and verify later) that the value function of a firm is linear in $\{k_t, b_t\}$:

$$V_t\left(k_t, b_t, \varepsilon_t\right) = \phi_t^K\left(\varepsilon_t\right) k_t + \phi_t^B\left(\varepsilon_t\right) b_t. \tag{B.1}$$

We then have

$$\mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(k_{t+1}, b_{t+1}, \varepsilon_{t+1}\right) = Q_t k_{t+1} + Q_t^B b_{t+1},$$

where

$$Q_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \phi_{t+1}^K\left(\varepsilon_{t+1}\right), \tag{B.2}$$

$$Q_t^B = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \phi_{t+1}^B\left(\varepsilon_{t+1}\right). \tag{B.3}$$

Substituting equation (B.1) into the LHS of equation (10), and substituting equations (2), (11), (B.2) and (B.3) into the RHS of equation (10) yields

$$\begin{aligned}
\phi_t^K(\varepsilon_t)k_t + \phi_t^B(\varepsilon_t)b_t &= b_t + R_t^K k_t + P_t^K\left(k_t^{g,s} + k_t^{l,s}\right) - P_t^B b_{t+1} - P_t^K k_t^b - i_t \\
&\quad + Q_t\left((1-\pi)\gamma k_t + \theta_t k_t^a - k_t^{g,s} + \varepsilon_t i_t\right) + Q_t^B b_{t+1} \\
&= \left(R_t^K + Q_t(1-\delta)\right)k_t + \left(P_t^K - Q_t^K\right)k_t^{g,s} + P_t^K k_t^{l,s} + \left(Q_t\varepsilon_t - 1\right)i_t + b_t, \quad \text{(B.4)}
\end{aligned}$$

where $Q_t^B = P_t^B$, $\theta_t = P_t^K/Q_t$, and capital's effective depreciation rate is $\delta_t = 1-(1-\pi_t)\gamma$. Keep in mind that we set $\gamma = 1$ for illustration purpose in the main context. We now derive the general results here under $\gamma \in [0,1]$.

Denote $\varepsilon_t^* = 1/Q_t$ and $\varepsilon_t^{**} = 1/q_t^K$. On the one hand, when $\varepsilon_t > \varepsilon_t^*$, the constraint (6) implies $d_t = 0$, and equation (11) suggests that

$$i_t = b_t + R_t^K k_t + P_t^K\left(k_t^{g,s} + k_t^{l,s}\right) - P_t^K k_t^a - P_t^B b_{t+1}.$$

Thus, the RHS of equation (B.4) can be further simplified to

$$\begin{aligned}
RHS &= \left(\frac{\varepsilon_t}{\varepsilon_t^*}R_t^K + Q_t(1-\delta)\right)k_t + \left(\frac{\varepsilon_t - \varepsilon_t^{**}}{\varepsilon_t^*}\right)P_t^K k_t^{g,s} + \frac{\varepsilon_t}{\varepsilon_t^*}P_t^K\pi k_t + \frac{\varepsilon_t}{\varepsilon_t^*}b_t \\
&= \left(Q_t\varepsilon_t R_t^K + Q_t(1-\delta)\right)k_t + \left(P_t^K\varepsilon_t - 1\right)Q_t k_t^{g,s} + Q_t\varepsilon_t P_t^K\pi k_t + Q_t\varepsilon_t b_t. \qquad \text{(B.5)}
\end{aligned}$$

Using constraints (6) and (12), we obtain $k_t^{l,s} = \bar{\omega}_t\pi k_t$, $k_t^b = 0$, and $b_{t+1} = 0$. Then $k_t^{g,s} = \bar{\omega}_t(1-\delta)k_t$ if and only if $\varepsilon_t > \varepsilon_t^{**}$. Due to adverse selection, we have $\theta_t = P_t^K/Q_t < 1$. Thus $P_t^K < Q_t$, and

$$\varepsilon_t^{**} = 1/P_t^K > 1/Q_t = \varepsilon_t^*. \qquad \text{(B.6)}$$

Substituting equation (B.5) into equation (B.1) suggests that, for any $\varepsilon_t > \varepsilon_t^*$, we have

$$\begin{aligned}
\phi_t^K(\varepsilon_t) &= Q_t(1-\delta) + Q_t\varepsilon_t r_t^K + \max\left(P_t^K\varepsilon_t - 1, 0\right)Q_t\bar{\omega}_t(1-\delta) + Q_t\varepsilon_t\bar{\omega}_t\pi q_t^K \\
&= \frac{\varepsilon_t}{\varepsilon_t^*}\left(R_t^K + \bar{\omega}_t\pi q_t^K\right) + \left(1 + \max\left(\frac{\varepsilon_t}{\varepsilon_t^{**}} - 1, 0\right)\right)\bar{\omega}_t(1-\delta)Q_t + (1-\delta)(1-\bar{\omega}_t)Q_t, \\
\phi_t^B(\varepsilon_t) &= Q_t\varepsilon_t = \frac{\varepsilon_t}{\varepsilon_t^*}.
\end{aligned}$$

On the other hand, when $\varepsilon_t \le \varepsilon_t^*$, constraint (6) implies $i_t = 0$. Thus the RHS of equation (B.4) can be rewritten as

$$RHS = \left(R_t^K + Q_t(1-\delta)\right)k_t + P_t^K\bar{\omega}_t\pi k_t + b_t, \qquad \text{(B.7)}$$

where $k_t^{g,s} = 0$ because $P_t^K < Q_t$, as proved above. Meanwhile, $k_t^{l,s} = \bar{\omega}_t \pi k_t$.Substituting equation (B.7) into (B.1) suggests that, for any $\varepsilon_t \leq \varepsilon_t^*$, we have

$$\phi_t^K(\varepsilon_t^*) = \left(R_t^K + \bar{\omega}_t \pi P_t^K\right) + Q_t(1-\delta),$$
$$\phi_t^B(\varepsilon_t^*) = 1.$$

To sum up, for $\forall \varepsilon_t \in \mathcal{E} = (\varepsilon_{\min}, \varepsilon_{\max})$, firms' policy functions are given by

$$d_t = \begin{cases} 0, & \text{if } \varepsilon_t \geq \varepsilon_t^* \\ R_t^K k_t + P_t^K \left(k_t^{s,g} + k_t^{s,b} - k_t^a\right), & \text{otherwise} \end{cases}$$

$$i_t = \begin{cases} R_t^K k_t + P_t^K \left(k_t^{s,g} + k_t^{s,b} - k_t^a\right), & \text{if } \varepsilon_t \geq \varepsilon_t^* \\ 0, & \text{otherwise} \end{cases}$$

$$k_t^a = \begin{cases} 0, & \text{if } \varepsilon_t \geq \varepsilon_t^* \\ \text{indeterminate}, & \text{otherwise} \end{cases}$$

$$k_t^{s,g} = \begin{cases} \bar{\omega}_t(1-\pi)k_t, & \text{if } \varepsilon_t \geq \varepsilon_t^{**} \\ 0, & \text{otherwise} \end{cases}$$

$$k_t^{s,b} = \bar{\omega}_t \pi k_t \text{ for } \forall\ \varepsilon_t.$$

where the two cut-offs $\{\varepsilon_t^*, \varepsilon_t^{**}\}$ are characterized by equation (B.6). As a result, we also have

$$\begin{aligned} \phi_t^K(\varepsilon_t) &= \left[1 + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right)\right]\left(R_t^K + \bar{\omega}_t \pi P_t^K\right) \\ &+ \left[1 + \max\left(\frac{\varepsilon_t}{\varepsilon_t^{**}} - 1, 0\right)\right]\bar{\omega}_t(1-\delta)Q_t + (1-\bar{\omega}_t)(1-\delta)Q_t, \\ \phi_t^B(\varepsilon_t) &= 1 + \max\left(\frac{\varepsilon_t}{\varepsilon_t^*} - 1, 0\right). \end{aligned}$$

Consequently, equations (B.2) and (B.3) can be rewritten, respectively, as

$$\begin{aligned} Q_t = \mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t}&\left[1 + \int_{\varepsilon_{t+1}^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon_{t+1}^*} - 1\right) dF(\varepsilon)\right]\left(R_{t+1}^K + \bar{\omega}_{t+1}\pi_{t+1}P_{t+1}^K\right) \\ + \mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t}&\left[1 + \int_{\varepsilon_{t+1}^{**}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon_{t+1}^{**}} - 1\right) dF(\varepsilon)\right]\bar{\omega}_{t+1}(1-\delta)Q_{t+1}^K \\ + \mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t}&(1-\bar{\omega}_{t+1})(1-\delta)Q_{t+1}, \end{aligned} \tag{B.8}$$

$$P_t^B = Q_t^B = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\varepsilon_{t+1}^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon_{t+1}^*} - 1 \right) dF \right].$$

It is worth noting that 1 unit of bond costs $P_t^B < 1$ (real) dollars today and its return tomorrow (at maturity) is 1 (real) dollar plus a premium measured in consumption goods, which is identical to the option value of output. Because bonds are not subject to resaleability constraint and adverse selection, they have the same option value as one unit of real output:

$$\left[1 + \int_{\varepsilon_{t+1}^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon_{t+1}^*} - 1 \right) dF(\varepsilon) \right] > 1.$$

This option value is larger than the option value of the marginal product of capital because of capital-market imperfections: resaleability constraints and asymmetric information. Hence, the spread between the rate of return to bonds and the marginal product of capital is the convenience yield (CY) and it contains a liquidity premium due to the resaleability constraint and a safety premium due to the information friction.

Note that the aggregate supply of good capital and that of lemon capital are given by $(1 - F(\varepsilon_t^{**}))(1-\pi)\gamma\omega K_t$ and $\pi\bar{\omega}K_t$, respectively. Meanwhile, the aggregate investment is given by

$$\begin{aligned} I_t = \int i_t &= \int_{\varepsilon_t > \varepsilon_t^*} \left(b_t + R_t^K k_t + P_t^K \left(k_t^{g,s} + k_t^{l,s} \right) - P_t^K k_t^a - P_t^B b_{t+1} \right) dF \\ &= \left(B_t + R_t^K K_t + \bar{\omega}_t \pi_t P_t^K K_t \right) (1 - F(\varepsilon_t^*)) + \bar{\omega}_t (1 - \delta_t) P_t^K K_t (1 - F(\varepsilon_t^{**})). \end{aligned}$$

The above equation shows that aggregate investment is financed through (1) bond returns plus the internal cash flow $R_t K_t$ and the resale value of lemon assets $\bar{\omega}\pi P^K K$, adjusted by the number of active firms $(1 - F(\varepsilon^*))$ (for inactive firms the proceeds of lemon assets go to dividends); and (2) the liquidation value of good assets $(1-\delta)\omega P^K K$ adjusted by the number of very productive firms who liquidate good assets $(1 - F(\varepsilon^{**}))$.

Therefore, the law of motion for aggregate capital stock is characterized by

$$K_{t+1} = (1 - \pi_t) K_t + \left(B_t + R_t^K K_t + \bar{\omega}_t \pi_t P_t^K K_t \right) \int_{\varepsilon_t^*}^{\varepsilon_{\max}} \varepsilon dF + \bar{\omega}_t (1 - \pi_t) \gamma P_t^K K_t \int_{\varepsilon_t^{**}}^{\varepsilon_{\max}} \varepsilon dF.$$

Similarly, we obtain

$$\varepsilon_t^* = \frac{1}{Q_t} = \theta_t \varepsilon_t^{**} = \frac{\theta_t}{P_t^K},$$

$$\theta_t = \frac{(1 - F(\varepsilon_t^{**}))(1-\pi)\gamma}{(1 - F(\varepsilon_t^{**}))(1-\pi)\gamma + \pi_t},$$

$$\omega_t^* = \left[\frac{\pi}{\pi + (1-\pi)\gamma} + \frac{(1-\pi)\gamma}{\pi + (1-\pi)\gamma} \frac{1 - F(\varepsilon_t^{**})}{1 - F(\varepsilon_t^*)} \right] \bar{\omega}_t,$$

Setting $\gamma = 1$ yields the results in Propositions 1, 2, and 3.

Proof of Proposition 4: As shown in equation (B.8), the shadow price of capital (aggregate Tobin's q) is determined by

$$\begin{aligned}Q_t = & \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(R^K_{t+1} + \bar{\omega}_{t+1} \pi_{t+1} P^K_{t+1} \right) \left[1 + \int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*_{t+1}} - 1 \right) dF \right] \\ & + \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{\omega}_{t+1} \left(1 - \delta \right) Q_{t+1} \left[1 + \int_{\varepsilon^{**}_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^{**}_{t+1}} - 1 \right) dF \right] \\ & + \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(1 - \bar{\omega}_{t+1} \right) \left(1 - \delta \right) Q_{t+1}, \end{aligned} \tag{B.9}$$

where the right-hand side is the payoff of newly installed capital in period $t+1$ (the expected present value of future cash flows generated from 1 unit of new capital). The term in the first pair of square brackets in equation (B.9) is the option value of 1 unit of output. Firms can opt not to undertake investment (or convert output into capital goods) because waiting has the following option value: If the efficiency shock is low (or $\varepsilon Q < 1$, which happens with $\Pr(\varepsilon < \varepsilon^*)$), 1 unit of real cash (output) can be kept in hand and its value remains the same (1 real dollar); but if the efficiency shock is high, one unit of output can be converted into ε units of capital goods, which is worth $\varepsilon Q = \frac{\varepsilon}{\varepsilon^*} > 1$ real dollars, which happens with probability $\Pr(\varepsilon \geq \varepsilon^*)$. So the option value of a real dollar (shadow value of output) is given by

$$\int_{\varepsilon < \varepsilon^*} dF + \int_{\varepsilon \geq \varepsilon^*} \frac{\varepsilon}{\varepsilon^*} dF = 1 + \int_{\varepsilon^* \geq \varepsilon^*} \left(\frac{\varepsilon}{\varepsilon^*} - 1 \right) dF > 1. \tag{B.10}$$

The term in the second pair of square brackets in the second row of equation (B.9) is the option value of used (high-quality) capital. First, one unit of used capital has a shadow value of Q if kept in hand. Second, it has a market value of P^K if sold to the asset market to finance investment, with a return of $P^K \varepsilon Q = \frac{\varepsilon}{\varepsilon^*} P^k$ units of real cash. Since a firm will liquidate good capital if and only if $\varepsilon \geq \varepsilon^{**}$, the expected option value of used (high-quality) capital is

$$\int_{\varepsilon < \varepsilon^{**}} Q dF + \int_{\varepsilon \geq \varepsilon^{**}} \frac{\varepsilon P^K}{\varepsilon^*} dF = Q + \int_{\varepsilon \geq \varepsilon^{**}} \left(\frac{\varepsilon P^K}{\varepsilon^*} - Q \right) dF = Q \left[1 + \int_{\varepsilon \geq \varepsilon^{**}} \left(\frac{\varepsilon}{\varepsilon^{**}} - 1 \right) dF \right], \tag{B.11}$$

where we used the relationships $Q = 1/\varepsilon^{**}$ and $P^K = 1/\varepsilon^*$ in deriving the last equality.

With these definitions and understanding of the option values of goods and used capital, the RHS of Equation (B.9) is interpreted as follows. The expected payoff from one unit of new capital in period $t+1$ consist of three parts, expressed in the three rows on the RHS of equation (B.9). The first row indicates that 1 additional unit of capital in the next period can yield marginal product R^K_{t+1} plus the market value of reselling the lemon (π) part of the used capital (subject to the resaleability constraint ω), times the option value of output, $\left[1+\int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*_{t+1}}-1\right)dF\right]$. The second row indicates the option value of $\omega(1-\delta)$ units of the non-depreciated used (high-quality) capital if it is sold to the asset market, $[\omega(1-\delta)Q_{t+1}]\left[1+\int_{\varepsilon^{**}_{t+1}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^{**}_{t+1}}-1\right)dF\right]$, where the cutoff is ε^{**} applies because good capital is liquidated only if $\varepsilon \geq \varepsilon^{**}$. The third row is the value of the remaining $(1-\omega)(1-\delta)$ units of unsold (high-quality) capital, whose shadow value is simply Q_{t+1}. In other words, the shadow value of capital is Q instead of $Q\left[1+\int_{\varepsilon\geq\varepsilon^{**}}\left(\frac{\varepsilon}{\varepsilon^{**}}-1\right)dF\right]$ if it is kept in hand.

Denote $R^K_t = \frac{r^k_{t+1}+(1-\delta)\Phi^k_{t+1}}{\Phi^k_t}$ as the rate of return to private assets and $R^B_t = 1/P^B_t$ as the rate of return to government bonds. Then the convenience yield is given by

$$CY_t = \mathbb{E}_t\frac{\beta\Lambda_{t+1}}{\Lambda_t}\left[1+\int_{\varepsilon_{t+1}}^{\varepsilon_{\max}}\left(\frac{\varepsilon_t}{\varepsilon^*_{t+1}}-1\right)dF\right]\left(R^K_t - R^B_t\right)$$
$$= r^\omega_t + r^\pi_t,$$

where (by using equation (B.8))

$$r^\omega_t \equiv \mathbb{E}_t\frac{\beta\Lambda_{t+1}}{\Lambda_t}\left[\int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*_{t+1}}-1\right)dF\right]\frac{(1-\delta)(1-\omega_{t+1})\Phi^K_{t+1}}{\Phi^K_t},$$

$$r^\pi_t \equiv \mathbb{E}_t\frac{\beta\Lambda_{t+1}}{\Lambda_t}\left[\int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*_{t+1}}-1\right)dF - \int_{\varepsilon^{**}_{t+1}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^{**}_{t+1}}-1\right)dF\right]\frac{(1-\delta)\omega_{t+1}\Phi^K_{t+1}}{\Phi^K_t}$$
$$- \mathbb{E}_t\frac{\beta\Lambda_{t+1}}{\Lambda_t}\left[1+\int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*_{t+1}}-1\right)dF\right]\frac{\omega_{t+1}\pi_{t+1}q^K_{t+1}}{\Phi^K_t}.$$

It is easy to show that $\lim_{\omega\to1} r^\omega_t = 0$ and $\lim_{\pi\to0} r^\pi_t = 0$. That is, the "liquidity premium" vanishes when $\omega = 1$ and the "quality premium" vanishes when $\pi = 0$.

Proof of Proposition 5: Given asset price P^K, all the other variables in the steady state can be solved *recursively* according to the following order (all the equations come from the dynamic system in Appendix C):

$$\varepsilon^{**} = \frac{1}{P^K},$$

$$\theta = \frac{(1-\delta)(1-F(\varepsilon^{**}))}{\pi+(1-\delta)(1-F(\varepsilon^{**}))},$$

$$Q=\frac{P^K}{\theta},$$

$$\varepsilon^*=\frac{1}{Q},$$

$$P^B=\beta\left(1+\int_{\varepsilon^*}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^*}-1\right)dF\right),$$

$$R^B=\frac{1}{P^B},$$

$$R^K=\left\{1-\beta\left(1-\delta\right)\left[\left(1+\int_{\varepsilon^{**}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\varepsilon^{**}}-1\right)dF\right)\omega+\left(1-\omega\right)\right]\right\}Q-\omega\pi P^K,$$

$$\frac{B}{K}=\frac{\delta}{\int_{\varepsilon^*}^{\varepsilon_{\max}}\varepsilon dF\left(\varepsilon\right)}-\left(r^K+\omega\pi P^K\right)-\left(1-\delta\right)\omega P^K\frac{\int_{\varepsilon^{**}}^{\varepsilon_{\max}}\varepsilon dF}{\int_{\varepsilon^*}^{\varepsilon_{\max}}\varepsilon dF},$$

$$\frac{I}{K}=\left(\frac{B}{K}+r^K+\omega\pi P^K\right)\left(1-F\left(\varepsilon^*\right)\right)+\left(1-\delta\right)\omega P^K\left(1-F\left(\varepsilon^{**}\right)\right),$$

$$\frac{Y}{K}=\frac{R^K}{\alpha},$$

$$\frac{C}{K}=\frac{Y}{K}-\frac{I}{K}-\left(1-P^B\right)\frac{B}{K},$$

$$N=\left(\frac{1-\alpha}{\chi}\frac{Y/K}{C/K}\right)^{\frac{1}{1+\xi}},$$

$$K=\left(\frac{\alpha A}{R^K}\right)^{\frac{1}{1-\alpha}}N.$$

Now it remains to pin down P^K. Denote $\bar{B}\geq 0$ as the aggregate supply of government bonds. Then the asset price P^K can be solved by the market clearing condition for bonds: $B\left(P^K\right)=\bar{B}$. More specifically, by

$$\bar{B}=\left(\frac{\delta}{\int_{\varepsilon^*}^{\varepsilon_{\max}}\varepsilon dF\left(\varepsilon\right)}-\left(R^K+\omega\pi P^K\right)-\left(1-\delta\right)\omega P^K\frac{\int_{\varepsilon^{**}}^{\varepsilon_{\max}}\varepsilon dF\left(\varepsilon\right)}{\int_{\varepsilon^*}^{\varepsilon_{\max}}\varepsilon dF\left(\varepsilon\right)}\right)\cdot K.$$

That is, since we have shown that both B/K and K are functions of P^K, the aggregate demand for public liquidity is given by $B\left(P^K\right)=\frac{B}{K}\cdot K$, which is also a function of P^K. Thus P^K is determined by the market-clearing condition for bonds, $B\left(P^K\right)=\bar{B}$. Note that the bond yield R^B is simply the inverse of P^K. Hence, we can also characterize the bond-market clearing condition as $B\left(R^B\right)=\bar{B}$.

C Dynamic System and Log-linearization

Dynamic System

Let $\delta_t = 1 - (1 - \pi_t)\gamma$. The dynamic system of the model is given by the following equations:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha},$$

$$R_t^K = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha},$$

$$\frac{W_t}{C_t} = \chi N_t^{\xi},$$

$$W_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha},$$

$$C_t + I_t + B_t - \frac{B_{t+1}}{R_t^B} = Y_t,$$

$$\varepsilon_t^* = \frac{1}{Q_t},$$

$$\varepsilon_t^{**} = \frac{1}{P_t^K},$$

$$\theta_t = \frac{(1-\delta)\left(1 - F\left(\varepsilon_t^{**}\right)\right)}{\pi_t + (1-\delta)\left(1 - F\left(\varepsilon_t^{**}\right)\right)},$$

$$R_t^B = \frac{1}{\mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t}\left[1 + \int_{\varepsilon_{t+1}^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon_{t+1}^*} - 1\right) dF(\varepsilon)\right]},$$

$$P_t^K = \theta_t Q_t,$$

$$I_t = B_t\left(1 - F\left(\varepsilon_t^*\right)\right) + \left[\left(R_t^K + \overline{\omega}_t \pi_t P_t^K\right)\left(1 - F\left(\varepsilon_t^*\right)\right) + (1-\delta)\,\overline{\omega}_t P_t^K \left(1 - F\left(\varepsilon_t^{**}\right)\right)\right] K_t,$$

$$K_{t+1} = (1-\delta) K_t + B_t \int_{\varepsilon_t^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)$$
$$+ \left[\left(R_t^K + \omega_t \pi_t P_t^K\right) \int_{\varepsilon_t^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) + \omega_t (1-\delta) P_t^K \int_{\varepsilon_t^{**}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)\right] K_t,$$

$$Q_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*_{t+1}} - 1 \right) dF(\varepsilon) \right] \left(R^K_{t+1} + \omega_{t+1} \pi_{t+1} P^K_{t+1} \right)$$

$$+ \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\varepsilon^{**}_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^{**}_{t+1}} - 1 \right) dF(\varepsilon) \right] \omega_{t+1} (1-\delta) Q^K_{t+1}$$

$$+ \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} (1-\omega_{t+1})(1-\delta) Q_{t+1},$$

$$r^\omega_t = \frac{\mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\varepsilon_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*_{t+1}} - 1 \right) dF(\varepsilon) \right] (1-\delta)(1-\omega_{t+1}) Q_{t+1}}{Q_t},$$

$$r^\pi_t = \frac{\mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[\int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*_{t+1}} - 1 \right) dF(\varepsilon) - \int_{\varepsilon^{**}_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^{**}_{t+1}} - 1 \right) dF(\varepsilon) \right] (1-\delta) \omega_{t+1} Q_{t+1}}{Q_t}$$

$$- \frac{\mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\varepsilon^*_{t+1}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*_{t+1}} - 1 \right) dF(\varepsilon) \right] \pi_{t+1} \omega_{t+1} P^K_{t+1}}{Q_t}.$$

Log-linearization

$$\hat{Y}_t = \hat{A}_t + \alpha \hat{K}_t + (1-\alpha) \hat{N}_t,$$

$$\hat{R}^K_t = \hat{A}_t - (1-\alpha) \hat{K}_t + (1-\alpha) \hat{N}_t,$$

$$(\alpha + \xi) \hat{N}_t = \hat{A}_t + \alpha \hat{K}_t - \hat{C}_t,$$

$$\hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{I}{Y} \hat{I}_t + \frac{B}{Y} \hat{B}_t - \frac{B}{R} \hat{B}_{t+1} + \frac{R}{R} \hat{R}^B_t,$$

$$\hat{\varepsilon}^*_t = -\hat{Q}_t,$$

$$\hat{\varepsilon}^{**}_t = -\hat{P}^K_t,$$

$$\hat{\theta}_t = -(1-\theta) \left(\hat{\pi}_t + f(\varepsilon^{**}) \varepsilon^{**} \hat{\varepsilon}^{**}_t \right),$$

$$\hat{C}_t - \hat{C}_{t+1} + \hat{R}^B_t - \left(\beta R \int_{\varepsilon^*}^{\varepsilon_{\max}} \frac{\varepsilon}{\varepsilon^*} dF(\varepsilon) \right) \mathbb{E}_t \hat{\varepsilon}^*_{t+1} = 0,$$

$$\hat{P}^K_t = \hat{\theta}_t + \hat{Q}_t,$$

$$
\begin{aligned}
\hat{Q}_t &= \hat{C}_t - \hat{C}_{t+1} \\
&\quad + \frac{R^K \varepsilon^*}{R^B} \hat{R}^K_{t+1} + \frac{\omega \pi \theta}{R^B} \left(\hat{\omega}_{t+1} + \hat{\pi}_{t+1} + \hat{P}^K_{t+1} \right) \\
&\quad - \beta \left(R^K + \omega \pi P^K \right) \left[\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF\left(\varepsilon\right) \right] \hat{\varepsilon}^*_{t+1} \\
&\quad + \beta \left(1 - \delta\right) \omega \left[1 + \int_{\varepsilon^{**}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^{**}} - 1 \right) dF\left(\varepsilon\right) \right] \left(\hat{\omega}_{t+1} + \hat{Q}_{t+1} \right) \\
&\quad - \beta \left(1 - \delta\right) \omega \left[\int_{\varepsilon^{**}}^{\varepsilon_{\max}} \frac{\varepsilon}{\varepsilon^{**}} dF\left(\varepsilon\right) \right] \hat{\varepsilon}^{**}_{t+1} \\
&\quad + \beta \left(1 - \delta\right) \left(1 - \omega\right) \left(\hat{Q}_{t+1} - \frac{\omega}{1 - \omega} \omega_{t+1} \right),
\end{aligned}
$$

$$
\begin{aligned}
\hat{I}_t &= \frac{B\left(1 - F\left(\varepsilon^*\right)\right)}{I} \hat{B}_t + \left(1 - \frac{B\left(1 - F\left(\varepsilon^*\right)\right)}{I} \right) \hat{K}_t \\
&\quad - \frac{\left(B + \left(R^K + \omega \pi P^K \right) K \right) f\left(\varepsilon^*\right) \varepsilon^*}{I} \hat{\varepsilon}^*_t - \frac{\left(1 - \delta\right) \omega_t P^K_t K f\left(\varepsilon^{**}\right) \varepsilon^{**}}{I} \hat{\varepsilon}^{**}_t \\
&\quad + \frac{R^K K \left(1 - F\left(\varepsilon^*\right)\right)}{I} \hat{R}^K_t + \frac{\omega \pi P^K K \left(1 - F\left(\varepsilon^*\right)\right)}{I} \hat{\pi}_t \\
&\quad + \frac{\left(\pi \left(1 - F\left(\varepsilon^*\right)\right) + \left(1 - \delta\right) \left(1 - F\left(\varepsilon^{**}\right)\right) \right) \omega P^K K}{I} \left(\hat{\omega}_t + \hat{P}^K_t \right),
\end{aligned}
$$

$$
\begin{aligned}
\hat{K}_{t+1} &= \left(1 - \delta\right) \hat{K}_t + \frac{B \int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF\left(\varepsilon\right)}{K} \hat{B}_t + \left(\delta - \frac{B \int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF\left(\varepsilon\right)}{K} \right) \hat{K}_t \\
&\quad - \left(\frac{B}{K} + \left(R^K + \omega \pi P^K \right) \right) f\left(\varepsilon^*\right) \left(\varepsilon^*\right)^2 \hat{\varepsilon}^*_t - \omega \left(1 - \delta\right) P^K f\left(\varepsilon^{**}\right) \left(\varepsilon^{**}\right)^2 \hat{\varepsilon}^{**}_t \\
&\quad + R^K \left(\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF\left(\varepsilon\right) \right) \hat{R}^K_t + \omega \pi P^K \left(\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF\left(\varepsilon\right) \right) \hat{\pi}_t \\
&\quad + \left(\pi \int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF\left(\varepsilon\right) + \left(1 - \delta\right) \int_{\varepsilon^{**}}^{\varepsilon_{\max}} \varepsilon dF\left(\varepsilon\right) \right) \omega P^K \left(\hat{\omega}_t + \hat{P}^K_t \right),
\end{aligned}
$$

$$
\hat{r}^\omega_t = \hat{C}_t - \hat{C}_{t+1} - \hat{Q}_t + \mathbb{E}_t \hat{Q}_{t+1} - \left(\beta R \int_{\varepsilon^*}^{\varepsilon_{\max}} \frac{\varepsilon}{\varepsilon^*} dF\left(\varepsilon\right) \right) \mathbb{E}_t \hat{\varepsilon}^*_{t+1} - \frac{\omega}{1 - \omega} \mathbb{E}_t \hat{\omega}_{t+1},
$$

$$
\begin{aligned}
\hat{r}^\pi_t &= \hat{C}_t - \hat{C}_{t+1} - \hat{Q}_t + \mathbb{E}_t \hat{Q}_{t+1} + \frac{X_1}{X} \left(\hat{\omega}_{t+1} + \hat{Q}_{t+1} \right) \\
&\quad - \frac{X_2}{X} \left(\mathbb{E}_t \hat{\pi}_{t+1} + \mathbb{E}_t \hat{\theta}_{t+1} - \left(\beta R \int_{\varepsilon^*}^{\varepsilon_{\max}} \frac{\varepsilon}{\varepsilon^*} dF\left(\varepsilon\right) \right) \mathbb{E}_t \hat{\varepsilon}^*_{t+1} \right),
\end{aligned}
$$

where

$$X_1 = \left[\int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) dF(\varepsilon) - \int_{\varepsilon^{**}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^{**}} - 1\right) dF(\varepsilon)\right] (1-\delta)\,\omega,$$

$$X_2 = \left[1 + \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon^*} - 1\right) dF(\varepsilon)\right] \pi\theta = \frac{\pi\theta}{\beta R},$$

$$X = X_1 - X_2 = \frac{r^\pi}{\beta},$$

D More Graphs

The model-implied time series for liquidity premium ($\hat{r}_t^\omega$) under each of the three shocks (dashed lines) along with the data counterpart (solid lines) are graphed in Figure C.1.

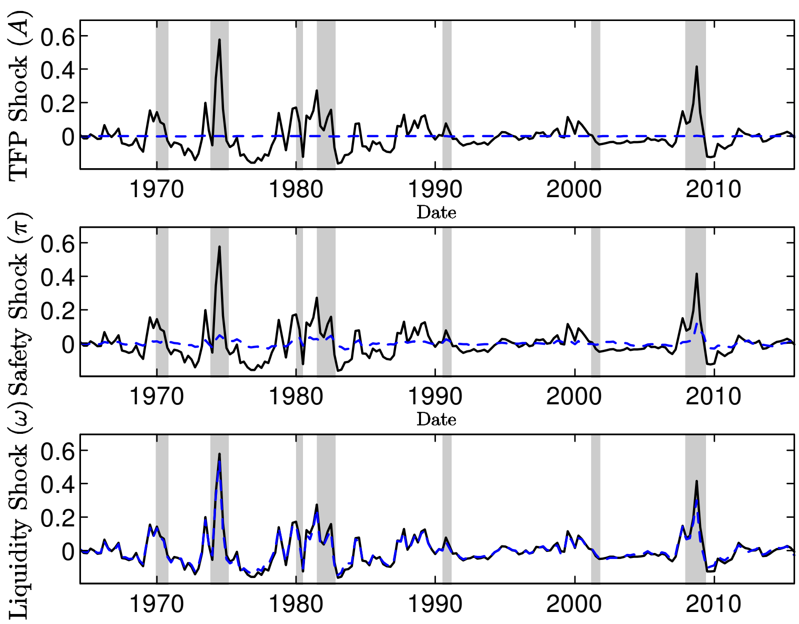

Figure C.1: Counterfactual Analysis for Liquidity Premium: (Data in Solid Line and Counterfactual Simulation in Dashed Line)

Under the unconventional policy, the law of motion of capital accumulated by all firms is modified to

$$K_{t+1}^F = (1-\pi_t) K_t^F + \left[\left(\frac{B_t}{K_t^F} + R_t^K + \omega_t \pi P_t^K\right) \int_{\varepsilon_t^*}^{\varepsilon_{\max}} \varepsilon dF + \omega_t (1-\delta) P_t^K \int_{\varepsilon_t^{**}}^{\varepsilon_{\max}} \varepsilon dF\right] K_t^F,$$

where $R_t^K = \alpha \frac{Y_t}{K_t^F}, \varepsilon_t^* = \frac{1}{P_t^K}, \varepsilon_t^{**} = \frac{1}{Q_t}, P_t^K = \theta_t Q_t$,and the Euler equation for Q_t remains the same as in equation (B.9) (the case without government interventions).

The effects of changing the debt-to-GDP ratio on welfare are shown in Figure C.2. Corresponding to section 5.1, the parameters used in generating the left and right panels in Figure C.2 come from Table 1 and footnote 13, respectively.

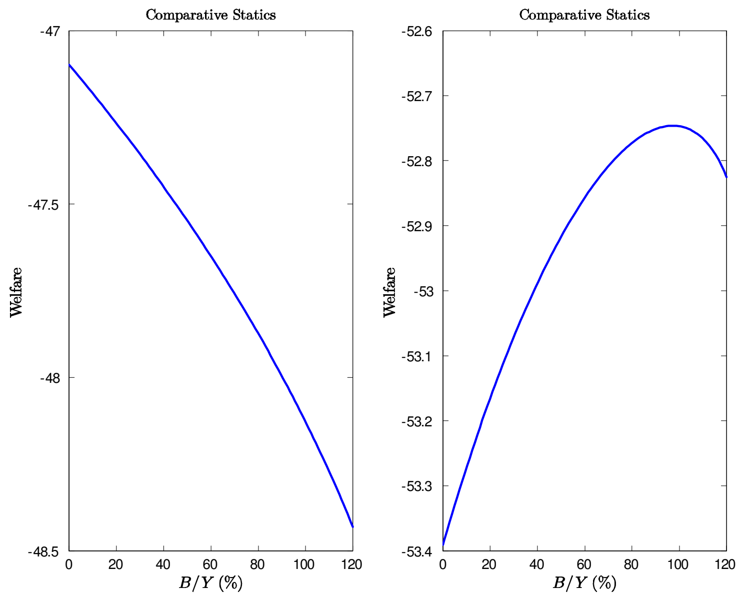

Figure C.2. Comparative Statics. (Parameters in left and right panels comes from Table 1 and footnote 12 respectively)