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Debt and Stabilization Policy: Evidence from a Euro Area FAVAR*

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Abstract

The Euro-area poses a unique problem in evaluating policy: a currency union with a shared monetary policy and country-specific fiscal policy. Analysis can be further complicated if high levels of public debt affect the performance of stabilization policy. We construct a framework capable of handling these issues with an application to Euro-Area data. In order to incorporate multiple macroeconomic series from each country but, simultaneously, treat country-specific fiscal policy, we develop a hierarchical factor-augmented VAR with zero restrictions on the loadings that yield country-level factors. Monetary policy, then, responds to area-wide conditions but fiscal policy responds only to its country-level conditions. We find that there is broad quantitative variation in different countries’ responses to area-wide monetary policy and both qualitative and quantitative variation in responses to country-specific fiscal policy. Moreover, we find that debt conditions do not diminish the effectiveness of policy in a significant manner, suggesting that any negative effects must come through other channels.

[JEL: C32, E58, E62]

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1 Introduction

Beginning with Reinhart and Rogoff (2010), a recent spate of literature has focused on the link between a country’s economic performance and its sovereign debt level. In particular, Reinhart and Rogoff (2011) argue that debt crisis can be a precursor to financial crisis, which, in turn, can be catastrophic for economic performance. A smaller literature has studied the performance of stabilization policy—both monetary and fiscal—in environments where debt levels are high.

Theory suggests that the level of sovereign debt can potentially affect the impact of government spending stimulus on the economy. If debt levels are high, then an increase in government spending signals that, due to debt stabilization motives, it might be followed shortly by fiscal tightening. If people expect increased future taxes, they reduce current consumption and the effects of stimulative spending can be subdued, or even have net contractionary effects. If, however, the agents in the economy expect debt stabilization through government spending—i.e., a spending reversal in the future—then there can be larger short-run positive effects on output.¹ Using a non-linear, neoclassical growth model, Bi, Shen, and Yang (2016) show that the effects of government policy can differ in high and low public debt environments. Moreover, it can depend on several factors, including wealth effects on labor supply and whether debt is stabilized using taxes or spending.

Existing empirical studies, however, find mixed results. For instance, Ilzetzki, Mendoza and Vegh (2013) consider a panel of 44 developing and low income countries, and find that, in episodes of high sovereign debt (above 60 percent of GDP), the effects of government stimulus are negligible or may even be negative in the long run. On the other hand, Corsetti, Meier and Muller (2012a) consider data on 17 OECD countries and find that while the impact response of GDP to a government spending shock is smaller during a high debt or weak public finance state, the differences in responses across states are not statistically significant.²

¹Corsetti, Meier and Muller (2012b) show that because of the combined effects of monetary and fiscal policy on long term rates, anticipated spending reversal can have large positive effects at shorter horizons. If households expect future decreased spending, then they expect it to reduce future inflation and thus future policy rates. This expectation leads to a decline in long-term real interest rates, and causes current private consumption to rise.

²They define countries as having weak public finances when period gross government debt exceeds 100
High levels of public indebtedness, as experienced in the Euro Area countries, have led to many concerns regarding debt sustainability and have contributed to low consumer confidence. This state of uncertainty, in turn, affects the transmission of monetary policy as well.\textsuperscript{3} A growing empirical literature has considered the relative effectiveness of monetary policy during the financial crisis (see e.g., Ciccarelli, Maddaloni and Peydro (2013); Jannsen, Potjagailo and Wolters (2015); and von Borstel, Eickmeier and Krippner (2015) for work on Euro Area countries and Hubrich and Tetlow (2015), who consider data from the United States). Less attention, however, has been devoted to explicitly modeling the interaction of public debt levels and the effects of monetary policy.

Our contribution in this paper is threefold. Firstly, we focus on \textit{jointly} analyzing the efficacy of both monetary and fiscal policy in the Euro Area. Secondly, we explore potential variations in the responses to both these policy shocks dependent on the public debt levels prevailing in a given country.\textsuperscript{4} Lastly, and perhaps most importantly, there is a methodological contribution with the econometric framework, where we show how to consider both monetary and fiscal policy shocks in a currency union. In particular, the Euro Area is an interesting case, since it is a currency union with a single monetary instrument but each country has an independent fiscal policy. Thus, fiscal stabilization policies can differ across countries—as can the debt level—but each country is subject to the same monetary policy. In addition, because monetary policy is centralized, it may or may not react to the idiosyncratic shocks within a single particular country.

We investigate the differences in the effects of shocks to stabilization instruments—both monetary and fiscal—for a large number of countries, in this currency union, each with independent fiscal policy and heterogeneous debt levels. For each country, we use a small number of GDP or if lagged net government borrowing exceeds 6 percent of GDP.

\textsuperscript{3}Dornbusch (1996) summarizes the role that public debt can potentially play in the transmission of monetary policy. The major concerns are related to how expansionary monetary policy could be seen as a way to inflate away debt. But on the other hand, monetary tightening with a rise in interest rates, could dramatically raise the interest burden of a high debt country.

\textsuperscript{4}Given recent sovereign debt issues in the Euro Area, note that our specific focus is on how the policy effects depend on government debt levels, but their effects may also depend on level of private debt. Bernardini and Peersman (2015) find evidence that government spending is more effective and has larger multipliers during periods of high private debt. On the other hand, Alpanda and Zubairy (2017) show that monetary policy shocks are less effective at stimulating the economy during periods of high household debt.
of economic indicators, including real output, employment, and inflation. To account for the large number of variables, we augment the standard monetary and fiscal VAR with a set of factors. We include one factor that represents area-wide economic activity and a factor for each country that represents the idiosyncratic economic conditions of that country relative to other countries in the currency union. The factors are related to a vector of other macroeconomic variables which can include the policy variables as well as other exogenous variables such as oil prices and the economic conditions of countries outside the currency union.\(^5\)

To streamline and interpret the model factors, we adopt exclusion restrictions on the factor loadings similar to those suggested in Kose, Otrok and Whiteman (2003). In particular, we assume that each country’s macroeconomic indicators load only on the area-wide factor and its own country factor. Thus, idiosyncratic economic conditions in one country do not spill over onto the factors of the other countries. We also assume that the fiscal authority reacts only to economic conditions in the home country and that monetary policy reacts only to area-wide fluctuations.

We find that the area-wide factor captures the common business cycle shared by nominal measures among the Euro-Area countries, explaining most of the fluctuations in cross-country inflation in our sample. Output fluctuations in some countries differ enough from an area-wide cycle that the country factor becomes more important for explaining the real side of the economy.

The FAVAR allows us to construct impulse responses of all macroeconomic data series for the countries in the sample. We find that shocks to the aggregate monetary policy instrument produce consistent responses across countries as production and employment both fall following a contractionary monetary policy shock. While we observe quantitative differences in the response to monetary policy across the various Euro-Area countries, we observe both quantitative and qualitative heterogeneity in the response of economic activity to country-specific fiscal policy. This suggests that while monetary policy seems to be a uniformly suitable sta-

\(^5\)A number of previous papers have employed FAVARs to assess the efficacy of monetary policy [see Boivin, Giannoni, and Mojon (2008); Barigozzi, Conti, and Lucian (2014); Soares (2013); Mandler, Scharnagl and Volz (2016); and Potjagailo (2017)]. The major methodological difference between these models and ours is that we identify country-level factors and impose restrictions on the fiscal policy’s response to other countries’ economic conditions.
obilization tool, the effects of fiscal policy really depend on the country under consideration. We find no evidence of significant differences in the effect of stabilization policy across high- and low-debt environments. Though, we do find suggestive evidence that monetary policy shocks may have slightly larger effects on output in some countries when facing a debt crisis. These larger effects of monetary policy during high-debt periods could reflect the exacerbated response of output due to a large drop in consumer and investor confidence. On the other hand, in the case of spending shocks, the median responses suggest mixed evidence of the effectiveness of government spending shocks in high-debt versus low-debt regimes: in some countries spending shocks are more effective in a high-debt state and vice versa in some others. This could reflect differences in expectations on how increased spending will be financed in the future.

The balance of the paper is constructed as follows: Section 2 presents the factor-augmented VAR model with the restrictions that identify country-level factors. We also outline the restrictions on the policy effects in the model—in particular, how area-wide monetary policy and country-specific fiscal policy interact with the country-level factors, and details of the identification of the shocks in the VAR. Section 3 presents our empirical strategy, describes the data and outlines the estimation. Sections 4 and 5 present the results for the baseline case and the case in which the level of sovereign debt influences the response to policy, respectively. Section 6 presents some robustness checks and specification analysis. Section 7 summarizes and concludes. Details of the estimation are included in a technical appendix.

2 Model

Countries in the European Monetary Union have a binding trade agreement, share a common monetary policy, but have independent fiscal policies. Perhaps because of this sovereign fiscal policy, in order to obtain membership in the EMU, potential members generally have been asked to demonstrate economic stability, abiding by standards for debt, spending, and taxes. These issues can complicate the evaluation of the effect of stabilization policy in the Euro area. First, monetary policy is common, responding to varying conditions in all of the countries
in the Euro area but, generally, not responding to conditions in a single country. Second, unlike in the U.S., fiscal policy is idiosyncratic to the country. Third, the efficacy of the stabilization policy may depend on both the economic and fiscal conditions in each country at the time of implementation.

2.1 The Baseline Model

Accounting for at least some of these issues requires a model that has multiple countries, a common monetary policy, and idiosyncratic fiscal policy. Additionally, the model should be tractable enough to obtain some inference. Suppose that $X_{nt}$ is a $(Q \times 1)$ vector of country-level economic indicators (e.g., GDP, national inflation, employment/unemployment) for each of $n = 1, ..., N$ countries. Let $X_t = [X_{1t}; ..., X_{Nt}]$ collect the $N$ vectors of national data. Define $Y_{nt}$ as an $(R \times 1)$ vector of nation $n$’s policy instruments (e.g., fiscal variables) and $Z_t$ as an $(M \times 1)$ vector of common macroeconomic variables, which may include the common monetary instrument or external international variables such as global economic conditions. The system would be of the order of $NQ + NR + M$ variables. While VARs are the workhorse model for evaluating the effects of policy, models of this size can be difficult to work with. Instead, we adopt a model that has both the flavor of the VAR combined with the dimension reduction of a factor model but imposes structure on the factors for identification and interpretation.

Let $W_t$ represent an area-wide factor that affects the macroeconomic conditions in all of the countries and let $F_{nt}$ represent a factor that affects only country $n$’s economic conditions (contemporaneously). $F_t = [F_{1t}, ..., F_{Nt}]'$ collects the set of $N$ country-level factors and $Y_y = [Y_{1t}', ..., Y_{Nt}']'$ collects the country-level macro policy variables. The national economic conditions data are related to the factors and the macroeconomic data through the measurement equation of the FAVAR:

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6 While states have their own fiscal policies, they are generally small compared to the federal fiscal variables. Moreover, state fiscal variables are not generally used for stabilization policy.

7 Bănăbura, Giannone, and Reichlin (2010) show that large VARs can be used for forecasting. Structural analysis, however, requires identifying restrictions that may not be obvious in these large VARs. Another alternative is the global VAR (see Pesaran, Schuermann, and Weiner, 2004) which imposes restrictions on the responses of variables across countries.
\[ X_{nt} = \Lambda_n^w W_t + \Lambda_n^f F_{nt} + \Gamma_n^y Y_{nt} + \Gamma_n^z Z_t + \varepsilon_{nt}, \]  

(1)

where deterministic components have been suppressed and \( \varepsilon_{nt} \sim N(0, \Omega_n) \). For simplicity and to be consistent with the extant literature, we assume that the \( \varepsilon_t \)'s are serially uncorrelated and that \( \Omega_n \) is diagonal. The latter assumption imposes that all of the cross-country and within-country–cross-series variation is assumed to result only from the factors or the macro variables. The matrices of factor loadings—\( \Lambda_n^w, \Lambda_n^f, \Gamma_n^y, \) and \( \Gamma_n^z \)—are \((Q \times 1), (Q \times 1), (Q \times R),\) and \((Q \times M)\), respectively.

We can rewrite (1) as a panel for all the countries:

\[ X_t = \Lambda F_t + \varepsilon_t, \]  

(2)

where \( F_t = [Y'_t, W_t, F'_t, Z'_t]' \), \( \Lambda \) is an \((NQ \times (M + 1 + N + NR))\) matrix that collects the factor loadings, \( \varepsilon_t = [\varepsilon'_1, \ldots, \varepsilon'_N]' \), \( \varepsilon_t \sim N(0, \Omega) \), and variance-covariance matrix \( \Omega \), like \( \Omega_n \), is diagonal.

Note that the generalized matrix equation (2) implies a number of zero restrictions on \( \Lambda \), the factor loadings, that help to identify and interpret the factors [see also Belviso and Milani (2006)]. Our approach has a similar flavor to the restrictions on the regional factors in Kose, Otrok, and Whiteman (2003). Moreover, the block restrictions in the matrix of factor loadings, in addition to some sign and scale restrictions, will be sufficient to rule out alternative rotations and allow us to identify the factors. The matrix of factor loadings \( \Lambda \) in our model has a block structure:

\[
\Lambda = \left[ \begin{array}{cccc}
\Gamma^y & \Lambda^w & \Lambda^f & \Gamma^z
\end{array} \right],
\]

where \( \Lambda^w = [\Lambda_1^w, \ldots, \Lambda_N^w]' \), \( \Gamma^z = [\Gamma_1^z, \ldots, \Gamma_N^z]' \), and the country-specific loadings
\[
\Lambda^f = \begin{bmatrix}
\Lambda_1^f & 0_{Q \times 1} & 0_{Q \times 1} \\
\vdots & \ddots & \vdots \\
0_{Q \times 1} & 0_{Q \times 1} & \Lambda_N^f
\end{bmatrix}
\]

and

\[
\Gamma^y = \begin{bmatrix}
\Gamma_1^y & 0_{Q \times R} & 0_{Q \times R} \\
\vdots & \ddots & \vdots \\
0_{Q \times R} & 0_{Q \times R} & \Gamma_N^y
\end{bmatrix}
\]

are block diagonal. In our model, the area-wide factor affects the economic conditions of all of the countries to a varying degree that depends on the magnitude of the factor loadings. Similarly, the area-wide macro variables are modeled as area-wide observable factors, affecting economic conditions directly contemporaneously but indirectly through the factors at a lag. The country-level factors and country-level macro variables affect contemporaneously only the economic conditions for its own country. They can, however, also affect the economic conditions of other countries at a lag. In order to identify the effect of the policy shocks, we may restrict the contemporaneous effects both through the variance-covariance matrix and through restrictions on the \( \Gamma_i \)’s.

The dynamics of the factors are described by a VAR. The reduced-form of (3) represents the transition equation of the FAVAR:

\[
F_t = \Phi(L) F_{t-1} + \varepsilon_t,
\]

where \( F_t = [Y_t, W_t, F_t, Z_t]^T \), \( \Phi(L) \) is a \((NR + 1 + N + M) K \times (NR + 1 + N + M) K\) matrix polynomial in the lag operator and \( \varepsilon_t \sim N(0, \Sigma) \).

Because only the sign of the loading times the factor is identified, we must place some additional restrictions on the sign and scale of the factors (or their loadings) for identification.

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8Initially, we restrict the loadings so that both \( \Gamma^y = 0 \) and \( \Gamma^x = 0 \). Thus, any comovement among all of the observable series in \( X \) is described by \( W \) and any comovement of the series in \( X_n \) is captured by the country-specific factor, \( F_n \). The measurement equation takes the form of the dynamic factor model of Stock and Watson (1998) or the FAVAR of Bernanke, Boivin, and Eliasz (2005) with zero observable factors.
Implementation is similar to that described in Kose, Otrok, and Whiteman (2003, 2008), setting the loading for one series in $X$ on the world factor to be strictly positive and the loading for the first series of $X_n$ for all $n$ to be strictly positive.

### 2.2 Identifying the VAR

The VAR requires the usual restrictions either on the short- or long-run effects to identify the structural shocks. We impose overidentifying restrictions on the matrix $A$ to achieve the goal of separately identifying aggregate monetary shocks and disaggregate fiscal spending shocks. To identify spending shocks, Blanchard and Perotti (2002) and Fatas and Mihov (2001) assume that, at a quarterly or higher frequency, government spending does not contemporaneously react to macroeconomic variables.\(^9\) We follow this approach. A shock to fiscal spending in country $n$ contemporaneously affects the factor for country $n$ only. In addition, consistent with Christiano, Eichenbaum, and Evans (1999), we assume that monetary policy responds contemporaneously to economic conditions but economic conditions respond only at a lag, providing a set of exclusion restrictions. We also assume that monetary policy does not respond to country-specific fiscal policy.

In order to identify both aggregate monetary policy and disaggregate fiscal policy shocks, we consider the following structural FAVAR relating the factors and the policy variables:

$$AF_t = B(L)F_{t-1} + u_t,$$

where $\Phi(L) = A^{-1}B(L)$, $\Sigma = (A'A)^{-1}$, $A$ describes the contemporaneous relationship between all components in the FAVAR, $B(L)$ is the $K$-lag polynomial, and $u_t$ are the structural shocks.\(^{10}\) The structural shocks can be identified by imposing short- or long-run restrictions on

\(^9\)Ramey (2011) highlights potential delays between announcements of fiscal plans and their implementation. These fiscal foresight issues are often addressed by accounting for news on spending/tax changes using narrative evidence or including many variables in a factor model [e.g., Forni and Gambetti, 2010]. Given that we want to identify disaggregated fiscal shocks in a very large number of countries along with a joint monetary policy shock, accounting for news is not feasible in our setup. In the robustness section, we consider augmenting the set of variables used to construct the country-specific factors with data on consumption and investment for each country.

\(^{10}\)The VAR in our model contains 26 series; thus, we require 325 restrictions to exactly identify the system. Our identification scheme produces an over-identified system that contains 222 over-identifying restrictions.
elements of the contemporaneous effects matrix \( A \). In particular, our identifying assumptions imply the following structure on \( A \):

\[
A = \begin{bmatrix}
    a_{yy} & 0 & 0 & 0 \\
    0 & a_{ww} & 0 & 0 \\
    a_{fy} & 0 & a_{ff} & 0 \\
    0 & a_{zw} & 0 & a_{zz}
\end{bmatrix},
\]

where \( a_{yy}, a_{fy}, \) and \( a_{ff} \) are all diagonal.

In addition to the contemporaneous restrictions, we also must impose some restrictions on the lagged dynamics of the elements of the FAVAR. A typical assumption is to assume that the factors are independent of each other, even at a lag. This assumption would manifest as block zero restrictions in the factor equations of the VAR:

\[
B(L) = \begin{bmatrix}
    b_{yy}(L) & b_{yw}(L) & b_{fy}(L) & 0 \\
    b_{wy}(L) & b_{ww}(L) & 0 & b_{wz}(L) \\
    b_{fz}(L) & 0 & b_{ff}(L) & b_{zf}(L) \\
    0 & b_{zw}(L) & 0 & b_{zz}(L)
\end{bmatrix},
\]

where \( b_{ff}(L) \) is diagonal.\(^{11}\) We further assume that the country-specific policy variables do not affect or respond to the factors for the other countries (e.g., Germany’s fiscal policy does not spill over to or respond to Austria’s economic conditions). These restrictions imply that \( b_{yy}(L), b_{fy}(L), \) and \( b_{yf}(L) \) are block diagonal. Also, we assume that country-level fiscal policy does not respond to area-wide monetary policy, implying that \( b_{yz}(L) = 0 \). Finally, we assume that Euro-area monetary policy responds only to the lags of the area-wide factor, not individual country conditions or country-level fiscal policy—i.e., \( b_{zy}(L) = b_{zf}(L) = 0 \).

Some of these restrictions may seem extreme; however, the restrictions that we impose which mostly restrict the contemporaneous cross-country effects of shocks.

\(^{11}\)Kose, Otrok, and Whiteman (2003) make the restriction that \( b_{ff} \) is diagonal and it has become common in these models. Kose, Otrok, and Prasad (2012) extend the model to allow for spillovers between the factors. Rather than considering the spillover across countries, Kose, Otrok, and Prasad focus more on the spillover from the global factor to the country factor and vice versa. Extension to allow spillovers is straightforward.
imply that there exists no ex ante systematic (lagged) response of one country’s macro aggregates to another country’s policy actions, no ex ante systematic response of one country’s fiscal policy to another, etc. Note that this does not prevent ex-post correlations of the country factors or the ex-post correlation of two countries’ policies. We relax some of these assumptions when we consider robustness of our results in Section 6.2.

2.3 Accounting for Debt

One of our main interests is to determine how the effect of stabilization policy changes with the level of debt of a country relative to its GDP. One hypothesis is that the debt-to-GDP ratio changes how monetary policy interacts with the macroeconomic indicators. In our model, this manifests in changes in the VAR coefficients on the policy variables in the factor equations when the debt-to-GDP changes.\(^{12}\)

We assume that the debt-to-GDP ratio affects the transmission of stabilization policy nonlinearly. At sufficiently low levels of debt-to-GDP, stabilization policy behaves normally. When debt-to-GDP becomes sufficiently high, the transmission of stabilization policy is altered. Let \(D_{nt}\) reflect country \(n\)’s level of debt-to-GDP ratio. Then, we can model the nonlinear effect of debt-to-GDP as:

\[
\begin{bmatrix}
Y_t \\
W_t \\
F_t \\
Z_t
\end{bmatrix} = \mathbf{A} \mathbf{S}_t (L) \begin{bmatrix} Y_{t-1} \\
W_{t-1} \\
F_{t-1} \\
Z_{t-1}
\end{bmatrix} + \begin{bmatrix} u_t^y \\
u_t^w \\
u_t^f \\
u_t^z
\end{bmatrix},
\]

(5)

where

\[
\mathbf{B}_t (L) = \begin{bmatrix}
b_{yy} (L) & b_{yw} (L) & b_{yf} (L) & 0 \\
b_{wy} (L) & b_{ww} (L) & 0 & b_{wz} (L) \\
b_{fy} (L) + \text{diag}(S_t) \odot \Delta b_{fy} (L) & 0 & b_{ff} (L) + S_t \Delta b_{ff} (L) & b_{zf} (L) + S_t \Delta b_{zf} (L) \\
0 & b_{zw} (L) & 0 & b_{zz} (L)
\end{bmatrix},
\]

\(S_t = [S_{1t}, ..., S_{Nt}]\) is a \((N \times 1)\) vector of 0’s and 1’s, such that

\(^{12}\)In principle, the effect of the level of public debt could also manifest in the policy variables in the measurement equation—i.e., in how policy is conducted.
\[ S_{nt} = \begin{cases} 
1 & \text{if } D_{nt} > D^* \\
0 & \text{otherwise} \end{cases} \]

\( D^* \) is a threshold debt-to-GDP ratio, and \( \text{diag}(S_t) \) is an \((N \times N)\) matrix with \( S_t \) on the diagonal and zeroes everywhere else.

The structure of the FAVAR coefficients on the right hand side implies particular assumptions about the effect of increasing a country’s debt. First, debt effects do not spill over from one country to another. This assumption is manifest in the diagonality of the \( \text{diag}(S_t) \odot \Delta b_{fy}(L) \) term and the fact that we do not allow country debt to affect the area-wide factor. Second, debt does not affect macroeconomic dynamics except through the effect of the policy variables on country-level macroeconomic activity. Debt also does not affect how policy behaves dynamically except through its response to macroeconomic conditions.

In addition to the restrictions we place on the effect of debt in the VAR, note that we also make the explicit assumption that debt affects the model non-linearly. That is, there are two states of the world: one in which debt is above the threshold and one in which debt is below the threshold. In each state, the model is conditionally linear but across the states, there is a discrete jump in the model parameters. Moreover, once the threshold is crossed, the level of debt relative to the threshold is irrelevant. For countries in which \( D_{nt} \) never exceeds the \( D^* \) threshold, we impose that \( \Delta b_{fy}, \Delta b_{ff}, \text{and } \Delta b_{fz} \) are zero. In doing so, only countries which experience the high debt state can exhibit variation in the dynamics of the model.

In order to assure tractability, we assume a particular form of the nonlinearity of debt, which is similar to the threshold non-linearity of fiscal policy in models such as Owyang, Ramey, and Zubairy (2013). We assume that when debt-to-GDP is sufficiently high—in this case, when \( D_{nt} > D^* - S_{nt} \) switches from 0 to 1 and the effect of lagged \( Y_{nt} \) and \( Z_t \) on the economic conditions factor changes. Additionally, when a country is in the high debt state, the autoregressive dynamics of its country-level macroeconomic activity may change. We explicitly assume here that debt in a single country does not alter how monetary policy affects the area-wide factor, does not alter how policy responds to itself, and does not alter...
how policy responds to economic conditions. Again, these assumptions are testable.

3 Econometric Implementation

3.1 Data

For each country, we use three monthly economic indicators: the log difference of employment, the log difference of gross domestic product (GDP), and inflation. GDP is deflated to 2010 euros and all the series are seasonally adjusted. Inflation is the log difference in the harmonized CPI for each country.

We have data for all 19 countries currently in the Euro Area from 1999:Q1 through 2015:Q4. However, not all of the countries were in the Euro Area for the full sample period, which would make the panel unbalanced. Seven of the countries obtained membership in 2007 or later. To achieve a balance between length of sample and size of the panel, we use the 11 original member countries (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain). We include Greece, who joined in 2001, and treat it as an original member in the sense that we do not include a country-specific Greek monetary policy for the period before it became an official member.\footnote{We do not have a full set of observations for the 1999:Q1-2015:Q4 period for one of the series in the panel. For these variables, we treat the quarters without data as missing observations in the Kalman filter when estimating the factors. This also requires an adjustment when estimating the factor loadings to include only the subsample of the estimated factors for which we have observable data.}

In addition to these three country-level variables, we include the short-term interest money market rate as the area-wide policy instrument. To represent the effective policy rate for the ECB, we use data on the Euro OverNight Index Average (EONIA). These data represent a market-determined, interbank rate that closely follows the ECB target rate. Finally, for each country, we include real per capita government consumption and the level of government debt. Government spending data are from the OECD Economic Outlook Database and we use the volume of final government consumption expenditure divided by the population, as measured by the National Accounts concept.\footnote{We follow the approach of Blanchard and Perotti (2002) and account for stochastic trends in the government consumption data. Thus, we take first differences of the log of each data series and subtract a time-varying mean. As a result, a shock to fiscal policy in our model represents a shock to the growth of government debt.} In order to measure the level government debt, we use...
data from the IMF World Economic Outlook on general government gross debt as a percent of GDP. These data on debt are available annually and we apply the annual value for all quarters within the year.

3.2 Estimation Overview

The Gibbs sampler consists of the following 4 blocks: (1) the structural VAR coefficients; (2) the factor loadings; (3) the measurement equation variances; and (4) the factors. The factor loadings and measurement variances have normal-inverse gamma priors. Conditional on the factors, we can draw the loadings (with the imposed zero restrictions) independently for each equation. In order to avoid sign flipping in the factors, we impose the sign restrictions as discussed in Section 2.1. This restriction amounts to imposing a positive loading for real GDP growth in Spain on the world factor and positive loadings on the country factor for employment growth in all countries. Then, conditional on the factors, we estimate the structural VAR in \([Y, W, F, Z]'\). Waggoner and Zha (2003) describe the procedure for estimating block (1) in detail. Finally, we draw the factors from the Kalman filter. We present results for 20,000 draws after discarding the first 10,000 draws.

For the model augmented with debt, we have the additional parameter, \(D^*\). As a first pass, we elect to apply \(D^* = 105\) as the critical threshold for debt, above which, stabilization policy may have differing effects. Using this value, there are enough observations for those countries with observations in both the high and low debt states to produce sensible estimates of the model parameters. Furthermore, \(D^* = 105\) is conservative enough to isolate troublesome levels of government debt. Table 2 describes the amount of time spent by each country in the high-debt environment in the baseline column for this debt threshold. Note, using this threshold, 5 of the 12 countries in the sample experience debt-crisis-conditions. Ireland and Portugal spend the least amount of time in the high-debt-state, 24% and 29% of the total time period, respectively. Belgium also spends less than half of the periods in the high debt

\[^{13}\text{To ensure this restriction is obtained, we adjust the prior on these coefficients to be a normal distribution centered around 1 rather than 0.}\]
\[^{16}\text{A brief overview is provided in the Appendix.}\]
\[^{17}\text{The table also shows two alternative lower thresholds that are explored in the robustness section.}\]
state, around 35%. Italy spends around half of the time in each state, with 53% of the observation in the high-debt environment. Greece has the largest number of observations in which the debt ratio exceeds the threshold, spending 59% of the sample in the high-debt state. Table 2 also provides the number of high-debt episodes observed for those countries which experience this regime. Few countries move between low-debt and high-debt regimes repeatedly, thus providing limited observations for the possibly differential dynamics under high-debt conditions. Belgium (2), Greece (3), and Italy (2) all have multiple high-debt episodes. Alternatively, Ireland and Portugal only experience the high-debt regime once, near the end of the sample.

We compute the impulse responses to the structural shocks at each iteration of the Gibbs sampler. The impulse responses of the country-specific macro aggregates such as GDP, are constructed as a sum of the responses of the factors, weighted by the factor loadings. Variation across countries, then, can be obtained through differences in either the response of the country factors or the factor loadings. For the model with debt, we compute two sets of impulse responses, each conditional on the level of debt [see Ehrmann, Ellison, and Valla (2003)]. That is, we compute one set of responses for which debt-to-GDP is above the threshold and one set of responses for which debt-to-GDP is below the threshold. Thus, we are computing impulse responses that implicitly assume that the stabilization policy cannot change the debt state, at least within the response horizon. We view this assumption as plausible, at least as a first pass, since debt-to-GDP ratio responds slowly to policy and economic conditions as evident by rather persistent debt-to-GDP regimes on average, with the critical issue being how quickly the level of GDP responds to stabilization policy.\footnote{One alternative to the regime-dependent response is the generalized impulse response (GIRF) of Koop, Pesaran and Potter (1996). The GIRF tends to be much more complicated, depending on both the level of $F_t$ at the time of the shock and the subsequent structural innovations [see, for example, Jackson, Owyang, and Soques (2016)].}

\footnote{Figure 9 plots the debt-to-GDP regimes for the relevant countries and shows that they tend to be rather persistent.}
4 Results

In this section, we present the results from the baseline model without debt, where we set the number of VAR lags, $K = 2$. We focus on the estimated factors (both area-wide and country-level) and the variance decompositions but do not report the estimates of factor loadings, variances, FAVAR coefficients, or the factor variance-covariance matrix. The full set of results is available from the authors upon request.

4.1 Variance Decompositions

In order to better understand the relative contributions of the area-wide and country factors in explaining variation in the observable data, we compute the variance decomposition of each series in $X$ attributed to either factor or the idiosyncratic, unexplained component. We calculate the variance decomposition for the area-wide and country factors at each iteration in the Gibbs sampler and report the posterior mean. The three left columns of Table 1 summarize these results for all 36 of the country-level macroeconomic series in the baseline model.

For 10 of the 12 countries (excluding Portugal and Spain), the idiosyncratic component explains the largest share of variation in GDP growth. For 9 of the 12 countries (excluding Finland, Greece, and the Netherlands), the area-wide factor explains the largest share of variation in inflation. Typically, the country-factor explains a majority of employment growth but also often explains a significant portion of GDP growth and inflation.

It is perhaps not surprising that fluctuations in inflation are correlated across a number of countries. Inflation is often viewed as primarily a monetary phenomenon and monetary policy is set for the entire area.\textsuperscript{20} While the area-wide factor captures comovement among inflation rate in many countries, the country factors capture much of the differentiation among real variables.

\textsuperscript{20}It may be useful to reiterate that Greece was not part of the Euro-area during the first few years of the sample. This may account for some, but not all, of the lack of explanatory power attributed to the area-wide factor for Greek inflation.
4.2 Factors

Figure 1 shows the posterior median of the area-wide factor extracted from the baseline model, along with the 16-percent and 84-percent quantiles. This factor clearly depicts the economic contraction of 2008 and 2009 during the global financial crisis. GDP and employment growth in almost all countries in the Euro Area decline substantially during this time, thus representative of a pervasive recession affecting all economies in the sample. The factor also appears to contract after 2011, suggesting a double dip in the Euro area.

Figure 2 depicts the posterior medians of the country-level factors and their respective 16-percent and 84-percent quantiles. All of the country-level factors are estimated precisely, as judged by the narrow bands produced by the posterior distributions. The country factors depict common economic fluctuations within a country after accounting for the area-wide factor and can reflect differences from the area-wide cycle in either timing or severity. Recall, though, that the area-wide factor accounts for much of the nominal fluctuations. Thus, we interpret the area-wide factor as a common nominal (monetary) cycle, with each country having a distinct real cycle.

Some countries exhibit common real cyclical behavior that is apparent in the shapes of their country-level factors. For example, during the global financial crisis many of the countries appeared to move together. Greece, Ireland, Portugal, and Spain have similar country factors (albeit different magnitudes), suggesting some comovement. However, this comovement was not consistent across the area as a whole. Belgium, France, Luxembourg, and the Netherlands appear to have very different shaped country factors that appear mostly to reflect differences in their labor market experiences during the global financial crisis.

4.3 Impulse Responses to a Monetary Shock

The FAVAR framework allows us to compute the impulse responses of all macroeconomic data series to shocks either at the country or area-wide level. Utilizing factors to propagate the effects of the shocks over time significantly reduces the number of parameters that must be estimated and produces a much more practical setting for analyzing the effects of stabilization
policy.

Figures 3-5 plot the median 16-percent, and 84-percent quantile responses of employment growth, GDP growth, and inflation to a monetary policy shock (a 25-basis-point increase in the policy rate) in each of the 12 countries. As discussed in Section 2.2, we identify monetary policy shocks by assuming that monetary policy can respond contemporaneously to aggregate shocks within the Euro Area, but macroeconomic aggregates in each country can only respond to monetary policy with a lag.

Figure 3 shows how the response of employment, while qualitatively similar across countries, differs quite substantially in magnitude. Employment growth in Spain and Ireland falls much more than in the other countries following a 25-basis-point increase in the policy rate. Interestingly, it appears as if employment growth in Greece rises slightly following a contractionary monetary policy shock, but the effect is insignificant. Greece likely has a rather distinct real cycle of its own as the employment data load negatively on the area-wide factor and the variance decompositions show that the country factor is very important. We find similar behavior of GDP growth in all countries. Figure 4 illustrates that the model produces reasonable responses of GDP growth in all countries in the sample. After a contractionary monetary policy shock, GDP growth falls in all countries and exhibits a hump-shaped response in the periods thereafter. We find larger responses to a monetary shock for Ireland and Spain, while the responses are rather muted in Greece, Germany, Italy and Austria.\footnote{Our responses for GDP are broadly consistent with the findings of others. Georgiadis (2015) considers a global VAR to study the effects of a common euro area monetary policy shock, identified using sign restrictions, and also finds relatively small trough responses for Portugal, Belgium, Austria and France and large effects in Ireland, similar to our paper. However, in departure from our findings, Georgiadis (2015) finds relatively modest effects for Spain, but his sample ends in 2009 and does not include the global financial crisis. Our results are also broadly in line with Barigozzi, Conti and Luciani (2014), who consider a structural dynamic factor model. However, they also do not find the large response of GDP in Spain and in their case also Ireland to a monetary policy shock. Their sample also ends earlier, in 2007.}

Quantitative variation across countries is evident in the response of inflation to monetary policy shocks but, qualitatively, the responses are quite similar.\footnote{The inflation responses do not exhibit the price puzzle as has been found in some other papers [e.g., Llaudes, 2007]. Llaudes (2007), however, estimates each country model separately.} In this case inflation falls most significantly in Belgium, Ireland, Luxembourg and Spain.\footnote{These results for Spain are consistent with findings of Mandler, Scharnagl and Volz (2016) who consider a large four-country Bayesian VAR for sample period of 1999-2014, and also find that inflation responds more}
4.4 Impulse Responses to Government Spending Shock

Figures 6-8 plot the median 16-percent, and 84-percent quantile responses of employment growth, GDP growth, and inflation to a shock to each country’s own government consumption spending. Again, we identify fiscal shocks by assuming that government spending in a given country has only a contemporaneous effect on the factor specific to that country. We further assume that the countries’ fiscal policies do not respond to or affect other countries on impact. This assumption imposes that a country’s independent fiscal policy may influence its own economy within the same period but that fiscal policy responds to its own macroeconomic conditions only at a lag.

Most of the dynamic effects occur within the first eight quarters. For 9 of 12 countries, we obtain the expected expansionary effects of fiscal policy: GDP growth, employment growth, and inflation all rise following a 1% increase in the home country’s government consumption. Note that in many countries the impulse responses are statistically significant for only a few quarters. Similar to the responses to monetary policy shocks, the magnitude of the response differs across countries, with Austria and Germany showing no significant effects but the boost to all variables in France persists for at least four quarters.

On the other hand, for Finland and Luxembourg, government consumption shocks have no statistically relevant effects on output or employment. Real variables in Portugal expand after a government consumption shock but only after a few quarters; on impact, the government spending shock is contractionary.

Overall, unlike the monetary policy responses, not only quantitative but also qualitative differences are observed across countries in response to an expansionary fiscal policy shock. This suggest that the fiscal policy responses tend to be much more country-dependent. The mute and sometimes negative response to an expansionary government spending shock in some Euro Area countries is consistent with the findings of Perotti (2005) who found that the effects of government spending have become substantially weaker over time.\footnote{This decline in Spain than France, Germany and Italy in response to a monetary policy shock.}

\footnote{In particular, in the post-1980 period, he finds a negative response of GDP to a positive spending shock in some countries, particularly at longer horizons.}
in fiscal policy effectiveness has been documented by others with a more updated data set as well, such as Kirchner, Cimamodo and Hauptmeier (2010) who consider a time varying parameter VAR for Euro area wide data.

5 Accounting for Debt

In this section, we augment the model to account for possible variation in the response of policy in high debt regimes. As detailed in Section 3.2 and Table 2, of the 12 countries in the sample, 7 never experience the high debt state. For these countries, we impose that all $\Delta \phi_{ij} = 0$, as we have no observations for $S_{nt} \neq 0$. Countries that do have a high debt experience for some share of the sample (in parentheses) are: Belgium (35%), Greece (59%), Ireland (24%), Italy (53%), and Portugal (29%). Figure 9 plots the indicator variable $S_{nt}$ for these countries.

Table 3 presents summary statistics for the country-level observable data series in both high- and low-debt environments during the sample. The data series exhibit similar behavior across regimes in Belgium. For all other countries, the mean employment growth and GDP growth rates are smaller and much more volatile in the high-debt regime compared with the low-debt regime. Employment growth is negative, on average, for Greece, Ireland, and Portugal in the high-debt regime. Similarly, GDP growth is negative for Ireland and Portugal when debt levels are high. Average inflation is lower in the high-debt regime for Greece, Ireland, and Portugal but comparable across regimes for Belgium and Italy.

5.1 Differences from the Baseline Model

Because we are estimating the factors using the Kalman filter, and the state space has changed, we consider the effect of adding the debt regime on the estimated factors and their associated variance decompositions. As expected, the posterior median of the area-wide factor extracted from the model in which we allow for variation in the model dynamics between low- and

\[19\]
high-debt states has almost identical shape to the factor estimated from the baseline model. The correlation between the posterior median of the two estimated series is 0.999.\footnote{This result is expected because, in both cases, the area-wide factor is nearly identical to the first principal component which is invariant to the change in the state space.}

We find very little difference in the estimates of the country factors when accounting for high-debt scenarios. The correlations between the posterior medians of each country factor in the baseline and debt models all exceed 0.99. The last three columns of Table 1 repeat the variance decomposition exercise for this model. We find almost identical results suggesting that the area-wide factor is important for inflation in most countries.

5.2 Impulse Responses

When accounting for potential variation in the macroeconomic dynamics of a country in debt crisis, we allow for the response of the country factors to fiscal and monetary policy to differ when debt exceeds a critical threshold.\footnote{This framework also allows us to compute impulse responses to any of the variables within the FAVAR: government spending shocks, area-wide activity shocks, country-specific shocks, etc. We do not report all of these results here but they are available from the authors upon request.} Figure 10 shows the impulse responses to monetary policy shocks for those countries which experience debt crises. We plot the median, 16-percent, and 84-percent quantiles in both the low- and high-debt states. The responses are almost indistinguishable in most cases, thus suggesting limited variation in the responses of the macro variables in our dataset. When comparing the median responses, we find evidence of a larger decline in GDP and employment growth in Belgium and Portugal during the high-debt state.

This result suggests that the impact of monetary policy shocks, at least for some countries, is larger for the high sovereign debt state. This finding lines up with earlier findings of Ciccarelli, Maddaloni and Peydro (2013) who document stronger effects of monetary policy on Euro Area countries at the height of the financial crisis, and even more for countries with elevated sovereign debt levels. As pointed out in their paper, during financial and sovereign stress periods, monetary transmission is stronger due to various credit channels: namely both the bank lending and non-financial borrower balance sheet channels, that decrease the supply and access to credit in the economy.
Figure 11 plots the impulse responses to government consumption shocks for the high-debt countries. In the case of Belgium and Greece, the median responses of the real variables are larger in the high-debt regime. For Ireland, the response of GDP and employment is smaller during the high-debt regime. However, the responses are typically not significantly different across regimes.

Overall, for most countries we find little evidence that fiscal stabilization effects depend on public debt levels. This finding is consistent with the findings of Corsetti, Meier and Muller (2012a), who consider data on 17 OECD countries and find that responses to fiscal policy are not statistically significantly different across debt levels.

There is some weak evidence that government spending is more effective during a high debt state for Belgium and Greece, whereas the reverse is true for Ireland. As mentioned in the introduction, if debt levels are high, then an increase in government spending signals that it might be followed by fiscal tightening. If people expect increased taxes in the near future, then the effects of stimulative spending can potentially be offset by these concerns. On the other hand, if the agents expect future spending decreases, then as shown by Corsetti, Meier and Muller (2012b), there can be short-run positive effects, consistent with ones seen in Belgium and Greece. The differences in the results across the various countries could be reflecting the difference in expectations on when and how the government will be conducting fiscal consolidations in the future.\textsuperscript{28}

6 Robustness

In this section, we consider a number of robustness checks to determine the sensitivity of our results to (1) our choice of variables, (2) the restrictions on the propagation of shocks, and (3) the specification of the debt ratio threshold. In each case, we discuss significant changes in the results or indicate whether the results were unchanged. In the interest of brevity, we do not present alternative results unless they are dramatically different from the baseline. All

\textsuperscript{28}See Bi, Shen and Yang (2016) for more details on how the effects of fiscal policy in a high and low debt environment depend on wealth elasticities of labor supply and how the government debt is retired: with tax increases or spending cuts in the context of a theoretical model.
results, however, are available upon request.

6.1 Alternative Data Series

In our baseline model we consider a limited set of variables for each country. In order to evaluate whether the inclusion of additional macroeconomic variables can affect the overall country-specific economic conditions or factors and the qualitative nature of the results, we added real personal consumption expenditures and real private investment to each country’s vector of variables in the measurement equation. There is essentially no change to the area-wide factor, and thus the effects of monetary policy shocks are unchanged. The addition of these real variables affects the country-specific factors at very high frequency only for Austria, Italy and Portugal, but overall the factors look similar to the baseline case. The responses of the original three variables to a government spending shock resemble those in the baseline, for the most part.\textsuperscript{29} Models studying the effects of fiscal policy often include consumption and investment in order to capture the effects of public spending on private activity to see if crowding out occurs. When we consider the responses of consumption to a government spending shock in our model, shown in Figure 12, we find evidence of consumption rising in Austria, Ireland, Netherlands and Spain. In many others countries, the response of consumption is statistically insignificant. There is evidence of consumption crowding-out in Germany, and a negative effect on consumption on impact in Greece, Italy and Portugal before turning positive. Figure 13 further illustrates that there is heterogeneity in the response of investment to a government spending shock as well. Investment rises in Belgium, Ireland, Netherlands and Spain on impact, and has negligible or negative response on impact in Austria, Finland, Italy, Luxembourg and Portugal.\textsuperscript{30}

Our sample period includes the aftermath of the financial crisis. During this period, the short rate reaches the zero lower bound (ZLB). We conduct two separate experiments to

\textsuperscript{29} The results for the debt-dependent model with these additional variables are also very similar to the baseline case.

\textsuperscript{30} Kirchner, Cimamodo and Hauptmeier (2010) consider a TVP-VAR considering the responses of government spending shock for the Euro Area. They do not distinguish between the response across different countries but find that consumption and investment have a positive response on impact for the Euro Area overall for the time invariant model. This is consistent with the positive or negligible response of consumption and investment in response to a spending shock that we find for most countries.
account for the ZLB period. First, we include government bond rates. Since these are different across countries but are policy rates, we include them in the VAR and order them in the last block with the short rate. Macroeconomic conditions and fiscal policy within each country responds to lags of its government bond rate in a similar manner to the dynamic restrictions on the response to aggregate monetary policy shocks. Then, we replace the short rate (our policy instrument) with the shadow short rate constructed by Wu and Xia (2017). The shadow short rate is an artificial unbounded short term rate that is designed to capture the stance of monetary policy when the short rate is at the ZLB. The rate is constructed from a modification of a Gaussian affine term structure model.

We analyze the response of employment growth, GDP growth, and inflation to an increase in the sovereign government bond rate in each country. Qualitatively, the results are comparable to the impulse responses when considering an aggregate monetary policy shock. For example, the direction of the response of real variables is the same in all countries but the responses are less significant than when using the ECB policy rate. Inflation in Finland and France rises slightly following a shock to government bond rates in those countries but the effect is not significant.

When we substitute the ECB shadow short rate for the area-wide monetary policy instrument, we find similar results to the original specification for our debt dependent model. The differences across debt regimes for Portugal are less pronounced but suggest a similar qualitative conclusion – here, real variables respond more to monetary policy shocks in the high-debt regime.

6.2 Propagation of Shocks

In order to evaluate the severity of our modeling assumptions, we consider a variety of alternative specifications regarding the restrictions on $B(L)$ which governs the dynamic propagation of shocks over time. First, we consider whether country fiscal policy responds to lagged monetary policy: $b_{yz} \neq 0$. In the model accounting for debt, the differences across debt regimes

\footnote{These data are available from the IMF International Financial Statistics. The data refer to all government bonds issued, weighted by the share of each bond within the total value of all government bonds in circulation.}
are amplified but do not differ substantially from our earlier results. The area-wide interest rate is not the primary rate influencing the sovereign debt burden of individual countries and, thus, fiscal policy likely does not respond directly to it.

Second, we consider aggregate spillovers to individual countries. One might assume that, if the Euro-Area economy expands as a whole, this might lead to strengthening among all individual members. Here, we relax the assumption on the dynamics of country factors and allow them to respond to lagged area-wide conditions: $b_{fw}(L) \neq 0$. When we allow for this additional dimension, the results change very little. If we assume that area-wide policy is set in response to area-wide economic conditions, the systematic response of the policy rate should capture some of these dynamics and country-level conditions already respond to area-wide policy.

Third, we restrict the response of area-wide conditions to no longer depend on lagged country-level fiscal policy: $b_{wy} = 0$. Since the area-wide factor does not depend on country-specific conditions via the country-level factors, perhaps one might consider that aggregate conditions also should not depend on country-specific policies. This restriction would limit any spillover from the country level to the aggregate level. The results are unchanged.

Fourth, we consider whether the scope for fiscal policy interventions changes when debt levels are high. We conduct two experiments. We allow for switching in the response of country-level fiscal policy to lagged area-wide and country factors: $b_{yw}(L) + S_t\Delta b_{yw}(L)$ and $b_{yf}(L) + S_t\Delta b_{yf}(L)$. Additionally, we consider a specification where we also allow switching in the response of the area-wide factor to lagged country-level fiscal policy: $b_{wy}(L) + S_t\Delta b_{wy}(L)$. In both cases, the differences across debt regimes get more significant for Greece, Ireland, Italy, and Portugal. However, the differences are diminished for Belgium.

### 6.3 The Debt Ratio Threshold

One caveat to our results is that the relatively small time sample does not allow us to easily test the debt ratio threshold. Our threshold was chosen such that some (but not all) countries experience time both in and out of the high-debt regime. Lowering the debt threshold
substantially would lead to some countries being in the high debt regime for the entire sample period. If countries such as Italy and Greece were in the high-debt regime for the entire sample period, it might explain why we observe little variation in the responses of policy across regimes for these countries.\footnote{Furthermore, the ECB has rather limited experience under such conditions with only a few recessions observed during the sample period and few instances of individual countries enduring multiple debt crises.}

We examine two alternative threshold levels for the debt ratio. Table 2 describes the amount of time spent in each debt environment for two alternative thresholds along with the baseline. First, in order to compare our results with those of Reinhart and Rogoff (2010), we consider a country to be in a high-debt environment if the debt-to-GDP ratio exceeds 60 percent. At such a low threshold, Austria, Belgium, Greece, and Italy spend the entire sample in the high-debt regime. Thus, the impulse responses for these countries are only identified for one regime and resemble those from the baseline model without switching. Spain and France both experience the high-debt episode briefly early on in the sample and then again in latter portion of the sample. The other countries which see the high-debt regime move into it once and then remain under these conditions for the remainder of the sample. Figures 14 and 15 shows the impulse responses for five countries which do move between both low- and high-debt regimes throughout the sample: France, Germany, Ireland, Portugal, and Spain.

In response to a contractionary monetary policy shock, Figure 14 highlights the consistency across countries that was evident in the version with a higher threshold. However, among this subset of countries, real variables in Germany, Ireland, and Spain exhibit a smaller response in the high-debt regime. This result would suggest that monetary policy has muted effects on these countries when debt levels are high. Figure 15 illustrates how the response to government consumption shocks varies at this debt threshold. For Germany and Portugal, the response of real variables is larger in the high-debt regime while the responses in Ireland are smaller, like in our original specification.

Finally, we consider a debt threshold of 95 percent, between our original specification and the Reinhart and Rogoff (2010) level. As shown in Table 2, Greece and Italy still spend the entire sample in the high-debt regime. Belgium is the only country to have more than one
high-debt experience. Therefore, we plot the responses of Belgium, France, Ireland, Portugal, and Spain in Figures 16 and 17. These countries experience both low- and high-debt regimes throughout our sample when the threshold is set to 95 percent. In response to contractionary monetary policy shock, Belgium and France exhibit behavior consistent with our other results: real variables fall by more in the high-debt regime. We find similar consistency in response to government spending shocks. Interestingly, with this threshold, employment and GDP growth in Spain demonstrate behavior comparable to Ireland in which the response is smaller in the high-debt regime. Note that, with the 105 percent threshold in the initial results, Spain does not experience the high-debt regime. Perhaps these results suggest that an interesting extension would be to consider debt thresholds that vary across countries. We leave this for future research.

7 Conclusion

We estimate a Bayesian factor-augmented VAR for the original countries in the Euro Area, allowing for comovement among multiple macroeconomic data series at the area-wide and country levels. Additionally, we allow for a potential nonlinearity in the effects of stabilization policy during low- and high-debt environments. We find that the area-wide factor is important for explaining the commonality of real output among all countries in the Euro Area. Furthermore, the country-specific factors capture much of the variation driven by the disaggregate inflation data.

The FAVAR allows for constructing impulse responses of a much larger set of data by imposing restrictions in the propagation of shocks to occur via a set of latent factors. We find that a contractionary monetary policy shock at the aggregate level leads to a decline in output and employment in all countries within the currency union. Furthermore, the aggregate monetary policy instrument rises significantly on impact following an expansionary shock to the area-wide activity factor. As economic activity in the Euro Area as a whole gains momentum, the systematic response of policy adjusts for this rapidly. While we do not find much significant difference in the responses across low- and high-debt states, we find
suggestive evidence that monetary policy shocks may have slightly larger effects on output in some countries when facing a debt crisis. These larger effects of monetary policy during high debt periods could reflect the exacerbated response of output due to a large drop in consumer and investor confidence.

We also find some amount of heterogeneity in the responses of economic activity in the countries in the union to country specific expansionary government spending shocks. In some countries government spending stimulates GDP and employment a great deal, while the responses are mute or even negative at some horizons for some other countries. In the case of spending shocks too, we do not find significant differences across high and low debt regimes, but the median responses suggest mixed evidence of the effectiveness of government spending shocks in high debt regime versus low-debt regime: in some countries spending shocks are more effective in a high debt state and vice versa in some others. This could reflect differences in expectations on how increased spending will be financed in the future.

Overall, the results suggest that monetary policy seems to be a uniformly suitable stabilization tool, but fiscal policy effects really depend on the country under consideration.
References


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<td>0.002</td>
<td>0.246</td>
<td>0.752</td>
<td>0.003</td>
<td>0.246</td>
<td>0.752</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.074</td>
<td>0.725</td>
<td>0.200</td>
<td>0.074</td>
<td>0.740</td>
<td>0.184</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Variance Decompositions. The first three columns show the percent of the variance in each of the three country variables—employment growth (EMP), output growth (GDP), and inflation (INF)—attributed to the Euro-area ( $W_t$), country-specific ( $F_t$), and idiosyncratic ( $\varepsilon_t$) factors in equation (1) for the baseline model. The next three columns show the percentage of the variance accounted for by the same three factors in the model with debt.
### Regime for Alternative Thresholds

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Baseline</th>
<th>60%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Share of Sample</td>
<td># Episodes</td>
<td>Share of Sample</td>
</tr>
<tr>
<td>Austria</td>
<td>0.00</td>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.35</td>
<td>2</td>
<td>1.00</td>
</tr>
<tr>
<td>Finland</td>
<td>0.00</td>
<td>0</td>
<td>0.06</td>
</tr>
<tr>
<td>France</td>
<td>0.00</td>
<td>0</td>
<td>0.88</td>
</tr>
<tr>
<td>Germany</td>
<td>0.00</td>
<td>0</td>
<td>0.76</td>
</tr>
<tr>
<td>Greece</td>
<td>0.59</td>
<td>3</td>
<td>1.00</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.24</td>
<td>1</td>
<td>0.41</td>
</tr>
<tr>
<td>Italy</td>
<td>0.53</td>
<td>2</td>
<td>1.00</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>0.00</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.00</td>
<td>0</td>
<td>0.29</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.29</td>
<td>1</td>
<td>0.65</td>
</tr>
<tr>
<td>Spain</td>
<td>0.00</td>
<td>0</td>
<td>0.41</td>
</tr>
</tbody>
</table>

**Table 2: Observations in the High-Debt Regimes.** The table shows the percentage of the total sample (1999:Q1 through 2015:Q4) that each country spends in the high-debt regime and the number of high-debt episodes observed in each country. Each pair of columns represents a different debt-to-GDP threshold, where the first pair of columns shows the baseline threshold.

<table>
<thead>
<tr>
<th>High-Debt Regime</th>
<th>Low-Debt Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMP</td>
<td>GDP</td>
</tr>
<tr>
<td>Belgium</td>
<td>Mean</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>0.19</td>
</tr>
<tr>
<td>Greece</td>
<td>Mean</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>1.22</td>
</tr>
<tr>
<td>Ireland</td>
<td>Mean</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>1.10</td>
</tr>
<tr>
<td>Italy</td>
<td>Mean</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>0.48</td>
</tr>
<tr>
<td>Portugal</td>
<td>Mean</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>0.89</td>
</tr>
</tbody>
</table>

**Table 3: Summary Statistics in Different Debt Environments.** The table shows the differences between the means and standard deviations of the three country variables—employment growth (EMP), output growth (GDP), and inflation (INF)—in the two debt regimes under the baseline debt-to-GDP threshold of 105 percent for the five countries that spend a non-zero portion of the sample in both regimes. For each of the variables, the means are in units of percentage points.
Figure 1: Area-Wide Factor. The figure shows the mean of the posterior distribution for the area-wide factor ($W_t$) over the sample period.

Figure 2: Country Factors. The figure shows the means of the posterior distributions for the country factors ($F_{ut}$) over the sample period.
Figure 3: Impulse Responses of EMP to Monetary Policy Shock. Responses of country-level employment growth to a 25-basis-point increase in the aggregate monetary policy rate. The solid dark line is the median response across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 4: Impulse Responses of GDP to Monetary Policy Shock. Responses of country-level GDP growth to a 25-basis-point increase in the aggregate monetary policy rate. The solid dark line is the median response across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.

Figure 5: Impulse Responses of INF to Monetary Policy Shock. Responses of country-level inflation to a 25-basis-point increase in the aggregate monetary policy rate. The solid dark line is the median response across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 6: Impulse Responses of EMP to Government Consumption Shock. Responses of country-level employment growth to a 1-percentage-point increase in the first difference of log real government consumption in the respective country. The solid dark line is the median response across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 7: Impulse Responses of GDP to Government Consumption Shock. Responses of country-level GDP growth a 1-percentage-point increase in the first difference of log real government consumption in the respective country. The solid dark line is the median response across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 8: Impulse Responses of INF to Government Consumption Shock. Responses of country-level inflation to a 1-percentage-point increase in the first difference of log real government consumption in the respective country. The solid dark line is the median response across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.

Figure 9: Indicator of High-Debt Periods. The figure shows the regime indicator ($S_{nt}$) for the five countries that spend time in both regimes over the sample period. When $S_{nt} = 1$, the country is in the high debt regime.
Figure 10: Impulse Responses to Monetary Policy Shock: Debt Crisis Countries.
Responses of country-level employment growth, GDP growth, and inflation in the debt-crisis-countries to a 25-basis-point increase in the aggregate monetary policy rate. The solid (dashed) dark line is the median response in the low-debt (high-debt) regime across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 11: Impulse Responses to Government Consumption Shock: Debt Crisis Countries. Reponses of country-level employment growth, GDP growth, and inflation in the debt-crisis-countries to a 1-percentage-point increase in the first difference of log real government consumption in the respective country. The solid (dashed) dark line is the median response in the low-debt (high-debt) regime across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 12: Impulse Responses of Consumption to Government Consumption Shock - Model with C&I Data. Including consumption and investment data: Responses of country-level consumption growth to a 1-percentage-point increase in the first difference of log real government consumption in the respective country. The solid dark line is the median response across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 13: Impulse Responses of Investment to Government Consumption Shock - Model with C&I Data. Including consumption and investment data: Responses of country-level investment growth to a 1-percentage-point increase in the first difference of log real government consumption in the respective country. The solid dark line is the median response across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 14: Impulse Responses to Monetary Policy Shock: Debt Crisis Countries - 60% Threshold. Responses of country-level employment growth, GDP growth, and inflation in the debt-crisis-countries to a 25-basis-point increase in the aggregate monetary policy rate. The high-debt regime is defined as a debt-to-GDP ratio above 60 percent. The solid (dashed) dark line is the median response in the low-debt (high-debt) regime across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 15: Impulse Responses to Government Consumption Shock: Debt Crisis Countries - 60% Threshold. Responses of country-level employment growth, GDP growth, and inflation in the debt-crises-countries to a 1-percentage-point increase in the first difference of log real government consumption in the respective country. The high-debt regime is defined as a debt-to-GDP ratio above 60 percent. The solid (dashed) dark line is the median response in the low-debt (high-debt) regime across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 16: Impulse Responses to Monetary Policy Shock: Debt Crisis Countries - 95% Threshold. Responses of country-level employment growth, GDP growth, and inflation in the debt-crisis-countries to a 25-basis-point increase in the aggregate monetary policy rate. The high-debt regime is defined as a debt-to-GDP ratio above 95 percent. The solid (dashed) dark line is the median response in the low-debt (high-debt) regime across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
Figure 17: Impulse Responses to Government Consumption Shock: Debt Crisis Countries - 95% Threshold. Responses of country-level employment growth, GDP growth, and inflation in the debt-crisis-countries to a 1-percentage-point increase in the first difference of log real government consumption in the respective country. The high-debt regime is defined as a debt-to-GDP ratio above 95 percent. The solid (dashed) dark line is the median response in the low-debt (high-debt) regime across 20,000 draws from the posterior distribution. The bands are the 16th and 84th percentiles of the posterior draws.
A Details of the Gibbs Sampler

Let $\Psi = \left\{ \tilde{W}_T, \tilde{F}_T, \Phi, \Sigma, \Lambda, \Omega \right\}$ collect all of the elements to be estimated, including the time series of the factors $\tilde{W}_T = [W_1, ..., W_T]'$ and $\tilde{F}_T = [F_1, ..., F_T]'$. Because the structural model is overidentiﬁed, it is convenient to estimate the model directly in the structural form by drawing $A'$ and $B'$ instead of drawing $\Phi$ and $\Sigma$. The set $\hat{\Psi} = \left\{ \tilde{W}_T, \tilde{F}_T \right\}, \{A, B\}, \Lambda, \Omega$ delineates the blocks for the sampler, with the first two elements drawn jointly in the Kalman ﬁlter and the next two elements drawn jointly as transformations of the structural parameters.\textsuperscript{33}

A.1 The Prior

Denote $a_i$ and $b_i$ as the $i$th columns of $A'$ and $B'$, respectively. This allows one to consider each equation of the structural VAR separately, as in Waggoner and Zha (2003). We utilize a variation of the prior proposed by Sims and Zha (1998). This prior imposes independence across structural equations and thus imposes a prior on the elements of $a_i$ to be jointly normal and mean zero. Additionally, the prior of $b_i$ is conditional on $a_i$. Following the general framework of the prior described in Litterman (1986), Sims and Zha (1998) impose that the conditional mean of the coefﬁcients in $b_i$ on the first lag term in the VAR is equal to $a_i$, while the coefﬁcients on the remaining lags have mean zero. In doing so, this assumes that the random walk is a reasonable approximation of the behavior of the variables within the VAR. In our VAR, we include the ﬁrst difference of log real government consumption, the factors, and the monetary policy rate. As discussed in Litterman (1986), if the variables within the VAR are growth rates, it may be more appropriate to specify a prior assuming some persistence but less than a random walk. Additionally, we wish to assign some meaningful role to the factors without violating stationarity of the transition equation in the state-space system.\textsuperscript{34} To accomplish this, we set the prior such as to impose the mean of the reduced-form coefﬁcient on the ﬁrst lag of the dependent variable is centered around 0.9 rather than

\textsuperscript{33}Because the model is heirarchical, we could draw the factors using the algorithm proposed in Otrok and Whiteman (1998). The model cannot, however, be estimated easily using principal components. Jackson, Kose, Otrok, and Owyang (2016) discuss the advantages and drawbacks of the three approaches.

\textsuperscript{34}This stability is important for the Kalman ﬁlter step in which we estimate the factors.
1. Ultimately, the prior on the VAR coefficients can be described as the following:

\[ a_i \sim N(0, \bar{S}_i), \]

\[ b_i | a_i \sim N(P_i a_i, H_i), \]

for \( i = 1, \ldots, M + K + 1 \).

Sims and Zha (1998) define a set of hyperparameters to control the tightness of various components of the prior. We adopt a subset of these hyperparameters necessary to implement the prior in our specification of the structural VAR model. These are defined in Table 4.

We construct \( \bar{S}_i \) as a diagonal matrix with the standard deviation for coefficients on elements in the \( j \)th row of the VAR defined as \( \frac{\lambda_0}{\sigma_j} \). The \( \sigma_j \) terms are scale factors that account for potential variation in the units of measure of the data. To aid in identification of the scale of the factors, we set \( \sigma_j = 1 \) for \( j = M + 1, \ldots, M + N \), all rows which correspond to the latent factors in the model. For \( j = 1, \ldots, M \) and \( j = M + N + 2 \), \( \sigma_j^2 \) is computed as the sample variance of residuals from a univariate autoregression of the government consumption and monetary policy rate series.

As discussed previously, we adjust the prior on the first lag to be less persistent than the random walk. The matrix \( P_i \) consists of 0.9 × \( I_{M+K+1} \) in the first \( M + N + 2 \) rows and zeros elsewhere, corresponding to all coefficients on lags greater than 1. We construct \( H_i \) as a diagonal matrix with the variance of coefficients on the \( p \)th lag of variable \( j \) as \( \frac{\lambda_0^2 \lambda_3^2}{\sigma_j^2 \lambda_3^3} \).

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Hyperparameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal distribution of factor loadings: ( \Lambda \sim N(m_0, M_0) )</td>
<td>( m_0 = 0 ) ( M_0 = I )</td>
</tr>
<tr>
<td>Gamma distribution of innovation variances: ( \Omega_{ii}^{-1} \sim \Gamma(\frac{\nu_0}{2}, \frac{\delta_0}{2}) )</td>
<td>( \nu_0 = 3 ) ( \delta_0 = 0.001 )</td>
</tr>
</tbody>
</table>

**Table 4: Details of the Prior.**
A.2 Drawing $\tilde{W}_T$, $\tilde{F}_T$ conditional on $\Psi_{\tilde{W}_T, \tilde{F}_T}$

Define the state vector $\xi_t = [F'_t, F'_{t-1}, ..., F'_{t-p}]'$. We use the Kalman filter to obtain the posterior distribution of the period-$t$ state vector conditional on the model parameters and the state vector at all other time periods. In particular, the joint distribution for the state variable would be

$$p\left(\xi_T | \tilde{Y}_T, \tilde{X}_T, \tilde{Z}_T\right) = p(\xi_T | \tilde{y}_T, \tilde{x}_T) \prod_{t=1}^{T-1} p(\xi_t | \xi_{t+1}, \tilde{y}_t, \tilde{x}_t).$$

Notice that, in our case,

$$E_t [\xi'_t \xi'_t] = \begin{bmatrix} \Sigma & 0_{\pi \times K \pi} \\ 0_{K \pi \times \pi} & 0_{K \pi \times K \pi} \end{bmatrix},$$

which is singular and where $\pi = NR + 1 + N + M$. This means we cannot condition on the full vector $\xi_{t+1}$. We can, however, condition on a subset of the elements of $\xi_{t+1}$. Define

$$\xi^*_t = \xi_t,$$

$$\Phi^* = \begin{bmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_K \end{bmatrix},$$

and,

$$Q^* = \Sigma.$$

Then, the joint density for the state vector becomes

$$p\left(\tilde{\xi}_T | \tilde{y}_T, \tilde{x}_T\right) = p(\xi_T | \tilde{y}_T, \tilde{x}_T) \prod_{t=1}^{T-1} p(\xi_t | \xi^*_{t+1}, \tilde{y}_t, \tilde{x}_t).$$

We can draw from this density by running the full Kalman filter (see the appendix), saving $\xi^*_{t|t}$ and $P^*_{t|t}$. Then, we can update $\xi^*_{t|t}$ and $P^*_{t|t}$ to condition on $\xi^*_{t+1}$:
\[ \xi_{t|t,\xi_{t+1}^*} = \xi_{t|t} + P_{t|t} \Phi^* \left( \Phi^* P_{t|t} \Phi + Q^* \right)^{-1} \left( \xi_{t+1}^* - \Phi^* \xi_{t|t} \right), \]

\[ P_{t|t,\xi_{t+1}^*} = P_{t|t} \Phi \left( \Phi^* P_{t|t} \Phi + Q^* \right)^{-1} \Phi^* P_{t|t}. \]

Given an arbitrary starting value \( \xi_{0|0} \), we draw \( \xi_t \) from

\[ \xi_t \sim \begin{cases} 
N \left( \xi_{T|T}, P_{T|T} \right) & \text{for } t = T \\
N \left( \xi_{t|t,\xi_{t+1}^*}, P_{t|t,\xi_{t+1}^*} \right) & \text{for } t = T - 1, \ldots, 1 
\end{cases} \]

### A.3 Drawing \( \Lambda \) conditional on \( \Psi_{-\Lambda} \)

Given the independent normal prior, \( N \left( m_0, M_0 \right) \), and the fact that we have assumed the variance-covariance matrix \( \Omega \) is diagonal, we can individually sample the \( i \)th row of \( \Theta \). Let \( \tilde{F}_T = [F_1, \ldots, F_T] \) collect the time series of state vectors. Conditional on \( \tilde{F}_T \), and the innovation variance \( \Omega_{ii} \), the \((i,i)\) element of \( \Omega \), the conjugate posterior distribution for the \( i \)th row of \( \Lambda \) is \( N \left( m_i, M_i \right) \), where

\[
M_i = M_{i0} + \Omega_{ii}^{-1} \tilde{F}_T \tilde{F}_T^T, \\
m_i = M_i^{-1} \left( m_{i0} M_{i0}^{-1} + \Omega_{ii}^{-1} \tilde{F}_T \tilde{F}_T^T \tilde{X}_{iT} \right),
\]

and \( \tilde{X}_{iT} = [X_{i1}, \ldots, X_{iT}]^T \). The prior explicitly imposes the zero restrictions discussed above.

### A.4 Drawing \( \Omega \) conditional on \( \Psi_{-\Omega} \)

Conditional on the set of factor loadings, the factors, and the macro variables, each \( X_{it} \)'s innovation variance can be drawn from as \( \Omega_{ii}^{-1} \sim \Gamma \left( \nu_i, \delta_i \right) \), where
\[ \nu_i = \nu_0 + \frac{1}{2}T, \]
\[ \delta_i = \delta_0 + \frac{1}{2} \varepsilon_i T, \]

and \( \bar{\varepsilon}_{iT} \) are the stacked fitted residuals for the \( i \)th series.

### A.5 Drawing \( A \) and \( B | A \) conditional on \( \Psi_A \)

To implement the draw for each of these columns, define \( c_i \) and \( d_i \) such that \( a_i = U_i c_i \) and \( b_i = V_i d_i \), where \( U_i \) and \( V_i \) are orthonormal rotation matrices that impose the identifying restrictions placed upon \( A \) and \( B \). Considering the normal prior for \( a_i \) and \( b_i \), the marginal posterior pdf of \( c_i \) and \( d_i \) take the form:

\[
p\left(c_1, \ldots, c_{M+N+2} | \bar{Y}_T, \bar{X}_T, \bar{Z}_T \right) \propto | \text{det} [U_1 c_1 | \ldots | U_{M+N+2} c_{M+N+2}] |^T \exp \left(-\frac{T}{2} \sum_{i=1}^{M+N+2} c_i S_i^{-1} c_i \right) \quad (6)
\]

\[
p \left(d_i | c_i, \bar{Y}_T, \bar{X}_T, \bar{Z}_T \right) = \phi \left(P_i c_i, H_i \right), \quad (7)
\]

where \( H_i, P_i, S_i \) are the parameters of the posterior based upon the transformations described in Waggoner and Zha (2003) and \( \phi \) is the pdf of the Normal distribution with mean \( P_i c_i \) and variance \( H_i \). The Gibbs sampler sequentially draws each \( c_i \), conditional on the other \( c_j \), \( j = 1, \ldots, i - 1, i + 1, \ldots, M + N + 2m \). Once we obtain a draw for \( c_i \), and thus \( a_i \), we use (7) to draw \( d_i \) and compute \( b_i \). Upon completion of the draws for \( a_i \) and \( b_i \) \( \forall i \), we construct the matrices \( A \) and \( B \) and convert them to the reduced-form expressions \( \Phi = A^{-1} B \) and \( \Sigma = (A' A)^{-1} \) for use in the Kalman filter.