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Optimal Taxes Under Private Information: The Role of the Inflation Tax

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Abstract

We consider an overlapping generation framework with search and private information to study optimal taxation. Agents sequentially trade in markets that are characterized by different frictions and trading protocols. In frictional decentralized markets, agents receive shocks that determine if they are going to be consumers or producers. Shocks are private information. Mechanism design is used to solve for the constrained optimal allocation. We then study whether a government can replicate the constrained optimal allocation with an array of policy instruments including fiat money. We show that if the government has a full set of non-linear taxes, then lump-sum taxes and inflation are irrelevant for the allocation. However, if the government is constrained to use linear taxes, then using the inflation tax is optimal even if lump-sum taxes are available.

JEL Codes: E52, E62, H21

Key words: Inflation, Monetary Policy, Fiscal Policy

1 Introduction

Is the inflation rate part of an optimal tax plan? Within the Ramsey tradition, for more than four decades the answer to this question has been explored in a variety of frameworks, ranging from the overlapping generation models to representative agent environments. Typically the results are that having a zero nominal interest rate is optimal when the fiscal authority has access to lump-sum and/or linear taxes.¹ Thus deviating from a zero nominal interest rate requires some imperfections, incompleteness or heterogeneity in the underlying economic environment. More precisely, the literature has found that it is optimal to observe positive nominal interest rates when the inflation tax is a substitute instrument for a missing tax.² Once additional heterogeneity is considered, deviations from the Friedman rule have been found to be optimal in overlapping generation frameworks and infinitely lived agent frameworks as it allows for the possibility of redistribution through wealth transfers.³

Implicit in the Ramsey approach to taxation is that the government, other than having enforcement powers, does not face any informational problems. This later assumption is quite restrictive for the taxing authority as it implicitly assumes that all realizations of the shocks in the economy can be observed by everyone. In this paper we relax this requirement and introduce an informational asymmetry. We do so in the spirit of Mirrlees (1971).⁴ The Mirrleesian approach to taxation incorporates heterogeneity and explicitly considers private information problems. The government chooses taxes and imposes no restrictions on the set of available tax instruments to maximize welfare subject to providing appropriate incentives for agents to reveal their private information. The robust result emerging from this literature is that non-linear taxes tend to be optimal.⁵ This is the case as they can help induce truth telling. Here we add to this literature by considering a monetary economy where agents face anonymity and search frictions so that a medium of exchange is essential.

In this paper we study optimal monetary and fiscal policy in an environment with private

¹For overlapping generation models we refer to Abel (1987), Freeman (1987), among others. For representative agent models Chari, Christiano and Kehoe (1996) show that the Friedman rule is optimal if preferences are homothetic and weakly separable in consumption and leisure (1996).

²Schmitt-Grohe and Uribe (2004a,b), for instance, show that a positive nominal interest rate can tax producers' monopoly profits, and Chugh (2006b) shows it can tax monopolistic labor suppliers' rents. These are examples of the Ramsey planner using a positive nominal interest rate to indirectly tax some rent, as a direct tax instrument is not available.

³For more on the non optimality of the Friedman rule in overlapping generation models we refer to Bhattacharya et al. (2005) and Gahvari (2007). For infinitely lived frameworks we refer to Levine (1991), Molico (2006) and Berentsen, Camera and Waller (2005), just to name a few.

⁴The New Dynamic Public Finance literature studies optimal taxation in a dynamic version of Mirrlees' (1971) model where agents are heterogeneous and agents face private information.

⁵We refer the reader to Golosov, Kocherlakota and Tysvinski (2003), Kocherlakota (2005b) and da Costa and Werning (2008), just to name a few, for more information.

information, search frictions and heterogeneous trading histories. One of the issues that we address is whether or not the inflation tax is needed to achieve the desired allocation. We build on Zhu's (2008) overlapping generation monetary search model. A new generation is born every other period. Young and old agents trade with each other and with the government in a frictionless Walrasian competitive market. Then the old die and the young become middle aged who trade amongst themselves. Middle aged agents receive idiosyncratic preference shocks making them producers or consumers. These preference shocks are private information.

To determine optimal allocations, we begin by using a mechanism design approach first with full information about the preference shocks and then with private information. With private information, the planner's allocation must be incentive compatible so that truthful revelation is possible. A key result we obtain is that the planner must create consumption risk for the old to induce agents to produce in middle age. Thus, the planner trades off risk sharing amongst the old against productive efficiency when middle-aged.⁶

We then study whether or not a government with a variety of policy instruments can replicate the planner allocation. Fiat money is one of the policy instruments under the control of the government and it plays three roles in our model. First, money serves as a store of value to allow young agents to move consumption to old age. Second, because the middle-aged trade in a decentralized anonymous market, fiat money serves as a medium of exchange. Finally, money serves as a substitute for record-keeping to induce truthful revelation of types in middle age.⁷ The government controls the real return on money via its choice of inflation. The return on money affects the average level of consumption in old age but it also affects the quantities traded in middle age, which generates a distribution of real balances and old age consumption.

Using this framework we address two questions. First, can the constrained planner allocation be implemented when the government has access to non-linear tax policies and fiat money? We show that the constrained optimum can be replicated with a non-linear consumption tax on old age consumption and zero lump-sum taxes. In this sense, the government has the option to use lump-sum taxation but decides not to do so. Instead it relies on distorting non-linear consumption taxes to induce truthful revelation of types. Achieving this allocation requires that the government can observe actual consumption in old age. Because the government has a full set of tax instruments, including non-linear and age specific taxes,

⁶An alternative approach would be to make money a part of the environment that gives agents an outside option to trade, as in Aiyagari and Williamson (2000). Since money creates a better outside option for agents and thus makes the planner's problem more difficult, a planner would never introduce money if it was under his control.

⁷Thus, money is memory as shown by Kocherlakota (1998).

the inflation tax is a redundant tax instrument. Hence, the optimal allocation is unaffected by the inflation rate.

Second, does the optimal fiscal policy require use of the inflation tax if non-linear taxes are unavailable? We consider the case where the government can observe *purchases* of consumption goods but not actual consumption. In this case a non-linear consumption tax system is not coalition proof. Hence, the government is limited to using linear and lump-sum taxes. We then show that the optimal policy dictates using the inflation tax.

2 Literature Review

This paper connects with two different literatures. One where monetary and fiscal policies have been analyzed in search theoretic models of money using the Ramsey approach. The other literature we relate to, is the one where monetary policy and fiscal policy have been analyzed in environments where agents face private information problems.

Within the literature on search theoretic models of money, such as Shi (1997) or Lagos and Wright (2005) (hereafter denoted LW), the policy maker does not face a trade-off between efficiency and risk sharing. Shi uses the large household assumption which creates consumption insurance for all members thereby eliminating any concern about risk sharing for the policymaker. Alternatively, LW assume quasi-linear utility in the centralized market, which means agents are risk neutral so there is no need for risk sharing. Furthermore, agents are unconstrained in the amount of labor they can supply in the centralized market, which effectively allows them to self-insure against idiosyncratic trading shocks. As a result, the Friedman rule is always the optimal policy. Both Shi and LW adopt these assumptions in order to make the distribution of money balances degenerate. Consequently, their models are analytically tractable. We use the overlapping generations framework as an alternative way to control the distribution of wealth (by having agents die). This allows us to obtain analytical results while still having a non-degenerate distribution of wealth in old age.

More recently, using the LW model, Wong (2016) assumes agents have concave utility over consumption and leisure in the centralized market. While one would think this opens the door to an efficiency/risk sharing trade-off, it does not. Wong shows that if preferences are of a particular class (e.g., they generate linear Engel curves in consumption and leisure) *and* there are no bounds on the amount of labor supplied, the distribution of money balances is still degenerate. He also shows that the Friedman rule is optimal despite the variance in CM consumption and leisure.⁸ Rocheteau et al (2015) study the optimality of inflation in

⁸As an aside, none of these authors examine other fiscal policy instruments that can achieve the first best.

the LW model when CM labor has a binding upper bound. They show that if this upper bound holds for some agents, then the distribution of money holdings is not degenerate and a one-time increase in the money supply can be welfare improving. However, they do not study how changes in the steady-state inflation rate affect welfare.

However, there are several papers in this literature that show the Friedman rule is not optimal. For instance, Gomis-Porqueras et al (2010) show that the Friedman rule is not optimal when the trading protocol is not monotonic in the buyer surplus. A combination of linear subsidies and sales tax rates are required to provide incentives to sellers to increase their production which require positive nominal interest rates. Once bonds and capital are also considered, Aruoba et al (2010) find that the Friedman Rule is typically not optimal, and the long-run capital income tax is not zero. The inflation tax is used because money has a rent associated with it and bargaining frictions lead to holdup problems in capital investment.⁹ Ritter (2010) uses the Shi model and finds that a departure from the Friedman rule is optimal whenever there is a thick market on the buyer's side. Having positive nominal interest rates changes the agents' bargaining position and allowing buyers to extract a larger fraction of the trade surplus. A common feature of all these papers is that agents do not face consumption risk in the centralized market due to quasi-linear utility or because of the large family assumption in the Shi model. So an efficiency/risk sharing trade-off does not exist. In our paper, this is the critical trade-off in determining the optimal inflation rate.

Within the literature based on search, the paper that is closest to ours, in terms of structure, is Zhu (2008) and in terms of the private information problem and inflation, Ennis (2008). Zhu (2008) allows for heterogeneity in the money holdings and the government does not face private information problems. By introducing the overlapping generation structure, the author can incorporate wealth effects which are not present in Lagos and Wright (2005). He shows that inflation is welfare improving. However, he never explores whether the inflation tax is optimal when other tax instruments are available. Furthermore, Zhu claims that this welfare improvement requires 'near' buyer-take-all' bargaining to determine the terms of trade among the middle-aged. We show that this result continues to hold even with competitive pricing in the decentralized market.¹⁰

Ennis (2008) studies the role of asymmetric information over buyers' tastes in the Lagos and Wright (2005) framework when the seller has all the bargaining power and the government has access to lump sum taxes. He shows that this information friction can endogenously

⁹The deviation from the Friedman Rule nor the non-zero capital tax arises due to any inability on the part of the government to create any wedges.

¹⁰Zhu (2008) does not allow for any other tax instruments hence, is subject to the Kocherlakota (2005a) critique.

limit the ability of sellers to extract surplus from buyers, which sustains monetary exchange.¹¹ When inflation increases, agents that are unlikely to benefit from monetary exchange decide to stop participating in those trades for which money is essential. Thus inflation can result in welfare costs by reducing the participation of agents in markets with monetary exchange. In our paper, fiat money can help induce truth telling and the fiscal authority has access to a richer set of instruments to design optimal policy.

Our paper is also related to the seminal paper of Mirrlees (1971) and the literature that followed. Mirrlees suggests a way to formalize the planner's problem that deals explicitly with unobserved heterogeneity among taxpayers. By recognizing unobserved heterogeneity, diminishing marginal utility of consumption and incentive effects, the Mirrlees approach formalizes the classic trade-off between equality and efficiency. The two main insights are that the optimal marginal tax rate schedules depend on the distribution of agents' ability and that the optimal marginal tax schedule could decline for high incomes. Subsequent work in dynamic settings, such as Chamley (1986) and Judd (1985), typically ignored uncertainty about individual earnings. Recent work on optimal taxation has considered shocks and have explored new tax policy designs. The main conclusions of this new literature has been that, except in special cases, optimal taxation in dynamic economies depends on the income histories of individuals and requires interactions between different types of taxation, such as taxes on capital and labor. Key contributions to this literature include Golosov, Kocherlakota, and Tsyvinski (2003), Albanesi and Sleet (2006), Kocherlakota (2005b), and Golosov, Tsyvinski, and Werning (2006), to name a few.

This previous literature is typically analyzed using real models. A notable exception is that of da Costa and Werning (2008) and Gahvaria and Micheletto (2014). da Costa and Werning (2008) use a Mirrlees approach to study inflation taxation by assuming labor productivity is heterogenous and private information. For fiat money to have a role in this economy, these authors consider a money-in-the utility framework and ask whether inflation can be Pareto improving. These authors assume that individual money balances are not observed by the government. Thus the taxation of labor income is unrestricted but the taxation of money is linear. Within this environment they find that the Friedman rule is optimal if money and work effort are complements and labor income is taxed in a non-decreasing manner. But, then why do we obtain different results than these authors? Consider the following interpretation of da Costa and Werning's results. Think of effort as being negative leisure and money as being merely a form of wealth, as opposed to an argument in the utility function. Then assuming complementarity between money and effort is implicitly assuming leisure is an inferior good. In particular, the more wealth one has, the less leisure an individ-

¹¹The information problem generates an endogenous extensive margin on the participation of agents.

ual has. In our framework, agents have to work today to acquire fiat money that can help finance future consumption. So a standard wealth effect is observed – if an agent has more money today, then he wants more leisure today (via lower effort/production). Consequently, to induce optimal effort/production, money is taxed via inflation even though lump-sum taxes are available.

Gahvaria and Micheletto (2014) consider a variant of the money in the utility function of da Costa and Werning (2008) framework. Rather than agents being infinitely lived, Gahvaria and Micheletto (2014) consider a two-period overlapping-generations economy where agents face a nonlinear income tax schedule and can engage in tax evasion. Contrary to da Costa and Werning (2008) findings, complementarity of real cash balances and labor supply alone does not guarantee the optimality of the Friedman rule. When agents have access to a misreporting technology, which allows them to shelter part of their earned income from the tax authority, monetary policy becomes another useful instrument for redistribution. Inflation is useful because it is the only instrument that can be used to tax informal activity. A critical difference between our approach and Gahvaria and Micheletto is that in their model, the government would like to abolish money since it allows agents to avoid taxation, whereas in our model the government wants money in the economy to induce truthful revelation of types.

3 Environment

The basic environment builds on the overlapping generations framework with search of Zhu (2008). Time is discrete and agents live for three periods: young, middle and old. A continuum of agents are born with unit measure *every other* period. Agents trade sequentially in markets that differ in terms of the frictions that agents face. Goods differ across markets and are perishable so there is no possibility to store goods across periods.

An agent born at date t is young and trades a homogenous good with the existing old. Trade in this market is done in a centralized manner. We label this market as the CM. After trade concludes in the CM, the old die and the young become middle-aged. Middle-aged agents trade amongst themselves in a decentralized market or DM. Trade in this market is characterized by stochastic trading opportunities, specialized goods and anonymity.¹² Agents discount time at rate $\beta \leq 1$.

Endowments and Production Technologies Young agents are endowed with labor at the start of each CM, while old agents are unable to work. Young agents in the CM have

¹²The CM/DM structure follows from Lagos and Wright (2005).

access to a linear production technology that converts one unit of labor into one unit of CM goods. Middle-aged agents have also a labor endowment at the start of each DM. These agents can operate a linear production technology that also converts one unit of labor into one unit of the DM specialized good.

Preferences Agents derive utility from consuming the CM and DM goods and receive disutility from effort when producing them. Throughout the rest of the paper we assume that agents' preferences are time separable and additively separable across consumption goods and labor. Let $U(C)$ denote the utility from consuming C units of CM goods and $v(h)$ represents the cost (in terms of utility) of working when young in the CM when exerting effort h . We further assume that preferences satisfy standard concavity assumptions such that $U', -U'', v', v'' > 0$ and $U'(0) \rightarrow +\infty$.

Preferences when middle-aged are such that agents derive $\epsilon_b u(q)$ utils from consuming q DM goods, while they experience disutility from producing these goods equal to $-\epsilon_s \psi(q)$; where ϵ_b and ϵ_s represent the preference shocks that determine whether they will be DM buyers or DM sellers. Middle-aged agents are DM buyers ($\epsilon_b = 1$ and $\epsilon_s = 0$) with probability σ_b , while with probability σ_s agents are DM producers ($\epsilon_b = 0$ and $\epsilon_s = 1$). Finally, with probability $1 - \sigma_b - \sigma_s$, agents will be inactive in this market ($\epsilon_b = 0$ and $\epsilon_s = 0$) and receive zero payoffs, which implies that $u(0) = \psi(0) = 0$. Throughout the rest of the paper we assume that $\sigma_b = \sigma_s = \sigma$ with $\sigma \leq 1/2$. We consider two cases: 1) these middle-aged preference shocks are public information and 2) they are private information.

An agent born at time $t - 1$ has a lifetime utility that is given by

$$W_{t-1} = U(C_{y,t-1}) - v(h_{t-1}) + V_t \quad (1)$$

where C_y denotes young consumption and V_t corresponds to the value function when becoming middle-aged that is given by

$$V_t = \sigma [u(q_{b,t}) - \psi(q_{s,t})] + \beta [\sigma U(C_{o,t+1}^b) + \sigma U(C_{o,t+1}^s) + (1 - 2\sigma) U(C_{o,t+1}^n)] \quad (2)$$

where q_b is consumption of DM goods by a buyer, q_s is the amount of DM goods produced if an agent is a seller and C_o^j is the old consumption when the agent's trading state in DM was j , which could have been either a buyer (b), a seller (s) or inactive (n).

Through the rest of the paper we consider economies with various informational structures. We then compare the resulting monetary equilibria and highlight how private information affects the design of optimal policies.

4 Social Planner's Problem

Before we characterize the equilibrium allocations that are obtained when agents trade in CM and DM, we first analyze the social planner's problem. To better understand the role of private information on the allocation. We first consider the case where the preference shocks are public information. We then consider the social planner's problem when these shock are private information.

4.1 Full Information

Consider a situation where the social planner has access to a record-keeping technology, can observe all of the DM preference shocks that middle-aged agents receive, can force exchange to occur ex-post and can commit to deliver any proposed consumption sequence to agents. In this environment, the record-keeping technology allows the planner to keep track of agents' histories. Conditional on the realization of the DM preference shock, the planner can give them a sequence of DM and CM consumptions as well as CM and DM effort to be exerted. In particular, if an agent reports himself as a buyer, the planner gives him q_b units of DM output and C_o^b units of CM good to consume when middle-aged and old respectively. If he reports himself as a producer, the agent has to deliver q_s units of DM output to the planner and consumes C_o^s units of CM goods when old. If the agent reports being idle, he neither consumes nor produces in the DM and receives C_o^m units of CM goods when old. Finally, young agents receive C_y units of consumption and provide h units of labor. Notice that since the realization of the DM preference shocks are public information, an agent can not misrepresent his type.

Let us characterize the allocation that the planner would propose to agents. More precisely, consider an economy that starts in period $t = -1$ with an initial generation of middle-aged agents.¹³ The planner's social welfare is given by

$$\begin{aligned} \mathcal{W} = & \sigma \lambda^{-1} [u(q_{b,-1}) - \psi(q_{s,-1})] + \beta \lambda^{-1} [\sigma U(C_{o,0}^b) + \sigma U(C_{o,0}^s) + (1 - 2\sigma) U(C_{o,0}^m)] \\ & + \sum_{t=0}^{\infty} \lambda^t \{U(C_{y,2t}) - v(h_{2t}) + \sigma [u(q_{b,2t+1}) - \psi(q_{s,2t+1})]\} \\ & + \sum_{t=0}^{\infty} \lambda^t \beta [\sigma U(C_{o,2t+2}^b) + \sigma U(C_{o,2t+2}^s) + (1 - 2\sigma) U(C_{o,2t+2}^m)], \end{aligned}$$

where λ^t is the weight assigned to a generation born at time $2t$ and the weight on the first

¹³An advantage of this structure is that by focusing on the middle-aged, the agents are all the same. This makes it easier to define the social optimum than in a standard OG model.

young generation is normalized to 1.

The planner chooses sequences of CM consumptions and CM effort $\{C_{y,2t}, h_{2t}, C_{o,2t}^b, C_{o,2t}^s, C_{o,2t}^n\}_{t=0}^\infty$ as well as DM consumption and production $\{q_{b,2t+1}, q_{s,2t+1}\}_{t=-1}^\infty$ to maximize a weighted average of current and future generations expected utilities subject to the resource constraint. Letting $\{C_y, h, C_o^j\}$ denote $\{C_{y,2t}, h_{2t}, C_{o,2t}^b, C_{o,2t}^s, C_{o,2t}^n\}_{t=0}^\infty$ and $\{q_b, q_s\}$ denotes $\{q_{b,2t+1}, q_{s,2t+1}\}_{t=-1}^\infty$ the social planner problem is then given by

$$\begin{aligned} \max_{\{C_y, h, C_o^j\}, \{q_b, q_s\}} \mathcal{W} \quad s.t. \quad & h_{2t} \geq \sigma C_{o,2t}^b + \sigma C_{o,2t}^s + (1 - 2\sigma) C_{o,2t}^n + C_{y,2t} \quad \forall t \geq 0, \quad (3) \\ & \sigma q_{s,2t+1} \geq \sigma q_{b,2t+1} \quad \forall t \geq 0, \end{aligned}$$

The optimal allocation satisfies the resource constraints and the following intra-temporal and inter-temporal conditions

$$u'(q_{2t+1}) = \psi'(q_{2t+1}) \quad t \geq 0, \quad (4)$$

$$U'(C_{y,2t}) = v'(h_{2t}) \quad t \geq 0, \quad (5)$$

$$U'(C_{o,2t}^b) = U'(C_{o,2t}^n) = U'(C_{o,2t}^s) \quad t \geq 0 \quad \Rightarrow C_{o,2t}^j = C_{o,2t} \quad \forall j, \quad (6)$$

$$U'(C_{y,0}) = \beta \lambda^{-1} U'(C_{o,0}), \quad (7)$$

$$U'(C_{y,2t}) = \beta \lambda^{-1} U'(C_{o,2t}) \quad t > 0, \quad (8)$$

$$U'(C_{y,2t}) = \frac{v'(h_{2t})}{v'(h_{2t+2})} \beta \lambda^{-1} \mathbb{E}[U'(C_{o,2t+2})] \quad (9)$$

where $\mathbb{E}[U'(C_{o,2t+2})] = [\sigma U'(C_{o,2t+2}^b) + \sigma U'(C_{o,2t+2}^s) + (1 - 2\sigma) U'(C_{o,2t+2}^n)]$

Lemma 1 *The unconstrained optimal social planner's allocation is such that the marginal benefit of consuming the DM good equates its marginal cost and old agents face no consumption risk in the CM so that $C_{o,2t}^b = C_{o,2t}^s = C_{o,2t}^n$ for all t .*

Note that the $t = 0$ old get the same allocation as future old. In short, the economy can start in a steady-state. Also, if the planner sets $\lambda = \beta$, then in a steady state (or with linear disutility of labor for the young), the planner equates the marginal utility of CM consumption across young and old at a point in time as well as across time for an individual generation. From now on we assume that $\lambda = \beta$.

Example Assume $U(C) = \ln C$, $v(h) = h$, $u(q) = 1 - \exp^{-q}$ and $\psi(q) = \rho q$ with $\rho < 1$. The unconstrained optimal allocation is given by

$$\begin{aligned} C_y^* &= C_o^b = C_o^s = C_o^n = 1 \\ h^* &= 2 \\ q^* &= -\ln \rho. \end{aligned}$$

4.2 Private Information

Let us now consider the case where the DM preference shock is private information. Under this new information structure, the unconstrained allocation is not feasible. Why? Because the allocation under public information is such that those agents who are producers in the DM get the same consumption in the next CM as everyone else. Hence these agents obtain no additional compensation for exerting effort when producing. Consequently, DM producers have an incentive to hide their true type from the planner. Note that even if the social planner is able to force agents to produce, this ability does not resolve the incentive problem. This is the case as the planner cannot identify who should be producing in DM. Thus, with private information, the planner is constrained to implement an allocation that is incentive compatible. The social planner has to make sure that middle-aged agents truthfully reveal the idiosyncratic shock they have received in the DM.

In order to induce truthful reporting of middle-aged types, the planner has to consider the following incentive constraints

$$u(q_{b,2t+1}) + \beta U(C_{o,2t+2}^b) - \beta U(C_{o,2t+2}^n) \geq 0, \quad (10)$$

$$-\psi(q_{s,2t+1}) + \beta U(C_{o,2t+2}^s) - \beta U(C_{o,2t+2}^n) \geq 0, \quad (11)$$

$$U(C_{o,2t+2}^n) - U(C_{o,2t+2}^b) \geq 0. \quad (12)$$

Note that the inequality constraints given by equations (10) and (11) require that buyers and sellers in the DM have no incentive to misreport themselves as inactive. The last inequality constraint makes sure that an inactive agent has no incentive to misreport himself as a buyer. It is important to highlight that buyers and idle agents would not want to misrepresent themselves as sellers. This is the case as they would be required to deliver $q_s > 0$ units of goods, which they cannot produce. Thus, incentive constraints for these cases can be ignored. However, an inactive agent can declare himself as a buyer and freely dispose of the goods. The inequality constraint given by equations (12) also ensures that a seller would rather report himself as inactive than as a buyer.

In general, there also must be a participation constraint on the young to induce them to produce output for both generations rather than go into autarky and produce only for themselves. Suppose a young agent decides not to participate in the social planner's scheme. The worst punishment the planner can impose is to force them into autarky. Participation by the young then requires the following incentive compatibility constraint

$$W^a \leq U(\bar{C}_{y,t}) - v(\bar{h}_t) + \beta\sigma [u(\bar{q}_{b,2t+1}) - c(\bar{q}_{s,2t+1})] + \beta\mathbb{E}(U(\bar{C}_{o,2t+2})),$$

where W^a denotes the value of autarky for a young agent, which is given by

$$W^a = U(C_y^a) - v(C_y^a) + \beta U(0)$$

with C_y^a solving $U'(C_y^a) = v'(C_y^a)$ and the bars on each variable denote the planner's allocation. Note that for CM preferences, such that $U(0)$ is sufficiently negative, the young buyer's participation will never bind. We assume this from here on.

The constrained planner's problem is given by

$$\begin{aligned} \max_{\{C_{y,h}, C_o^j\}_{\{q_b, q_s\}}} \mathcal{W} \quad s.t. \quad & h_{2t} \geq \sigma C_{o,2t}^b + \sigma C_{o,2t}^s + (1-2\sigma)C_{o,2t}^n + C_{y,2t} \quad \forall t \geq 0, \\ & \sigma q_{s,2t+1} \geq \sigma q_{b,2t+1} \quad \forall t \geq 0, \\ & u(q_{b,2t+1}) + \beta U(C_{o,2t+2}^b) - \beta U(C_{o,2t+2}^n) \geq 0, \\ & -\psi(q_{s,2t+1}) + \beta U(C_{o,2t+2}^s) - \beta U(C_{o,2t+2}^n) \geq 0, \\ & U(C_{o,2t+2}^m) - U(C_{o,2t+2}^b) \geq 0 \end{aligned} \quad (13)$$

where the constrained optimal allocation satisfies the following marginal and inter-temporal conditions

$$\psi(q_{2t+1}) = \beta U(C_{o,2t+2}^s) - \beta U(C_{o,2t+2}^m) \quad \text{for } t \geq -1, \quad (14)$$

$$\frac{u'(q_{2t+1})}{\psi'(q_{2t+1})} = \frac{U'(C_{o,2t+2}^b)}{\sigma U'(C_{o,2t+2}^b) + (1-\sigma)U'(C_{o,2t+2}^s)} \geq 1 \quad \text{for } t \geq -1, \quad (15)$$

$$U'(C_{y,2t}) = v'(h_{2t}) \quad t \geq 0 \quad (16)$$

$$C_{o,2t+2}^b = C_{o,2t+2}^m < C_{o,2t+2}^s \quad \text{for } t \geq 0, \quad (17)$$

$$U'(C_{y,0}) = \left[\frac{\sigma}{U'(C_{o,0}^s)} + \frac{\sigma}{U'(C_{o,0}^b)} + \frac{1-2\sigma}{U'(C_{o,0}^n)} \right]^{-1}, \quad (18)$$

$$U'(C_{y,2t+2}) = \left[\frac{\sigma}{U'(C_{o,2t+2}^s)} + \frac{\sigma}{U'(C_{o,2t+2}^b)} + \frac{1-2\sigma}{U'(C_{o,2t+2}^n)} \right]^{-1} \quad \text{for } t > 0. \quad (19)$$

Lemma 2 *The social planner's allocation under private information makes the marginal benefit of consuming the DM good greater than its marginal cost and old agents face consumption risk in the CM.*

It is worth emphasizing three key properties of the planner's allocation under private information. First, sellers' incentive constraints bind so they get no trade surplus in the DM. It then follows that, for any $q_{2t+1} > 0$, DM sellers have to receive more old age consumption than inactive agents in order to compensate them for producing in DM. Second, the inactive DM agents' incentive constraints bind. As a result they get the same old age consumption as DM buyers. Thus, $C_{o,2t+2}^s > C_{o,2t+2}^n = C_{o,2t+2}^b$ for all t . It then follows that there is incomplete risk-sharing in old age consumption all for $t \geq 0$. This feature is a direct consequence of the private information problem. Finally, because the planner must accept incomplete risk-sharing among the old, in order to induce DM sellers to produce, DM output is low relative to the full-information case. Thus, because of private information, the planner trades off efficiency in DM against old age risk-sharing. The allocation between the initial old and the initial young is equivalent to that arising in later generations. i.e., the planner can start the economy off in a steady state.

The constrained optimal allocation has also to satisfy the inter-temporal Euler equation which is given by

$$U'(C_{y,2t}) = \frac{v'(h_{2t})}{v'(h_{2t+2})} \left[\frac{\sigma}{U'(C_{o,2t+2}^s)} + \frac{\sigma}{U'(C_{o,2t+2}^b)} + \frac{(1-2\sigma)}{U'(C_{o,2t+2}^n)} \right]^{-1}$$

which can be rewritten

$$U'(C_{y,t}) = \frac{v'(h_{2t})}{v'(h_{2t+2})} \left\{ \mathbb{E} \left[\frac{1}{U'(C_{o,2t+2})} \right] \right\}^{-1}. \quad (20)$$

Note that the constrained efficient allocation needs to equate the marginal utility of young consumption to the *harmonic* mean of old age marginal utility of consumption. This is in contrast to the case with public information, equation (9), where the planner equates the marginal utility of young consumption to the *arithmetic* mean of marginal utility when old. This is similar to the Euler equation that arises in the New Dynamic Public Finance literature.

A steady-state constrained allocation is a list $\{C_y, h, q, C_o^b, C_o^s, C_o^n\}$ that solves the fol-

lowing optimal conditions

$$C_o^m = C_o^b \quad (21)$$

$$h = C_y + \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^m \quad (22)$$

$$U'(C_y) = v'(h) \quad (23)$$

$$\frac{u'(q)}{\psi'(q)} = \frac{U'(C_o^b)}{(1 - \sigma)U'(C_o^s) + \sigma U'(C_o^b)} > 1 \quad (24)$$

$$\psi(q) = \beta [U(C_o^s) - U(C_o^m)] \quad (25)$$

$$U'(C_y) = \left[\sigma \frac{1}{U'(C_o^s)} + \sigma \frac{1}{U'(C_o^b)} + (1 - 2\sigma) \frac{1}{U'(C_o^m)} \right]^{-1} \quad (26)$$

Note that if $\sigma = 0$ the information problem is effectively eliminated as there is no trade in DM and the only state in this market is inactive. The unconstrained allocation (C_o, C_y) can then be implemented subject to the young agents' participation constraint being satisfied. This is the case as trading in DM goods q is irrelevant in this case.

Example For the same preferences as before, the constrained optimal allocation is given by (derivations are in the appendix):

$$\begin{aligned} \bar{C}_y &= 1, \quad \bar{C}_o^s > 1, \quad \bar{C}_o^m = \bar{C}_o^b = \frac{1 - \sigma \bar{C}_o^s}{1 - \sigma} < 1 \\ \bar{h} &= 2 \\ \bar{q} &= q^* - \ln \bar{C}_o^s < q^* \end{aligned}$$

where $1 < \bar{C}_o^s < 1/\sigma$ and solves

$$-\rho \ln \rho - \beta \ln(1 - \sigma) = (\beta + \rho) \ln \bar{C}_o^s - \beta \ln(1 - \sigma \bar{C}_o^s).$$

For $\rho = \sigma = 1/2$, $\beta = .9$ then $\bar{C}_o^s = 1.152$, $\bar{C}_o^m = \bar{C}_o^b = 0.848$ and $\bar{q} = 0.55165$.

From this example we see that the constrained optimal allocation has lower production in the middle age trading round, some consumption risk in old age but the *average* level of old age consumption is the same as in the full-information planner solution. Hence, the constrained planner is trading off DM efficiency against CM consumption risk.

5 Monetary Economy

We now examine whether a benevolent government can replicate the constrained optimal allocation by using a variety of tax instruments along with fiat money. Money is the only

durable object. The government has a limited record-keeping technology since it cannot observe types or trades in the DM. However, as pointed out by Kocherlakota (1998), fiat money is a useful tool for conveying information as it is a substitute for record-keeping. Here we also highlight the fact that fiat money is also able to induce truthful telling.

More precisely, a seller is happy to reveal his DM type as long as the buyer is willing to exchange fiat money for DM goods. By revealing his true type and producing DM goods, a seller can generate future CM consumption by exchanging fiat currency for DM goods. The fact that fiat money can store value, prevents idle agents from mimicking buyers. This is the case as they do not derive utility in DM and they have to give up fiat money to induce sellers to produce DM goods. By doing so they reduce their future CM consumption, which they value. Hence, money can be thought as an imperfect substitute to dynamic contracts.

CM goods that are sold in a perfectly competitive market. Given the linear production technology, the real wage is 1. Young agents sell their output at the nominal price $\mathcal{P} = 1/\phi$ where ϕ is the goods price of money in terms of the CM good. It then follows that the gross real rate of return on money from the CM in $t - 1$ to the CM in $t + 1$ is $R_m = \phi_{+1}/\phi_{-1}$.

In the DM, agents are anonymous and face stochastic trading opportunities. Preference shocks create a double coincidence of wants problem while anonymity rules out trade credit, making fiat money the only incentive compatible means for exchange in the DM. After the preference shock is realized, buyers give up money for goods while sellers increase their holdings of money by selling goods. Thus, after agents exit the DM, there is a non-degenerate distribution of money balances.¹⁴ This DM/CM structure gives money a ‘store of value’ role from young to old age and a ‘medium of exchange’ function when trading during the middle-aged period.¹⁵

We assume that the stock of money grows by lump-sum injections given to middle-aged agents at the beginning of the period. As will be shown, since they all leave the CM with the same amount of money, the lump-sum injection has equal value to all young agents. This transfer scheme eliminates welfare gains from inflation due to a non-degenerate distribution of money balances within members of the same generation as in Levine (1991), Molico (2006), Berentsen, Camera and Waller (2005) and Rocheteau et al (2015), just to name a few. Consequently, we can better isolate the effects of private information on the design of monetary policy.

Since monetary injections occur every other period, fiat currency evolves according to $M_{t+2} = \gamma_t M_t$, where M_{t+2} denotes the money supply at time $t + 2$ and $\gamma_t = 1 + \mu_t$ denotes

¹⁴The fact that agents have finite lives greatly reduces the complexities of keeping track of evolution of this distribution which depends on the sequence of preference shock each agent has received.

¹⁵In contrast to most overlapping generation models, the medium exchange role occurs within members of the same generation, rather than across generations.

the gross growth rate of the money supply from t to $t + 2$ and μ_t is the net growth rate from t to $t + 2$. From here on, the t subscript is suppressed for notational ease so that $+1$ denotes $t + 1$ and so on. For $\gamma_t < 1$ we assume that the government can impose lump-sum taxes on middle-aged agents cash holdings.

Government A benevolent government is able to observe an agent's age, hours worked in CM, CM consumption and the money holdings of each agent when entering CM. Other than issuing fiat money, the government can also impose and enforce lump sum taxes/transfers by age, implement and enforce distortionary labor taxes on the young when working in CM and levy (and enforce) a non-linear consumption tax on CM consumption which can be made age dependent.¹⁶ Since the old cannot work, all old age consumption (net of transfers) is financed by holdings of cash which means that the consumption tax is equivalent to a tax on real monetary wealth. Thus by looking at the money holdings of agents in CM, the government can infer an agent's type in the previous DM.¹⁷

The government's budget constraint is given by

$$T_o + T_y = \tau^h h + \tau^c C_y + \sigma \eta (\phi_{-1} m_{-1}^b) + \sigma \eta (\phi_{-1} m_{-1}^s) + (1 - 2\sigma) \eta (\phi_{-1} m_{-1}^n), \quad (27)$$

where T_y is a lump-sum transfer of CM goods to the young, T_o represents the lump-sum transfer to the old, τ^h is the tax rate on real labor CM income, τ^c is the consumption tax rate applied to young agents' CM consumption, $\eta (\phi_{-1} m_{-1}^j)$ denotes the market consumption tax collected from an old agent who had $\phi_{-1} m_{-1}^j$ units of goods in CM, which depends on their DM type j .¹⁸ As we will show, assuming linear tax rates on the young's work and consumption is without loss of generality since the young do not have private information over their actions.

Finally, lump-sum monetary transfers are given to middle-aged agents at the beginning of DM so that

$$\phi(M - M_{-2}) = T_{MA},$$

where T_{MA} denotes the corresponding lump sum monetary transfers.

¹⁶The consumption tax on the old is collected in the form of consumption goods rather than in cash. The use of age-dependent distortionary taxes has been studied in an overlapping generations framework by Erosa and Gervais (2002).

¹⁷This feature permits the implementation of state contingent old CM consumption taxes.

¹⁸Old agents total consumption is $C_o^j = C_o^{CM} + T_o$ where C_o^{CM} is the consumption of goods acquired in CM.

CM Problem

At the start of the DM, buyers and sellers enter this anonymous market with potentially different money holdings. The resulting expected utility of a middle-aged agent with m units of fiat money is then given by

$$\begin{aligned}
 V(m) &= \sigma \int \{u[q_b(m, m^s)] + \beta U[C_{o,+1}^b(m, m^s)]\} dF(m^s) \\
 &+ \sigma \int \{-\psi[q_s(m^b, m)] + \beta U[C_{o,+1}^s(m^b, m)]\} dF(m^b) \\
 &+ \beta(1 - 2\sigma) U[C_{o,+1}^o(m)],
 \end{aligned} \tag{28}$$

where $q_b(m, m^s)$ and $C_{o,+1}^b(m, m^s)$ represent the quantities of an agent who consumes when he is a buyer in DM and CM respectively, while holding m units of money as a buyer and his matched seller has m^s money balances. Similarly, $q_s(m^b, m)$ and $C_{o,+1}^s(m^b, m)$ denote the quantities an agent produces in DM and consumes in the CM when he holds m units of money and is matched in the DM with a buyer with m^b units of money.

A young agent born at time $t - 1$ in CM chooses $C_{y,-1}$, h_{-1} and m in order to maximize his expected lifetime utility. Formally, we have

$$\begin{aligned}
 &\max_{C_{y,-1}, h_{-1}, m} U(C_{y,-1}) - v(h_{-1}) + V_t(m) \\
 \text{s.t.} \quad &(1 + \tau^c)(C_{y,-1} - T_y) + \phi_{-1}m = (1 - \tau^h)h_{-1},
 \end{aligned}$$

where m_t denotes the amount of fiat money the agent acquires in the CM to be used in the ensuing DM. The corresponding first order conditions yield

$$U'(C_{y,-1}) = \frac{1 + \tau^c}{1 - \tau^h} v'(h_{-1}), \tag{29}$$

$$\frac{\phi_{-1}}{1 + \tau^c} U'(C_{y,-1}) = V'(m). \tag{30}$$

Since the optimal decisions are the same for all young agents, they leave the CM with the same amount of real money balances. Moreover, since CM consumption and the acquisition of money balances are financed by their labor, the labor tax rate does not directly appear in equation (30).

Having characterized the young's CM problem, let us consider the decision problem for old agents. Since they cannot produce CM goods, they have to use the real balances acquired in the previous DM to fund their CM consumption and pay their tax liabilities. Since old agents die after trade takes place in the CM, they spend all of their wealth. Thus the optimal

decision of old agents is given by

$$\begin{aligned} C_{o,-1}^b &= T_o + \phi_{-1} m_{-1}^b - \eta(\phi_{-1} m_{-1}^b), \\ C_{o,-1}^s &= T_o + \phi_{-1} m_{-1}^s - \eta(\phi_{-1} m_{-1}^s), \\ C_{o,-1}^n &= T_o + \phi_{-1} m_{-1}^n - \eta(\phi_{-1} m_{-1}^n). \end{aligned}$$

It is easy to check that the marginal effect of one additional unit of fiat currency to old consumption is given by

$$\frac{dC_{o,-1}^j}{dm_{-1}^j} = [1 - \eta'(\phi_{-1} m_{-1}^j)] \phi_{-1} \quad \text{for } j = b, s, n$$

where $\eta'(\phi_{-1} m_{-1}^j)$ is the marginal tax rate on real money holdings $\phi_{-1} m_{-1}^j$. To fully characterize the CM problem we need to determine how the expected utility of a middle-aged agent varies with money balances; i.e, $V'(m)$. To do so we need to solve the DM agent's problem.

DM Problem

At the beginning of the second period agents obtain their lump sum monetary transfers, receive their idiosyncratic preference shocks, are bilaterally matched and trade according a bargaining protocol. Once a buyer is randomly matched with a seller, agents bargain over the DM terms of trade (q, d) , where q denotes the quantity of DM goods exchanged while d represents the corresponding monetary payment. Assuming a buyer take-it-or-leave-it offer, the buyer then solves the following problem

$$\begin{aligned} \max_{q,d} & u(q) + \beta U \left[(C_{o,+1}^b) - U(\tilde{C}_{o,+1}^b) \right] \\ \text{s.t.} & \quad d \leq m^b, \\ & -\psi(q) + \beta \left[U(\check{C}_{o,+1}^s) - U(\tilde{C}_{o,+1}^s) \right] \geq 0, \end{aligned}$$

where

$$\begin{aligned} C_{o,+1}^b &= \phi_{+1} (m - d) [1 - \eta'(\phi_{+1} m_{+1}^b)] + T_o \\ \check{C}_{o,+1}^s &= [\tilde{C}_{o,+1}^s + \phi_{+1} d] [1 - \eta'(\phi_{+1} m_{+1}^s)] + T_o \end{aligned}$$

and $\tilde{C}_{o,+1}^b$ and $\tilde{C}_{o,+1}^s$ are the buyer's and seller's old age consumption threat points if they do not trade. The FOC are

$$\begin{aligned} q &: u'(q) - \lambda_s \psi(q) = 0 \\ d &: -\beta U'(C_{o,+1}^b) [1 - \eta'(\phi_{+1} m_{+1}^b)] \phi_{+1} + \lambda_s \beta U'(\check{C}_{o,+1}^s) [1 - \eta'(\phi_{+1} m_{+1}^b)] \phi_{+1} - \lambda_b = 0 \end{aligned}$$

where λ_b is the multiplier on the buyer's cash constraint and λ_s is the multiplier on the seller's surplus constraint. As will be shown later, it is important to note that the buyer sets

$$\frac{\partial \tilde{C}_{o,+1}^b}{\partial d} = \frac{\partial \tilde{C}_{o,+1}^s}{\partial d} = 0.$$

In short, the buyer's decision to spend an additional unit of real balances does not affect the reservation payoff for either agent should they walk away from the trade.

We have

$$\lambda_b = \lambda_s \beta U'(\check{C}_{o,+1}^s) [1 - \eta'(\phi_{+1} m_{+1}^s)] \phi_{+1} - \beta U'(C_{o,+1}^b) [1 - \eta'(\phi_{+1} m_{+1}^b)] \phi_{+1}.$$

For $\lambda_b > 0$ we need

$$\frac{\lambda_s U'(\check{C}_{o,+1}^s) [1 - \eta'(\phi_{+1} m_{+1}^s)]}{U'(T_0) [1 - \eta'(0)]} > 1.$$

As $T_0 \rightarrow 0$ the numerator converges to a positive finite number. However, as long as Inada conditions apply to $U(C)$, then the denominator goes to infinity and this condition is violated.

Thus, if T_0 is sufficiently small, then $\lambda_b = 0$, and the optimal q and d satisfy the following

$$\begin{aligned} \frac{u'(q)}{\psi'(q)} &= \frac{U'(C_{o,+1}^b) [1 - \eta'(\phi_{+1} m_{+1}^b)]}{U'(\check{C}_{o,+1}^s) [1 - \eta'(\phi_{+1} m_{+1}^s)]} \\ \psi(q) &= \beta U(\check{C}_{o,+1}^s) - \beta U(C_{o,+1}^b). \end{aligned} \tag{31}$$

However, if T_0 is sufficiently large, then $\lambda_b = 0$, and the optimal q and d satisfy the following

$$\begin{aligned} d &= m^b \\ \psi(q) &= \beta U(\check{C}_{o,+1}^s) - \beta U(C_{o,+1}^b). \end{aligned}$$

The resulting envelope condition for the DM value function can be rewritten as follows

$$V'_t(m) = \sigma \left[u' [q_b(m, m^s)] \frac{\partial q_b}{\partial m} + \beta U' [C_{o,t+1}^b(m, m^s)] [1 - \eta'(\phi_{+1} m_{+1}^b)] \phi_{+1} \left(1 - \frac{\partial d_{+1}}{\partial m} \right) \right] + \beta (1 - \sigma) U' [C_{o,t+1}^n(m)] [1 - \eta'(\phi_{+1} m_{+1}^n)] \phi_{+1}.$$

Depending whether the cash constraint binds in the bargaining game, we are going to observe different envelope conditions. Suppose it does not bind. Then differentiating the seller's surplus we have

$$\frac{\partial q}{\partial m} = \frac{\beta U' (\tilde{C}_{o,+1}^s) [1 - \eta'(\phi_{+1} m_{+1}^s)]}{\psi'(q)} \phi_{+1} \frac{\partial d}{\partial m} = \frac{\beta U' (C_{o,+1}^b) [1 - \eta'(\phi_{+1} m_{+1}^b)]}{u'(q)} \phi_{+1} \frac{\partial d}{\partial m} \quad (32)$$

where the seller's old age consumption threat point, $\tilde{C}_{o,+1}^s$, is unaffected by the buyer's choice of money holdings.

If $\lambda_b = 0$, then, in a symmetric equilibrium we have that the envelope condition satisfies

$$V'_t(m) = \phi_{+1} \beta [\sigma U' (C_{o,+1}^b) [1 - \eta'(\phi_{+1} m_{+1}^b)] + (1 - \sigma) U' (C_{o,+1}^n) [1 - \eta'(\phi_{+1} m_{+1}^n)]] . \quad (33)$$

The inter-temporal condition is then given by

$$\frac{U'(C_{y,-1})}{1 + \tau^c} = \frac{\phi_{+1}}{\phi_{-1}} \beta [\sigma U' (C_{o,+1}^b) [1 - \eta'(\phi_{+1} m_{+1}^b)] + (1 - \sigma) U' (C_{o,+1}^n) [1 - \eta'(\phi_{+1} m_{+1}^n)]] . \quad (34)$$

Note that the term in brackets in equation (34) is not the expected marginal utility from consuming when old because $U' (C_{o,+1}^s)$ does not appear in the expression.¹⁹ Thus, when choosing money balances, young agents ignore any old age consumption value from a marginal unit of money should they be a seller in the DM. Since a buyer does not spend all of his money balances, DM consumption does not appear even though money is essential for trade in this market. In short, the marginal liquidity value of money in the DM is zero.

¹⁹This is the case as the buyer makes a take-it-or-leave-it offer. Using the optimal terms of trade from the bargaining solution, the previous inter-temporal conditions can be rewritten as follows

$$\frac{U'(C_{y,-1})}{1 + \tau^c} = R_m \beta \left[\frac{\sigma U' (C_{o,+1}^s)}{1 + \eta' (C_{o,+1}^s - T_o)} \frac{u'(q)}{\psi'(q)} + \frac{(1 - \sigma) U' (C_{o,+1}^n)}{1 + \eta' (C_{o,+1}^n - T_o)} \right].$$

However, if the cash constraint does bind then, in a symmetric equilibrium, we have

$$\frac{U'(C_{y,-1})}{1 + \tau^c} = \frac{\phi_{+1}}{\phi_{-1}} \beta \left[\sigma U'(C_{o,+1}^s) [1 - \eta'(\phi_{+1} m_{+1}^s)] \frac{u'(q)}{\psi'(q)} + (1 - \sigma) U'(C_{o,+1}^n) [1 - \eta'(\phi_{+1} m_{+1}^n)] \right]. \quad (35)$$

5.1 Stationary Equilibria

Consider a symmetric monetary steady state where fiat money is such that $m^b = m^s = m^n = M_{+1}$, real balances in the CM are constant across time so that $\phi_{+1} M_{+1} = \phi_{-1} M_{-1} = z$, the gross rate of return on money is constant so that $\phi_{+1}/\phi_{-1} = 1/\gamma = R_m$ and real spending (measured in the next CM goods price) in the DM is constant so that $\phi_{+1} d = \delta$. Furthermore, in a symmetric equilibrium, the threat-points of no trade correspond to the consumption of the idle DM agents. Thus we have that $\tilde{C}_{o,+1}^b = \tilde{C}_{o,+1}^s = C_{o,+1}^n$.

A steady state equilibrium with a non-binding cash constraint is an allocation of CM and DM consumption and effort profiles as well as real balances $\{z, q, \delta, C_o^b, C_o^s, C_o^n, C_y, h\}$ that satisfy the following

$$C_o^s = z + \delta - \eta(z + \delta) + T_o, \quad (36)$$

$$C_o^b = z - \delta - \eta(z - \delta) + T_o, \quad (37)$$

$$C_o^n = z - \eta(z) + T_o, \quad (38)$$

$$h = C_y + \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^n, \quad (39)$$

$$U'(C_y) = \frac{1 + \tau^c}{1 - \tau^h} v'(h), \quad (40)$$

$$\psi(q) = \beta [U(C_o^s) - U(C_o^n)], \quad (41)$$

$$\frac{u'(q)}{\psi'(q)} = \frac{U'(C_o^b) [1 - \eta'(z - \delta)]}{U'(C_o^s) [1 - \eta'(z + \delta)]}, \quad (42)$$

$$\frac{U'(C_y)}{(1 + \tau^c)} = R_m \beta \{ \sigma U'(C_o^b) [1 - \eta'(z - \delta)] + (1 - \sigma) U'(C_o^n) [1 - \eta'(z)] \}, \quad (43)$$

where (39) is the CM aggregate resource constraint.

A similar set of stationary equilibrium conditions are obtained when the cash constraint is binding. The only difference relative to the previous conditions is that $z = \delta$ in all equations, (42) is no longer valid and (43) is replaced by

$$\frac{U'(C_{y,-1})}{1 + \tau^c} = R_m \beta \left[\sigma U'(C_{o,+1}^s) [1 - \eta'(2z)] \frac{u'(q)}{\psi'(q)} + (1 - \sigma) U'(C_{o,+1}^n) [1 - \eta'(z)] \right].$$

Having characterized the stationary monetary equilibria, we now explore whether monetary

and fiscal policies can replicate the constrained efficient allocation.

Throughout the rest of the paper we focus on equilibria where the cash constraint does not bind which requires that $U'(T_o)$ to be sufficiently large. Since the CM utility function satisfies the Inada condition, the non binding cash constraint equilibria can always be achieved by setting T_o sufficiently low (or even zero). As we show below, this is without loss of generality.

5.2 Optimal Policies

Under the assumption that a constrained optimal allocation exists, we now ask whether or not it can be replicated in a monetary economy through an appropriate design of fiscal and monetary policies. We consider two different tax schemes that differ on the type of instruments the fiscal authority has at its disposal.

Non-linear Old Consumption and Linear Labor Taxes

Let us consider a situation where the government is able to observe an agent's age, hours worked in CM, CM consumption and the money holdings of each agent when entering CM. As a result, non-linear consumption taxes are feasible. In particular, these taxes can be age specific. Note that this consumption tax is equivalent to a non-linear tax on old agents' monetary wealth, which can be easily seen by the taxing authority when the government has a lot of information when agents enter and trade in CM. Comparing equations (21)-(26) to equations (37)-(43), we see that in order to replicate the constrained optimal allocation, we need

$$\tau^h = -\tau^c \quad (44)$$

$$\eta(z) = \delta + \eta(z - \delta), \quad (45)$$

$$\frac{1 - \eta'(z - \delta)}{1 - \eta'(z + \delta)} = \frac{U'(\bar{C}_o^s)}{(1 - \sigma)U'(\bar{C}_o^s) + \sigma U'(\bar{C}_o^b)} < 1 \Rightarrow \eta'(z - \delta) > \eta'(z + \delta) \quad (46)$$

$$\frac{U'(\bar{C}_o^s)}{\sigma U'(\bar{C}_o^b) + (1 - \sigma)U'(\bar{C}_o^s)} = (1 + \tau^c) R_m \beta [1 - \sigma \eta'(z - \delta) - (1 - \sigma) \eta'(z)] < 1. \quad (47)$$

where \bar{C}_o^j are the constrained optimal values for $j = b, s, n$.

It is worth highlighting that equation (44) ensures that the young agents' consumption/work decision is not distorted – a positive consumption tax on the young must be offset with a subsidy on their work effort. This result is essentially an application of the uniform taxation result of Atkinson and Stiglitz (1980). Equation (45) is needed to ensure $\bar{C}_o^b = \bar{C}_o^n$ so that partial risk-sharing is possible. Equation (46) ensures that the DM quantities are

constrained optimal. Finally, equation (47) ensures that the optimal amount of risk-sharing occurs.

Given that we are considering more tax instruments $(\tau^c, \tau^h, \eta(z), \eta(z - \delta), \eta(z + \delta), T_o, R_m)$ than relevant wedges faced by agents, there are several tax schemes that can implement the constrained optimal allocation.²⁰ As we can see from (45), some rprogressivity in the tax system is required to replicate the constrained efficient allocation. However, from (46), the tax system is regressive since the ‘rich’ old agents (those who sold in the DM) face a lower tax rate than the ‘poorest’ agents who were the buyers in the DM. This is needed to induce sellers to produce in the DM. This implies that when agents have private information, a linear consumption (or wealth) tax cannot replicate the constrained optimal allocation.

Rewriting equation (45) as

$$\frac{\eta(z) - \eta(z - \delta)}{\delta} = 1$$

shows that the ‘marginal’ tax rate from holding an additional δ units of real balances is 100%. In short, in order to equate $\bar{C}_o^b = \bar{C}_o^n$, the difference in real balances must be taxed away. This implies that if $\eta'(z - \delta), \eta'(z) < 1$ then $\eta'(x) > 1$ for some values of $x \in (0, z)$.

With a full set of tax instruments considered in this section, the real return on money R_m can be greater than one, one or less than one. Nevertheless, the gross return on money has a lower bound. This is the case as a young agent could work more and take in $z + \delta$ units of money into the DM, then choose not to work if he is a seller and has $z + \delta$ or z when old. This incentive to misreport type in DM can be avoided by raising the real value of money and thus δ . This in turn makes it more costly for the young agent to acquire the additional δ units of real balances. Consequently, R_m cannot be too low. The optimal policy in this setting with private information is to reduce the after tax return on money in order to reduce consumption risk when old.

Finally, the transfers T_o have no effect on the allocation. Why? If $T_o > 0$, young agents anticipate that they will be receiving a transfer of goods when old. This transfer lowers the marginal value of carrying a unit of money into old age and therefore decreases the demand for fiat money when young. It then follows that the goods price of money in the CM, ϕ , decreases thereby reducing the equilibrium value of real balances by exactly the value of the transfer. Consequently, consumption when old is unaffected by the transfer. In other words, real balances for old age consumption and transfers are perfect substitutes. While the current young have to work more to provide the transfer to the current old, they work less to earn the needed real balances. On net, labor hours are unchanged. So setting $T_o = T_y = 0$

²⁰One can determine the completeness of the tax system by counting the relevant margins of adjustment (using standard concepts of marginal rate of substitution and marginal rate of transformation) and the number of instruments.

is consistent with maximizing steady state welfare. As a result, the policymaker can always ensure that the cash constraint does not bind if the Inada condition holds for $U(C)$.

Example Suppose we have the same preferences as before. Consider the following tax policy: $T_o = T_y = 0$, $\tau^c = -\tau^h = \tau$ and $\eta'(z + \delta) = 0$. Equations (46)-(47) yield

$$\begin{aligned}\eta'(z - \delta) &= 1 - \bar{C}_o^b \\ \eta'(z) &= 1 - \bar{C}_o^b \left[\frac{1 - \sigma(1 + \tau) R_m \beta}{(1 - \sigma)(1 + \tau) R_m \beta} \right] \equiv 1 - \bar{C}_o^b f(\tau, R_m).\end{aligned}$$

We have $0 < \eta'(z - \delta) < 1$ and $0 < \eta'(z - \delta) \leq \eta'(z) < 1$ if

$$\frac{1}{1 + \tau} \leq R_m \beta.$$

Consider the piecewise linear tax function

$$\eta'(x) = \begin{cases} 1 - \bar{C}_o^b & \text{for } 0 \leq x \leq 1 \\ \mu \bar{C}_o^b & \text{for } 1 \leq x \leq y_2 \\ 1 - \bar{C}_o^b f(\tau, R_m) & \text{for } y_2 \leq x \leq y_3 \\ 0 & y_3 < x \end{cases}.$$

where the tax parameter μ is chosen such that $\mu \bar{C}_o^b > 1$. Conjecture an equilibrium where $z - \delta < 1$, $y_2 \leq z \leq y_3$ and $y_3 < z + \delta$ (derivations are in the appendix). Integrating over this tax function and using the old age budget constraints gives us the following solutions for z , δ and tax payments:

$$\begin{aligned}z &= y_2 + \frac{(\mu \bar{C}_o^b - 1)(y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)}, & \delta &= (y_2 - 1) \left[1 + \frac{(\mu \bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] \\ \eta(z - \delta) &= (1 - \bar{C}_o^b)(z - \delta) \\ \eta(z) &= (1 - \bar{C}_o^b)y_1 + \mu \bar{C}_o^b(y_2 - y_1) + [1 - \bar{C}_o^b f(\tau, R_m)](z - y_2) \\ \eta(z + \delta) &= (1 - \bar{C}_o^b) + \mu \bar{C}_o^b(y_2 - 1) + [1 - \bar{C}_o^b f(\tau, R_m)](y_3 - y_2).\end{aligned}$$

and the tax parameters $\mu, y_2, y_3, f(\tau, R_m)$ must satisfy

$$\bar{C}_o^s = \bar{C}_o^b + (y_2 - 1) \left[2 - \mu \bar{C}_o^b + 2 \frac{(\mu \bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] - [1 - \bar{C}_o^b f(\tau, R_m)](y_3 - y_2).$$

We establish existence via a numerical example. Suppose we set

$$y_3 = y_2 + \frac{(\mu \bar{C}_o^b - 1)(y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)} + (1 - \lambda)(y_2 - 1) \left[1 + \frac{(\mu \bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right]$$

with $0 < \lambda < 1$. This ensures that $z + \delta > y_3 > z$. Suppose we set $\lambda = 1/2$, $\mu \bar{C}_o^b = 1.0$ and $f(\tau, R_m) = 1$ which implies $(1 + \tau) R_m \beta = 1$. Using the numerical values from the constrained planner problem, $(\bar{C}_o^s = 1.152, \bar{C}_o^n = \bar{C}_o^b = 0.848)$, we obtain the following numerical solutions:

$$\begin{aligned} z &= 1.2027, \quad \delta = 0.2027, \quad z + \delta = 1.4054 \\ y_2 &= 1.1275, \quad y_3 = 1.304. \end{aligned}$$

Linear Tax Rates and Lump-sum Taxes

While allowing for nonlinear taxes is possible using the Mirrlees approach, it assumes the government is able to observe many individual actions in the CM, which may be infeasible in practice. For example, the government can typically see *purchases* of consumption goods but cannot observe agent's *actual* consumption of the goods. In this case, with a non-linear tax system, a tax arbitrage opportunity exists – once they get to the CM, the DM buyers and DM non-active agents can give their funds to the DM sellers and let them make all the purchases of consumption goods. Since the DM sellers pay the lowest marginal tax rate on purchases of consumption goods, this reduces the total taxes paid to the government. This can easily be seen in the example above – DM buyers and non-active agents can lower their tax bills to zero by giving their money balances to the DM sellers while the DM sellers' tax liability is a constant. Thus, the non-linear tax structure is not coalition proof. In this case, a linear consumption tax is the only coalition proof tax structure.

In this section, consider a situation where the government only has access to a linear consumption tax rate that is age independent. This implies $1 - \eta'(x) = (1 + \tau^c)^{-1}$. The government still has access to age dependent lump sum taxes/transfers (T_y, T_o) and the inflation rate (R_m) as policy instruments. It is clear that this set of tax instruments cannot replicate the constrained optimal allocation since it is impossible to obtain $\bar{C}_o^b = \bar{C}_o^n$. Furthermore, comparing (24) and (42) it is clear that q will differ across the two allocations even if $C_o^b = \bar{C}_o^b$ and $C_o^s = \bar{C}_o^s$. We now want to determine if will be optimal to use the inflation tax by setting $R_m < 1/\beta$.

With linear consumption taxes (36)-(43) can be written as

$$C_o^s = \frac{z + \delta}{1 + \tau^c} + T_o \quad (48)$$

$$C_o^b = \frac{z - \delta}{1 + \tau^c} + T_o \quad (49)$$

$$C_o^n = \frac{z}{1 + \tau^c} + T_o \quad (50)$$

$$h = C_y + \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^n \quad (51)$$

$$U'(C_y) = \frac{1 + \tau^c}{1 - \tau^h} v'(h) \quad (52)$$

$$\psi(q) = \beta [U(C_o^s) - U(C_o^n)] \quad (53)$$

$$\frac{u'(q)}{\psi'(q)} = \frac{U'(C_o^b)}{U'(C_o^s)} \quad (54)$$

$$U'(C_y) = R_m \beta [\sigma U'(C_o^b) + (1 - \sigma) U'(C_o^n)] \quad (55)$$

With $\lambda = \beta$ welfare is given by

$$\begin{aligned} \mathcal{W} &= \frac{\sigma}{\beta(1 - \beta)} [u(q) + \beta U(C_o^b)] + \frac{\sigma}{\beta(1 - \beta)} [-\psi(q) + \beta U(C_o^s) - \beta U(C_o^n)] \\ &\quad + \frac{\beta(1 - \sigma)}{\beta(1 - \beta)} U(C_o^n) + \frac{1}{1 - \beta} [U(C_y) - v(h)]. \end{aligned}$$

With buyer-take-all bargaining, the seller's surplus is zero. So we have

$$(1 - \beta) \mathcal{W} = \frac{\sigma}{\beta} u(q) + \sigma U(C_o^b) + (1 - \sigma) U(C_o^n) + U(C_y) - v(h).$$

Maximize with respect to R_m :

$$(1 - \beta) d\mathcal{W} = \frac{\sigma}{\beta} u'(q) \frac{dq}{dR_m} + \sigma U'(C_o^b) \frac{dC_o^b}{dR_m} + (1 - \sigma) U'(C_o^n) \frac{dC_o^n}{dR_m} + U'(C_y) \frac{dC_y}{dR_m} - v'(h) \frac{dh}{dR_m} = 0.$$

Using (48)-(51) and (53) to solve for the derivatives (see appendix) we have

$$\begin{aligned} 0 &= \left\{ \sigma U'(C_o^b) \left[2 - \sigma \frac{U'(C_o^n)}{U'(C_o^s)} \right] + (1 - \sigma) U'(C_o^n) - v'(h) \right\} \left(\frac{1}{1 + \tau^c} \right) \frac{dz}{dR_m} \\ &\quad + [U'(C_y) - v'(h)] \frac{dC_y}{dR_m}. \end{aligned}$$

It follows that the policymaker wants

$$U'(C_y) = v'(h) \rightarrow \frac{1 + \tau^c}{1 - \tau^h} = 1 \quad (56)$$

$$v'(h) = \sigma U'(C_o^b) \left[2 - \sigma \frac{U'(C_o^n)}{U'(C_o^s)} \right] + (1 - \sigma) U'(C_o^m). \quad (57)$$

Let $\hat{h}, \hat{C}_y, \hat{C}_o^j$ denote the solutions to the policymaker's problem above. Equate (57) to (55) evaluated at the optimal values

$$\sigma U'(\hat{C}_o^b) \left[2 - \frac{U'(\hat{C}_o^n)}{U'(\hat{C}_o^s)} \right] + (1 - \sigma) U'(\hat{C}_o^m) = R_m \beta \left[\sigma U'(\hat{C}_o^b) + (1 - \sigma) U'(\hat{C}_o^m) \right]$$

solving for R_m yields

$$R_m = \frac{1}{\beta} \frac{\sigma U'(\hat{C}_o^b) \left[2 - \frac{U'(\hat{C}_o^n)}{U'(\hat{C}_o^s)} \right] + (1 - \sigma) U'(\hat{C}_o^m)}{\sigma U'(\hat{C}_o^b) + (1 - \sigma) U'(\hat{C}_o^m)} < \frac{1}{\beta}$$

since $U'(\hat{C}_o^m) > U'(\hat{C}_o^s)$. So the policymaker uses the inflation tax when constrained to use linear consumption taxes. Note that if $\sigma = 0$, the private information problem goes away and it is optimal to set $R_m = 1/\beta$.

Why does the policymaker resort to the inflation tax? When determining how much additional q the DM buyer gets from increasing d , his choice does not affect the seller's consumption threat point \check{C}_o^s (the real balances he brings into the match). More explicitly, from (32) the buyer uses

$$\begin{aligned} \frac{\partial q}{\partial m} &= \frac{\beta}{\psi'(q)} \left[U'(C_{o,+1}^s) \frac{\partial C_o^s}{\partial d} - U'(\check{C}_o^s) \frac{\partial \check{C}_o^s}{\partial d} \right] \frac{\partial d}{\partial m} \\ &= \frac{\beta}{\psi'(q)} U'(C_{o,+1}^s) \frac{\partial C_o^s}{\partial d} \frac{\partial d}{\partial m} \end{aligned}$$

when determining his choice of real balances when young. However, the policymaker takes into account how changes in R_m affects the seller's consumption threat point. The policymaker uses the total derivative of (53)

$$\frac{\partial q}{\partial m} \frac{dm}{dR_m} = \frac{\beta}{\psi'(q)} \left[U'(C_o^s) \frac{\partial C_o^s}{\partial d} - U'(C_o^n) \frac{\partial C_o^n}{\partial d} \right] \frac{\partial d}{\partial m} \frac{dm}{dR_m}.$$

Since an increase in R_m raises the value of the seller's threat point, this will lower q in equilibrium and worsen the terms of trade for the buyer. Since they do not account for this general equilibrium effect when making their individual choice of real balances, young agents overvalue a unit of real balances. As a result, δ is too high which makes old age consumption more disperse and creates old age consumption risk. By lowering R_m below $1/\beta$ the policymaker makes money less valuable, lowers δ and lowers old age consumption risk.

Competitive pricing The result above seems to be driven largely by the buyer-take-all bargaining structure. This is Zhu's (2008) explanation for his result that inflation is welfare improving. However, as we show above, it is the result of agents not accounting for general equilibrium effects on the terms of trade when R_m changes. Does this intuition apply to other pricing protocols such as competitive pricing in the DM? In this section we study the optimal policy setting for R_m under competitive pricing in the DM.

With competitive pricing, (48)-(52) and (54) are unchanged while (53) and (55) are replaced by

$$\delta = p_1 q = (1 + \tau^c) q \psi'(q) / \beta U'(C_o^s) \quad (58)$$

$$U'(C_y) = R_m \beta [\sigma U'(C_o^b) + \sigma U'(C_o^s) + (1 - 2\sigma) U'(C_o^o)]. \quad (59)$$

The first expression is the DM seller's FOC for determining how much to supply at the market price p_1 . The second equation is the Euler equation for consumption when seller's receive non-zero surplus from exchange. The RHS of (59) is the marginal benefit of carrying an additional unit of real balances to old age when they do not spend everything in the DM if they are a buyer. In this case, agents assume that an additional unit of real balances brought into the DM is simply carried over to old age in all states. This is due to the fact that they are price takers in the DM – when choosing their real balances, the young take DM prices as given. However, as can be seen by (58), the equilibrium DM price is affected by old age consumption of the sellers. So once again, young agents do not take into account the general equilibrium effects of their actions on the DM equilibrium price.

In this case, the policymaker wants to maximize

$$\beta(1 - \beta) \mathcal{W} = \sigma [u(q) + \beta U(C_o^b)] + \sigma [-\psi(q) + \beta U(C_o^s)] + (1 - 2\sigma) \beta U(C_o^o) + \beta [U(C_y) - v(h)].$$

The FOC wrt R_m is

$$0 = \sigma \left[u'(q) \frac{\partial q}{\partial R_m} + \beta U'(C_o^b) \frac{\partial C_o^b}{\partial R_m} \right] + \sigma \left[-\psi'(q) \frac{\partial q}{\partial R_m} + \beta U'(C_o^s) \frac{\partial C_o^s}{\partial R_m} \right] + (1 - 2\sigma) \beta U'(C_o^o) \frac{\partial C_o^o}{\partial R_m} \\ + \beta U'(C_y) \frac{\partial C_y}{\partial R_m} - \beta v'(h) \frac{\partial h}{\partial R_m}.$$

Define $\varepsilon \equiv \frac{\delta}{C_o^o(1+\tau^c)}$ which gives us $C_o^s = (1 + \varepsilon) C_o^o$ and $C_o^b = (1 - \varepsilon) C_o^o$. Using the expressions above and setting $\psi''(q) = 0$ without loss of generality, we obtain

$$\begin{aligned} \frac{\partial C_o^o}{\partial R_m} &= \frac{1}{1 + \tau^c} \frac{\partial z}{\partial R_m} \\ \frac{\partial C_o^s}{\partial R_m} &= (1 + \Delta) \frac{\partial C_o^o}{\partial R_m} + (1 + \Delta) \frac{\phi_{+1} p_1}{(1 + \tau^c)} \frac{\partial q}{\partial R_m} \\ \frac{\partial C_o^b}{\partial R_m} &= (1 - \Delta) \frac{\partial C_o^o}{\partial R_m} - (1 + \Delta) \frac{\phi_{+1} p_1}{(1 + \tau^c)} \frac{\partial q}{\partial R_m} \\ \frac{\partial q}{\partial R_m} &= \frac{q [(1 + \varepsilon) R(C_o^b) (1 - \Delta) - (1 - \varepsilon) R(C_o^s) (1 + \Delta)]}{[R(C_o^s) + R(C_o^b)] (1 + \Delta) \varepsilon + (1 - \varepsilon^2) C_o^o R(q)} \frac{\partial C_o^o}{\partial R_m} \\ \frac{\partial h}{\partial R_m} &= \frac{\partial C_y}{\partial R_m} + \frac{\partial C_o^o}{\partial R_m} \end{aligned}$$

where

$$\Delta = \frac{\varepsilon R(C_o^s)}{1 + \varepsilon [1 - R(C_o^s)]}$$

and $R(\cdot)$ is the coefficient of relative risk aversion.

In general, due to the impact of wealth effects on the agent's decisions from changes in R_m , we cannot obtain general results on the optimal value of R_m . However, for $U(C) = \ln C$, we have $R(C) = 1$ which implies $\Delta = \varepsilon$ and $\frac{\partial q}{\partial R_m} = 0$. So q is unaffected by the return on real balances. It then follows from (54) that

$$\frac{u'(q)}{\psi'(q)} = \frac{1 + \varepsilon}{1 - \varepsilon}.$$

Since q is independent of R_m then ε is as well. We can then write the policymaker's FOC as

$$0 = \beta \left[\sigma U'(C_o^b) (1 - \varepsilon) + \sigma U'(C_o^s) (1 - \varepsilon) + (1 - 2\sigma) U'(C_o^o) - v'(h) \right] \frac{\partial C_o^o}{\partial R_m} \\ + \beta [U'(C_y) - v'(h)] \frac{\partial C_y}{\partial R_m}.$$

So the policymaker wants

$$U'(C_y) = v'(h) \rightarrow \frac{1 + \tau^c}{1 - \tau^h} = 1 \quad (60)$$

$$U'(C_y) = \sigma U'(C_o^b)(1 - \varepsilon) + \sigma U'(C_o^s)(1 + \varepsilon) + (1 - 2\sigma)U'(C_o^o). \quad (61)$$

Comparing (59) and (61) we see that the planner puts less weight on $U'(C_o^b)$ and more weight on $U'(C_o^s)$ than the young agent does. The reason is as follows. As R_m increases, young agents are willing to acquire additional real balances. Since they do not spend all of their money in the DM, the young agents assume that this additional amount of real balances will be used to finance old age consumption. However, if C_o^s increases a DM seller values of unit of real balances less on the margin and thus δ increases. Since q is unchanged when R_m increases, this implies that a buyer has to pay more for the same amount of q due to changes in the equilibrium price of the DM good. Consequently, the DM buyer will only carry $1 - \varepsilon$ of the additional real balances into old age. A similar argument holds for DM sellers – by carrying in additional real balances, the young do not account for the fact that in equilibrium they will get $1 + \varepsilon$ units of real balances if they are a DM seller. As a result, the spread between consumption of a DM seller and a DM buyer in old age is wider than expected. As a result, young agents under-estimate the amount of old age consumption risk that they face.

Let \hat{C}_o^j denote the solutions to the policymaker's problem. Equate (59) and (61) evaluated at the optimal values to obtain

$$R_m = \frac{1}{\beta} \frac{\sigma U'(\hat{C}_o^b)(1 - \varepsilon) + \sigma U'(\hat{C}_o^s)(1 + \varepsilon) + (1 - 2\sigma)U'(\hat{C}_o^o)}{\sigma U'(\hat{C}_o^b) + \sigma U'(\hat{C}_o^s) + (1 - 2\sigma)U'(\hat{C}_o^o)} < \frac{1}{\beta}.$$

As was the case with buyer-take-all bargaining, the policymaker wants to lower the return on real balances in order to discourage over-accumulation of real balances and reduce old age consumption risk.

Discussion Unlike Shi (1997) and Lagos and Wright (2005) we allow agents to face consumption risk after they have completed decentralized trades. We have shown that the Friedman rule is not optimal. This result stands in contrast to Wong (2016). He shows that the Friedman rule is optimal even if agents face consumption risk after completing decentralized trades. This is true as long as preferences generate linear Engel curves for consumption and leisure. So why do our results differ? It is due to different assumptions regarding restrictions on the number of hours that the old agents can work. In Wong's paper, agents can

work *unlimited* amounts of hours in the CM. If there are limits on agents' CM work hours and they bind, then his result goes away.²¹ We can show that, using Wong's preferences and allowing the old to work unlimited hours, the policymaker would set $R_m = 1/\beta$. However, if we have binding limits on hours of work for the old, then even with Wong's preferences, it is optimal to set $R_m < 1/\beta$.

6 Conclusion

In this paper we have analyzed the optimal tax plan in an environment where finitely lived agents face private information as well as search and bargaining frictions. Agents sequentially trade in two markets that are characterized by different frictions and trading protocols. In frictional decentralized and anonymous markets, agents receive preference shocks that determine if they are going to be consuming (producing) in exchange for fiat money.

In an environment where the government is able to observe an agent's age, hours worked in CM, CM consumption and the money holdings of each agent when entering CM, non-linear taxes are able to implement the constrained efficient allocation. In particular, there is some progressivity in the tax system and some regressivity. Thus, a linear consumption (or wealth) tax cannot replicate the constrained optimal allocation. So, without a full set of non-linear tax rates, the government uses the inflation tax as part of the optimal tax policy.

²¹The same is true for Lagos and Wright – if agents are constrained in the amount of hours that they can work in the CM, then they cannot rebalance their money holdings. This implies the distribution of money holdings is not degenerate and a policymaker would be confronting an efficiency/risk sharing tradeoff similar to what we study. See Rocheteau et al (2015) for a related restriction on hours worked in the LW framework.

References

- [1] Abel, A. (1987) “Optimal Monetary Growth,” *Journal of Monetary Economics*, 437-450.
- [2] Albanesi, S. and C. Sleet, (2006). “Dynamic Optimal Taxation with Private Information,” *Review of Economic Studies*, 73,1-30.
- [3] Andolfatto, D. (2010) “Essential Interest-Bearing Money,” *Journal of Economic Theory*, 145, 1495-507.
- [4] Aiyagari, S.R. and S. Williamson (2000). “Money and Dynamic Credit Arrangements with Private Information,” *Journal of Economic Theory* 91, 248-279.
- [5] Atkinson, A. and J. Stiglitz (1980). *Lectures on Public Economics*. New York: McGraw Hill.
- [6] Aruoba, S.B. and S. Chugh (2010). “Optimal Fiscal and Monetary Policy when Money is Essential,” *Journal of Economic Theory*, 145, 1618-1647.
- [7] Bhattacharya, J., J. Haslag, and S. Russell (2005) “The Role of Money in Two Alternative Models: When is the Friedman Rule Optimal, and Why?,” *Journal of Monetary Economics* 52, 1401-1433.
- [8] Berentsen, A., G. Camera and C. Waller (2005). “The Distribution of Money Balances and the Non-Neutrality of Money,” *International Economic Review* 46, 465-487.
- [9] Chamley, C. (1986). “Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives,” *Econometrica*, 54, 607-622.
- [10] Chari, V.V., L. Christiano and P. Kehoe (1996). “Optimality of the Friedman Rule in Economies with Distorting Taxes,” *Journal of Monetary Economics* 37, 203-223.
- [11] Chari, V.V., L. Christiano and P. Kehoe (1999). “Optimal Fiscal and Monetary Policy,” *Handbook of Macroeconomics*, vol. 1c, J. Taylor and M. Woodford, eds., Amsterdam: Elsevier Press.

Chugh, S. K. (2007a) “Optimal Inflation Persistence: Ramsey Taxation with Capital and Habits,” *Journal of Monetary Economics* 54, 1809-1836.
- [12] Chugh, S. K. (2006b) “Optimal Fiscal and Monetary Policy with Sticky Wages and Sticky Prices,” *Review of Economic Dynamics* 9, 683-714.

- [13] da Costa, C. and I. Werning (2008). “On the Optimality of the Friedman Rule with Heterogeneous Agents and Non-Linear Income Taxation,” *Journal of Political Economy* 116, 82-112.
- [14] Ennis, H. (2008). “Search, Money, and Inflation under Private Information,” *Journal of Economic Theory*, 138, 101-131.
- [15] Erosa, A. and M. Gervais (2002). “Optimal Taxation in Life-Cycle Economies,” *Journal of Economic Theory* 105, 338-369.
- [16] Freeman, S. (1987) “Reserve Requirements and Optimal Seignorage,” *Journal of Monetary Economics*, 19 , 307-314.
- [17] Gahvari, F. (2007) “The Friedman Rule: Old and New,” *Journal of Monetary Economics* 54, 581-589.
- [18] Gahvari, F. and L. Micheletto (2014) “The Friedman Rule in an Overlapping-generations Model with Nonlinear Taxation and Income Misreporting,” *Journal of Public Economics*, 119, 10-23.
- [19] Golosov, M., N. Kocherlakota and A. Tsyvinski (2003). “Optimal Indirect and Capital Taxation,” *Review of Economic Studies* 70, 569-587.
- [20] Golosov, M., A. Tsyvinski, and I. Werning (2006). “New Dynamic Public Finance: A User’s Guide,” *NBER Macroeconomic Annual* 2006.
- [21] Gomis-Porqueras, P. and A. Peralta-Alva (2010). “Optimal Monetary and Fiscal Policies in a Search Theoretic Model of Monetary Exchange,” *European Economic Review*, 54(3), 331-344.
- [22] Gomis-Porqueras, P. and D. Sanches (2013) “Optimal Monetary Policy in a Model of Money and Credit,” *Journal of Money, Credit and Banking*, 45(4), 701-730.
- [23] Judd, K. (1985). “Redistributive Taxation in a Simple Perfect Foresight Model,” *Journal of Public Economics*, 28, 59-83.
- [24] Kareken, J. and N. Wallace (1981) “On the Indeterminacy of Equilibrium Exchange Rates,” *Quarterly Journal of Economics*, 96 (2), 207-22.
- [25] Kocherlakota, N. (1998). “Money is Memory,” *Journal of Economic Theory* 81, 232-251.
- [26] Kocherlakota, N. (2005a). “Optimal Monetary Policy: What We Know and What We Don’t,” *International Economic Review* 46, 715-729.

- [27] Kocherlakota, N. (2005b). “Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation,” *Econometrica* 73, 1587-1621.
- [28] Lagos, R. and R. Wright (2005). “A Unified Framework for Monetary Theory and Policy Evaluation,” *Journal of Political Economy* 113, 463-484.
- [29] Levine, D. (1991). “Asset Trading Mechanisms and Expansionary Policy,” *Journal of Economic Theory* 54, 148-164.
- [30] Mirrlees, J. (1971). “An Exploration in the Theory of Optimal Income Taxation,” *Review of Economic Studies* 38, 175-208.
- [31] Molico, M. (2006). “The Distribution of Money and Prices in Search Equilibrium,” *International Economic Review* 47, 701-722.
- [32] Rocheteau, G., P. Weill and R. Wong (2015). “Working Through the Distribution: Money in the Short and Long Run,” Working paper, UC-Irvine.
- [33] Ritter, M. (2010). “The Optimum Quantity of Money Revisited: Distortionary Taxation in a Search Model of Money,” *B.E. Journal of Macroeconomics*, 10, 1-26.
- [34] Schmitt-Grohe, S. and M. Uribe (2004a) “Optimal Fiscal and Monetary Policy Under Imperfect Competition,” *Journal of Macroeconomics*, 26, 183-209.
- [35] Schmitt-Grohe, S. and M. Uribe (2004b) “Optimal Fiscal and Monetary Policy Under Sticky Prices,” *Journal of Economic Theory*, 114, 198-230.
- [36] Schmitt-Grohe, S. and M. Uribe (2005) “Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model,” *NBER Macroeconomics Annual* 2005.
- [37] Shi, S. (1997) “A Divisible Model of Fiat Money,” *Econometrica*, 65, 75-102.
- [38] Wallace, N. (2001). “Whither Monetary Economics?” *International Economic Review*, 42, 847-869.
- [39] Wong, R. (2016). “A Tractable Monetary Model under General Preference,” *Review of Economic Studies* 83, 402-420.
- [40] Zhu, T. (2008). “An Overlapping Generations Model with Search,” *Journal of Economic Theory* 142, 318-331.

Derivation of example for constrained planner. Not to be included for publication.

We have $U'(C_y) = 1$ and $C_o^b = C_o^n$ which implies $U'(C_o^b) = U'(C_o^n)$ so the Euler equation yields

$$\begin{aligned}
1 &= \left[\frac{\sigma U'(C_o^b) + (1 - \sigma) U'(C_o^s)}{U'(C_o^s) U'(C_o^b)} \right]^{-1} \\
1 &= \frac{U'(C_o^s) U'(C_o^b)}{\sigma U'(C_o^b) + (1 - \sigma) U'(C_o^s)} \\
\frac{1}{U'(C_o^s)} &= \frac{U'(C_o^b)}{\sigma U'(C_o^b) + (1 - \sigma) U'(C_o^s)} \\
C_o^s &= \frac{\frac{1}{C_o^b}}{\sigma \frac{1}{C_o^b} + (1 - \sigma) \frac{1}{C_o^s}} \\
1 &= \frac{1}{\sigma C_o^s + (1 - \sigma) C_o^b} \\
C_o^b &= \frac{1 - \sigma C_o^s}{1 - \sigma}
\end{aligned}$$

From the FOC in the DM we then have

$$\begin{aligned}
\frac{\exp^{-q}}{\rho} &= \frac{U'(C_o^b)}{\sigma U'(C_o^b) + (1 - \sigma) U'(C_o^s)} = C_o^s \\
\exp^q &= \frac{1}{\rho C_o^s} \\
q &= -\ln \rho - \ln C_o^s
\end{aligned}$$

From the zero surplus equation we get

$$\begin{aligned}
\rho [-\ln \rho - \ln C_o^s] &= \beta [\ln C_o^s - \ln C_o^n] = \beta [\ln C_o^s - \ln C_o^b] \\
\rho [-\ln \rho - \ln C_o^s] &= \beta \ln C_o^s - \beta \ln \frac{1 - \sigma C_o^s}{1 - \sigma} \\
\rho [-\ln \rho - \ln C_o^s] &= \beta \ln C_o^s - \ln(1 - \sigma C_o^s) + \beta \ln(1 - \sigma) \\
-\rho \ln \rho - \beta \ln(1 - \sigma) &= (\beta + \rho) \ln \bar{C}_o^s - \beta \ln(1 - \sigma \bar{C}_o^s)
\end{aligned}$$

The LHS is a positive number. The RHS is monotonically increasing in \bar{C}_o^s . At $\bar{C}_o^s = 0$ the RHS is negative infinity. At $\bar{C}_o^s = 1$ the RHS is less than the LHS. As $\bar{C}_o^s \rightarrow 1/\sigma$ the RHS goes to positive infinity. So there is a unique value \bar{C}_o^s solving this expression that lies between 1 and $1/\sigma$. Note that as $\sigma \rightarrow 0$, $\bar{C}_o^s \rightarrow \left(\frac{1}{\rho}\right)^{\frac{\rho}{\rho+\beta}} > 1$.▲

Derivation of example for non-linear taxes

From the FOC in the DM we have the equality

$$\frac{u'(q)}{\psi'(q)} = \frac{U'(C_o^b)}{(1-\sigma)U'(C_o^s) + \sigma U'(C_o^b)} = \frac{U'(C_o^b) [1 - \eta'(z - \delta)]}{U'(C_o^s) [1 - \eta'(z + \delta)]}$$

which implies

$$\frac{1 - \eta'(z - \delta)}{1 - \eta'(z + \delta)} = \frac{U'(C_o^s)}{(1-\sigma)U'(C_o^s) + \sigma U'(C_o^b)}$$

Rearrange and use the functional forms to obtain:

$$\begin{aligned} \frac{1 - \eta'(z - \delta)}{1 - \eta'(z + \delta)} &= \frac{\frac{1}{\bar{C}_o^s}}{\frac{(1-\sigma)\bar{C}_o^b + \sigma\bar{C}_o^s}{\bar{C}_o^s\bar{C}_o^b}} \\ &= \frac{\bar{C}_o^b}{(1-\sigma)\bar{C}_o^b + \sigma\bar{C}_o^s} \end{aligned}$$

Sub in the solution for $\bar{C}_o^b = \frac{1-\sigma\bar{C}_o^s}{1-\sigma}$ to get

$$\begin{aligned} \frac{1 - \eta'(z - \delta)}{1 - \eta'(z + \delta)} &= \frac{\frac{1-\sigma\bar{C}_o^s}{1-\sigma}}{(1-\sigma)\frac{1-\sigma\bar{C}_o^s}{1-\sigma} + \sigma\bar{C}_o^s} \\ &= \frac{1 - \sigma\bar{C}_o^s}{1 - \sigma} = \bar{C}_o^b \end{aligned}$$

Now equate the Euler equations and impose $C_o^b = C_o^m$ to obtain

$$\begin{aligned} \frac{U'(C_o^s)U'(C_o^b)}{\sigma U'(C_o^b) + (1-\sigma)U'(C_o^s)} &= (1 + \tau^c) R_m \beta U'(C_o^b) \{1 - \sigma\eta'(z - \delta) - (1 - \sigma)\eta'(z)\} \\ \frac{U'(C_o^s)}{\sigma U'(C_o^b) + (1-\sigma)U'(C_o^s)} &= (1 + \tau^c) R_m \beta [1 - \sigma\eta'(z - \delta) - (1 - \sigma)\eta'(z)] \end{aligned}$$

Using the expression above we have

$$\frac{1 - \eta'(z - \delta)}{1 - \eta'(z + \delta)} = (1 + \tau^c) R_m \beta [1 - \sigma\eta'(z - \delta) - (1 - \sigma)\eta'(z)]$$

Consider the following tax policy: $T_0 = 0$ and $\tau^c = -\tau^h = \tau$. Now impose $\eta'(z + \delta) = 0$. It then follows from the first equation that

$$\eta'(z - \delta) = 1 - \bar{C}_o^b > 0.$$

Substitute into the third equation and rearrange to obtain

$$\begin{aligned}
\bar{C}_o^b &= (1 + \tau^c) R_m \beta [1 - \sigma (1 - \bar{C}_o^b) - (1 - \sigma) \eta'(z)] \\
\bar{C}_o^b &= (1 + \tau^c) R_m \beta [1 - \sigma + \sigma \bar{C}_o^b - (1 - \sigma) \eta'(z)] \\
\bar{C}_o^b [1 - \sigma (1 + \tau^c) R_m \beta] &= (1 + \tau^c) R_m \beta (1 - \sigma) [1 - \eta'(z)] \\
1 - \eta'(z) &= \bar{C}_o^b \left[\frac{1 - \sigma (1 + \tau^c) R_m \beta}{(1 - \sigma) (1 + \tau^c) R_m \beta} \right] \\
\eta'(z) &= 1 - \bar{C}_o^b \left[\frac{1 - \sigma (1 + \tau^c) R_m \beta}{(1 - \sigma) (1 + \tau^c) R_m \beta} \right] \equiv 1 - \bar{C}_o^b f(\tau, R_m)
\end{aligned}$$

To ensure that $\eta'(z) \geq \eta'(z - \delta)$ we need $f(\tau^c, R_m) \leq 1$ which requires

$$\begin{aligned}
\bar{C}_o^b \frac{1 - \sigma (1 + \tau^c) R_m \beta}{(1 - \sigma) (1 + \tau^c) R_m \beta} &< 1 \\
\bar{C}_o^b - \bar{C}_o^b \sigma (1 + \tau^c) R_m \beta &< (1 - \sigma) (1 + \tau^c) R_m \beta \\
1 &< (1 - \sigma) (1 + \tau^c) R_m \beta + \sigma (1 + \tau^c) R_m \beta \\
1 &< (1 + \tau^c) R_m \beta
\end{aligned}$$

Finally, we need to satisfy the second equation. Since $\eta'(z)$ and $\eta'(z - \delta)$ are constants, the total tax payments for the DM buyers is given by

$$\eta(x) = \int_0^{z-\delta} \eta'(x) dx$$

Assume $\eta'(x) = 1 - \bar{C}_o^b$ for the relevant range $[0, y_1]$ where $0 < z - \delta \leq y_1$. Then we have

$$\begin{aligned}
\eta(z - \delta) &= (1 - \bar{C}_o^b) \int_0^{z-\delta} dx \\
&= (1 - \bar{C}_o^b) x \Big|_0^{z-\delta} \\
&= (1 - \bar{C}_o^b) (z - \delta)
\end{aligned}$$

Substitute into the DM buyer's old age budget constraint

$$\begin{aligned}
\bar{C}_o^b &= z - \delta - \eta(x) \\
&= z - \delta - (1 - \bar{C}_o^b) (z - \delta) \\
&= [1 - (1 - \bar{C}_o^b)] (z - \delta) \\
&= \bar{C}_o^b (z - \delta) \\
1 &= z - \delta
\end{aligned}$$

which requires that $y_1 \geq 1$.

Let the total tax payments by a non-active agent be given by

$$\eta(z) = \int_0^{y_1} \eta'(x) dx + \int_{y_1}^{y_2} \eta'(x) dx + \int_{y_2}^z \eta'(x) dx$$

Assume that $\eta'(x) = 1 - \bar{C}_o^b$ for the range $0 < x \leq y_1$, $\eta'(x) = \mu\bar{C}_o^b > 1$ over the range $y_1 < x < y_2$ and $\eta'(x) = 1 - \bar{C}_o^b f(\tau^c, R_m)$ over the range $y_2 \leq x \leq z$ where $y_2 < z$. Then we have

$$\begin{aligned} \eta(z) &= (1 - \bar{C}_o^b) \int_0^{y_1} dx + \int_{y_1}^{y_2} \mu\bar{C}_o^b dx + [1 - \bar{C}_o^b f(\tau^c, R_m)] \int_{y_2}^z dx \\ \eta(z) &= (1 - \bar{C}_o^b) y_1 + \mu\bar{C}_o^b (y_2 - y_1) + [1 - \bar{C}_o^b f(\tau, R_m)] (z - y_2) \end{aligned}$$

We need

$$\begin{aligned} \eta(z) &= \delta + \eta(z - \delta) \\ (1 - \bar{C}_o^b) y_1 + \mu\bar{C}_o^b (y_2 - y_1) + [1 - \bar{C}_o^b f(\tau, R_m)] (z - y_2) &= \delta + (1 - \bar{C}_o^b) (z - \delta) \\ (1 - \bar{C}_o^b) y_1 + \mu\bar{C}_o^b (y_2 - y_1) + [1 - \bar{C}_o^b f(\tau, R_m)] (z - y_2) &= z - \bar{C}_o^b (z - \delta) \\ y_1 - \bar{C}_o^b y_1 + \mu\bar{C}_o^b (y_2 - y_1) + z - \bar{C}_o^b f(\tau, R_m) z - y_2 + \bar{C}_o^b f(\tau, R_m) y_2 &= z - \bar{C}_o^b \\ -\bar{C}_o^b y_1 + (\mu\bar{C}_o^b - 1) (y_2 - y_1) - \bar{C}_o^b f(\tau, R_m) z + \bar{C}_o^b f(\tau, R_m) y_2 &= -\bar{C}_o^b \\ \bar{C}_o^b y_1 - (\mu\bar{C}_o^b - 1) (y_2 - y_1) + \bar{C}_o^b f(\tau, R_m) z - \bar{C}_o^b f(\tau, R_m) y_2 &= \bar{C}_o^b \\ y_1 - \left(\mu - \frac{1}{\bar{C}_o^b}\right) (y_2 - y_1) + f(\tau, R_m) z - f(\tau, R_m) y_2 &= 1 \end{aligned}$$

So

$$\begin{aligned} z &= \frac{1 - y_1 + \left(\mu - \frac{1}{\bar{C}_o^b}\right) (y_2 - y_1) + f(\tau, R_m) y_2}{f(\tau, R_m)} \\ z &= y_2 + \frac{1 - y_1 + \left(\mu - \frac{1}{\bar{C}_o^b}\right) (y_2 - y_1)}{f(\tau, R_m)} \end{aligned}$$

With $y_1 = 1$ we have

$$z = y_2 + \frac{(\mu\bar{C}_o^b - 1) (y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)}$$

We need $z > y_2$ which requires $y_2 > 1$ and $\mu\bar{C}_o^b > 1$. We then have

$$\begin{aligned}\delta &= z - 1 \\ &= y_2 + \frac{(\mu\bar{C}_o^b - 1)(y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)} - 1 \\ \delta &= (y_2 - 1) \left[1 + \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right]\end{aligned}$$

The last thing we need is to determine the taxes paid by the DM sellers. So we have

$$\begin{aligned}\eta(z + \delta) &= \int_0^{y_1} \eta'(x) dx + \int_{y_1}^{y_2} \eta'(x) dx + \int_{y_2}^{y_3} \eta'(x) dx + \int_{y_3}^{\infty} \eta'(x) dx \\ \eta(z + \delta) &= (1 - \bar{C}_o^b) y_1 + \mu\bar{C}_o^b (y_2 - y_1) + [1 - \bar{C}_o^b f(\tau, R_m)] (y_3 - y_2)\end{aligned}$$

Where we have imposed $\eta'(x) = 0$ for $z + \delta > y_3$. If we set $y_1 = 1$ we need

$$\begin{aligned}z + \delta &= y_2 + \frac{(\mu\bar{C}_o^b - 1)(y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)} + (y_2 - 1) \left[1 + \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] > y_3 > y_2 + \frac{(\mu\bar{C}_o^b - 1)(y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)} = z \\ z + \delta &= y_2 + \left\{ 1 + 2 \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right\} (y_2 - 1) > y_3 > y_2 + \frac{(\mu\bar{C}_o^b - 1)(y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)} = z\end{aligned}$$

The DM seller's taxes are thus:

$$\eta(z + \delta) = (1 - \bar{C}_o^b) + \mu\bar{C}_o^b (y_2 - 1) + [1 - \bar{C}_o^b f(\tau, R_m)] (y_3 - y_2)$$

Using the old age budget constraint of the DM seller (setting $z - \delta = 1$)

$$\begin{aligned}\bar{C}_o^s &= z + \delta - \eta(z + \delta) \\ \bar{C}_o^s &= 1 + 2\delta - \eta(z + \delta) \\ \bar{C}_o^s &= 1 + 2(y_2 - 1) \left[1 + \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] - (1 - \bar{C}_o^b) - \mu\bar{C}_o^b (y_2 - 1) - [1 - \bar{C}_o^b f(\tau, R_m)] (y_3 - y_2) \\ &= \bar{C}_o^b + 2(y_2 - 1) \left[1 + \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] - \mu\bar{C}_o^b (y_2 - 1) - [1 - \bar{C}_o^b f(\tau, R_m)] (y_3 - y_2) \\ \bar{C}_o^s &= \bar{C}_o^b + (y_2 - 1) \left[2 - \mu\bar{C}_o^b + 2 \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] - [1 - \bar{C}_o^b f(\tau, R_m)] (y_3 - y_2)\end{aligned}$$

The parameters $\mu, y_2, y_3, f(\tau, R_m)$ are chosen to satisfy this expression. Suppose we set

$$y_3 = y_2 + \frac{(\mu\bar{C}_o^b - 1)(y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)} + (1 - \lambda)(y_2 - 1) \left[1 + \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right]$$

with $0 < \lambda < 1$. This ensures that $z + \delta > y_3 > z$. We then have

$$\begin{aligned} \bar{C}_o^s &= \bar{C}_o^b + (y_2 - 1) \left[2 - \mu\bar{C}_o^b + 2\frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] - [1 - \bar{C}_o^b f(\tau, R_m)] \frac{(\mu\bar{C}_o^b - 1)(y_2 - 1)}{\bar{C}_o^b f(\tau, R_m)} \\ &\quad + (1 - \lambda)(y_2 - 1) \left[1 + \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] \\ \frac{\bar{C}_o^s - \bar{C}_o^b}{(y_2 - 1)} &= 2 - \mu\bar{C}_o^b + 2\frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} - [1 - \bar{C}_o^b f(\tau, R_m)] \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \\ &\quad + 1 - \lambda + (1 - \lambda) \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \\ \frac{\bar{C}_o^s - \bar{C}_o^b}{(y_2 - 1)} &= 2 - \mu\bar{C}_o^b + 2\frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} - \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} + \bar{C}_o^b f(\tau, R_m) \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \\ &\quad + 1 - \lambda + \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} - \lambda \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \\ \frac{\bar{C}_o^s - \bar{C}_o^b}{(y_2 - 1)} &= 2 - \mu\bar{C}_o^b + 2\frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} + \mu\bar{C}_o^b - 1 + 1 - \lambda - \lambda \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \\ \frac{\bar{C}_o^s - \bar{C}_o^b}{(y_2 - 1)} &= 2 - \lambda + (2 - \lambda) \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \\ \bar{C}_o^s - \bar{C}_o^b &= \left[1 + \frac{(\mu\bar{C}_o^b - 1)}{\bar{C}_o^b f(\tau, R_m)} \right] (2 - \lambda)(y_2 - 1) \end{aligned}$$

Suppose we set $f(\tau, R_m) = 1$ which implies $(1 + \tau)R_m\beta = 1$, and $\mu\bar{C}_o^b = 1.05$ and $\lambda = 1/2$.

Then we have

$$\begin{aligned} \bar{C}_o^s - \bar{C}_o^b &= \left(1 + \frac{0.5}{\bar{C}_o^b} \right) (1.5)(y_2 - 1) \\ \bar{C}_o^s - \bar{C}_o^b &= \left(1.5 + \frac{0.75}{\bar{C}_o^b} \right) (y_2 - 1) \\ y_2 &= 1 + \frac{\bar{C}_o^s - \bar{C}_o^b}{1.5 + \frac{0.75}{\bar{C}_o^b}} \end{aligned}$$

Using the numerical values from the constrained planner problem we had $\bar{C}_o^s = 1.152$,

$\bar{C}_o^m = \bar{C}_o^b = 0.848$. Substitute in to get

$$y_2 = 1 + \frac{1.152 - 0.848}{1.5 + \frac{0.75}{0.848}} = 1.1275$$

It then follows that

$$\begin{aligned} z &= 1.1275 + \frac{(0.5)(0.1275)}{0.848} = 1.2027 \\ \delta &= (0.1275) \left(1 + \frac{0.5}{0.848} \right) = 0.2027 \\ z + \delta &= 1.2027 + 0.20268 = 1.4054 \\ y_3 &= 1.1275 + \frac{(0.5)(0.1275)}{0.848} + (0.5)(0.1275) \left(1 + \frac{0.5}{0.848} \right) = 1.304 \end{aligned}$$

■

Derivation of optimal R_m with linear taxes.

The policymaker's FOC is:

$$(1 - \beta) d\mathcal{W} = \frac{\sigma}{\beta} u'(q) \frac{dq}{dR_m} + \sigma U'(C_o^b) \frac{dC_o^b}{dR_m} + (1 - \sigma) U'(C_o^n) \frac{dC_o^n}{dR_m} + U'(C_y) \frac{dC_y}{dR_m} - v'(h) \frac{dh}{dR_m} = 0$$

Using (48)-(51) we have

$$\begin{aligned} \frac{dC_o^s}{dR_m} &= \left(\frac{1}{1 + \tau^c} \right) \left[\frac{dz}{dR_m} + \frac{d\delta}{dR_m} \right] \\ \frac{dC_o^b}{dR_m} &= \left(\frac{1}{1 + \tau^c} \right) \left[\frac{dz}{dR_m} - \frac{d\delta}{dR_m} \right] \\ \frac{dC_o^n}{dR_m} &= \left(\frac{1}{1 + \tau^c} \right) \frac{dz}{dR_m} \\ \frac{dh}{dR_m} &= \frac{dC_y}{dR_m} + \sigma \frac{dC_o^b}{dR_m} + \sigma \frac{dC_o^s}{dR_m} + (1 - 2\sigma) \frac{dC_o^n}{dR_m} \\ &= \frac{dC_y}{dR_m} + \left(\frac{1}{1 + \tau^c} \right) \frac{dz}{dR_m} \end{aligned}$$

Finally, use (53) to get

$$\frac{dq}{dR_m} = \frac{\beta}{\psi'(q)} \left[U'(C_o^s) \frac{dC_o^s}{dR_m} - U'(C_o^n) \frac{dC_o^n}{dR_m} \right]$$

Substituting into the policymaker FOC and rearranging yields

$$\begin{aligned} 0 &= \left\{ \sigma U'(C_o^b) \left[2 - \sigma \frac{U'(C_o^n)}{U'(C_o^s)} \right] + (1 - \sigma) U'(C_o^n) - v'(h) \right\} \left(\frac{1}{1 + \tau^c} \right) \frac{dz}{dR_m} \\ &\quad + [U'(C_y) - v'(h)] \frac{dC_y}{dR_m}. \end{aligned}$$