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Authors	Pere Gomis-Porqueras, and Christopher J. Waller
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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Optimal Taxes Under Private Information: The Role of Inflation*

Pedro Gomis-Porqueras
Deakin University

Christopher Waller
Federal Reserve Bank of St. Louis
and Deakin University

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Abstract

Once explicit information available to the taxing authority and incentives associated with private information problems are explicitly taken into account, only a limited set of policy instruments are consistent with agents not engaging in side trades. Within this spirit, we study the role of taxes and inflation in minimizing the dead-weight loss associated with the provision of good incentives and other market imperfections when fiat money is essential. When the government is able to observe an agent's age, consumption and efforts in all markets, the constrained efficient allocation can be achieved with affine history-dependent consumption taxes. Thus, monetary policy is redundant. In contrast, when the government observes sales, rather than actual consumption, a tax arbitrage exists. Uniform affine consumption tax rates prevent side trades and a deviation from the Friedman rule is optimal. This is the case as it helps induce truth-telling and alleviate a pecuniary externality. However, optimal monetary and fiscal policies in this setting cannot implement the constrained efficient allocation.

JEL Codes: E40, H21
Keywords: Fiat Money, Taxation, Private information

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1 Introduction

The standard theory of optimal taxation posits that a tax system should be chosen to maximize a social welfare function, while taking into account private agents' behavior. Within this spirit, Mirrlees (1971) emphasized the importance of explicitly taking into account the informational asymmetries between different tax payers and the fiscal authority. This is the case as optimal tax schemes generically involve redistribution, which relies on individuals' information. However, private agents have an incentive to conceal information in order to reduce their tax liabilities. Once these incentives problems are recognized, it is clear that only a limited set of fiscal instruments are coalition proof.[1] Thus, given a level of enforcement, what can be observed by the taxing authority is key in restricting the policy instruments that prevent side trades between different tax payers. When taking these important considerations into account in an environment where fiat money is essential, the goal of this paper is to design monetary and fiscal policies that minimize the dead-weight loss associated with the provision of good incentives.[2] In particular, we want to provide answers to the following questions. When the taxing authority faces private information problems, what sort of tax schemes can reproduce the constrained efficient social planner's allocation? Is the inflation part of the optimal tax plan? When the government has a more restricted information set, are positive nominal interest rates part of the optimal tax plan? In this scenario, does the inflation tax lead to constrained efficiency?

To answer these questions, we enrich Zhu's (2008) monetary environment by considering asymmetric information and different government information sets. Agents live for three periods: young, middle-aged and old. Agents can work when young and middle-aged, but retire when old. Over their lifetime, agents trade sequentially two perishable goods in two different competitive markets. Young and old agents trade a perishable good in a frictionless market, where the government can enforce taxes and transfers. In contrast, the other perishable good is only traded by middle-aged agents. Moreover, in this frictional market, middle-aged agents experience shocks, making them either producers or consumers with different marginal utilities. As in Mirrlees (1971), the realization of these shocks is private information. Moreover, the government can not enforce any taxes/transfers in this market. Finally, middle-aged agents are also anonymous to each other and face limited commitment. These frictions make fiat money an essential medium of exchange.[3] As a result, in this frictional market, trade occurs only when fiat money is exchanged for goods.

Within this environment, we examine how different government information sets affect the

[1] We refer to Hammond (1971, 1981) for more on this important observation.

[2] Essentially of money refers to a situation where its existence increases the consumption possibilities of agents.

[3] These additional frictions and shock structures make credit not incentive compatible. These features of the environment are absent in Golosov et al. (2006) and the majority of papers in the new public finance literature.

optimal tax plan. In particular, conditional on what can be observed by the taxing authority, we restrict attention to policy instruments that are coalition proof. In the rich information set, the government observes an agent's age, savings and all trading information when participating in goods and financial markets. This allows the government to implement affine history-dependent consumption taxes and age specific lump-sum taxes. However, in the more realistic scenario, where government can observe an agent's age, individual *purchases* and old agent's wealth, the previous tax scheme is not implementable.[4] In this more restricted informational environment, only affine uniform taxes that are neither type nor history-dependent are coalition proof. Otherwise, agents that face the lowest after-tax prices would purchase the goods on behalf of other agents and subsequently split the goods and consume them, obtaining higher welfare.

In this paper we find three two of wedges. This is the case regardless of the government's information set that is considered. One of them arise when the private information between tax payers and the taxing authority exist. The first wedge is the inter-temporal one, which has been extensively analyzed by new dynamic public finance literature, as in Kocherlakota (2005) and Golosov and Tsyvinski (2006), among others. This inter-temporal wedge takes into account the various consumption/production margins that middle-aged agents face under private information that need to consider when trading in the frictional market.[5] Finally, our environment has also an additional wedge that is a direct consequence of market imperfections not arising from informational problems. In particular, when making their individual portfolio choices, agents ignore the general equilibrium effects they have on equilibrium prices.[6]

The social planner's optimal stationary allocation in our environment is one where he creates old-aged consumption risk in order to induce middle-aged agents to produce.[7] To decentralize such allocation in the rich information set, a large set of taxes is available to the government. When the taxing authority implements age and history-dependent lump-sum taxes and history-dependent linear consumption taxes, it can correct all wedges. Thus, this tax scheme delivers the social planner's optimal stationary allocation. As a result, monetary policy is a redundant instrument. This is the case as the government can always re-adjust the old consumption tax rate to correct

[4]It is important to highlight that the monitoring requirements needed for the government to observe actual consumption are quite large. In practice, what government observe are sales in formal markets but no who consumes the purchased goods or services.

[5]This is a unique feature of our environment not present in the new dynamic public finance literature.

[6]The real environment with private information of Golosov and Tsyvinski (2007) also emphasizes the additional distortionary aspect of taxation. These authors show that while the markets can provide a significant amount of insurance, there is still a role for welfare improving distortionary fiscal taxes or subsidies to correct an externality that arises in the dynamic provision of insurance.

[7]An alternative approach would be to make money a part of the environment that gives agents an outside option to trade, as in Aiyagari and Williamson (2000). Since money creates a better outside option for agents and thus makes the planner's problem more difficult, a planner would never introduce money if it was under his control.

for the inter-temporal wedge. In contrast, in the more realistic government's information scenario, only affine tax functions that are neither type nor history-dependent are coalition proof. These uniform linear consumption taxes cannot alter all relevant wedges. Thus, a constrained efficient allocation can not be replicated. Other than serving as a medium of exchange and store of value, fiat money also conveys some information about past trades. However, in contrast to Kocherlakota (1998), fiat money is not equivalent to perfect memory. Note that by simply observing real balances when old, the government cannot infer in which market the agent has produced. We also find that a deviation from the Friedman rule is optimal. This is the case as it can help induce truth-telling and alleviate the pecuniary externality by reducing old-age consumption volatility.

The rest of the paper is organized as follows. Section 2 provides a review of the existing literature, highlighting our contribution to previous work. Section 3 describes the economic environment and Section 4 solves the social planner's problem. Section 5 characterizes the monetary equilibrium under the various government information sets and Section 6 offers some concluding remarks.

2 Literature Review

This paper connects with two different strands of literature. One where monetary and fiscal policy have been analyzed in environments where agents face wealth effects and fiat money is essential. The other one studies optimal monetary and fiscal polices when agents face private information problems.

The literature so far has considered two frameworks that incorporate wealth effects in environments where fiat money is essential. One approach is that of Zhu (2008), who combines features of overlapping generation models (OLG) with those search frameworks along the lines of Lagos and Wright (2005). In Zhu (2008) agents do not know their type when they can accumulate assets and agents trade fiat money for perishable goods in stochastic and bilateral trades. In terms of policy instruments, the government only considers golden-rule rate monetary transfers. Within this environment, Zhu (2008) finds that the Friedman rule can be sub-optimal, for some parameters, as it can make saving relatively cheap. As a result, it reduces the sellers' willingness to produce in the frictional and decentralized market. Within the same framework, Hiraguchi (2017) extends Zhu (2008) by introducing capital, which can not be used as a medium of exchange, and shows that the Friedman rule remains sub-optimal. Using a different OLG search model where fiat money and savings are essential, Altermatt (2019) shows that an optimal policy is one away from the Friedman rule and where fiscal policy increases the amount of outstanding government debt.[8]

[8]Using a similar environment Altermatt et al. (2020) when capital investment along fiat money co-exists, the authors find that the Friedman rule is optimal if and only if there is no Mundell-Tobin effect.

Within the same spirit, Huber and Kim (2020) show that the Friedman rule can be sub-optimal if old agents face a higher disutility of labor than young agents. When agents are infinitely lived, Rocheteau et al. (2018) consider households facing a binding labor supply within the Lagos and Wright (2005) framework. To implement policy, the authors assume that the government has only access to uniform lump-sum taxes. Within this environment, Rocheteau et al. (2018) show that the distribution of money holdings is not degenerate and that a one-time increase in the money supply can be welfare improving.

The robust finding of all these papers is that when wealth effects are possible, a deviation from the Friedman rule is optimal.[9] Relative to these, we analyze when and how monetary policy can induce truth-telling and help mitigate additional distortions, while considering different government information sets. Deviations from the Friedman rule are optimal when the government uses uniform affine consumption taxes in order to avoid side trades between private agents.

This paper also relates to the literature that studies optimal policy design under private information. The robust result emerging from the static and real Mirrleesian literature is that non-linear taxes tend to be optimal and can lead to constrained efficient allocations.[10] When dynamic real economies are considered, optimal taxation depends on the income histories of individuals and requires interactions between different types of taxation, such as taxes on capital and labor.[11] However, much less work has analyzed how non-linear taxes shape optimal monetary policies. Notable exceptions are the work of da Costa and Werning (2008) and Gahvaria and Micheletto (2014), who consider a money-in-the-utility framework and analyze optimal monetary and fiscal policies.[12] Other than money being a store of value and an object that yields direct utility, da Costa and Werning (2008) additionally assume that real balances and work effort are complements. Thus, high productivity agents have incentives to exert less effort than is socially optimal. The authors show that the Friedman rule is Pareto efficient when combined with a non-decreasing labor income tax.[13] Within the same environment, Gahvaria and Micheletto (2014) allow for tax evasion. When agents have access to a misreporting technology, monetary policy becomes another useful instrument for redistribution. A deviation from the Friedman rule is optimal. The authors also show

[9] In environments where fiat money is not essential, a deviation from the Friedman rule is typically neither optimal nor efficient. For overlapping generation models we refer to Abel (1987) and Freeman (1987), among others. For representative agent models, Chari, Christiano and Kehoe (1996).

[10] Green (1987) and others have shown, when shocks are i.i.d., the constrained-efficient allocation can be implemented by a mechanism that is recursive in promised utilities.

[11] Key recent references in this literature that study real economies include Golosov, Kocherlakota, and Tsyvinski (2003), Albanesi and Sleet (2006), Kocherlakota (2005), Golosov and Tsyvinski (2006), and Golosov, Tsyvinski, and Werning (2006), just to name a few.

[12] The social planner's problem is not well defined as an element of the allocation contains market prices.

[13] When money and effort are complements, the deviating and non-deviating agents share the same before- and after-tax income, but the one under-reporting productivity requires less work effort and demands the same or less money.

that even in the absence of tax evasion, optimality requires differential commodity taxation.

Relative to the private information literature, we consider life cycle features and various perishable goods. The fact that old agents cannot partially insure themselves by working is an important aspect that is typically not found in the new public finance literature. This is not an innocuous, as the social planner has to trade off efficiency versus insurance. In addition to the private information problem, our environment has also agents that are anonymous and face limited commitment in frictional markets.[14] As a result, the only way agents trade in these markets is if fiat money is exchanged for goods. In contrast to da Costa and Werning (2008) and Gahvaria and Micheletto (2014), money in our environment is a pure fiat object and plays three different roles. First, fiat money serves as a store of value. Second, fiat money is a useful medium of exchange, enlarging the set of feasible trades. Third, when the government can only observe purchases, fiat money also conveys some useful information about past trades.

3 Environment

The basic environment builds on the overlapping generations and matching framework of Zhu (2008) by incorporating a private information problem between agents and the taxing authority. We also consider a different trading protocol and different government information sets.

Our environment is one where time is discrete and a continuum of agents of unit mass live for three consecutive periods, while discounting at rate $\beta \leq 1$. In the first stage of their life cycle, agents are labeled as young. In the second stage, they are referred to as middle-aged, and in the last one they are labeled as old. A new generation of agents is born every other period. So at time t a new generation of young agents are born and trade with old agents in a competitive frictionless market. Consequently, all trade in this market is *inter-generational*. Then in $t+1$ the old die and young agents become middle-aged. During this stage of their lives, these agents trade among themselves in a frictional competitive market. Thus, this trade is *intra-generational*. Then at $t+2$ these middle-aged agents become old and trade with the next generation of young agents. From now on, we refer to the market where young and old trade as CM and the market where middle-aged agents trade among themselves as DM.

Agents can work when young and middle-aged, but retire when old. Inter-temporal contracts are not feasible between the young and old, since the old die immediately after trade. Middle-aged agents can potential engage in inter-temporal contracts with each other. However, middle-aged

[14] These additional frictions and shock structures are absent in Golosov and Tsyvinski (2006) and the majority of papers in the new public finance literature.

agents agents are "anonymous" to each other.[15] Anonymity eliminates credit between middle-aged agents and makes a medium of exchange essential for trade. Note though that there is another dimension of anonymity that has to do with the taxing authority.[16] It is important to highlight that having the government observe trades in frictional markets, where private agents do not know the identity of their trading partner, is not inconsistent with fiat money being a useful medium of exchange.[17]

Production Technologies: At the beginning of each CM, young agents have a labor endowment and access to a linear production technology that converts one unit of labor into one unit of CM goods. In contrast, old agents cannot produce CM goods.[18] Finally, depending on the realization of shocks, which we describe below, some middle-aged agents can produce the other perishable good, while others derive utility from consuming it. This good is also produced with a linear technology that converts one unit of labor into one unit of DM goods.

Preferences and Shocks: Agents derive utility from consuming in CM and DM and experience disutility from effort. Let $U(C)$ denote old and young utility from consuming C units of CM goods and $-\upsilon(h)$ represent young disutility of exerting effort, h, when producing CM goods. Preferences satisfy standard concavity assumptions and Inada conditions.

At the beginning of their middle-aged period, agents experience shocks that determine whether they will be DM-consumers or DM-producers. With probability $1-\kappa$, a middle-aged agent can produce DM goods, but does not get utility when consuming them. We refer these agents as DM-producers, who experience disutility $-\psi(q)$ when producing q units of DM goods. We assume that $\psi'(q) > 0, \psi''(q) \geq 0$. In contrast, with probability κ, a middle-aged agent can obtain utility from consuming DM goods, but is unable to produce them. In addition to this preference shock, these agents face another shock, which we denote by ϵ_j. This shock determines the marginal utility of consuming q units of DM goods. The payoff of an agent that has an ϵ_j-shock is then $\epsilon_j u(q)$, where $j = \{L, H\}$, $\epsilon_H > \epsilon_L$, $u'(q) > 0$ and $u''(q) \leq 0$. The shock ϵ_j has an associated probability p_j that satisfies the monotone hazard rate property. Finally, all shocks are independent across agents and their realization is private information.[19] However, the shock structure and the probability

[15] This is a standard and key feature of New Monetarist models of fiat money.

[16] This is an aspect much less explored in New Monetarist models of fiat money.

[17] In order for fiat money to not be essential in an environment where private agents are anonymous and cannot send public signals, the government would also need to have the ability to tax and redistribute DM goods as well as induce production. These enforcement powers are not available to the government in this frictional market.

[18] This is an important aspect as access to future labor income can be used to smooth shocks during middle age. This feature is not present in Golosov et al. (2006) or most papers in the new public finance.

[19] These features are in sharp contrast to disability insurance framework of Golosov et al. (2006), where agents can always consume and have the same marginal value of consumption in all states of the world. Moreover, the

distribution are common knowledge.

Taking into account all pay-offs and shocks, an agent born at time $t-1$ has the following lifetime utility:

$$W_{t-1} = U\left(C_{y,t-1}\right) - \upsilon\left(h_{t-1}\right) + \beta\, V_t \tag{1}$$

where C_y denotes young consumption and V_t corresponds to the expected value function when becoming middle-aged, which is given by:

$$V_t = \kappa \sum_j p_j\, \epsilon_j\, u\left(q_t^{\epsilon_j}\right) - (1-\kappa)\, \psi\left(q_t^s\right) + \beta\left[\kappa \sum_j p_j\, U\left(C_{o,t+1}^{\epsilon_j}\right) + (1-\kappa)\;\; U\left(C_{o,t+1}^{s}\right)\right] \tag{2}$$

where q^{ϵ_j} is the consumption of an ϵ_j DM-consumer, q^s denotes a seller's DM production and C_o^ℓ represents old-aged consumption, which depends on the DM agent's type ℓ, where $\ell = \{\epsilon_j, s\}$ with $j = \{L, H\}$.[20]

Government: A benevolent government issues fiat money, the only durable object in the economy, and can collect taxes and provide transfers. These fiscal powers, however, are only enforceable in the frictionless centralized market, CM, where young and old trade.[21] As in Mirrlees (1971), the government can not observe the realization of agents' DM preference shocks.

The government information set is key in determining optimal monetary and fiscal policies.[22] This is the case as redistribution among individuals relies upon individuals' information. Thus, private agents have an incentive to conceal their tax liabilities. Once these incentives problems are recognized and the government information set is explicit, only a limited set of fiscal instruments can be implemented, ruling out side trades. To highlight this, we consider different government information sets. In the rich information environment, the taxing authority observes an agent's age, savings and all trading information when participating in goods and financial markets.[23] We also consider a more realistic and poorer information environment, where the government can only observe *purchases*, rather than agents' consumption, and old agents' wealth. In this new setting, the fiscal instruments used in the rich information set allow for tax arbitrages. Thus, the set of

shock structure and the additional frictions in DM make fiat money an essential medium of exchange.

[20]Note that in this environment when in CM, being able to know the agent's DM type is equivalent to learning about their history.

[21]In DM, as in Gomis-Porqueras et al. (2014), agents require fiat money to trade and the government cannot enforce taxes, which can be viewed as an informal market.

[22]We refer to Mirrlees (1971), Hammond (1971, 1981), among others, for more details.

[23]It is important to highlight that having the government observe trades in frictional markets is not inconsistent with fiat money being a useful medium of exchange. In order for fiat money to not be essential in an environment where private agents are anonymous and cannot send public signals, the government would also need to have the ability to tax, redistribute DM goods and induce voluntary trade.

feasible fiscal instruments consistent with agents truthfully revealing their DM type is restricted.

4 Social Planner's Problem

Consider a situation where the social planner can commit to deliver any proposed consumption plan to agents. Moreover, he knows an agent's age and all DM and CM quantities produced and consumed. The social planner, however, can not observe the realization of shocks experienced by middle-aged agents, relying on their report. Then, conditional on an agent's message, the planner can provide a sequence of DM and CM consumption as well as CM and DM effort to be exerted that maximizes social welfare. Since we focus on stationary equilibria, we consider a situation where the planner only cares about steady state allocations. The corresponding welfare function is then given by:[24]

$$\begin{aligned}\mathcal{W} = U\left(C_y\right) - v\left(h\right) + \beta\left[\kappa \sum_j p_j\, \epsilon_j\, u\left(q^{\epsilon_j}\right) - (1-\kappa)\, \psi\left(q^s\right)\right] + \\ +\beta^2\left[\kappa \sum_j p_j\, U\left(C_o^{\epsilon_j}\right) + (1-\kappa)\ U\left(C_o^s\right)\right] .\end{aligned} \tag{3}$$

When proposing the consumption and effort plans to agents, the social planner also needs to take into account the overall resources in the economy, which requires that:

$$(1-\kappa)\, q^s \geq \kappa \sum_j p_j\, q^{\epsilon_j} \tag{4}$$

$$h \geq \kappa \sum_j p_j\, C_o^{\epsilon_j} + (1-\kappa)\ C_o^s + C_y\ . \tag{5}$$

Under full information the social planner's allocation would be one where agents face no consumption risk in CM. However, when the realization of shocks is private information, such an allocation cannot be sustained. This is the case as middle-aged DM-producers obtain the same old age consumption as everyone else. Thus, middle-aged DM-producers are not compensated for their DM effort. As a result, they have an incentive to misrepresent their type. Similarly, the planner has to consider the possibility that DM-consumers misrepresent their type. To ensure truth-telling, the social planner's allocation also has to consider additional incentive constraints

[24] Note that this problem is equivalent to one where the planner maximizes lifetime utility of a cohort born into the stationary equilibrium.

for all DM-consumers and DM-producers. These are given by:

$$\epsilon_j\, u\left(q^{\epsilon_j}\right) + \beta\, U\left(C_o^{\epsilon_j}\right) - \left[\epsilon_j\, u\left(q^{\epsilon_k}\right) + \beta\, U\left(C_o^{\epsilon_k}\right)\right] \geq 0 \quad \forall\ \ k \neq j \tag{6}$$

$$-\psi\left(q^s\right) + \beta\, U\left(C_o^s\right) - \beta\, U\left(C_o^{\epsilon_j}\right) \quad \geq 0\ \ \forall\ j. \tag{7}$$

It is important to highlight that condition (6) ensures that an ϵ_j DM-consumer does not find it profitable to misrepresent his type by mimicking up or down. Finally, equation (7) ensures that DM-producers have no incentive to misreport their type by claiming to be a DM-consumer of type ϵ_j. Note that inequality (7) captures the fact that the seller always has access to free disposal of DM goods and does not get utility from consuming them. It is worth highlighting that DM-consumers cannot claim to be DM-producers. This is the case as they cannot produce DM goods. Thus, these incentive constraints are not needed.

In general, there is also a participation constraint on the young that needs to be considered. This will ensure production in CM and DM, rather than young agents producing only for themselves. Suppose a young agent decides not to participate in the social planner's scheme. A planner can then impose zero consumption when old. The participation constraint for the young is then given by:

$$W^a \leq U\left(\bar{C}_y\right) - \upsilon\left(\bar{h}\right) + \beta\ \left[\kappa\ \sum_j p_j\ \epsilon_j\ u\left(\bar{q}^{\epsilon_j}\right) - (1-\kappa)\ c\left(\bar{q}^s\right)\right] + \beta^2\ \left[\kappa\ \sum_j p_j\ \ U\left(\bar{C}_o^{\epsilon_j}\right) + (1-\kappa)\ \ U\left(\bar{C}_o^s\right)\right]$$

where the bars denote the social planner's optimal allocation and W^a represents the value of a young deviating agent. This value is given by:

$$W^a = U\left(C_y^a\right) - \upsilon\left(C_y^a\right) + \beta^2\, U\left(0\right)$$

where C_y^a solves the following implicit equation $U'\left(C_y^a\right) = \upsilon'\left(C_y^a\right)$. Given our Inada assumptions on $U\left(x\right)$, the young's participation constraint is always satisfied.

Having considered all relevant constraints, we can formally describe the constrained planner's problem as follows:

$$\max_{\left\{C_y, h, q^{\epsilon_j}, C_o^{\epsilon_j}, q^s, C_o^s\right\}} \mathcal{W}\ \ s.t.\ (4), (5), (6) \text{ and } (7). \tag{8}$$

To simplify exposition, throughout the rest of the paper, we assume that there is equal probability to be a DM buyer or seller, κ=0.5, and preferences are such that $u(x) = U(x) = \ln x$ and

$\psi(y) = v(y) = y$.[25] With this specification, we can establish the following result. All proofs can be found in Appendix A.

Proposition 1 *At most one allocation solves the constrained social planner's problem.*

To induce truth-telling, the planner's optimal allocation is one where the high marginal DM-consumer does not pretend to be a low marginal utility DM-consumer, a seller is indifferent between producing and claiming to be a low DM-consumer and a low DM-consumer is indifferent between his own type and the high one. This allocation delivers more CM consumption to agents that have been DM-producers and the least amount of CM goods to the high marginal utility DM-consumers. Such scheme creates old CM consumption risk, which is required to induce middle-aged agents to produce in DM.

To show that the parameter space consistent with Proposition 1 is non-empty, we provide a numerical example. Consider an economy where each period represents 20 years, which implies a discount rate equal to β= 0.6676.[26] Let us further assume that the marginal preference shocks are such that ϵ_H= 3 and ϵ_L= 1.4, with a probability of experiencing a low marginal DM utility shock being p_L= 0.85. Under this parametrization, it is easy to compute the corresponding planner's optimal and unique allocation. This and the allocation under full information are reported in Table 1.

	Private Info	Full Info
C_y	1	1
C_o^s	0.736	0.6676
$C_o^{\epsilon_2}$	0.1	0.6676
$C_o^{\epsilon_1}$	0.166	0.6676
q^{ϵ_2}	1.16	3
q^{ϵ_1}	0.966	1.4

Table 1: Planner's Optimal Allocation

As stated in Proposition 1, the planner creates old CM consumption risk. DM consumption is increasing in the size of the shock. As we can see from Table 1, middle-aged high marginal utility agents consume 20% more than the low ones. In contrast, when old, the highest marginal DM buyer consumes 40% less.

[25] In previous work, for instance, Kocherlakota (2005), Golosov, Kocherlakota and Tsyvinski (2003), and Golosov and Tsyvinski (2006), among others, authors assume the existence of an optimal social planner's allocation and then design a tax plan that implements such allocation.

[26] This corresponds to an economy where the young generation starts from 20 to 40, middle-aged from 40 to 60 and old-age from 60 to 80.

5 Monetary Economy

We now examine whether a benevolent government can replicate the constrained planner's optimal allocation under different information structures. In the decentralized environment, the government issues fiat money, the only durable object in the economy, which cannot be counterfeited. The stock of money grows by lump-sum injections given to the young after all trades have taken place and before the realization of middle-aged shocks are revealed. As it will be shown, since all young agents leave the CM with the same amount of fiat money, the lump-sum injection has equal value to all middle-aged agents. This transfer scheme eliminates welfare gains from inflation due to a non-degenerate distribution of money balances within members of the same generation.[27] Consequently, we can better isolate the effects of private information on the design of monetary policy. From here on, the t subscript is suppressed for notational ease. Moreover, we have that $+1$ denotes $t+1$ and so on.

Since monetary injections occur every other period, fiat currency evolves according to $M_{+2} = \gamma M$, where M_{+2} denotes the money supply at time $t+2$, and $\gamma = 1+\pi$ denotes the gross growth rate of the money supply from t to $t+2$ and π is the net inflation rate from t to $t+2$. Lump-sum monetary transfers, which we denote by T_{MA}, are given to young after all trades have taken place and before the realization of middle-aged shocks are revealed. Thus we have that:

$$\phi(M - M_{-2}) = T_{MA}$$

where T_{MA} denotes the corresponding lump-sum monetary transfers and ϕ is the good's price of money in terms of the CM good.[28]

To illustrate the money injection, assume that at time $t-1$ a young agent is in CM. Thus, at period t he will be trading in DM, and in period $t+2$ he will be trading in CM when old. The timing of money injection implies the following money stocks:

$$\begin{aligned} at \;\; t-1: &\quad M_{t-1} \\ at \;\; t: &\quad M_t = M_{t-1} + \tau M_{t-1} \\ at \;\; t+1: &\quad M_{t+1} = M_t. \end{aligned}$$

It then follows that the gross real rate of return on money from the CM at time t to the CM in $t+2$ is $R_m = \phi_{+2}/\phi$.

[27] We refer to Levine (1991), Molico (2006), Berentsen, Camera and Waller (2005), Zhu (2008) and Rocheteau et al. (2018), just to name a few, for the redistributive properties of inflation.

[28] When the money growth rate is positive the central bank gives out fiat money; when is negative it takes it away from agents and removes it from circulation.

It is important to emphasize that in our environment we find two different concepts of constrained efficiency.[29] The first notion is the one used by the Mirrleesian literature. This is a direct consequence of having a private information problem. In such settings, the social planner takes into account incentive compatibility constraints when proposing allocations. The other notion of constrained efficiency is the one that comes from the incomplete markets literature.[30] When agents make their consumption plans, the social planner takes into account trading frictions and market imperfections to induce desired allocations.

In what follows, given a government's information set, we design monetary and fiscal policies in order to minimize the dead-weight loss associated with the provision of good incentives and other market imperfections.

5.1 Rich Information Set

We first consider a situation where the government can observe an agent's age, effort in DM and CM, young savings, consumption of all perishable goods and an agent's money holdings. This rich information set allows the government to implement a rich set of CM tax schemes.

Given the various observables available to the taxing authority, lump-sum taxes/transfers and non-linear consumption taxes that can be conditioned by age, type and individual history are implementable in CM.[31] The corresponding government's budget constraint, in terms of the CM good, is then given by:

$$T_y + \frac{1}{2}\left(T_o^s + \sum_j p_j T_o^j\right) = \frac{1}{2}\left(\sum_j p_j\, \eta\left(C_{o,-1}^{\epsilon_j}\right) + \eta\left(C_{o,-1}^{s}\right)\right) \tag{9}$$

where T_y (T_o^ℓ) represents lump-sum taxes/transfers of CM goods to young (to old, which can depend on their DM type, where $\ell = \{\epsilon_j, s\}$), and $\eta\left(C_{o,-1}^{\ell}\right)$ denotes CM consumption tax revenues collected from an old-agent who consumed $C_{o,-1}^{\ell}$ units of CM goods.[32] In addition to these fiscal tools, changes in the money supply can also be used as an additional policy instrument.

[29] We refer to Farhi and Werning (2012) for more on this issue.

[30] Diamond (1967) and Geanakoplos and Polemarchakis (1986) introduced the notion of constrained efficiency in environments with incomplete markets. We refer the reader to these authors for more on this issue.

[31] The use of age-dependent distortionary taxes has been studied in an overlapping generations framework by Erosa and Gervais (2002).

[32] The consumption tax on the old is collected in the form of consumption goods rather than in cash.

CM Problem

A young agent born at time $t-1$ chooses young consumption, CM effort and money holdings in order to maximize his expected lifetime utility. Formally, we have that:

$$\max_{C_{y,-1},h_{-1},m} \quad U\left(C_{y,-1}\right)-\upsilon\left(h_{-1}\right)+\beta\, V_t\left(m\right) \quad s.t. \quad C_{y,-1}+\phi_{-1}m=h_{-1}+T_y$$

where m denotes the amount of fiat money a young agent acquires in CM to be used in the ensuing DM. The corresponding first-order conditions yield the following:

$$U'\left(C_{y,-1}\right)= \ \upsilon'\left(h_{-1}\right) \tag{10}$$

$$\phi_{-1}\, U'\left(C_{y,-1}\right)=\beta\, V'\left(m\right). \tag{11}$$

Since optimal decisions are the same for all young agents, they leave CM with the same amount of real money balances.

Let us now consider the CM decision problem for old agents. Since they cannot produce CM goods, they use their real balances to fund CM consumption and pay their tax liabilities. As old agents die after trade takes place in CM, it is optimal for them to spend all of their wealth. In $t-1$, old agents come into the period with the entire money stock. In particular, we have that old consumption is given by

$$C^s_{o,-1}=T_s+\phi_{-1}m^s_{-1}-\eta(C^s_{o,-1})=T_s+\phi_{-1}m_{-2}+\phi_{-1}p_{t-2}q^s_{-2}-\eta(C^s_{o,-1})$$
$$C^{\epsilon_j}_{o,-1}=T_{\epsilon_j}+\phi_{-1}m^{\epsilon_j}_{-1}-\eta(C^{\epsilon_j}_{o,t-1})=T_{\epsilon_j}+\phi_{-1}m_{-2}-\phi_{-1}p_{-2}q^{\epsilon_j}_{-2}-\eta(C^{\epsilon_j}_{o,-1})$$

where m_{-2} is the amount of money they had at the start of the previous DM. In equilibrium we have that $M_{-3}+\tau M_{-3}=M_{-2}=M_{-1}$.

To fully characterize the CM problem, we need to determine how the expected utility of a middle-aged agent varies with money balances; i.e, $V'\left(m\right)$. To do so we need to solve the agent's DM problem.

DM Problem

At the end of the young-aged period, after CM trade has taken place, agents obtain their lump-sum monetary transfers. Then they experience idiosyncratic preference shocks, which are private information. After the various shocks are realized, middle-aged agents trade in a competitive market where DM goods are exchanged for fiat money. The expected utility of a middle-aged agent with m units of fiat money is given by:

$$V(m) = \frac{1}{2}\left\{\sum_j p_j\ \epsilon_j\ u\left(q^{\epsilon_j}\right) + \beta\ \sum_j p_j\ U\left(C_{o,+1}^{\epsilon_j}\right) - \psi\left(q^s\right) + \beta\ U\left(C_{o,+1}^{s}\right)\right\}$$

where q^{ϵ_j} and q^s represent the DM quantities consumed by an ϵ_j DM-consumer and by a DM-producer, respectively. Similarly, $C_{o,+1}^{\epsilon_j}$ and $C_{o,+1}^{s}$ represent the CM consumption of an ϵ_j DM-consumer and DM-producers, respectively. By feasibility, a DM-consumer cannot hand in more real balances than the ones he has brought into DM. As a result, a buyer faces the following constraint $pq^{\epsilon_j} \leq m$, where p is the nominal price of the DM good.

After the shock is realized, if an agent is an ϵ_j DM-consumer, his problem is given by:

$$\max_{q^{\epsilon_j}}\left\{\epsilon_j u\left(q^{\epsilon_j}\right) + \beta U\left(C_{o,+1}^{\epsilon_j}\right)\right\} \quad s.t.\ pq^{\epsilon_j} \leq m.$$

Optimality requires that:

$$\epsilon_j u'\left(q^{\epsilon_j}\right) - \frac{\beta U'\left(C_{o,+1}^{\epsilon_j}\right)}{1+\eta'\left(C_{o,+1}^{\epsilon_j}\right)}\phi_{+1}p - p\lambda^{\epsilon_j} = 0$$

where λ^{ϵ_j} corresponds to the cash constraint. Similarly, the DM-producer's problem is given by:

$$\max_{q^s}\left\{-\psi\left(q^s\right) + \beta U\left(C_{o,+1}^{s}\right)\right\}$$

resulting in the following first-order condition:

$$-\psi'\left(q^s\right) + \frac{\beta U'\left(C_{o,+1}^{s}\right)}{1+\eta'\left(C_{o,+1}^{s}\right)}\phi_{+1}p = 0.$$

In a stationary equilibria where the buyer's money constraint does not bind, the optimal DM-consumer and DM-producer choices imply the following conditions:

$$\frac{\epsilon_j u'\left(q^{\epsilon_j}\right)\left[1+\eta'\left(C_{o,+1}^{\epsilon_j}\right)\right]}{\beta U'\left(C_{o,+1}^{\epsilon_j}\right)} = p\phi_{+1} = \frac{\psi'\left(q^s\right)\left[1+\eta'\left(C_{o,+1}^{s}\right)\right]}{\beta U'\left(C_{o,+1}^{s}\right)} \tag{12}$$

$$\frac{\epsilon_j u'\left(q^{\epsilon_j}\right)\left[1+\eta'\left(C_{o,+1}^{\epsilon_j}\right)\right]}{\beta U'\left(C_{o,+1}^{\epsilon_j}\right)} = \frac{\epsilon_k u'\left(q^{\epsilon_k}\right)\left[1+\eta'\left(C_{o,+1}^{\epsilon_k}\right)\right]}{\beta U'\left(C_{o,+1}^{\epsilon_k}\right)}. \tag{13}$$

These wedges reflect the various consumption/production margins that agents need to consider when trading in the frictional market.[33] As we can see, marginal taxes (which are given by

[33]This is a unique feature of our environment not present in Kocherlakota (2005) and Golosov and Tsyvinski

$\eta'\left(C^{\ell}_{o,+1}\right)$) directly affect these wedges.

Having characterized the optimal DM responses, we can determine the marginal DM value of bringing an additional unit of fiat money. This is given by:

$$V'(m) = \frac{1}{2}\left\{\beta\sum_{\epsilon_j} p_{\epsilon_j}\frac{U'\left(C^{\epsilon_j}_{o,+1}\right)}{1+\eta'\left(C^{\epsilon_j}_{o,+1}\right)}\phi_{+1} + \beta\frac{U'\left(C^{s}_{o,+1}\right)}{1+\eta'\left(C^{s}_{o,+1}\right)}\phi_{+1}\right\}$$

which allows us to rewrite the young's first-order condition for effort and money holdings as follows:

$$U'(C_{y,-1}) = \upsilon'(h_{-1}) \tag{14}$$

$$U'(C_{y,-1}) = \frac{\beta^2}{2}\frac{\phi_{+1}}{\phi_{-1}}\left\{\sum_{\epsilon_j} p_{\epsilon_j}\frac{U'\left(C^{\epsilon_j}_{o,+1}\right)}{1+\eta'\left(C^{\epsilon_j}_{o,+1}\right)} + \frac{U'\left(C^{s}_{o,+1}\right)}{1+\eta'\left(C^{s}_{o,+1}\right)}\right\}. \tag{15}$$

As we can see from (15), the inter-temporal wedge is affected by monetary and fiscal policies. In particular, both the return on fiat money, $\frac{\phi_{+1}}{\phi_{-1}}$, and the different marginal taxes on old consumption, $\eta'\left(C^{\ell}_{o,+1}\right)$, impact the agent's inter-temporal decision.[34] The government has more instruments than wedges. Thus, monetary policy might be a redundant policy instrument.

Having characterized the optimal agent's decisions, we can define a stationary monetary equilibrium for this economy.

Definition 2 *Given monetary and fiscal policy instruments, a stationary equilibrium is an n-tuple* $\left\{h, C_y, C^{\ell}_o, q^{\epsilon_j}, q^s, \phi m^{\epsilon_j}, p\right\}$ *that satisfies equations (14), (15), (12) and (13), and all markets clear where* $\ell = \{\epsilon_j, s\}$ *and* $j = \{L, H\}$.

As in Kocherlakota (2005) and the dynamic contract literature, from now we focus on affine history-dependent consumption taxes. In particular, consider the following old consumption marginal tax rate $\eta'\left(C^{\ell}_{o,+1}\right) = \tau^{\ell}$, where τ^{ℓ} is constant and $\ell = \{\epsilon_j, s\}$.[35] It is easy to see that with these instruments, the government's optimal tax problem is differentiable for any old consumption profile. Moreover, a joint deviation, where an agent accumulates more real balances when young and misrepresents his type when middle-aged is never profitable.[36]

(2006), among others.

[34]As opposed to Golosov and Tsyvinski (2006), it is fiat money not capital that allows agents to store value across periods.

[35]Notice that in this environment the DM type and old history-dependent taxes are equivalent.

[36]Golosov and Tsyvinski (2006), Kocherlakota (2005) and Albanesi and Sleet (2006) analyze joint deviations in an environment with a poor information set. In our setting the government can observe all trades and young savings. Thus, the government can impose a tax structure when agents are young that rules out over saving. Under such scheme a young deviating agent would have zero old consumption. Given our Inada assumptions, deviating is never profitable.

When the planner's solution is unique, one can derive the optimal tax plan by imposing the planner's allocation into the decentralized solution to the corresponding first-order conditions and back out the relevant taxes.

Proposition 3 *Linear history-dependent consumption taxes and type-specific lump-sum taxes can deliver the constrained planner's optimal allocation. Under such tax scheme, the inflation tax is redundant.*

To satisfy the incentive constraints, the decentralized allocation needs to deliver the largest amount of CM goods to agents that have been DM-producers and the smallest to the ϵ_H DM-consumers. This is required to induce middle-aged agents to produce in DM and prevent ϵ_L DM-consumers from mimicking ϵ_H DM-consumers. The tax scheme in Proposition 3 is able to deliver the planner's constrained efficient allocation. Note that the optimal tax plan is such that the inflation tax is redundant. This is the case as the government is able to replicate the planner's allocation with different inflation rates by appropriately adjusting taxes on old consumption. This is the case as agents face three wedges (two corresponding to DM, equations (12) and (13), and the inter-temporal one, equation (15)), while the government has four instruments that can affect them. As in Diamond and Mirrlees (1978), the government uses distortionary taxation to obtain the desired planner's optimal marginal conditions and uses lump-sum taxes/transfers to get the consumption levels right to satisfy the feasibility constraints.

It is worth emphasizing that not all returns on fiat money, R_m, are feasible. Nominal interest rates have to be consistent with marginal tax rates not exceeding one hundred percent. Using the parametrization in the previous example, it is easy to compute the optimal tax scheme that delivers the social planner's optimal allocation. The corresponding taxes are reported in Table 2.

τ^H	0.8
τ^L	0.94
τ^s	-0.317
T_o^H	-0.011
T_o^L	0
T_o^s	-0.495

Table 2: Optimal Taxes when R_m=1.79

As we can see from Table 2, the optimal old linear consumption tax rate plan has a hump shape. The fiscal authority wants to subsidize the high marginal DM-consumers relative to the low types. This allows the old consumption profile to be more compressed. Finally, the government subsidizes the old CM consumption of DM-producers so more production takes place when middle-aged. To further reduce consumption volatility, the optimal lump-sum plans are such that the low marginal DM-consumers face the lowest lump-sum tax, while DM-producers face the highest.

5.2 A More Realistic Government Information Set

In practice the taxing authorities observe sales, but not who is actually consuming the purchased goods or services. Moreover, agents might be able to hide some savings from the taxing authority. These seem to be rather technical issues, but as we will show they have important implications for the design of optimal policies.

To emphasize these realistic features, we consider an environment where the government observes an agent's age, young effort, *purchases* of CM goods and old agent's wealth. It is important to highlight that if we consider type or history-dependent taxes, a tax arbitrage opportunity would exist. This is the case as agents that face the lowest after-tax prices would purchase the goods on behalf of other agents. After these have been made, agents would voluntarily divide the goods among themselves and consume them, increasing their welfare. Thus, nonlinear or history-dependent CM taxes are not coalition proof. To avoid any side trades, the government needs to restrict attention to uniform affine history-independent consumption taxes in CM.[37] In particular, it can use the following coalition proof taxes: (i) uniform lump-sum taxes/transfers ($T_y = T_o^\ell = T \ \forall\ell$), (ii) linear uniform CM consumption taxes ($\eta'\left(C_y\right) = \eta'\left(C_o^\ell\right) = \tau \ \forall\ell$), (iii) young labor taxes (τ^h) and (iv) the inflation rate (R_m), where $\ell = \{\epsilon_j, s\}$.[38] Under this new tax scheme, the young and old budget constraints are given by:

$$(1+\tau)C_{y,-1} + \phi_{-1}m = (1-\tau^h)h_{-1} + T$$

$$(1+\tau)C_{o,-1}^\ell = T + \phi_{-1}m_{-1}^\ell \qquad \text{for } \ell = \{\epsilon_j, s\}.$$

It is clear that uniform linear consumption taxes cannot affect the different DM wedges given by equations (12) and (13). Thus, the resulting decentralized equilibrium cannot replicate the planner's optimal allocation. Moreover, it is worth highlighting that fiat money is the only useful tool that can provide some information about past trades. However, in contrast to Kocherlakota (1998), fiat money is not equivalent to perfect memory. Note that by simply observing an old-agent's fiat money when entering CM, the government cannot infer what type of shock this agent has received when middle-aged. Observing these holdings when old only tells us that an agent has

[37] Hammond (1987) shows that the constraint imposed on the mechanism by the presence of side markets finds its counterpart in the tax system when restricting the taxation of goods traded in these markets to be linear.

[38] It is worth noticing that when uniform linear consumption taxes are in place, the government is able to determine an agent's actual consumption.

produced at some point in time, but not in which market. Thus, fiat money acts as an imperfect record-keeping device.

It is well known in the new dynamic public finance that when the government is able to observe individual's savings, a joint deviation is profitable.[39] To prevent this, the tax system needs to be enrich by conditioning tax rates by agent's wealth. In this environment, however, a joint deviation is not profitable. This is the case as deviating and non-deviating agents face exactly the same budget constraints. In other words, an agent that saves more and lies about his type, does not have a different set of taxes when compared to an agent that tells the truth.

Proposition 4 *Given uniform lump-sum taxes and uniform constant tax rates on CM consumption and effort, a deviation from the Friedman rule is optimal but does not deliver the constrained efficient allocation.*

It is worth highlighting that when making their individual portfolio choices, agents ignore the general equilibrium effect they have on equilibrium prices. In particular, agents over-estimate how much old CM consumption they will get from acquiring one more unit of real balances when young. As a result, they over-accumulate real balances. When richer fiscal instruments are available, this externality and incentive problems associated with the private information problem can be corrected without monetary policy. However, in this more realistic scenario this is not the case. Moving away from the Friedman rule is a useful policy instrument. By doing so, the government can generate some redistribution and reduce old-age consumption volatility. Having positive nominal interest rates are also useful in affecting the inter-temporal wedge, which is required to induce truth-telling. In this setting, monetary policy becomes an essential instrument as it can help minimize the dead-weight loss associated with both the provision of good incentives and the externality.

6 Conclusion

Once explicit information available to the taxing authority and incentives associated with private information problems are explicitly taken into account, only a limited set of fiscal instruments are consistent with private agents not engaging in side trades. Within this spirit, we study the role of inflation in minimizing the dead-weight loss associated with the provision of good incentives and other market imperfections. To do so, we consider an overlapping generations and matching framework where the government faces a private information problem between agents and

[39] A joint deviation is a situation where an agent saves more and misrepresent his type as in Kocherlakota (2005b) and Golosov and Tsyvinski (2006), among others.

the taxing authority. We then allow different government information sets, restricting the set of implementable taxes.

When the government has a rich information set, the taxing authority can implement age and history-dependent lump-sum taxes and history-dependent linear consumption taxes. This tax scheme delivers the social planner's optimal stationary allocation. As a result, monetary policy is a redundant instrument. However, when the taxing authority observes purchases rather than actual consumption, only affine tax functions that are neither type nor history-dependent are coalition proof. These uniform linear consumption taxes cannot alter all relevant wedges, inducing an inefficient allocation. In this scenario, a deviation from the Friedman rule is optimal. This is the case as it helps alleviate a pecuniary externality and ensures truth-telling as coalition proof fiscal policies cannot achieve the required redistribution.

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Appendix A

Proof of Proposition 1

A planner's allocation where a low marginal utility DM buyer is indifferent between telling the truth or claiming to be the higher type and a seller being indifferent between producing and claiming to be the low marginal DM utility buyer implies the following incentive conditions

$$\epsilon_L(u\left(q^H\right) - u\left(q^L\right)) = \beta(U\left(C_o^L\right) - U\left(C_o^H\right))$$

$$\psi\left(q^s\right) = \beta\left[U\left(C_o^s\right) - U\left(C_o^L\right)\right].$$

From the DM-producers constraint we can conclude that $C_o^s > C_o^L$ whenever there is trade in DM. Since $u(\cdot)$ and $U(\cdot)$ are increasing functions, in order to satisfy the low DM-consumer incentive it has to be the case that $q^H > q^L$ and $C_o^L > C_o^H$. Thus a necessary condition for optimality requires that $C_o^s > C_o^L > C_o^H$ and $q^H > q^L$ for the incentive constraints to be satisfied.

To determine the optimal allocation, we now examine the first-order conditions (FOC), which imply the following non-linear equations

$$\begin{aligned}
U'\left(C_y\right) &= \upsilon'\left(h\right) = \lambda_{CM} \\
u'\left(q^H\right) &= \frac{p_H \lambda_{DM}}{p_H \epsilon_H + p_H \lambda_1 \epsilon_H - p_L \lambda_2 \epsilon_L} \\
u'\left(q^L\right) &= \frac{p_L \lambda_{DM}}{p_L \epsilon_L - p_H \lambda_1 \epsilon_H + p_L \lambda_2 \epsilon_L} \\
\psi'\left(q^s\right) &= \frac{\lambda_{DM}}{1 + \lambda_s} \\
\beta^2 U'\left(C_o^H\right) &= \frac{p_H \lambda_{CM}}{p_H + p_H \lambda_1 - p_L \lambda_2} \\
\beta^2 U'\left(C_o^L\right) &= \frac{p_L \lambda_{CM}}{p_L - p_H \lambda_1 + p_L \lambda_2 - \lambda_s} \\
\beta^2 U'\left(C_o^s\right) &= \frac{\lambda_{CM}}{1 + \lambda_s}
\end{aligned}$$

where $\frac{\beta p_H}{2}\lambda_1$ ($\frac{\beta p_L}{2}\lambda_2$) and $\frac{\beta}{2}\lambda_s$ correspond to the Lagrange multipliers of the incentive constraints of the high (low) marginal utility DM-consumer not to misrepresent the low (high) type and the seller, respectively. $\frac{\beta \lambda_{DM}}{2}$ and $\lambda_{CM}/2$ represent the feasibility constraints for DM and CM, respectively.

It is easy to check that the only case that does not violate the various incentive constraints is the situation where $\lambda_s > 0$, $\lambda_2 > 0$ and $\lambda_1 = 0$. After repeated substitution, the planner's optimal solution is an allocation $\left\{h, C_y, C_o^H, C_o^L, C_o^s, q^H, q^L, q^s\right\}$ given by the solution to the following system of non-linear equations

$$U'(C_y) = \upsilon'(h) \tag{16}$$

$$\epsilon_L u\left(q^L\right) + \beta U\left(C_o^L\right) = \epsilon_L u\left(q^H\right) + \beta U\left(C_o^H\right) \tag{17}$$

$$\psi(q^s) = \beta\left[U(C_o^s) - U\left(C_o^L\right)\right] \tag{18}$$

$$q^s = p_H q^H + p_L q^L \tag{19}$$

$$h = C_y + \frac{1}{2}\left[p_L C_o^L + p_H C_o^H + C_o^s\right] \tag{20}$$

$$U'(C_y) = \left\{\frac{1}{2\beta^2}\left[p_L \frac{1}{U'\left(C_o^L\right)} + p_H \frac{1}{U'\left(C_o^H\right)} + \frac{1}{U'\left(C_o^s\right)}\right]\right\}^{-1} \tag{21}$$

$$\frac{\epsilon_H u'\left(q^H\right)}{\epsilon_L u'\left(q^L\right)} = \frac{1 + \frac{p_H}{p_L}\left[1 - \frac{U'(C_y)}{\beta^2 U'(C_o^H)}\right]}{1 - \frac{\epsilon_L}{\epsilon_H}\left[1 - \frac{U'(C_y)}{\beta^2 U'(C_o^H)}\right]} \tag{22}$$

$$\frac{\epsilon_L u'(q_L)}{\psi'(q^s)} = \frac{U'(C_y)}{\beta^2 U'(C_o^s)\left\{1 + \frac{p_H}{p_L}\left[1 - \frac{U'(C_y)}{\beta^2 U'(C_o^H)}\right]\right\}}. \tag{23}$$

Under our utility and disutility specifications, it is easy to see that $C_y = 1$. Moreover, the effort when young under full insurance is such that $h = 1 + \beta^2$. When the problem of private information exists, the split of old consumption over states changes but not the overall amount of resources produced. After repeated substitutions and imposing our functional forms, the optimal planner's allocation can be written in terms of old-aged consumption for the seller. In particular, the optimal solution solves the following fixed point equation in C_o^s

$$\epsilon_L \ln\left(\frac{\left[1 + p_L \exp^{\frac{-\bar{\epsilon}\beta}{C_o^s}}\right]\frac{\epsilon_L}{p_L}C_o^s - \frac{\epsilon_L}{p_L}\beta^2}{\beta^2\epsilon_H + \epsilon_L\beta^2\left(\frac{1+p_L}{p_H}\right) - \left[1 + p_L \exp^{\frac{-\bar{\epsilon}\beta}{C_o^s}}\right]\frac{\epsilon_L}{p_H}C_o^s}\right) - \frac{\bar{\epsilon}\beta^2}{C_o^s} = \beta \ln\left(\frac{\frac{1}{p_H}2\beta^2 - \left[1 + p_L \exp^{\frac{-\bar{\epsilon}\beta}{C_o^s}}\right]\frac{1}{p_H}C_o^s}{C_o^s}\right) \tag{24}$$

where $\bar{\epsilon} = p_L\epsilon_L + p_H\epsilon_H$.

Let us now define $LHS(C_o^s)$ and $RHS(C_o^s)$ as the functions corresponding to the left and right hand sides of equation (24), respectively. It worth noting that in order for an allocation to exist, the arguments inside the logarithm have to be positive. This restriction implies that the optimal old-aged consumption for sellers is bounded. From now on, we focus on economies where the underlying structural parameters are such that they satisfy the following condition

$$\frac{p_H\beta^2}{\epsilon_L}\left(\epsilon_H + \epsilon_L\frac{1+p_L}{p_H}\right) > 2\beta^2.$$

It then follows that an optimal C_o^s has to satisfy the following conditions

$$2\beta^2 \geq \left(1 + p_L \exp^{\frac{-\bar{\epsilon}\beta}{C_o^s}}\right) C_o^s \geq \beta^2.$$

For ease of notation, let us relabel C_o^s as x and define $\mathcal{F}(x) = \left(1 + p_L \exp^{\frac{-\bar{\epsilon}\beta}{x}}\right) x$. Let us define the lower and upper bounds for old-age seller's consumption as solutions to the following implicit equations

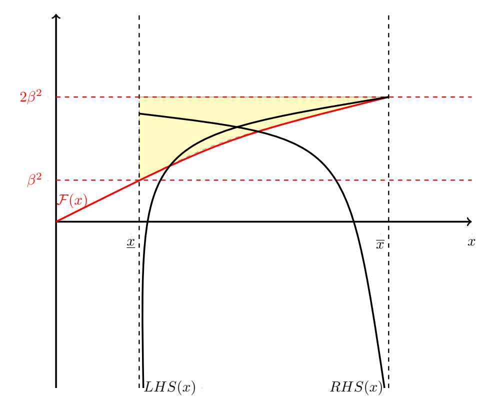
$$\beta^2 = \left(1 + p_L \exp^{\frac{-\bar{\epsilon}\beta}{\underline{x}}}\right) \underline{x}$$

$$2\beta^2 = \left(1 + p_L \exp^{\frac{-\bar{\epsilon}\beta}{\overline{x}}}\right) \overline{x}.$$

It is easy to check that $RHS(\underline{x}) > 0$ and $RHS(\overline{x}) \to -\infty$. Moreover, $RHS(x)$ is an increasing function of $x \in (\underline{x}, \overline{x})$. In contrast, we have that $LHS(x)$ is a decreasing function of $x \in (\underline{x}, \overline{x})$, $LHS(\underline{x}) \to -\infty$ and $LHS(\overline{x}) = \epsilon_L \ln\left(\frac{\epsilon_L}{p_L}\right) - \epsilon_L \ln\left(\epsilon_H - \epsilon_L\right) - \frac{\bar{\epsilon}\beta^2}{\overline{x}}$. Note that a necessary condition for existence is then that

$$2\beta^2 \geq LHS(\overline{x}) \geq \beta^2.$$

Thus under the previous parameter restrictions, the $LHS(\cdot)$ and $RHS(\cdot)$ at most will cross once, yielding a unique C_o^s when that occurs. The existence of the unique equilibria is illustrated below, where the yellow region depicts the parameter space where all inequalities are satisfied.



Proof of Proposition 3

Let us consider linear tax dependent old consumption taxes so that $1 + \eta'\left(C_o^L\right) = (1 + \tau^L)$, $1 + \eta'\left(C_o^H\right) = (1 + \tau^H)$ and $1 + \eta'\left(C_o^s\right) = (1 + \tau^s)$. Under such fiscal scheme, it is easy to check that under our preference assumptions, the corresponding decentralized stationary equilibrium is characterized by the following equations

1

$$
\begin{aligned}
\frac{1}{C_y} &= 1 \\
h &= \frac{1}{2}\left[p_H C_o^H + p_L C_o^L + C_o^s\right] + C_y \\
C_o^H &= \frac{\beta}{(1+\tau^H)(\beta+\epsilon_H)}\left[T_o^H + z\right] \\
C_o^L &= \frac{\beta}{(1+\tau^L)(\beta+\epsilon_L)}\left[T_o^L + z\right] \\
C_o^s &= \frac{1}{1+\tau^s}\left[T_o^s + T_o^H \frac{p_H\epsilon_H}{\beta+\epsilon_H} + T_o^L \frac{p_L\epsilon_L}{\beta+\epsilon_L} + z\Delta\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{C_y} = \frac{\beta^2 R_m}{2}\left\{\frac{p_H}{(1+\tau^H)C_o^H} + \frac{p_L}{(1+\tau^L)C_o^L} + \frac{1}{(1+\tau^s)C_o^s}\right\} \\
\frac{\epsilon_L}{q^L} = \frac{C_o^s}{C_o^L}\frac{(1+\tau^s)}{(1+\tau^L)} \\
\frac{\epsilon_H q^L}{\epsilon_L q^H} = \frac{C_o^L}{C_o^H}\frac{(1+\tau^L)}{(1+\tau^H)}
\end{aligned}
$$

where $\phi m = z$ and $\Delta = 1 + \frac{p_H\epsilon_H}{(\beta+\epsilon_H)} + \frac{p_L\epsilon_L}{(\beta+\epsilon_L)}$.

Assuming the planner's optimal allocation exists and comparing the decentralized FOCs (given by the previous equations) with the planner's ones (given by equations (16)-(23)), we can determine the optimal tax plan.[40] This yields the following restrictions on the linear old consumption tax rates

$$
\frac{C_o^s}{C_o^L}\frac{(1+\tau^s)}{(1+\tau^L)} = \frac{C_o^s}{C_y}\frac{1}{\beta^2\left\{1+\frac{p_H}{p_L}\left[1-\frac{C_o^H}{\beta^2 C_y}\right]\right\}} \tag{25}
$$

$$
\frac{(1+\tau^L)}{(1+\tau^H)} = \frac{C_o^H}{C_L}\frac{1+\frac{p_H}{p_L}\left[1-\frac{C_o^H}{\beta^2 C_y}\right]}{1-\frac{\epsilon_L}{\epsilon_H}\left[1-\frac{C_o^H}{\beta^2 C_y}\right]} \tag{26}
$$

$$
\frac{\beta^2 R_m}{2}\left\{\frac{p_H}{(1+\tau^H)C_o^H} + \frac{p_L}{(1+\tau^L)C_o^L} + \frac{1}{(1+\tau^s)C_o^s}\right\} = 1. \tag{27}
$$

It is easy to check that from equations (28) and (27) we have the following

$$
\tau^L = B(1+\tau^H) - 1
$$

$$
\tau^s = A\, B(1+\tau^H) - 1
$$

where $A = \frac{C_o^L}{\beta^2\left\{1+\frac{p_H}{p_L}\left[1-\frac{C_o^H}{\beta^2 C_y}\right]\right\}}$ and $B = \frac{C_o^H}{C_L}\frac{1+\frac{p_H}{p_L}\left[1-\frac{C_o^H}{\beta^2 C_y}\right]}{1-\frac{\epsilon_L}{\epsilon_H}\left[1-\frac{C_o^H}{\beta^2 C_y}\right]}$ and $C_y = 1, C_o^H, C_o^L$ and C_o^s are solutions to the planner's optimal allocation. Substituting these linear tax relations into (29), we can determine

[40] This approach of looking only at the FOC is valid as long as the tax functions do not have any kinks.

the tax rate for high marginal DM-consumers

$$\tau^H = \frac{\beta^2 R_m}{2} \left\{ \frac{p_H}{C_o^H} + \frac{p_L}{B\, C_o^L} + \frac{1}{A\, B\, C_o^s} \right\} - 1.$$

It is clear that the government can sustain several inflation rates (different values of R_m) by adjusting the linear consumption tax rate.

To determine the appropriate lump-sum taxes that deliver the constrained efficient allocation, let us consider the case where $T_o^L = 0$. Then the equilibrium real balances are given by

$$z = C_o^L (1 + \tau^L) \left(1 + \frac{\epsilon_L}{\beta}\right).$$

The remaining lump-sum taxes are then

$$T_o^H = (1 + \tau^H) C_o^H \left(1 + \frac{\epsilon_H}{\beta}\right) - z$$

$$T_o^s = (1 + \tau^s) C_o^s - T_o^H \frac{p_H \epsilon_H}{\beta + \epsilon_H} - z\Delta.$$

Proof of Proposition 4

Let us consider the scenario where $j = 2$. In this case, the policymaker wants to maximize

$$\mathcal{W} = U\left(C_y\right) - v\left(h\right) + \frac{\beta}{2}\left[p_H\, \epsilon_H\, u\left(q^H\right) + p_L\, \epsilon_L\, u\left(q^L\right) - q^s\right] + \\ + \frac{\beta^2}{2}\left[p_H\, U\left(C_o^H\right) + p_L\, U\left(C_o^L\right) + U\left(C_o^s\right)\right]$$

subject to the agent's optimal behavior. It is easy to show that the decentralized equilibrium is such that

$$\frac{1}{C_y(1+\tau)} = \frac{1}{1-\tau_h}$$

$$C_o^L = \frac{\beta\left(T+z\right)}{(\beta+\epsilon_L)(1+\tau)}$$

$$C_o^H = \frac{\beta\left(T+z\right)}{(\beta+\epsilon_H)(1+\tau)}$$

$$C_o^s = \frac{\Delta\left(T+z\right)}{1+\tau}$$

$$h = \frac{1}{2}\left[p_H C_o^H + p_L C_o^L + C_o^s\right] + C_y$$

$$\frac{1}{C_y} = \frac{\beta^2 R_m}{2}\left(\frac{p_L}{C_o^L} + \frac{p_H}{C_o^H} + \frac{1}{C_o^s}\right)$$

$$\frac{\epsilon_L}{q^L} = \frac{C_o^s}{C_o^L}$$

$$\frac{\epsilon_L q^H}{\epsilon_H q^L} = \frac{C_o^L}{C_o^H}$$

where $\phi m = z$ and $\Delta = 1 + \frac{p_H \epsilon_H}{(\beta+\epsilon_H)} + \frac{p_L \epsilon_L}{(\beta+\epsilon_L)}$.

At an optimum, the inflation rate is such that

$$\left(\frac{\partial \mathcal{W}}{\partial R_m}\right) = \left[\frac{1}{C_y}\frac{\partial C_y}{\partial R_m} - \frac{\partial h}{\partial R_m}\right] + \frac{\beta}{2}\left[\frac{p_H \epsilon_H}{q^H}\frac{\partial q^H}{\partial R_m} + \frac{p_L \epsilon_L}{q^L}\frac{\partial q^L}{\partial R_m} - \frac{\partial (p_H q^H + p_L q^L)}{\partial R_m}\right] + \\ + \frac{\beta^2}{2}\left[\frac{p_H}{C_o^H}\frac{\partial C_o^H}{\partial R_m} + \frac{p_L}{C_o^L}\frac{\partial C_o^L}{\partial R_m} + \frac{1}{C_o^s}\frac{\partial C_o^s}{\partial R_m}\right] = 0.$$

It is easy to check that

$$\begin{aligned}
\frac{\partial h}{\partial R_m} &= \frac{\partial C_y}{\partial R_m} + \frac{p_H}{2}\frac{\partial C_o^H}{\partial R_m} + \frac{p_L}{2}\frac{\partial C_o^L}{\partial R_m} + \frac{1}{2}\frac{\partial C_o^s}{\partial R_m} \\
\frac{\partial C_o^s}{\partial R_m} &= \frac{\Delta}{(1+\tau)}\frac{\partial z}{\partial R_m} \\
\frac{\partial C_o^L}{\partial R_m} &= \frac{\beta}{(1+\tau)(\beta+\varepsilon_L)}\frac{\partial z}{\partial R_m} \\
\frac{\partial C_o^H}{\partial R_m} &= \frac{\beta}{(1+\tau)(\beta+\varepsilon_H)}\frac{\partial z}{\partial R_m}
\end{aligned}$$

so

$$\frac{\partial h}{\partial R_m} = \frac{\partial C_y}{\partial R_m} + \frac{1}{(1+\tau)}\frac{\partial z}{\partial R_m}.$$

The effects on DM are

$$\frac{\partial q_H}{\partial R_m}C_o^s + q_H \frac{\partial C_o^s}{\partial R_m} = \frac{\varepsilon_H}{C_o^s}\frac{\partial C_o^H}{\partial R_m} - \frac{\varepsilon_H \beta}{C_o^s \Delta(\beta+\varepsilon_H)}\frac{\partial C_o^s}{\partial R_m}$$

$$\frac{\partial q_L}{\partial R_m} = \frac{\varepsilon_L}{C_o^s}\frac{\partial C_o^L}{\partial R_m} - \frac{\varepsilon_L \beta}{C_o^s \Delta(\beta+\varepsilon_L)}\frac{\partial C_o^s}{\partial R_m}.$$

After substituting these partial derivatives, it is easy to see that $\left(\frac{\partial \mathcal{W}}{\partial R_m}\right)$ yields the following

$$1 = \frac{\beta^2}{2}\left(\frac{\beta p_L \epsilon_L}{(\beta+\epsilon_L)C_o^L} + \frac{\beta p_H \epsilon_H}{(\beta+\epsilon_H)C_o^H} + \frac{\Delta}{C_o^s}\right). \tag{28}$$

We know that at the constrained efficient planners allocation $C_y = 1$. Thus from now, we choose taxes that satisfy

$$\frac{(1-\tau_h)}{(1+\tau)} = 1.$$

Under such policy, the agent's intertemporal condition is given

$$1 = \frac{\beta^2 R_m}{2}\left(\frac{p_L}{C_o^L} + \frac{p_H}{C_o^H} + \frac{1}{C_o^s}\right).$$

Substituting this expression into (28), we have that the optimal return on money is given by

$$R_m = \frac{\Delta(1+\bar{\epsilon})\beta}{\Delta(\beta+\bar{\epsilon})+\beta}$$

where $\bar{\epsilon} = p_H\epsilon_H + p_L\epsilon_L$. It is easy to check that $R_m\beta^2 < 1$, so that a deviation from the Friedman rule is optimal.

Appendix B: Intermediate Information Set

We now consider a situation where the government observes an agent's age, CM consumption and old money holdings when entering CM. The government can also observe whether an agent has produced or consumed in frictional markets. However, it can neither determine the amounts of DM goods that are consumed or produced nor observe young savings.

Under this environment, the government can implement the same instruments as in the rich information setting. In particular, lump-sum taxes/transfers by age, linear history-dependent old consumption taxes and monetary policy are feasible. However, in contrast to the rich information setting, joint deviations are possible.

Proposition 5 *When j=2 and the government implements the same optimal policies as in the rich information setting, the constrained planner's optimal allocation cannot be achieved as joint deviations are profitable. However, when the government augments its tax plan, the constrained efficient allocation can be achieved.*

When an agent considers changing his labor supply, then, in general, he also considers changing his savings. The tax scheme in the rich information set is not sufficient to implement the constrained planner's allocation. Recall that in the rich information setting, when $j = 2$, the optimal tax plan is such that $\tau^L > \tau^H > \tau^s$ while lump-sum transfers were $T_o^L < T_o^H < T_o^s$. This tax plan is designed to preclude single deviations. Given that an agent tells the truth, the rich information tax scheme guarantees that an agent chooses the correct amount of money holdings when young. Given that a young agent chooses the correct amount of savings, an agent decides to tell the truth. However, a joint deviation in which an agent decides to both lie about his type and save more when young is profitable.[41] Under the optimal rich information tax plan, this is the case as deviating and non-deviating agents face different budget constraints. It turns out that under such tax scheme, the marginal value of holding an extra dollar is always higher when jointly deviating. In our environment there are two different joint deviations that are possible.[42] One where an agent accumulates more real balances and ϵ_L pretends to be an ϵ_H agent. The other profitable deviation is one where an agent saves more and the DM-producer pretends to be a ϵ_L DM buyer. In order to make such deviations not profitable, the rich information tax plan needs to be augmented. One way to do so is to consider old consumption tax rates that are conditioned on the money holdings that agents enter in CM when old-aged.[43] In particular, let us consider the following old consumption

[41] Golosov and Tsyvinski (2006), Kocherlakota (2005) and Albanesi and Sleet (2006) showed that such joint deviations would give an agent a higher utility than the utility from the socially optimal allocations.

[42] In Golosov and Tsyvinski (2006), Kocherlakota (2005) and Albanesi and Sleet (2006) there is only one type of joint deviation.

[43] Such tax scheme is similar to Golosov and Tsyvinski's (2006) means-tested disability insurance program.

tax scheme

$$\mathcal{T}^{\ell} = \begin{cases} \tau^{\epsilon_j} & \text{if an agent has consumed in DM and reports to be } \epsilon_j \\ \tau^{s} & \text{if an agent has produced in DM} \\ \bar{\tau}^{s} > \tau^{s} \ \&\ \bar{T}^{s}_{o} < T^{s}_{o} & \text{if an agent has produced and } m \geq m^{s} \\ \bar{\tau}^{\epsilon_1} > \tau^{\epsilon_1} \ \&\ \bar{T}^{\epsilon_1}_{o} < T^{\epsilon_1}_{o} & \text{if an agent has consumed and } m > z \end{cases} \tag{29}$$

where τ^{ℓ} and T^{ℓ}_{o} are the optimal taxes and lump-sum transfers in the rich information setting; m^s and z are the equilibrium amount of real balances sellers bring into CM when old under the optimal tax plan in the rich information setting. It is important to highlight that this augmented tax scheme aims to only punish the deviating agent. Altering the rate of return on fiat money, R_m, is not a useful policy as it changes the incentives of both non and deviating young agents. When such an augmented tax system is implemented, agents choose not to do a joint deviation as they would be worse off.

Proof of Proposition 5

The optimal condition for real balances for a non-deviating agent is given by

$$\frac{\phi}{\phi_{+2}} = \frac{\beta^2}{2}\left(\frac{p_H}{(1+\tau^H)C^H_{o,+2}} + \frac{p_L}{(1+\tau^L)C^L_{o,+2}} + \frac{1}{(1+\tau^s)C^s_{o,+2}}\right) \tag{30}$$

where $C^{\ell}_{o,+2}$ and τ^{ℓ} are solutions to the rich information set that deliver the constrained efficient allocation.

In this new environment where the government cannot see young savings and actual DM consumption and production, there are two different joint deviations we need to consider. One such deviation involves the DM-producer imitating the ϵ_L agent and saving more than what he would do under the rich information setting. The problem of this deviating agent is given by

$$\max\left\{\ln\tilde{C}_y - h + \frac{\beta}{2}\left(p_H\epsilon_H\ln\tilde{q}^H + \beta\ln\tilde{C}^H_{o,+2}\right) + \frac{\beta}{2}\left(p_L\epsilon_L\ln\tilde{q}^L + \beta\ln\tilde{C}^L_{o,+2}\right) + \frac{\beta^2}{2}\ln\tilde{C}^L_{o,+2}\right\} \quad s.t.$$

$$\tilde{C}_y + \phi\tilde{m} = \tilde{h}, \quad (1+\tau^L)\tilde{C}^L_{o,+2} = \phi_{+2}(\tilde{m} - p\tilde{q}^L + T^L_o), \quad (1+\tau^H)\tilde{C}^H_{o,+2} = \phi_{+2}(\tilde{m} - p\tilde{q}^H + T^H_o).$$

It is easy to check that the first-order condition on fiat money for the deviating agent satisfies the following

$$\frac{\beta^2}{2}\left(\frac{p_H}{(1+\tau^H)C^H_{o,+2}} + \frac{p_L}{(1+\tau^L)C^L_{o,+2}} + \frac{1}{(1+\tau^s)C^s_{o,+2}}\right) \leq \frac{\beta^2}{2}\left(\frac{p_H}{(1+\tau^H)\tilde{C}^H_{o,+2}} + \frac{1+p_L}{(1+\tau^L)\tilde{C}^L_{o,+2}}\right) \tag{31}$$

as $C^H_{o,} < \tilde{C}^H_{o}$, $C^L_{o,} < \tilde{C}^L_{o}$, $C^H_{o} < C^L_{o} < C^s_{o}$, and taxes are such that $\tau^L > \tau^H > \tau^s$. Thus bringing additional fiat money into CM for the deviating agent is more valuable than for the non-deviating agent. Thus there exists a profitable deviation.

The other deviation involves the ϵ_L imitating an ϵ_H agent and saving more than what he would do under the rich information setting. The problem of this deviating agent is given by

$$\max\left\{\ln\tilde{C}_y - h + \frac{\beta}{2}\left(p_H\epsilon_H\ln\tilde{q}^H + \beta\ln\tilde{C}^H_{o,+2}\right) + \frac{\beta}{2}\left(p_L\epsilon_L\ln\tilde{q}^H + \beta\ln\tilde{C}^H_{o,+2}\right) + \frac{\beta}{2}\left(-\tilde{q}^s + \beta\ln\tilde{C}^s_{o,+2}\right)\right\} \quad s.t.$$

$$\tilde{C}_y + \phi\tilde{m} = \tilde{h}, \quad (1+\tau^H)\tilde{C}^H_{o,+2} = \phi_{+2}(\tilde{m} - p\tilde{q}^H + T^H_o), \quad (1+\tau^s)\tilde{C}^s_{o,+2} = \phi_{+2}(\tilde{m} + p\tilde{q}^s + T^s_o).$$

It is easy to check that the first-order condition on fiat money is for the deviating agent satisfies the following

$$\frac{\beta^2}{2}\left(\frac{p_H}{(1+\tau^H)C^H_{o,+2}} + \frac{p_L}{(1+\tau^L)C^L_{o,+2}} + \frac{1}{(1+\tau^s)C^s_{o,+2}}\right) \leq \frac{\beta^2}{2}\left(\frac{1}{(1+\tau^H)\tilde{C}^H_{o,+2}} + \frac{1}{(1+\tau^s)\tilde{C}^s_{o,+2}}\right) \tag{32}$$

as $C^H_{o,} < \tilde{C}^H_o$, $C^s_{o,} < \tilde{C}^s_o$, $C^H_o < C^L_o < C^s_o$, and taxes are such that $\tau^L > \tau^H > \tau^s$. Thus bringing additional fiat money into CM is more valuable for the deviating than the non-deviating agent. Thus agents have an incentive to increase their savings.

To prevent such deviations, the government has to augment the tax scheme in the rich information. Given the available information to the government, it can implement the following tax plan

$$\mathcal{T}^\ell = \begin{cases} \tau^{\epsilon_j} & \text{if an agent has consumed in DM and reports to be } \epsilon_j, \\ \tau^s & \text{if an agent has produced in DM,} \\ \bar{\tau}^s > \tau^s \ \&\ \bar{T}^s_o < T^s_o & \text{if an agent has produced and } m \geq m^s, \\ \bar{\tau}^L > \tau^L \ \&\ \bar{T}^L_o < T^L_o & \text{if an agent has consumed and } m > z \end{cases} \tag{33}$$

where the new tax rates $\bar{\tau}^s$ and $\bar{T}^s_o$ are such that satisfy the following conditions

$$\frac{\beta^2 R_m}{2(1+\bar{\tau}^s)}\left(\frac{1+\bar{\tau}^s}{1+\tau^H}\frac{1}{\tilde{C}^H_o} + \frac{1}{\tilde{C}^s_o}\right) = \Omega$$

where $\tilde{C}^L_o$ and $\tilde{C}^H_o$ solve the problem of the DM-producer deviating agent and Ω is given by

$$\Omega = \frac{\beta^2}{2}\left(\frac{p_H}{(1+\tau^H)C^H_{o,+2}} + \frac{p_L}{(1+\tau^L)C^L_{o,+2}} + \frac{1}{(1+\tau^s)C^s_{o,+2}}\right)$$

evaluated at the optimal tax scheme in the rich information setting. Similarly, $\bar{\tau}^L$ and $\bar{T}^L_o$ satisfy the following equations

$$\frac{\beta^2 R_m}{2(1+\bar{\tau}^L)}\left(\frac{1+\bar{\tau}^L}{1+\tau^H}\frac{1}{\tilde{C}^H_o} + \frac{1+P_L}{\tilde{C}^L_o}\right) = \Omega$$

where $\tilde{C}^s_o$ and $\tilde{C}^H_o$ solve the problem of the DM-consumer deviating agent. Note that with such an augmented tax scheme, there is no joint profitable deviation as agents following that strategy would be worse off.